

# Mathematics 18.08.22 (1) Notes

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## Section 1 Revision

### §1.1 Numbers

**Problem 1.1.** Determine whether  $-3$  is in  $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ .

**Solution.**  $-3 \notin \mathbb{N}, -3 \in \mathbb{Z}, -3 \in \mathbb{Q}, -3 \in \mathbb{R}$ .

**Problem 1.2.** Prove that  $7.385 \in \mathbb{Q}$ .

**Solution.**  $7.385 = 7385/1000$ .

**Problem 1.3.** Prove that  $0.\dot{7}1\dot{5} \in \mathbb{Q}$ .

**Solution.**  $0.\dot{7}1\dot{5} = 715/999$ .

### §1.2 Factorization, lcms and hcfs

**Problem 1.4.** Factorize 432.

**Solution.**

$$\begin{aligned}
 432 &= 2 \times 216 \\
 &= 2^2 \times 108 \\
 &= 2^3 \times 54 \\
 &= 2^4 \times 27 \\
 &= 2^4 \times 3 \times 9 \\
 &= 2^4 \times 3^2 \times 3 \\
 &= 2^4 \times 3^3.
 \end{aligned}$$

Therefore  $432 = 2^4 \times 3^3$ .

**Problem 1.5.** Factorize 900.

**Solution.**

$$\begin{aligned}
 900 &= 2 \times 450 \\
 &= 2^2 \times 225 \\
 &= 2^2 \times 3 \times 75 \\
 &= 2^2 \times 3^2 \times 25 \\
 &= 2^2 \times 3^2 \times 5^2.
 \end{aligned}$$

Therefore  $900 = 2^2 \times 3^2 \times 5^2$ .

**Problem 1.6.** Calculate the HCF of 432 and 900.

**Solution.** Common factors:  $2^2, 3^2$ . 432-Only factors:  $2^2, 3$ . 900-Only factors:  $5^2$ .  
 $\text{hcf}(432, 900) = 2^2 \times 3^2 = 36$ .

**Problem 1.7.** Calculate the LCM of 432 and 900.

**Solution.** Common factors:  $2^2, 3^2$ . 432-Only factors:  $2^2, 3$ . 900-Only factors:  $5^2$ .  
 $\text{lcm}(432, 900) = 2^4 \times 3^3 \times 5^2 = 10800$ .

## Section 2 Fractions

### §2.1 Fraction Calculation

**Example 2.1. (Faction Calculation)**

$$\begin{aligned}
 \frac{a}{432} - \frac{b^2}{900} &\xrightarrow[\text{\tiny \times Other-Only Factor}]{\text{\tiny LCM Common Denominator}} \frac{a \cdot 5^2}{10800} - \frac{b^2 \cdot 2^2 \cdot 3}{10800} \\
 &= \frac{25a - 12b^2}{10800}.
 \end{aligned}$$

**Problem 2.2.** Simplify

$$\frac{a}{c} \times b - \frac{e}{3f}.$$

**Solution.**

$$\begin{aligned}
 \frac{a}{c} \times b - \frac{e}{3f} &= \frac{ab}{c} - \frac{e}{3f} \\
 &= \frac{3abf}{3cf} - \frac{ce}{3cf} \\
 &= \frac{3abf - ce}{3cf}.
 \end{aligned}$$

**Problem 2.3.** Simplify

$$\frac{b}{8e} \div \frac{2a}{c}.$$

**Solution.**

$$\begin{aligned}
 \frac{b}{8e} \div \frac{2a}{c} &= \frac{b}{8e} \times \frac{c}{2a} \\
 &= \frac{bc}{16ae}.
 \end{aligned}$$

**Problem 2.4.** Simplify

$$4 - \frac{3}{a}.$$

**Solution.**

$$\begin{aligned}
 4 - \frac{3}{a} &= \frac{4a}{a} - \frac{3}{a} \\
 &= \frac{4a - 3}{a}.
 \end{aligned}$$

**Problem 2.5.** Simplify

$$\frac{8}{3} \div 7a.$$

**Solution.**

$$\begin{aligned}
 \frac{8}{3} \div 7a &= \frac{8}{3} \times \frac{1}{7a} \\
 &= \frac{8}{21a}.
 \end{aligned}$$

**Problem 2.6.** Simplify

$$3c \div \frac{276}{e^5}.$$

**Solution.**

$$\begin{aligned}
 3c \div \frac{276}{e^5} &= 3c \times \frac{e^5}{276} \\
 &= \frac{3ce^5}{276} \\
 &= \frac{ce^5}{92}.
 \end{aligned}$$

**Problem 2.7.** Simplify

$$\frac{9a^2}{10bc} \times \frac{12b}{6a^3}.$$

**Solution.**

$$\begin{aligned} \frac{9a^2}{10bc} \times \frac{12b}{6a^3} &= \frac{9a^2 \cdot 12b}{10bc \cdot 6a^3} \\ &= \frac{9}{5ac}. \end{aligned}$$

## §2.2 Fraction Comparison

**Example 2.8. (Fraction Comparison)** Compare  $11/432$  and  $23/900$ .  $11/432 = 275/10800$ .  $23/900 = 276/10800$ .  $11/432 < 23/900$ .

## §2.3 Concepts of Fractions

**Definition 2.9. (Mixed Fraction)** e.g.  $7\frac{2}{11}$ .

**Definition 2.10. (Vulger Fraction)** e.g.  $\frac{300}{40}$ .

**Example 2.11. (Fractions)** Convert  $7\frac{2}{11}$  into a vulger fraction.  $7\frac{2}{11} = \frac{79}{11}$ .

**Example 2.12. (Fractions)** Convert  $\frac{300}{40}$  into a mixed fraction.  $\frac{300}{40} = \frac{15}{2} = 7\frac{1}{2}$ .

## §2.4 Percentages

**Problem 2.13.** Express  $32/73$  as a %.

**Solution.**  $32/73 = 43.836\%$ .

**Problem 2.14.** Find 17% of 453.

**Solution.**  $453 \times 17\% = 77.01$ .

**Problem 2.15.** Increase 82 by 61%.

**Solution.**  $82 \times (1 + 61\%) = 132.02$ .

**Problem 2.16.** A phone marked £370 need to be reduced by 15%. Find the now price.

**Solution.**  $(1 - 15\%) \times £370 = £314.5$ .

**Problem 2.17.** A coat priced at £120 need a 20% sales tax to be added to the price. Find the new price.

**Solution.**  $£120 \times (1 + 20\%) = £144$ .

**Problem 2.18.** After a 20% tax has been added to a bag's price is £45. Find the pre-tax price.

**Solution.** Let  $p$  be the pre-tax price. Therefore

$$(1 + 20\%)p = £45,$$

Simplify, we have  $p = £37.5$ .

**Problem 2.19.** After a 35% discount a shop price is £795. Find the previous price.

**Solution.** Let  $p$  be the previous price. Therefore

$$(1 - 35\%)p = £795$$

Simplify, we have  $p = £1223.08$ .

**Problem 2.20.** £12000 is invested in a 2.75% amount for 20 years.

Find the value at the end of the investment saving.

**Solution.**

(1) Single interest.  $\pounds 12000 \times 2.75\% = \pounds 330$ .  $\pounds 330 \times 20 = \pounds 6600$ .  $\pounds 12000 + \pounds 6600 = \pounds 18600$ .

(2) Compound interest.  $\pounds 12000 \times (1 + 2.75\%)^{20} = \pounds 20645.141$ .

**Problem 2.21.** A \$30,000 car depreciate at 15% per year. What is the value by 12 years?

**Solution.**  $\$30000 \times (1 - 15\%)^{12} = \$4267.253$ .

## Section 3 Standard Form

**Definition 3.1. (Standard Form)**  $a \times 10^n$ ,  $n \in \mathbb{Z}$ ,  $1 \leq |a| < 10$ .

**Example 3.2. (Standard Form)** Write 280700 in standard form.  $280700 = 2.807 \times 10^5$ .

**Example 3.3. (Standard Form)** Write  $7.09 \times 10^{-5}$  as a half decimal.  $7.09 \times 10^{-5} = 0.0000709$ .

**Problem 3.4.** Simplify  $4.7 \times 10^8 + 9.4 \times 10^8$

**Solution.**

$$\begin{aligned} 4.7 \times 10^8 + 9.4 \times 10^8 &= 14.1 \times 10^8 \\ &= 1.41 \times 10^9. \end{aligned}$$

**Problem 3.5.** Simplify  $4.07 \times 10^5 - 2.1 \times 10^4$

**Solution.**

$$\begin{aligned} 4.07 \times 10^5 - 2.1 \times 10^4 &= 4.07 \times 10^5 - 0.21 \times 10^5 \\ &= 3.86 \times 10^5. \end{aligned}$$

**Problem 3.6.** Simplify  $5 \times 10^6 \times 9 \times 10^{-3}$ .

**Solution.**

$$\begin{aligned} 5 \times 10^6 \times 9 \times 10^{-3} &= 45 \times 10^3 \\ &= 4.5 \times 10^4. \end{aligned}$$

**Problem 3.7.** Simplify  $1.6 \times 10^5 \div (2.4 \times 10^{-7})$ .

**Solution.**

$$\begin{aligned} 1.6 \times 10^5 \div (2.4 \times 10^{-7}) &= 0.667 \times 10^{12} \\ &= 6.67 \times 10^{11}. \end{aligned}$$

## Section 4 Tax Rates

**Example 4.1. (UK Tax Rates)**

Parts	Tax Rate
Up to $\pounds 12570$	No Tax
$\pounds 12570$ to $\pounds 37700$	20% Tax
$\pounds 37700$ to $\pounds 150000$	40% Tax
$\pounds 150000$ —	45% Tax

**Example 4.2. (Tax Rates)** For  $\pounds 12000$ , no tax.

**Example 4.3. (Tax Rates)** For  $\pounds 13000$ , no tax on  $\pounds 12570$ ,  $20\% \times (\pounds 13000 - \pounds 12570) = \pounds 86$  tax.

**Problem 4.4.** For £36570, calculate the tax.

**Solution.**  $£24000 \times 0.2 = £4800$ .

**Problem 4.5.** For £80000, calculate the tax.

**Solution.**  $£25200 \times 0.2 + £42300 \times 0.4 = £21960$ .

**Problem 4.6.** For £200000, calculate the tax.

**Solution.**  $£25200 \times 0.2 + £112300 \times 0.4 + £50000 \times 0.45 = £72460$ .