

# Mathematics 16.08.22 Notes

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## Section 1 General Introduction of the Course

### Intro 1.1. (Course Structure)

- (1) IGCSE Maths, CIE, 0580, A\*-E;
- (2) IGCSE Additional Maths, CIE, 0606, A\*-E.

### Intro 1.2. (Maths) Core and Extended, Aug - Oct.

- (1) Paper 2: 90 mins, 70 pts, 23 Short Qs;
- (2) Paper 4: 150 mins, 130 pts, 12 Longer Qs.

### Intro 1.3. (Additional Maths) Nov - Mar.

- (1) Paper 1: 120 mins, 80 pts;
- (2) Paper 2: 120 mins, 80 pts.

### Intro 1.4. (Important Documents) Syllabus (Changed in 2020 and 2022); 2017- Practice Problems.

### Intro 1.5. (Curriculum) Number 15-20%, Algebra 35-40%, Geometry 30-35%, Stat/Prob 10-15%.

## Section 2 Numbers

**Definition 2.1. (Integers)**  $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$ . All whole numbers including 0. Symbol:  $\mathbb{Z}$ .

**Definition 2.2. (Natural Numbers)**  $1, 2, 3, \dots$ . All positive whole numbers without 0. Symbol:  $\mathbb{N}$  or  $\mathbb{Z}^+$ .

**Definition 2.3. (Rational Numbers)** Can be written as  $\frac{a}{b}$  where  $a, b \in \mathbb{Z}$ . Any number with terminating or repeating decimals. Symbol:  $\mathbb{Q}$ .

**Definition 2.4. (Irrational Numbers)** Real numbers excluding rational numbers.

**Example 2.5. (Irrational Numbers)**  $\pi, \sqrt{2}, \sqrt{3}, \sqrt{9.2}, \sqrt[5]{19}, e$ .

**Definition 2.6. (Real Numbers)** All numbers. Symbol:  $\mathbb{R}$ .

**Definition 2.7. (Factors)** Factors of a number  $n$ :  $a$  where  $\frac{n}{a} \in \mathbb{N}$ .

**Example 2.8. (Factors)** Factors of 12: 1, 2, 3, 4, 6, 12.

**Definition 2.9. (Multiples)** Multiples of a number  $n$ :  $an$  where  $a \in \mathbb{N}$ .

**Example 2.10. (Multiples)** Multiples of 12: 12, 24, 36,  $\dots$ .

**Definition 2.11. (Prime Numbers)** 2, 3, 5, 7, 11,  $\dots$ . Exactly two factors.

**Method 2.12. (Determine whether a number  $n$  is Prime)** Check every Prime up to  $\sqrt{n}$ .

**Problem 2.13.** Determine whether 149 is a prime.

**Solution.**  $\sqrt{149} \approx 12.02$ , check 2, 3, 5, 7, 11.

(1)  $149 \equiv 1 \pmod{2}$ ,

(2)  $149 \equiv 2 \pmod{3}$ ,

(3)  $149 \equiv 4 \pmod{5}$ ,

(4)  $149 \equiv 2 \pmod{7}$ ,

(5)  $149 \equiv 6 \pmod{11}$ .

Therefore 149 is a prime.

**Definition 2.14. (Reciprocal):** Reciprocal of a non-zero number  $n$  is  $\frac{1}{n}$ .

**Method 2.15. (Highest Common Factor, HCF)** Factorization. Do a Venn Diagram of factors. Multiple ONLY the common factors.

**Problem 2.16.** Find the HCF of 156 and 72.

**Solution.**

$$\begin{aligned} 156 &= 2 \times 78 \\ &= 2 \times 2 \times 39 \\ &= 2 \times 2 \times 3 \times 13 \\ &= 2^2 \times 3 \times 13; \\ 72 &= 2 \times 36 \\ &= 2 \times 2 \times 18 \\ &= 2 \times 2 \times 2 \times 9 \\ &= 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^3 \times 3^2. \end{aligned}$$

Common Factors:  $2^2, 3$ ; 156-Only Factors: 13; 72-Only Factors: 2, 3.

$$\text{hcf}(156, 72) = 2^2 \times 3 = 12.$$

**Method 2.17. (Least Common Multiple, LCM)** Factorization. Do a Venn Diagram of factors. Multiple ALL factors (only multiple the common ones once).

**Problem 2.18.** Find the LCM of 156 and 72.

**Solution.** Common Factors:  $2^2, 3$ ; 156-Only Factors: 13; 72-Only Factors: 2, 3.

$$\text{lcm}(156, 72) = 2^3 \times 3^2 \times 13 = 936.$$