

# Mathematics 18.08.22 (2) Notes

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## Section 1 Sequence

**Example 1.1. (Sequence)** Find the next two items: 2.1, 2.01, 2.001, 2.0001,  $\dots$  2.000001, 2.0000001.

**Example 1.2. (Sequence)** Find the next two items: 1,  $1/2$ ,  $1/3$ ,  $1/4$ ,  $1/5$ ,  $\dots$   $1/6$ ,  $1/7$ .

**Problem 1.3.** Find the next two items:  $2/64$ ,  $5/32$ ,  $8/16$ ,  $11/8$ ,  $\dots$ .

**Solution.** The numerator  $+3$ , the denominator  $\div 2$ . Therefore the answer is:  $14/4$ ,  $17/2$ .

**Problem 1.4.** Find the next two items: 1, 1, 2, 3, 5, 8, 13, 21,  $\dots$ .

**Solution.** Every item is the sum of the previous two itmes. Therefore the answer is:  $13+21 = 34$ ,  $21+34 = 55$ .

### §1.1 Mathematical Notation

**Problem 1.5.**  $u_n = 4n^2 - 1$  defines a sequence.

(1) Find  $u_1, u_2, u_3$ .

**Solution.**  $u_1 = 4 \times 1^2 - 1 = 3$ ,  $u_2 = 4 \times 2^2 - 1 = 15$ ,  $u_3 = 4 \times 3^2 - 1 = 35$ .

(2) What is  $n$  when  $u_n = 1295$ ?

**Solution.** We have  $u_n = 4n^2 - 1 = 1295$ , thus  $n^2 = 324$ , and  $n = 18$ .

(3) Show 900 is not in the sequence.

**Solution.**  $u_n = 900 = 4n^2 - 1 \Leftrightarrow 4n^2 = 901 \Leftrightarrow n^2 = 225.25 \notin \mathbb{N}$ , while  $n \in \mathbb{N}$ .

## §1.2 Other Way Around

**Problem 1.6.** Find the  $n$ th term for 2, 16, 54, 128,  $\dots$ .

**Solution.**

$$2, 16, 54, 128, \dots \xrightarrow{\div 2} 1, 8, 27, 64, \dots \\ = n^3.$$

$$u_n = 2 \times n^3.$$

**Method 1.7. (Important Sequences)** Remember these sequences:

$n$	1	2	3	4	5
Square Numbers $n^2$	1	4	9	16	25
Cube Numbers $n^3$	1	8	27	64	125
Triangular Numbers $n(n+1)/2$	1	3	6	10	15
All Below: Exponential $2^n$	2	4	8	16	32
$3^n$	3	9	27	81	243
$0.1^n$	0.1	0.01	0.001	0.0001	0.00001
$(-1)^n$	-1	1	-1	1	-1

**Problem 1.8.** Find the  $n$ th term for 0, 2, 5, 9, 14,  $\dots$ .

**Solution.**

$$0, 2, 5, 9, 14, \dots \xrightarrow{+1} 1, 3, 6, 10, 15, \dots \\ = \frac{n(n+1)}{2}.$$

$$u_n = n(n+1)/2 - 1 = (n^2 + n - 2)/2.$$

**Problem 1.9.** Find the  $n$ th term for 4, 9, 16, 25, 36,  $\dots$ .

**Solution.**  $u_n = (n-1)^2 = n^2 - 2n + 1.$

**Problem 1.10.** Find the  $n$ th term for 8, 16, 24, 32, 40,  $\dots$ .

**Solution.**

$$8, 16, 24, 32, 40, \dots \xrightarrow{\div 8} 1, 2, 3, 4, 5, \dots \\ = n.$$

$$u_n = 8n.$$

**Problem 1.11.** Find the  $n$ th term for 3, 12, 27, 48, 75,  $\dots$ .

**Solution.**

$$3, 12, 27, 48, 75, \dots \xrightarrow{\div 3} 1, 4, 9, 16, 25, \dots \\ = n^2.$$

$$u_n = 3n^2.$$

**Problem 1.12.** Find the  $n$ th term for 1, 0.1, 0.01, 0.001, 0.0001,  $\dots$ .

**Solution.**  $u_n = 10 \times 0.1^n = 0.1^{n-1} = 10^{-n+1}.$

**Problem 1.13.** Find the  $n$ th term for 5, 0.5, 0.05, 0.005,  $\dots$ .

**Solution.**  $u_n = 50 \times 0.1^n = 5 \times 0.1^{n-1} = 5 \times 10^{-n+1}$ .

**Problem 1.14.** Find the  $n$ th term for 3, 4, 11, 30, 67,  $\dots$ .

**Solution.**

$$3, 4, 11, 30, 67, \dots \xrightarrow{-3} 0, 1, 8, 27, 64 \\ = (n-1)^3.$$

$$u_n = (n-1)^3 + 3 = n^3 - 3n^2 + 3n + 2.$$

**Method 1.15. (Trying Out)** Try out after writing the formulae.

**Problem 1.16.** Find the  $n$ th term for 5, 20, 45, 80, 125,  $\dots$ .

**Solution.**

$$5, 20, 45, 80, 125, \dots \xrightarrow{\div 5} 1, 4, 9, 16, 25, \dots \\ = n^2.$$

$$u_n = 5n^2.$$

**Problem 1.17.** Find the  $n$ th term for 1, 15, 53, 127, 249,  $\dots$ .

**Solution.**

$$1, 15, 53, 127, 249, \dots \xrightarrow{+1} 2, 16, 54, 128, 250, \dots \\ \xrightarrow{\div 2} 1, 8, 27, 64, 125, \dots \\ = n^3.$$

$$u_n = 2n^3 - 1.$$

**Problem 1.18.** Find the  $n$ th term for 3, 5, 9, 17, 33,  $\dots$ .

**Solution.**

$$3, 5, 9, 17, 33, \dots \xrightarrow{-1} 2, 4, 8, 16, 32, \dots \\ = 2^n.$$

$$u_n = 2^n + 1.$$

**Problem 1.19.** Find the  $n$ th term for 2, 6, 12, 20, 30,  $\dots$ .

**Solution.**

$$2, 6, 12, 20, 30, \dots \xrightarrow{\div 2} 1, 3, 6, 10, 15, \dots \\ = \frac{n(n+1)}{2}.$$

$$u_n = n(n+1) = n^2 + n.$$

**Problem 1.20.** Find the  $n$ th term for  $1/3, 2/7, 4/11, 8/15, 16/19, \dots$ .

**Solution.** Numerator:  $2^n$ ; Denominator:  $4n-1$ .  $u_n = 2^n/(4n-1)$ .

**Problem 1.21.** Find the  $n$ th term for 0.1,  $-0.8, 2.7, -6.4, 12.5, \dots$ .

**Solution.**

$$0.1, -0.9, 2.7, -6.4, 12.5, \dots \xrightarrow{\times 10} 1, -9, 27, -64, 125, \dots$$

$$\xrightarrow{\div (-1)^{n+1}} 1, 9, 27, 64, 125, \dots$$

$$u_n = 0.1(-1)^{n+1}n^3.$$

**Problem 1.22.** Find the  $n$ th term for  $-2, 4, -8, 16, -32, \dots$ .

**Solution.**

$$-2, 4, -8, 16, -32, \dots \xrightarrow{\div (-1)^n} 2, 4, 8, 16, 32, \dots$$

$$= 2^n.$$

$$u_n = (-2)^n.$$

**Problem 1.23.** Find the  $n$ th term for  $0.01, 0.04, 0.09, 0.16, 0.25, \dots$ .

**Solution.**

$$0.01, 0.04, 0.09, 0.16, 0.25, \dots \xrightarrow{\times 100} 1, 4, 9, 16, 25$$

$$= n^2.$$

$$u_n = 0.01n^2.$$

## §1.3 Using Difference

**Method 1.24. (Using Difference)**

- (1) Constant difference  $\Rightarrow$  Highest power is one;
- (2) Difference is constant difference  $\Rightarrow$  Highest power is two;
- (3) Third difference is same  $\Rightarrow$  Highest power is three;
- (4)  $n$ th difference is same  $\Rightarrow$  Highest power is  $n$ .

**Problem 1.25.** Find the  $n$ th term for  $0, 6, 24, 60, 120, \dots$ .

**Solution.** First Difference: 6, 18, 36, 60. Second Difference: 12, 18, 24. Third Difference: 6, 6. Therefore it is cubic.

Study sequence  $n^3$ . 1, 8, 27, 64, 125. First Difference: 7, 19, 37, 61. Second Difference: 12, 18, 24. Same as the sequence. Therefore the second highest is linear.

$$0, 6, 24, 60, 120, \dots \xrightarrow{-n^3} -1, -2, -3, -4, -5, \dots$$

$$\xrightarrow{\times -1} 1, 2, 3, 4, 5, \dots$$

$$= n.$$

$$u_n = n^3 - n.$$