

Mathematics 08.09.22 (1) Notes

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Section 1 Index

Example 1.1. (Fraction Indices) $3^4 = 3 \times 3 \times 3 \times 3 = 81, 9^{1/2} = \sqrt[2]{9} = 3, 2^{-3} = 1/8, 100^{3/2} = 1000, 32^{1/5} = \sqrt[5]{32} = 2, (2^3)^4 = 2^{3 \times 4} = 2^{12} = 4096.$

Method 1.2. (P=Index Rules)

(1) $a^m \times a^n = a^{m+n};$

(2) $a^m \div a^n = a^{m-n};$

(3) $a^{-n} = 1/a^n;$

(4) $a^0 = 1(a \neq 0);$

(5) $(a^m)^n = a^{mn};$

(6) $a^{1/n} = \sqrt[n]{a};$

(7) $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}.$

Example 1.3. (Indices) $(3/2)^4 = (3^4/2^4) = 81/16.$

Problem 1.4. (Indices Equations)

(1) $2^{7x+2} \times 8^{3-2x} = 4^{x+1}.$

Solution.

$$2^{7x+2} \times 8^{3-2x} = 4^{x+1}$$

$$2^{7x+2} \times 2^{9-6x} = 2^{2x+2}$$

$$2^{7x+2+9-6x} = 2^{2x+2}$$

$$7x + 2 + 9 - 6x = 2x + 2$$

$$x = 9.$$

(2) $2^{7x+y} \times 3^{x-2y+2} = 12^{y-1}$.

Solution.

$$\begin{aligned} 2^{7x+y} \times 3^{x-2y+2} &= 12^{y-1} \\ &= (2^2 \times 3)^{y-1} \\ 2^{7x+y} \times 3^{x-2y+2} &= 2^{2y-2} \times 3^{y-1} \\ 7x + y &= 2y - 2 \\ x - 2y + 2 &= y - 1 \\ x &= -0.05 \\ y &= 0.45. \end{aligned}$$

Section 2 Factorisation

Example 2.1. (Number Factorisation) $12 = 2^2 \times 3$.

Example 2.2. (Algebraic Factorisation) $15x - 10 = 5(3x - 2)$.

Problem 2.3. Factorise $6x^3 - 4x^5$.

Solution.

$$\begin{aligned} 6x^3 - 4x^5 &= 2x^3(3 - 2x^2) \\ &= 2x^3(\sqrt{3} - \sqrt{2}x)(\sqrt{3} + \sqrt{2}x). \end{aligned}$$

Problem 2.4. Factorize $2x^2y^3z - 8xyz^2 - 7y^2z^2$.

Solution.

$$2x^2y^3z - 8xyz^2 - 7y^2z^2 = yz(2x^2y^2 - 8xz - 7yz).$$

Problem 2.5. Factorise $2(x + 10)^3 + y(x + 10)^2$.

Solution.

$$2(x + 10)^3 + y(x + 10)^2 = (x + 10)^2(2x + y + 20).$$

Section 3 Rearrangement

Problem 3.1. Rearrange to make x the subject: $10x + a = -3x + 7$.

Solution. $x = (7 - a)/13$.

Problem 3.2. Rearrange ...: $2/x = a/b$.

Solution. $x = 2b/a$.

Problem 3.3. Rearrange ...: $(3x + c)/[d(x - 5)] = 2$.

Solution.

$$\begin{aligned} \frac{3x + c}{d(x - 5)} &= 2 \\ 3x + c &= 2dx - 10d \\ (3 - 2d)x &= c - 10d \\ x &= \frac{c - 10d}{3 - 2d}. \end{aligned}$$

Problem 3.4. Rearrange ...: $1 + \sqrt{4x - 5} = 2d$.

Solution.

$$\begin{aligned}1 + \sqrt{4x - 5} &= 2d \\ \sqrt{4x - 5} &= 2d - 1 \\ 4x - 5 &= 4d^2 - 4d + 1 \\ x &= d^2 - d + \frac{3}{2}.\end{aligned}$$

Problem 3.5. Rearrange ...: $2 - 7/(3x^2 - 1) = k$.

Solution.

$$\begin{aligned}2 - \frac{7}{3x^2 - 1} &= k \\ 2 - k &= \frac{7}{3x^2 - 1} \\ \frac{1}{2 - k} &= \frac{3x^2 - 1}{7} \\ 3x^2 - 1 &= \frac{7}{2 - k} \\ 3x^2 &= \frac{9 - k}{2 - k} \\ x^2 &= \frac{9 - k}{6 - 3k} \\ x &= \pm \sqrt{\frac{9 - k}{6 - 3k}}.\end{aligned}$$