Mathematics 23.08.22 Notes

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Contents

Section 1 Converting Between Decimals and Fractions

Example 1.1. Write -7 as a fraction. $-7 = -7/1$.
Example 1.2. Write 2.9 as a fraction. $2.9 = 29/10$.
Example 1.3. Write -37.081 as a fraction. $-37.081 = -37081/1000$.
Example 1.4. Write $0.\dot{3}$ as a fraction. $0.\dot{3} = 3/9 = 1/3$.
Example 1.5. Write $0.\dot{1}$ as a fraction. $0.\dot{1} = 1/9$.

Problem 1.6. Write $0.\dot{7}$ as a fraction.
Solution. $0.\dot{7} = 7/9$.

Method 1.7. (Converting decimals to fractions) Let $x = 0.\dot{3}$, thus $10x = 3.\dot{3}$. Therefore $10x - x = 3.\dot{3} - 0.\dot{3} = 3 = 9x$, and $x = 3/9 = 1/3$.

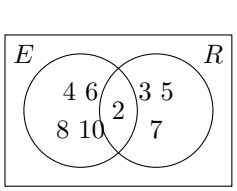
Problem 1.8. Show that $2.1\dot{5}\dot{7} \in \mathbb{Q}$.
Solution. Let x be $2.1\dot{5}\dot{7}$, therefore $10x = 21.\dot{5}\dot{7}, 1000x = 2157.\dot{5}\dot{7}$.
$1000x - 10x = 2136 = 990x, x = 2136/990 = 178/75 \in \mathbb{Q}$.

Problem 1.9. Express $x = -62.\dot{0}3\dot{2}$ as a fraction.
Solution. $1000x = -62032.\dot{0}3\dot{2}, 1000x - x = 999x = -61970, x = 61970/999$.

Problem 1.10. Express $x = 82.\dot{8}57142\dot{8}$ as a frat ion.
Solution. $x = 82\frac{6}{7} = 580/7$.

Section 2 Sets

Example 2.1. Let Universal Set be $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let E be the set of evens, R be the set of primes, D be the set of odds, $T := \{2, 3\}$. Write in a Venn Diagram. $E = \{2, 4, 6, 8, 10\}; R = \{2, 3, 5, 7\}; D = \{1, 3, 5, 7, 9\}$.



Example 2.2. (Intersection) $E \cap R = \{2\}$.
Example 2.3. (Union) $E \cup R = \{2, 4, 6, 8, 10, 3, 5, 7\}$.
Example 2.4. (Complement) $R' = \{4, 6, 8, 10, 1, 9\}$.
Example 2.5. (Empty Set) $D \cap E = \emptyset$.
Example 2.6. (In) $3 \in R$ is True. 3 is an element of R.
Example 2.7. (Not In) $4 \notin R$ is True. 4 is not an element of R.
Example 2.8. (Proper Subset) $T \subset R (T \not\in R)$.
Example 2.9. (Subset) $R \subseteq R (R \not\subset R)$.
Example 2.10. (Number) $\mathrm{n}(R) = 4$. Number of elements in R is 4.

Problem 2.11. Simplify $R \cup E'$.
Solution.

$$\begin{aligned} R \cup E' &= \{2, 3, 5, 7\} \cup \{2, 4, 6, 8, 10\}' \\ &= \{2, 3, 5, 7\} \cup \{1, 3, 5, 7, 9\} \\ &= \{1, 9, 3, 5, 7, 2\}. \end{aligned}$$

Problem 2.12. Simplify $T' \cup E$.
Solution.

$$\begin{aligned} T' \cup E &= \{2, 3\}' \cup \{2, 4, 6, 8, 10\} \\ &= \{1, 4, 5, 6, 7, 8, 9, 10\} \cup \{2, 4, 6, 8, 10\} \\ &= \{1, 2, 4, 5, 6, 7, 8, 9, 10\}. \end{aligned}$$

Problem 2.13. Simplify $T' \cap E$
Solution.

$$\begin{aligned} T' \cap E &= \{2, 3\}' \cap \{2, 4, 6, 8, 10\} \\ &= \{1, 4, 5, 6, 7, 8, 9, 10\} \cap \{2, 4, 6, 8, 10\} \\ &= \{4, 6, 8, 10\}. \end{aligned}$$

Problem 2.14. Simplify $\mathrm{n}(T' \cap R)$.
Solution.

$$\begin{aligned} \mathrm{n}(T' \cap R) &= \mathrm{n}(\{2, 3\}' \cap \{2, 3, 5, 7\}) \\ &= \mathrm{n}(\{1, 4, 5, 6, 7, 8, 9, 10\} \cap \{2, 3, 5, 7\}) \\ &= \mathrm{n}\{5, 7\} \\ &= 2. \end{aligned}$$

Problem 2.15. Simplify $R' \cup (E \cap T)$.
Solution.

$$\begin{aligned} R' \cup (E \cap T) &= \{2, 3, 5, 7\} \cup (\{2, 4, 6, 8, 10\} \cap \{2, 3\}) \\ &= \{1, 4, 6, 8, 9, 10\} \cup \{2\} \\ &= \{1, 4, 6, 8, 9, 10, 2\}. \end{aligned}$$

Problem 2.16. Simplify $(E \cup T)'$.
Solution.

$$\begin{aligned}(E \cup T)' &= (\{2,4,6,8,10\} \cup \{2,3\})' \\ &= \{2,3,4,6,8,10\}' \\ &= \{1,5,7,9\}.\end{aligned}$$

Problem 2.17. Simplify $R' \cup T$.
Solution.

$$\begin{aligned}R' \cup T &= \{1,4,6,8,9,10\} \cup \{2,3\} \\ &= \{1,2,3,4,6,8,9,10\}.\end{aligned}$$