Mathematics 25.08.22 (1) Notes

Eason Shao, Mr Finch-Noyes

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Contents

Section 1 Sets

Problem 1.1. In a group of 20 students, 12 study History, 10 study Geography, 3 study neither. How many study both?

Solution. Let x be the number of students who study both History and Geography. According to the **Inclusion-Exclusion Principle**

$$\mathrm{n}(A \cup B) = \mathrm{n}(A) + \mathrm{n}(B) - \mathrm{n}(A \cap B),$$

where $\mathrm{n}(A \cup B) = 20 - 3 = 17, \mathrm{n}(A) = 12, \mathrm{n}(B) = 10, \mathrm{n}(A \cap B) = x$.
Simplify, we have

$$17 = 12 + 10 - x,$$

therefore we have $x = 5$, which means 5 people study both.

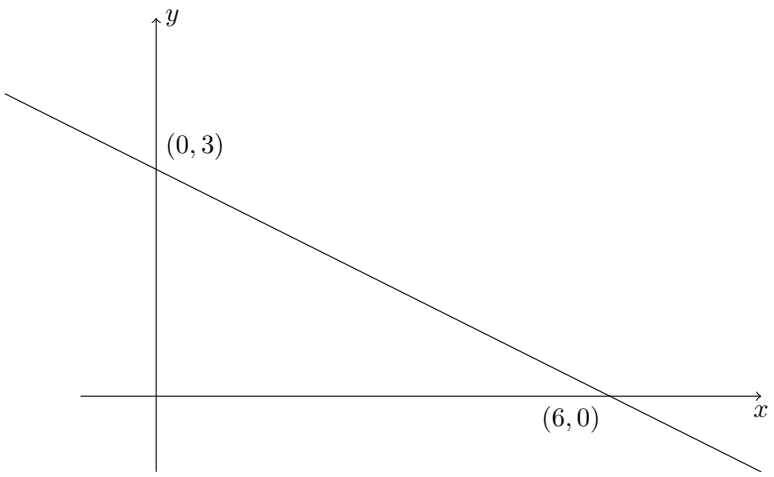
Definition 1.2. (Set Builder Notation) The notation $A = \{x : P\}$ means that the set A includes and only includes all possible x which satisfies P.

Example 1.3. (Set Builder Notation) What does $\{x : 1 \leq x \leq 6, x \in \mathbb{N}\}$ mean? All values (a set). How is it different to $x \in \{1, 2, 3, 4, 5, 6\}$? One value of x.

Problem 1.4. What is the difference between $\{x : 1 < x < 2\}$ and $x \in (1, 2)$?

Solution. The first one is all possible values, while the second one is a notation which means a single x is within the interval $(1, 2)$.

Example 1.5. (Set of Points) Find the equation of the following line. Describe the set of coordinates of points on this line in set build notation.



The line is $y = -\frac{1}{3}x + 3$; the set will be $\left\{(x, y) : y = -\frac{1}{3}x + 3\right\}$.

Section 2 Unit Conversion

§2.1 General Unit Conversion

Example 2.1. (Unit Conversion, Length) $1\,\text{m} = 1 \times 10^2\,\text{cm} = 1 \times 10^3\,\text{mm}$.
Example 2.2. (Unit Conversion, Squared, Hectares) $1\,\text{m}^2 = 1{\times}10^4\,\text{cm}^2 = 1{\times}10^6\,\text{mm}^2 = 1{\times}10^{-4}\,\text{ha}$.
Example 2.3. (Unit Conversion, Cubed) $1\,\text{m}^3 = 1 \times 10^6\,\text{cm}^3 = 1 \times 10^9\,\text{mm}^3$.
Example 2.4. (Unit Conversion, Km) $1\,\text{km} = 1 \times 10^3\,\text{m}$.
Example 2.5. (Unit Conversion, Grams) $1\,\text{kg} = 1 \times 10^3\,\text{g} = 1 \times 10^{-3}\,\text{t}$.
Example 2.6. (Unit Conversion, Litres) $1\,\text{L} = 1 \times 10^{-3}\,\text{m}^3 = 1 \times 10^3\,\text{mL}1 \times 10^2\,\text{cL}$.

§2.2 Time Conversion

Problem 2.7. A plane leaves London at 21 : 15. It lands in Singapore at 17 : 45. Given Singapore is 7 hours ahead, find the flight time.
Solution. $21 : 15_{\text{London}} = 28 : 15_{\text{Singapore}} = 4 : 15_{\text{Singapore, }+1}$.
$17 : 45_{\text{Singapore}} - 4 : 15_{\text{Singapore=13:30}}$. 13 hours and 30 minutes.

§2.3 Bounds

Example 2.8. (Significant Figures) $x = 47$ to 2 significant figures, so we have $x \in [46.5, 47.5)$.
Definition 2.9. (Lower Bound) The lower bound for the previous example is 46.5.
Definition 2.10. (Upper Bound) The upper bound for the previous example is 47.5.

Problem 2.11. $a = 23.10$ to 2 decimal points. Bounds.
Solution. Lower bound is 23.095, upper bound is 23.105.

Problem 2.12. $b = 700$ to 2 significant figures. Bounds.
Solution. Lower bound is 695, upper bound is 705.

Problem 2.13. State the upper and lower bounds of

(1) $a + b$;

Solution. Lower: $23.095 + 695$; Upper: $23.105 + 705$.

(2) $b - a$;

Solution. Lower: $695 - 23.105$; Upper: $705 - 23.095$.

(3) ab;

Solution. Lower: 695×23.095; Upper: 705×23.105.

(4) $\frac{b}{a}$.

Solution. Lower: $695/23.105$; Upper: $705/23.095$.