Mathematics 08.09.22 (2) Notes

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08.09.22 (2)

Contents

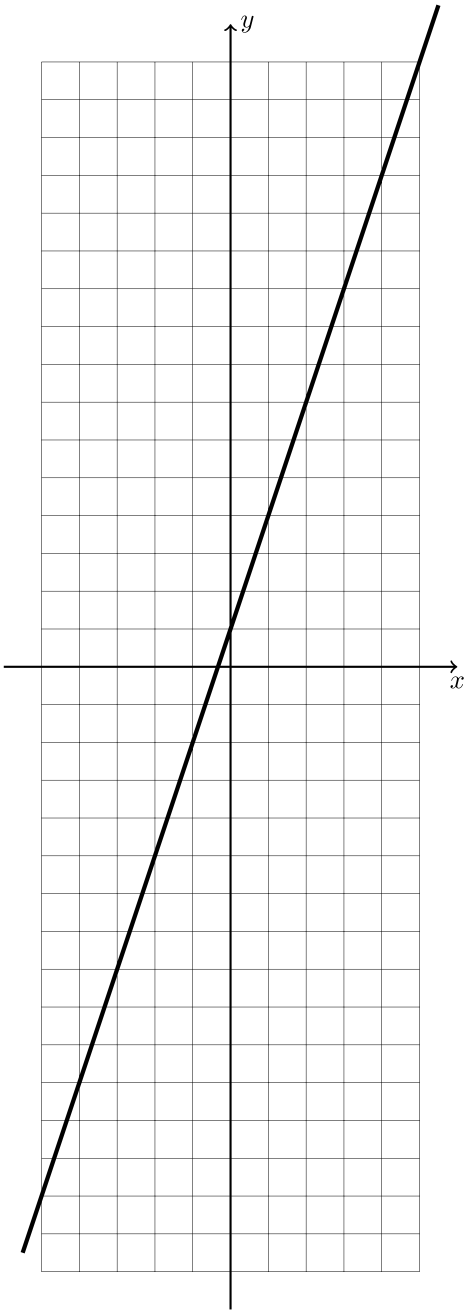
Section 1 Linear Graph

§1.1 Sketching a Graph

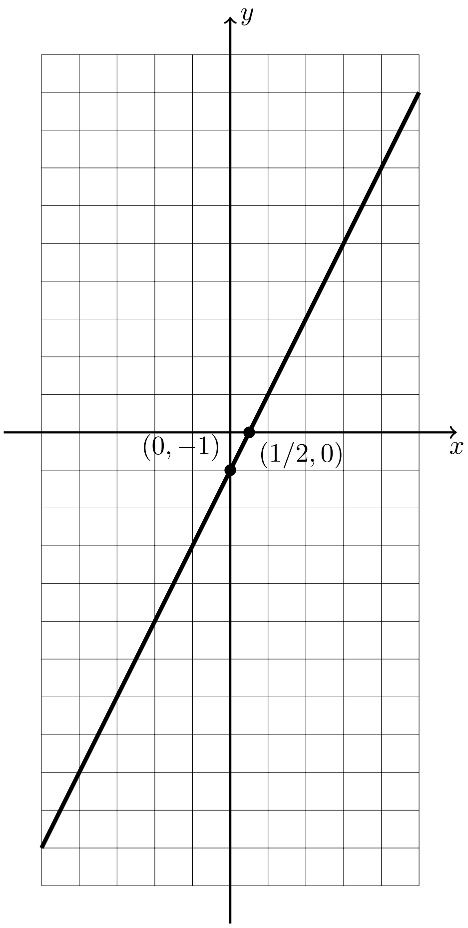
Definition 1.1. (Plot and Sketch) Plot needs to be accurate (e.g. scale), **Sketch** can be not so accurate (Focus on Shape).

Example 1.2. (Plot) Plot $y = 3x + 1$.

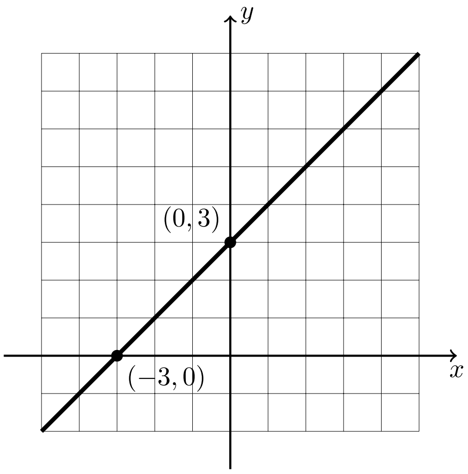
x	5	0	1	-2
y	16	1	4	-5



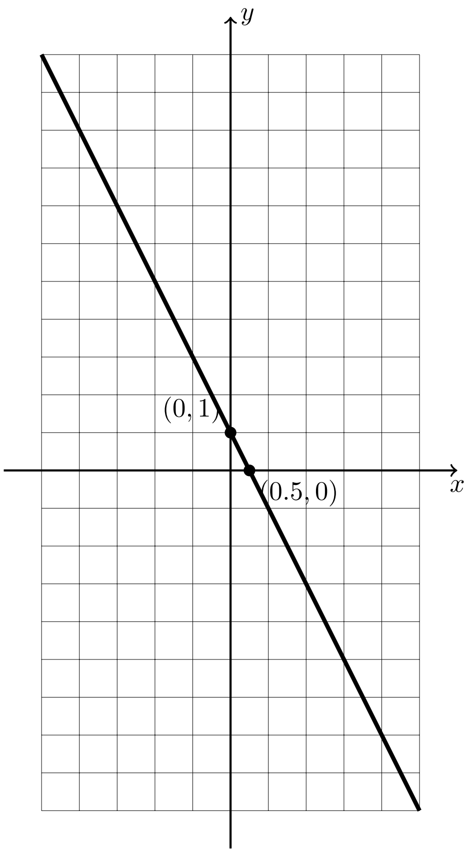
Example 1.3. (Sketch) Sketch $y = 2x - 1$. Meet x axis at $(1/2, 0)$; Meets y axis at $(0, -1)$.



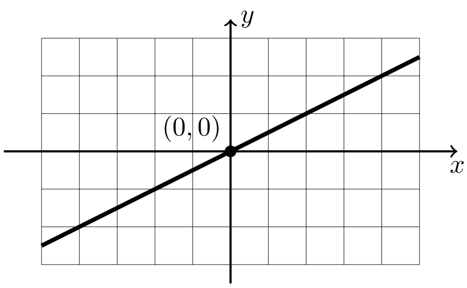
Problem 1.4. Sketch $y = x + 3$.
Solution. Meet x axis at $(-3, 0)$; Meets y axis at $(0, 3)$.



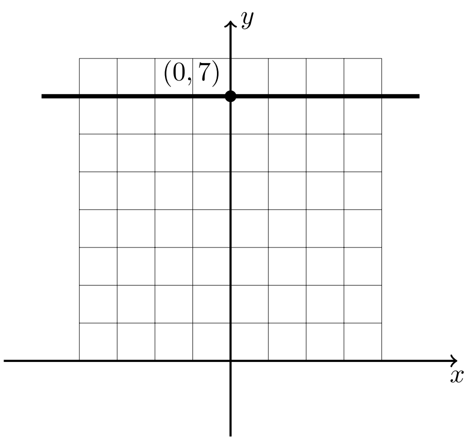
Problem 1.5. Sketch $y = -2x + 1$.
Solution. Meet x axis at $(1/2, 0)$; Meets y axis at $(0, 1)$.



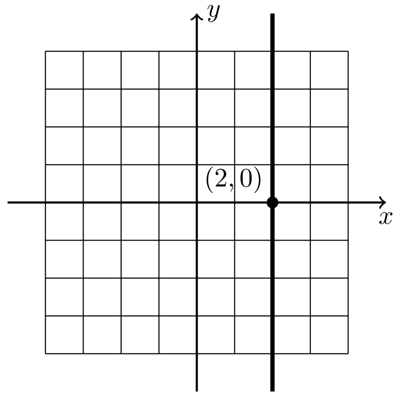
Problem 1.6. Sktech $y = 1/2x$.
Solution. Meet x and y axis at Origin $(0, 0)$.



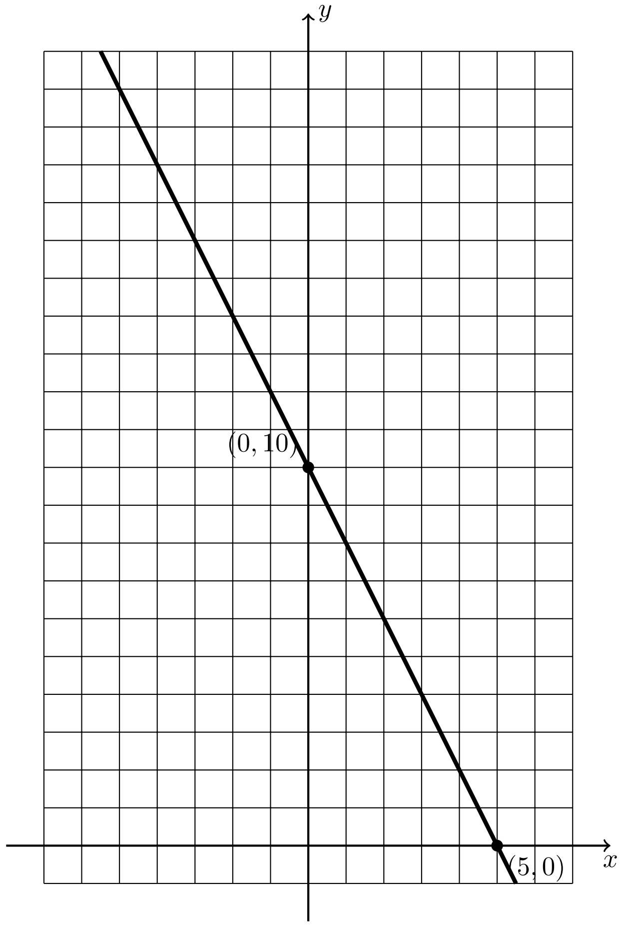
Problem 1.7. Sketch $y = 7$.
Solution.



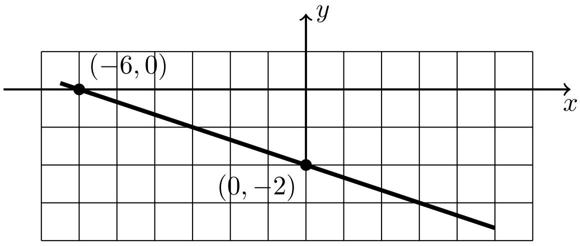
Problem 1.8. Sketch $x = 2$.
Solution.



Problem 1.9. Sketch $2x + y = 10$.
Solution. Meet x axis at $(5, 0)$; Meet y axis at $(0, 10)$.



Problem 1.10. Sketch $x + 3y + 6 = 0$.
Solution. Meet x axis at $(-6, 0)$; Meet y axis at $(0, -2)$.



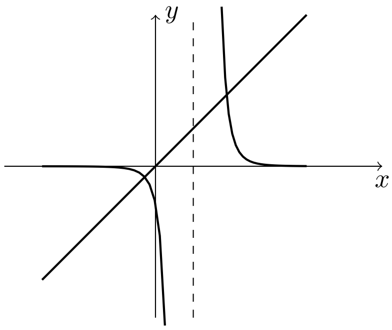
Problem 1.11. Sketch $x = 0$.
Solution. y axis. **Not x axis!**
Problem 1.12. Sketch $y = 0$.
Solution. x axis. **Not y axis!**
Problem 1.13. Sketch $x + y = 0$.
Solution. $y = -x$.

§1.2 Use of Graphs

Method 1.14. (Using Graphs to Solve Equations) A equation $f(x) = g(x)$ can be solved using plotting graphs of $f(x)$ and $g(x)$ and checking the intersectinos.
Problem 1.15. Determine the number of solutions to the equation

$$\frac{1}{(x-1)^5} = x,$$

and determine the approximate values of each solution.
Solution. Sketch the graphs:

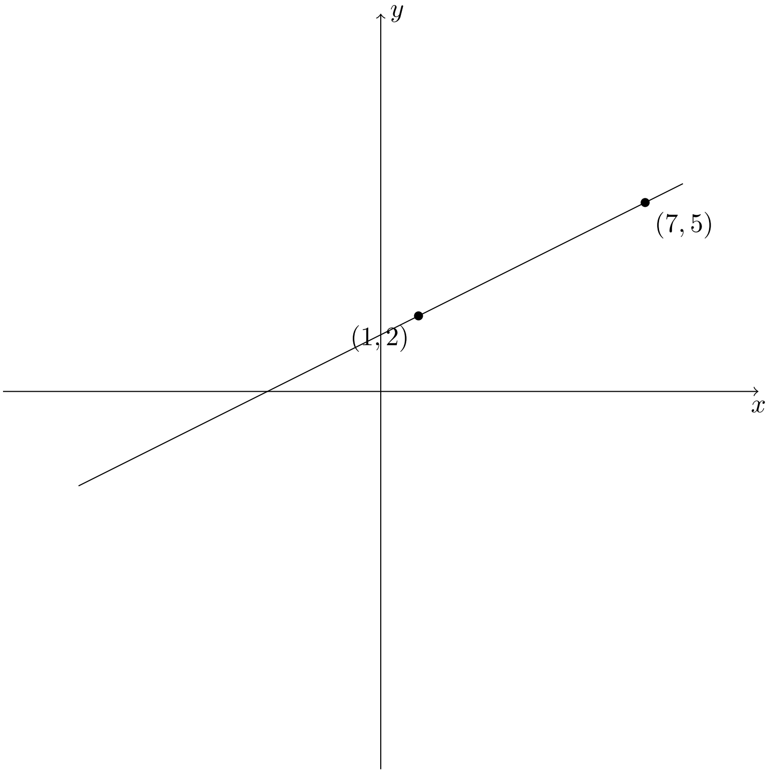


Two solutions (two intersections), one slightly smaller than 0, one bigger than 1 and close to 2.

§1.3 Gradient

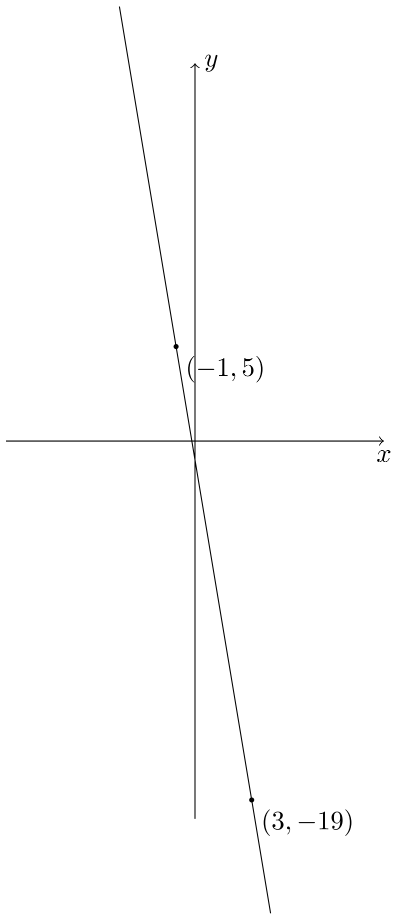
Definition 1.16. (Gradient in a Linear Graph) Change in y when x increases by 1.
Example 1.17. (Gradient) Work out the gradient of the line below.
Example 1.18. (Gradient) Work out the gradient of the line below.
Problem 1.19. Calculate the following four gradients.

(1)



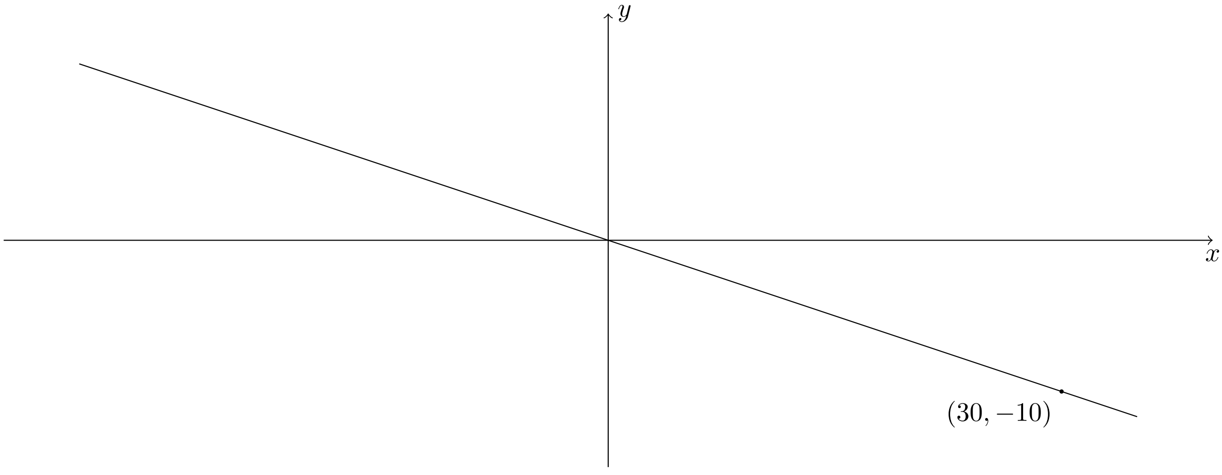
Solution. $m = (5 - 2)/(7 - 1) = 1/2$.

(2)



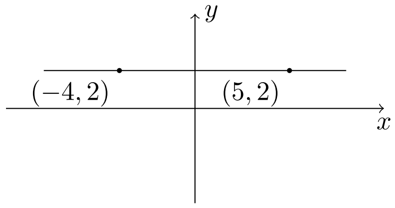
Solution. $m = (-19 - 5)/(3 - -1) = -6.$

(3)



Solution. $m = (-10)/30 = -1/3.$

(4)



Solution. $m = 0$.

Method 1.20. (Calculating Gradient) Gradient $m = (y_2 - y_1)/(x_2 - x_1)$

§1.4 Writing an Equation

Definition 1.21. (Linear Equation) $y = mx + C$. m is the **Gradient** and C is the **y-intercept**.
Problem 1.22. Write the equations of the previous four lines.
Solution.

(1) $y = 1/2x + C$, let $(x, y) = (1, 2)$, $c = 3/2$, $y = 1/2x + 3/2$.

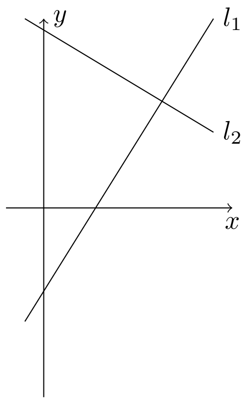
(2) $y = -6x + C$, let $(x, y) = (-1, 5)$, $c = -1$, $y = -6x - 1$.

(3) $y = -1/3x + C$, let $(x, y) = (0, 0)$, $c = 0$, $y = -1/3x$.

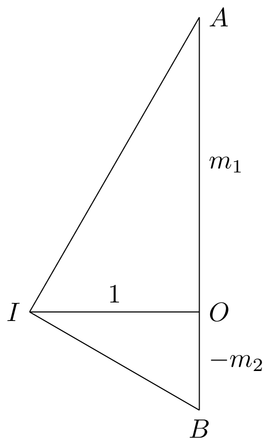
(4) $y = C$, $y = 2$.

§1.5 Parallel and Perpendicular

Method 1.23. (Parallel) Two **parallel** lines have the same gradient or if they both have undefined gradient (i.e. verticle). That is, for $l_1 : y = m_1x + c_1, l_2 : y = m_2x + c_2, m_1 = m_2 \Leftrightarrow l_1 \parallel l_2$.
Method 1.24. (Perpendicular) Two **perpendicular** lines have gradient with the product of -1 or one is 0 and one is undefined. That is, for $l_1 : y = m_1x + c_1, l_2 : y = m_2x + c_2, m_1 \times m_2 = -1 \Leftrightarrow l_1 \perp l_2$.
Problem 1.25. Prove the relationship between the product of gradients and being perpendicular.
Solution. Let $l_1 : y = m_1x + c_1, l_2 : y = m_2x + c_2$. It is obvious that $m_1 \times m_2 < 0$, thus we could draw the following shape:



We could then go 1 unit positive to the right of the intersection point, where l_1 will increase for m_1 and l_2 will decrease for m_2. This is shown in the following diagram: (Several Extra Letters are marked):



It is obvious (due to the nature of the xOy plane that $x \perp y$) that all triangles are right-angled, therefore similar to each other.

$$\tan \angle A = \frac{m_1}{1} = \tan \angle OIB = \frac{1}{-m_2},$$

thus

$$m_1 \times m_2 = -1.$$

$\square$

Problem 1.26.

(1) Find equation of the straight lines through $A(1,7)$ and are perpendicular/parallel to $y = 3x - 2$.
Solution. $\parallel$: $y = 3x + 4$; $\perp$: $y = -1/3x + 22/3$.

(2) The lines found in **(1)** meet the y-axis at points B and C. Find the area of triangle ABC.
Solution. $B(0,4), C(0,22/3)$. Area $= 1/2 \times 10/3 \times 1 = 5/3$.

§1.6 Length of Sector and Midpoint

Definition 1.27. (Distance between Two Points/Length of Sector) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Definition 1.28. (Midpoint of Two Points/A Sector) $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.