Mathematics 16.08.22 Notes

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Contents

Section 1   General Introduction of the Course

Intro 1.1. (Course Structure)

(1) IGCSE Maths, CIE, 0580, A*-E;

(2) IGCSE Additional Maths, CIE, 0606, A*-E.

Intro 1.2. (Maths) Core and Extended, Aug - Oct.

(1) Paper 2: 90 mins, 70 pts, 23 Short Qs;

(2) Paper 4: 150 mins, 130 pts, 12 Longer Qs.

Intro 1.3. (Additional Maths) Nov - Mar.

(1) Paper 1: 120 mins, 80 pts;

(2) Paper 2: 120 mins, 80 pts.

Intro 1.4. (Important Documents) Syllabus (Changed in 2020 and 2022); 2017- Practice Problems.

Intro 1.5. (Curriculum) Number 15-20%, Algebra 35-40%, Geometry 30-35%, Stat/Prob 10-15%.

Section 2   Numbers

Definition 2.1. (Integers) $\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$. All whole numbers including 0. Symbol: $\mathbb{Z}$.

Definition 2.2. (Natural Numbers) $1, 2, 3, \cdots$. All positive whole mumbers without 0. Symbol: $\mathbb{N}$ or $\mathbb{Z}^+$.

Definition 2.3. (Rational Numbers) Can be written as $\frac{a}{b}$ where $a, b \in \mathbb{Z}$. Any number with terminating or repeating decimals. Symbol: $\mathbb{Q}$.

Definition 2.4. (Irrational Numbers) Real numbers excluding rational numbers.

Example 2.5. (Irrational Numbers) $\pi, \sqrt{2}, \sqrt{3}, \sqrt{9.2}, \sqrt[5]{19}, \mathrm{e}$.

Definition 2.6. (Real Numbers) All numbers. Symbol: $\mathbb{R}$.

Definition 2.7. (Factors) Factors of a number n: a where $\frac{n}{a} \in \mathbb{N}$.

Example 2.8. (Factors) Factors of 12: $1, 2, 3, 4, 6, 12$.
Definition 2.9. (Multiples) Multiples of a number n: an where $a \in \mathbb{N}$.
Example 2.10. (Multiples) Multiples of 12: $12, 24, 36, \cdots$.
Definition 2.11. (Prime Numbers) $2, 3, 5, 7, 11, \cdots$. Exactly two factors.
Method 2.12. (Determine whether a number n is Prime) Check every Prime up to $\sqrt{n}$.

Problem 2.13. Determine whether 149 is a prime.
Solution. $\sqrt{149} \approx 12.02$, check $2, 3, 5, 7, 11$.

(1) $149 \equiv 1 \pmod 2$,

(2) $149 \equiv 2 \pmod 3$,

(3) $149 \equiv 4 \pmod 5$,

(4) $149 \equiv 2 \pmod 7$,

(5) $149 \equiv 6 \pmod{11}$.

Therefore 149 is a prime.

Definition 2.14. (Reciprocal): Reciprocal of a non-zero number n is $\frac{1}{n}$.
Method 2.15. (Highest Common Factor, HCF) Factorization. Do a Venn Diagram of factors. Multiple ONLY the common factors.

Problem 2.16. Find the HCF of 156 and 72.
Solution.

$$
\begin{aligned}
156 &= 2 \times 78 \\
&= 2 \times 2 \times 39 \\
&= 2 \times 2 \times 3 \times 13 \\
&= 2^2 \times 3 \times 13; \\
72 &= 2 \times 36 \\
&= 2 \times 2 \times 18 \\
&= 2 \times 2 \times 2 \times 9 \\
&= 2 \times 2 \times 2 \times 3 \times 3 \\
&= 2^3 \times 3^2.
\end{aligned}
$$

Common Factors: $2^2, 3$; 156-Only Factors: 13; 72-Only Factors: $2, 3$.
$\mathrm{hcf}(156, 72) = 2^2 \times 3 = 12$.

Method 2.17. (Least Common Multiple, LCM) Factorization. Do a Venn Diagram of factors. Multiple ALL factors (only multiple the common ones once).

Problem 2.18. Find the LCM of 156 and 72.
Solution. Common Factors: $2^2, 3$; 156-Only Factors: 13; 72-Only Factors: $2, 3$.
$\mathrm{lcm}(156, 72) = 2^3 \times 3^2 \times 13 = 936$.