Mathematics 18.08.22 (1) Notes

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Contents

Section 1 Revision

§1.1 Numbers

Problem 1.1. Determine whether -3 is in $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$.
Solution. $-3 \notin \mathbb{N}, -3 \in \mathbb{Z}, -3 \in \mathbb{Q}, -3 \in \mathbb{R}$.

Problem 1.2. Prove that $7.385 \in \mathbb{Q}$.
Solution. $7.385 = 7385/1000$.

Problem 1.3. Prove that $0.\dot{7}1\dot{5} \in \mathbb{Q}$.
Solution. $0.\dot{7}1\dot{5} = 715/999$.

§1.2 Factorization, lcms and hcfs

Problem 1.4. Factorize 432.
Solution.

$$\begin{aligned}432 &= 2 \times 216\\ &= 2^2 \times 108\\ &= 2^3 \times 54\\ &= 2^4 \times 27\\ &= 2^4 \times 3 \times 9\\ &= 2^4 \times 3^2 \times 3\\ &= 2^4 \times 3^3.\end{aligned}$$

Therefore $432 = 2^4 \times 3^3$.

Problem 1.5. Factorize 900.
Solution.

$$\begin{aligned}900 &= 2 \times 450\\ &= 2^2 \times 225\\ &= 2^2 \times 3 \times 75\\ &= 2^2 \times 3^2 \times 25\\ &= 2^2 \times 3^2 \times 5^2.\end{aligned}$$

Therefore $900 = 2^2 \times 3^2 \times 5^2$.

Problem 1.6. Calculate the HCF of 432 and 900.
Solution. Common factors: $2^2, 3^2$. 432-Only factors: $2^2, 3$. 900-Only factors: 5^2. $\text{hcf}(432, 900) = 2^2 \times 3^2 = 36$.

Problem 1.7. Calculate the LCM of 432 and 900.
Solution. Common factors: $2^2, 3^2$. 432-Only factors: $2^2, 3$. 900-Only factors: 5^2. $\text{lcm}(432, 900) = 2^4 \times 3^3 \times 5^2 = 10800$.

Section 2 Fractions

§2.1 Fraction Calculation

Example 2.1. (Faction Calculation)

$$\frac{a}{432} - \frac{b^2}{900} \xrightarrow[\times\text{Other-Only Factor}]{\text{LCM Common Denominator}} \frac{a \cdot 5^2}{10800} - \frac{b^2 \cdot 2^2 \cdot 3}{10800} = \frac{25a - 12b^2}{10800}.$$

Problem 2.2. Simplify

$$\frac{a}{c} \times b - \frac{e}{3f}.$$

Solution.

$$\begin{aligned}\frac{a}{c}\times b-\frac{e}{3f}&=\frac{ab}{c}-\frac{e}{3f}\\&=\frac{3abf}{3cf}-\frac{ce}{3cf}\\&=\frac{3abf-ce}{3cf}.\end{aligned}$$

Problem 2.3. Simplify

$$\frac{b}{8e}\div\frac{2a}{c}.$$

Solution.

$$\begin{aligned}\frac{b}{8e}\div\frac{2a}{c}&=\frac{b}{8e}\times\frac{c}{2a}\\&=\frac{bc}{16ae}.\end{aligned}$$

Problem 2.4. Simplify

$$4-\frac{3}{a}.$$

Solution.

$$\begin{aligned}4-\frac{3}{a}&=\frac{4a}{a}-\frac{3}{a}\\&=\frac{4a-3}{a}.\end{aligned}$$

Problem 2.5. Simplify

$$\frac{8}{3}\div 7a.$$

Solution.

$$\begin{aligned}\frac{8}{3}\div 7a&=\frac{8}{3}\times\frac{1}{7a}\\&=\frac{8}{21a}.\end{aligned}$$

Problem 2.6. Simplify

$$3c\div\frac{276}{e^5}.$$

Solution.

$$\begin{aligned}3c\div\frac{276}{e^5}&=3c\times\frac{e^5}{276}\\&=\frac{3ce^5}{276}\\&=\frac{ce^5}{92}.\end{aligned}$$

Problem 2.7. Simplify

$$\frac{9a^2}{10bc} \times \frac{12b}{6a^3}.$$

Solution.

$$\begin{aligned}\frac{9a^2}{10bc} \times \frac{12b}{6a^3} &= \frac{9a^2 \cdot 12b}{10bc \cdot 6a^3} \\ &= \frac{9}{5ac}.\end{aligned}$$

§2.2 Fraction Comparison

Example 2.8. (Fraction Comparison) Compare 11/432 and 23/900. $11/432 = 275/10800$. $23/900 = 276/10800$. $11/432 < 23/900$.

§2.3 Concepts of Fractions

Definition 2.9. (Mixed Fraction) e.g. $7\frac{2}{11}$.
Definition 2.10. (Vulger Fraction) e.g. $\frac{300}{40}$.
Example 2.11. (Fractions) Convert $7\frac{2}{11}$ into a vulger fraction. $7\frac{2}{11} = \frac{79}{11}$.
Example 2.12. (Fractions) Convert $\frac{300}{40}$ into a mixed fraction. $\frac{300}{40} = \frac{15}{2} = 7\frac{1}{2}$.

§2.4 Percentages

Problem 2.13. Express 32/73 as a %.
Solution. $32/73 = 43.836\%$.

Problem 2.14. Find 17% of 453.
Solution. $453 \times 17\% = 77.01$.

Problem 2.15. Increase 82 by 61%.
Solution. $82 \times (1 + 61\%) = 132.02$.

Problem 2.16. A phone marked £370 need to be reduced by 15%. Find the now price.
Solution. $(1 - 15\%) \times £370 = £314.5$.

Problem 2.17. A coat priced at £120 need a 20% sales tax to be added to the price. Find the new price.
Solution. $£120 \times (1 + 20\%) = £144$.

Problem 2.18. After a 20% tax has been added to a bag's price is £45. Find the pre-tax price.
Solution. Let p be the pre-tax price. Therefore

$$(1 + 20\%)p = £45,$$

Simplify, we have $p = £37.5$.

Problem 2.19. After a 35% discount a shop price is £795. Find the previous price.
Solution. Let p be the previous price. Therefore

$$(1 - 35\%)p = £795$$

Simplify, we have $p = £1223.08$.

Problem 2.20. £12000 is invested in a 2.75% amount for 20 years.
Find the value at the end of the investment saving.
Solution.

(1) Single interest. £12000 × 2.75% = £330. £330 × 20 = £6600. £12000 + £6600 = £18600.

(2) Compound interest. £12000 × $(1 + 2.75\%)^{20}$ = £20645.141.

Problem 2.21. A \$30,000 car depreciate at 15% per year. What is the value by 12 years?
Solution. \$30000 × $(1 - 15\%)^{12}$ = \$4267.253.

Section 3 Standard Form

Definition 3.1. (Standard Form) $a \times 10^n, n \in \mathbb{Z}. 1 \leq |a| < 10$.
Example 3.2. (Standard Form) Write 280700 in standard form. $280700 = 2.807 \times 10^5$.
Example 3.3. (Standard Form) Write 7.09×10^{-5} as a half decimal. $7.09 \times 10^{-5} = 0.0000709$.

Problem 3.4. Simplify $4.7 \times 10^8 + 9.4 \times 10^8$
Solution.

$$\begin{aligned} 4.7 \times 10^8 + 9.4 \times 10^8 &= 14.1 \times 10^8 \\ &= 1.41 \times 10^9. \end{aligned}$$

Problem 3.5. Simplify $4.07 \times 10^5 - 2.1 \times 10^4$
Solution.

$$\begin{aligned} 4.07 \times 10^5 - 2.1 \times 10^4 &= 4.07 \times 10^5 - 0.21 \times 10^5 \\ &= 3.86 \times 10^5. \end{aligned}$$

Problem 3.6. Simplify $5 \times 10^6 \times 9 \times 10^{-3}$.
Solution.

$$\begin{aligned} 5 \times 10^6 \times 9 \times 10^{-3} &= 45 \times 10^3 \\ &= 4.5 \times 10^4. \end{aligned}$$

Problem 3.7. Simplify $1.6 \times 10^5 \div (2.4 \times 10^{-7})$.
Solution.

$$\begin{aligned} 1.6 \times 10^5 \div (2.4 \times 10^{-7}) &= 0.667 \times 10^{12} \\ &= 6.67 \times 10^{11}. \end{aligned}$$

Section 4 Tax Rates

Example 4.1. (UK Tax Rates)

Parts	Tax Rate
Up to £12570	No Tax
£12570 to £37700	20% Tax
£37700 to £150000	40% Tax
£150000–	45% Tax

Example 4.2. (Tax Rates) For £12000, no tax.
Example 4.3. (Tax Rates) For £13000, no tax on £12570, 20% × (£13000 − £12570) = £86 tax.

Problem 4.4. For £36570, calculate the tax.
Solution. £24000 × 0.2 = £4800.

Problem 4.5. For £80000, calculate the tax.
Solution. £25200 × 0.2 + £42300 × 0.4 = £21960.

Problem 4.6. For £200000, calculate the tax.
Solution. £25200 × 0.2 + £112300 × 0.4 + £50000 × 0.45 = £72460.