SES 2024届高一下数学测验（5）22.03.30

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一、填空题：（本大题共12小题，每题5分，共60分）

1	1　$\{x \mid x=2k\pi+\frac{\pi}{2}, k\in\mathbb{Z}\}$	2	2　$\{x \mid x=2k\pi, k\in\mathbb{Z}\}$
3	$4k\pi,\ k\in\mathbb{Z}, k\neq 0$	4	$6+6\sqrt{3}$
5	$\sqrt{21}$ or $\sqrt{61}$	6	$\bigcup\limits_{k\in\mathbb{Z}}\left[k\pi-\frac{\pi}{12},\ k\pi+\frac{5}{12}\pi\right]$
7	$[-6,4]$	8	$[1,2]$
9	$\sqrt{2}$	10	14
11	-7	12	$\left[-1,\frac{\sqrt{2}}{2}\right]$

二、解答题：（12+14+14=40分）

							12	11	10	9	8	7	6	5	4	3	2	1	0	

13.

$\cos B = 2\cos^2\frac{B}{2}-1 = 2\times\left(\frac{2\sqrt{5}}{5}\right)^2-1=\frac{3}{5}>0 \Rightarrow B\in(0,\frac{\pi}{2})$

$\Rightarrow \sin B = \sqrt{1-\cos^2 B} = \frac{4}{5}$.

$C=\frac{\pi}{4}$, $A+B+C=\pi \Rightarrow \sin A=\sin(\pi-B-C)=\sin(\frac{3}{4}\pi-B)=\frac{7\sqrt{2}}{10}$

$\frac{a}{\sin A}=\frac{c}{\sin C} \Rightarrow \frac{2}{\frac{7\sqrt{2}}{10}}=\frac{c}{\frac{\sqrt{2}}{2}} \Rightarrow c=\frac{10}{7}$

$\Rightarrow S_\triangle=\frac{1}{2}ac\sin B=\frac{1}{2}\times 2\times\frac{10}{7}\times\frac{4}{5}=\frac{8}{7}$

					14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	

14. (1) $\because c=2,\ C=\frac{\pi}{3}$

$\therefore S_\triangle=\frac{1}{2}ab\sin\frac{\pi}{3}=\sqrt{3}$

$\therefore ab=4$

有 $\cos\frac{1}{3}\pi=\frac{a^2+b^2-4}{2ab}$

$\Rightarrow a=b=2$

(2) $\because A+B+C=\pi$

$\therefore \sin C=\sin(A+B)$

$\therefore \sin(A+B)+\sin(B-A)$

$=\sin A\cos B+\sin B\cos A+\sin B\cos A-\sin A\cos B$

$=2\sin A\cos A$

$\Rightarrow 2\sin B\cos A=2\sin A\cos A$

$\Rightarrow \cos A=0$ or $\sin B=\sin A$

$\Rightarrow A=\frac{\pi}{2}$ or $A=B$

$\Rightarrow \triangle ABC$为 $Rt\triangle$ or 等腰$\triangle$.

					14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	

15.

(1) $f(x)=2-\sin(2x+\frac{\pi}{6})-2\sin^2x$

$=2-(\sin2x\cos\frac{\pi}{6}+\cos2x\sin\frac{\pi}{6})-(1-\cos2x)$

$=1+\cos2x-(\frac{\sqrt{3}}{2}\sin2x+\frac{1}{2}\cos2x)$

$=\frac{1}{2}\cos2x-\frac{\sqrt{3}}{2}\sin2x+1$

$=\cos(2x+\frac{\pi}{3})+1$

$\Rightarrow \min\limits_{T>0} T=\frac{2\pi}{2}=\pi$

(2) 有$2x+\frac{\pi}{3}\in\bigcup\limits_{k\in Z}[2k\pi-\pi,2k\pi]$

$\Rightarrow 2x\in\bigcup\limits_{k\in Z}[2k\pi-\frac{4}{3}\pi,2k\pi-\frac{\pi}{3}]$

$\Rightarrow x\in\bigcup\limits_{k\in Z}[k\pi-\frac{2}{3}\pi,k\pi-\frac{\pi}{6}]$

∴单调增区间为$\bigcup\limits_{k\in Z}[k\pi-\frac{2}{3}\pi,k\pi-\frac{\pi}{6}]$

(3) $f(\frac{B}{2})=1$

$\Rightarrow \cos(B+\frac{\pi}{3})+1=1$

$\Rightarrow \cos(B+\frac{\pi}{3})=0$

$\Rightarrow B+\frac{\pi}{3}=k\pi+\frac{\pi}{2}, k\in Z$

又$B\in(0,\pi)\Rightarrow B+\frac{\pi}{3}\in(\frac{\pi}{3},\frac{4}{3}\pi)$

$\Rightarrow k=0, B=\frac{\pi}{6}$.

$\because b=1, c=\sqrt{3}$, $\frac{b}{\sin B}=\frac{c}{\sin C}$

$\therefore \sin C=\frac{\sqrt{3}}{2}$

$\therefore C=\frac{\pi}{3}$ or $\frac{2}{3}\pi$.

当$C=\frac{\pi}{3}$, $A=\frac{\pi}{2}$, $a=\sqrt{b^2+c^2}=2$

当$C=\frac{2}{3}\pi$, $A=\frac{\pi}{6}$, $B=\frac{\pi}{6}\Rightarrow a=b=1$

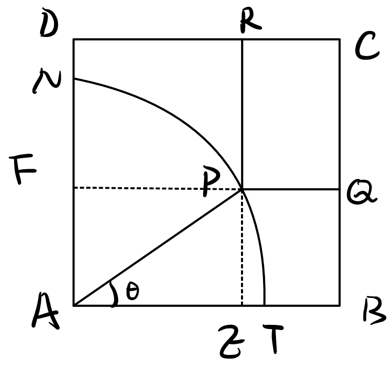
$\therefore a=1$ or 2

三、附加题

									10	9	8	7	6	5	4	3	2	1	0	

16.

(1) 延长RP交AB于E，延长QP交AD于F



$\left.\begin{array}{l}\text{正方形}ABCD\\ \text{矩形}PRCQ\end{array}\right\}\Rightarrow PE\perp AB, PF\perp AD$

$\left.\begin{array}{l}\angle TAP=\theta\\ AT=AN=6\end{array}\right\}\Rightarrow\begin{cases}ZP=6\cos\theta\\ FP=6\sin\theta\end{cases}$

$\Rightarrow\begin{cases}PR=7-6\sin\theta\\ PQ=7-6\cos\theta\end{cases}$

有$S=PR\cdot PQ=(7-6\sin\theta)(7-6\cos\theta)$

$=49-42(\sin\theta+\cos\theta)+36\sin\theta\cos\theta$, $\theta\in[0,\frac{\pi}{2}]$

(2) 令$\sin\theta+\cos\theta=t$. 有$\sin\theta\cos\theta=\frac{t^2-1}{2}$. $t=\sqrt{2}\sin(\theta+\frac{\pi}{4})$

$\therefore S=49-42t+18(t^2-1)=18t^2-42t+31$

又有$\theta\in[0,\frac{\pi}{2}]$，可知$\theta+\frac{\pi}{4}\in[\frac{\pi}{4},\frac{3}{4}\pi]$

有$t=\sqrt{2}\sin(\theta+\frac{\pi}{4})\in[1,\sqrt{2}]$

$\therefore S=S(t)=18t^2-42t+31, t\in[1,\sqrt{2}]$

$=18(t-\frac{7}{6})^2+\frac{13}{2}$

$\therefore \max\limits_{t\in[1,\sqrt{2}]}S=S|_{t=\sqrt{2}}=67-42\sqrt{2}$，此时$\theta=\frac{\pi}{4}$