Physics Problem Solving

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Physics Problem Solving Society
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16.09.2024

Table of Contents

Table of Contents

Reflection of Waves

The diagram shows the effect of touching the centre of the surface of water in a glass.

Qu (3) a, b: [2 + 1]

1. Draw a series of four sketches to show the motion of a single circular ripple, viewed from above, from the time it is generated at the centre to the time it returns to the centre. Include arrows to show the direction of the ripple.
2. Explain why the amplitude of the ripple reduces as the ripple travels outwards and its radius increases.

Figure: Surface of water in a glass disturbed by touching by a finger.

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and that it is also proportional to its frequency squared:

$$I \propto f^2.$$

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$$F = kx^2,$$

and the proportionality to the amplitude squared is now demonstrated. □

Table of Contents

Convex Lens and Curvature

This situation is analogous to the focussing effect of a circular mirror. Here the object is the initial contact with the water and the (real) image is at the final point of convergence of the ripple.
Another, familiar situation in which image distance equals object distance is shown. (The lens is assumed to be thin for the purpose of drawing rays.)

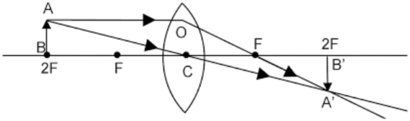

Figure: Ray diagram for a thin converging lens.

Qu (4) c: [2]

Explain how this lens behaviour resembles the reflecting behaviour of a circular mirror and use your explanation to state the relationship between the focal length of a mirror and its radius of curvature.

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Consider thin convex lens with focal distance f and object distance u. Draw diagrams for the following relations. Think about whether the image is: (1) real or virtual, (2) upright or inverted, (3) enlarged or diminished:

- $0 < u < f$,
- $u = f$,
- $f < u < 2f$,
- $u = 2f$,
- $2f < u$.

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Note that, by convention, concave lens have $f < 0$ and convex lens have $f > 0$; real images have $v > 0$ and virtual images have $v < 0$.

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$M > 0$ for upright images, and $M < 0$ for inverted images.

Table of Contents

Parabola and Light

A better focussing geometry is given by a parabolic mirror.
The standard equation of a parabola is

$$y^2 = 4ax$$

where a is the so-called focal distance of the parabola.

Qu (4) d, e: [2 + 6]

1. Sketch a graph of the function $y^2 = 4ax$. This must occupy at least half a page. Add to your sketch a light ray parallel to the x–axis, meeting the parabola at the general point (X, Y).
2. Show mathematically that this arbitrary ray is reflected through the point $(a, 0)$ and so justify describing a as the focal distance.

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- *Rays emitted from one of the foci of a hyperbola will end up emmiting from the other focus, after exactly one reflection on either branch of the hyperbola.* ▸ +ve Branch ▸ -ve Branch