

Physics Problem Solving

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1 Fluid Pressure

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About Fluid Pressure

We have previously discussed (in our first session) that the static pressure that fluid exerts only depends on gravitational acceleration g , fluid density ρ , and the depth h (to the free surface).

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Formula for Fluid Pressure

The pressure exerted by a certain liquid with density ρ at depth h is given by the following formula:

$$p = \rho gh.$$

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Remark

Static fluid pressure does not depend on the shape of the container, the total mass, or the surface area of the liquid.

About Fluid Pressure

However, it is not intuitive why the pressure exerted by the liquid at the bottom of the liquid, times by the area (which is the force exerted by the liquid), is different from the weight of the liquid:

$$F = pA = \rho hAg \neq W = mg = \rho Vg$$

for containers which does not satisfy $V = Ah$.

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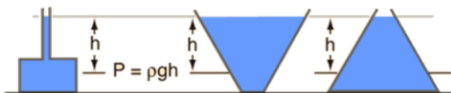


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First Container

- The liquid would have exerted pressure on the top flat surface, and hence by N3 will receive a reaction from the container, downwards.
- To let the resultant force be equal to zero, not only does the container has to provide an upward force equal to the weight at the bottom, but also some extra to compensate for the downwards force.

About Fluid Pressure

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Pressure in a Non-Column Container

Fluid exerts static pressure in a non-column container, as if a column of liquid with the same depth is on the top of the bottom of the container.

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Eason's Question

There is a step of height h , with a smooth pulley attached on top. A block of mass M rests on a smooth ground, with distance x from the step. There is a light string connecting the mass to the pulley, and there is a constant force F applied to the string (which is passed on to the mass by the tension).

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Find the work done by the tension in the string as it pulls the mass to just below the step, assuming that the mass does not lift off from the ground (i.e. $F \leq Mg$).

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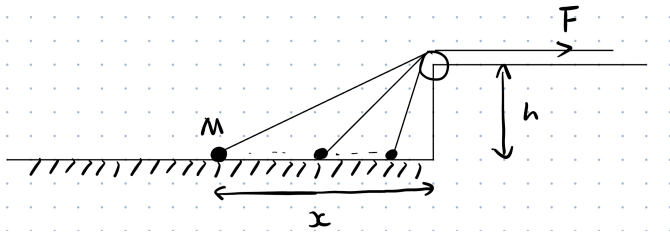


Figure: Diagram for Question

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Work Done

Work done W by a force \mathbf{F} over displacement \mathbf{x} is defined by

$$W = \int \mathbf{F} \cdot d\mathbf{x} = \int F \cos \theta dx.$$

Solutions

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Work done is $W = F \left(\sqrt{x^2 + h^2} - h \right)$.

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Q1 (2016 R1 q)

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A particle, mass m , slides down the smooth track, from a height H under gravity. It is to complete a circular trajectory of radius R when reaching its lowest point. Determine the smallest value of H .

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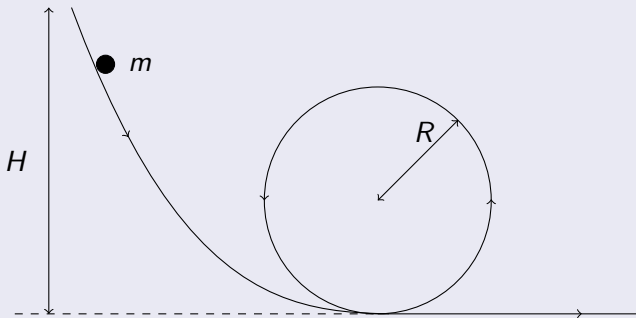


Figure: 2016 R1 q

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The centripetal force must be equal to the forces acting on the mass in order for its path to be circular, so

$$\frac{mv^2}{R} = N + mg.$$

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$$\begin{aligned}\frac{mv^2}{R} &\geq mg, \\ v^2 &= Rg.\end{aligned}$$

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