Entropy – From Physics, and Beyond

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Physics Problem Solving Society
St Paul's School

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The increase of disorder or entropy is what distinguishes the past from the future, giving a direction to time.

— *Stephen Hawking (1988), A Brief History of Time*

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- The **multiplicity** of a **macrostate**, Ω, is the number of microstates corresponding to the given macrostate.

Macrostate and Microstate

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Remark

For a gas which is not mono-molecular, there is an extra component in each microstate, which is the orientation of the molecule.

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and entropy is additive!

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Example

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Entropy is a measure of disorder.

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Therefore, $\Omega_B > \Omega A$, and $S_B > S_A$, entropy increases.

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Proof.

At absolute zero, the only possible configuration for a crystal is for all atoms to stay in place and have no vibration. Therefore, $\Omega = 1$, and $S = 0$. $\square$

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This means the new multiplicity of the system is $\Omega = N$, and therefore the new entropy is simply

$$S = k_B \ln \Omega.$$

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Definition (Gibbs Entropy)

$$S_\mathrm{G} = -k_\mathrm{B} \sum p_i \ln p_i,$$

where p_i is the probability of obtaining a certain microstate.

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Solution

When all p_is are equal, we have $\sum p_i = 1$, therefore $p_i = \frac{1}{\Omega}$. Plugging this in gives us

$$\begin{aligned} S_G &= -k_B \sum p_i \ln p_i \\ &= -k_B \cdot \Omega \cdot \left(\frac{1}{\Omega} \cdot \ln \frac{1}{\Omega}\right) \\ &= -k_B(-\ln \Omega) \\ &= k_B \ln \Omega. \end{aligned}$$

Information Theory

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Compare this to Gibbs Entropy: $S = -k_{\mathrm{B}} \sum p_i \ln p_i$.

Significance of Shannon Entropy

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Theorem (Shannon's Source Coding Theorem)

The limit of reliable compression of N i.i.d. random variables $X_1, X_2, \ldots, X_n \sim X$ each with entropy $H(X)$ is $N \cdot H(X)$ bits.

Data Compression – Example

Problem

There are four horses, A, B, C and D in a particular series of races for horses. The probabilities of individual horses winning, respectively, is as in the table below.

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- *Why is $H(X)$ a fraction number of bits, how does it make sense?*

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The entropy **disappeared**.

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$$S_{\mathrm{BH}} = \frac{k_{\mathrm{B}} A}{4 l_{\mathrm{P}}^2}.$$

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- Why is the Maxwell-Boltzmann Distribution the one we see?

Just as the constant increase of entropy is the basic law of the universe, so it is the basic law of life to be ever more highly structured and to struggle against entropy.
— Vaclav Havel (1986), Czech playwright, Letter to Dr. Gustav Husak, Living in Truth

References I

[1] ACuriousMind. *What is the conceptual difference between Gibbs and Boltzmann entropies?* URL: https://physics.stackexchange.com/questions/141321/what-is-the-conceptual-difference-between-gibbs-and-boltzmann-entropies.

[2] Nilanjana Datta and UK Maths Trust. *Entropy – ubiquitous, enigmatic & unique, UKMT IMO Celebration Lecture 2024*. URL: https://youtu.be/VQ-RoiTU2ns.

[3] Patch Thomas. *When are all microstates equally probable?* URL: https://physics.stackexchange.com/questions/317563/when-are-all-microstates-equally-probable.

[4] Mozibur Ullah. *Why did people think that black holes have zero entropy before the works of Bekenstein and Hawking?* URL: https://physics.stackexchange.com/q/667962.

References II

[5] Wikipedia. *Black hole thermodynamics*. URL: https://en.wikipedia.org/wiki/Black_hole_thermodynamics.

[6] Wikipedia. *Boltzmann's entropy formula*. URL: https://en.wikipedia.org/wiki/Boltzmann%27s_entropy_formula.

[7] Wikipedia. *Entropy*. URL: https://en.wikipedia.org/wiki/Entropy.

[8] Wikipedia. *Entropy (Statistical Thermodynamics)*. URL: https://en.wikipedia.org/wiki/Entropy_(statistical_thermodynamics).

[9] Wikipedia. *Second law of thermodynamics*. URL: https://en.wikipedia.org/wiki/Second_law_of_thermodynamics.

[10] Wikipedia. *Shannon's source coding theorem*. URL: https://en.wikipedia.org/wiki/Shannon%27s_source_coding_theorem.