Physics Problem Solving

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Physics Problem Solving Society
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22.04.2024

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About Fluid Pressure

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The pressure exerted by a certain liquid with density ρ at depth h is given by the following formula:

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Remark

Static fluid pressure does not depend on the shape of the container, the total mass, or the surface area of the liquid.

About Fluid Pressure

However, it is not intuitive why the pressure exerted by the liquid at the bottom of the liquid, times by the area (which is the force exerted by the liquid), is different from the weight of the liquid:

$$F = pA = \rho hAg \neq W = mg = \rho Vg$$

for containers which does not satisfy $V = Ah$.

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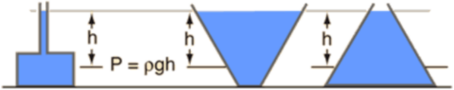

Figure: Some Containers

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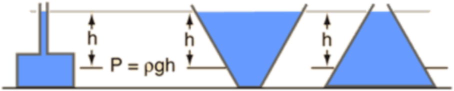

Figure: Some Containers

First Container

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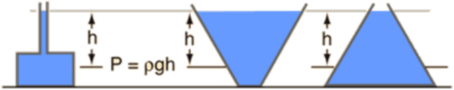

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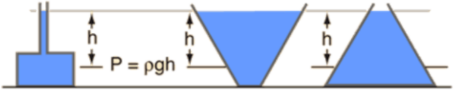

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First Container

- The liquid would have exerted pressure on the top flat surface, and hence by N3 will receive a reaction from the container, downwards.
- To let the resultant force be equal to zero, not only does the container has to provide an upward force equal to the weight at the bottom, but also some extra to compensate for the downwards force.

About Fluid Pressure

Similar explanations can be given for the other two containers, where the side walls either give support reactions for the weight, or give reactions downwards which requires more force exerted at the bottom.

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Pressure in a Non-Column Container

Fluid exerts static pressure in a non-column container, as if a column of liquid with the same depth is on the top of the bottom of the container.

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Eason's Question

There is a step of height h, with a smooth pulley attached on top. A block of mass M rests on a smooth ground, with distance x from the step. There is a light string connecting the mass to the pulley, and there is a constant force F applied to the string (which is passed on to the mass by the tension).

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Find the work done by the tension in the string as it pulls the mass to just below the step, assuming that the mass does not lift off from the ground (i.e. $F \leq Mg$).

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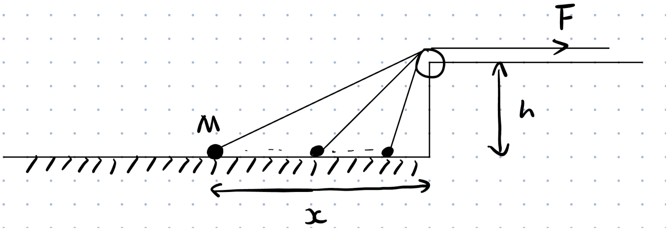

Figure: Diagram for Question

Solutions

As Eason mentioned in Google Classroom, in fact there are two very different solutions to this question.

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Work Done

Work done W by a force $\mathbf{F}$ over displacement $\mathbf{x}$ is defined by

$$W = \int \mathbf{F} \cdot \mathrm{d}\mathbf{x} = \int F \cos\theta \mathrm{d}x.$$

Solutions

If we set l as the distance of mass M from the step, and θ as the angle between the string and the horizontal ground, we will have

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We may notice that

$$I = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}l}(h^2 + l^2).$$

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Q1 (2016 R1 q)

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A particle, mass m, slides down the smooth track, from a height H under gravity. It is to complete a circular trajectory of radius R when reaching its lowest point. Determine the smallest value of H.

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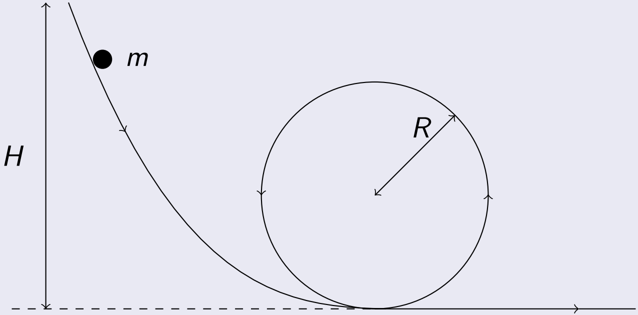

Figure: 2016 R1 q

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$$mg(H - 2R) = \frac{1}{2}mv^2.$$

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