Solutions

Question 1

Circular track [2016r1q]

If the mass is to complete the circular path, then at the top of the circle we require the reaction force from the constraining circle, N, to satisfy $N \geq 0$.

The centripetal force must be equal to the forces acting on the mass in order for its path to be circular, so

$$\frac{mv^2}{R} = N + mg.$$

The smallest value for v while still satisfying $N \geq 0$ occurs when $N = 0$. Therefore,

$$\frac{mv^2}{R} \geq mg, \qquad \textbf{[1]}$$
$$v^2 = Rg.$$

By the conservation of energy, the gravitational potential energy lost by the particle from the top of the track to the top of the circle is equal to the kinetic energy gained (recall that the track is smooth so no work done against friction).

$$mg(H - 2R) = \frac{1}{2}mv^2 \qquad \textbf{[1]}$$

Substituting the expression we have for v^2 from above,

$$mg(H - 2R) = \frac{1}{2}mRg,$$
$$H - 2R = \frac{1}{2}R,$$
$$H = \frac{5}{2}R. \qquad \textbf{[1]}$$

{**3**}

Question 2

Pulley system [2012r1b]

(i) W will move upwards because the force pulling section A of rope downwards, must act upwards on section D of the rope. **[1]**

(ii) Rope D moves upwards with speed V because we assume that the rope is inextensible and ropes A and D are attached. **[1]**
However, we cannot say that rope B moves downwards with speed V. Since the two pulleys of radii a and b are rigidly fixed together, they must rotate with the same angular velocity, $\omega_a = \omega_b$. Using $v = \omega r$,

$$V = \omega_a a \;\Rightarrow\; \omega_a = \frac{V}{a} \quad \text{and} \quad V_B = \omega_b b.$$

Substituting $\omega_a = \frac{V}{a}$ into the expression for V_B using $\omega_a = \omega_b$,

$$V_B = \omega_b b = \omega_a b = \frac{V}{a} b.$$

[1]

W rises as a result of the difference in speeds of ropes D and B since a greater length of rope D is pulled than length of rope B is pushed in a given time. Therefore, the centre of P and W rise with speed V_W,

$$V_W = \frac{1}{2}\left(V - \frac{b}{a}V\right)$$

[1]

$$V_W = \frac{V}{2}\left(1 - \frac{b}{a}\right)$$

$$V_W = V\left(\frac{a-b}{2a}\right).$$

[1]

The factor of a half is required because length of rope on both sides of W must decrease by a length l for W to rise a length W.

$\{\mathbf{5}\}$

Question 3

Converging boats [2013r1l]

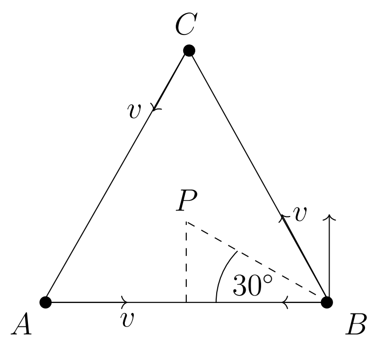


Figure 9: Boats arranged at corners of an equilateral triangle

(i) The boats spiral towards the centre of the triangle. By symmetry, the boats will meet at the centre of the triangle, P. **[1]**

(ii) The velocity of the boats in the direction of P is given by

$$v_P = v\cos 30^\circ = \frac{\sqrt{3}}{2}v$$

[1]

by using trigonometry on **Figure 9**. Resolving in perpendicular directions, the component of velocity in the direction B towards A is $v_{BA} = v\sin 30^\circ = \frac{v}{2}$. The component of velocity perpendicular to this is $v_{\perp BA} = v\cos 30^\circ = \frac{\sqrt{3}}{2}v$. (These are shown by arrows in **Figure 9**). **[1]**

(iii) When the boats meet, they must all have travelled a distance d to the middle of the triangle. d can be found using **Figure 9**, where l is the side length of the triangle,

$$d = \frac{l}{2}\frac{1}{\cos 30^\circ}.$$

[1]

Substituting values,

$$d = \frac{50}{2}\frac{2}{\sqrt{3}} = 28.9\,\mathrm{km}.$$

[1]

Then by considering the component of velocity of each boat towards P (which was found in the previous part of the question), T_M can be found. From $v = \frac{s}{t}$,

$$T_M = \frac{d}{v_P} = \frac{28.9}{30\frac{\sqrt{3}}{2}} = 1.11\,\mathrm{h}.$$

[2]

(Note that the distance was given in km and the speed in $\mathrm{km\,h^{-1}}$ so T_M must be in hours.)

(iv) The total distance, D, travelled by each boat is found using $s = vt$ and considering the overall speed of the boat, $30\,\mathrm{km\,h^{-1}}$. The boats travel for a total time T_M so

$$D = vT_M = 30 \times 1.11 = 33.3\,\mathrm{km}.$$

$$D = 28.9\,\mathrm{km}.$$

[1]

{8}

Question 4

Water tank with hole [2012r1l]

(i) Water is pushed out of the hole due to the weight of water above. The velocity of water exiting the hole can be found using energy conservation, where gravitational potential energy is transferred to kinetic energy,

$$mgh = \frac{1}{2}mv^2$$

[1]

$$v^2 = 2gh.$$

Substituting values, with the height of water above the hole $h = 20\,\mathrm{cm}$,

$$v = \sqrt{2 \times 9.81 \times 0.2} = 1.98\,\mathrm{m\,s^{-1}}.$$

[1]

(ii) Water exits the hole in the tank horizontally. It can then be modelled as a projectile as it falls to the ground. Considering the vertical motion of the water, constant acceleration formula $s = ut + \frac{1}{2}at^2$ can be used to find the time, t, taken to reach the floor. Since the vertical velocity of the water is initially zero,

$$s = \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2s}{g}}.$$

[1]

Substituting values,

$$t = \sqrt{\frac{2 \times 0.8}{9.81}} = 0.404\,\mathrm{s}.$$

[1]

Now considering the horizontal motion of the water, $s = vt$ can be used to find the distance travelled horizontally by the water,

$$d = 1.98 \times 0.404 = 0.80\,\mathrm{m} = 80\,\mathrm{cm}.$$

[1]

{**5**}

Question 5

Copper and Tungsten [2014r1e]

The force due to the mass is

$$F = mg = 100g.$$

If δl_c and δl_t are the extensions of the copper and tungsten wires respectively, we can write out the Young's moduli, since $E = \dfrac{Fl}{A\delta l}$, as following.

$$12.4 \times 10^{10} = \frac{100g \times 1.00}{\left(\pi\left[\frac{0.500\times10^{-3}}{2}\right]^2\right)\delta l_c}$$ **[1]**

$$35.5 \times 10^{10} = \frac{100g \times 1.00}{\left(\pi\left[\frac{d}{2}\right]^2\right)\delta l_t}$$ **[1]**

Rearranging both for δl,

$$\delta l_c = \frac{100g \times 1.00}{\left(\pi\left[\frac{0.500\times10^{-3}}{2}\right]^2\right) \times 12.4 \times 10^{10}},$$
$$\delta l_t = \frac{100g \times 1.00}{\left(\pi\left[\frac{d}{2}\right]^2\right) \times 35.5 \times 10^{10}}.$$

We also know that the combined extension of the two lengths of wire is $6.00\,\mathrm{cm}$. Therefore,

$$6.00 \times 10^{-2} = \delta l_c + \delta l_t.$$ **[1]**

Substituting for δl quantities,

$$6.00 \times 10^{-2} = \frac{100g \times 1.00}{\left(\pi\left[\frac{0.500\times10^{-3}}{2}\right]^2\right) \times 12.4 \times 10^{10}} + \frac{100g \times 1.00}{\left(\pi\left[\frac{d}{2}\right]^2\right) \times 35.5 \times 10^{10}}.$$ **[1]**

Evaluating to simplify,

$$6.00 \times 10^{-2} = 4.029 \times 10^{-2} + \frac{3.518 \times 10^{-9}}{d^2}.$$

It is d that we wish to find. Rearranging for d,

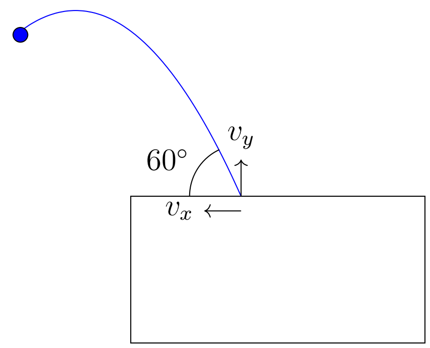
$$d^2 = \frac{3.518 \times 10^{-9}}{6.00 \times 10^{-2} - 4.029 \times 10^{-2}},$$
$$d = \sqrt{\frac{3.518 \times 10^{-9}}{6.00 \times 10^{-2} - 4.029 \times 10^{-2}}},$$
$$d = 4.23 \times 10^{-4}\,\mathrm{m}.$$ working and result **[2]**

{**6**}

Question 6

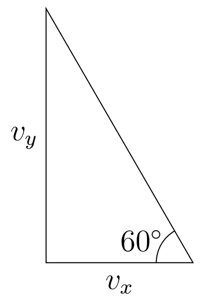
Train and ball [2014r1b]

Relative to the man on the moving train, the trajectory of the ball appears as following.



When the horizontal component of the velocity of the ball in the backward direction is equal to the forward speed of the train, $v_x = 10\,\mathrm{m\,s^{-1}}$, the stationary woman observes only vertical motion. Relative to her, the x-component is zero. **[1]**

To find the height that the ball rises, we need to find v_y (which is the same in each of their reference frames because neither observer is moving in the y direction). **[1]**
Given that we know v_x and the angle at which the ball was thrown, we have enough information to find v_y.



$$\tan 60^\circ = \frac{v_y}{v_x}$$
$$v_y = v_x \tan 60^\circ$$

Substituting $v_x = 10\,\mathrm{m\,s^{-1}}$,

$$\begin{aligned} v_y &= 10\tan 60^\circ, \\ &= 10\sqrt{3}\,\mathrm{m\,s^{-1}}. \end{aligned}$$ **[1]**

Now we can apply the constant acceleration formula vertically, $v^2 - u^2 = 2as$. When the ball reaches maximum height, the vertical speed is instantaneously zero. Substituting $v = 10\sqrt{3}\,\mathrm{m\,s^{-1}}$, $u = 0$ and $a = g$,

$$\begin{aligned} \left(10\sqrt{3}\right)^2 - 0^2 &= 2gh_{\text{max}}, \\ h_{\text{max}} &= \frac{\left(10\sqrt{3}\right)^2}{2g}, \\ h_{\text{max}} &= 15.3\,\mathrm{m}. \end{aligned}$$ **[1]**

This is the same for both the man and the woman. **[1]**

{**5**}

Question 7

Internal resistance [2012r1j]

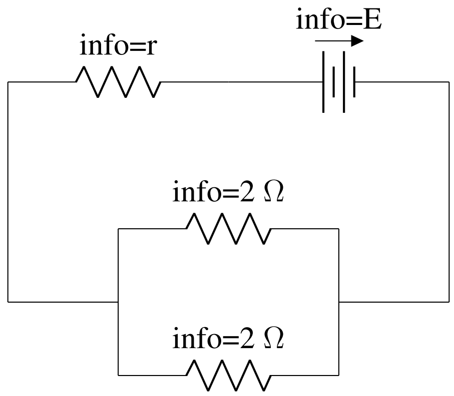


Figure 10

For the two resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{2} + \frac{1}{2} \;\Rightarrow\; R_T = 1.$$

The total resistance in the circuit, R_{C1}, is found by adding the internal resistance in series to the resistors in parallel,

$$R_{C1} = 1 + r.$$

Using $V = IR$, the emf is then given by

$$E = 3(r + 1).$$

[1]

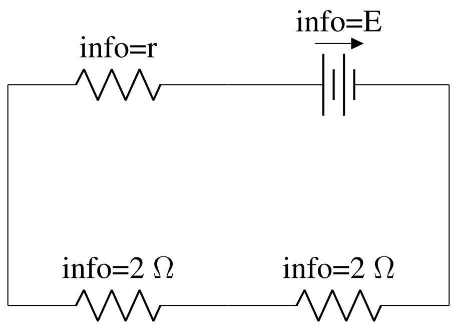


Figure 11

The total resistance of this circuit, R_{C2}, is found by adding all of the resistances in series,

$$R_{C2} = 2 + 2 + r = 4 + r.$$

Using $V = IR$ for this circuit,

$$E = 1.2(4 + r).$$

[1]

Two expressions containing E and r have been found so can be solved simultaneously. Equating the two expressions found for the emf,

$$3(1 + r) = 1.2(4 + r).$$

$$(3 - 1.2)r = 4.8 - 3 \;\Rightarrow\; r = 1.0\,\Omega.$$

Then E is

$$E = 3(1 + 1) = 6\,\mathrm{V}.$$

[1]

For the series circuit, power dissipated in each resistor is calculated most easily using $P = I^2R$,

$$W_s = 1.2^2 \times 2 = 2.88\,\mathrm{W}.$$

[1]

In the parallel circuit, half of the overall current flows in each resistor from Kirchhoff's current law. Therefore, the power through each resistor can also be found using $P = I^2R$,

$$W_p = \left(\frac{3}{2}\right) \times 2 = 4.5\,\mathrm{W}.$$

[1]
$\{\mathbf{5}\}$

Question 8

Resistor combinations [2015r1h]

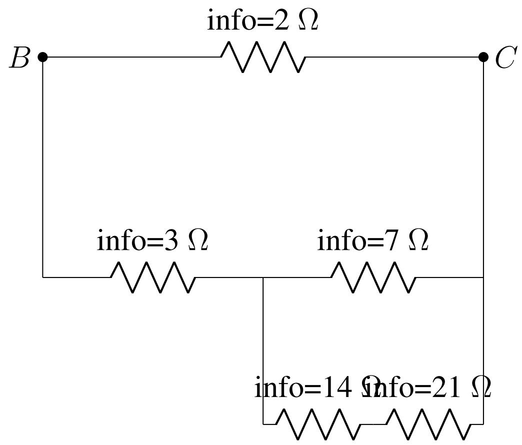


Figure 12: Redrawn circuit between B and C

We can redraw the resistor circuit to show a more familiar format. The overall resistance of the $7\,\Omega$, $14\,\Omega$ and $21\,\Omega$ resistors can be found by adding $14\,\Omega$ and $21\,\Omega$ in series then considering two branches in parallel. Using $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$ for resistors in parallel,

$$\frac{1}{R_P} = \frac{1}{7} + \frac{1}{(14+21)}$$
$$\frac{1}{R_P} = \frac{6}{35} \;\Rightarrow\; R_P = \frac{35}{6}\,\Omega.$$

R_P is in series with the $3\,\Omega$ resistor, and resistors add in series to give

$$R_S = \frac{35}{6} + 3 = \frac{53}{6}\,\Omega.$$

The $2\,\Omega$ resistor is in parallel with R_S so we consider the resistors in parallel to find the total resistance across BC,

$$\frac{1}{R_{TBC}} = \frac{1}{2} + \frac{6}{53} = \frac{65}{106}$$
$$R_{TBC} = \frac{106}{65} = 1.63\,\Omega.$$

In a similar way, we can redraw the resistor circuit to find the resistance between DB. The overall resistance of the $7\,\Omega$, $14\,\Omega$ and $21\,\Omega$ resistors can be found by considering two branches in parallel in the same way,

$$\frac{1}{R_P} = \frac{1}{7} + \frac{1}{(14+21)}$$
$$\frac{1}{R_P} = \frac{6}{35} \;\Rightarrow\; R_P = \frac{35}{6}\,\Omega.$$

R_P is in series with the $2\,\Omega$ resistor, and resistors in series add to give

$$R_S = \frac{35}{6} + 2 = \frac{47}{6}\,\Omega.$$

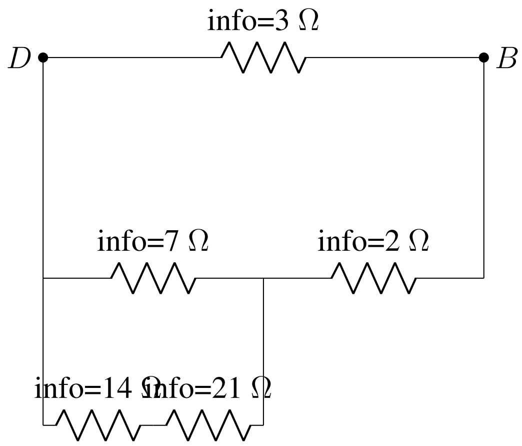


Figure 13: Redrawn circuit between D and B

The $3\,\Omega$ resistor is in parallel with R_S so we consider the resistors in parallel to find the total resistance across BC,

$$\frac{1}{R_{TBD}} = \frac{1}{3} + \frac{6}{47} = \frac{65}{141}$$

$$R_{TBD} = \frac{141}{65} = 2.17\,\Omega.$$

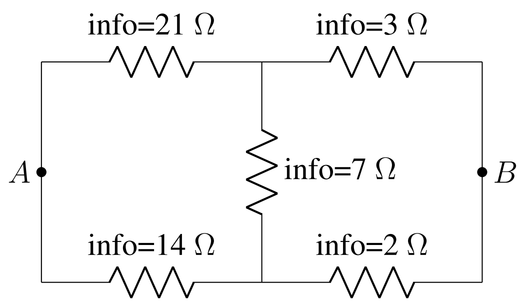


Figure 14: Redrawn circuit between A and B

Between A and B, we have a resistance bridge. We notice that no current flows through the $7\,\Omega$ resistor because the bridge is balanced since $\frac{21}{3}$ and $\frac{14}{2}$ are in the same ratio. This result can be derived by considering the case when there is no potential difference between the top and bottom branch and using Kirchhoff's voltage and current laws.
Therefore, the $7\,\Omega$ resistor can be disregarded and the top and bottom branches considered in parallel to find R_{TBA},

$$\frac{1}{R_{TBA}} = \frac{1}{21+3} + \frac{1}{14+2} = \frac{1}{24} + \frac{1}{16} = \frac{5}{48}$$

$$R_{TBA} = \frac{48}{5} = 9.60\,\Omega.$$

[1]
{**7**}

Question 9

Resistance with temperature [2015r1b]

Given that L_c is the length of constantan wire and L_m is the length of manganin wire, a $5\,\Omega$ wire requires that

$$6.3L_c + 5.3L_m = 5.$$

[1]

Dimensions can be used to check that $6.3L_c$ and $5.3L_m$ have units of resistance.
Next we consider the change in resistance with temperature. The relative change in resistance for constantan is $-3.0 \times 10^{-5}\,\theta$ and for manganin is $1.4 \times 10^{-5}\,\theta$ at temperature θ. Note that this is a relative change in resistance, not change in resistance, because the unit of temperature coefficient is $^\circ\,\mathrm{C}^{-1}$.
Then at temperature θ, we require

$$6.3L_c(1 - 3.0 \times 10^{-5}\,\theta) + 5.3L_m(1 + 1.4 \times 10^{-5}\,\theta) = 5.$$

We now have two equations that can be solved simultaneously to find L_c and L_m. Substituting the first equation into the second,

$$6.3L_c(1 - 3.0 \times 10^{-5}\,\theta) + 5.3L_m(1 + 1.4 \times 10^{-5}\,\theta) = 6.3L_c + 5.3L_m$$

[1]

$$6.3L_c(3.0 \times 10^{-5}\,\theta) = 5.3L_m(1.4 \times 10^{-5}\,\theta).$$

From this, the ratio between the two lengths required for constant temperature can be found,

$$L_m = \frac{6.3(3.0 \times 10^{-5}\,\theta)}{5.3(1.4 \times 10^{-5}\,\theta)} \cdot L_c = 2.55L_c.$$

[1]

This can be substituted back into the first equation,

$$6.3L_c + 5.3(2.55L_c) = 5 \;\rightarrow\; L_c = 0.25\,\mathrm{m}.$$

[1]

$$\frac{6.3}{2.55}L_m + 5.3L_m = 5 \;\rightarrow\; L_m = 0.64\,\mathrm{m}.$$

[1]
{**5**}

Question 10

Unknown resistance [2014r1a]

(i) To solve this problem, we use $V = IR$. When R is placed into the circuit, the resistance is $(R + R_0)$ because the components are in series, and we are given that the current is αI_0. Hence,

$$E = \alpha I_0(R + R_0).$$

The question also gives us another equation for E, when R is not included in the circuit: $E = I_0 R_0$. Substituting,

$$\begin{aligned} I_0 R_0 &= \alpha I_0 (R + R_0), \\ R_0 &= \alpha (R + R_0), \\ \frac{R_0}{\alpha} &= R + R_0, \\ R &= \frac{R_0(1-\alpha)}{\alpha}. \end{aligned}$$ **[1]**

We can find the ranges of validity by ensuring that resistance remains positive, because negative resistance does not exist. Hence,

$$0 \leq R \leq \infty,$$

and, looking at the equation for R above, this can only hold for

$$0 \leq \alpha \leq 1.$$ both ranges **[1]**

(ii) This can be solved similarly, but this time we add resistances in parallel:

$$\begin{aligned} \frac{1}{R_T} &= \frac{1}{R} + \frac{1}{R_0}, \\ \frac{1}{R_T} &= \frac{R + R_0}{R_0 R}, \\ R_T &= \frac{R_0 R}{R + R_0}. \end{aligned}$$

With the current still αI_0, we can substitute into $V = IR$.

$$E = \alpha I_0 \frac{R_0 R}{R + R_0}$$

Substituting $E = I_0 R_0$,

$$\begin{aligned} I_0 R_0 &= \alpha I_0 \frac{R_0 R}{R + R_0}, \\ 1 &= \alpha \frac{R}{R + R_0}, \\ R + R_0 &= \alpha R, \\ R &= \frac{R_0}{\alpha - 1}. \end{aligned}$$ **[1]**

For the range of validity, we have again that

$$0 \leq R \leq \infty,$$

and from the equation, this only holds when

$$1 \leq \alpha \leq \infty.$$ both **[1]**

{**4**}

Question 11

Resistor chain [2013r1a]

(i) The three resistors that make up the additional unit (which add in series to $3R$) can be considered in parallel to the middle resistor of the first unit. The total resistance due to these four resistors is R_P. For resistors in parallel, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$ so

$$\frac{1}{R_P} = \frac{1}{3R} + \frac{1}{R} \;\Rightarrow\; R_P = \frac{3}{4}R.$$

Then R_2 can be found by adding R_P in series with the two other resistors in the first unit,

$$R_2 = R_P + 2R = \frac{3}{4}R + 2R = \frac{11}{4}R.$$

[1]

Now R_3 can be found by considering the R_2 in parallel with the middle resistor in the third unit (in a similar way to the previous calculation),

$$\frac{1}{R_M} = \frac{1}{R_2} + \frac{1}{R} = \frac{4}{11R} + \frac{1}{R} \;\Rightarrow\; R_M = \frac{11}{15}R.$$

Adding R_M in series to the two remaining resistors,

$$R_3 = R_M + 2R = \frac{11}{15}R + 2R = \frac{41}{15}R.$$

[1]

(ii) Using the same reasoning as the previous part of the question, we can consider the resistor chain, R_T, in parallel to the middle resistor of the additional unit. Then the total resistance, R_{total}, is found by adding the other two resistors in series. Since the total resistance is unaltered here, $R_{\text{total}} = R_T$. Mathematically,

$$R_T = \left(\frac{1}{R_T} + \frac{1}{R}\right)^{-1} + 2R$$

[1]

$$R_T = \frac{RR_T}{R_T + R} + 2R$$
$$R_T^2 + RR_T = 3RR_T + 2R^2$$
$$R_T^2 - 2RR_T - 2R^2 = 0.$$

[1]

Solving with the quadratic formula,

$$R_T = \frac{2R \pm \sqrt{4R^2 + 8R^2}}{2} = R \pm \sqrt{3}R.$$

[1]

Since the resistance can only be positive,

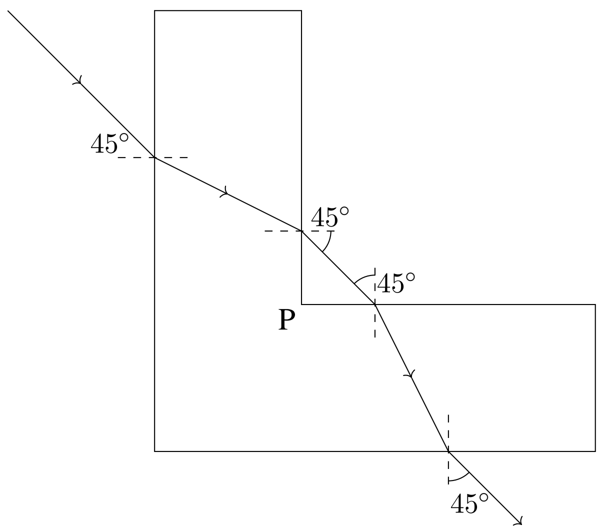
$$R_T = (1 + \sqrt{3})R.$$

[1]
$\{$**6**$\}$

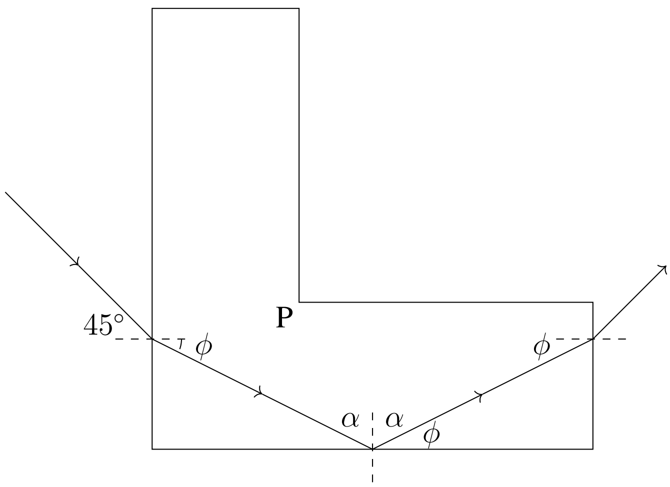
Question 12

Glass L [2014r1c]

(i) The beam will emerge from the vertical section of the glass block parallel to the initial beam as the angles of refraction at both surfaces are equal. Similarly for the horizontal section, the angle of incidence is $45°$ and consequently the angle of emergence is $45°$. Overall, the beam emerges from the 'L' shaped block parallel to the initial beam. **[2]**



(ii)



Applying Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, for the incident beam,

$$1 \sin 45^\circ = 1.5 \sin\phi,$$
$$\phi = \arcsin \frac{1}{1.5} \sin 45^\circ,$$
$$\phi = 28.13^\circ.$$

Hence the angle of incidence for the second surface, α, is

$$\alpha = 90^\circ - \phi = 61.87^\circ.$$

If we plug this into the equation for Snell's law, we obtain

$$1.5 \sin 61.87^\circ = 1 \sin\theta,$$

where θ is the angle of refraction. However, there is no value of θ that satisfies the equation. This means that refraction does not happen. It must be totally internally reflected! To confirm, we can find the critical angle:

$$1.5 \sin\theta_c = 1 \sin 90^\circ,$$

where 90° is the maximum angle of refraction possible.

$$\theta_c = \arcsin \frac{1}{1.5}$$
$$= 41.81^\circ.$$

Indeed, the angle of incidence of 61.87° is greater than the critical angle.

In the diagram above, we have shown one total internal reflection. But if the block were longer, or the beam entered at a lower point (but still at the same angle), then there may have been more total internal reflections from the parallel horizontals.

total internal reflection analysis **[3]**

Finally, we must determine what happens once the beam hits a vertical wall. By drawing out the diagram, we can see that by symmetry, the beam must exit the glass at an angle of 45°.

beam exit **[1]**

$\{\mathbf{6}\}$

Question 13

Light passing through a prism [2015r1f]

(i) From Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, and using **Figure 15**, **[1]**

$$\sin 48.59 = 1.500 \sin\theta_2 \;\Rightarrow\; \theta_2 = 30.0^\circ.$$

[1]

Then using the dotted blue kite, angle $A = 180 - 60 = 120^\circ$. The angle θ_3 can be found by using the fact that angles in a triangle add to 180°,

$$\theta_3 = 180 - \theta_2 - A = 180 - 30 - 120 = 30.0^\circ.$$

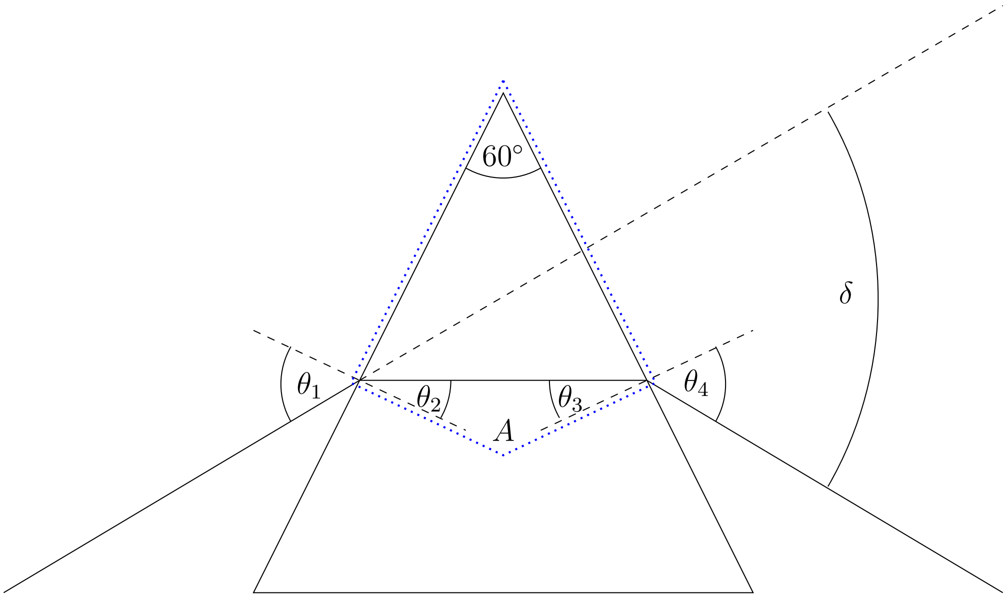


Figure 15: Path of light ray passing through the prism

By symmetry, we can say that $\theta_4 = \theta_1 = 48.59°$ because the same numbers are substituted into Snell's law again. **[1]**

(ii) By vertically opposite angles,

$$\begin{aligned}\delta &= \theta_1 - \theta_2 + \theta_4 - \theta_3 \\ &= \theta_1 + \theta_4 - (\theta_2 + \theta_3).\end{aligned}$$

Since $\theta_2 = \theta_3$ and $\theta_1 = \theta_4$,

$$= 2\theta_1 - 2\theta_2 \quad \textbf{[1]}$$
$$= 2 \times 48.59 - 2 \times 30$$
$$= 37.18\,°. \quad \textbf{[1]}$$

{**5**}

Question 14

Fixed and rotating mirrors [2014r1i]

The light must follow the path shown in the diagram (source $\rightarrow$ rotating mirror $\rightarrow$ fixed mirror $\rightarrow$ rotating mirror $\rightarrow$ receiver).
In the time taken for light to travel from the rotating mirror to the fixed mirror and back, Δt, the rotating mirror must have rotated through half the angle subtended by the source and the receiver at the rotating mirror, $\Delta\theta$. **[2]**

First the time:

$$\begin{aligned}\Delta t &= \frac{0.30 \times 10^3 \times 2}{c} \\ &= \frac{0.60 \times 10^3}{3.0 \times 10^8} \\ &= 2.0 \times 10^{-6}\,\text{s}.\end{aligned}$$

And the angle through which the mirror needs to rotate, by trigonometry:

$$\begin{aligned}\sin(\Delta\theta) &= \frac{\frac{1}{2} \times 0.60}{0.30 \times 10^3}, \\ \Delta\theta &= \arcsin \frac{0.30}{0.30 \times 10^3}, \\ \Delta\theta &= 1.0 \times 10^{-3}\,\text{rad}.\end{aligned}$$

Hence, the angular speed of the mirror,

$$\begin{aligned}\omega &= \frac{\Delta\theta}{\Delta t} \\ &= \frac{1.0 \times 10^{-3}}{2.0 \times 10^{-6}} && \text{working } \mathbf{[1]} \\ &= 500\,\text{rad}\,\text{s}^{-1} && \text{result } \mathbf{[1]}\end{aligned}$$

{**4**}

Question 15

Closest approach of accelerated proton [2015r1d]

First, the energy gained by accelerated protons can be found using the potential which is given by $V = \frac{E}{q}$,

$$E = Vq = (2.0 \times 10^6)e \text{ J}.$$

[1]

The closest approach occurs when all kinetic energy of the photon is transferred to electric potential energy. So by conservation of energy,

$$Vq = \frac{Qq}{4\pi\epsilon_0 r}$$

[1]

$$r = \frac{Q}{4\pi\epsilon_0 V}.$$

[1]

Substituting the potential and the charge of the gold nucleus, $79e$ (since it has an atomic number of 79),

$$r = \frac{79e}{4\pi\epsilon_0 \times 2.0 \times 10^6} = 5.7 \times 10^{-14}\text{ m}.$$

[1]

{**4**}

Question 16

Photoelectric effect [2012r1f]

For the photoelectric effect,

$$hf = W + E_{K\max}$$

[1]

where the Planck relation, $E = hf$, is used to find the energy of the incident photons.
The further loss of electrons is prevented when the the potential of the metal is such that electrons can't escape the metal. Since the energy required to overcome potential V is eV, this occurs when $eV = E_{K\max}$. Substituting,

$$hf = W + eV.$$

[1]

Now we can rearrange for the potential V at which the further loss of electrons is prevented,

$$V = \frac{hf - W}{e}.$$

[1]

Substituting values,

$$V = \frac{(6.63 \times 10^{-34} \times 6.3 \times 10^{14}) - 3.7 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.298 = 0.30\,\mathrm{V}.$$

[1]
$\{$**4**$\}$

Question 17

Scale pan [2016r1h]

The solution to this problem is quickly found after noticing that the acceleration of the plate must be g for the mass to just leave the scale pan. The acceleration in shm is not constant, however, so the amplitude at which this first starts happening is when the *maximum* acceleration is g. For an amplitude A, the maximum acceleration is $\omega^2 A$, so

$$\omega^2 A = g.$$ **[1]**

Given that $\omega = \dfrac{2\pi}{T}$,

$$\left(\frac{2\pi}{T}\right)^2 A = g.$$ **[1]**

$$A = \frac{gT^2}{(2\pi)^2)},$$

$$A = 0.0621\,\mathrm{m} = 6.21\,\mathrm{cm}.$$ **[1]**

Question 18

Calorimeter [2016r1g]

The amount of heat absorbed by the water and the calorimeter, according to $Q = mc\Delta\theta$ and $Q = C\Delta\theta$ for the calorimeter, where $\Delta\theta$ for both is $(25 - 15)$, is

$$\begin{aligned} Q_1 &= 0.80 \times 4200 \times (25 - 15) + 42.8 \times (25 - 15) \\ &= 34\,028\,\text{J}. \end{aligned}$$ **[1]**

Now if we let T_i be the initial temperature of the lead, then the heat lost to the water and calorimeter just before solidification is

$$\begin{aligned} Q_2 &= 0.40 \times 158 \times (T_i - 327) \\ &= 63.2(T_i - 327). \end{aligned}$$ **[1]**

And the latent heat ($Q = mL$) released by the lead during freezing:

$$\begin{aligned} Q_3 &= 0.40 \times 2.323 \times 10^4 \\ &= 9292\,\text{J}. \end{aligned}$$ **[1]**

The heat lost by lead after solidification until $25°$:

$$\begin{aligned} Q_4 &= 0.40 \times 137 \times (327 - 25) \\ &= 16\,549.6\,\text{J}. \end{aligned}$$ **[1]**

All of the energy lost by lead is gained by the water and calorimeter, so

$$Q_1 = Q_2 + Q_3 + Q_4.$$

Substituting,

$$\begin{aligned} 34028 &= 63.2(T_i - 327) + 9292 + 16549.6, \\ T_i &= 327 + \frac{34028 - 9292 - 16549.6}{63.2}, \\ T_i &= 457°. \end{aligned}$$ **[1]**

[5]

Question 19

Specific heat capacity of moving liquid [2012r1g]

We can consider $E = mc\Delta T$. Differentiating,

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}(mc\Delta T)}{\mathrm{d}t}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}t} c\Delta T.$$

[1]

Notice that power is given by $P = \frac{\mathrm{d}E}{\mathrm{d}t}$, so we have related power to the mass flow rate given in the question, as well as specific heat capacity and change in temperature.
The mass flow rate of the liquid can be converted from $\mathrm{kg\,min^{-1}}$ to $\mathrm{kg\,s^{-1}}$,

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{0.060}{60} = 0.0010\,\mathrm{kg\,s^{-1}}.$$

Substituting power, mass flow rate, change in temperature and specific heat capacity s into the equation found above,

$$12 = 0.0010 \times s \times 2.0$$
$$s = \frac{12}{0.0010 \times 2.0} = 6.0 \times 10^3\ \mathrm{J\,kg^{-1}}.$$

[1]
{**2**}

Question 20

Bicycle pump [2013r1m]

Air in the bicycle pump is at atmospheric pressure P_a and holds n moles of air in a volume V_a. Using the ideal gas equation, we can write

$$P_a V_a = nRT.$$

[1]

Now, after x strokes of the bicycle pump, the initially empty tyre holds xn moles of air (since each stroke puts n moles of air into the tyre) which is at pressure P_T in volume V_T. Similarly, we can use the ideal gas law to write

$$P_T V_T = xnRT.$$

[1]

Then x can be found by dividing the two ideal gas law expressions,

$$\frac{xnRT}{nRT} = \frac{P_T V_T}{P_a V_a}$$
$$x = \frac{P_T V_T}{P_a V_a}.$$

[1]

Substituting the pressure and volume values given in the question,

$$x = \frac{3.0 \times 10^5 \times 1.20 \times 10^{-3}}{1.00 \times 10^5 \times 9.0 \times 10^{-5}} = 40\ \text{ strokes.}$$

[1]
{**4**}