SI Units and Dimensions

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Some Obsession ...

- I aim to typeset this slides properly, using the correct conventions as defined in the 2019 (9^{th}) SI Brochure[1] ...
- So the siunitx package is used.
- In some places CIE Syllabus[2]/what we got taught slightly differs from the SI Brochure[1] ...
- And I will make it clear when they come up.

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SI Base Units

There are 7 *SI Base Units*, each corresponding to a *Dimension*.

Quantity	Name	Unit	Dimension
Time	second	s	T
Length	metre	m	L
Mass	kilogram	kg	M
Electric Current	ampere	A	I
Amount of Substance	mole	mol	N
Thermodynamic Temperature	kelvin	K	Θ
Luminous Intensity	candela	cd	J

Table: SI Base Units

SI Derived Units

There are *Derived Units*, represented as multiple of powers of the base units, perhaps with a numeric factor.
Those without are called *Coherent Derived Units*.[1,3]
The list of Named SI Coherent Derived Units that we are required to know:

Quantity	Name	Unit	Dimension
Plane Angle	radian	$1\,\mathrm{rad} = 1\,\mathrm{m\,m^{-1}}$	1
Frequency	hertz	$1\,\mathrm{Hz} = 1\,\mathrm{s^{-1}}$	T^{-1}
Activity	becquerel	$1\,\mathrm{Bq} = 1\,\mathrm{s^{-1}}$	T^{-1}
Force	newton	$1\,\mathrm{N} = 1\,\mathrm{kgm/s^2}$	MLT^{-2}
Pressure	pascal	$1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$	$\mathsf{ML}^{-1}\mathsf{T}^{-2}$
Energy	joule	$1\,\mathrm{J} = 1\,\mathrm{N\,m}$	$\mathsf{ML}^{2}\mathsf{T}^{-2}$
Power	watt	$1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$	$\mathsf{ML}^{2}\mathsf{T}^{-3}$
Celsius Temp.	degree Celsius	$t/{}^\circ\mathrm{C} = T/\mathrm{K} - 273.15$	Θ

Table: Some Named SI Coherent Derived Units

And some more named coherent units ...

Quantity	Name	Unit	Dimension
Electric Charge	coulomb	$1\,\mathrm{C} = 1\,\mathrm{A\,s}$	IT
Electric P.D. (Voltage)	volt	$1\,\mathrm{V} = 1\,\mathrm{J\,C^{-1}}$	$\mathsf{ML^2T^{-3}I^{-1}}$
Capacitance	farad	$1\,\mathrm{F} = 1\,\mathrm{C\,V^{-1}}$	$\mathsf{M^{-1}L^{-2}T^4I^2}$
Electrical Resistance	ohm	$1\,\Omega = 1\,\mathrm{V\,A^{-1}}$	$\mathsf{ML^2T^{-3}I^{-2}}$
Magnetic Flux	weber	$1\,\mathrm{Wb} = 1\,\mathrm{V\,s}$	$\mathsf{ML^2T^{-2}I^{-1}}$
Magnetic Flux Density	tesla	$1\,\mathrm{T} = 1\,\mathrm{Wb/m^2}$	$\mathsf{MT^{-2}I^{-1}}$

Table: Some More Named SI Coherent Derived Units

Non-SI Units

There are some units that are not part of the SI system, but are still widely used in the scientific community.
The ones we should be aware of include:

Quantity	Name	Unit	Dimension
Time	minute	$1\,\text{min} = 60\,\text{s}$	T
Time	hour	$1\,\text{h} = 60\,\text{min}$	T
Time	day	$1\,\text{d} = 24\,\text{h}$	T
Plane Angle	degree	$1^\circ = \frac{\pi}{180}\,\text{rad}$	1
Energy	electronvolt	$1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$	$\mathsf{ML^2T^{-2}}$
Volume	litre	$1\,\text{L} = 0.001\,\text{m}^3$	$\mathsf{L^3}$
Mass	tonne	$1\,\text{t} = 1000\,\text{kg}$	M
Mass	atomic mass unit	$1\,\text{u} = 1.66 \times 10^{-27}\,\text{kg}$	M

Table: Some non-SI units accepted for use with SI units

Dimension and Units

When I first learnt physics, my dad told me that we could only add/subtract quantities with the same units ... but we can actually add quantities with the same dimensions.
To actually find the value, we have to convert them to the same units, and adjust the values of the quantities.

- A *dimension* is a property of a physical quantity, and
- A *unit* is a standard for measuring that quantity.

A unit incorporates the idea of a *scale* to the value measured.
CIE: **All physical quantities consist of a numerical magnitude value and a unit.**[2]

Dimensional Analysis

- CIE: *Use SI base units to check the homogeneity of physical equations.*[2] *Dimensional* analysis isn't exactly mentioned.
- *Note:* For a quantity Q, The SI Brochure[1] uses $\dim Q$ for the dimension of Q, $[Q]$ for the unit of Q, and $\{Q\}$ for the numerical value of Q. $Q = \{Q\}[Q]$.
- *Dimensional Analysis* is checking the *homogeneity* of an equation, by comparing the dimensions of the quantities on both sides of the equation.
- Dimensional Analysis does not allow us to find the dimensionless constants involved, but it can help us find the form of the equation, and we can find the constant by some other means (e.g. experiment).

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Overview

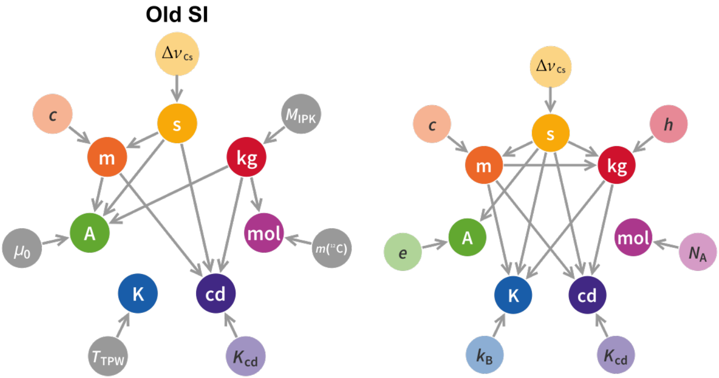

Figure: The Definitions of Units[4,5]

The second, the metre and the candela remained unchanged.

The Kilogram

The old SI relies on the *International Prototype of the Kilogram* (IPK) as the standard for the kilogram. But ...

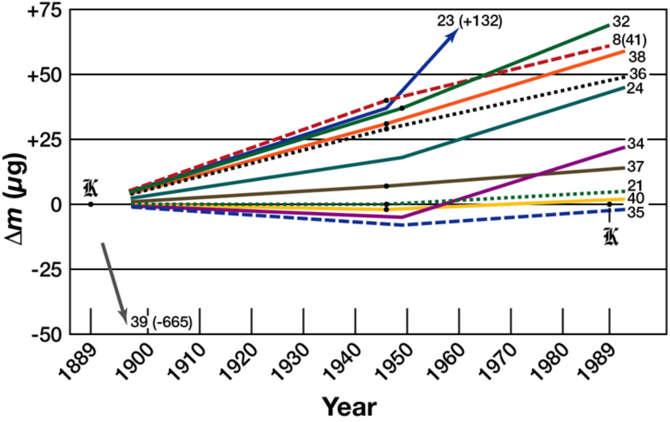

Figure: The Mass Drifts of the National Prototypes[6,7]

The Kilogram

Considering that mass and *energy* are equivalent, we can use some energy to define mass! (Since speed of light is exactly $c = 299\,792\,458\,\mathrm{m\,s^{-1}}$, and $E = mc^2$.)
We already have the second and the metre defined.
So we could use the energy of a photon with a certain frequency! $E = hf$.

$$h = 6.626\,070\,15 \times 10^{-34}\,\mathrm{J\,s}.$$

The Ampere

The old SI relies on the force between parallel conductors as the standard for the ampere:[5]

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}.$$

This relies on force, which has dimensions MLT^{-2}.
Another constant that we could use is:

$$e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C}.$$

So now the Ampere still depends on the second, but is no longer dependent on mass or length![8]
Notice that how the ampere is defined in terms of the coulomb, but is still the base unit.

The Kelvin

The old SI relies on the triple point of water as 273.16 K. (The zero is fixed by the nature of thermodynamics.)

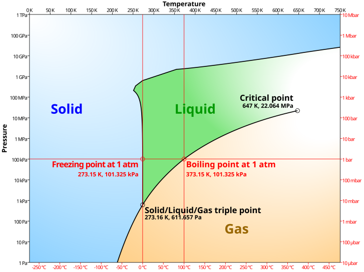

Figure: The Phase Diagram of Water[9]

To remove the need to accurately measure the triple point of water, they choose to let it depend on the Boltzmann constant k_B.[10]

$$k_\mathrm{B} = 1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}.$$

The Mole

This might be the most controversial unit, but it also brings the most controversial change in the SI Unit System.
Previously, the mole was defined as the number of atoms in $12\,\mathrm{g}$ of $^{12}\mathrm{C}$.
Similar to the kelvin, this relies on the measurement of some physical stuff (a sphere of atoms to be measured). So we chose to some constant, the Avogadro constant N_A:

$$N_\mathrm{A} = 6.022\,140\,76 \times 10^{23}\,\mathrm{mol}^{-1}.$$

This brings a problem: one of these two has to be demolished:[5]

- The mass of a $^{12}\mathrm{C}$ atom is exactly $12\,\mathrm{Da}$.
- The number of dalton in a gram is exactly N_A.

The Mole

The SI Brochure chose to preserve the former and demolish the latter:

> *The dalton (*Da*) and the unified atomic mass unit (*u*) are alternative names (and symbols) for the same unit, equal to* $1/12$ *of the mass of a free carbon 12 atom, at rest and in its ground state.*[1]

This is just simply bad – because the ratio of one gram to one dalton is no longer perfectly the Avogadro's constant – or is it?

> *According to the present definition* $M(^{12}\mathrm{C})$ *is no longer known exactly and must be determined experimentally. The value chosen for* N_A *is such that at the time of adopting the present definition of the mole,* $M(^{12}\mathrm{C})$ *was equal to* $0.012\,\mathrm{kg\,mol^{-1}}$ *with a relative standard uncertainty of* 4.5×10^{-10}*.*[1]

The Constants

$$
\begin{aligned}
h &= 6.626\,070\,15 \times 10^{-34}\,\mathrm{kg\,m^2\,s^{-1}} \\
e &= 1.602\,176\,634 \times 10^{-19}\,\mathrm{A\,s} \\
k_\mathrm{B} &= 1.380\,649 \times 10^{-23}\,\mathrm{kg\,m^2\,K^{-1}\,s^{-2}} \\
N_\mathrm{A} &= 6.022\,140\,76 \times 10^{23}\,\mathrm{mol^{-1}} \\
c &= 299\,792\,458\,\mathrm{m\,s^{-1}} \\
\Delta\nu_\mathrm{Cs} &= 9\,192\,631\,770\,\mathrm{s^{-1}} \\
K_\mathrm{cd} &= 683\,\mathrm{cd\,sr\,s^3\,kg^{-1}\,m^{-2}}
\end{aligned}
$$

Why we need units, and the SI?

Physics, and natural science in general, is a reasonable enterprise based on valid experimental evidence, criticism, and rational discussion.[11]

References I

[1] International Bureau of Weights and Measures. *Le Système international d'unités / The International System of Units ('The SI Brochure')*. Ninth. 2019. ISBN: 978-92-822-2272-0. URL: https://www.bipm.org/en/publications/si-brochure/.

[2] Cambridge International Education. *Cambridge International AS and A Level Physics (9702)*. URL: https://www.cambridgeinternational.org/programmes-and-qualifications/cambridge-international-as-and-a-level-physics-9702/ (visited on 08/03/2025).

[3] Wikipedia, The Free Encyclopedia. *International System of Units*. 8th Mar. 2025. URL: https://en.wikipedia.org/wiki/International_System_of_Units (visited on 08/03/2025).

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[4] E. Pisanty. *SI Unit Relations*. 18th Nov. 2018. URL: https://github.com/episanty/SI-unit-relations (visited on 08/03/2025).

[5] Wikipedia, The Free Encyclopedia. *2019 revision of the SI*. 3rd Mar. 2025. URL: https://en.wikipedia.org/wiki/2019_revision_of_the_SI (visited on 08/03/2025).

[6] G. Girard. 'The Third Periodic Verification of National Prototypes of the Kilogram (1988-1992)'. In: *Metrologia* 31.4 (Jan. 1994), pp. 317–336. ISSN: 1681-7575. DOI: 10.1088/0026-1394/31/4/007.

[7] Wikipedia, The Free Encyclopedia. *International Prototype of the Kilogram*. 24th Feb. 2025. URL: https://en.wikipedia.org/wiki/International_Prototype_of_the_Kilogram (visited on 08/03/2025).

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[11] A. Franklin and S. Perovic. 'Experiment in Physics'. In: *The Stanford Encyclopedia of Philosophy*. Ed. by E. N. Zalta and U. Nodelman. Fall 2023. Metaphysics Research Lab, Stanford University, 2023. URL: `https://plato.stanford.edu/archives/fall2023/entries/physics-experiment/` (visited on 08/03/2025).