

Q1

The homogenous form of $x - 3y + 4 = 0$ is $x_1 - 3x_2 + 4x_3 = 0$, which is $(1, -3, 4)^T$.

And the homogenous coordinate of the infinity line is $(0, 0, 1)^T$.

Then we can calculate the infinity point of the line by

get its intersection with the infinity line.

$$\begin{aligned}(1, -3, 4)^T \times (0, 0, 1)^T &= \begin{vmatrix} i & j & k \\ 1 & -3 & 4 \\ 0 & 0 & 1 \end{vmatrix} \\ &= \begin{vmatrix} -3 & 4 \\ 0 & 1 \end{vmatrix} i + \begin{vmatrix} 4 & 1 \\ 1 & 0 \end{vmatrix} j + \begin{vmatrix} 1 & -3 \\ 0 & 0 \end{vmatrix} k \\ &= -3i - j + 0k\end{aligned}$$

So the the homogeneous coordinate of the infinity point of this line is $(-3, -1, 0)$.

Q2

Let $A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3), D(x_4, y_4, z_4)$.

They form three vectors which are $a(x_2 - x_1, y_2 - y_1, z_2 - z_1), b(x_3 - x_1, y_3 - y_1, z_3 - z_1), c(x_4 - x_1, y_4 - y_1, z_4 - z_1)$.

We know that:

a, b, c are coplanar $\Leftrightarrow (a, b, c) = 0$

What is more:

$$(a, b, c) = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix}$$

So we have:

$$a, b, c \text{ are coplanar} \Leftrightarrow \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0$$

We have:

$$\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 & 0 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 & 0 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 & 0 \end{vmatrix} \\
= (-1)^5 * \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} \\
= - \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} \\
\text{So } a, b, c \text{ are coplanar} \Leftrightarrow \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

Q3

Based on the conditions, the process of the caculation is as follow:

$$\begin{aligned}
\frac{dp_d}{dp_n^T} &= \begin{bmatrix} \frac{\partial x_d}{\partial x} & \frac{\partial x_d}{\partial y} \\ \frac{\partial y_d}{\partial x} & \frac{\partial y_d}{\partial y} \end{bmatrix} \\
r^2 &= x^2 + y^2 \\
\frac{\partial r^2}{\partial x} &= 2x \\
\frac{\partial r^2}{\partial y} &= 2y
\end{aligned}$$

(1):

$$\begin{aligned}
\frac{\partial x_d}{\partial x} &= \frac{\partial [x + k_1 r^2 x + k_2 (r^2)^2 x + 2\rho_1 xy + \rho_2 r^2 + 2\rho_2 x^2 + k_3 x (r^2)^3]}{\partial x} \\
&= 1 + k_1 (r^2 + 2x^2) + k_2 [r^4 + x(2r^2 * 2x)] + 2\rho_1 y + 2\rho_2 x + 4\rho_2 x + k_3 [r^6 + x(3r^4 * 2x)] \\
&= 1 + (r^2 + 2x^2)k_1 + (r^4 + 4r^2 x^2)k_2 + 2\rho_1 y + 6\rho_2 x + (r^6 + 6r^4 x^2)k_3
\end{aligned}$$

(2):

$$\begin{aligned}
\frac{\partial x_d}{\partial y} &= \frac{\partial [x + k_1 r^2 x + k_2 (r^2)^2 x + 2\rho_1 xy + \rho_2 r^2 + 2\rho_2 x^2 + k_3 x (r^2)^3]}{\partial y} \\
&= 2k_1 xy + k_2 x * 2r^2 * 2y + 2\rho_1 x + 2\rho_2 y + k_3 x * 3r^4 * 2y \\
&= 2xyk_1 + 4r^2 xyk_2 + 2\rho_1 x + 2\rho_2 y + 6xyr^4 k_3
\end{aligned}$$

(3):

$$\begin{aligned}
\frac{\partial y_d}{\partial x} &= \frac{\partial [y + k_1 r^2 y + k_2 (r^2)^2 y + 2\rho_2 xy + \rho_1 r^2 + 2\rho_1 y^2 + k_3 y (r^2)^3]}{\partial x} \\
&= 2k_1 xy + k_2 y * 2r^2 * 2x + 2\rho_1 y + 2\rho_2 x + k_3 y * 3r^4 * 2x \\
&= 2xyk_1 + 4r^2 xyk_2 + 2\rho_1 y + 2\rho_2 x + 6xyr^4 k_3
\end{aligned}$$

(4):

$$\begin{aligned}
\frac{\partial y_d}{\partial y} &= \frac{\partial[y + k_1 r^2 y + k_2 (r^2)^2 y + 2\rho_2 xy + \rho_1 r^2 + 2\rho_1 y^2 + k_3 y(r^2)^3]}{\partial y} \\
&= 1 + k_1(r^2 + 2y^2) + k_2[r^4 + y(2r^2 * 2y)] + 2\rho_2 x + 2\rho_1 y + 4\rho_1 y + k_3[r^6 + y(3r^4 * 2y)] \\
&= 1 + (r^2 + 2y^2)k_1 + (r^4 + 4r^2 y^2)k_2 + 2\rho_2 x + 6\rho_1 y + (r^6 + 6r^4 y^2)k_3
\end{aligned}$$

So

$$\frac{dp_d}{dp_n^T} = \begin{bmatrix} 1 + (r^2 + 2x^2)k_1 + (r^4 + 4r^2 x^2)k_2 + 2\rho_1 y + 6\rho_2 x + (r^6 + 6r^4 x^2)k_3 & 2xyk_1 + 4r^2 xyk_2 + 2\rho_1 x + 2\rho_2 y + 6xyr^4 k_3 \\ 2xyk_1 + 4r^2 xyk_2 + 2\rho_1 y + 2\rho_2 x + 6xyr^4 k_3 & 1 + (r^2 + 2y^2)k_1 + (r^4 + 4r^2 y^2)k_2 + 2\rho_2 x + 6\rho_1 y + (r^6 + 6r^4 y^2)k_3 \end{bmatrix}$$

Q4

Because n is a 3D unit vector and $r = \theta n = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$, we have:

$$\begin{aligned}
n_1^2 + n_2^2 + n_3^2 &= 1 \\
r_1^2 + r_2^2 + r_3^2 &= \theta^2
\end{aligned}$$

Based on Rodrigues formula:

$$R = \cos\theta + (1 - \cos\theta)nn^T + \sin\theta n^\wedge$$

Let $\sin\theta = \alpha, \cos\theta = \beta, 1 - \cos\theta = \gamma$, we have:

$$\begin{aligned}
R &= \beta + \gamma nn^T + \alpha n^\wedge \\
&= \begin{bmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \beta \end{bmatrix} + \begin{bmatrix} \gamma n_1^2 & \gamma n_1 n_2 & \gamma n_1 n_3 \\ \gamma n_1 n_2 & \gamma n_2^2 & \gamma n_2 n_3 \\ \gamma n_1 n_3 & \gamma n_2 n_3 & \gamma n_3^2 \end{bmatrix} + \begin{bmatrix} 0 & -\alpha n_3 & \alpha n_2 \\ \alpha n_3 & 0 & -\alpha n_1 \\ -\alpha n_2 & \alpha n_1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} \beta + \gamma n_1^2 & \gamma n_1 n_2 - \alpha n_3 & \gamma n_1 n_3 + \alpha n_2 \\ \alpha n_3 + \gamma n_1 n_2 & \beta + \gamma n_2^2 & \gamma n_2 n_3 - \alpha n_1 \\ \gamma n_1 n_3 - \alpha n_2 & \gamma n_2 n_3 + \alpha n_1 & \beta + \gamma n_3^2 \end{bmatrix}
\end{aligned}$$

Let $u = (R_{11}, R_{12}, R_{13}, R_{21}, R_{22}, R_{23}, R_{31}, R_{32}, R_{33})$, we have:

$$\frac{du}{dr^T} = \begin{bmatrix} \frac{\partial R_{11}}{\partial r_1} & \frac{\partial R_{11}}{\partial r_2} & \frac{\partial R_{11}}{\partial r_3} \\ \dots & \dots & \dots \\ \frac{\partial R_{33}}{\partial r_1} & \frac{\partial R_{33}}{\partial r_2} & \frac{\partial R_{33}}{\partial r_3} \end{bmatrix}_{9 \times 3}$$

(1):

$$\begin{aligned}
\frac{\partial R_{11}}{\partial r_1} &= \frac{\partial[\beta + \gamma \frac{r_1^2}{\theta^2}]}{\partial r_1} \\
&= (\frac{2\gamma}{\theta} - \alpha)n_1(n_2^2 + n_3^2)
\end{aligned}$$

(2):

$$\begin{aligned}
\frac{\partial R_{11}}{\partial r_2} &= \frac{\partial[\beta + \gamma \frac{r_1^2}{\theta^2}]}{\partial r_2} \\
&= \alpha n_2(n_1^2 - 1) - \frac{2\gamma n_1^2 n_2}{\theta}
\end{aligned}$$

(3):

$$\begin{aligned}\frac{\partial R_{11}}{\partial r_3} &= \frac{\partial[\beta + \gamma \frac{r_1^2}{\theta^2}]}{\partial r_3} \\ &= \alpha n_3 (n_1^2 - 1) - \frac{2\gamma n_1^2 n_3}{\theta}\end{aligned}$$

(4):

$$\begin{aligned}\frac{\partial R_{12}}{\partial r_1} &= \frac{\partial[\gamma \frac{r_1 r_2}{\theta^2} - \alpha \frac{r_3}{\theta}]}{\partial r_1} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1^2 n_2 + \gamma n_2 - (\beta\theta - \alpha)n_1 n_3]\end{aligned}$$

(5):

$$\begin{aligned}\frac{\partial R_{12}}{\partial r_2} &= \frac{\partial[\gamma \frac{r_1 r_2}{\theta^2} - \alpha \frac{r_3}{\theta}]}{\partial r_2} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_2^2 n_1 + \gamma n_1 - (\beta\theta - \alpha)n_2 n_3]\end{aligned}$$

(6):

$$\begin{aligned}\frac{\partial R_{12}}{\partial r_3} &= \frac{\partial[\gamma \frac{r_1 r_2}{\theta^2} - \alpha \frac{r_3}{\theta}]}{\partial r_3} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1 n_2 n_3 - (\beta\theta - \alpha)n_3^2 - \alpha]\end{aligned}$$

(7):

$$\begin{aligned}\frac{\partial R_{13}}{\partial r_1} &= \frac{\partial[\gamma \frac{r_1 r_3}{\theta^2} + \alpha \frac{r_2}{\theta}]}{\partial r_1} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1^2 n_3 + n_3 \gamma + (\beta\theta - \alpha)n_1 n_2]\end{aligned}$$

(8):

$$\begin{aligned}\frac{\partial R_{13}}{\partial r_2} &= \frac{\partial[\gamma \frac{r_1 r_3}{\theta^2} + \alpha \frac{r_2}{\theta}]}{\partial r_2} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1 n_2 n_3 + (\beta\theta - \alpha)n_2^2 + \alpha]\end{aligned}$$

(9):

$$\begin{aligned}\frac{\partial R_{13}}{\partial r_3} &= \frac{\partial[\gamma \frac{r_1 r_3}{\theta^2} + \alpha \frac{r_2}{\theta}]}{\partial r_3} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_3^2 n_1 + n_1 \gamma + (\beta\theta - \alpha)n_2 n_3]\end{aligned}$$

We can find some patterns in the process of the calculation, so we can easily get:

(10):

$$\begin{aligned}
\frac{\partial R_{21}}{\partial r_1} &= \frac{\partial[\gamma \frac{r_1 r_2}{\theta^2} + \alpha \frac{r_3}{\theta}]}{\partial r_1} \\
&= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1^2 n_2 + n_2 \gamma + (\beta\theta - \alpha)n_1 n_3]
\end{aligned}$$

(11):

$$\begin{aligned}
\frac{\partial R_{21}}{\partial r_2} &= \frac{\partial[\gamma \frac{r_1 r_2}{\theta^2} + \alpha \frac{r_3}{\theta}]}{\partial r_2} \\
&= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_2^2 n_1 + n_1 \gamma + (\beta\theta - \alpha)n_2 n_3]
\end{aligned}$$

(12):

$$\begin{aligned}
\frac{\partial R_{21}}{\partial r_3} &= \frac{\partial[\gamma \frac{r_1 r_2}{\theta^2} + \alpha \frac{r_3}{\theta}]}{\partial r_3} \\
&= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1 n_2 n_3 + (\beta\theta - \alpha)n_3^2 + \alpha]
\end{aligned}$$

(13):

$$\begin{aligned}
\frac{\partial R_{22}}{\partial r_1} &= \frac{\partial[\beta + \gamma \frac{r_2^2}{\theta^2}]}{\partial r_1} \\
&= \alpha n_1 (n_2^2 - 1) - \frac{2\gamma n_2^2 n_1}{\theta}
\end{aligned}$$

(14):

$$\begin{aligned}
\frac{\partial R_{22}}{\partial r_2} &= \frac{\partial[\beta + \gamma \frac{r_2^2}{\theta^2}]}{\partial r_2} \\
&= (\frac{2\gamma}{\theta} - \alpha)n_2 (n_1^2 + n_3^2)
\end{aligned}$$

(15):

$$\begin{aligned}
\frac{\partial R_{22}}{\partial r_3} &= \frac{\partial[\beta + \gamma \frac{r_2^2}{\theta^2}]}{\partial r_3} \\
&= \alpha n_3 (n_2^2 - 1) - \frac{2\gamma n_2^2 n_3}{\theta}
\end{aligned}$$

(16):

$$\begin{aligned}
\frac{\partial R_{23}}{\partial r_1} &= \frac{\partial[\gamma \frac{r_3 r_2}{\theta^2} - \alpha \frac{r_3}{\theta}]}{\partial r_1} \\
&= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1 n_2 n_3 - (\beta\theta - \alpha)n_1^2 - \alpha]
\end{aligned}$$

(17):

$$\begin{aligned}\frac{\partial R_{23}}{\partial r_2} &= \frac{\partial[\gamma \frac{r_3 r_2}{\theta^2} - \alpha \frac{r_1}{\theta}]}{\partial r_2} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_2^2 n_3 + \gamma n_3 - (\beta\theta - \alpha)n_2 n_1]\end{aligned}$$

(18):

$$\begin{aligned}\frac{\partial R_{23}}{\partial r_3} &= \frac{\partial[\gamma \frac{r_3 r_2}{\theta^2} - \alpha \frac{r_1}{\theta}]}{\partial r_3} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_3^2 n_2 + \gamma n_2 - (\beta\theta - \alpha)n_1 n_3]\end{aligned}$$

(19):

$$\begin{aligned}\frac{\partial R_{31}}{\partial r_1} &= \frac{\partial[\gamma \frac{r_1 r_3}{\theta^2} - \alpha \frac{r_2}{\theta}]}{\partial r_1} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1^2 n_3 + \gamma n_3 - (\beta\theta - \alpha)n_1 n_2]\end{aligned}$$

(20):

$$\begin{aligned}\frac{\partial R_{31}}{\partial r_2} &= \frac{\partial[\gamma \frac{r_1 r_3}{\theta^2} - \alpha \frac{r_2}{\theta}]}{\partial r_2} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1 n_2 n_3 - (\beta\theta - \alpha)n_2^2 - \alpha]\end{aligned}$$

(21):

$$\begin{aligned}\frac{\partial R_{31}}{\partial r_3} &= \frac{\partial[\gamma \frac{r_1 r_3}{\theta^2} - \alpha \frac{r_2}{\theta}]}{\partial r_3} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_3^2 n_1 + \gamma n_1 - (\beta\theta - \alpha)n_2 n_3]\end{aligned}$$

(22):

$$\begin{aligned}\frac{\partial R_{32}}{\partial r_1} &= \frac{\partial[\gamma \frac{r_2 r_3}{\theta^2} + \alpha \frac{r_1}{\theta}]}{\partial r_1} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_1 n_2 n_3 + (\beta\theta - \alpha)n_1^2 + \alpha]\end{aligned}$$

(23):

$$\begin{aligned}\frac{\partial R_{32}}{\partial r_2} &= \frac{\partial[\gamma \frac{r_2 r_3}{\theta^2} + \alpha \frac{r_1}{\theta}]}{\partial r_2} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_2^2 n_3 + n_3 \gamma + (\beta\theta - \alpha)n_1 n_2]\end{aligned}$$

(24):

$$\begin{aligned}\frac{\partial R_{32}}{\partial r_3} &= \frac{\partial[\gamma \frac{r_2 r_3}{\theta^2} + \alpha \frac{r_1}{\theta}]}{\partial r_3} \\ &= \frac{1}{\theta}[(\alpha\theta - 2\gamma)n_3^2 n_2 + n_2 \gamma + (\beta\theta - \alpha)n_1 n_3]\end{aligned}$$

(25):

$$\begin{aligned}\frac{\partial R_{33}}{\partial r_1} &= \frac{\partial[\beta + \gamma \frac{r_3^2}{\theta^2}]}{\partial r_1} \\ &= \alpha n_1 (n_3^2 - 1) - \frac{2\gamma n_3^2 n_1}{\theta}\end{aligned}$$

(26):

$$\begin{aligned}\frac{\partial R_{33}}{\partial r_2} &= \frac{\partial[\beta + \gamma \frac{r_3^2}{\theta^2}]}{\partial r_2} \\ &= \alpha n_2 (n_3^2 - 1) - \frac{2\gamma n_3^2 n_2}{\theta}\end{aligned}$$

(27):

$$\begin{aligned}\frac{\partial R_{33}}{\partial r_3} &= \frac{\partial[\beta + \gamma \frac{r_3^2}{\theta^2}]}{\partial r_3} \\ &= (\frac{2\gamma}{\theta} - \alpha) n_3 (n_2^2 + n_1^2)\end{aligned}$$

So:

$$\frac{du}{dr^T} = \begin{bmatrix} (\frac{2\gamma}{\theta} - \alpha) n_1 (n_2^2 + n_3^2) & \alpha n_2 (n_1^2 - 1) - \frac{2\gamma n_1^2 n_2}{\theta} & \alpha n_3 (n_1^2 - 1) - \frac{2\gamma n_1^2 n_3}{\theta} \\ \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1^2 n_2 + \gamma n_2 - (\beta\theta - \alpha) n_1 n_3] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_2^2 n_1 + \gamma n_1 - (\beta\theta - \alpha) n_2 n_3] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1 n_2 n_3 - (\beta\theta - \alpha) n_3^2 - \alpha] \\ \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1^2 n_3 + n_3 \gamma + (\beta\theta - \alpha) n_1 n_2] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1 n_2 n_3 + (\beta\theta - \alpha) n_2^2 + \alpha] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_3^2 n_1 + n_1 \gamma + (\beta\theta - \alpha) n_2 n_3] \\ \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1^2 n_2 + n_2 \gamma + (\beta\theta - \alpha) n_1 n_3] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_2^2 n_1 + n_1 \gamma + (\beta\theta - \alpha) n_2 n_3] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1 n_2 n_3 + (\beta\theta - \alpha) n_3^2 + \alpha] \\ \alpha n_1 (n_2^2 - 1) - \frac{2\gamma n_2^2 n_1}{\theta} & (\frac{2\gamma}{\theta} - \alpha) n_2 (n_1^2 + n_3^2) & \alpha n_3 (n_2^2 - 1) - \frac{2\gamma n_2^2 n_3}{\theta} \\ \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1 n_2 n_3 - (\beta\theta - \alpha) n_1^2 - \alpha] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_2^2 n_3 + \gamma n_3 - (\beta\theta - \alpha) n_2 n_1] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_3^2 n_2 + \gamma n_2 - (\beta\theta - \alpha) n_1 n_3] \\ \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1^2 n_3 + \gamma n_3 - (\beta\theta - \alpha) n_1 n_2] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1 n_2 n_3 - (\beta\theta - \alpha) n_2^2 - \alpha] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_3^2 n_1 + \gamma n_1 - (\beta\theta - \alpha) n_2 n_3] \\ \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_1 n_2 n_3 + (\beta\theta - \alpha) n_1^2 + \alpha] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_2^2 n_3 + n_3 \gamma + (\beta\theta - \alpha) n_1 n_2] & \frac{1}{\theta} [(\alpha\theta - 2\gamma) n_3^2 n_2 + n_2 \gamma + (\beta\theta - \alpha) n_1 n_3] \\ \alpha n_1 (n_3^2 - 1) - \frac{2\gamma n_3^2 n_1}{\theta} & \alpha n_2 (n_3^2 - 1) - \frac{2\gamma n_3^2 n_2}{\theta} & (\frac{2\gamma}{\theta} - \alpha) n_3 (n_2^2 + n_1^2) \end{bmatrix}_{9 \times 3}$$