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Abstract:

There are two different methods you can use XPP AUTO to draw bifurcation diagram. The first method is starting from steady state (or equilibrium point) and continue with AUTO. The second method is starting from periodic oscillations and continue with AUTO. There is little information or example on how to draw bifurcation diagram from periodic oscillations. Thus, this tutorial is to explain how to draw bifurcation diagram from periodic oscillations. Since AUTO is well-known as a tricky software and to be able to draw bifurcation diagram starting from periodic oscillations depends on your knowledge of the dynamical system under study and also your experience of using AUTO. It requires skills and knowledge of dynamical system to draw a correct bifurcation diagram. Below we will show you the detailed steps on how to draw bifurcation diagram starting from periodic oscillations by going through an example from a modified Morris-Lecar model.
keywords: XPP AUTO; bifurcation diagram; periodic oscillations; neuronal spikes; dynamical systems

Introduction

Bifurcation diagram is a diagram that often use by modeler to explore the dynamical behavior of biological systems over a range of parameter value changes[1]. A saddle-node bifurcation can be used to describe biological switches[2]. On the other hand, a Hopf bifurcation can be used to explain biological oscillations[1]. The purpose of this tutorial is to explain how to draw bifurcation using XPP AUTO[3] with computer simulation of a concrete example from Ratté et al.(2018), which is a modified Morris-Lecar model[4]. For an introduction of bifurcation analysis with AUTO we refer readers to Marc R. Roussel's lecture[5]. As Roussel addmitted that AUTO is a tricky software[5], we prefer to explain how to draw bifurcation diagram by doing hands on computer simulation with the codes supplied. **XPP ode code: Ratte2018JNeurosci.ode**

Step by step instructions to draw bifurcation diagram

Step 1.

Set the parameter value and initial conditions as given in the XPP ode file. Open the XPP ode file and initiate the XPP GUI interface. Then Integrate -> Go and you should get a time-course simulation that shows limit cycle oscillations or persistent spiking (see Fig. 1).

Step 2.

After the time course simulation ended at t = 100000, open the Data Viewer by clicking the Data button or icon. First, we need to find the period before we can continue with AUTO. In the Data Viewer click the End button to move the current row to the end of the time-course data. Then, we need to use the Page Up (PgUp) button to go one page up or continue page up until we see the voltage (V the variable of the second column) close to 0. Let us assumed this point as the end point of a limit cycle.

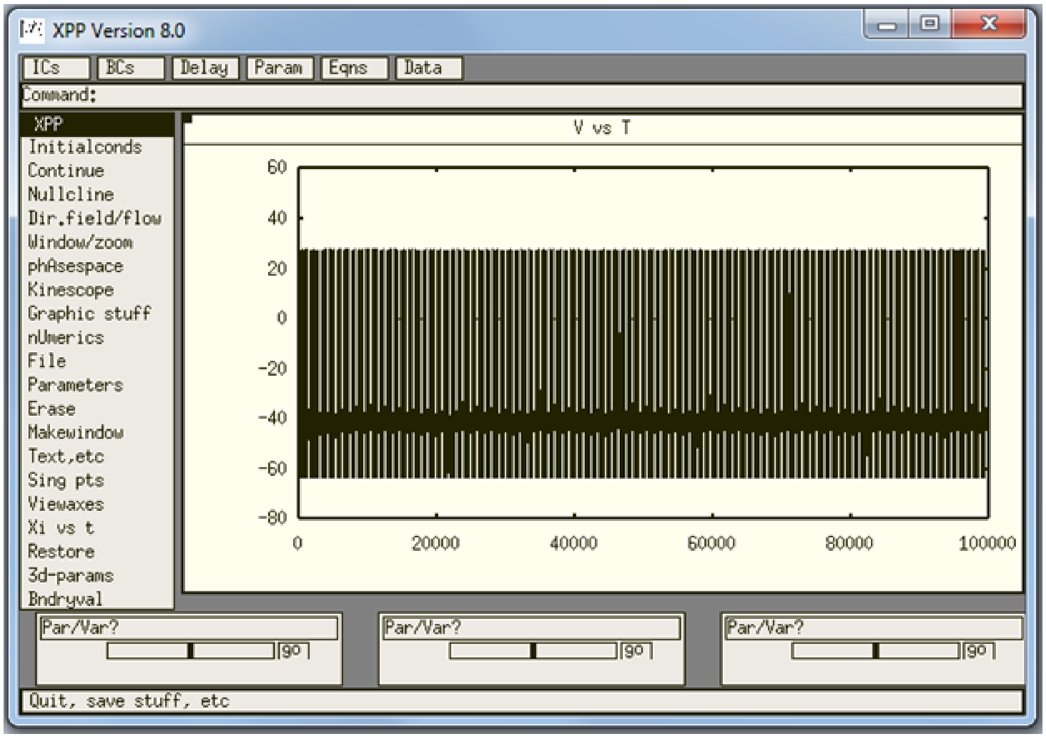
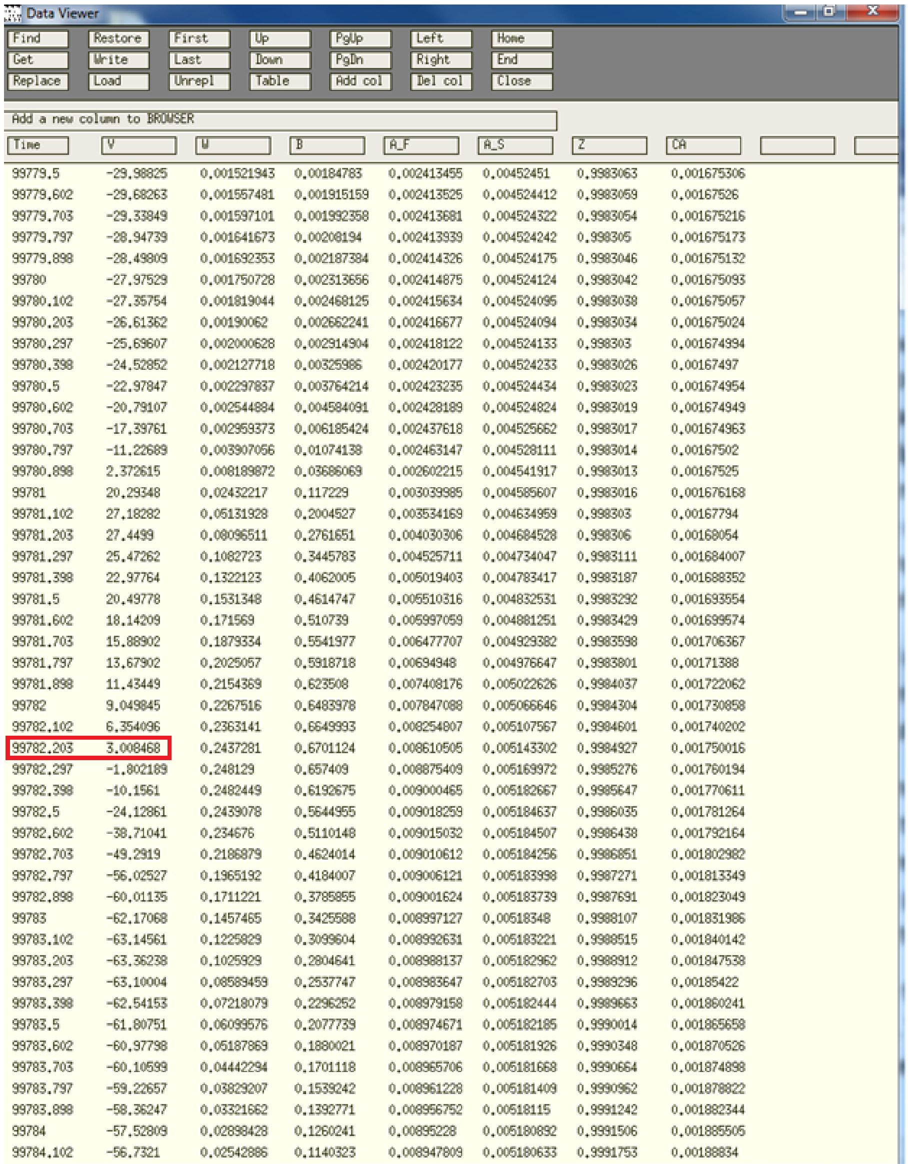


Figure 1: XPP GUI interface that shows the voltage time-course simulation of approaching stable limit cycle oscillations or persistent spiking.

Step 3.

Record the time t_2 when V is close to zero. In our example, the end point of the limit cycle we got $t_2 = 99782.203$ when $V = 3.008$ (see Fig. 2). Your results may be different slightly. Then scroll the page up (and you may need to use the down or up to navigate the row down or up) until we see the voltage V reach close to 0 again which mean one full cycle. The starting point of the limit cycle. In our example, we got $t_1 = 99485.602$ when $V = 2.004$ (see Fig. 3).

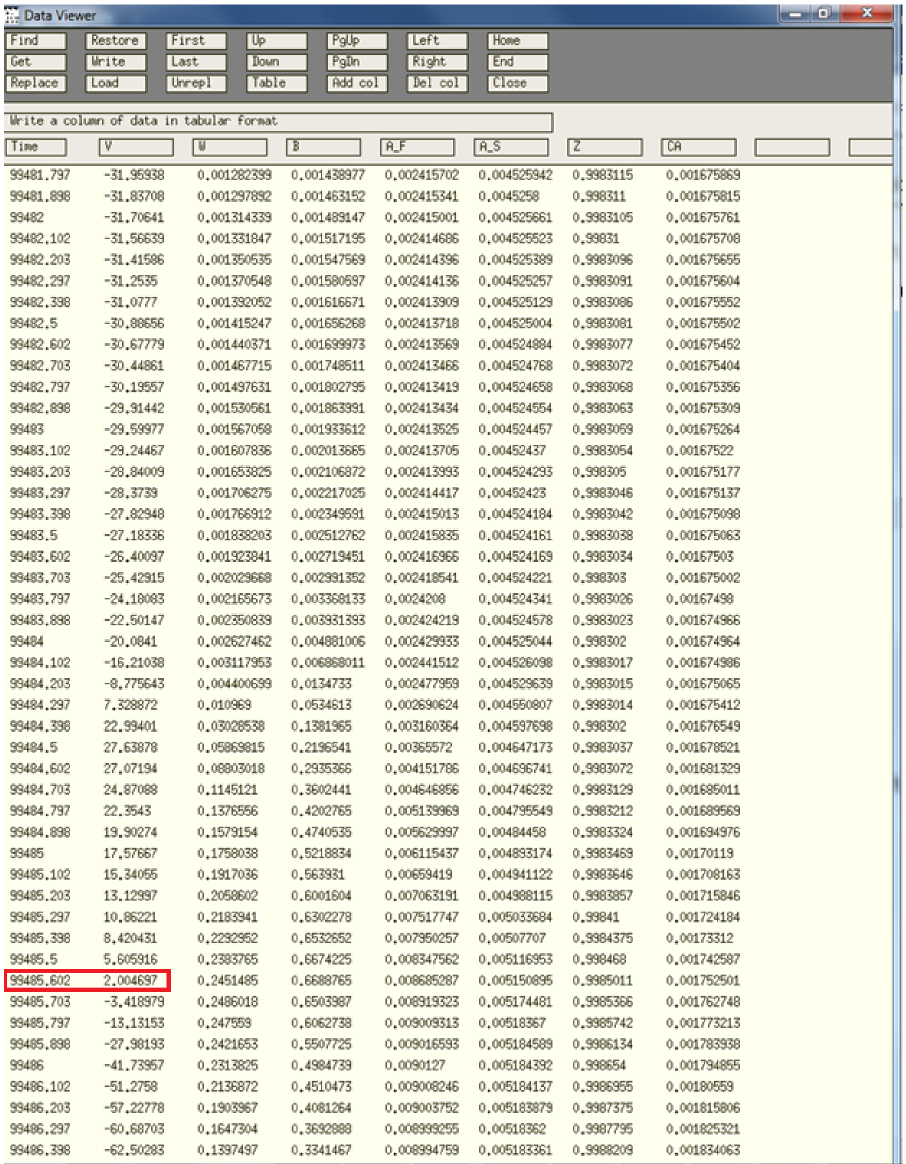


Time	V	W	B	A_F	A_S	Z	CA
99779,5	-29,98825	0,001521943	0,00184783	0,002413455	0,00452451	0,9983063	0,001675306
99779,602	-29,68263	0,001557481	0,001915159	0,002413525	0,004524412	0,9983059	0,00167526
99779,703	-29,33849	0,001597101	0,001992358	0,002413681	0,004524322	0,9983054	0,001675216
99779,797	-28,94739	0,001641673	0,00208194	0,002413939	0,004524242	0,998305	0,001675173
99779,898	-28,49809	0,001692353	0,002187384	0,002414326	0,004524175	0,9983046	0,001675132
99780	-27,97529	0,001750728	0,002313656	0,002414875	0,004524124	0,9983042	0,001675093
99780,102	-27,35754	0,001819044	0,002468125	0,002415634	0,004524095	0,9983038	0,001675057
99780,203	-26,61362	0,00190062	0,002662241	0,002416677	0,004524094	0,9983034	0,001675024
99780,297	-25,69607	0,002000628	0,002914904	0,002418122	0,004524133	0,998303	0,001674994
99780,398	-24,52852	0,002127718	0,00325986	0,002420177	0,004524233	0,9983026	0,00167497
99780,5	-22,97847	0,002297837	0,003764214	0,002423235	0,004524434	0,9983023	0,001674954
99780,602	-20,79107	0,002544884	0,004584091	0,002428189	0,004524824	0,9983019	0,001674949
99780,703	-17,39761	0,002959373	0,006185424	0,002437618	0,004525662	0,9983017	0,001674963
99780,797	-11,22689	0,003907056	0,01074138	0,002463147	0,004528111	0,9983014	0,00167502
99780,898	2,372615	0,008189872	0,03686069	0,002602215	0,004541917	0,9983013	0,00167525
99781	20,29348	0,02432217	0,117229	0,003039985	0,004585607	0,9983016	0,001676168
99781,102	27,18282	0,05131928	0,2004527	0,003534169	0,004634959	0,998303	0,00167794
99781,203	27,4499	0,08096511	0,2761651	0,004030306	0,004684528	0,998306	0,00168054
99781,297	25,47262	0,1082723	0,3445783	0,004525711	0,004734047	0,9983111	0,001684007
99781,398	22,97764	0,1322123	0,4062005	0,005019403	0,004783417	0,9983187	0,001688352
99781,5	20,49778	0,1531348	0,4614747	0,005510316	0,004832531	0,9983292	0,001693554
99781,602	18,14209	0,171569	0,510739	0,005997059	0,004881251	0,9983429	0,001699574
99781,703	15,88902	0,1879334	0,5541977	0,006477707	0,004929382	0,9983598	0,001706367
99781,797	13,67902	0,2025057	0,5918718	0,00694948	0,004976647	0,9983801	0,00171388
99781,898	11,43449	0,2154369	0,623508	0,007408176	0,005022626	0,9984037	0,001722062
99782	9,049845	0,2267516	0,6483978	0,007847088	0,005066646	0,9984304	0,001730858
99782,102	6,354096	0,2363141	0,6649993	0,008254807	0,005107567	0,9984601	0,001740202
99782,203	3,008468	0,2437281	0,6701124	0,008610505	0,005143302	0,9984927	0,001750016
99782,297	-1,802189	0,248129	0,657409	0,008875409	0,005169972	0,9985276	0,001760194
99782,398	-10,1561	0,2482449	0,6192675	0,009000465	0,005182667	0,9985647	0,001770611
99782,5	-24,12861	0,2439078	0,5644955	0,009018259	0,005184637	0,9986035	0,001781264
99782,602	-38,71041	0,234676	0,5110148	0,009015032	0,005184507	0,9986438	0,001792164
99782,703	-49,2919	0,2186879	0,4624014	0,009010612	0,005184256	0,9986851	0,001802982
99782,797	-56,02527	0,1965192	0,4184007	0,009006121	0,005183998	0,9987271	0,001813349
99782,898	-60,01135	0,1711221	0,3785855	0,009001624	0,005183739	0,9987691	0,001823049
99783	-62,17068	0,1457465	0,3425588	0,008997127	0,00518348	0,9988107	0,001831986
99783,102	-63,14561	0,1225829	0,3099604	0,008992631	0,005183221	0,9988515	0,001840142
99783,203	-63,36238	0,1025929	0,2804641	0,008988137	0,005182962	0,9988912	0,001847538
99783,297	-63,10004	0,08589459	0,2537747	0,008983647	0,005182703	0,9989296	0,00185422
99783,398	-62,54153	0,07218079	0,2296252	0,008979158	0,005182444	0,9989663	0,001860241
99783,5	-61,80751	0,06099576	0,2077739	0,008974671	0,005182185	0,9990014	0,001865658
99783,602	-60,97798	0,05187869	0,1880021	0,008970187	0,005181926	0,9990348	0,001870526
99783,703	-60,10599	0,04442294	0,1701118	0,008965706	0,005181668	0,9990664	0,001874898
99783,797	-59,22657	0,03829207	0,1539242	0,008961228	0,005181409	0,9990962	0,001878822
99783,898	-58,36247	0,03321662	0,1392771	0,008956752	0,00518115	0,9991242	0,001882344
99784	-57,52809	0,02898428	0,1260241	0,00895228	0,005180892	0,9991506	0,001885505
99784,102	-56,7321	0,02542886	0,1140323	0,008947809	0,005180633	0,9991753	0,00188834

Figure 2: Data Viewer that shows the voltage approaching 0 (V = 3.008) say at t_2 = 99782.203 (see a red box). Assumed as the end point of the limit cycle.

Step 4.

Calculate the period using period = t_2 - t_1. Period = 99782.203 - 99485.602 = 296.600 (since the value for V is 2.004 at t_1 and 3.008 at t_2 the period should



Time	V	W	B	A_F	A_S	Z	CA
99481.797	-31.95938	0.001282399	0.001438977	0.002415702	0.004525942	0.9983115	0.001675869
99481.898	-31.83708	0.001297892	0.001463152	0.002415341	0.0045258	0.998311	0.001675815
99482	-31.70641	0.001314339	0.001489147	0.002415001	0.004525661	0.9983105	0.001675761
99482.102	-31.56639	0.001331847	0.001517195	0.002414686	0.004525523	0.99831	0.001675708
99482.203	-31.41586	0.001350535	0.001547569	0.002414396	0.004525389	0.9983096	0.001675655
99482.297	-31.2535	0.001370548	0.001580597	0.002414136	0.004525257	0.9983091	0.001675604
99482.398	-31.0777	0.001392052	0.001616671	0.002413909	0.004525129	0.9983086	0.001675552
99482.5	-30.88656	0.001415247	0.001656268	0.002413718	0.004525004	0.9983081	0.001675502
99482.602	-30.67779	0.001440371	0.001699973	0.002413569	0.004524884	0.9983077	0.001675452
99482.703	-30.44861	0.001467715	0.001748511	0.002413466	0.004524768	0.9983072	0.001675404
99482.797	-30.19557	0.001497631	0.001802795	0.002413419	0.004524658	0.9983068	0.001675356
99482.898	-29.91442	0.001530561	0.001863991	0.002413434	0.004524554	0.9983063	0.001675309
99483	-29.59977	0.001567058	0.001933612	0.002413525	0.004524457	0.9983059	0.001675264
99483.102	-29.24467	0.001607836	0.002013665	0.002413705	0.00452437	0.9983054	0.00167522
99483.203	-28.84009	0.001653825	0.002106872	0.002413993	0.004524293	0.998305	0.001675177
99483.297	-28.3739	0.001706275	0.002217025	0.002414417	0.00452423	0.9983046	0.001675137
99483.398	-27.82948	0.001766912	0.002349591	0.002415013	0.004524184	0.9983042	0.001675098
99483.5	-27.18336	0.001838203	0.002512762	0.002415835	0.004524161	0.9983038	0.001675063
99483.602	-26.40097	0.001923841	0.002719451	0.002416966	0.004524169	0.9983034	0.00167503
99483.703	-25.42915	0.002029668	0.002991352	0.002418541	0.004524221	0.998303	0.001675002
99483.797	-24.18083	0.002165673	0.003368133	0.0024208	0.004524341	0.9983026	0.00167498
99483.898	-22.50147	0.002350839	0.003931393	0.002424219	0.004524578	0.9983023	0.001674966
99484	-20.0841	0.002627462	0.004881006	0.002429933	0.004525044	0.998302	0.001674964
99484.102	-16.21038	0.003117953	0.006868011	0.002441512	0.004526098	0.9983017	0.001674986
99484.203	-8.775643	0.004400699	0.0134733	0.002477959	0.004529639	0.9983015	0.001675065
99484.297	7.328872	0.010969	0.0534613	0.002690624	0.004550807	0.9983014	0.001675412
99484.398	22.99401	0.03028538	0.1381965	0.003160364	0.004597698	0.998302	0.001676549
99484.5	27.63878	0.05869815	0.2196541	0.00365572	0.004647173	0.9983037	0.001678521
99484.602	27.07194	0.08803018	0.2935366	0.004151786	0.004696741	0.9983072	0.001681329
99484.703	24.87088	0.1145121	0.3602441	0.004646856	0.004746232	0.9983129	0.001685011
99484.797	22.3543	0.1376556	0.4202765	0.005139969	0.004795549	0.9983212	0.001689569
99484.898	19.90274	0.1579154	0.4740535	0.005629997	0.00484458	0.9983324	0.001694976
99485	17.57667	0.1758038	0.5218834	0.006115437	0.004893174	0.9983469	0.00170119
99485.102	15.34055	0.1917036	0.563931	0.00659419	0.004941122	0.9983646	0.001708163
99485.203	13.12997	0.2058602	0.6001604	0.007063191	0.004988115	0.9983857	0.001715846
99485.297	10.86221	0.2183941	0.6302278	0.007517747	0.005033684	0.99841	0.001724184
99485.398	8.420431	0.2292952	0.6532652	0.007950257	0.00507707	0.9984375	0.00173312
99485.5	5.605916	0.2383765	0.6674225	0.008347562	0.005116953	0.998468	0.001742587
99485.602	2.004697	0.2451485	0.6688765	0.008685287	0.005150895	0.9985011	0.001752501
99485.703	-3.418979	0.2486018	0.6503987	0.008919323	0.005174481	0.9985366	0.001762748
99485.797	-13.13153	0.247559	0.6062738	0.009009313	0.00518367	0.9985742	0.001773213
99485.898	-27.98193	0.2421653	0.5507725	0.009016593	0.005184589	0.9986134	0.001783938
99486	-41.73957	0.2313825	0.4984739	0.0090127	0.005184392	0.998654	0.001794855
99486.102	-51.2758	0.2136872	0.4510473	0.009008246	0.005184137	0.9986955	0.00180559
99486.203	-57.22778	0.1903967	0.4081264	0.009003752	0.005183879	0.9987375	0.001815806
99486.297	-60.68703	0.1647304	0.3692888	0.008999255	0.00518362	0.9987795	0.001825321
99486.398	-62.50283	0.1397497	0.3341467	0.008994759	0.005183361	0.9988209	0.001834063

Figure 3: Data Viewer that shows the voltage approaching 0 (V = 2.004) say at t_1 = 99485.602 (see a red box). Assumed as the starting point of the limit cycle.

be 296. We purposely use 296.6 to let you alert that when error happens it might be your estimate of the period is not good for AUTO to start with the calculations.)

Step 5.

Go to the XPP GUI interface and Select nUmerics (or press U), click Total then set the total to 296.6 and press Enter. Then press Esc button in your keyboard to exit nUmerics menu.

Step 6.

Create a new window by Selecting Makewindow -> Create. Select Viewaxes -> 2D to pop up the 2D View page and set the X-axis: w; Xmin: -1; Ymin:-80; Xmax:1; Ymax:60; Xlabel: w; Ylabel: V and then click OK (see Fig. 4).

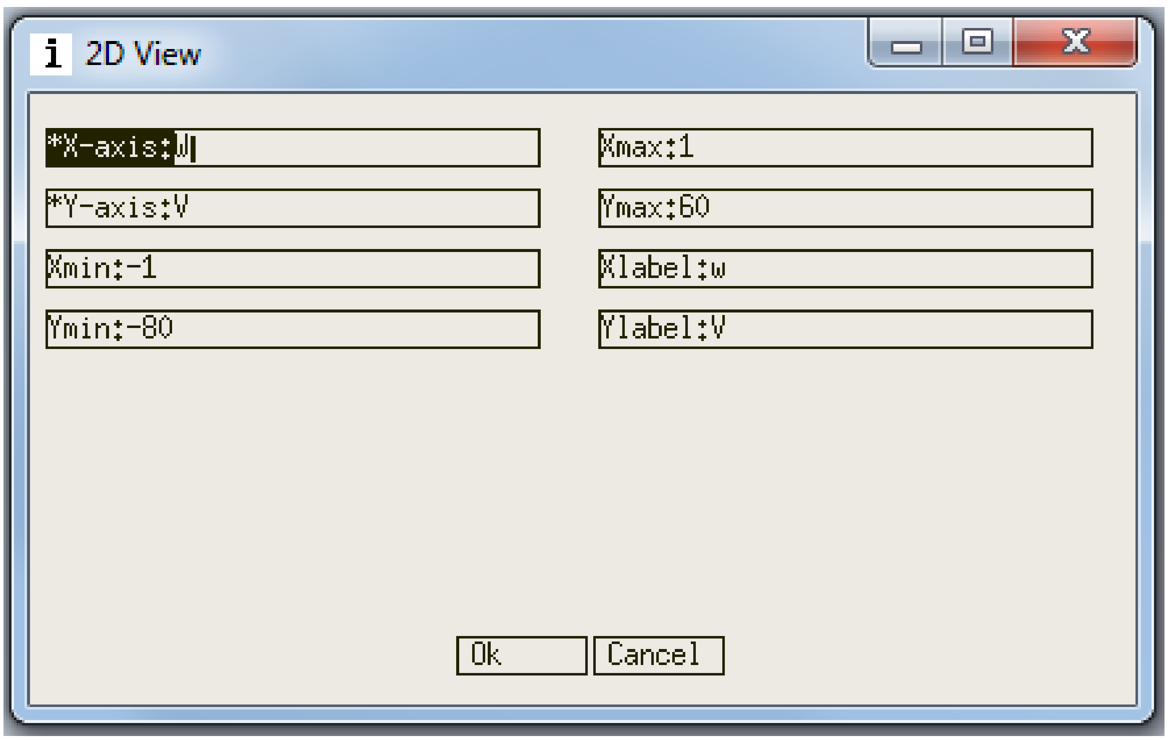


Figure 4: 2D View setting page that shows the values you should enter to be able to see a limit cycle in the V vs w phase plane.

Step 7.

Go to the XPP GUI interface, delete the graph by selecting Erase. To plot the limit cycle, select InitialConds -> Last. You should get a limit cycle (see Fig. 5).

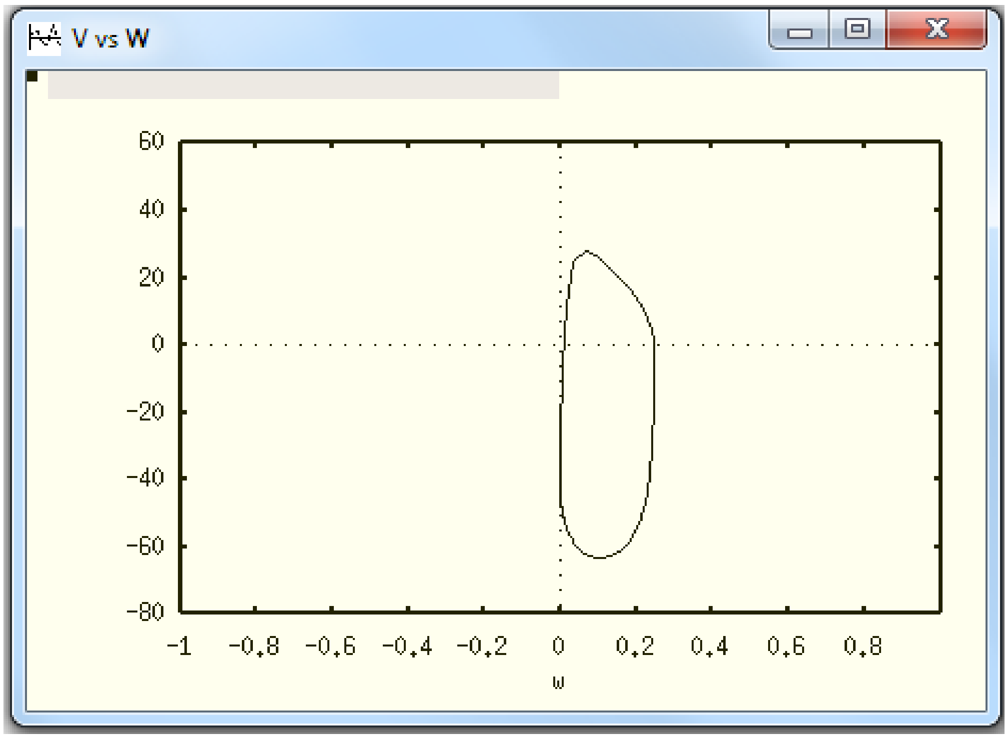


Figure 5: Phase plane view shows a limit cycle (one full cycle to demonstrate that the period is correct or not) in the V vs W phase plane.

Step 8.

Then we can go to AUTO by clicking File -> Auto. You should get the Auto page pop up (see Fig. 6). Then, go to step 9.

Note:

For your own model, you may click the numerics to change some numerics setting for plotting bifurcation diagram (see Fig. 7). Please refer to XPP documentation "xpp_doc.pdf" page 67/97 or Section 10.4 Numerical parameters for the meaning of the numerical parameters. If you cannot get to draw a bifurcation diagram you may need to change these parameter values. Most likely you need to adjust: Ntst sets to a larger value; Nmax sets to a larger value; Npr sets to a smaller value but not so crucial; Ds sets to a negative value to continue with a different direction.

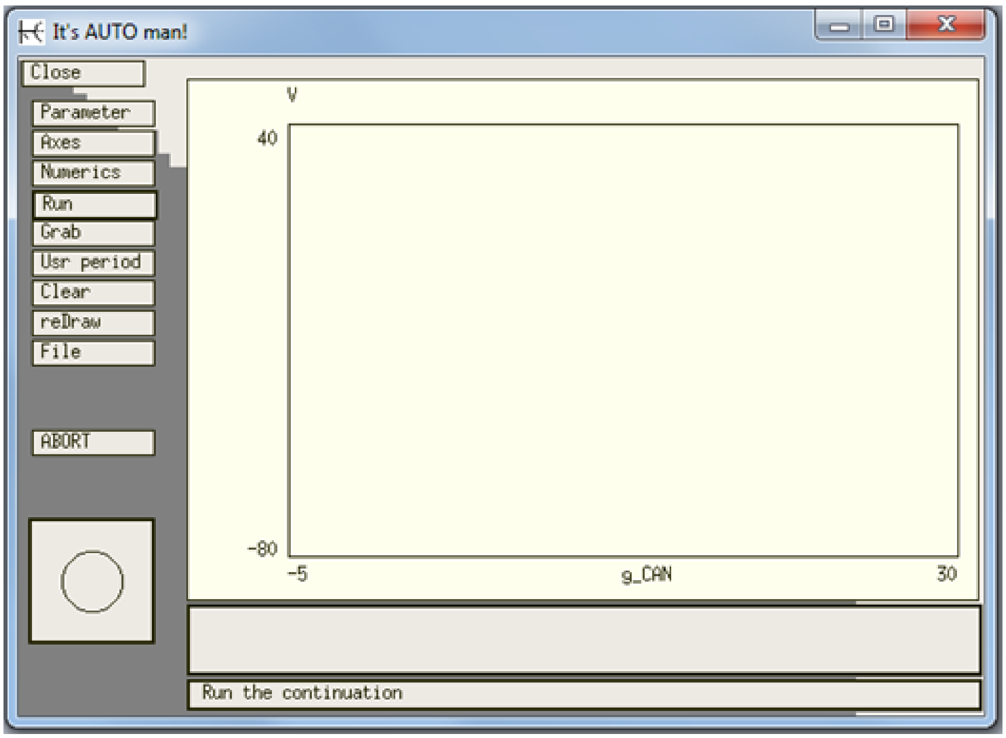


Figure 6: Auto page view open after you select File -> Auto.

Step 9.

To draw bifurcation diagram, select Run -> Periodic (see Fig. 8). For the case if Auto cannot draw bifurcation diagram (see Fig. 9), you may need to change the value of the period or adjust the numerical parameter in step 8. For this tutorial we let you start with a wrong period (296.6) where AUTO cannot draw a bifurcation diagram. In our case, the output shows MX means there is an error (see Fig. 10). The points at t_2, V = 3.008 and at t_1, V = 2.004 (try to get say 3.008 to 3 so that it is one close loop of limit cycle). It shows that the time for period is a little bit too long, so let us try period =

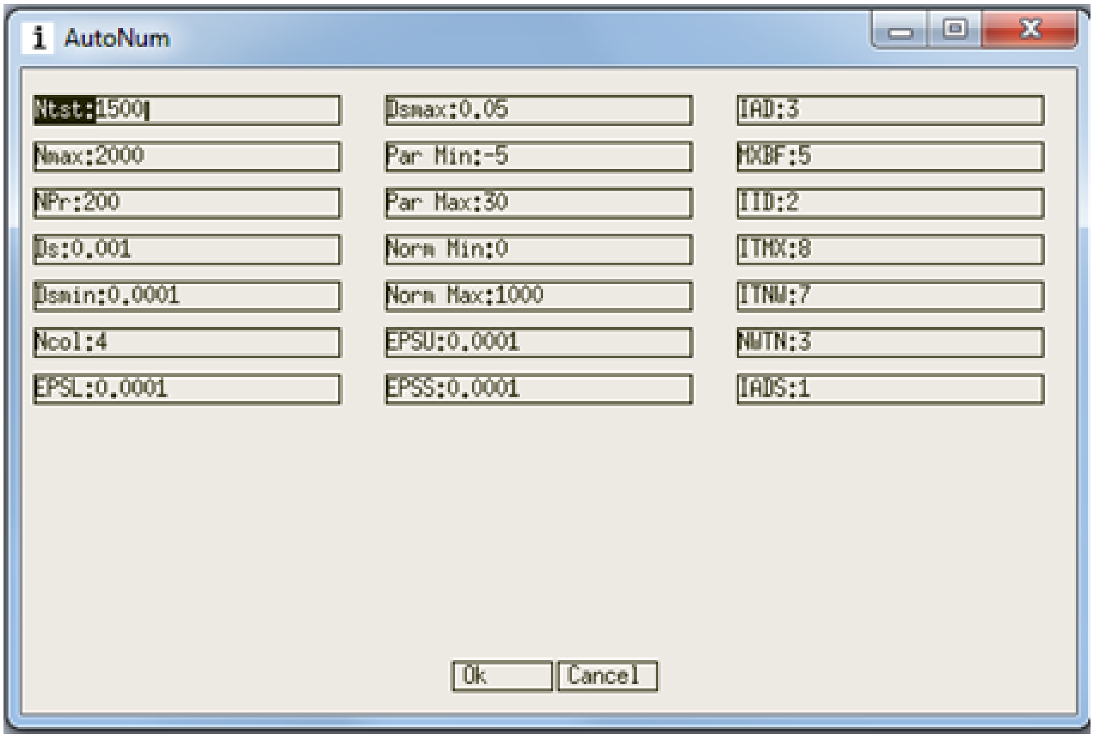


Figure 7: Numerics setting page open after you click Numerics. These setting were done using the Auto setting in the end of the ode file (for your convenience). You may manually key in these values one at a time.

296. For your own model, if you did not get bifurcation diagram try adjust the period until you input a good approximate time for period in order to draw a bifurcation diagram. This is what drawing bifurcation diagram in AUTO starting with periodic oscillation get to work properly.

Step 10.

So, in order to attempt to draw bifurcation diagram for the second time you should repeat the same steps from step 1 to step 9. For our case we know we need to adjust the value for period to 296, so you don't need to record the times and V values. Sets the period to 296 and follow the steps shown in Fig. 11 to Fig. 24. We need to reset the parameter value g_CAN the bifurcation parameter and the initial conditions to the original values

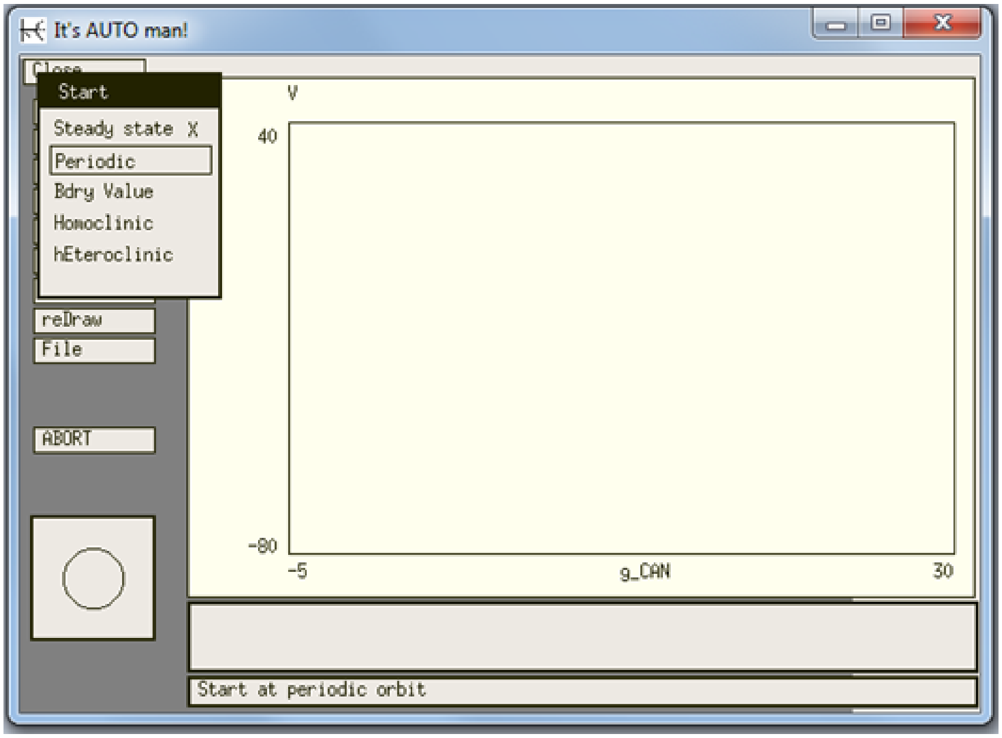


Figure 8: Select Run -> Periodic to draw bifurcation diagram.

because these values have been changed by Auto in step 9. At the end, you should be able to draw bifurcation diagram as shown in Fig. 24.

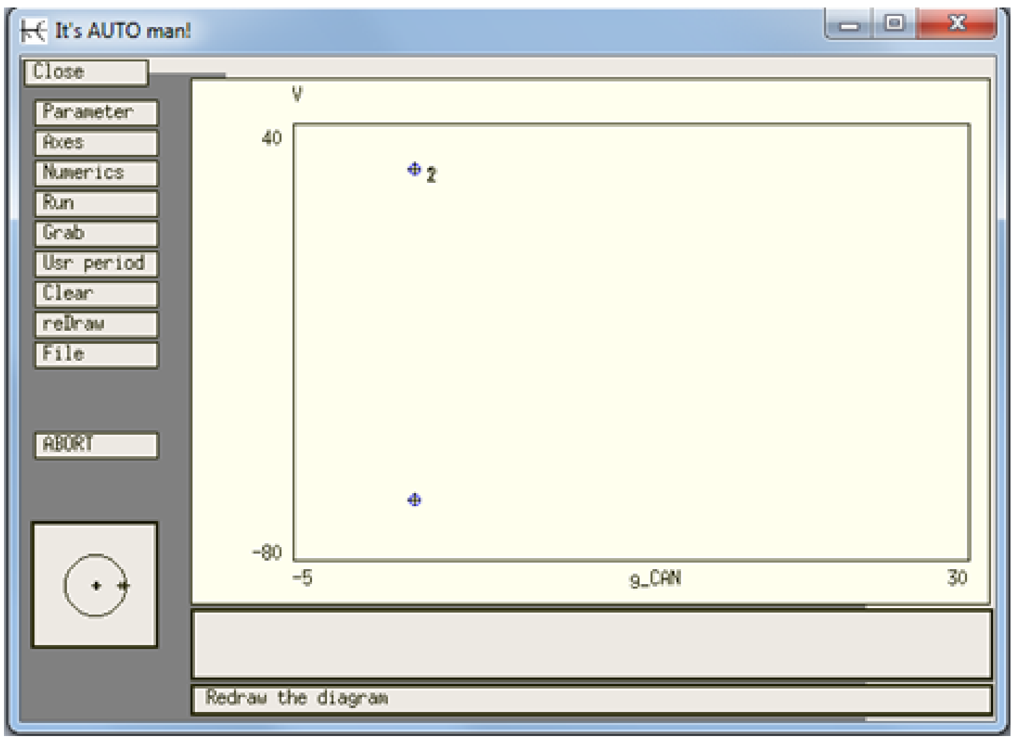


Figure 9: Auto was not able to draw a bifurcation diagram. Something wrong, so we need to adjust the value for period or numerical parameter values.

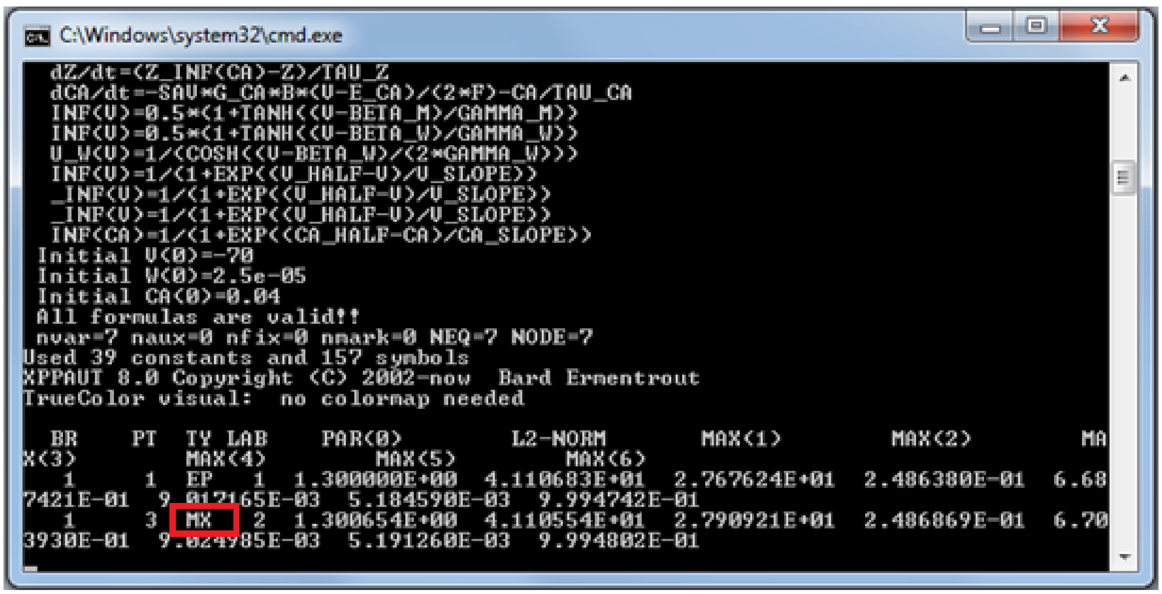


Figure 10: The label MX (red box) means Auto got error probably the calculation cannot be done because the period 296.6 is a little to big. The points at t2 V = 3.008 and at t1 V = 2.004 (try to get say 3.008 to 3) show that the time for period is a little bit too long, so let us try period = 296. If you did not get bifurcation diagram try adjust the period until you input a good approximate time for period you may be able to draw a bifurcation diagram.

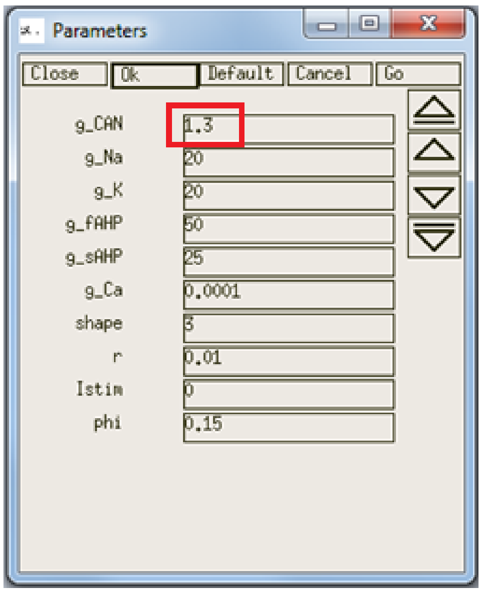


Figure 11: Click Param button, a Parameters page will pop up. Reset the g_CAN to 1.3 because during the drawing of bifurcation diagram this value has been changed.

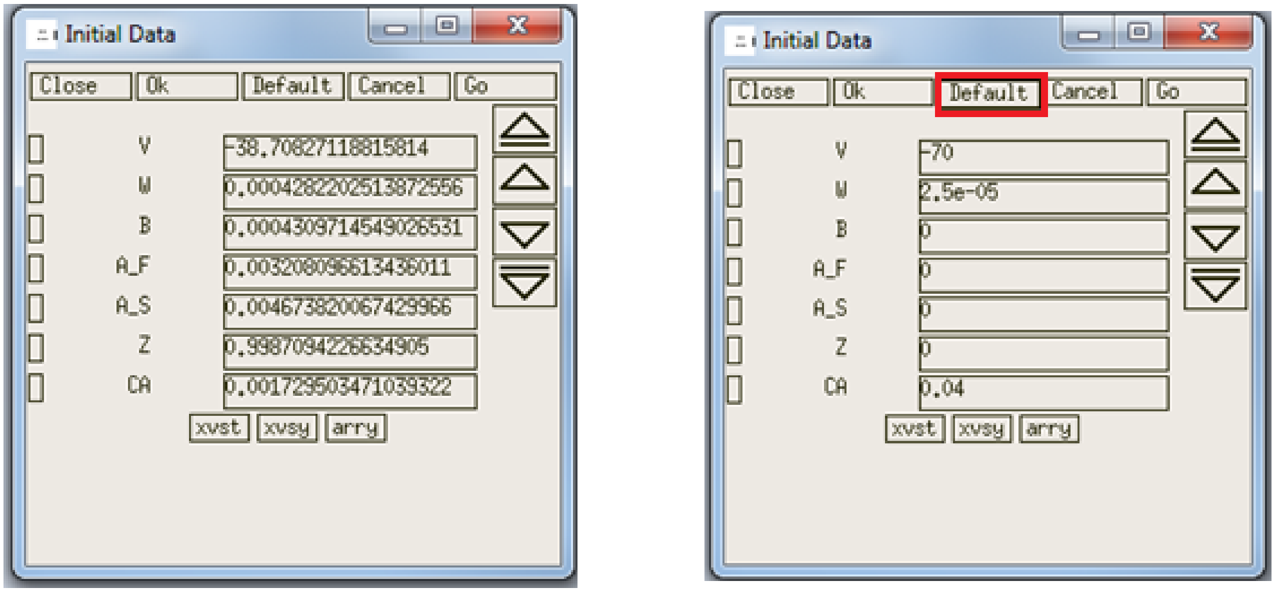


Figure 12: Click ICs button, an Initial Data page will pop up. Reset the initial value to default values by clicking the Default button (red box).

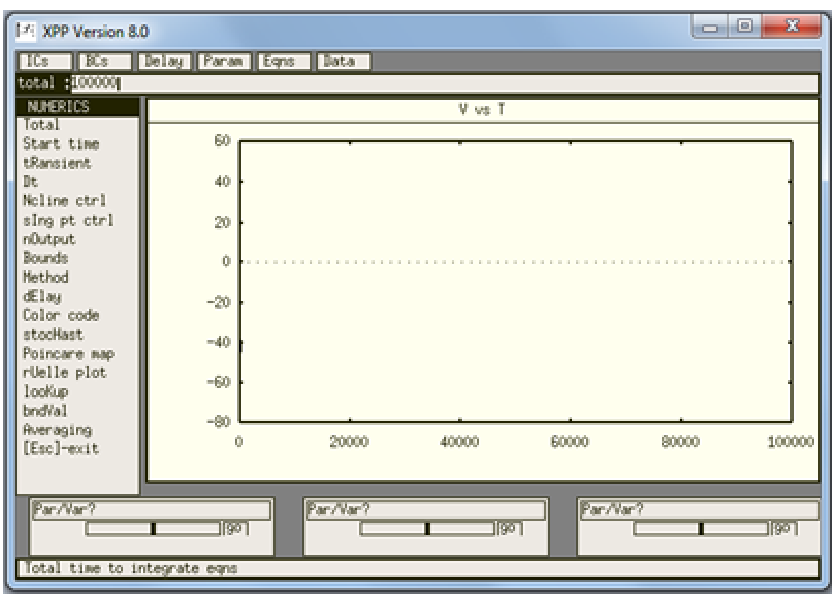


Figure 13: Reset the Total time span to 100000.

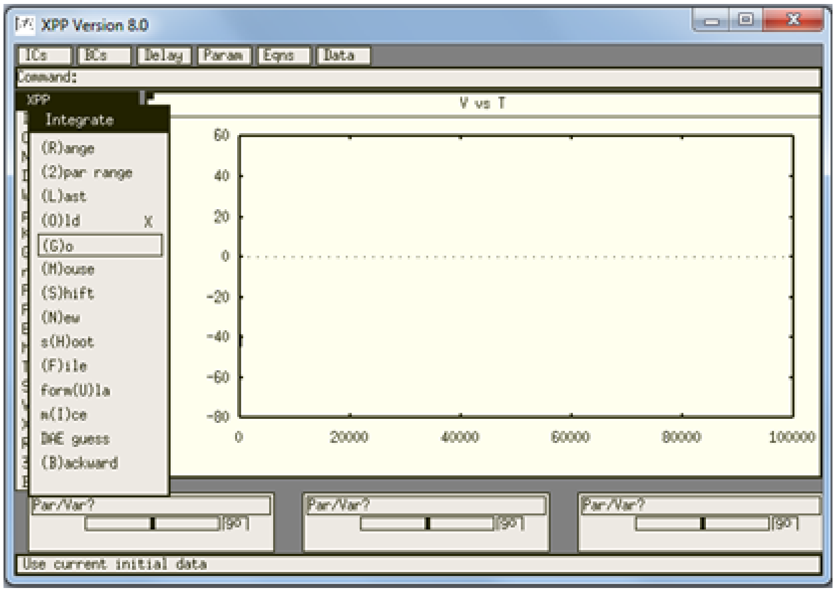


Figure 14: Integrate using Initial -> Go.

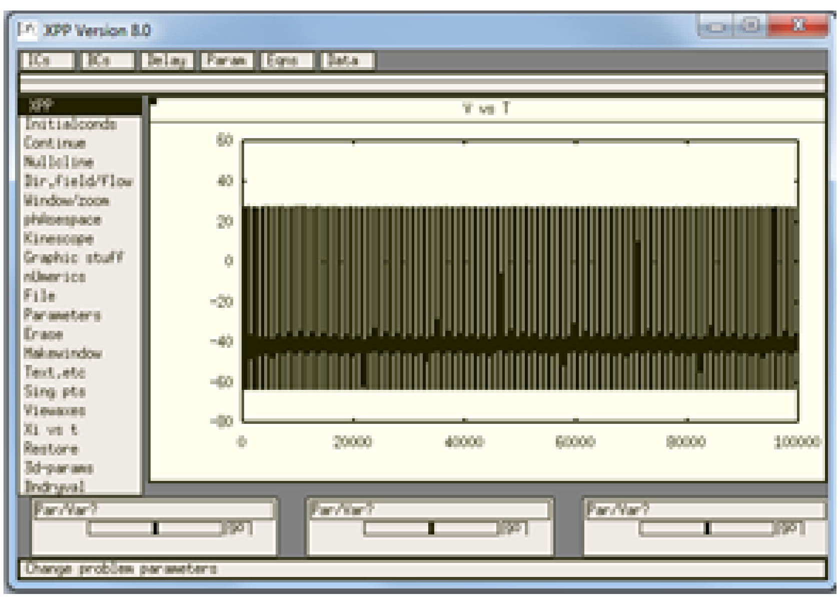


Figure 15: Let the time-course simulation run until 100000.

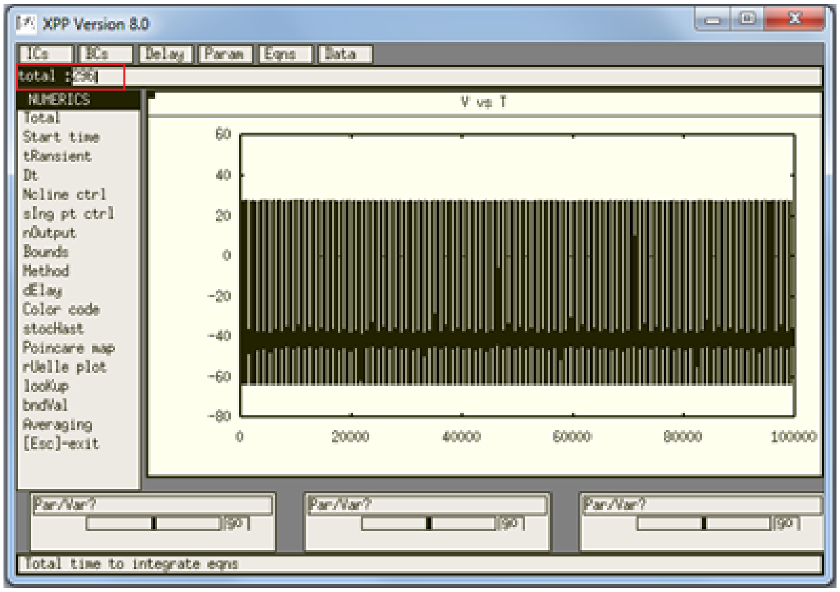


Figure 16: Use nUmerics -> Total and set the total to 296.

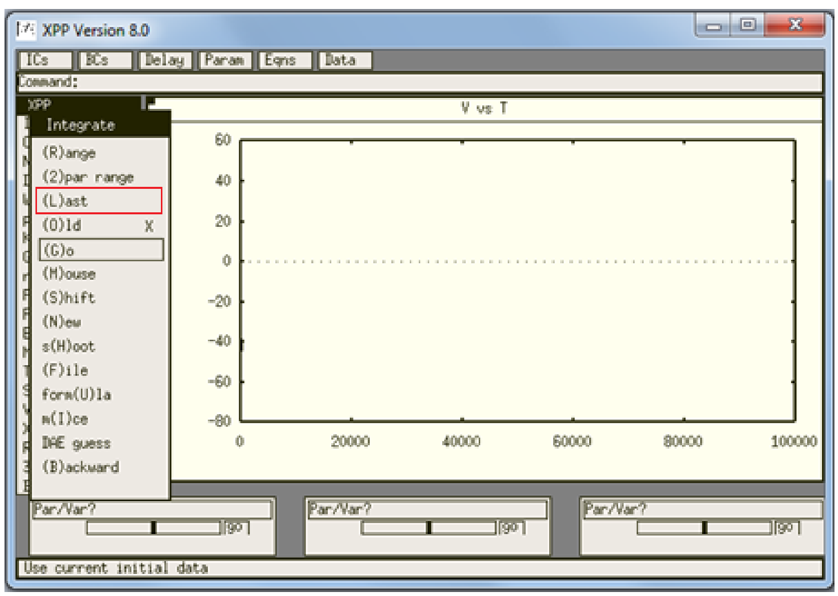


Figure 17: Activate the w vs V screen by clicking the pop up phase plane. Use Initial -> Last (red box) to integrate using the last initial conditions from 100000 end point calculated earlier.

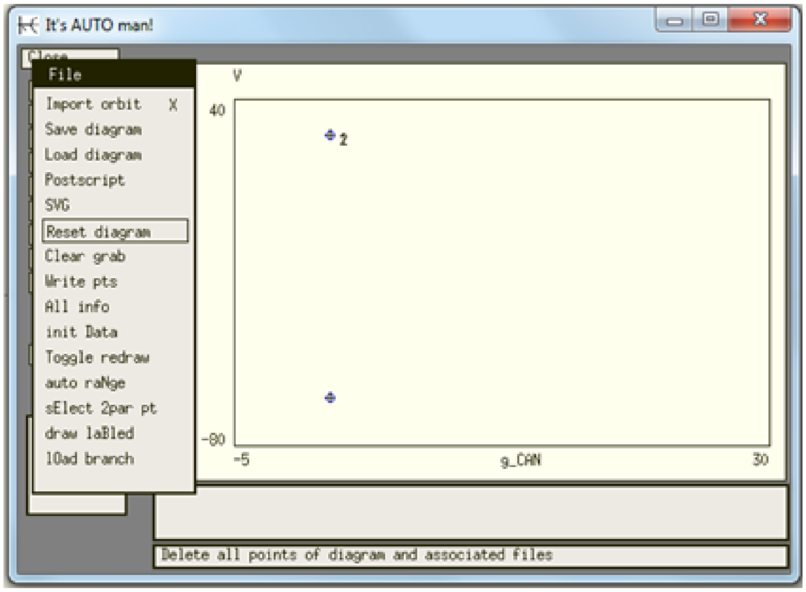


Figure 18: In the Auto Screen. Reset diagram by using File -> Reset diagram. There is a prompt message to ask you want to destroy the data, just answer Yes or click YES (see below). This will clear all previous bifurcation data.

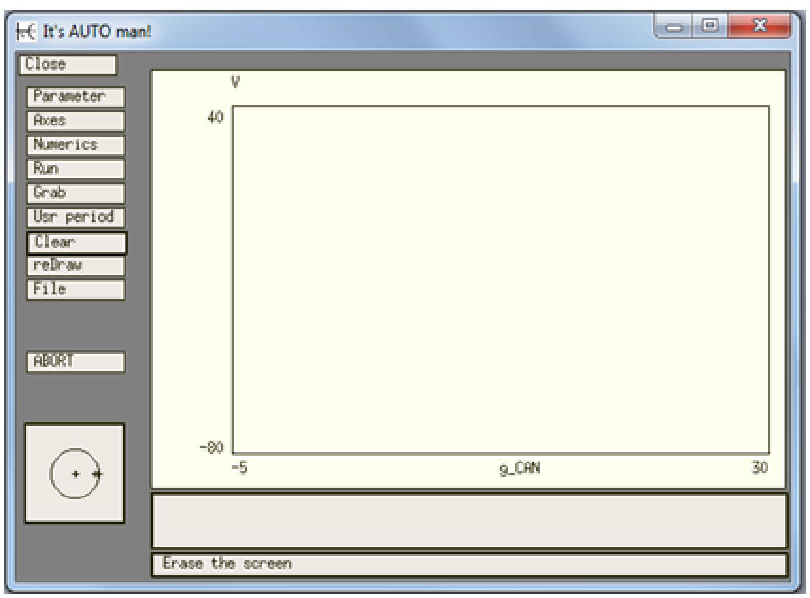


Figure 19: Click Clear button if the two points were not clear.

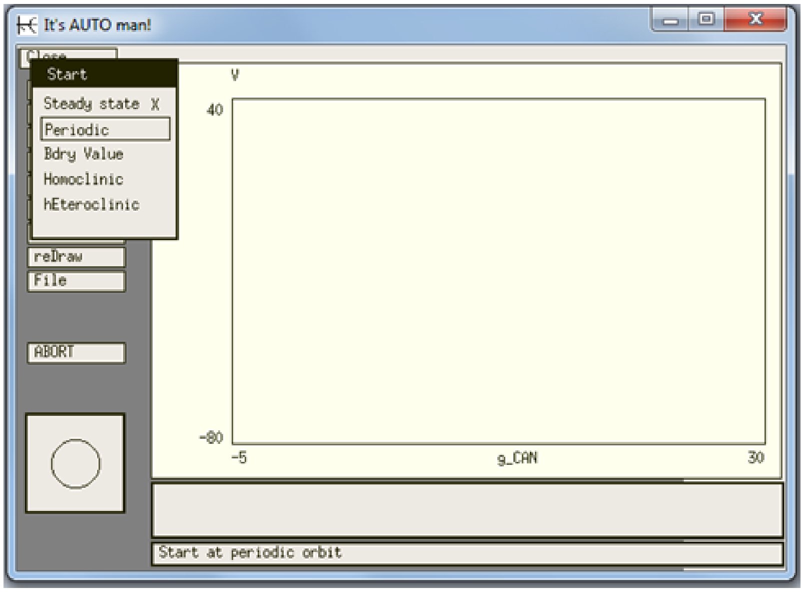


Figure 20: Click Run -> Select Periodic to run the drawing of bifurcation diagram from periodic solutions.

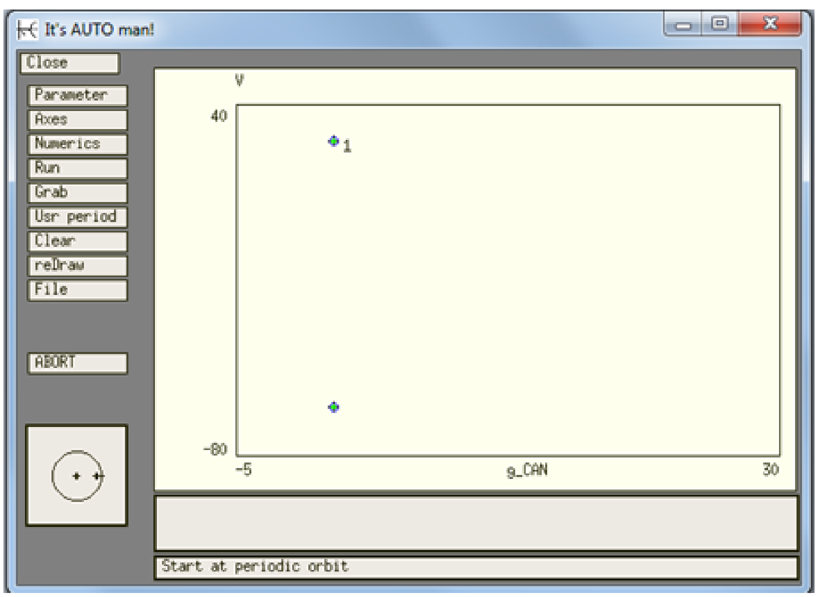


Figure 21: If everything you have done is correct this time the bifurcation diagram will be plotted.

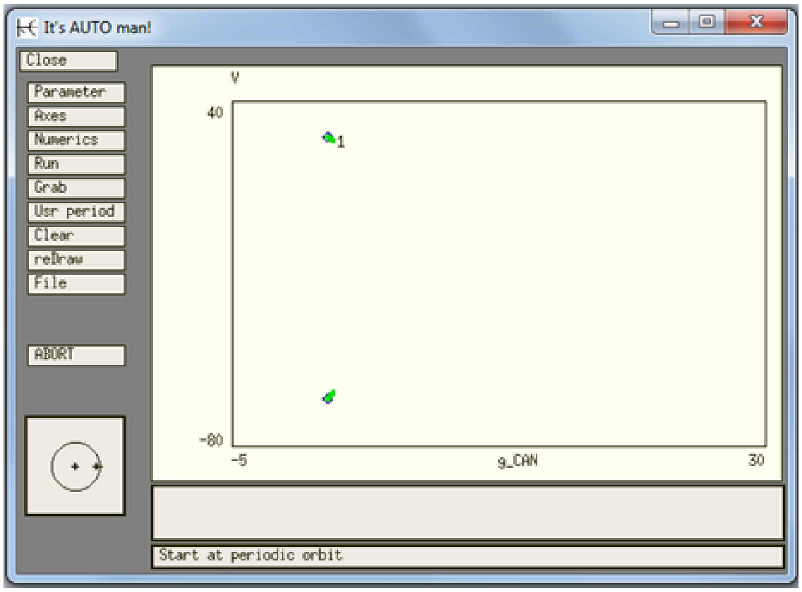


Figure 22: This diagram shows the bifurcation diagram is plotted (green points show stable limit cycle).

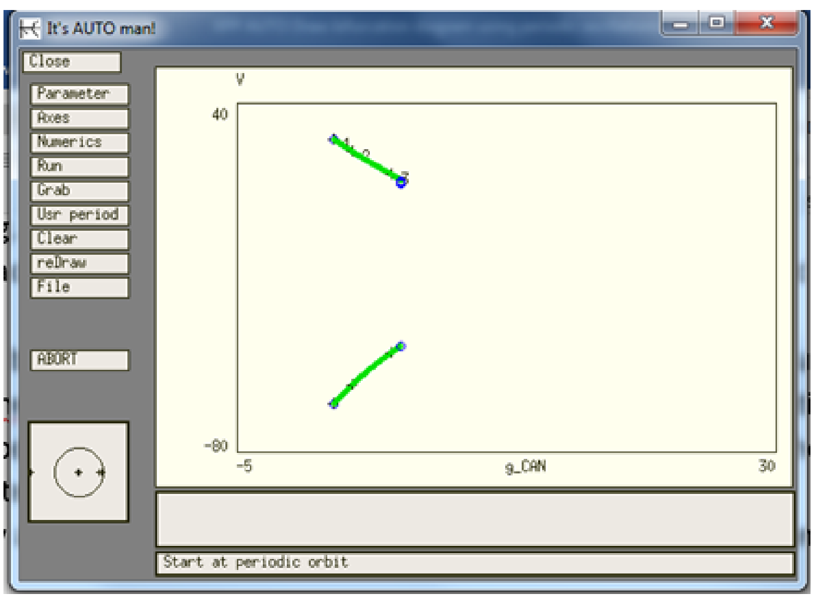


Figure 23: The bifurcation diagram was successfully plotted.

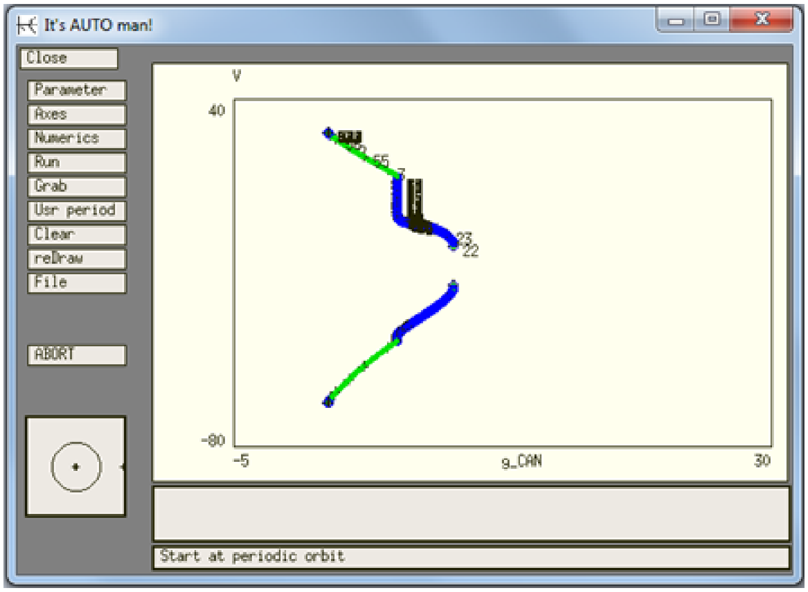


Figure 24: At the end of the plotting of the bifurcation diagram you should get the results as above. You can save the diagram using File -> Postscript and key in a filename to save the bifurcation diagram.

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