Data-driven model of port-Hamiltonian Systems with algebraic constraints

Binh Nguyen[1], Truong X. Nghiem[1]

***Abstract*—**

I. Introduction

Basic Introduction Port-Hamiltonian Systems (PHS) has been given in [1]. Survey papers on learning control techniques including reinforcement learning (RL), iterative control and for PHSs [2]–[4].

Reduced-order PHs [5], [6]

Gassian Process approach for modelling PHSs [7], [8] and then Bayesian Control for PHSs [9].

The first attemp using neural network to describe Hamiltonian systems [10]. Following the concept, [11]–[14] have applied physics-informed neural network (PINN) with well-chosen learning biases for modelling PHSs has established foundation of port-Hamiltonian neural network (PHNN).

Data-driven identificaiton of PHSs [15], [16]

A. Related Works

[16] **Pros:** can work to input-output data, interconnection of port-Hamiltonian systems (composite learning or identificaiton). **Cons:** No uncertatinty quantification. **Framework:** PHNN + composite PHSs

[11] **Pros:** can work to interconnection of port-Hamiltonian systems. **Cons:** Use state variables and no uncertainty quantification. **Framework: Framework:** PHNN + composite PHSs

[7] **Pros:** uncertatinty quantification with noised data. **Cons:** Use state variables, prior GP assumption. **Framework: Framework:** GP-PHSs

[17] focuses only on nonhomolomic systems, no constraint-preservation is given

[18], [19] Lagrangian constraints and Dirac constraints.

All above works do not consider constrained port-Hamiltonian systems.

B. Contribution

II. Constrained PHs

A. Problem statement

Definition 2.1 (Constant Dirac Structure [1]): A Dirac structure is a subspace $\mathcal{D} \subset \mathcal{S} \times \mathcal{S}^\star$ such that $\mathcal{D} = \mathcal{D}^{\perp\!\!\!\perp}$, where $^{\perp\!\!\!\perp}$ denotes the orthogonal companion with respect to the bilinear form $\langle \cdot, \cdot \rangle_+$

Definition 2.2 (Generalized pH DAE System [18]): Consider a Dirac structure $\mathcal{D} \subset \mathcal{S} \times \mathcal{S}^\star$ and a Lagrangian subspace $\mathcal{L} \subset \mathcal{S} \times \mathcal{S}^\star$. This defines the generalized

[1]The Department of Electrical and Computer Engineering, College of Engineering and Computer Science, University of Central Florida, Orlando, FL 32816, USA

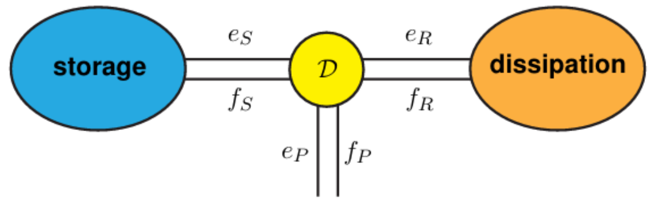

Fig. 1: Structure of Port-Hamiltonian system

port-Hamiltonian DAE system (briefly, gpH DAE system) $(\mathcal{D}, \mathcal{L})$, with dynamics given by

$$(\dot{x}, e) \in \mathcal{D}, (x, e) \in \mathcal{L} \tag{1}$$

By letting $e_S = \frac{\partial H}{\partial \mathbf{x}}$, $f_S = -\dot{\mathbf{x}}$, Consider input-state model of pHs as follows

$$\dot{\mathbf{x}} = (J - R)\frac{\partial H}{\partial \mathbf{x}} + G\mathbf{u} + \begin{bmatrix} 0 \\ A\boldsymbol{\lambda} \end{bmatrix} \tag{2}$$

$$A^\top \frac{\partial H}{\partial \mathbf{x}} = 0. \tag{3}$$

Where $\mathbf{x} \in \mathbb{R}^n$, $J = J^\top$ is a skew matrix, $R = R^\top$ stands for positive. As a special case of input-output algebraic constraints [1, Chapter 8].

From (9), $\frac{\partial H}{\partial \mathbf{x}}$ must be lie in the raw space of A, let us define a constraint set $\mathcal{X}$ as a set of all $\mathbf{x}$ satisfied (9). Let us select c basic vectors $\mathbf{s}_i$ of $\mathcal{X}$, and define matrix $B = [s_1^\top; \ldots; s_c^\top] \in \mathbb{R}^{n \times c}$ which satisfies (4).

$$BA = 0. \tag{4}$$

Problem 1: For given dataset $(\mathbf{x}, \mathbf{u})$ and assume that $A(\mathbf{x})$ is known, estimate $H(\mathbf{x})$ and parameters ϕ_J, ϕ_R, ϕ_G coressponding to matrices J, R, G such that the structure (2) and constraint (3) are presevered.

B. Proposed approach

Neural ODE with specific structures.

Define an approximation of Hamiltonian function

$$H(\mathbf{x}) = \sum_i^c NN_i(\mathbf{x})\mathbf{s}_i^\top \tag{5}$$

Design c neural network $NN_i(\mathbf{x})$ such that with $[NN_i(\mathbf{x})]_j s_{ij} \geq 0$ where $\mathbf{s}_i = [s_{ij}]_{1 \leq j \leq c}$.

III. NONHOMOLOMIC EULER-LAGRANGIAN SYSTEMS

Let us consider a mechanical system with n degrees of freedom, described by n measurable configuration variables $\mathbf{q} = [q_1, q_2, \ldots, q_n]^\top$. Accordingly, kinetic energy of the mechanical system is given by $K(\dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^\top M(\mathbf{q})\dot{\mathbf{q}}$ where $M(\mathbf{q}) = [M_{ij}(\mathbf{q})] \in \mathbb{R}^{n\times n}$ denotes the generalized mass matrix. In the light of [], we define the Lagrangian function $L(\mathbf{q}, \dot{\mathbf{q}})$ in the following form

$$L(\mathbf{q}, \dot{\mathbf{q}}) = K(\dot{\mathbf{q}}) - U(\mathbf{q}) \tag{6}$$

where $U(\mathbf{q})$ stands for potential energy of the mechanical system. This paper assume that there are nonhomolomic constraints of velocity $\dot{\mathbf{q}}$ as

$$A^\top(\mathbf{q})\dot{\mathbf{q}} = 0 \tag{7}$$

Euler-Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial L}{\partial \mathbf{q}} = A(\mathbf{q})\boldsymbol{\lambda} + B(\mathbf{q})\mathbf{u} \tag{8}$$

Defining the generalized momenta:

$$\mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{q}}} = M(\mathbf{q})\dot{\mathbf{q}} \tag{9}$$

Constrained Port-Hamiltonian is given by

$$\dot{\mathbf{q}} = \frac{\partial H}{\partial \mathbf{p}}, \tag{10}$$

$$\dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{q}} + A(\mathbf{q})\boldsymbol{\lambda} + B(\mathbf{q})\mathbf{u}, \tag{11}$$

$$0 = A^\top(\mathbf{q})\frac{\partial H}{\partial \mathbf{p}}(\mathbf{p}, \mathbf{q}) \tag{12}$$

with $H(\mathbf{q}, \mathbf{p}) = \frac{1}{2}\mathbf{p}^\top M^{-1}(\mathbf{q})\mathbf{p} + U(\mathbf{q})$.

Assumption:

Problem 2: From data $\{\mathbf{q}^\tau\}_{\tau\geq 0}$ and $\{\dot{\mathbf{q}}^\tau\}_{\tau\geq 0}$ up to time t ($\tau \leq t$), learn parameters ϕ_M in matrix M and provides predictions (for $\tau > t$) such that the nonhomolomic constraints (9) is satisfied.

IV. APPROACH

REFERENCES

[1] A. van der Schaft and D. Jeltsema, "Port-Hamiltonian Systems Theory: An Introductory Overview," *Foundations and Trends® in Systems and Control*, vol. 1, no. 2-3, pp. 173–378, Jun. 2014.

[2] S. P. Nageshrao, G. A. D. Lopes, D. Jeltsema, and R. Babuška, "Port-Hamiltonian Systems in Adaptive and Learning Control: A Survey," *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1223–1238, May 2016.

[3] R. Rashad, F. Califano, A. J. van der Schaft, and S. Stramigioli, "Twenty years of distributed port-Hamiltonian systems: A literature review," *IMA Journal of Mathematical Control and Information*, vol. 37, no. 4, pp. 1400–1422, Dec. 2020.

[4] K. Cherifi, "An overview on recent machine learning techniques for Port Hamiltonian systems," *Physica D: Nonlinear Phenomena*, vol. 411, p. 132620, Oct. 2020.

[5] Y. Wu, B. Hamroun, Y. Le Gorrec, and B. Maschke, "Reduced Order LQG Control Design for Infinite Dimensional Port Hamiltonian Systems," *IEEE Transactions on Automatic Control*, vol. 66, no. 2, pp. 865–871, Feb. 2021.

[6] P. Schwerdtner, T. Moser, V. Mehrmann, and M. Voigt, "Optimization-based model order reduction of port-Hamiltonian descriptor systems," *Systems & Control Letters*, vol. 182, p. 105655, Dec. 2023.

[7] T. Beckers, J. Seidman, P. Perdikaris, and G. J. Pappas, "Gaussian Process Port-Hamiltonian Systems: Bayesian Learning with Physics Prior," in *2022 IEEE 61st Conference on Decision and Control (CDC)*, Dec. 2022, pp. 1447–1453.

[8] T. Beckers, T. Z. Jiahao, and G. J. Pappas, "Learning Switching Port-Hamiltonian Systems with Uncertainty Quantification," *IFAC-PapersOnLine*, vol. 56, no. 2, pp. 525–532, Jan. 2023.

[9] T. Beckers, "Data-Driven Bayesian Control of Port-Hamiltonian Systems," in *2023 62nd IEEE Conference on Decision and Control (CDC)*, Dec. 2023, pp. 8708–8713.

[10] S. Greydanus, M. Dzamba, and J. Yosinski, "Hamiltonian Neural Networks," in *Advances in Neural Information Processing Systems*, vol. 32. Curran Associates, Inc., 2019.

[11] C. Neary and U. Topcu, "Compositional Learning of Dynamical System Models Using Port-Hamiltonian Neural Networks," in *Proceedings of The 5th Annual Learning for Dynamics and Control Conference*. PMLR, Jun. 2023, pp. 679–691.

[12] S. Eidnes, A. J. Stasik, C. Sterud, E. Bøhn, and S. Riemer-Sørensen, "Pseudo-Hamiltonian Neural Networks with State-Dependent External Forces," Jan. 2023.

[13] S. A. Desai, M. Mattheakis, D. Sondak, P. Protopapas, and S. J. Roberts, "Port-Hamiltonian neural networks for learning explicit time-dependent dynamical systems," *Physical Review E*, vol. 104, no. 3, p. 034312, Sep. 2021.

[14] T. Duong, A. Altawaitan, J. Stanley, and N. Atanasov, "Port-Hamiltonian Neural ODE Networks on Lie Groups for Robot Dynamics Learning and Control," *IEEE Transactions on Robotics*, vol. 40, pp. 3695–3715, 2024.

[15] J. Rettberg, J. Kneifl, J. Herb, P. Buchfink, J. Fehr, and B. Haasdonk, "Data-driven identification of latent port-Hamiltonian systems," Aug. 2024.

[16] G. J. E. van Otterdijk, S. Moradi, S. Weiland, R. Tóth, N. O. Jaensson, and M. Schoukens, "Learning Subsystem Dynamics in Nonlinear Systems via Port-Hamiltonian Neural Networks," Nov. 2024.

[17] A. Altawaitan, J. Stanley, S. Ghosal, T. Duong, and N. Atanasov, "Hamiltonian Dynamics Learning from Point Cloud Observations for Nonholonomic Mobile Robot Control," in *2024 IEEE International Conference on Robotics and Automation (ICRA)*, May 2024, pp. 16 937–16 944.

[18] A. van der Schaft and B. Maschke, "Generalized port-Hamiltonian DAE systems," *Systems & Control Letters*, vol. 121, pp. 31–37, Nov. 2018.

[19] ——, "Dirac and Lagrange Algebraic Constraints in Nonlinear Port-Hamiltonian Systems," *Vietnam Journal of Mathematics*, vol. 48, no. 4, pp. 929–939, Dec. 2020.