Lecture 15 – Thresholding（阈值处理）

This lecture will cover:

– Global thresholding (全局阈值处理)

- Basic global thresholding
- Optimum global thresholding using Otsu's method
- Improve global thresholding by using image smoothing
- Improve global thresholding by using edges
- Multiple thresholds

– Variable thresholding（可变阈值处理）

- Image partitioning（图像分块）
- Variable thresholding based on local image properties
- Using moving average（移动平均）

Foundation

➢ **Definition**

$$g(x,y)=\begin{cases}1, & f(x,y)>T \quad \text{(object points)}\\ 0, & f(x,y)\le T \quad \text{(background points)}\end{cases}$$

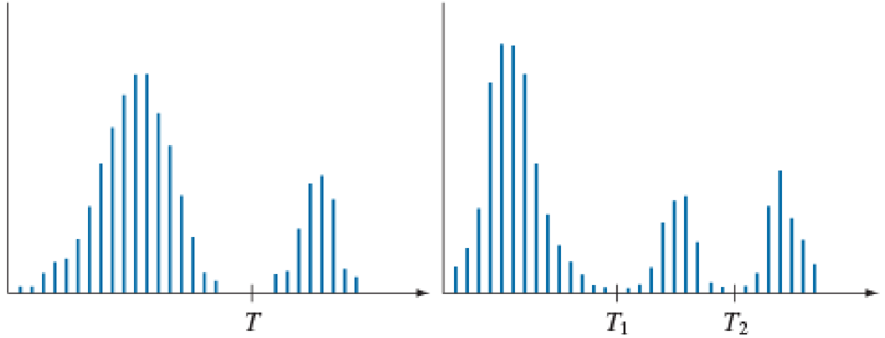
Where

- *Global Thresholding* **(全局阈值处理)** if T is constant over an entire image
- *Variable/Local/Regional Thresholding* **(可变/局域/区域阈值处理)** if T changes over an image
- *Dynamic/Adaptive Thresholding* **(动态/自适应阈值处理)** if T depends on spatial coordinates (x,y)
- *Multiple Thresholding* **(多阈值处理):**

$$g(x,y)=\begin{cases}a, & f(x,y)>T_2\\ b, & T_1<f(x,y)\le T_2\\ c, & f(x,y)\le T_1\end{cases}$$

a b

FIGURE 10.32 Intensity histograms that can be partitioned (a) by a single threshold, and (b) by dual thresholds.

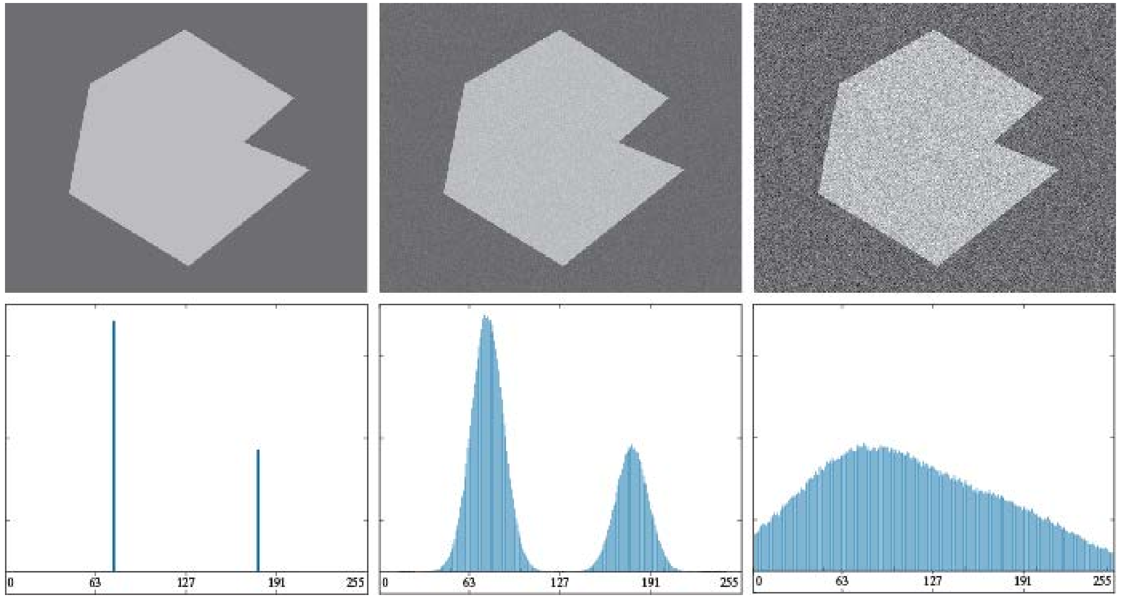


➢ **Matlab function:** BW = im2bw(I,level)

Influence on thresholding

Key factors affecting the properties of the intensity valley which separate the histogram modes

- The separation between peaks
- The noise content in the image
- The relative sizes of objects and background
- The uniformity of illumination source
- The uniformity of reflectance properties of the image

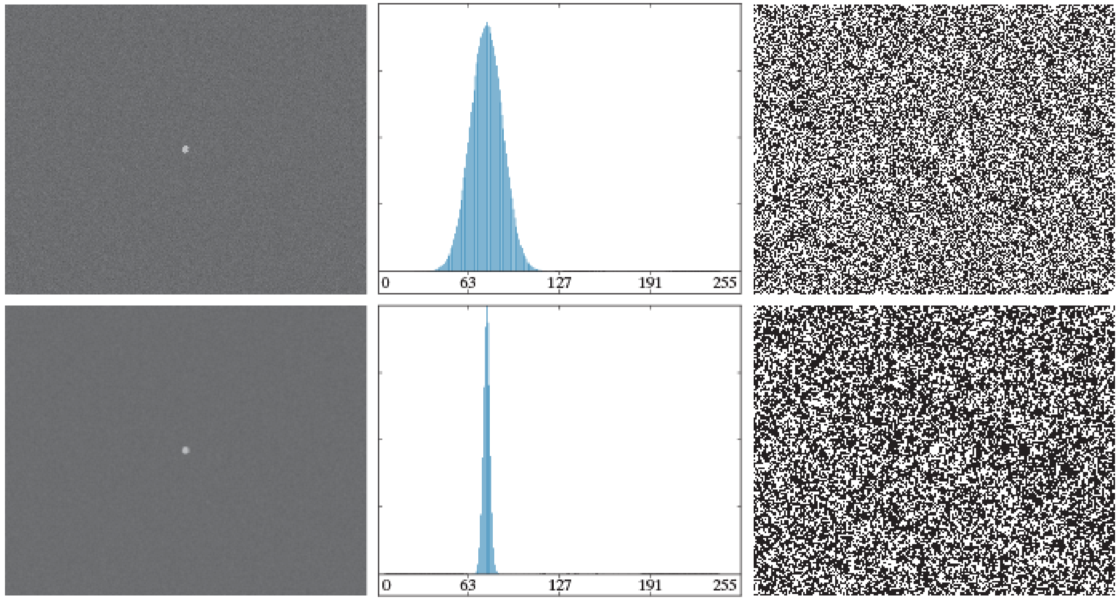


a b c
d e f

FIGURE 10.33 (a) Noiseless 8-bit image. (b) Image with additive Gaussian noise of mean 0 and standard deviation of 10 intensity levels. (c) Image with additive Gaussian noise of mean 0 and standard deviation of 50 intensity levels. (d) through (f) Corresponding histograms.

Influence on thresholding

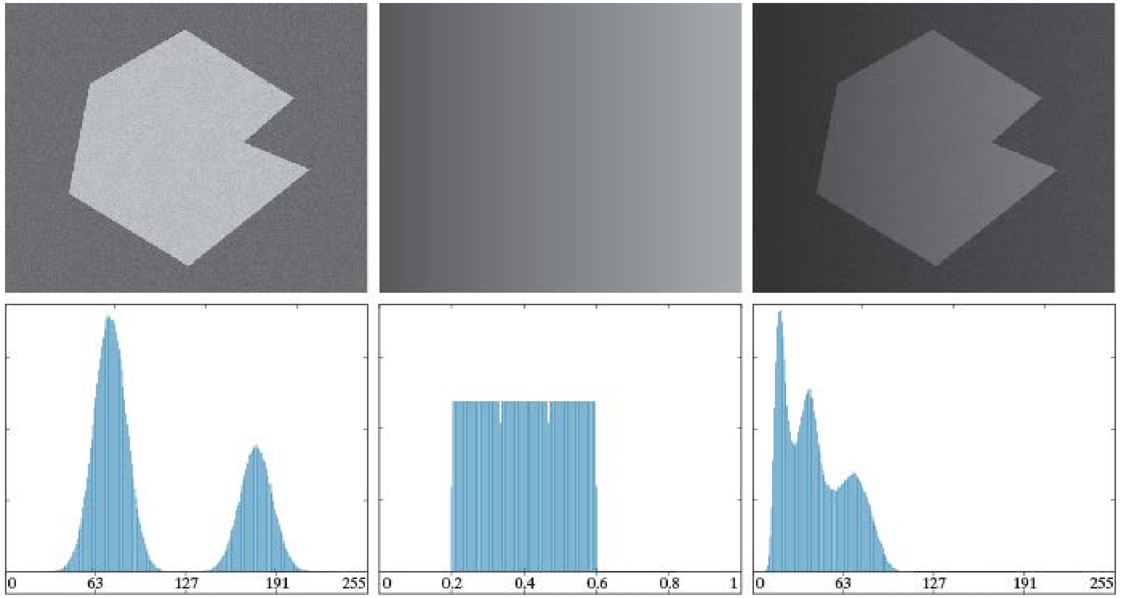
- Objects and background sizes



a b c
d e f

FIGURE 10.38 (a) Noisy image and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5×5 averaging kernel and (e) its histogram. (f) Result of thresholding using Otsu's method. Thresholding failed in both cases to extract the object of interest. (See Fig. 10.39 for a better solution.)

- Illumination and Reflection



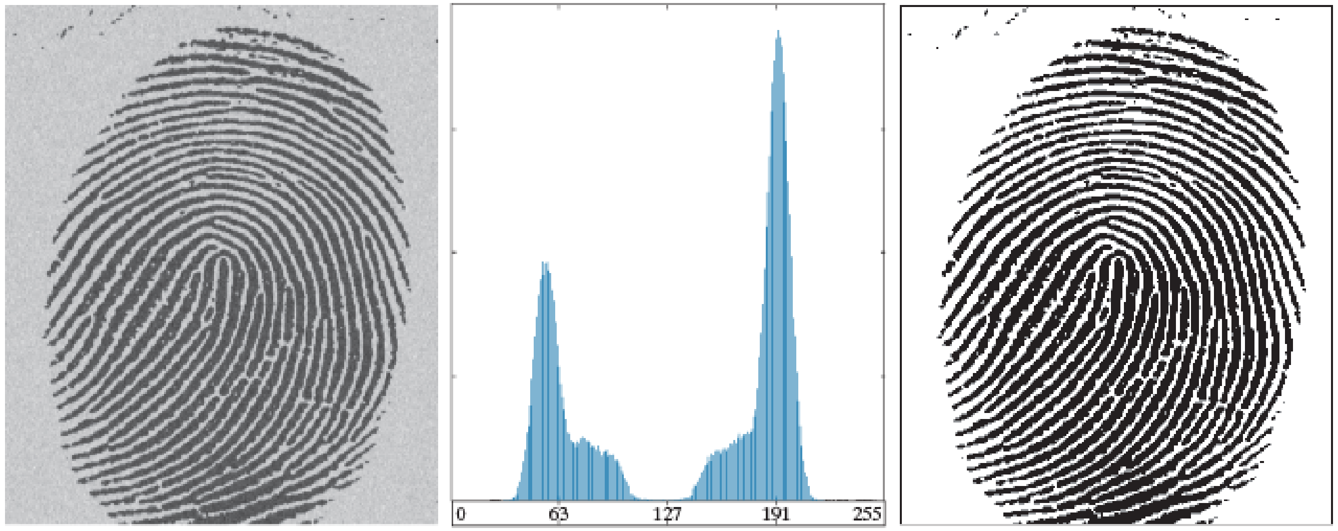
a b c
d e f

FIGURE 10.34 (a) Noisy image. (b) Intensity ramp in the range [0.2, 0.6]. (c) Product of (a) and (b). (d) through (f) Corresponding histograms.

Basic Global Thresholding

- **Steps:**

1. Select an initial estimate of the global threshold **T**;
2. Segment the image using **T** to two groups $G_1(>T)$ and $G_2(\leqslant T)$;
3. Compute average intensity m_1 and m_2 for G_1 and G_2 respectively;
4. Compute new threshold $\mathbf{T}=(m_1 + m_2)/2$;
5. Repeat 2-4 until the difference between **T** in successive iteration is smaller than requirement.



a b c

FIGURE 10.35 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (thin image border added for clarity). (Original image courtesy of the National Institute of Standards and Technology.).

Otsu's Method

- **Between-class variance (类间方差):**

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 = P_1P_2(m_1 - m_2)^2 = \frac{(m_G P_1 - m)^2}{P_1(1 - P_1)}$$

Where

m_1: the mean intensity up to level k

m_G: average intensity of entire image, and $m_G = P_1 m_1 + P_2 m_2$

P_1: the cumulative probability of all pixels in the intensity range [0,k], and $P_1 + P_2 = 1$

m : the accumulative mean up to level k, $m = P_1 m_1$

- **Otsu's Method**
  - The best threshold giving the best separation, i.e. the maximum between-class variance;
  - Performed on the histogram of an image

- **Matlab function:** [level,EM] = graythresh(I)

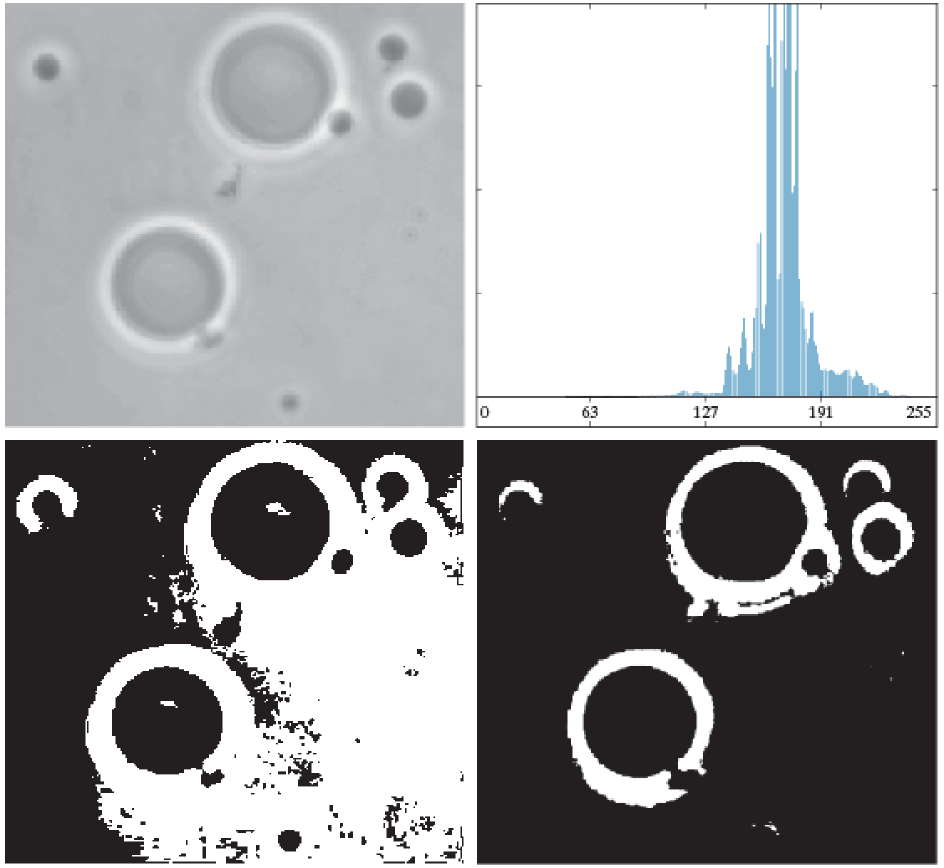
Otsu's Method

- **Algorithm summary:**

1. compute the normalized histogram of the input image p_i;
2. compute the cumulative sums $P_1(k) = \sum_{i=0}^{k} p_i$;
3. compute the cumulative means $m(k) = \sum_{i=0}^{k} ip_i$
4. compute the global intensity mean $m_G = \sum_{i=0}^{L-1} ip_i$
5. compute between-class variance σ_B^2 for $k = 0,1,\cdots,L-1$;

$$\sigma_B^2 = \frac{(m_G P_1 - m)^2}{P_1(1-P_1)}$$

6. obtain the Otsu threshold k^* when $\sigma_B^2(k^*)$ is the maximum of all k value
7. obtain the separability measure $\eta^* = \frac{\sigma_B^2(k^*)}{\sigma_G^2}$

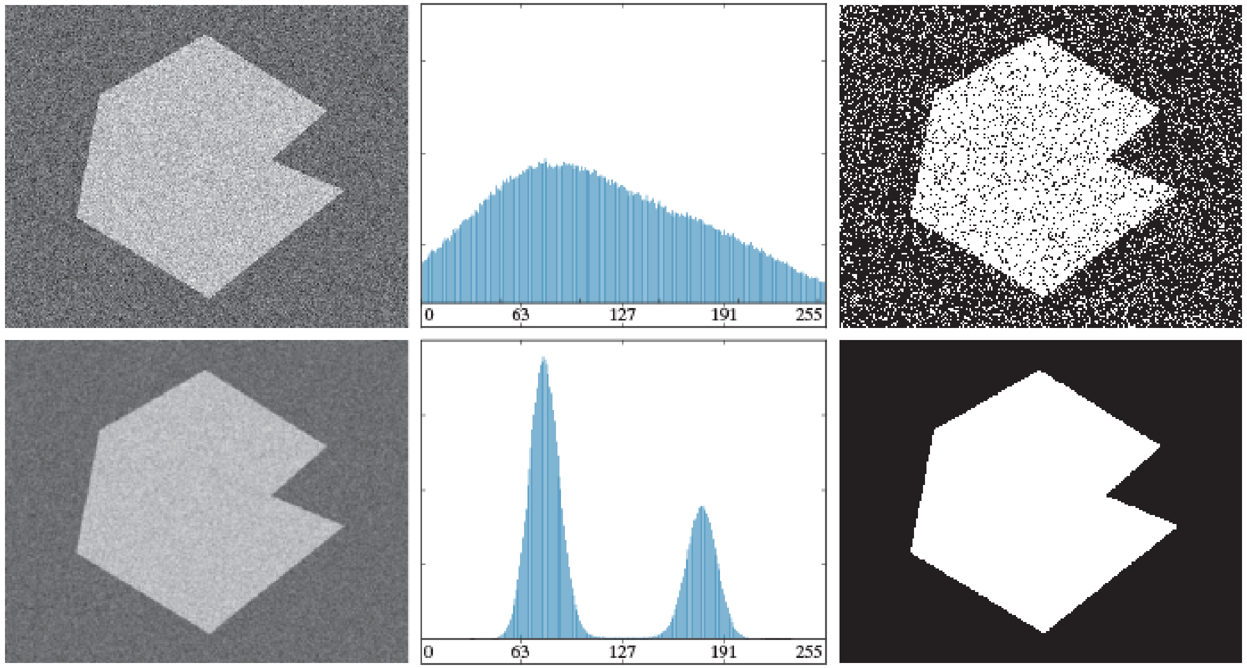


a b
c d

FIGURE 10.36 (a) Original image. (b) Histogram (high peaks were clipped to highlight details in the lower values). (c) Segmentation result using the basic global algorithm from Section 10.3. (d) Result using Otsu's method. (Original image courtesy of Professor Daniel A. Hammer, the University of Pennsylvania.)

Improve Global Thresholding

- **Using image smoothing:**
  - Smoothing the image prior to thresholding to reduce the noise;
  - The more aggressively smoothing the image, the more boundary errors.



a b c
d e f

FIGURE 10.37 (a) Noisy image from Fig. 10.33(c) and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging kernel and (e) its histogram. (f) Result of thresholding using Otsu's method.

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Improve Global Thresholding

- **Using edges:**

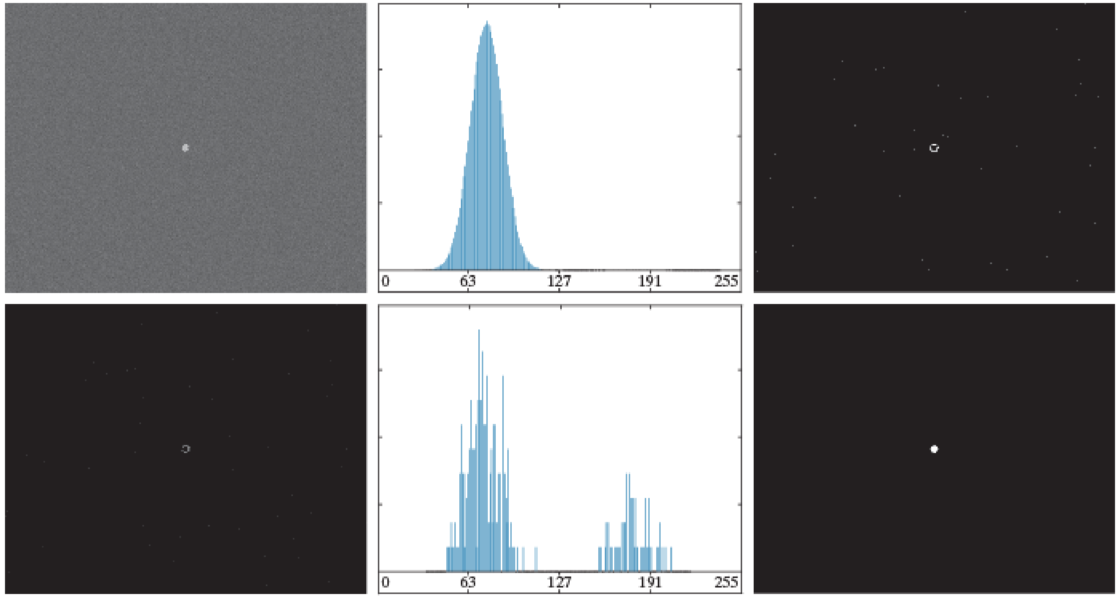
1. compute an edge image from the input image $f(x,y)$ using any edge detector;

2. specify a threshold value T;

3. Threshold the edge image using T to produce a binary image $g_T(x,y)$

4. compute a histogram using only the pixels in $f(x,y)$ that correspond to the locations of the 1-valued pixels in $g_T(x,y)$

5. use the histogram to segment $f(x,y)$;

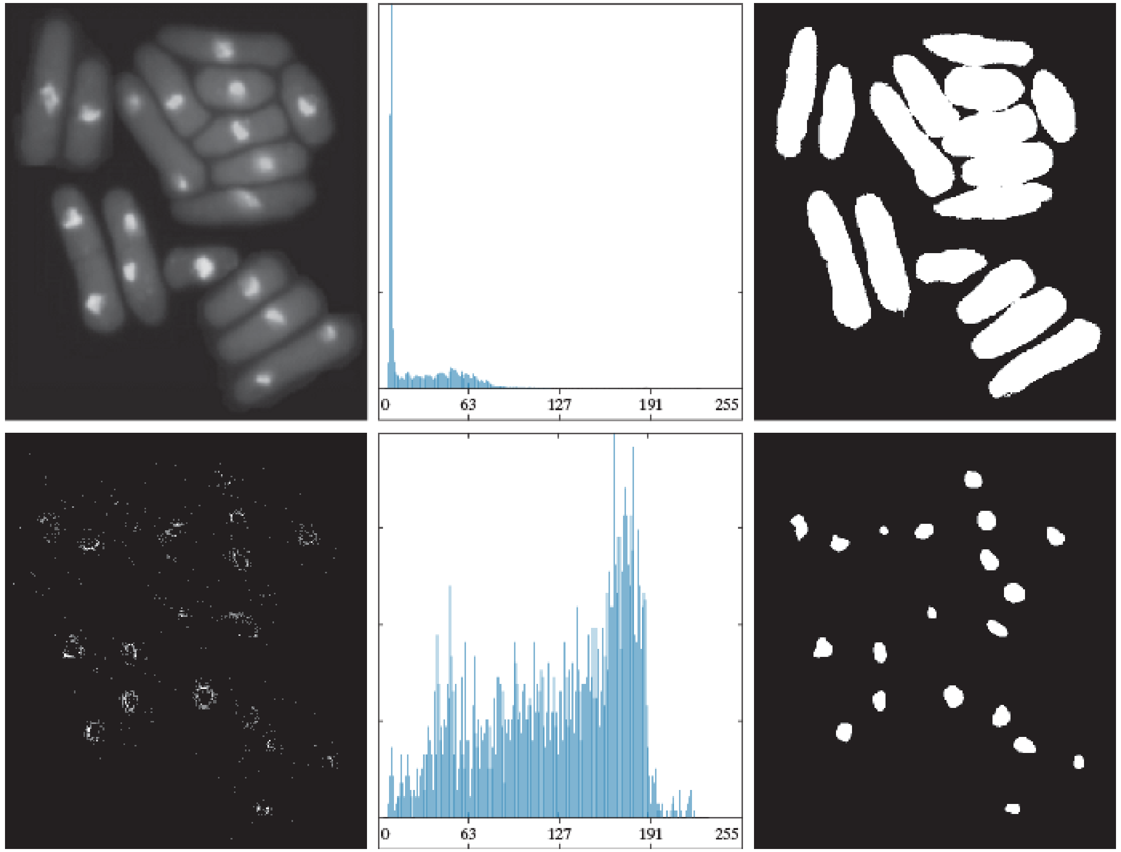


a b c
d e f

FIGURE 10.39 (a) Noisy image from Fig. 10.38(a) and (b) its histogram. (c) Mask image formed as the gradient magnitude image thresholded at the 99.7 percentile. (d) Image formed as the product of (a) and (c). (e) Histogram of the nonzero pixels in the image in (d). (f) Result of segmenting image (a) with the Otsu threshold based on the histogram in (e). The threshold was 134, which is approximately midway between the peaks in this histogram.

Improve Global Thresholding

- **Using edges:** apply the Laplacian edge detector



a b c
d e f

FIGURE 10.40 (a) Image of yeast cells. (b) Histogram of (a). (c) Segmentation of (a) with Otsu's method using the histogram in (b). (d) Mask image formed by thresholding the absolute Laplacian image. (e) Histogram of the non-zero pixels in the product of (a) and (d). (f) Original image thresholded using Otsu's method based on the histogram in (e). (Original image courtesy of Professor Susan L. Forsburg, University of Southern California.)

Multiple thresholds

- **Between-class variance (类间方差):**

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 + P_3(m_3 - m_G)^2$$

Where

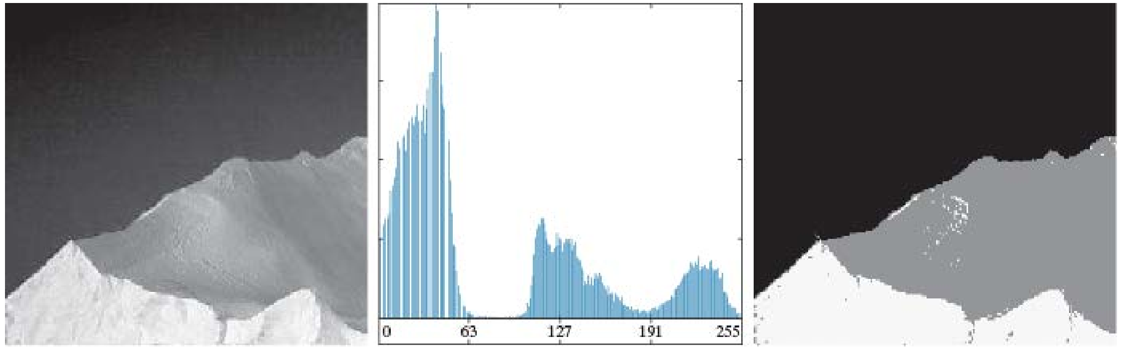
$$P_1 = \sum_{i=0}^{k_1} p_i \qquad P_2 = \sum_{i=k_1+1}^{k_2} p_i \qquad P_3 = \sum_{i=k_2+1}^{L-1} p_i$$

$$m_1 = \sum_{i=0}^{k_1} ip_i \qquad m_2 = \sum_{i=k_1+1}^{k_2} ip_i \qquad m_3 = \sum_{i=k_2+1}^{L-1} ip_i$$

$$P_1 m_1 + P_2 m_2 + P_3 m_3 = m_G \qquad P_1 + P_2 + P_3 = 1$$

The two optimum thresholds k_1^* and k_2^* are the values that maximize $\sigma_B^2(k_1, k_2)$, then

$$g(x,y) = \begin{cases} a, & f(x,y) \le k_1^* \\ b, & k_1^* < f(x,y) \le k_2^* \\ c, & f(x,y) > k_2^* \end{cases} \quad \text{and} \quad \eta(k_1^*, k_2^*) = \frac{\sigma_B^2(k_1^*, k_2^*)}{\sigma_G^2}$$

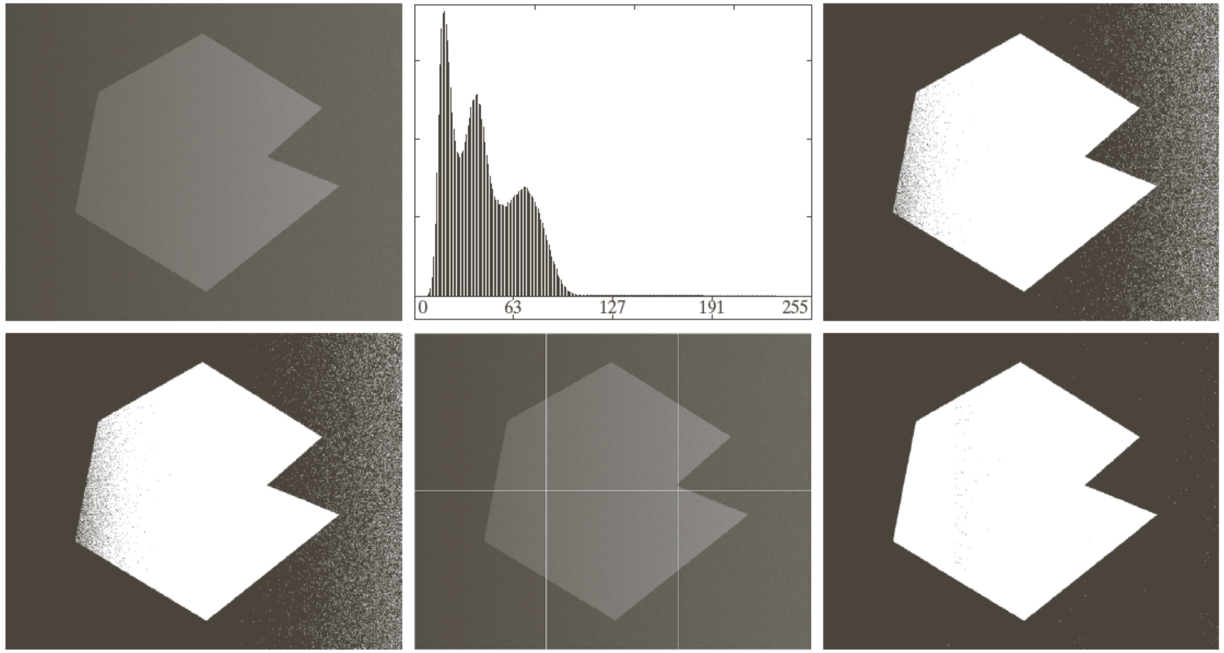


a b c

FIGURE 10.42 (a) Image of an iceberg. (b) Histogram. (c) Image segmented into three regions using dual Otsu thresholds. (Original image courtesy of NOAA.)

Variable thresholding

- **Image partitioning（图像分块）**
  - Subdivide an image into non-overlapping rectangles;
  - To compensate for non-uniformities in illumination and/or reflectance;
  - Small rectangles so that the illumination of each is approximately uniform;



a b c
d e f

FIGURE 10.46 (a) Noisy, shaded image and (b) its histogram. (c) Segmentation of (a) using the iterative global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (e) Image subdivided into six subimages. (f) Result of applying Otsu's method to each subimage individually.

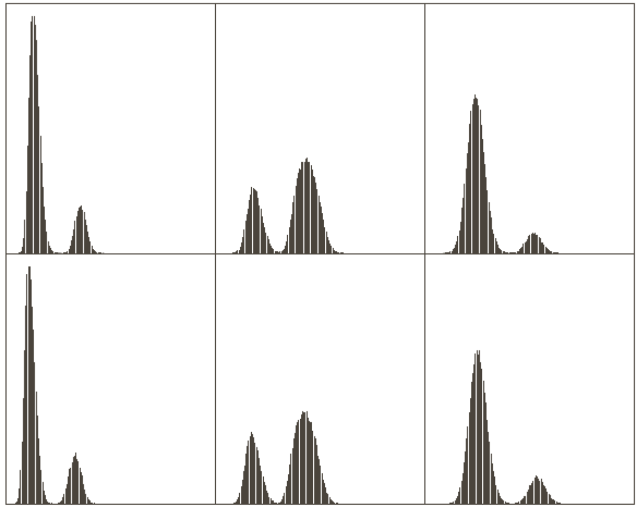


FIGURE 10.47 Histograms of the six subimages in Fig. 10.46(e).

Variable thresholding

- **Based on local image properties**
  - **Algorithm**

$$T_{xy} = a\sigma_{xy} + bm_{xy} \quad \text{or} \quad T_{xy} = a\sigma_{xy} + bm_G$$

where a, b: non-negative constants

m_{xy}: mean of pixel values in a neighborhood S_{xy}

σ_{xy}: STD of pixel values in a neighborhood S_{xy}

m_G: global image mean

The segmented image is computed as

$$g(x,y) = \begin{cases} 1, & f(x,y) > T_{xy} \\ 0, & f(x,y) \le T_{xy} \end{cases}$$

  - **Matlab function**

g = stdfilt(f, nhood);

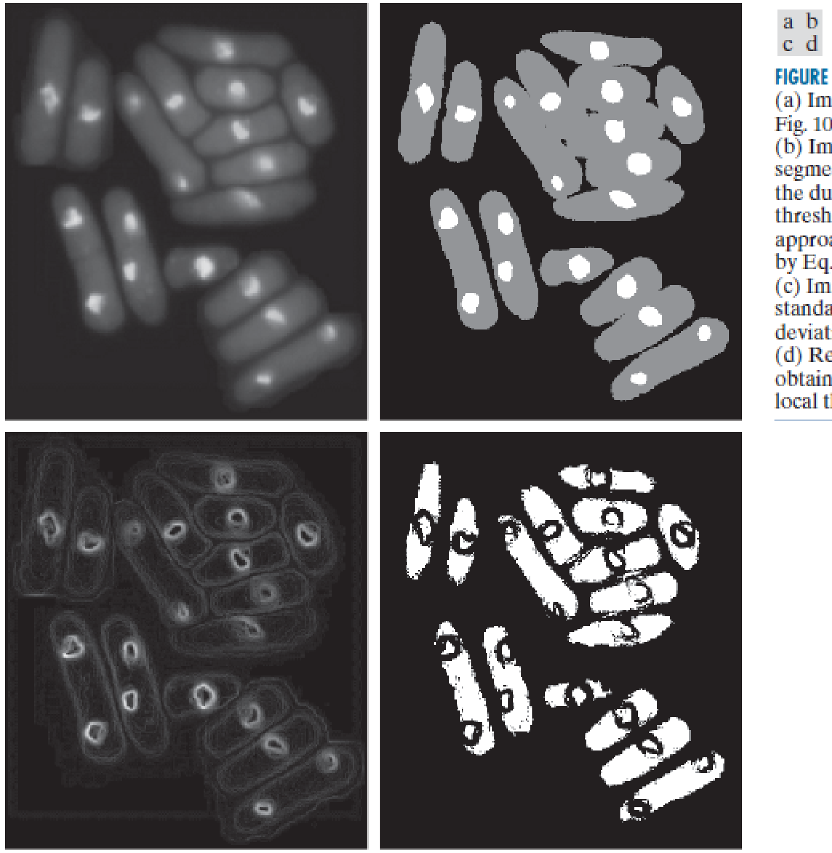


FIGURE 10.43 (a) Image from Fig. 10.40. (b) Image segmented using the dual thresholding approach given by Eq. (10-76). (c) Image of local standard deviations. (d) Result obtained using local thresholding.

Variable thresholding

➢ **Moving average（移动平均）**

- Algorithm

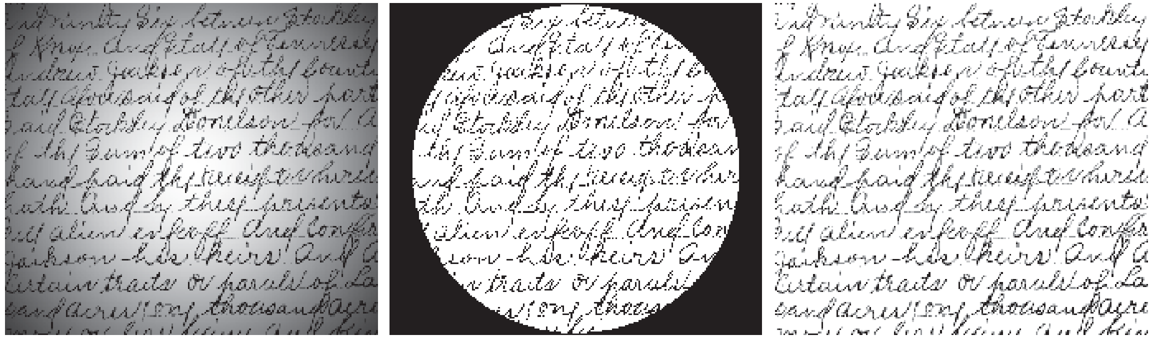
$$m(k+1) = \frac{1}{n} \sum_{i=k+2-n}^{k+1} z_i$$
$$= m(k) + \frac{1}{n}(z_{k+1} - z_{k-n})$$

Where

z_{k+1}: the intensity of the point encountered in the scanning sequence at step $k+1$

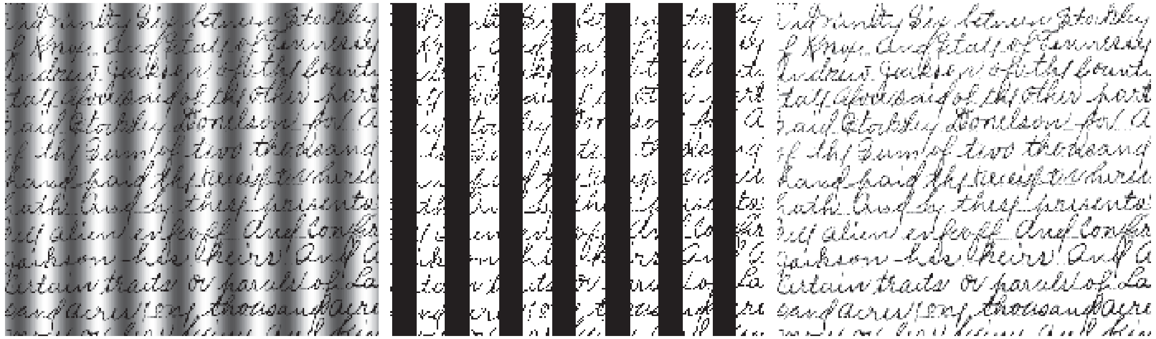
n: the number of points used in computing the average

- Applied to the case that the objects of interest are small with respect to the image size.



a b c

FIGURE 10.44 (a) Text image corrupted by spot shading. (b) Result of global thresholding using Otsu's method. (c) Result of local thresholding using moving averages.



a b c

FIGURE 10.45 (a) Text image corrupted by sinusoidal shading. (b) Result of global thresholding using Otsu's method. (c) Result of local thresholding using moving averages..

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