Lecture 6 – Histogram （直方图）

This lecture will cover:

– Basis of histogram processing

– Histogram processing (直方图处理)

- ✓ Histogram Equalization (直方图均衡)
- ✓ Histogram Matching (直方图匹配)
- ✓ Exact histogram Matching (精确直方图匹配)

– Local histogram processing (局部直方图处理)

– Histogram Statistics for Image Enhancement (直方图统计)

– Color histogram processing (彩色直方图处理)

Definition

$$h(r_k) = n_k$$

Where r_k : the *k*th intensity value in the level range of [0, L-1]

n_k : the number of pixels in the image with intensity r_k

Normalized Histogram（归一化直方图）

$$p(r_k) = \frac{n_k}{MN}$$

Where $p(r_k)$: the probability of occurrence of intensity r_k in an image

M, N: the row and column dimensions of the image

Basic Image Type

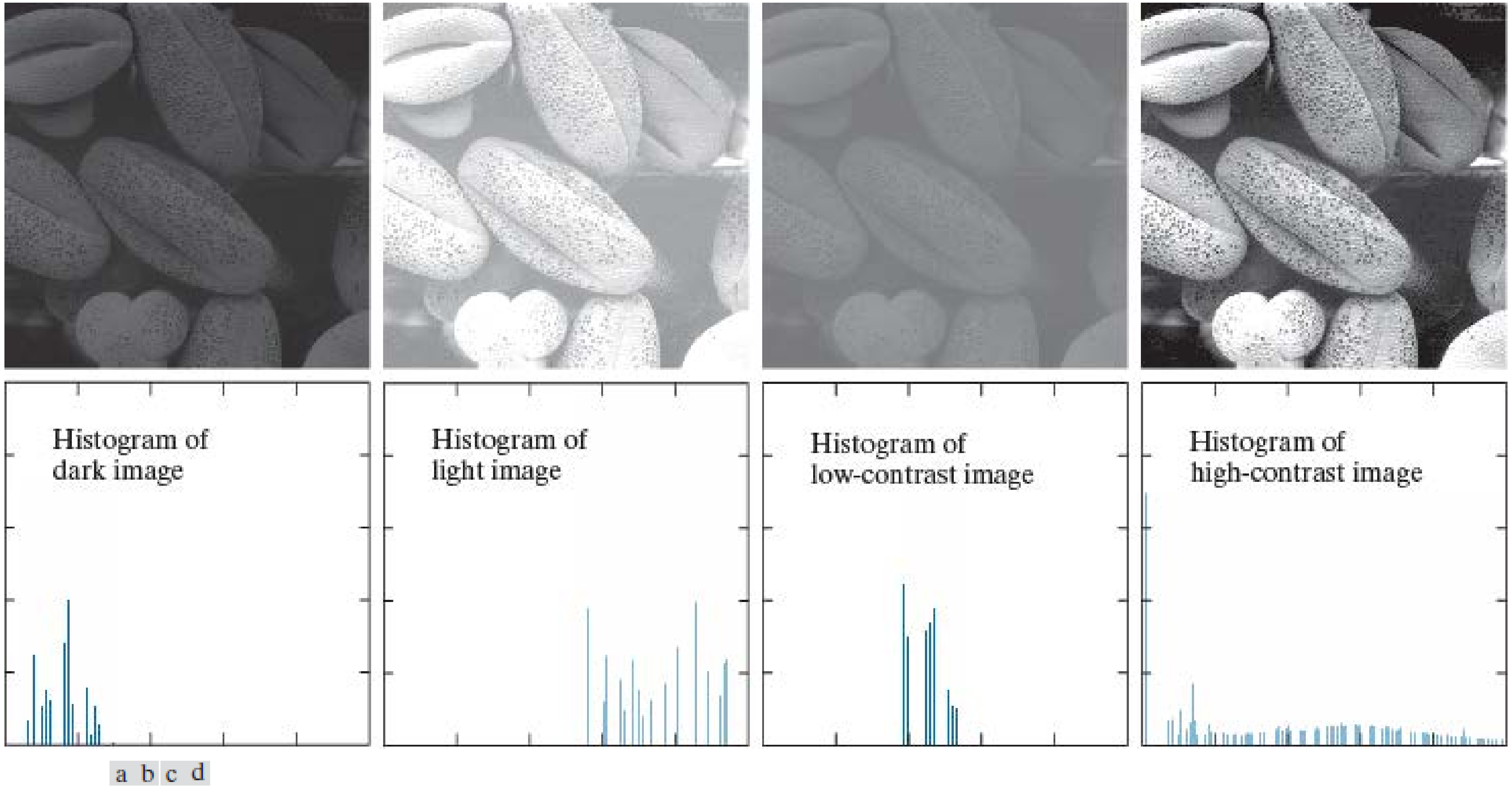


FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

Properties

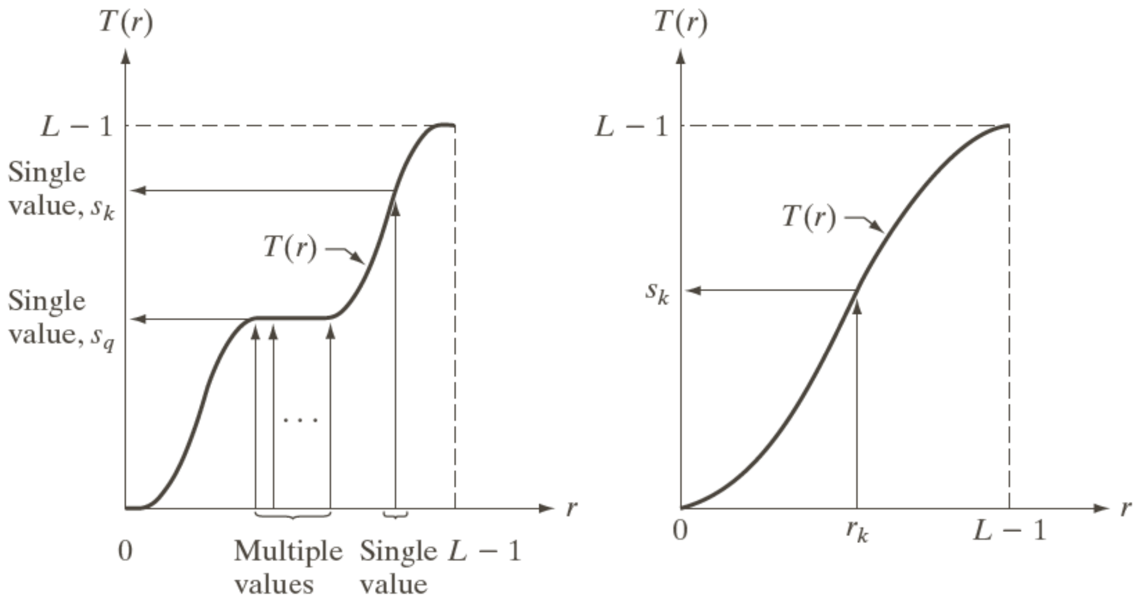
The histogram of an image

- describe the number or probability of intensity **(NO location (spatial) information)**
- can be same as other images
- $\sum_0^{L-1} n_k = M \cdot N$ or $\sum_0^1 p(r_k) = 1$
- If Region C=A∪B, A and B are disjoint, $H_C = H_A + H_B$

Basis of Histogram Processing

Given intensity transformation $s = T(r)$, where $T(r)$

- $T(r)$ is strictly monotonically increasing function （严格单调递增函数， $T(r_2) > T(r_1)$ if $r_2 > r_1$） in the interval $0 \le r \le L-1$
- $0 \le T(r) \le L-1$ for $0 \le r \le L-1$



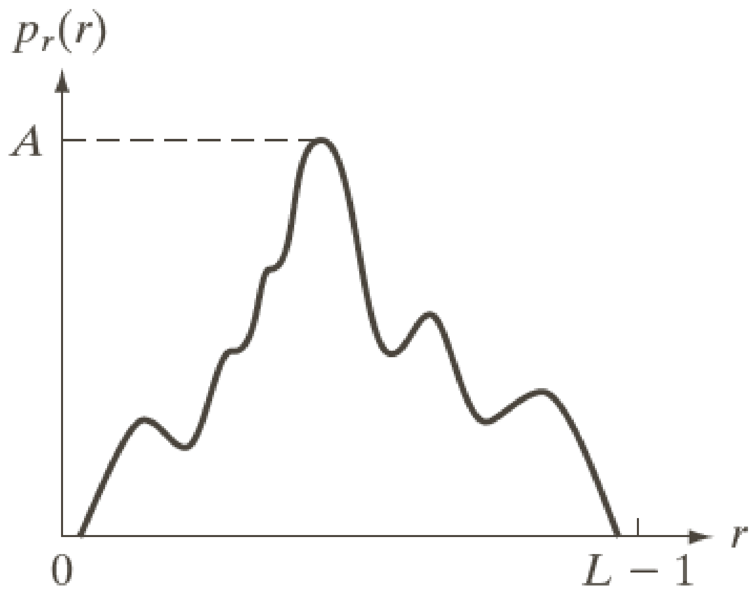
Then

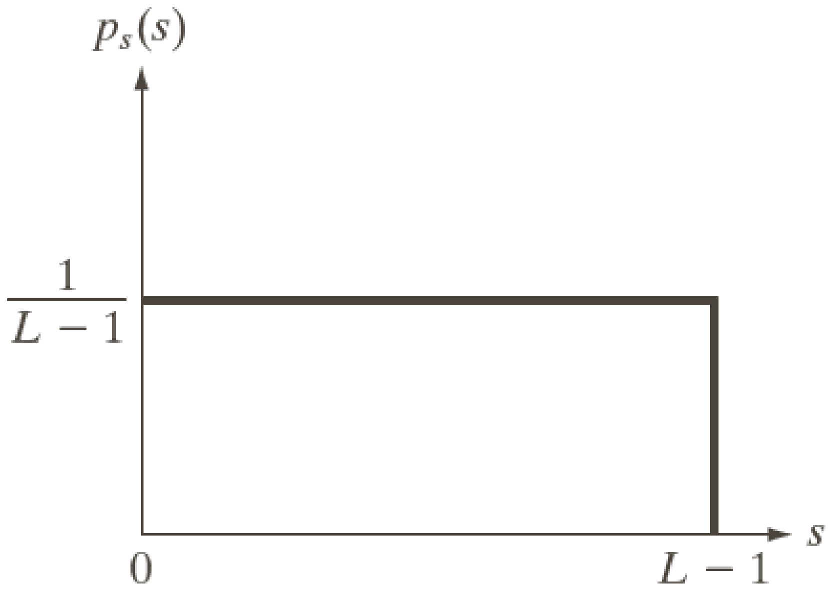
- ➢ The inverse transform $r = T^{-1}(s)$
- ➢ The probability density function (PDF, 概率密度函数) of s is

$$p_s(s) = p_r(r) \cdot \frac{dr}{ds}$$

Histogram Equalization

- **Transformation function :** $s = T(r) = (L-1)\int_0^r p_r(w)dw$
- **Uniform Probability density function :** $p_s(s) = \frac{1}{L-1}$





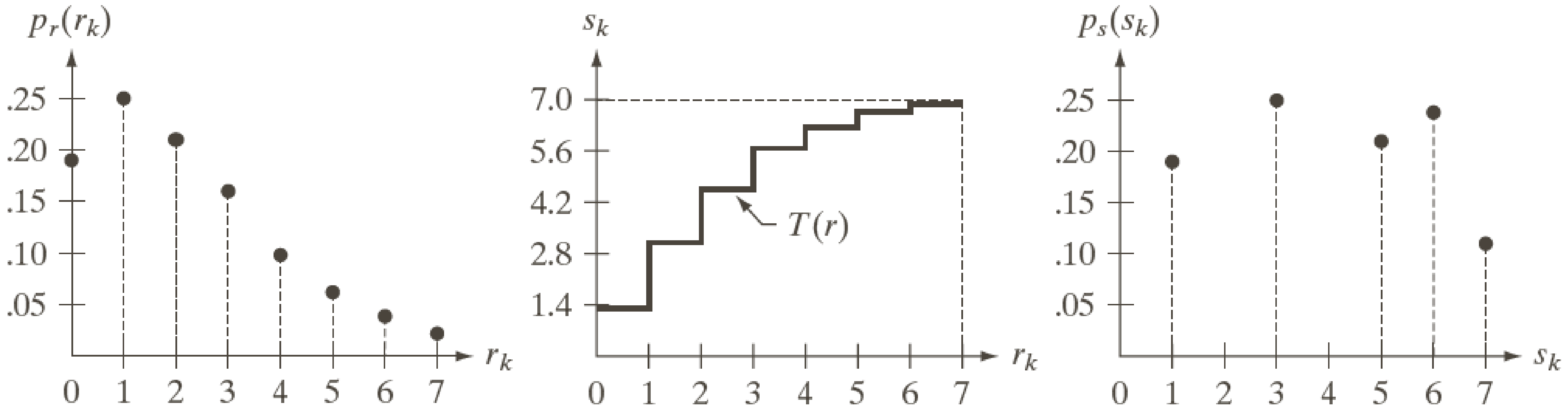
Histogram Equalization

$$s = T(r) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k}\frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j$$

K	r_k	n_k	$p(r_k)$	s_k		s_k	$p(s_k)$
0	0	790	0.19	1.33	1	0	0
1	1	1023	0.25	3.08	3	1	0.19
2	2	850	0.21	4.55	5	2	0
3	3	656	0.16	5.67	6	3	0.25
4	4	329	0.08	6.23	6	4	0
5	5	245	0.06	6.65	7	5	0.21
6	6	122	0.03	6.86	7	6	0.24
7	7	81	0.02	7.00	7	7	0.11

上海科技大学 ShanghaiTech University

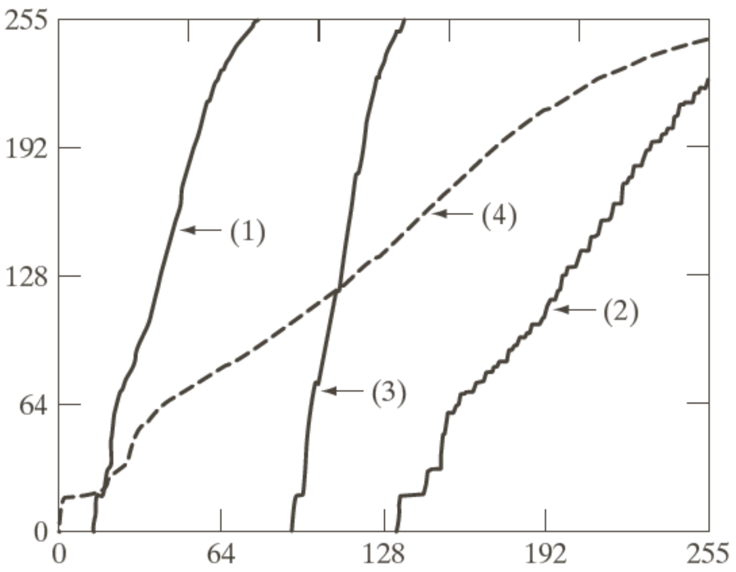
Example



$p_r(r_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
r_k
s_k
7.0
5.6
4.2
2.8
1.4
$T(r)$
0 1 2 3 4 5 6 7
r_k
$p_s(s_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
s_k

Example

FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained using Eq. (3-15) and the histograms of the images on the left column of Fig. 3.20. Mapping of one intensity value r_k in image 1 to its corresponding value s_k is shown.



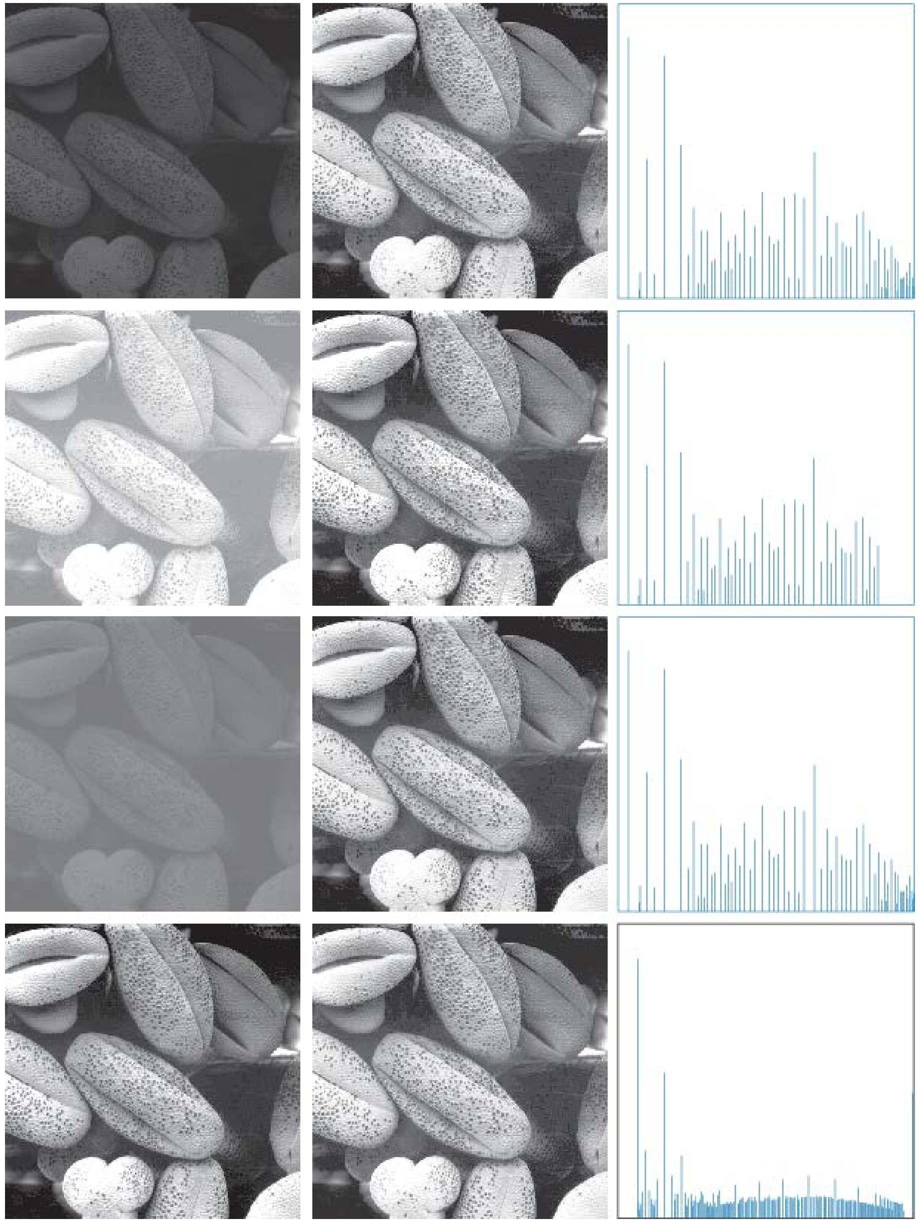


FIGURE 3.20 Left column: Images from Fig. 3.16. Center column: Corresponding histogram-equalized images. Right column: histograms of the images in the center column (compare with the histograms in Fig. 3.16).

Histogram Matching

Generate a processed image with a specified histogram

For input :

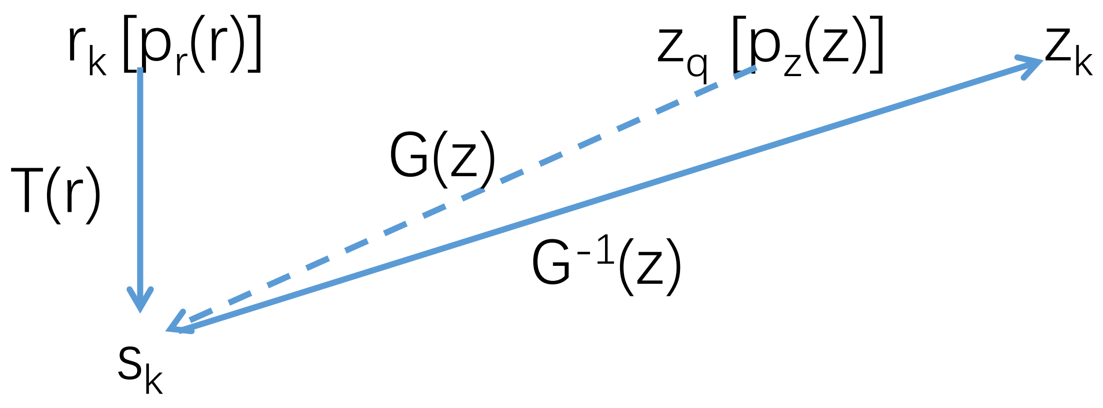
$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

For output :

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

Therefore

$$z = G^{-1}(s) = G^{-1}[T(r)]$$



Typical Transformation Function

Probability Density Function of Output Image		Transformation Function
Uniform	$P_g(g) = \dfrac{1}{g_{max} - g_{min}}, g_{max} \geq g \geq g_{min}$	$g = [g_{max} - g_{min}]C(f) + g_{min}$
Exponential	$P_g(g) = \alpha \exp[-\alpha(g - g_{min})], g \geq g_{min}$	$g = g_{min} - \dfrac{1}{\alpha}\ln[1 - C(f)]$
Raleigh	$P_g(g) = \dfrac{g - g_{min}}{\alpha^2}\exp[-\dfrac{(g - g_{min})^2}{2\alpha^2}], g \geq g_{min}$	$g = g_{min} + \{2\alpha^2 \ln[\dfrac{1}{1 - C(f)}]\}^{1/2}$
Hyperbolic (Cubic)	$P_g(g) = \dfrac{1}{3}[\dfrac{g^{-2/3}}{g^{1/3}{}_{max} - g^{1/3}{}_{min}}], g_{max} \geq g \geq g_{min}$	$g = \{[g^{1/3}{}_{max} - g^{1/3}{}_{min}]C(f) + g^{1/3}_{min}\}^3$
Hyperbolic (Logarithmic)	$P_g(g) = \dfrac{1}{g[\ln(g_{max}) - \ln(g_{min})]}, g_{max} \leq g \leq g_{min}$	$g = g_{min}\left[\dfrac{g_{max}}{g_{min}}\right]^{C(f)}$

Histogram Matching

Mapping: $r_k \longrightarrow s_k = G(z_q) \longrightarrow z_q \longrightarrow z_k$

r_k	$p(r_k)$	s_k	z_q	$p(z_q)$	$G(z_q)$	z_k	z_k	$p(z_k)$
0	0.19	1	0	0	0	3	0	0
1	0.25	3	1	0	0	4	1	0
2	0.21	5	2	0	0	5	2	0
3	0.16	6	3	0.15	1	6	3	0.19
4	0.08	6	4	0.20	2	6	4	0.25
5	0.06	7	5	0.30	5	7	5	0.21
6	0.03	7	6	0.20	6	7	6	0.24
7	0.02	7	7	0.15	7	7	7	0.11

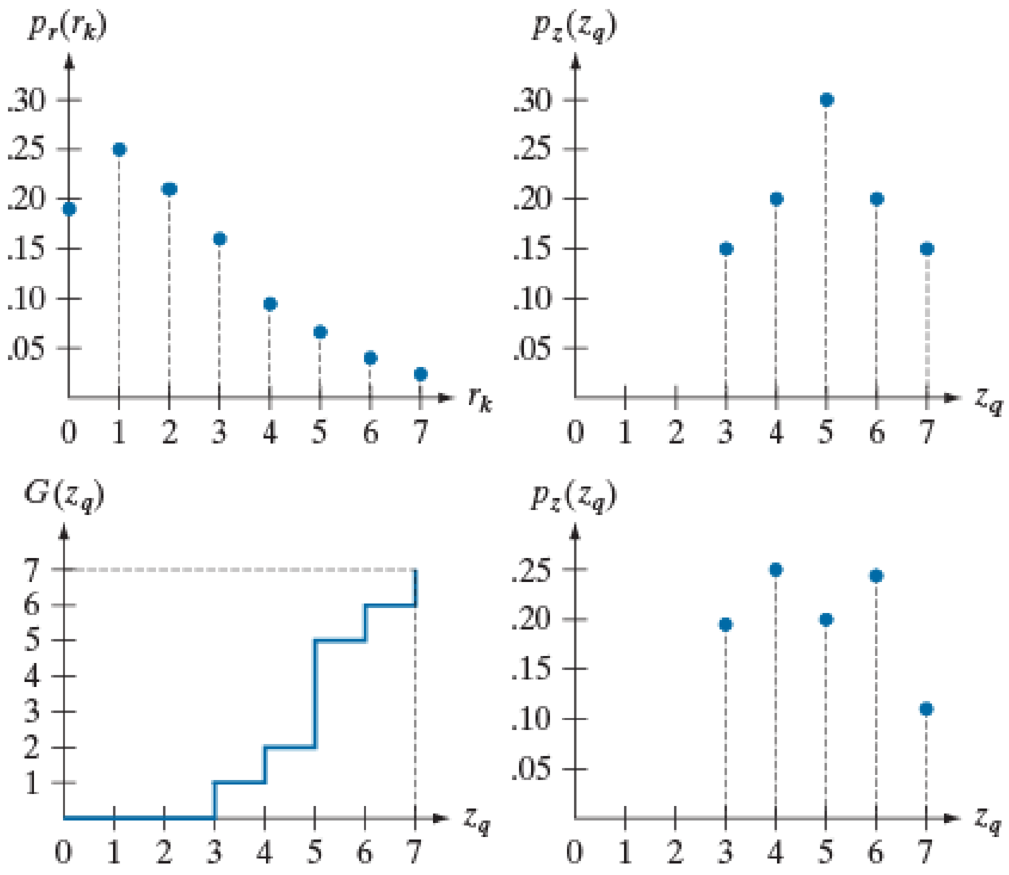
Note: (1) $s_k \nleftrightarrow G(z_q)$, find z_q at which $|G(z_q) - s_k|$ is smallest;

(2) $G(z_q)$ is mapping to multiple z_q, select the smallest z_q;

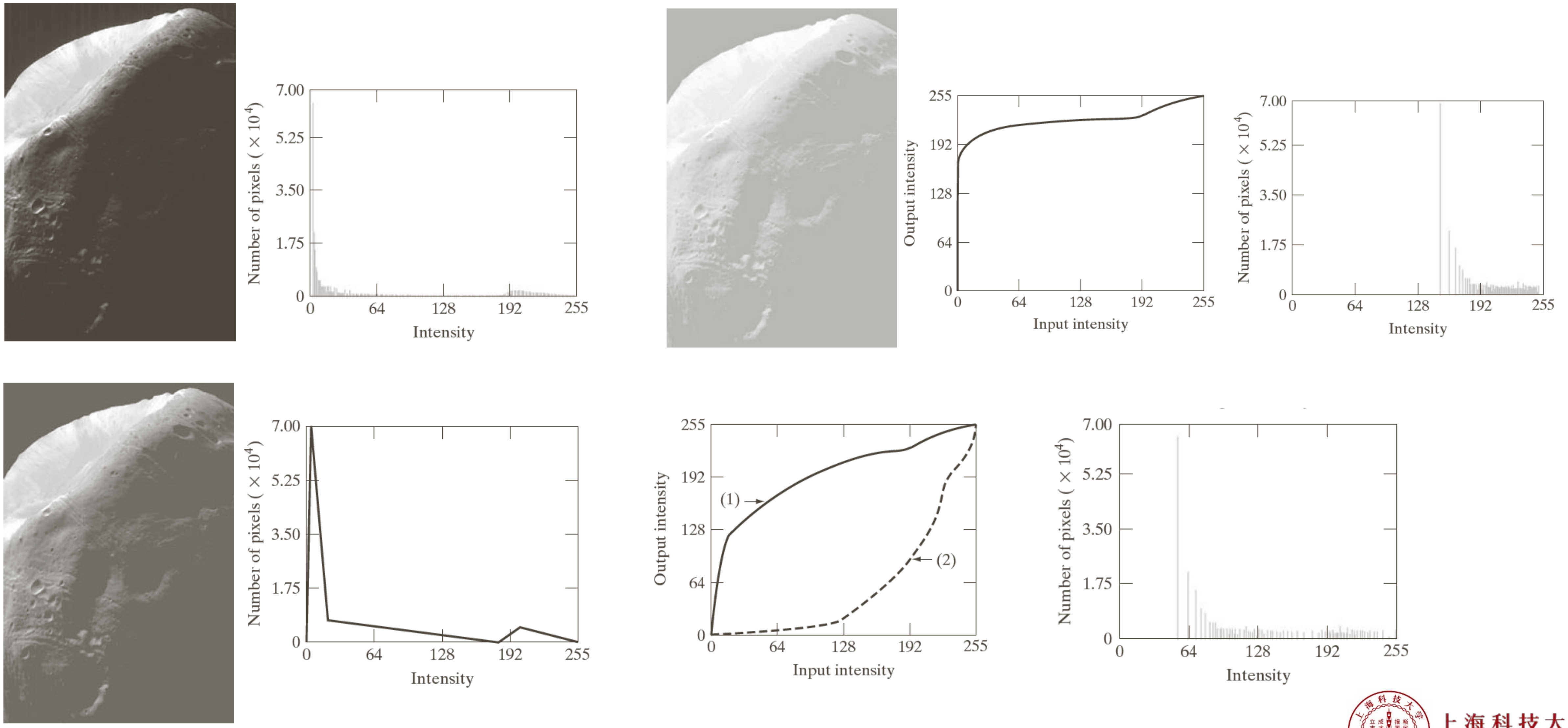
Histogram Matching

a b
c d

FIGURE 3.23 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of histogram specification. Compare the histograms in (b) and (d).



Equalization *vs* Matching



Exact Histogram Matching

- **To generate a image whose histogram truly match specified shapes**
  - Normalize large image data sets used for testing and validation
  - Establish a set of “golden images” for calibrating imaging system
  - Establish a norm for consistent image analysis and interpretation by humans, etc
- **To find a way to change and redistribute the intensities of the pixels of the input image to create the specify shapes, following the steps:**
  1. Order the image pixels according to a predefined criterion, i.e. K-tuples of extracting
  2. Split the ordered pixels into L groups, such that group j has $h(j)$ pixels;
  3. Assign intensity value j to all pixels in group j

Exact Histogram Matching

- Computing the neighborhood averages and extracting the *K*-tuple, resulting in a matrix of size *MN∗K*, and ordering by *K*-tuples

$$w_1 = [1] \qquad w_2 = \frac{1}{5}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad w_3 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$w_4 = \frac{1}{13}\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad w_5 = \frac{1}{21}\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \qquad w_6 = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

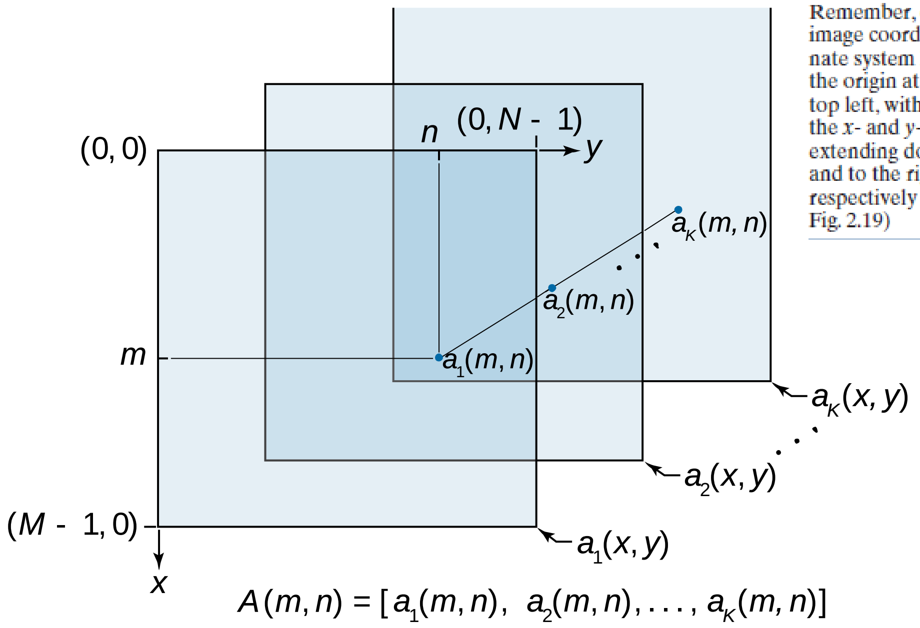
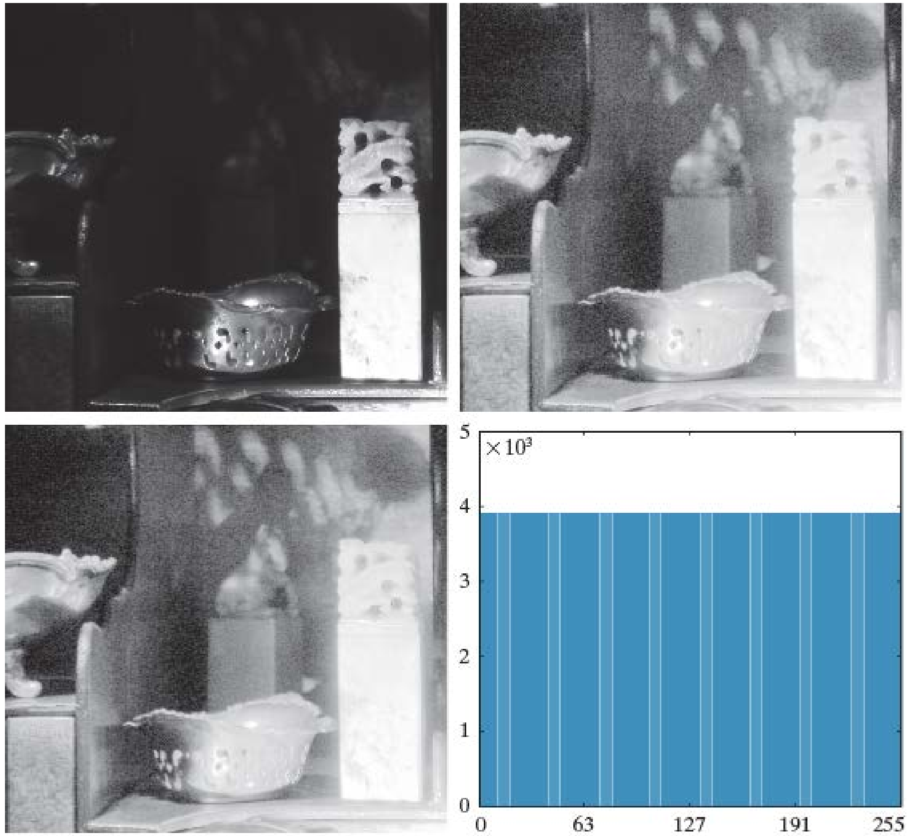


FIGURE 3.27 Extracting a *K*-tuple *A* at coordinates (m, n) of the processed image data. Remember, our image coordinate system has the origin at the top left, with the *x*- and *y*-axis extending down and to the right, respectively (see Fig. 2.19)

Equalization *vs* Exact matching

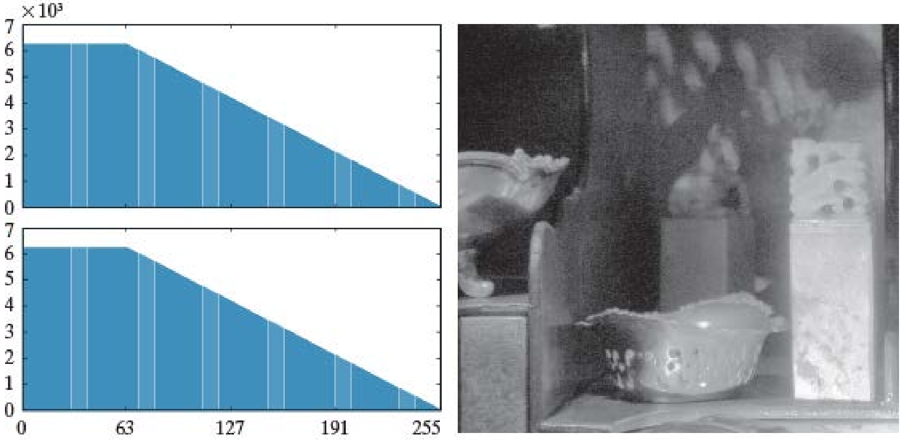
a b
c d

FIGURE 3.28 (a) Original image. (b) Result of histogram equalization. (c) Result of exact histogram specification with a uniform histogram. (d) Histogram of image (c).



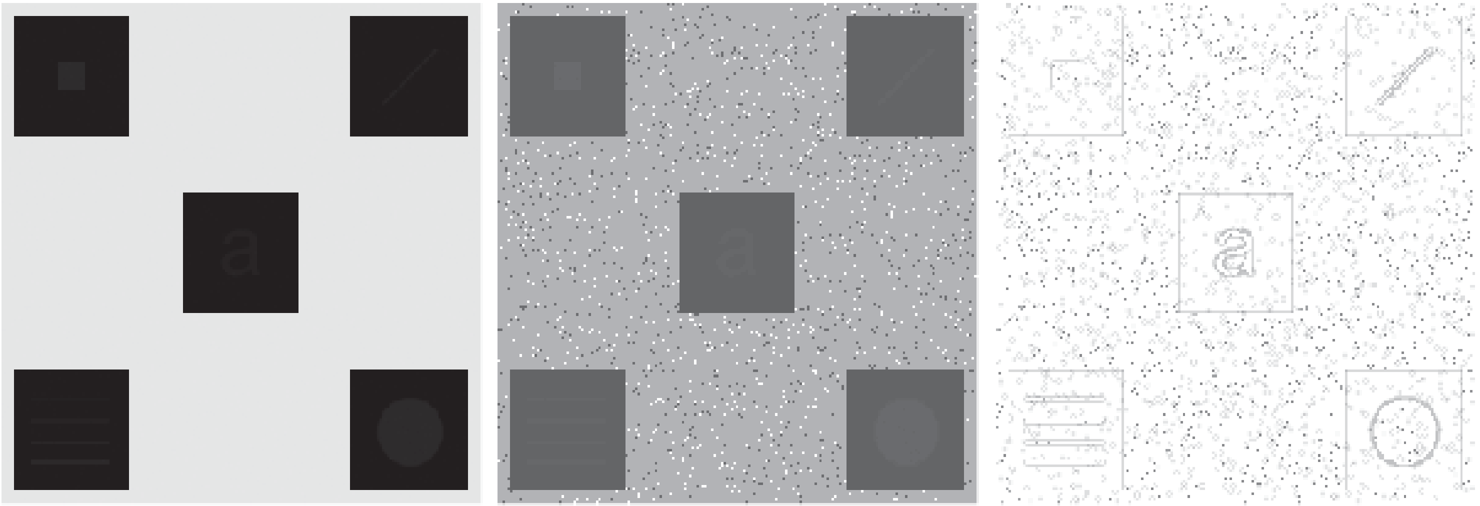
a b
c

FIGURE 3.29 (a) Specified histogram. (b) Result of exact histogram specification. (c) Histogram of (b).



Local Histogram Processing

- The objective is to enhance details over small area in an image.
- Devise transformation functions based the intensity distribution of pixel neighborhoods.
- All global histogram processing method are adapted.
- Define a neighborhood and apply histogram processing, then move the center of the neighborhood from pixel to pixel in a horizontal or vertical direction.



a b c

FIGURE 3.32 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization.

上海科技大学 ShanghaiTech University

Histogram Statistics

- **Can be applied for both global and local process**
- **Statistical parameters:**
  - Average Intensity (平均灰度)

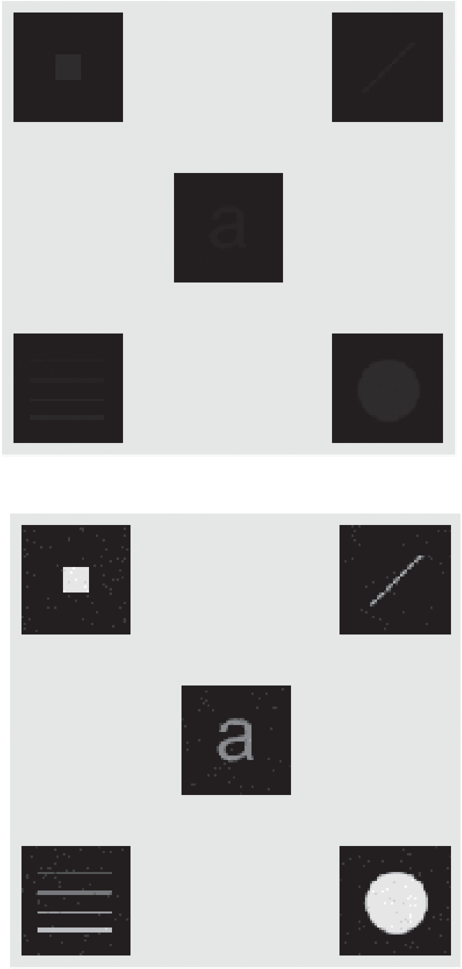
$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$$

  - Intensity Variance (灰度方差)

$$\sigma^2 = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - m]^2$$

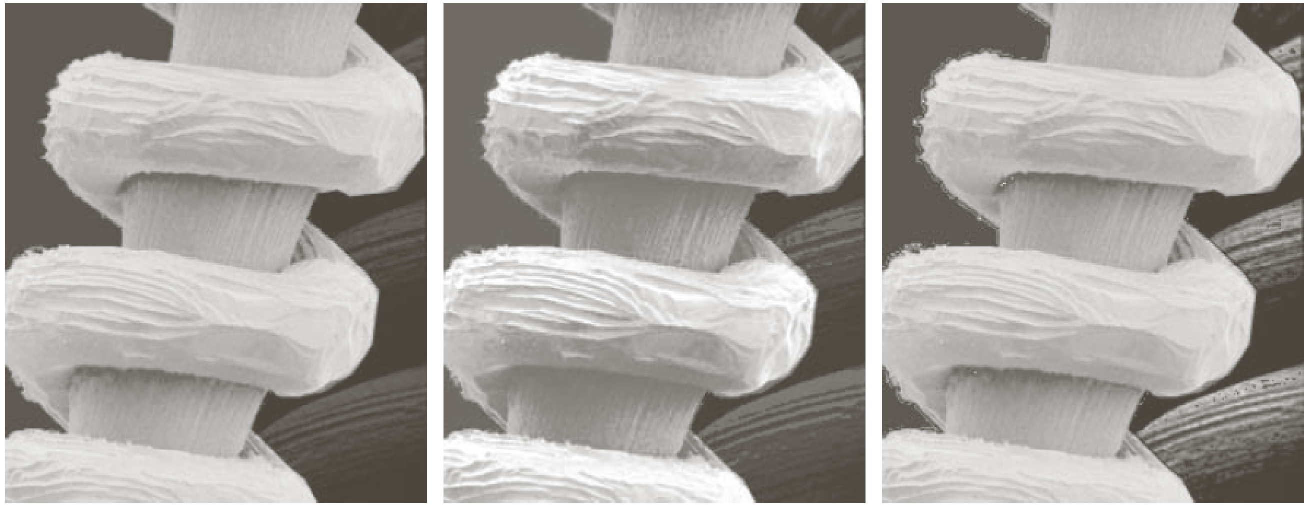
a b

FIGURE 3.33 (a) Original image. (b) Result of local enhancement based on local histogram statistics. Compare (b) with Fig. 3.32(c).



Histogram Statistics

$$g(x,y)=\begin{cases} E\cdot f(x,y), & \text{if } m_{S_{xy}}\le k_0 m_G \text{ AND } k_1\sigma_G\le\sigma_{S_{xy}}\le k_2\sigma_G \\ f(x,y), & \text{otherwise}\end{cases}$$



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Color Histogram

- Modify the color intensity and keep the hue unchanged in HSI model, instead of processing color component in RGB model.
- Increasing saturation component can partially correct the relative appearance of colors.

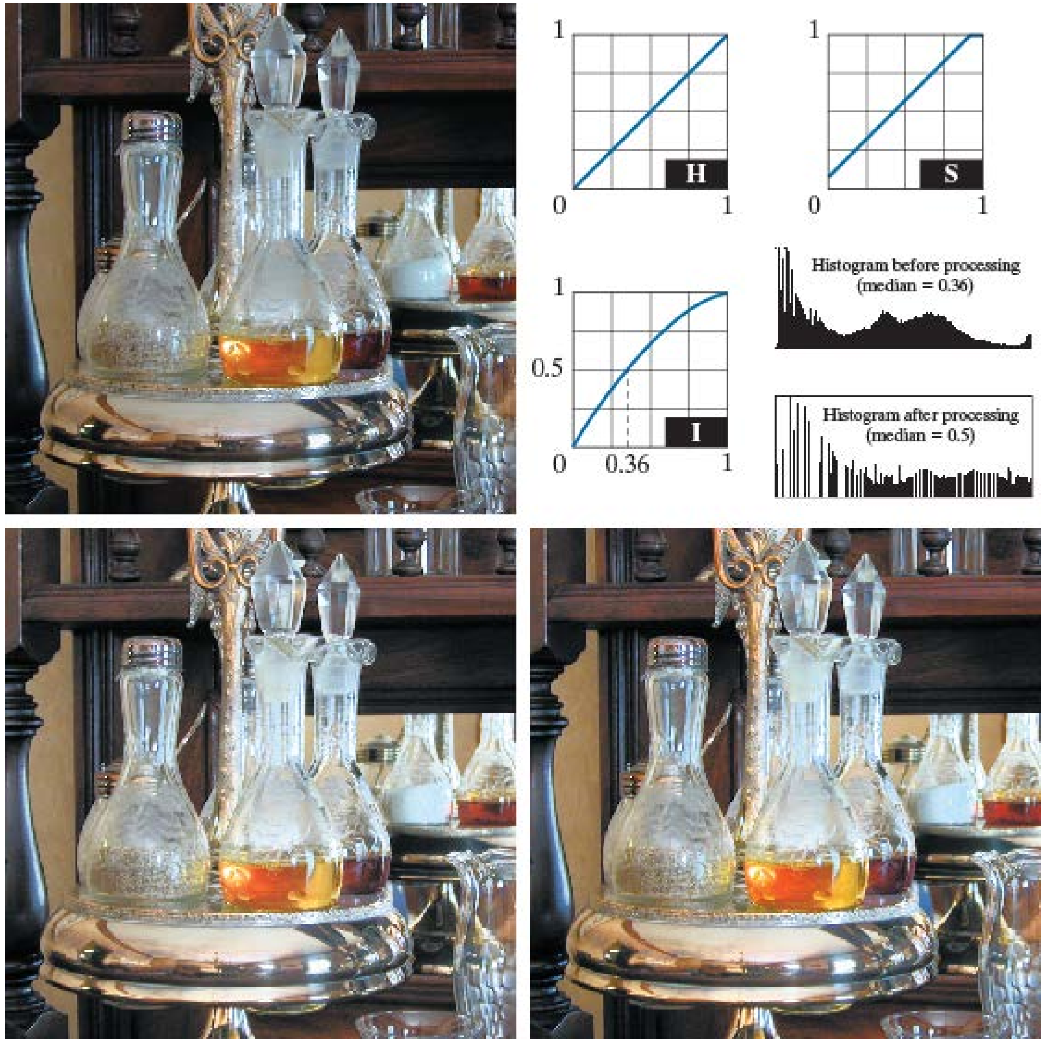


FIGURE 7.35 Histogram equalization (followed by saturation adjustment) in the HSI color space.

上海科技大学 ShanghaiTech University