Lecture 17 – Morphological Image Processing (形态学图像处理)

This lecture will cover:

- Preliminaries
- Morphological operation
  - Erosion and Dilation
  - Opening and Closing
  - The Hit-or-Miss Transform
- Basic morphological algorithms
- Morphological reconstruction
- Segmentation using Morphological Watersheds（形态学分水岭分割）

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Mathematical Morphology

- **Language： Set theory （集合论）**
  - Objects: sets of foreground (Image) pixels
  - Structuring element (SE, 结构元)
- **Advantages comparing to other spatial or frequency domain methods**
  - Keep more information from image;
  - Insensitive to noise
  - Continuous skeletons
  - Parallel processing
  - Smooth edge
- **Key operations**
  - HMT (Hit or Miss Transformation, 击中与击不中变换)
  - Dilation (膨胀) and Erosion (腐蚀)
- **Steps: to analyze image with mathematical morphological methods**

  ① Specify the geometrical structural pattern of the object;

  ② Choose structuring element based on the pattern

  ③ Proceed HMT with the selected SE to acquire characteristic image of the object

  ④ Emphasize the desired information to extract attributes

Structuring Element（结构元）

- **Structuring Element (SE) ---** small sets or subimages used to probe an image under study for properties of interest**.**
- **SE Selection**
  - Simpler than the image
  - With boundary
  - Convex
- **Structures**
  - Origin
  - Rectangular
  - Consist of foreground and background pixels, sometimes “don’t care” elements (无关像素);
- **Matlab Function:**

  se = strel(shape, parameters)

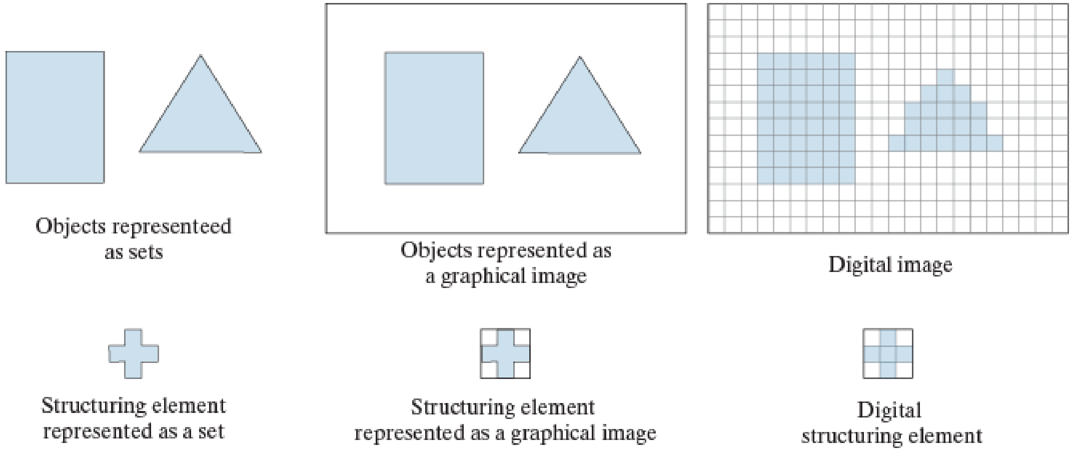
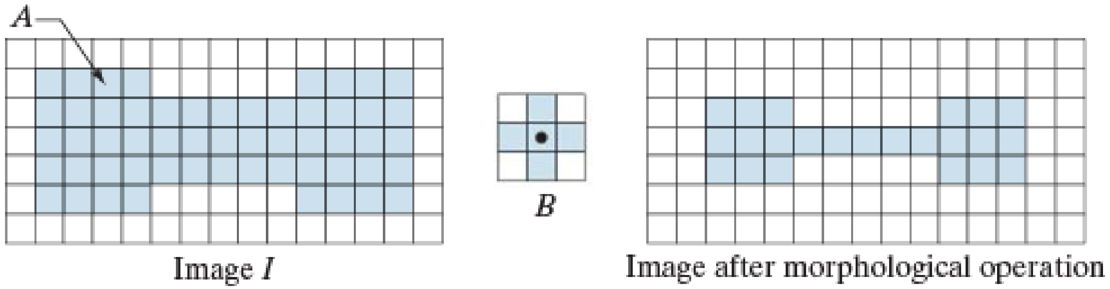


FIGURE 9.1 Top row. *Left:* Objects represented as graphical sets. *Center:* Objects embedded in a background to form a graphical image. *Right:* Object and background are digitized to form a digital image (note the grid). Second row: Example of a structuring element represented as a set, a graphical image, and finally as a digital SE.



a b c

FIGURE 9.3 (a) A binary image containing one object (set), *A*. (b) A structuring element, *B*. (c) Image resulting from a morphological operation (see text).

Preliminaries --- Set Operation

An digital image $\boldsymbol{f(x,y)}$ can be considered as a set $\boldsymbol{A}$, if $w=(x,y)$ in 2D integer space $\boldsymbol{Z^2}$, Then

- $\boldsymbol{w} \in \boldsymbol{A}$: w is an element of $\boldsymbol{A}$
- $\boldsymbol{w} \notin \boldsymbol{A}$: w is not an element of $\boldsymbol{A}$
- $\boldsymbol{B} = \{\boldsymbol{w} \mid \text{condition}\}$: all elements which meet the specified condition
  - $\boldsymbol{A} \cup \boldsymbol{B} = \{\boldsymbol{w} \mid \boldsymbol{w} \in \boldsymbol{A} \text{ or } \boldsymbol{w} \in \boldsymbol{B}\}$: union (并集)
  - $\boldsymbol{A} \cap \boldsymbol{B} = \{\boldsymbol{w} \mid \boldsymbol{w} \in \boldsymbol{A} \text{ and } \boldsymbol{w} \in \boldsymbol{B}\}$: intersection (交集
  - $\boldsymbol{A^c} = \{\boldsymbol{w} \mid \boldsymbol{w} \notin \boldsymbol{A}\}$: complement (补集)
  - $\boldsymbol{A} - \boldsymbol{B} = \{\boldsymbol{w} \mid \boldsymbol{w} \in \boldsymbol{A}, \boldsymbol{w} \notin \boldsymbol{B}\}$: difference (差集)
- **Reflection (反射) :** $\hat{\boldsymbol{B}} = \{\boldsymbol{w} \mid \boldsymbol{w} = -\boldsymbol{b}, \boldsymbol{b} \in \boldsymbol{B}\}$
- **Translation (平移):** $(\boldsymbol{B})_{\boldsymbol{z}} = \{\boldsymbol{w} \mid \boldsymbol{w} = \boldsymbol{b} + \boldsymbol{z}, \boldsymbol{b} \in \boldsymbol{B}\}$, where $z = (z_1, z_2)$

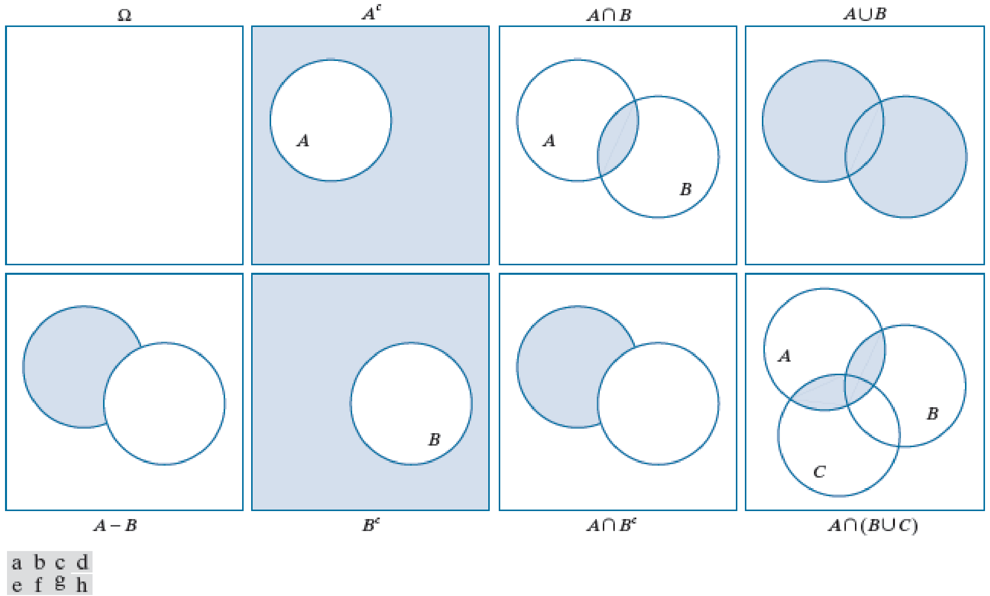


FIGURE 2.35 Venn diagrams corresponding to some of the set operations in Table 2.1. The results of the operations, such as A^c, are shown shaded. Figures (e) and (g) are the same, proving via Venn diagrams that $A - B = A \cap B^c$ [see Eq. (2-40)].

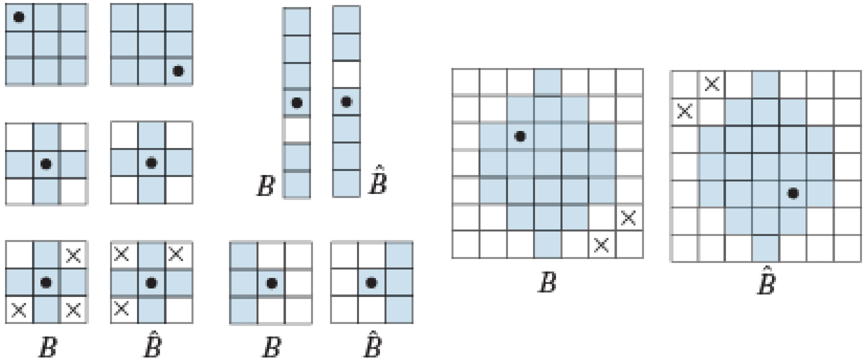


FIGURE 9.2 Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.

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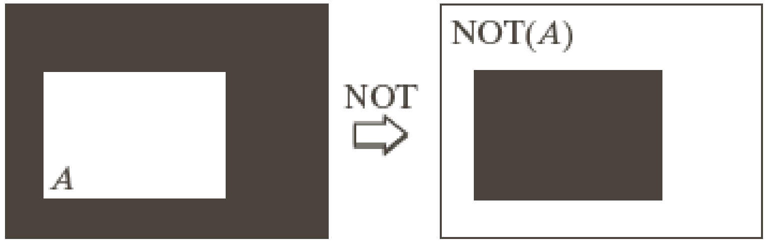
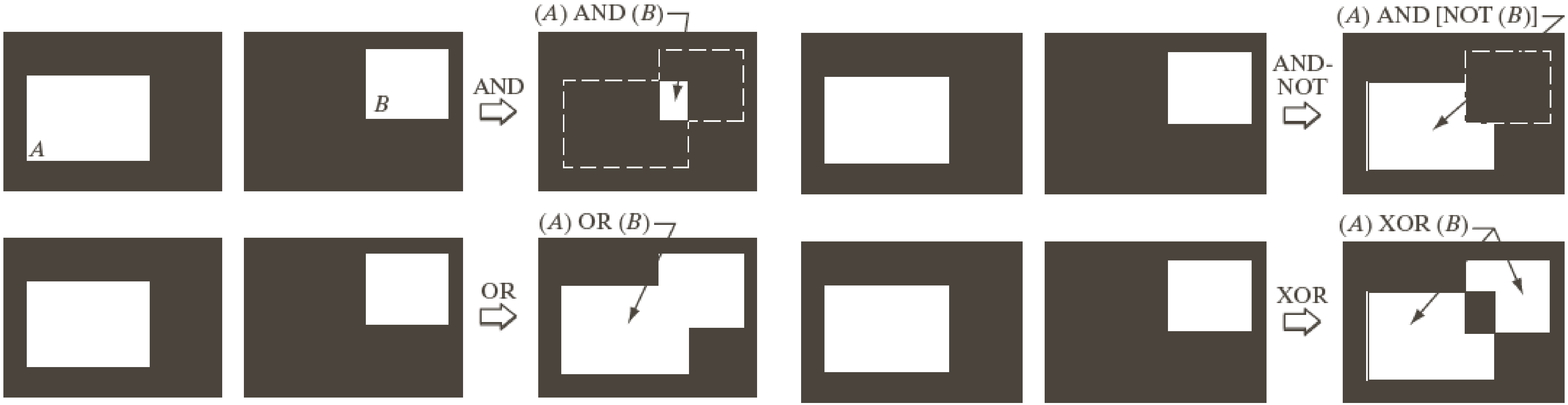
Preliminaries --- Logical Operation



Matlab Function:

- ~A / not(A)
- A&B / and(A,B)
- A|B / or(A,B)
- xor(A,B)



Erosion (腐蚀)

- **Definition:** $A \ominus B = \{z \mid (B)_z \subseteq A\}$ **or** $A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$
- **Matlab Function:** J = imerode(I,SE)

a b c
d e

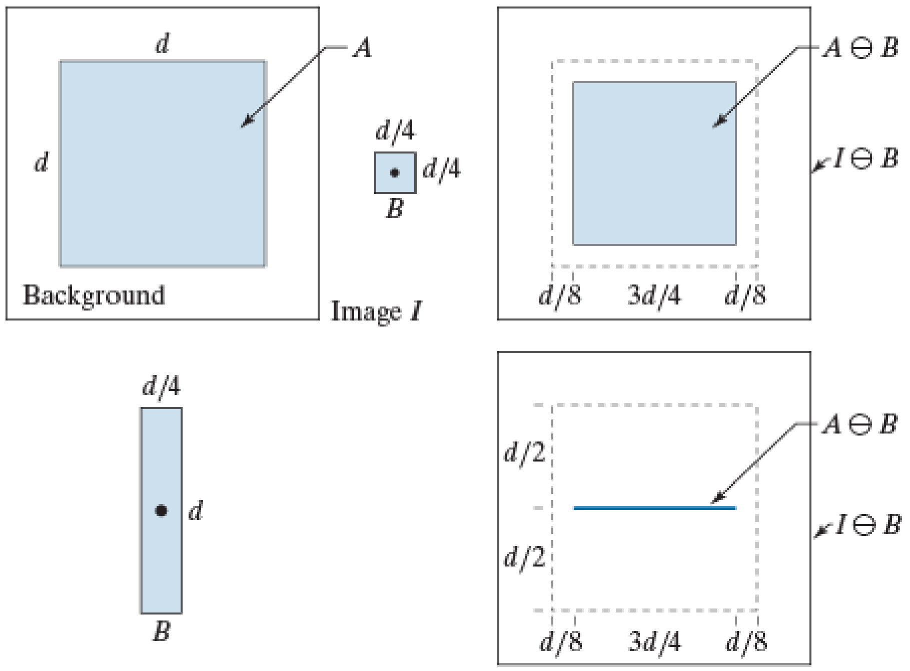
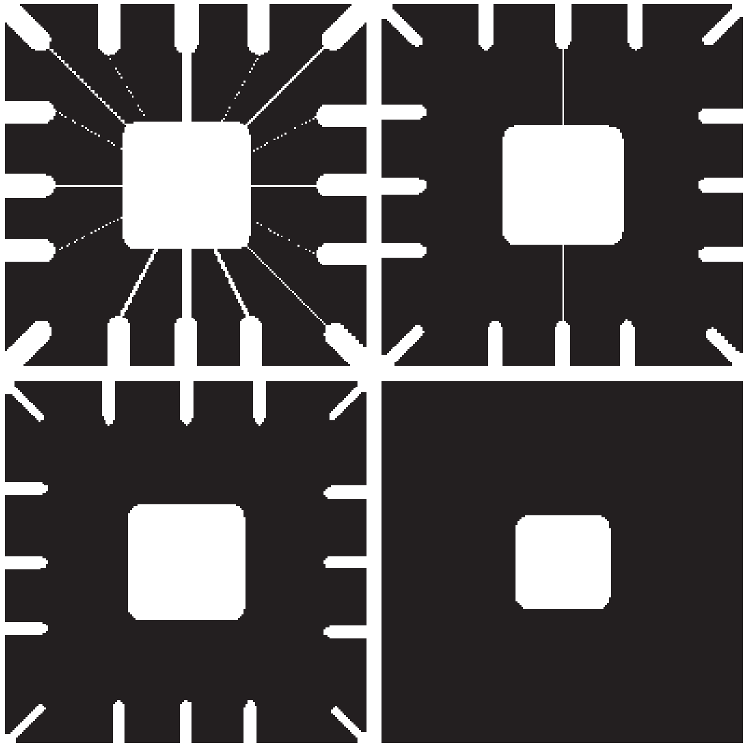


FIGURE 9.4 (a) Image I, consisting of a set (object) A, and background. (b) Square SE, B (the dot is the origin). (c) Erosion of A by B (shown shaded in the resulting image). (d) Elongated SE. (e) Erosion of A by B. (The erosion is a line.) The dotted border in (c) and (e) is the boundary of A, shown for reference.



a b
c d

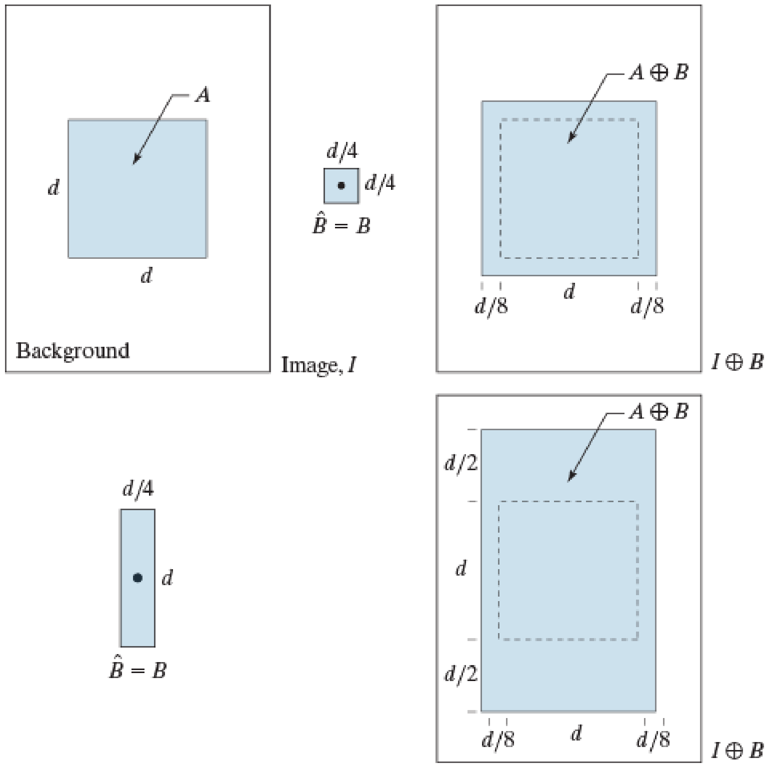
FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45 elements, respectively, all valued 1.

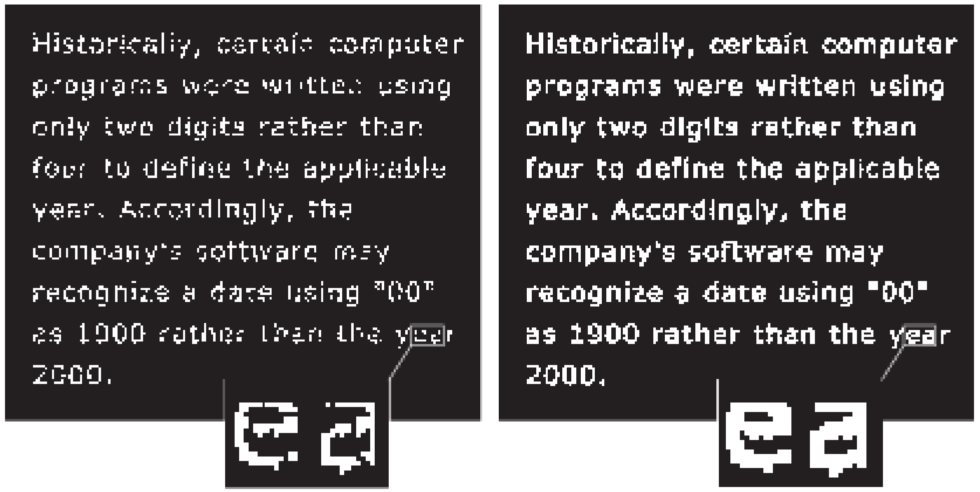
Dilation (膨胀)

- **Definition:** $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$ **or** $A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$
- **Matlab Function:** J = imdilate(I,SE)

a b c
d e

FIGURE 9.6
(a) Image I, composed of set (object) A and background.
(b) Square SE (the dot is the origin).
(c) Dilation of A by B (shown shaded).
(d) Elongated SE.
(e) Dilation of A by this element. The dotted line in (c) and (e) is the boundary of A, shown for reference.





1	1	1
1	1	1
1	1	1

a c
b

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Properties

- **Duality (对偶性)**
  - $(A \ominus B)^c = (A)^c \oplus \hat{B}$
  - $(A \oplus B)^c = (A)^c \ominus \hat{B}$
- **Associativity (结合律)**

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

or

$$(A \oplus B) = A \oplus (B_1 \oplus B_2) = (A \oplus B_1) \oplus B_2$$

Opening (开操作)

- **Definition:** $A \circ B = (A \ominus B) \oplus B$ **or** $A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$
- **Matlab Function:** J = imopen(I,SE)
- **Properties:**

  ① $\mathrm{A} \circ \mathrm{B}$ is a subset (subimage) of A

  ② If C is a subset of D, the $\mathrm{C} \circ \mathrm{B}$ is a subset of $\mathrm{D} \circ \mathrm{B}$

  ③ $(\mathrm{A} \circ \mathrm{B}) \circ \mathrm{B} = \mathrm{A} \circ \mathrm{B}$

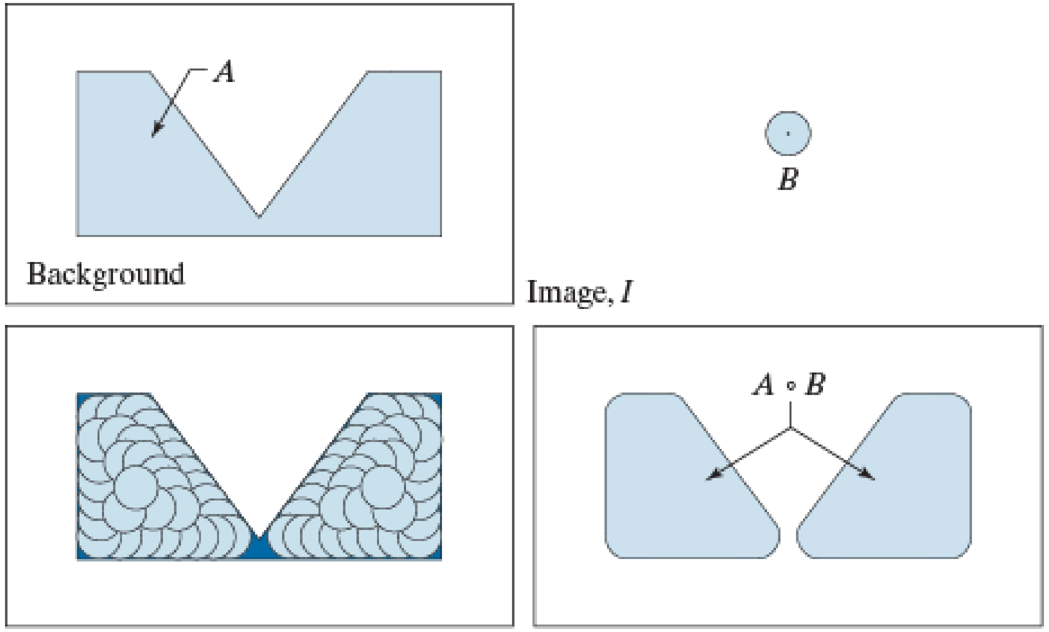


FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.

Closing (闭操作)

- **Definition:** $A \bullet B = (A \oplus B) \ominus B$
- **Matlab Function:** J = imclose(I,SE)
- **Properties:**

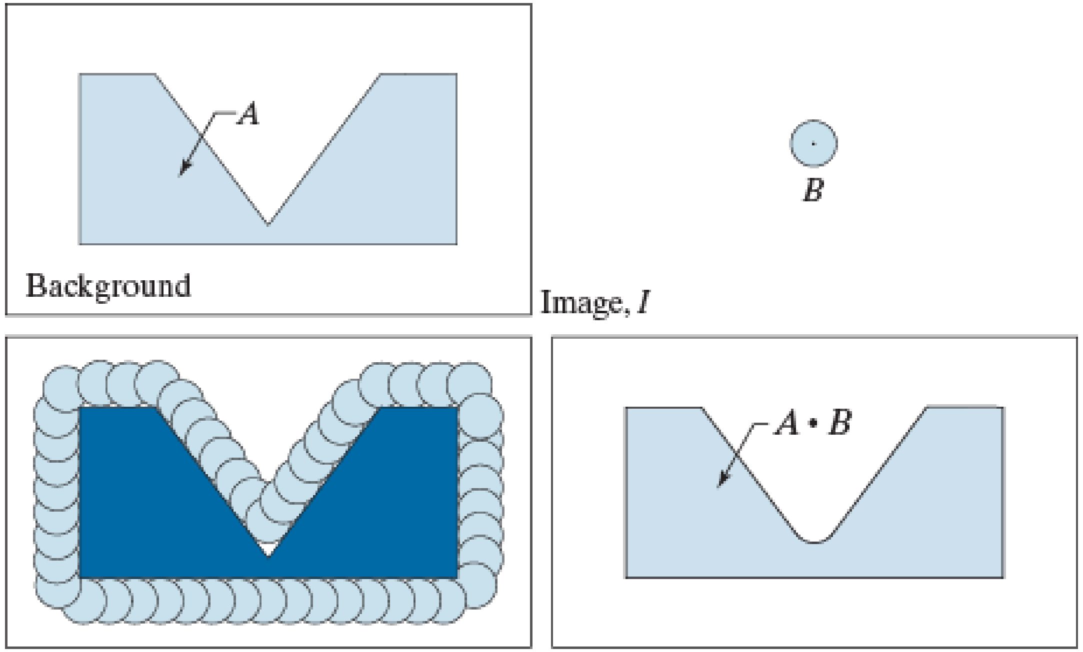
  ① A is a subset (subimage) of $\mathrm{A \bullet B}$

  ② If C is a subset of D, the $\mathrm{C \bullet B}$ is a subset of $\mathrm{D \bullet B}$

  ③ $\mathrm{(A \bullet B) \bullet B = A \bullet B}$

a b
c d

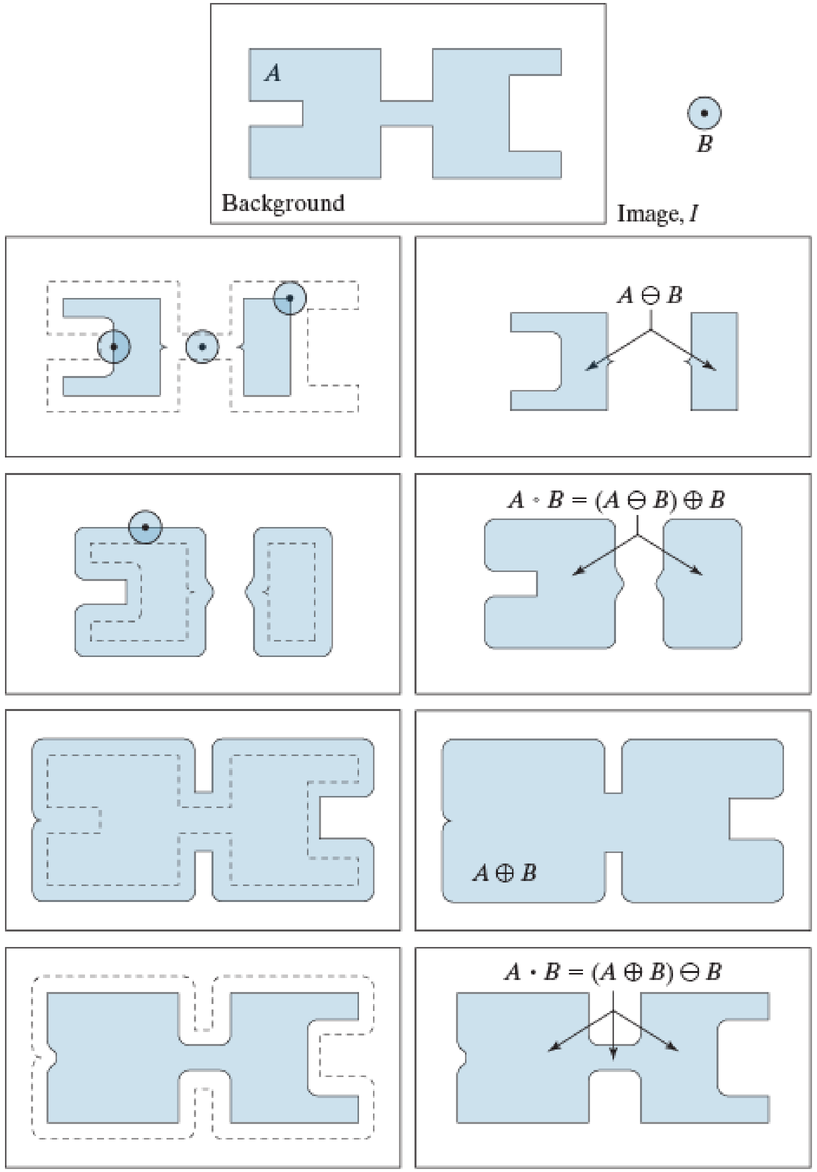
FIGURE 9.9
(a) Image I, composed of set (object) A, and background.
(b) Structuring element B.
(c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.)
(d) Closing of A by B.



Opening & Closing (开操作和闭操作)

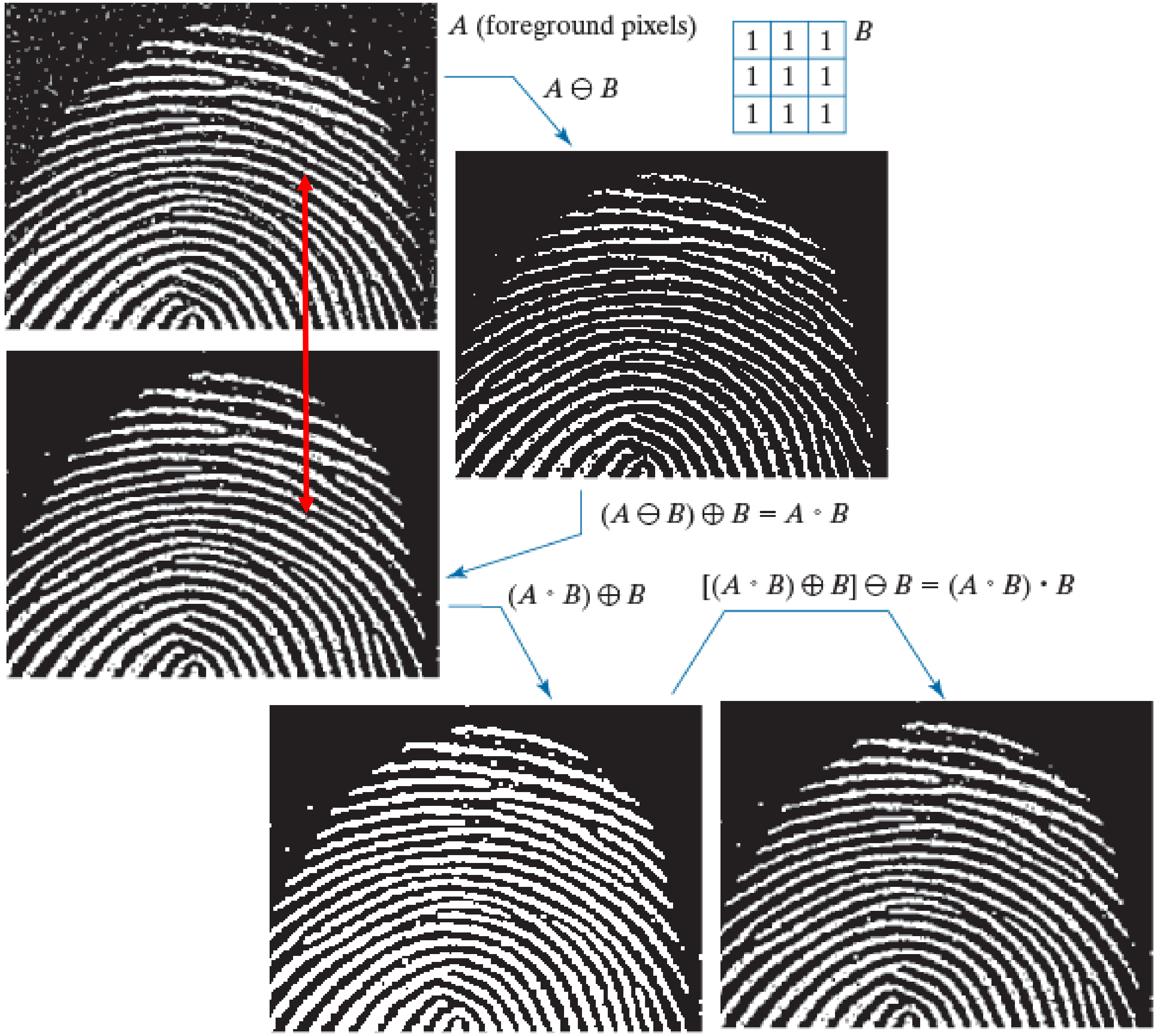
a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. (a) Image I, composed of a set (object) A and background; a solid, circular structuring element is shown also. (The dot is the origin.) (b) Structuring element in various positions. (c)-(i) The morphological operations used to obtain the opening and closing.



- **Comparison between Opening and Closing**
  - **Opening（开操作）**
    - ✓ Smooth the contour of an object
    - ✓ Break narrow isthmuses
    - ✓ Eliminate thin protrusions
  - **Closing（闭操作）**
    - ✓ Smooth the contour of an object
    - ✓ Fuse narrow breaks and long thin gulfs
    - ✓ Eliminate small holes
    - ✓ Fill gaps in the contour
- **Duality (对偶性)**
  - $(A \bullet B)^c = (A)^c \circ \hat{B}$
  - $(A \circ B)^c = (A)^c \bullet \hat{B}$

Morphological Filtering (形态学滤波)



a b
d c
e f

FIGURE 9.11
(a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Dilation of the erosion (opening of A). (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

The Hit-or-Miss Transform (击中或击不中变换)

Definition: $A \circledast B_{1,2} = \{z|(B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c\} = (A \ominus B_1) \cap (A^c \ominus B_2)$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	1	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0
0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

B_1

$$\begin{array}{ccc} & 1 & \\ 1 & \boxed{1} & 1 \\ & 1 & \end{array}$$

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	0	0	0	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	0	0	1	1
1	1	0	1	1	1	1	1	1	1	1	1	0	0	0	1
1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1
1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

B_2

$$\begin{array}{ccc} 1 & & 1 \\ & \boxed{} & \\ 1 & & 1 \end{array}$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	1
1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	1	1	1	1	0	0	0	0	1
1	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0
1	1	1	1	0	1	0	1	1	1	1	1	0	1	0	1
1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1
1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The Hit-or-Miss Transformation (击中或击不中变换)

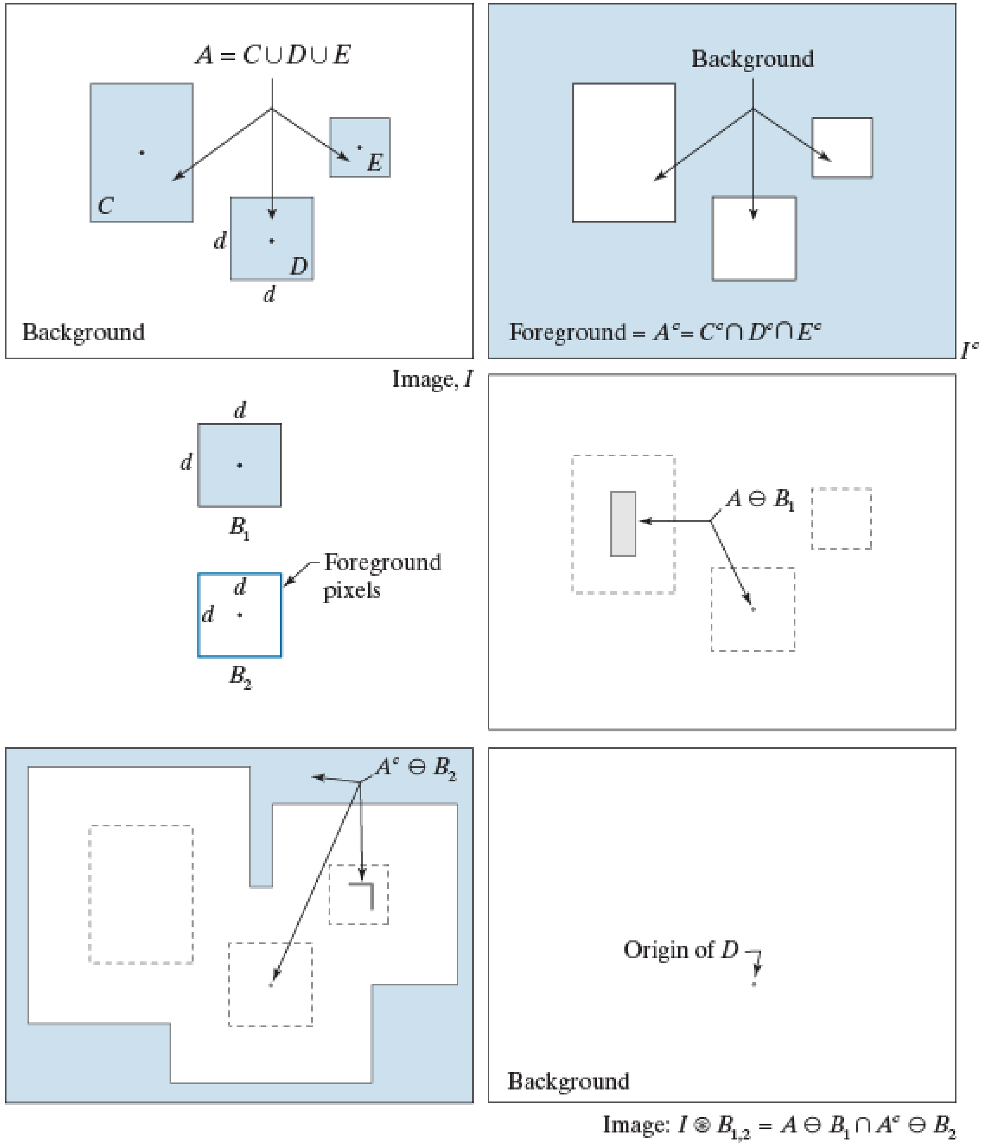


FIGURE 9.12 (a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's. (b) Image with its foreground defined as A^c. (c) Structuring elements designed to detect object D. (d) Erosion of A by B_1. (e) Erosion of A^c by B_2. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.

- $A \circledast B_{1,2} = (A \ominus B_1) \cap (A^c \ominus B_2)$

where

B_1 is associated with the object;

B_2 is associated with the corresponding background.

- $A \circledast D = (A \ominus D) \cap [A^c \ominus (W - D)]$
- $A \circledast B = (A \ominus B_1) - \left(A \oplus \hat{B}_2\right)$

where

$$\left(A \oplus \hat{B}_2\right)^c = (A^c \ominus B_2)$$

and $M - N = M \cap N^c$

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The Hit-or-Miss Transformation (击中或击不中变换)

➢ **To detect the object directly in image using a single structure element**

$$A \circledast B = \{z | (B)_z \subseteq A\}$$

Where $\boldsymbol{B}$ is a structuring element identical to object, but having in addition a border of background elements with a width of one pixel.

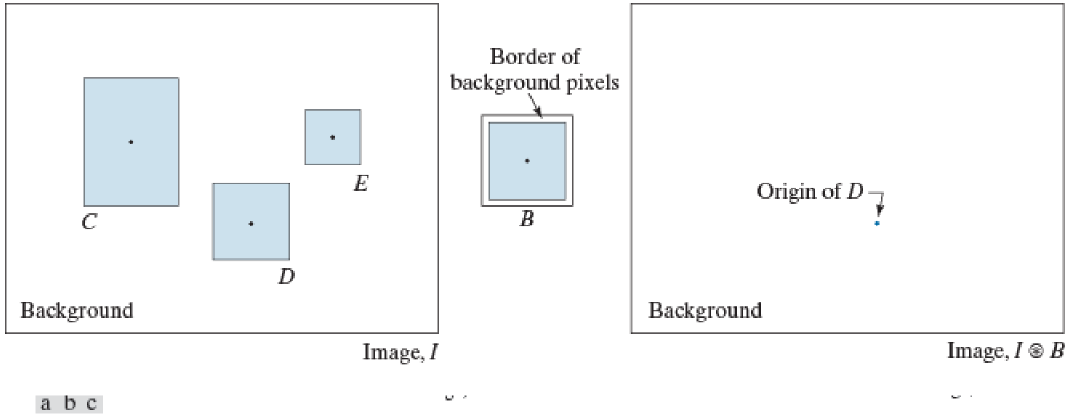

FIGURE 9.13 Same solution as in Fig. 9.12, but using Eq. (9-17) with a single structuring element.

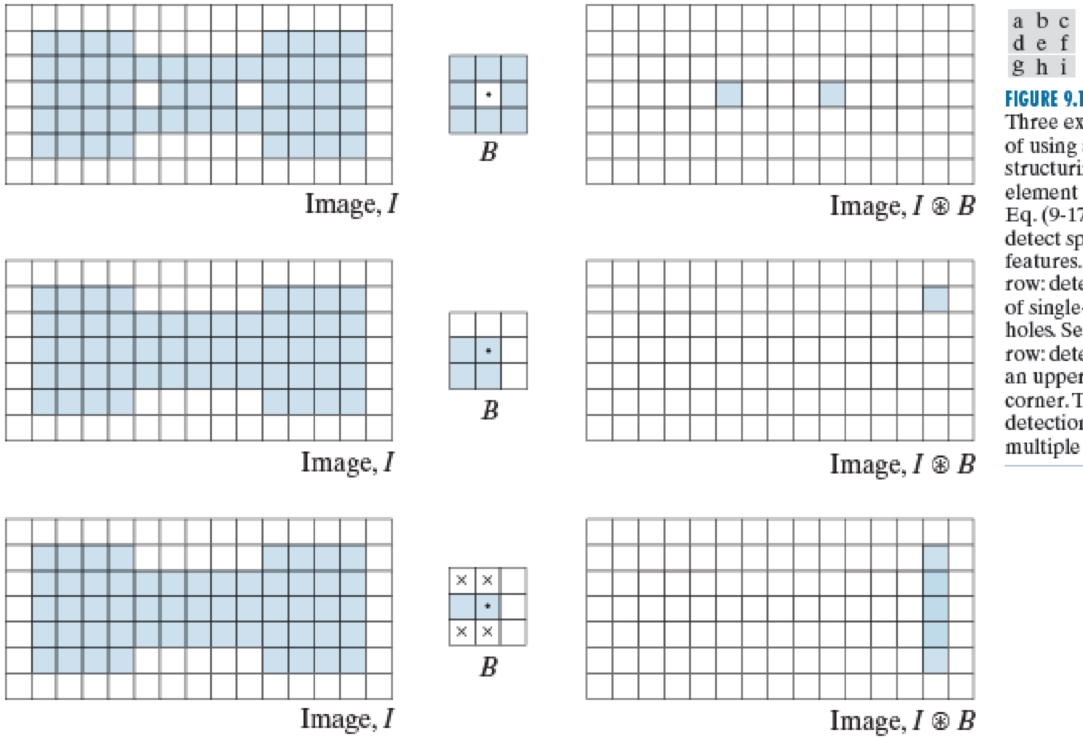

FIGURE 9.14 Three examples of using a single structuring element and Eq. (9-17) to detect specific features. First row: detection of single-pixel holes. Second row: detection of an upper-right corner. Third row: detection of multiple features.

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The Hit-or-Miss Transformation (击中或击不中变换)

- **Matlab Function:** J = bwhitmiss(BW,SE1,SE2)

$$B_1 = \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix} \qquad B_2 = \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}$$

- **Look-up Table (查找表)**
  - Condition: Small SE (2*2, 3*3)
  - For a 3*3 SE, an index matrix as below can be used as

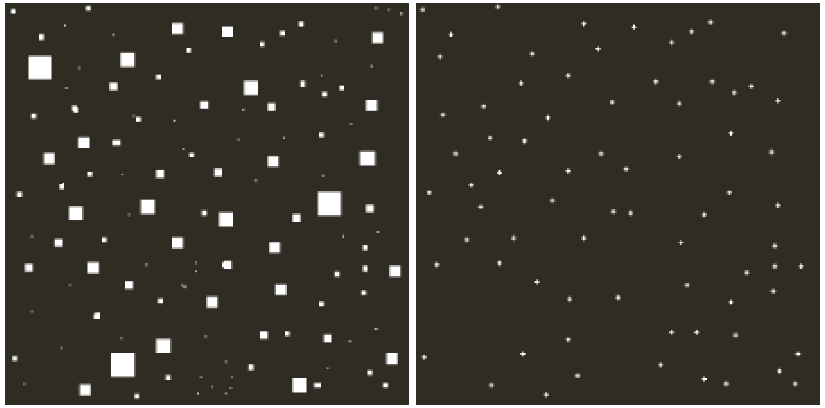
$$\begin{matrix} 1 & 8 & 64 \\ 2 & 16 & 128 \\ 4 & 32 & 256 \end{matrix}$$

  - Matlab Function:

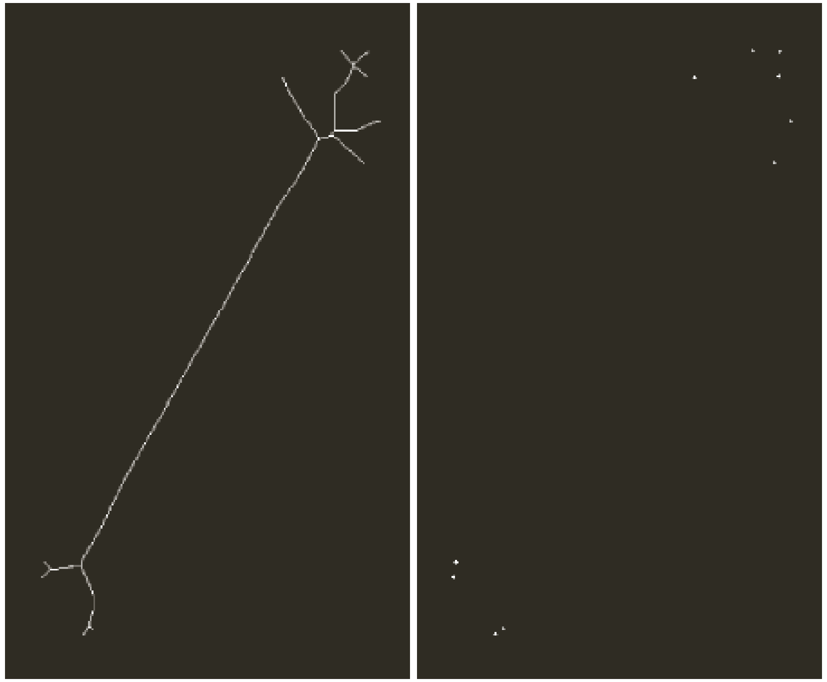
    lut = makelut(fun,n);

    g = applylut(f, lut); or

    g = bwlookup(f, lut);



a b
FIGURE 9.13 (a) Original image. (b) Result of applying the hit-or-miss transformation (the dots shown were enlarged to facilitate viewing).



a b
FIGURE 9.14 (a) Image of a morphological skeleton. (b) Output of function `endpoints`. The pixels in (b) were enlarged for clarity.

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Basic Morphologic Algorithms

- Boundary Extraction (边界提取)
- Hole Filling (孔洞填充)
- Extraction of Connected components (连通分量提取)
- Convex Hull (凸壳)
- Thinning (细化)
- Thickening (粗化)
- Skeleton (骨架)
- Pruning (裁剪)

Boundary Extraction (边界提取)

- **Morphological algorithm**: $\beta(A) = A - (A \ominus B)$

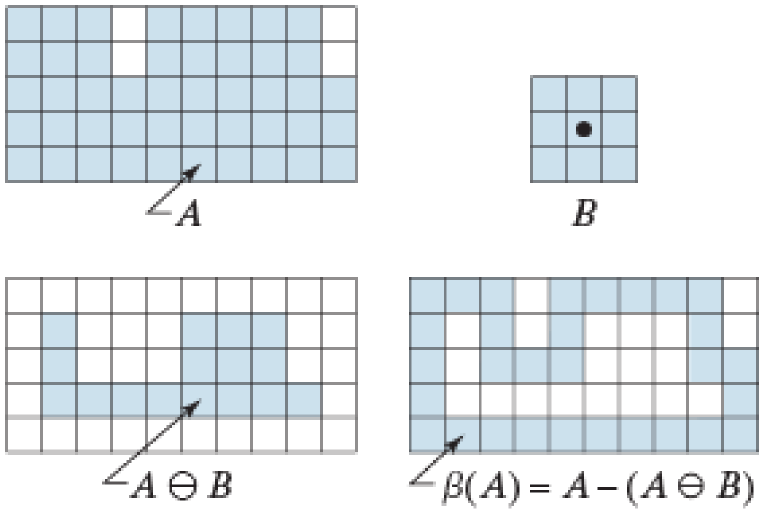
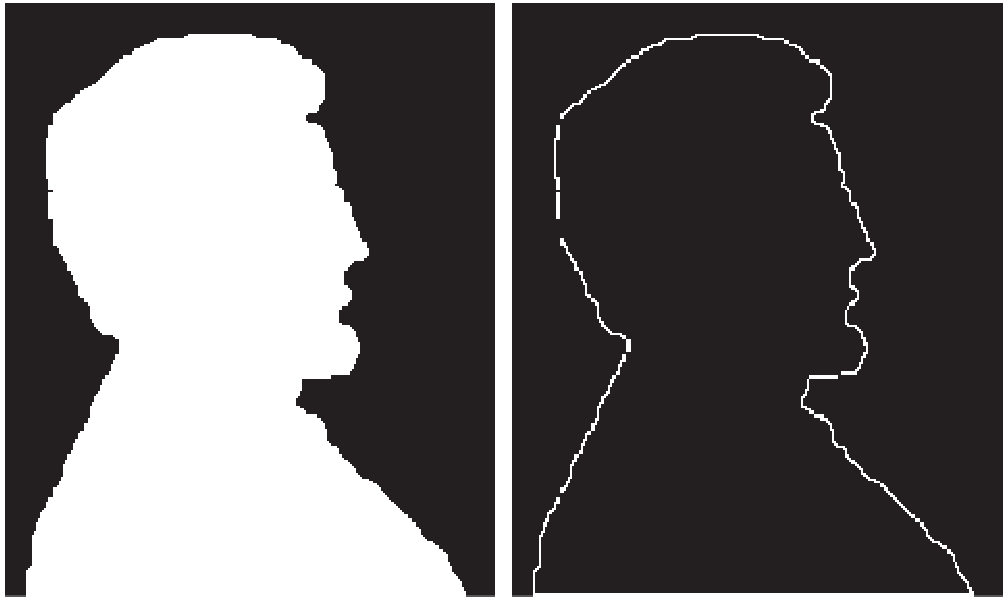
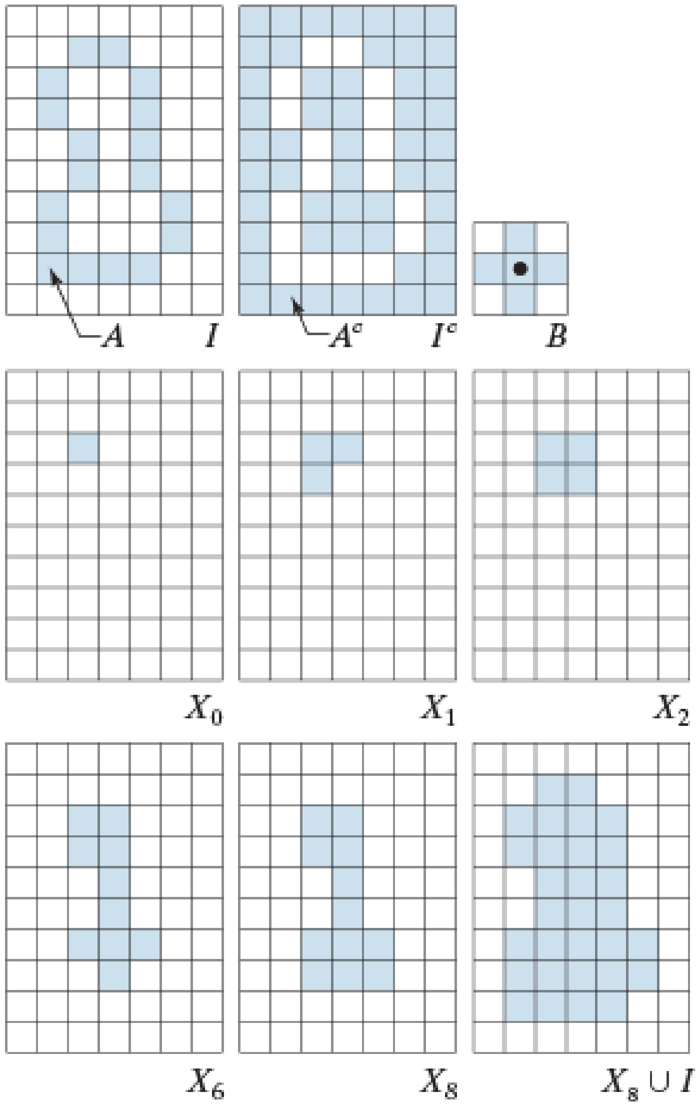


FIGURE 9.15
(a) Set, A, of foreground pixels.
(b) Structuring element.
(c) A eroded by B.
(d) Boundary of A.



a b
FIGURE 9.16
(a) A binary image.
(b) Result of using Eq. (9-18) with the structuring element in Fig. 9.15(b).

Hole Filling (孔洞填充)



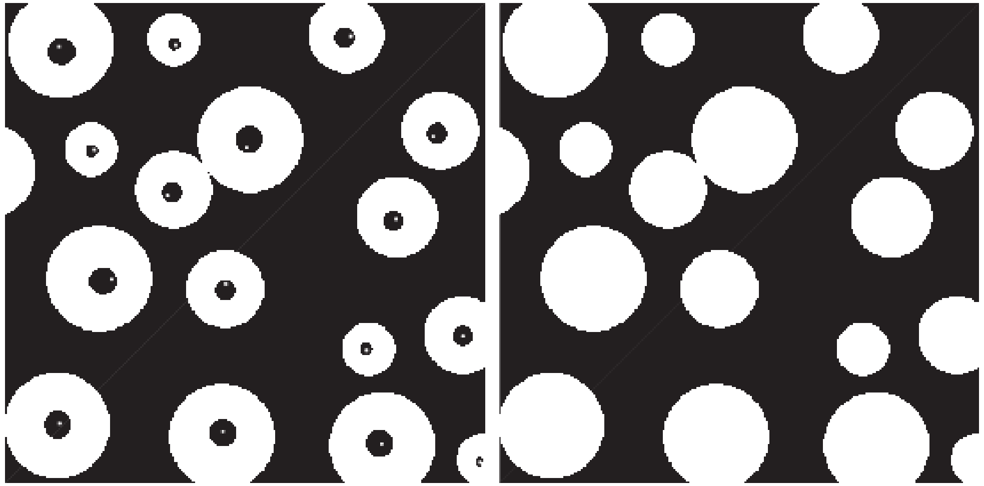
a b c
d e f
g h i

FIGURE 9.17 Hole filling. (a) Set *A* (shown shaded) contained in image *I*. (b) Complement of *I*. (c) Structuring element *B*. *Only* the foreground elements are used in computations (d) Initial point inside hole, set to 1. (e)–(h) Various steps of Eq. (9-19). (i) Final result [union of (a) and (h)].

- **Morphological algorithm:**

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

Stop when $X_k = X_{k-1}$



a b

FIGURE 9.18 (a) Binary image. The white dots inside the regions (shown enlarged for clarity) are the starting points for the hole-filling algorithm. (b) Result of filling all holes.

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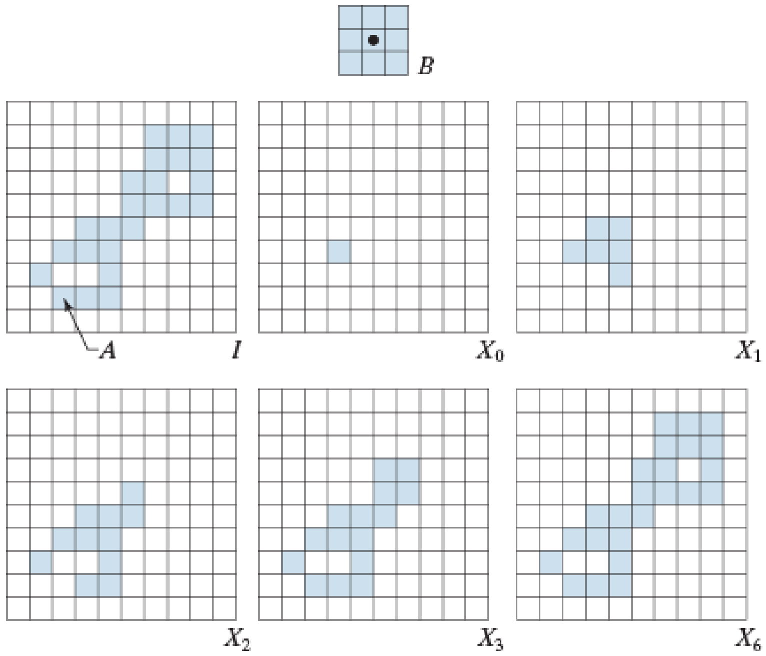
Extraction of Connected components (连通分量提取)

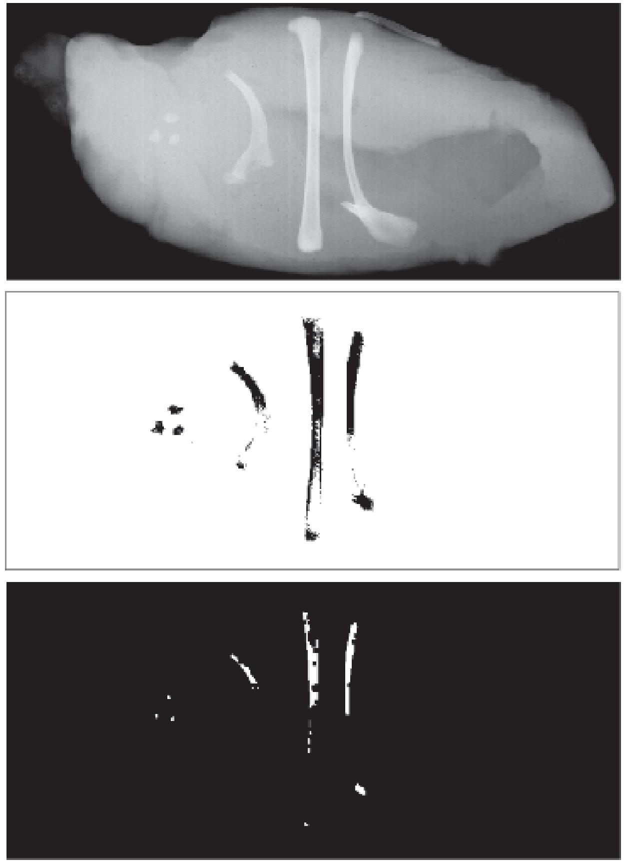
➢ **Morphological algorithm:**

$$X_k = (X_{k-1} \oplus B) \cap A$$



a
b c d
e f g

FIGURE 9.19
(a) Structuring element.
(b) Image containing a set with one connected component.
(c) Initial array containing a 1 in the region of the connected component.
(d)–(g) Various steps in the iteration of Eq. (9-20)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.20
(a) X-ray image of a chicken filet with bone fragments.
(b) Thresholded image (shown as the negative for clarity).
(c) Image eroded with a 5×5 SE of 1's.
(d) Number of pixels in the connected components of (c).
(Image (a) courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Convex Hull (凸壳)

➢ **Morphological algorithm:**

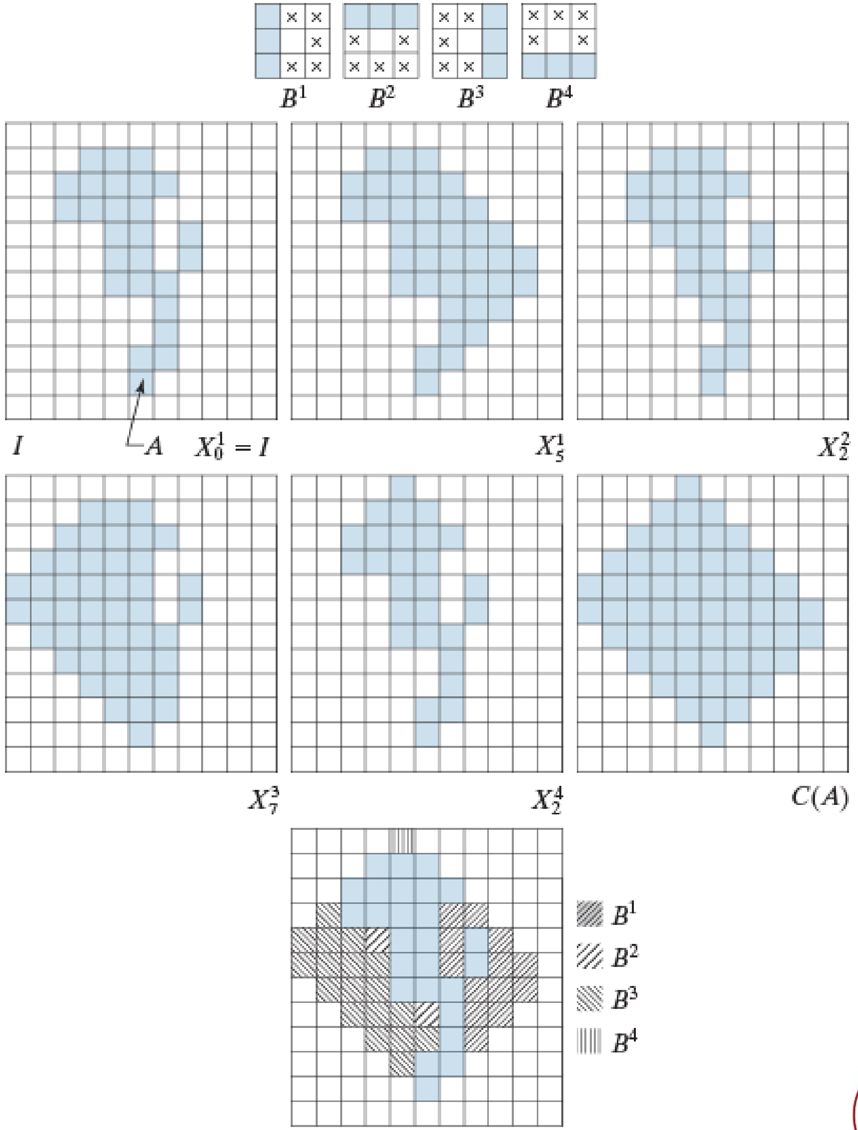
$$C(A) = \bigcup_{i=1}^{4} D^i$$

Where B^i: structuring elements

$$X_0^i = A$$

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$$

$$D^i = X_k^i \text{ when } X_k^i = X_{k-1}^i$$

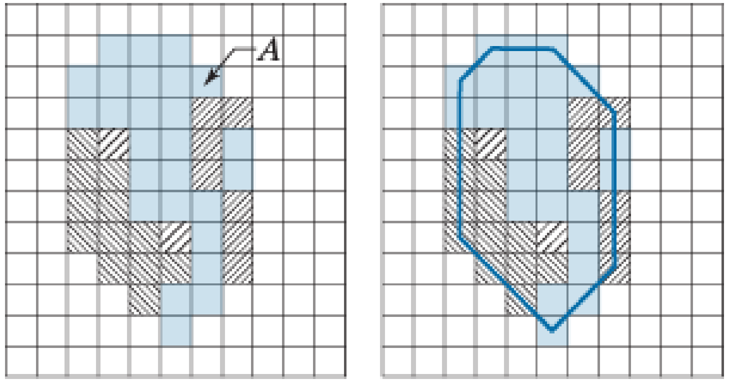


a
b c d
e f g
h

FIGURE 9.21 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

a b

FIGURE 9.22 (a) Result of limiting growth of the convex hull algorithm. (b) Straight lines connecting the boundary points show that the new set is convex also.



Thinning (细化)

- **Morphological algorithm**:

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

Let $B = \{B^1, B^2, B^3 \cdots, B^n\}$

$$A \otimes \{B\} = ((\cdots((A \otimes B^1) \otimes B^2) \cdots) \otimes B^n)$$

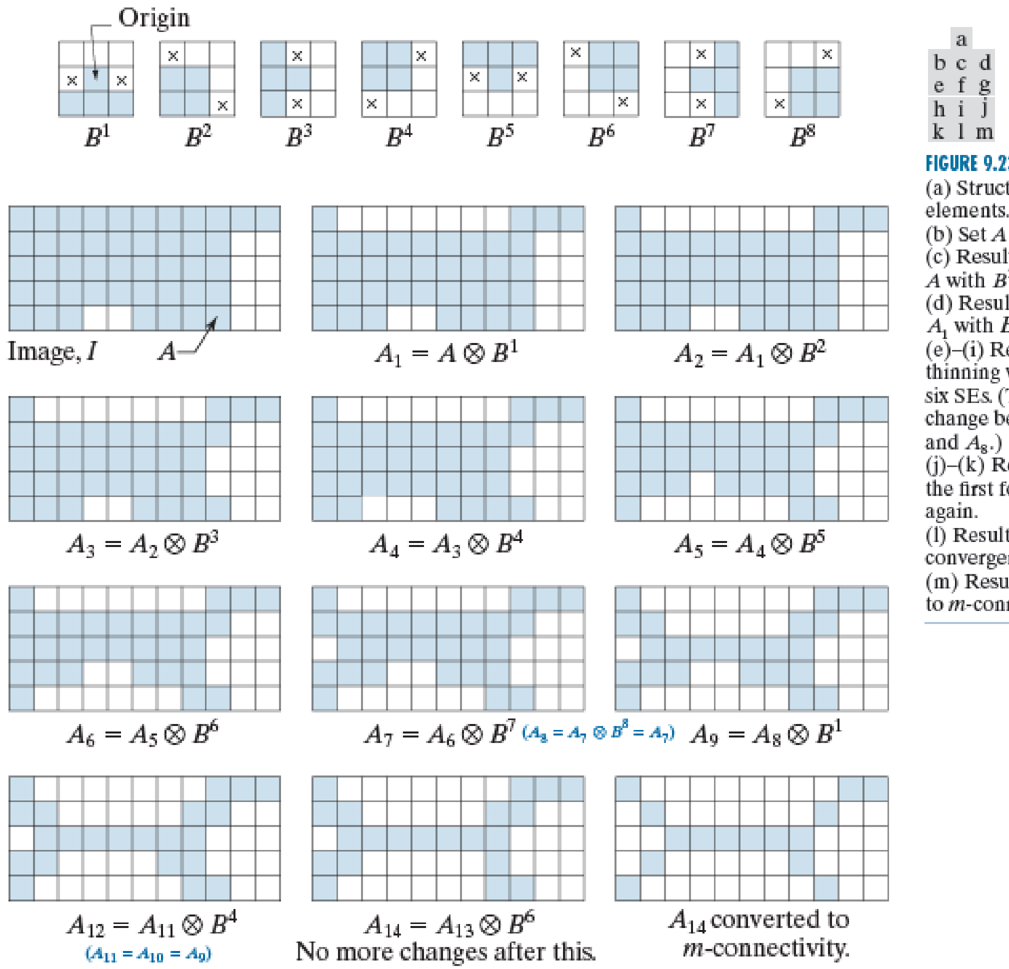


FIGURE 9.23 (a) Structuring elements. (b) Set A. (c) Result of thinning A with B^1 (shaded). (d) Result of thinning A_1 with B_2. (e)–(i) Results of thinning with the next six SEs. (There was no change between A_7 and A_8.) (j)–(k) Result of using the first four elements again. (l) Result after convergence. (m) Result converted to m-connectivity.

Thickening (粗化)

- **Morphological algorithm:**

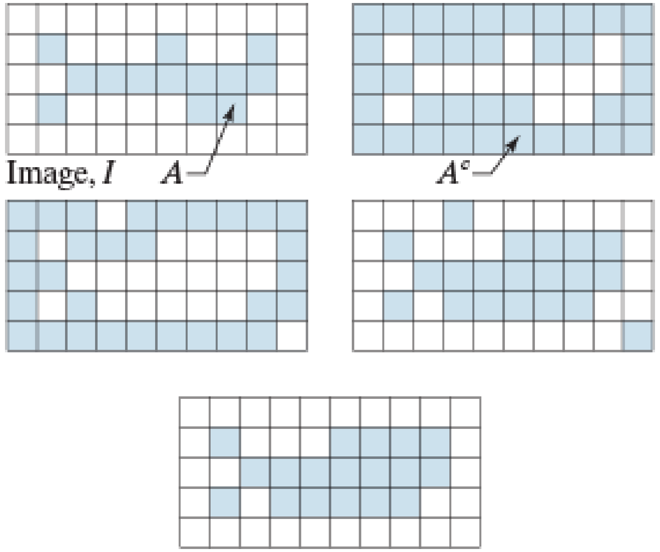
$$A \odot B = A \cup (A \circledast B)$$

Let $B = \{B^1, B^2, B^3 \cdots, B^n\}$

$$A \odot \{B\} = ((\cdots((A \odot B^1) \odot B^2) \cdots) \odot B^n)$$

- **Alternative operation**

1. Thinning the background
2. Complement the results

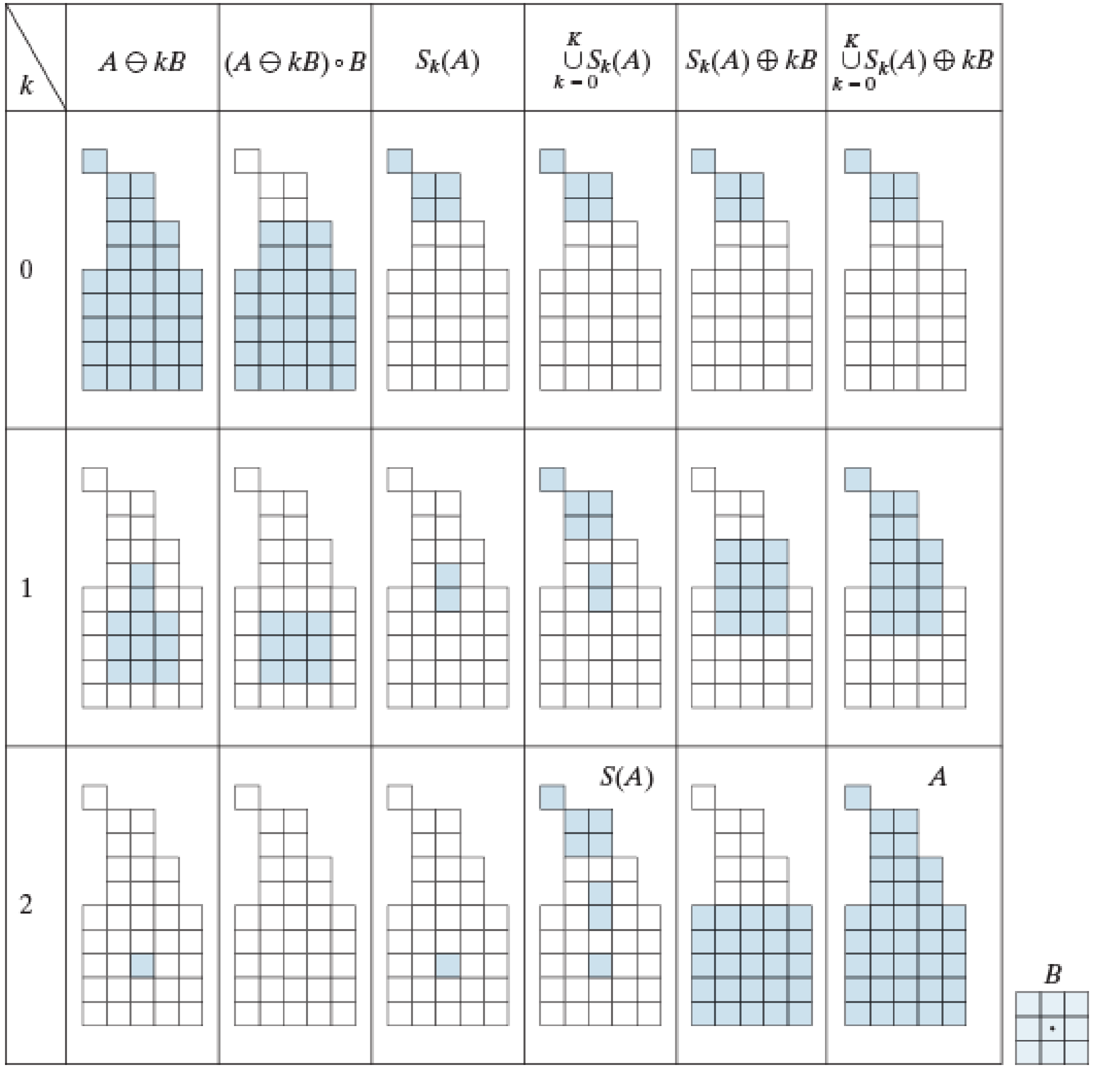


a b
c d
e

FIGURE 9.24
(a) Set A.
(b) Complement of A.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points.

Skeleton (骨架)

FIGURE 9.26 Implementation of Eqs. (9-28) through (9-33). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.



- **Morphological algorithm:**

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

Where

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = ((\cdots((A \ominus B) \ominus B)\cdots) \ominus B)$$

$$K = \max\{k | A \ominus kB \neq \emptyset\}$$

A can be reconstructed by

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

Where $(A \oplus kB) = ((\cdots((A \oplus B) \oplus B)\cdots) \oplus B)$

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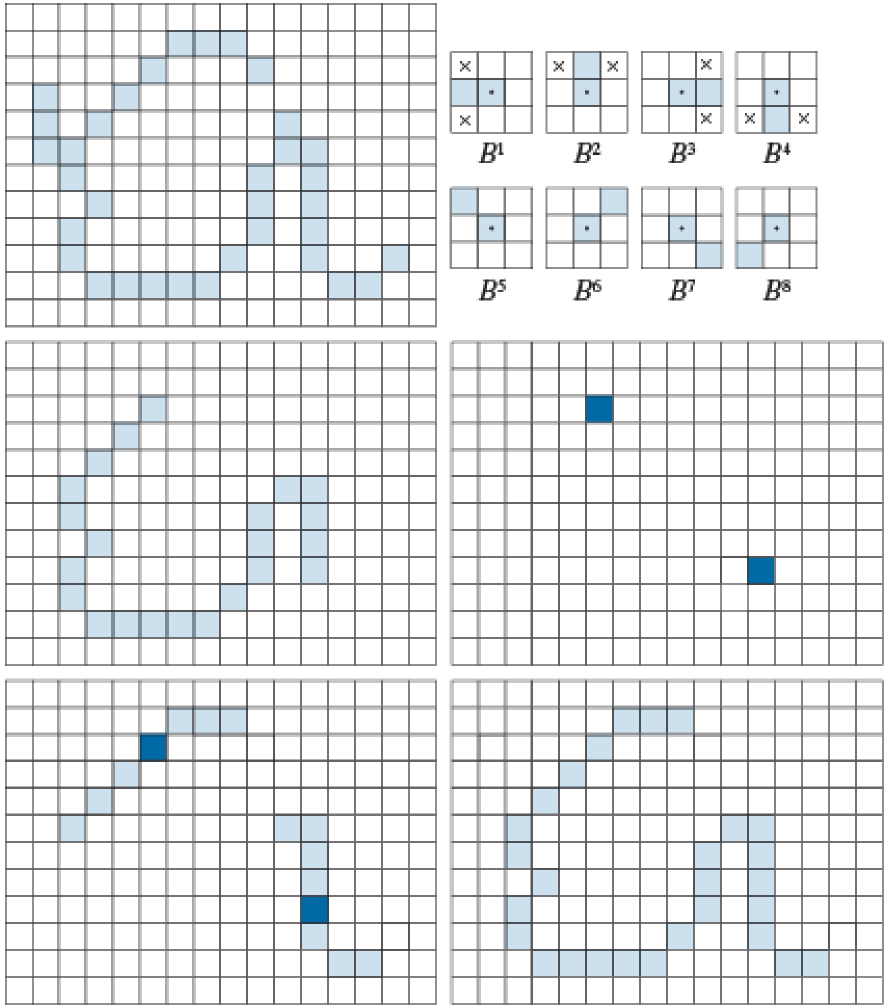
Pruning (裁剪)



a b
c d
e f

FIGURE 9.27 (a) Set A of foreground pixels (shaded). (b) SEs used for deleting end points. (c) Result of three cycles of thinning. (d) End points of (c). (e) Dilation of end points conditioned on (a). (f) Pruned image.

- **Morphological algorithm:**

1. Thinning: $X_1 = A \otimes \{B\}$
2. Finding endpoints: $X_2 = \bigcup_{k=1}^{8}(X_1 \circledast B^k)$
3. Dilating from endpoints: $X_3 = (X_2 \oplus H) \cap A$
4. $P = X_3 \cup X_1$

Morphological reconstruction

- **Involving two images (marker and mask) :**
  - Marker (F): the starting points for reconstruction
  - Mask (G): constrain the reconstruction (conditions)
  - A structuring element (B): to define connectivity
- **Geodesic Dilation (测地膨胀)**
  - Size 1 of the marker image with respect to the Mask

$$D_G^{(1)}(F) = (F \oplus B) \cap G \quad \text{where } F \subseteq G$$

  - Size n of F with respect to G

$$D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$$

- **Geodesic Erosion (测地腐蚀)**
  - Size 1 of F with respect to G

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

  - Size n of F with respect to G

$$E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$$

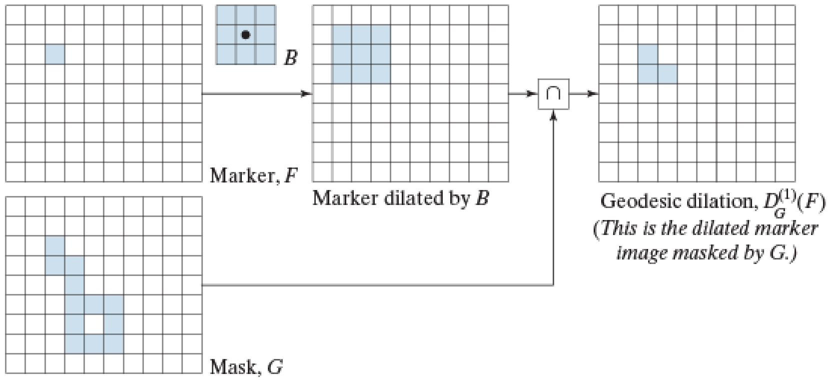


FIGURE 9.28 Illustration of a geodesic dilation of size 1. Note that the marker image contains a point from the object in G. If continued, subsequent dilations and maskings would eventually result in the object contained in G.

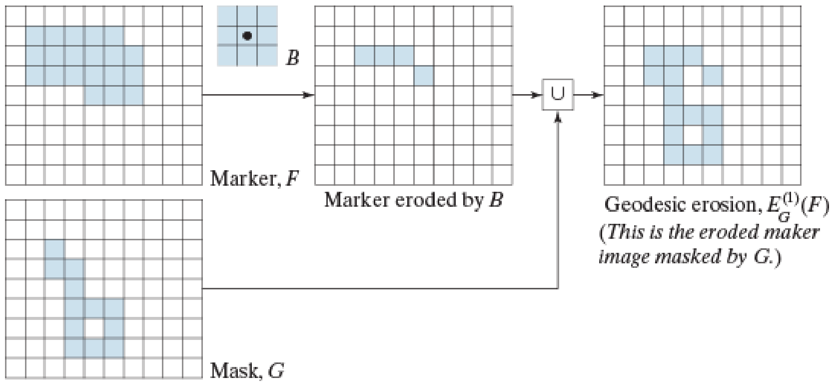


FIGURE 9.29 Illustration of a geodesic erosion of size 1.

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Morphological reconstruction

- **By dilation**

$$R_G^D(F) = D_G^{(k)}(F) \quad \text{with } k \text{ such that } D_G^{(k)}(F) = D_G^{(k+1)}(F)$$

- **By erosion**

$$R_G^E(F) = E_G^{(k)}(F) \quad \text{with } k \text{ such that } E_G^{(k)}(F) = E_G^{(k+1)}(F)$$

- **Examples**
  - Opening by reconstruction (重建开操作)

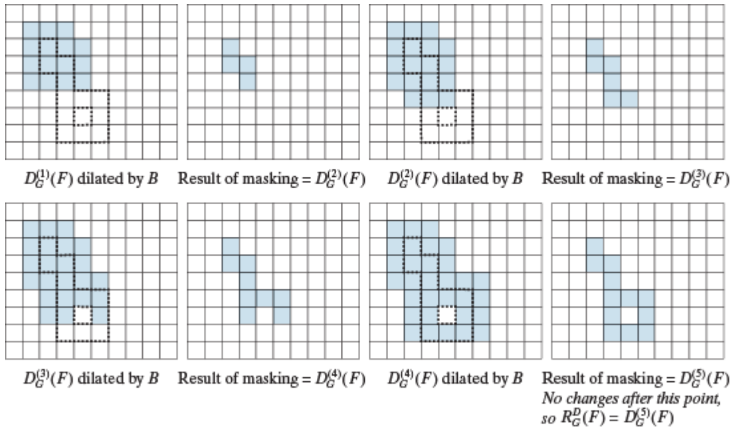
$$O_R^n(F) = R_F^D(F \ominus nB)$$

  - Automatic Algorithm for Filing holes (填充孔洞)

$$H = \left(R_{I^c}^D(F)\right)^c \text{ where } F(x) = \begin{cases} 1 - I(x,y), & \text{if } (x,y) \text{ is on the border of } I \\ 0, & \text{otherwise} \end{cases}$$

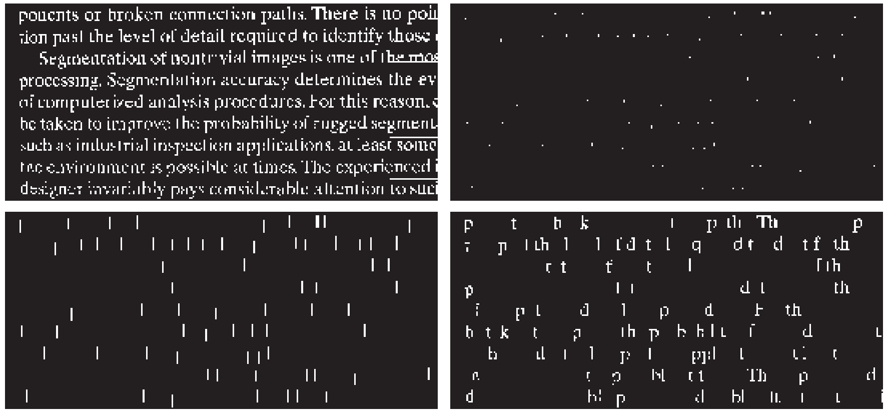
  - Border clearing (边界清除)

$$X = I - R_I^D(F) \text{ where } F(x) = \begin{cases} 1 - I(x,y), & \text{if } (x,y) \text{ is on the border of } I \\ 0, & \text{otherwise} \end{cases}$$



a b c d
e f g h

FIGURE 9.30 Illustration of morphological reconstruction by dilation. Sets $D_G^{(1)}(F)$, G, B and F are from Fig. 9.28. The mask (G) is shown dotted for reference.



a b
c d

FIGURE 9.31 (a) Text image of size 918×2018 pixels. The approximate average height of the tall characters is 51 pixels. (b) Erosion of (a) with a structuring element of size 51×1 elements (all 1's). (c) Opening of (a) with the same structuring element, shown for comparison. (d) Result of opening by reconstruction.

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Summary

FIGURE 9.35 Five basic types of structuring elements used for binary morphology.

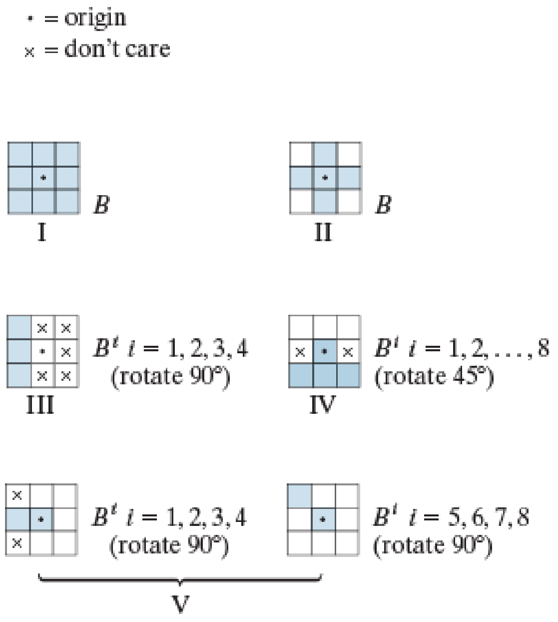


TABLE 9.1 Summary of binary morphological operations and their properties. A is a set of foreground pixels contained in binary image I, and B is a structuring element. I is a binary image (containing A), with 1's corresponding to the elements of A and 0's elsewhere. The Roman numerals refer to the structuring elements in Fig. 9.35.

Operation	Equation	Comments
Translation	$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$	Reflects B about its origin.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points in A, but not in B.
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	Erodes the boundary of A. (I)
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	Dilates the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$I \circledast B = \{z \mid (B)_z \subseteq I\}$	Finds instances of B in image I. B contains *both* foreground and background elements.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap I^c$ $k = 1, 2, 3, \ldots$	Fills holes in A. X_0 is of same size as I, with a 1 in each hole and 0's elsewhere. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap I$ $k = 1, 2, 3, \ldots$	Finds connected components in I. X_0 is a set, the same size as I, with a 1 in each connected component and 0's elsewhere. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup X_{k-1}^i$; $i = 1, 2, 3, 4 \quad k = 1, 2, 3, \ldots$ $X_0^i = I$; $D^i = X_{conv}^i$; $C(A) = \bigcup_{i=1}^{4} D^i$	Finds the convex hull, $C(A)$, of a set, A, of foreground pixels contained in image I. X_{conv}^i means that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$	Thickens set A using a sequence of structuring elements, as above. Uses (IV) with 0's and 1's reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements (V) are used for the first two equations. In the third equation H denotes structuring element. (I)
Geodesic dilation–size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and the *mask* images, respectively. (I)
Geodesic dilation–size n	$D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$	Same comment as above.
Geodesic erosion–size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	Same comment as above.
Geodesic erosion–size n	$E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$	Same comment as above.
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	With k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$.

Function for Morphological Algorithms

- **Matlab Function:**

BW2 = bwmorph(BW,operation,n)

Operation	Description
bothat	"Bottom-hat" operation using a 3 × 3 structuring element; use imbothat (see Section 9.6.2) for other structuring elements.
bridge	Connect pixels separated by single-pixel gaps.
clean	Remove isolated foreground pixels.
close	Closing using a 3 × 3 structuring element of 1s; use imclose for other structuring elements.
diag	Fill in around diagonally-connected foreground pixels.
dilate	Dilation using a 3 × 3 structuring element of 1s; use imdilate for other structuring elements.
erode	Erosion using a 3 × 3 structuring element of 1s; use imerode for other structuring elements.
fill	Fill in single-pixel "holes" (background pixels surrounded by foreground pixels); use imfill (see Section 10.1.2) to fill in larger holes.
hbreak	Remove H-connected foreground pixels.
majority	Make pixel p a foreground pixel if at least five pixels in $N_8(p)$ (see Section 9.4) are foreground pixels; otherwise make p a background pixel.
open	Opening using a 3 × 3 structuring element of 1s; use function imopen for other structuring elements.
remove	Remove "interior" pixels (foreground pixels that have no background neighbors).
shrink	Shrink objects with no holes to points; shrink objects with holes to rings.
skel	Skeletonize an image.
spur	Remove spur pixels.
thicken	Thicken objects without joining disconnected 1s.
thin	Thin objects without holes to minimally-connected strokes; thin objects with holes to rings.
tophat	"Top-hat" operation using a 3 × 3 structuring element of 1s; use imtophat (see Section 9.6.2) for other structuring elements.

- Thinning

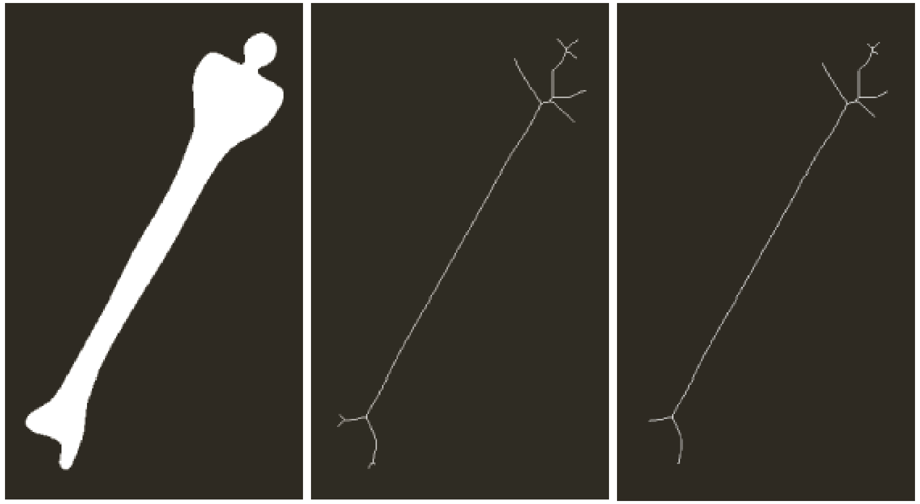
```
>> g1 = bwmorph(f, 'thin', 1);
>> g2 = bwmorph(f, 'thin', 2);
>> ginf = bwmorph(f, 'thin', Inf);
```

a b c

FIGURE 9.15 (a) Fingerprint image from Fig. 9.11(c) thinned once. (b) Image thinned twice. (c) Image thinned until stability.

- Skeleton

```
>> fs = bwmorph(f, 'skel', Inf);
```



a b c

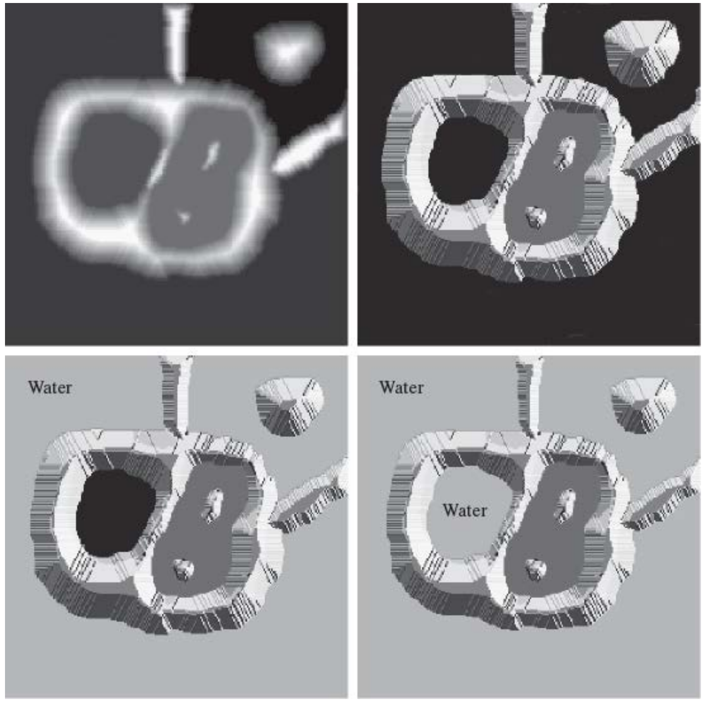
FIGURE 9.16 (a) Bone image. (b) Skeleton obtained using function bwmorph. (c) Resulting skeleton after pruning with function endpoints.

Morphological Watersheds

- **“Topographic” interpretation:**
  - **Catchment Basin(汇水盆地) or Watershed(分水岭)**: points of minimum and falling with certainty to a single minimum;
  - **Divided Line(分割线) or Watershed Line(分水线):** points equally likely falling to more than one minimum.
- **Often applied to the gradient of images**

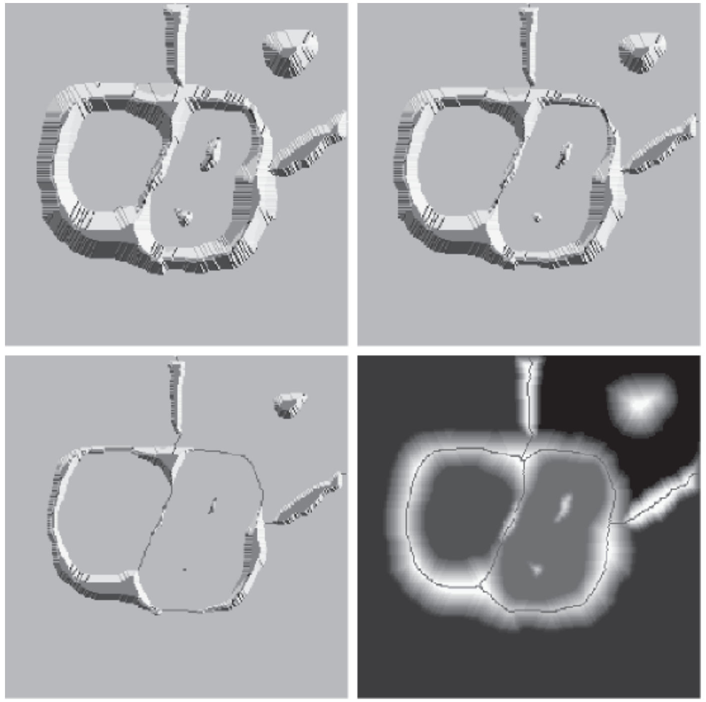
a c
b d

FIGURE 10.60 (a) Original image. (b) Topographic view. Only the background is *black*. The basin on the left is slightly lighter than black. (c) and (d) Two stages of flooding. All constant dark values of gray are intensities in the original image. Only constant *light gray* represents “water.” (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.) (*Continued on next page.*)



e f
g h

FIGURE 10.60 (*Continued*) (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines superimposed on the original image. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)



Dam Construction

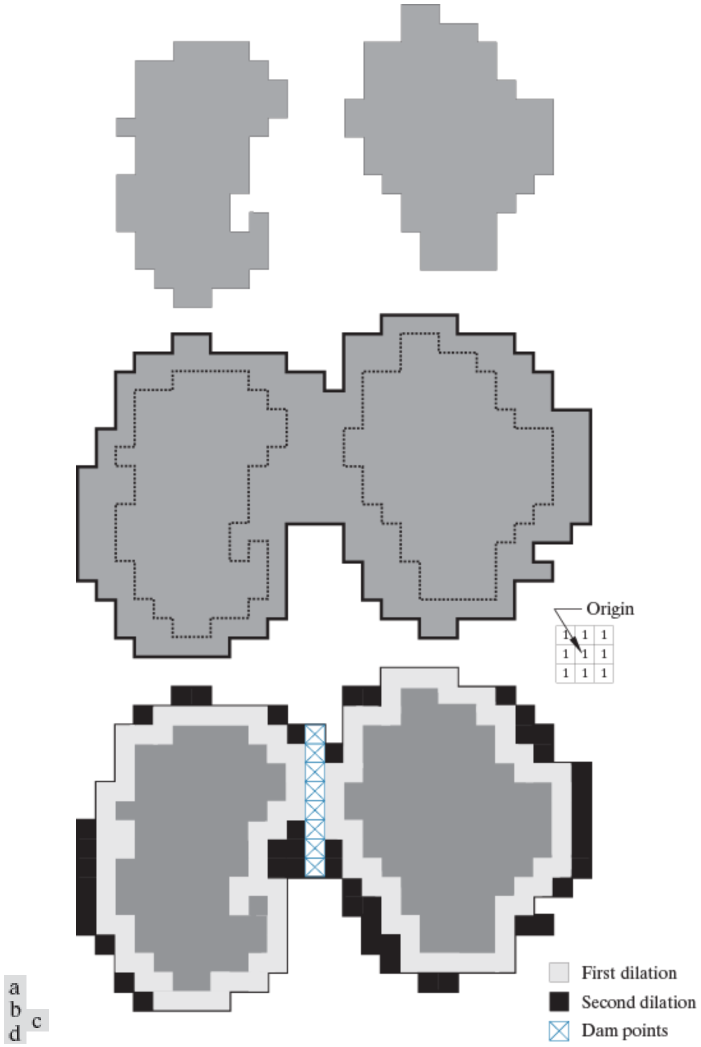


FIGURE 10.61 (a) Two partially flooded catchment basins at stage $n-1$ of flooding. (b) Flooding at stage n, showing that water has spilled between basins. (c) Structuring element used for dilation. (d) Result of dilation and dam construction.

Using morphological dilation

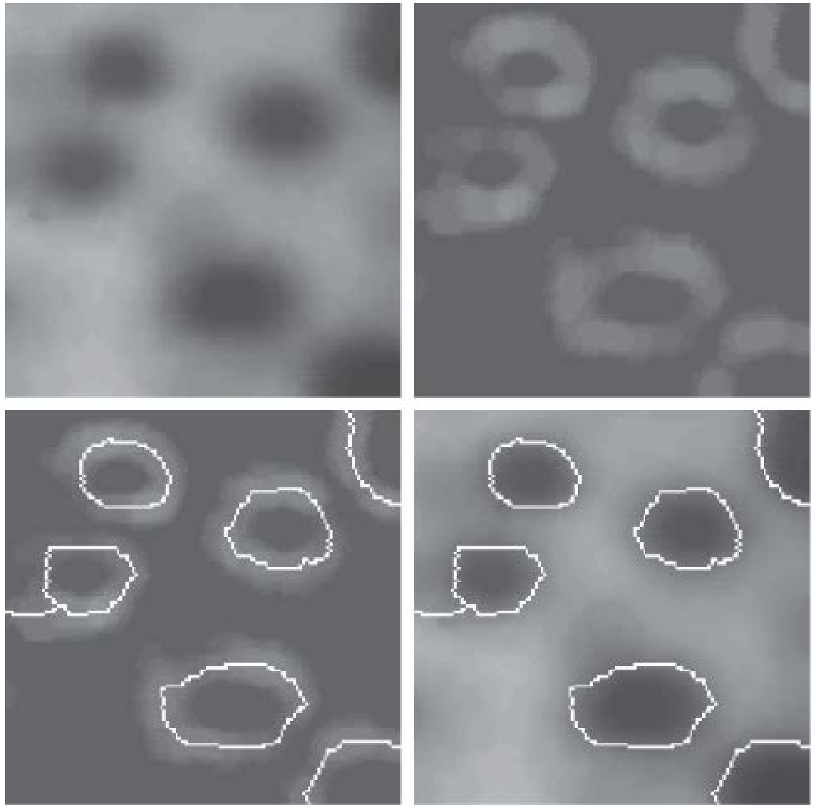
- Subject to two conditions:
  1. The dilation has to be constrained to *q*;
  2. The dilation cannot be performed on points that would cause the sets being dilated to merge;
- Points in *q* that satisfy the two conditions describe the one-pixel-thick separating dam;

Where *q* is the connected component after two catchment basins has merged.

Matlab function: L = watershed(A, conn)

Watershed segmentation

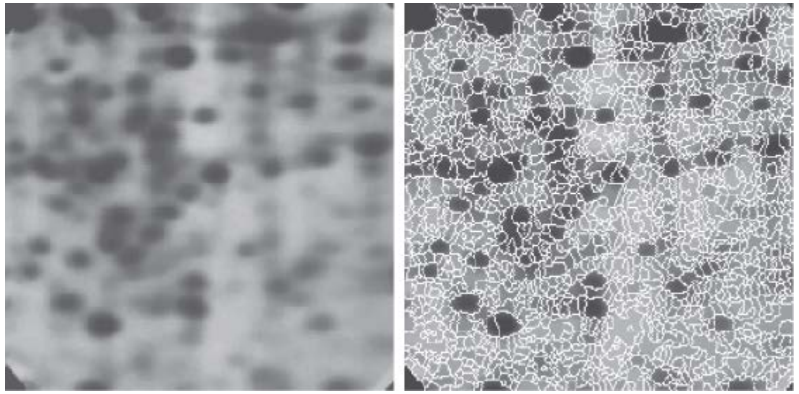
- Using Gradient operators



a b
c d

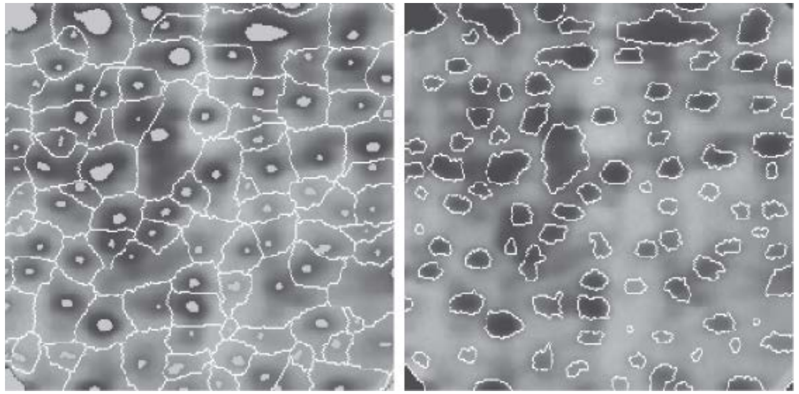
FIGURE 10.62 (a) Image of blobs. (b) Image gradient. (c) Watershed lines, superimposed on the gradient image. (d) Watershed lines superimposed on the original image. (Courtesy of Dr. S. Beucher, CMM/ Ecole des Mines de Paris.)

- The Use of Markers



a b

FIGURE 10.63 (a) Electrophoresis image. (b) Result of applying the watershed segmentation algorithm to the gradient image. Over-segmentation is evident. (Courtesy of Dr. S. Beucher, CMM/ Ecole des Mines de Paris.)



a b

FIGURE 10.64 (a) Image showing internal markers (light gray regions) and external markers (watershed lines). (b) Result of segmentation. Note the improvement over Fig. 10.63(b). (Courtesy of Dr. S. Beucher, CMM/ Ecole des Mines de Paris.)