Lecture 18 – Motion & Active contours

This lecture will cover:

- Active Contours (活动轮廓)
  - Image Segmentation Using Snakes
  - Segmentation Using Level Sets (水平集)
- The use of motion in segmentation
  - Spatial techniques
  - Frequency domain techniques

Segmentation – Active Contours

- **Active contours**
  - Deformable models confined to the plane
  - Approaching segmentation from a "modeling" point of view
  - Being attracted to region boundaries under the influence of forces extracted typically from an image being segmented
- **Two approaches**
  - **Snakes** – explicit (parametric) representation of segmentation curves
    - ✓ Parametric curves that seek the boundary of a region by minimizing an energy functional
    - ✓ Confined by internal forces and external forces (image forces)
  - **Level Sets** – implicit representation of curves
    - ✓ Following fronts propagating through a medium
    - ✓ The intersection of a 3-D surface with a plane

Segmentation using Snakes

- **The energy associated with a snake curve**

$$E(\boldsymbol{c}) = E_{internal} + E_{external} \quad \Rightarrow \quad E\big(\boldsymbol{c}(s)\big) = \frac{\alpha}{2}\int_0^1 \|\boldsymbol{c}'(s)\|^2 ds + \frac{\beta}{2}\int_0^1 \|\boldsymbol{c}''(s)\|^2 ds + \int_0^1 E_{image}\big(\boldsymbol{c}(s)\big) ds$$

Where $\boldsymbol{c}(s)$: parameterized (typically closed) curve and $\boldsymbol{c}(s) = \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}$

$E_{internal}$: internal energy

- Associated with the snake itself
- Including elastic energy related to $\boldsymbol{c}'(s)$ and bending energy related to $\boldsymbol{c}''(s)$,
- To keep the curve smooth and not stretching

$E_{external}$: external energy

- Obtained from the image being segmented related to $E_{image}\big(\boldsymbol{c}(s)\big)$
- To force the snake to areas of interest in segmentation

- **The solution of minimum energy:**

$$\alpha \boldsymbol{c}''(s) - \beta \boldsymbol{c}''''(s) + \boldsymbol{F}\big(\boldsymbol{c}(s)\big) = \boldsymbol{0} \qquad \text{Where} \quad \nabla E_{image} = -\boldsymbol{F}$$

External force of Snakes - MOG

- **The magnitude of the image gradient (MOG)**

$$E_{image}(x,y) = -\|\nabla f(x,y)\|^2$$

$$= -\left[\left(\frac{\partial f(x,y)}{\partial x}\right)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2\right]$$

Where $f(x,y)$ is an $M \times N$ image,

therefore the external energy

$$\boldsymbol{F} = -\nabla E_{image} = \boldsymbol{\nabla}[\|\nabla f(x,y)\|^2]$$

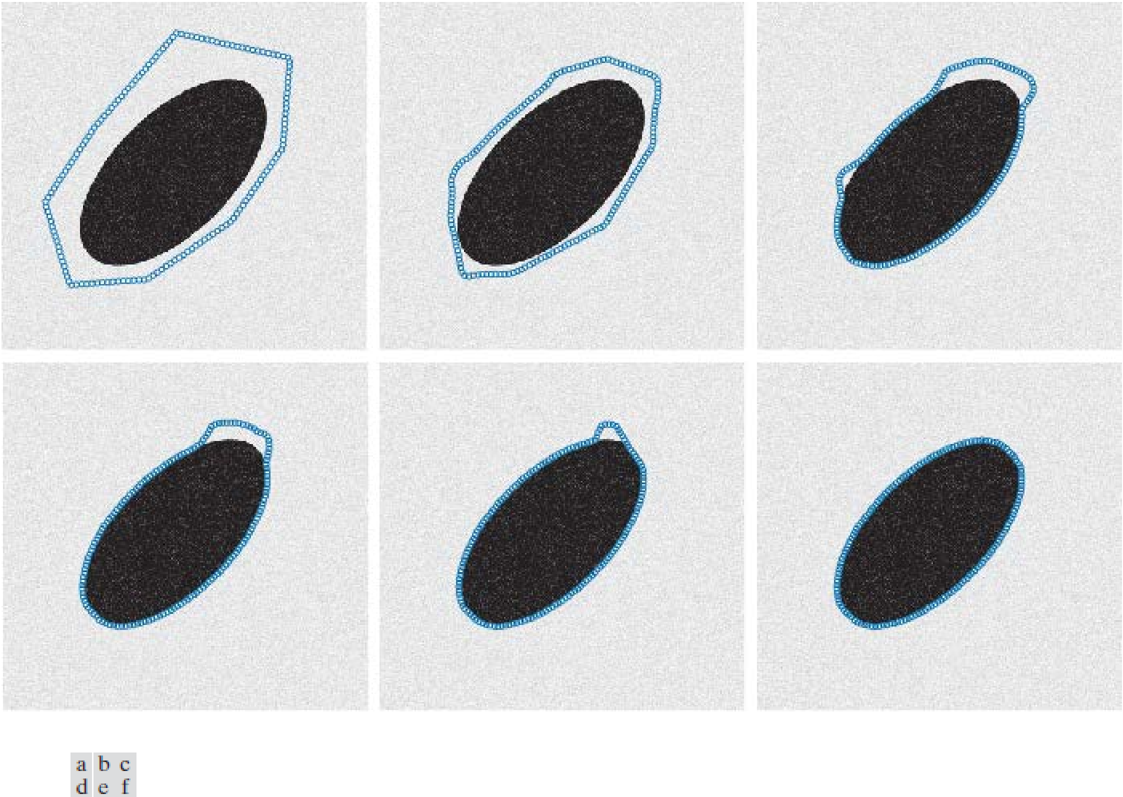
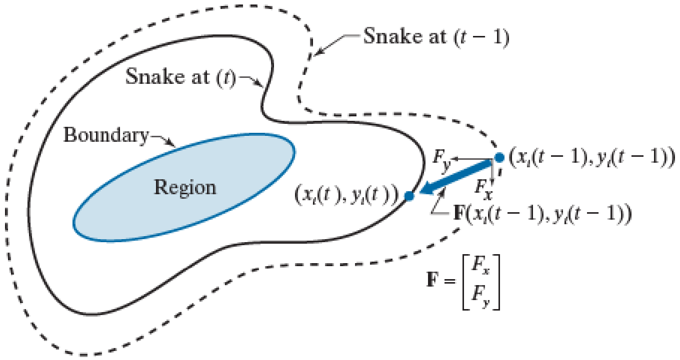


FIGURE 11.2 (a) Image and initial snake (the snake points are enclosed by small circles to make them easier to see). (b) Result after 10 iterations of Eq. (11-46) with $\alpha = 0.5$, $\beta = 0$, and $\gamma = 0.6$. Note how the snake is beginning to become smooth. (c) through (f) Results after 50, 100, 150, and 200 iterations, respectively.

FIGURE 11.1 Illustration of how a snake transitions from one time step to the next. Only two corresponding snake points are shown for clarity. The snake consists of K such points, each influenced by a different component of the external force.



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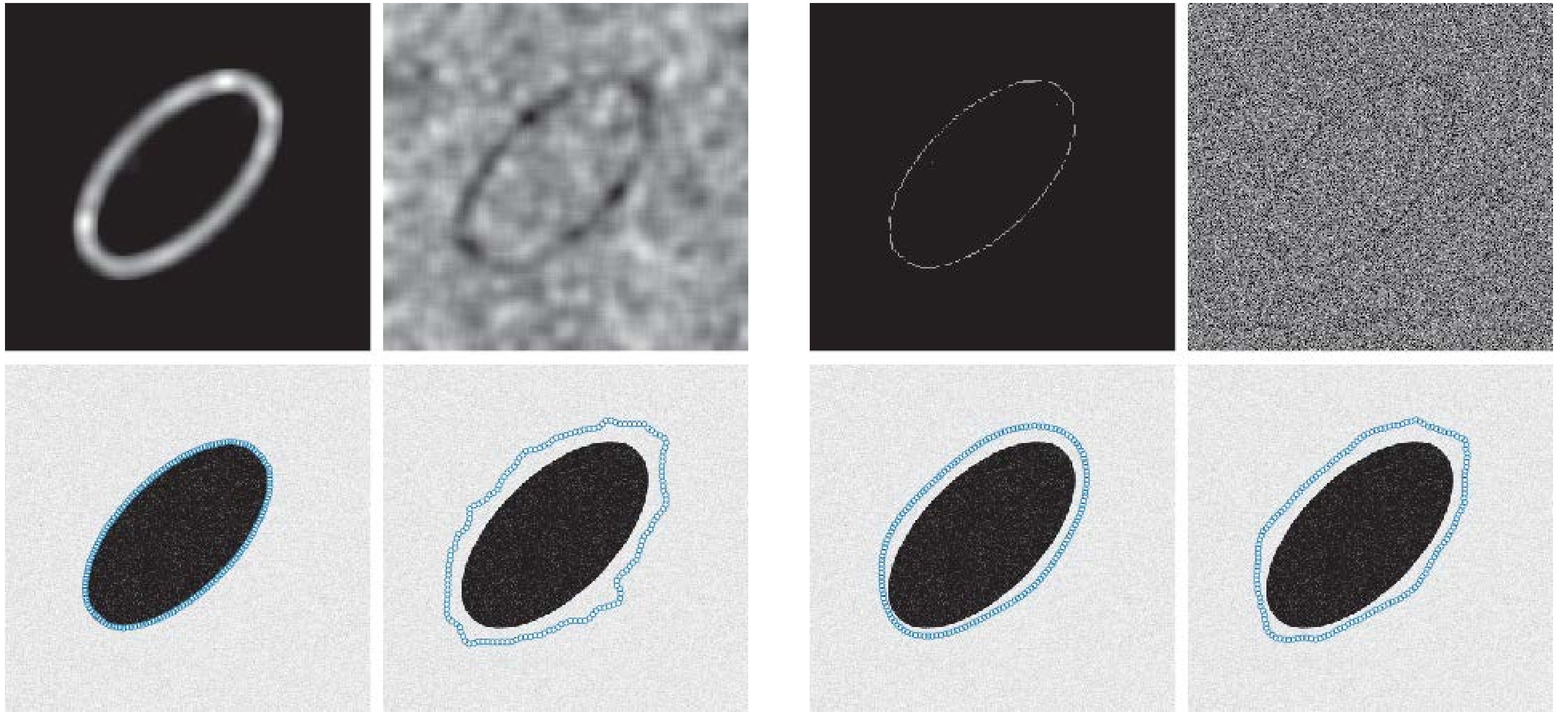
External force of Snakes - MOG

- “Spread out” the influence of the edge by blurring the edge map before using it to compute the force.

a b
c d

FIGURE 11.3
(a) Edge map used to generate the results in Fig. 11.2.
(b) Edge map with only the MOG filtered and then thresholded.
(c) Result after 200 iterations using the forces based on (a).
(d) Result after 200 iterations using the forces based on (b).
The initial snake is shown in Fig. 11.2(a).
(Continued)



e f
g h

FIGURE 11.3
(Continued)
(e) Edge map with the image filtered and MOG thresholded (but not filtered).
(f) Edge map with no filtering and the MOG thresholded.
(g) Result after 200 iterations using the forces based on (e).
(h) Result after 200 iterations using the forces based on (f).
The initial snake is shown in Fig. 11.2(a).

External force of Snakes - GVF

- **A gradient vector flow (GVF) field is defined as**

$$\varepsilon = \iint \mu\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial y}\right)^2 + \left(\frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2\right] + \|\nabla e\|^2 \|\mathbf{v} - \nabla e\|^2 \, dxdy$$

Where $\mathbf{v}(x, y)$: the vector field, $\mathbf{v}(x, y) = \begin{bmatrix} v_x(x, y) \\ v_y(x, y) \end{bmatrix}$

μ: a constant

$\|\nabla e\|^2$: the MOG of the edge map

- **Properties of the gradient vector flow (GVF) field**

  - When $\|\nabla e\|$ is small, yielding a slow-varying field; when $\|\nabla e\|$ is large, the energy is dominated by the image gradient (edge);

  - Keeping $\mathbf{v}$ nearly equal to the gradient of the edge map when the gradient is large, but forcing the field to be slow-varying in areas of nearly constant intensity.

GVF vs MOG

a b
c d

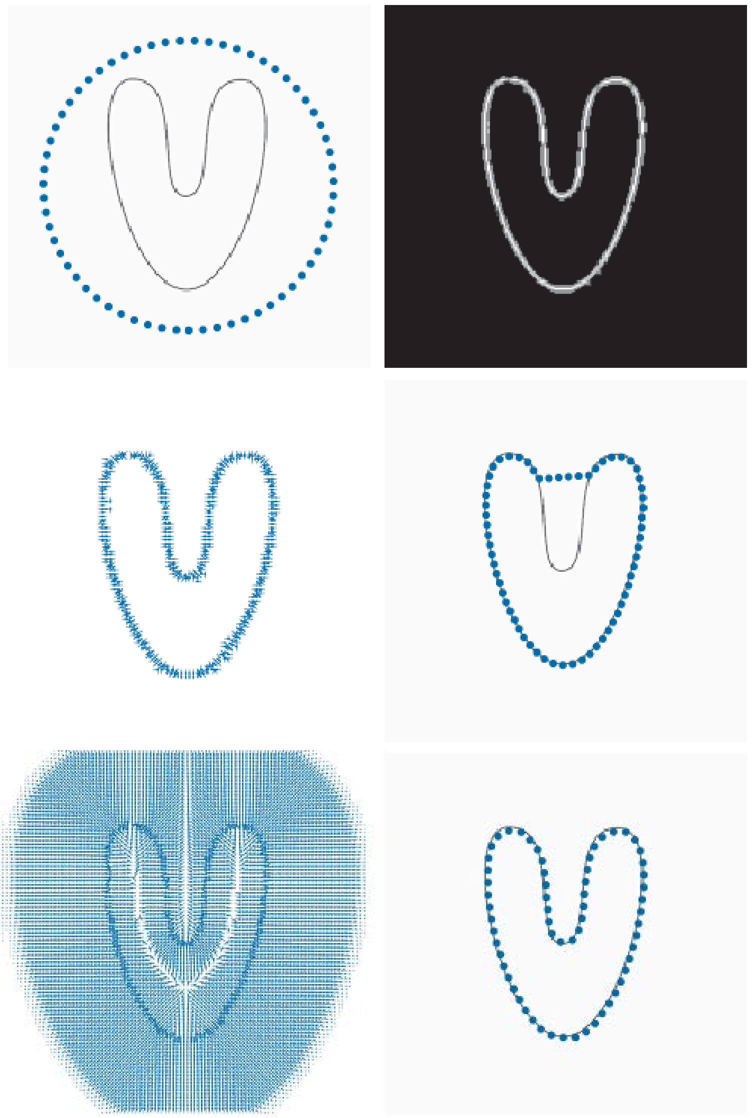


FIGURE 11.9 (a) A 200×200 image with intensities in the range [0,1], and an initial 64-point snake. (b) Edge map smoothed after computing the gradient using a Gaussian kernel of size $\sigma \times \sigma$ with $\sigma = 3$ (the smoothed gradient was thresholded at a value of 0.001). (c) Normalized force field based on the MOG. (d) Snake obtained after 1000 iterations using $\alpha = 0.06$, $\beta = 0$, and $\gamma = 1$, shown superimposed on the image.

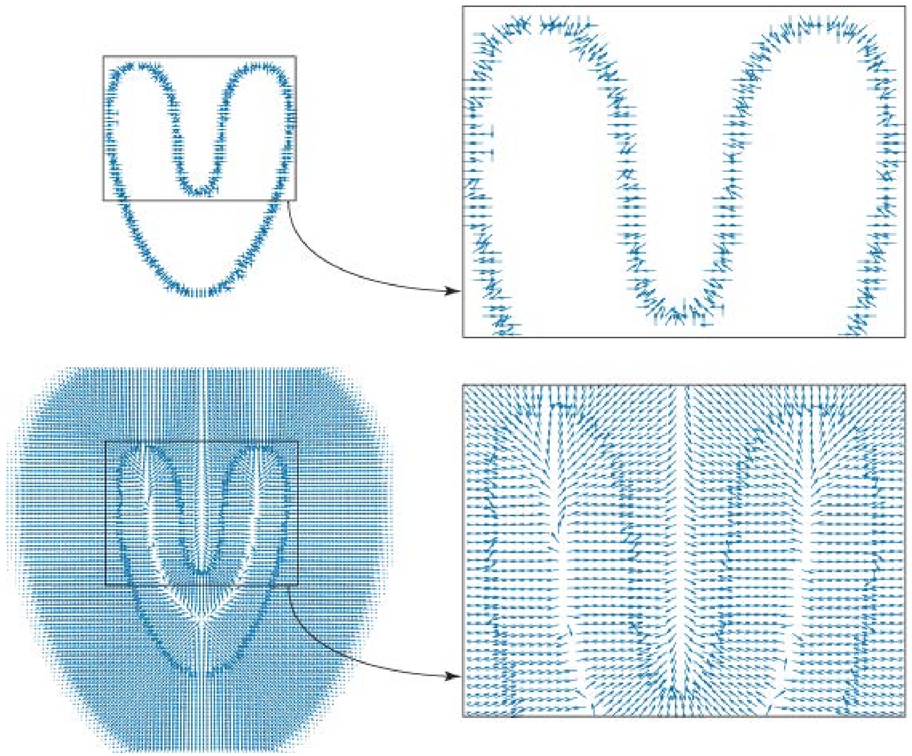


FIGURE 11.11 *Top:* Magnified section of the MOG force field in Fig. 11.9(c). *Bottom:* Corresponding magnified section from the GVF force field in Fig. 11.10(a).

a b

FIGURE 11.10 (a) Normalized GVF force field based on the edge map in Fig. 11.9(b), and obtained using Eq. (11-55) with $\mu = 0.25$ and 80 iterations. (b) Snake after 400 iterations using the same parameters as in Fig. 11.9(d). The GVF snake converged to the object's contour all the way into the deep concavity.

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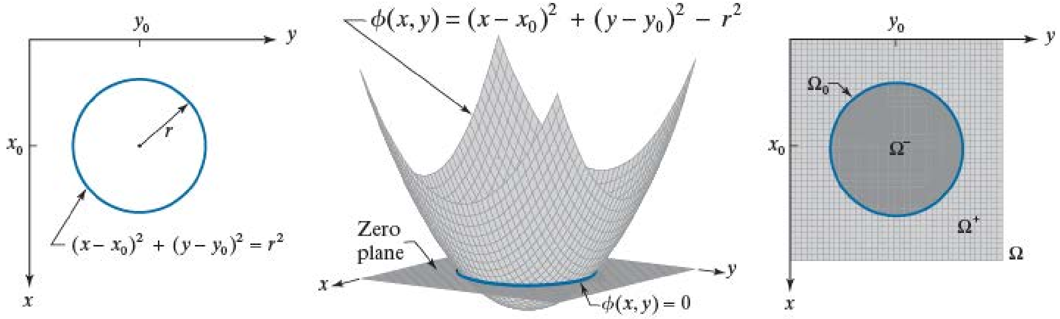
Segmentation using Level Set

- **Level set** – the set of points in the intersection of a place and a 3-D surface
- **The level set curve**

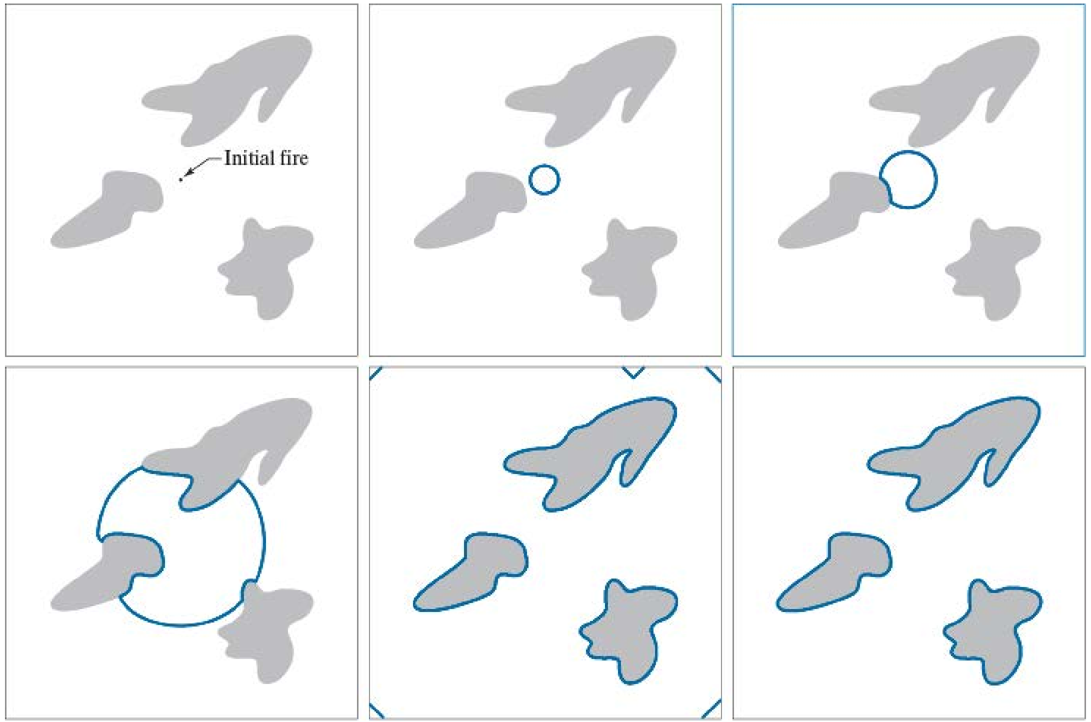
$$C = \{(x, y) | \phi(x, y) = 0\}$$

$$\text{Where} \quad \phi(x, y) = \begin{cases} > 0, & \text{for } (x, y) \in \Omega^+ \\ = 0, & \text{for } (x, y) \in \Omega_0 \\ < 0, & \text{for } (x, y) \in \Omega^- \end{cases}$$



a b c

FIGURE 11.14 (a) Cartesian equation of a circle and its corresponding plot. (b) The same circle, obtained as the level set curve $\phi(x, y) = 0$ (i.e., the intersection of $\phi(x, y)$ and the zero plane). (c) Top view of (b); the dark area enclosed by the circle is the visible section of the zero-plane. The symbols Ω_0, Ω^-, and Ω^+, are the sets of points (*on the plane*) that are on, inside, and outside the boundary, respectively, while Ω represents the entire image plane.



a b c
d e f

FIGURE 11.15 (a) Conceptual image of a dry grass field containing three lakes. The point represents an initial fire. (b) Fire front expanding uniformly at some later time. (c) The fire front encounters a lake shore, causing it to burn around the edge of the lake. (d) and (e) Results after further burning. (f) Result after the fire has completely burned out. This simple concept is the foundation of interface boundary evolution based on level sets.

Level Set Equation

- **The level set equation**

$$\frac{\partial \phi}{\partial t} = -F\|\nabla \phi\|$$

Where ϕ: surface scalar function $\phi = \phi(x(t), y(t), t)$

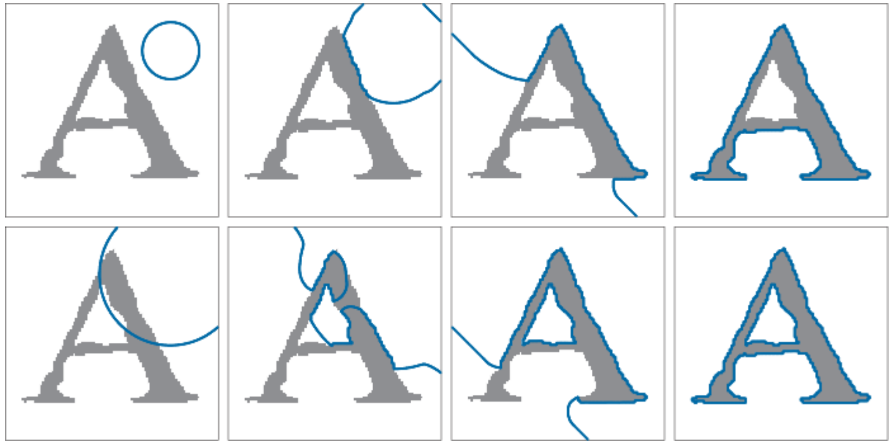
F: a force (speed) function acting in the direction normal to the curve at that point

- **Apply the level set equation to image segmentation**

1. Specify a suitable initial form for ϕ

The signed distance functions: $\phi(x,y) = \begin{cases} D(x,y) > 0, & \text{if } (x,y) \in \Omega^+ \\ 0, & \text{if } (x,y) \in \Omega_0 \\ -D(x,y) < 0, & \text{if } (x,y) \in \Omega^- \end{cases}$

2. Formulate F as a scalar field containing properties of interest in segmentation (e.g. edge content)
3. Solve the level set equation to find a ϕ that satisfies the equation
4. Extract the segmentation contour as the zero level set of ϕ



a b c d
e f g h

FIGURE 11.18 (a) Character image and initial zero level set boundary (gray = 0 and white = 1). (b)–(d) Results after 100, 400, and 900 iterations of Eq. (11-82) with $a = 1$ and $b = 0$ in the force definition. Only the outer boundary was detected. (e) A different initial level set function. (f)–(h) Results after 100, 400, and 900 iterations with $a = 1$ and $b = -1$ in the force definition. Both outer and inner boundaries were detected. (All curves are closed, but their values in (a), (d), and (h) are outside the confines of the image area.)

Force Function

Based only on image properties (evolve outward)

- **Characteristics**
  - Depending on pixel intensity values;
  - Calculated only once for a given image;
  - The force value acting on a point on the interface is determined completely by the location of that point;
- **Force function for grayscale images**

$$F(x,y) = \frac{1}{1 + \lambda \|\nabla[G_\sigma(x,y) \star f(x,y)]\|^p}$$

Where

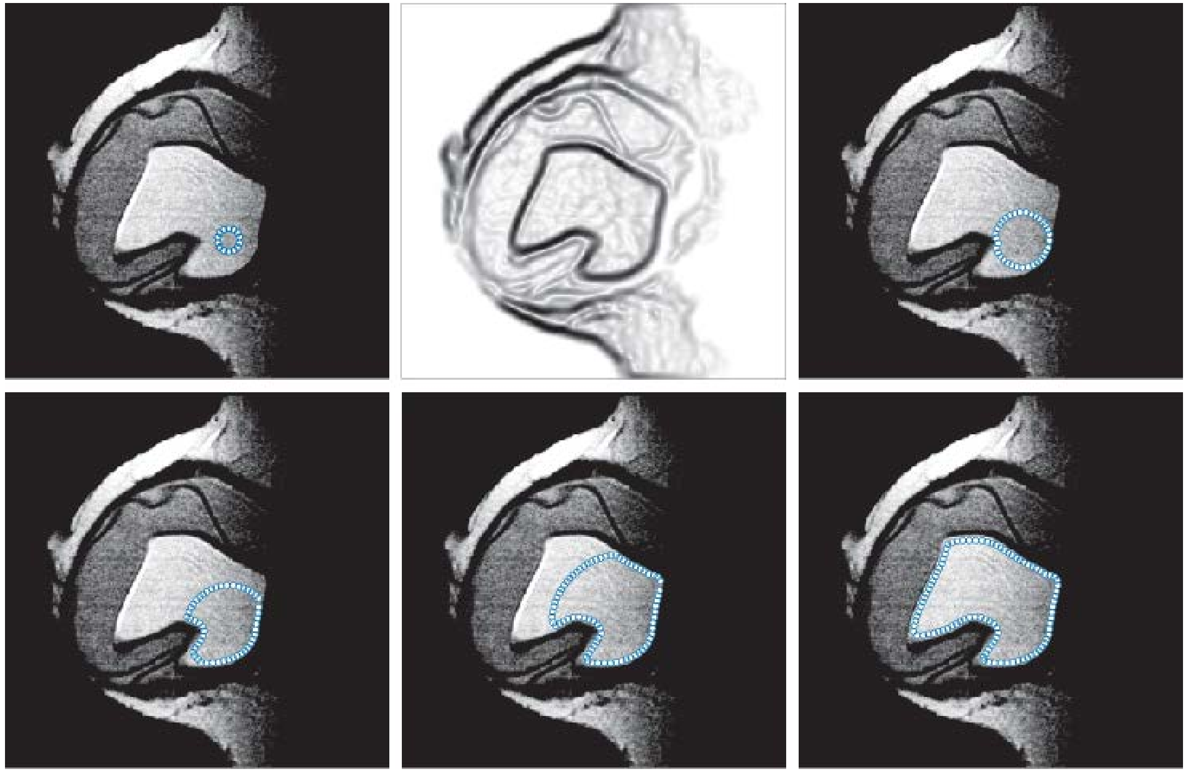
$f(x,y)$: the image

$G_\sigma(x,y)$: the smoothing Gaussian kernel

∇: the gradient operator

λ: a positive constant

p: a power constant, typically 1 or 2



a b c
d e f

FIGURE 11.22 (a) 586×600 MRI image of a breast implant and initial level set curve. (b) Force field displayed as an image. Results after: (c) 50 iterations, (d) 100 iterations, (e) 200 iterations, and (f) 400 iterations. (Original image courtesy of NIH/National Library of Medicine.)

Force Function

Edge/Curvature-Based Force (evolve inward)

- **Characteristics**
  - Based on the properties of image edge and level set curvature;
  - Force is calculated at every step because the level set function changes during iterations;

- **A geodesic curves**

$$E(\boldsymbol{c}(s)) = \frac{\alpha}{2}\int_0^1 \|\boldsymbol{c}'(s)\|^2 ds + \lambda \int_0^1 W(\|\nabla f(\boldsymbol{c}(s))\|)^2 ds$$

- **The level set equation**

$$\frac{\partial \phi}{\partial t} = (c+\kappa)W\|\nabla\phi\| + \nabla\phi \cdot \nabla W$$

Where W: stopping or edge-indication function $W = \frac{1}{1+\lambda\|\nabla \bar{f}\|^p}$

∇: the gradient operator

c: a constant

κ: the curvature of ϕ, $\kappa = \operatorname{div}\left(\frac{\nabla\phi\,(x,y)}{\|\nabla\phi\,(x,y)\|}\right) = \nabla \cdot \left(\frac{\nabla\phi\,(x,y)}{\|\nabla\phi\,(x,y)\|}\right)$

- **The force function:** $F = -\nabla \cdot \left(W \frac{\nabla\phi}{\|\nabla\phi\,\|}\right) - cW$

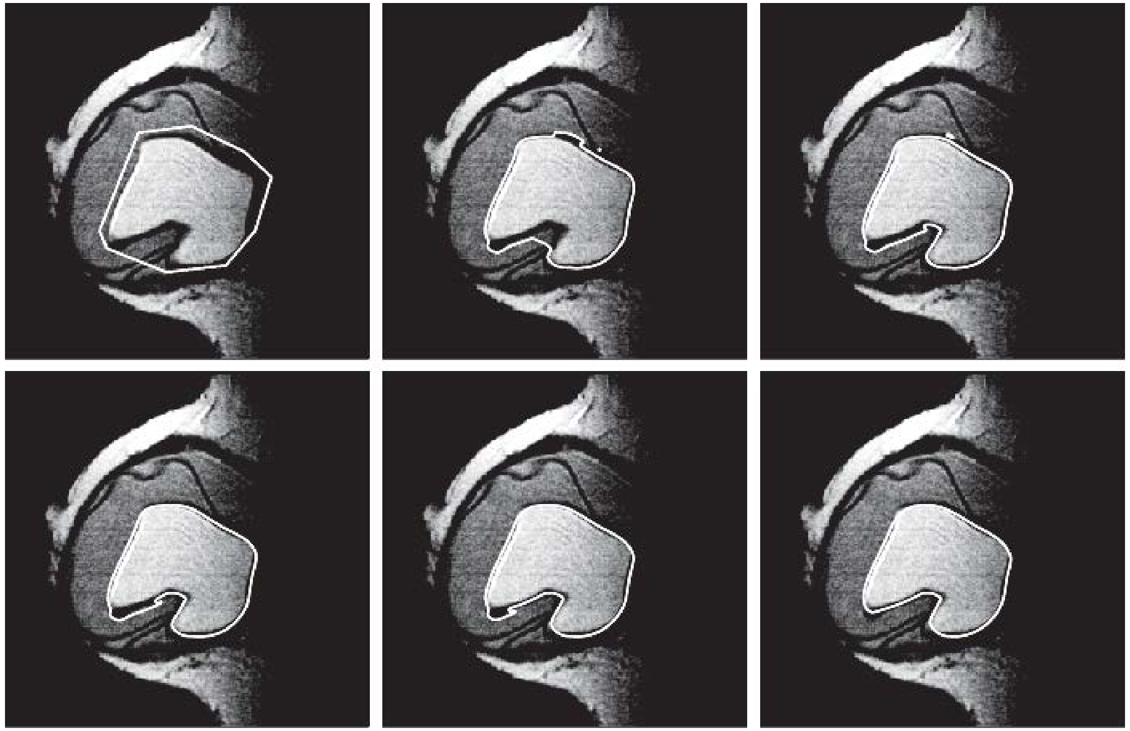


FIGURE 11.24 (a) 586 × 600 MRI image of a human breast and initial level set function. Segmentation results after: (b) 200, (c) 400, (d) 600, (e) 1000, and (f) 1500 iterations, respectively. Equations (11-82) and (11-102) were used. (Original image courtesy of NIH/National Library of Medicine.)

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Force Function

Region/Curvature-Based Force (evolve inward and outward)

- **Characteristics**
  - Based on the properties of region, and finding the boundary between regions;
  - Force is calculated at every step because the level set function changes during iterations;
- **Define a function for image with boundary C and regions inside and outside C**

$$E(C, c_1, c_2) = \mu \text{length}(C) + \nu \text{area}(inside(C))$$

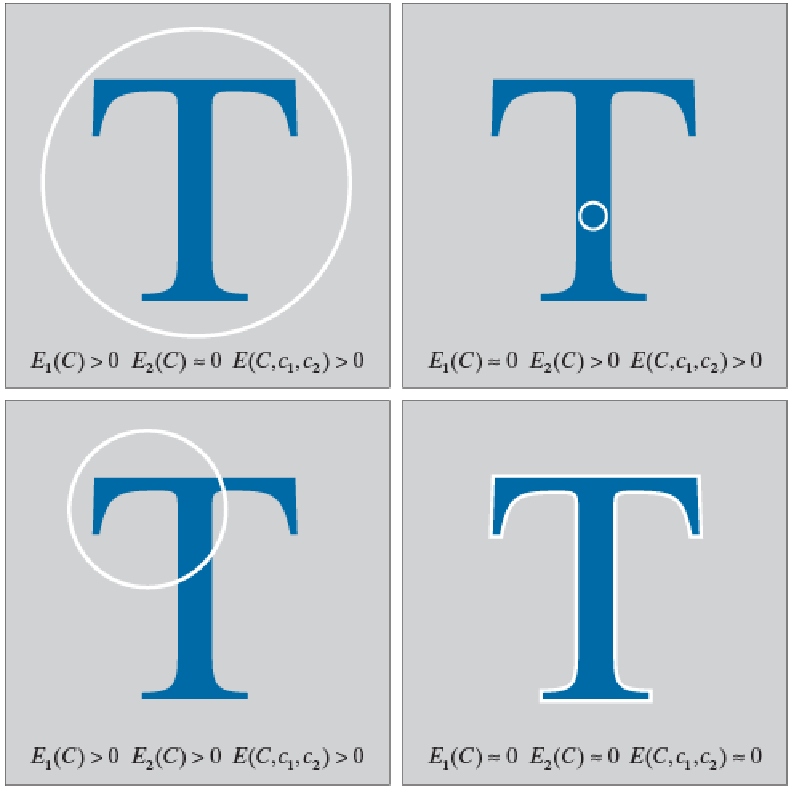
$$+\lambda_1 \int_{inside(C)} (f(x,y) - c_1)^2 dxdy$$

$$+\lambda_2 \int_{outside(C)} (f(x,y) - c_2)^2 dxdy$$

Where $\mu > 0, \nu \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$: fixed scalar parameters

c_1 : the average value of $f(x,y)$ inside C

c_2: the average value of $f(x,y)$ outside C



a b
c d

FIGURE 11.30 All four possible conditions for the terms in Eq. (11-104). Although all regions appear constant, they are slightly corrupted by noise, thus our use of "approximately equal" instead of "equal."

Force Function

Region/Curvature-Based Force (evolve inward and outward)

- **The level set equation**

$$\frac{\partial \phi}{\partial t} = \|\nabla \phi\| \left[\mu \nabla \cdot \left(\frac{\nabla \phi}{\|\nabla \phi\|} \right) - \nu - \lambda_1 (f - c_1)^2 + \lambda_2 (f - c_2)^2 \right]$$

Where

$\mu > 0, \nu \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$: fixed scalar parameters

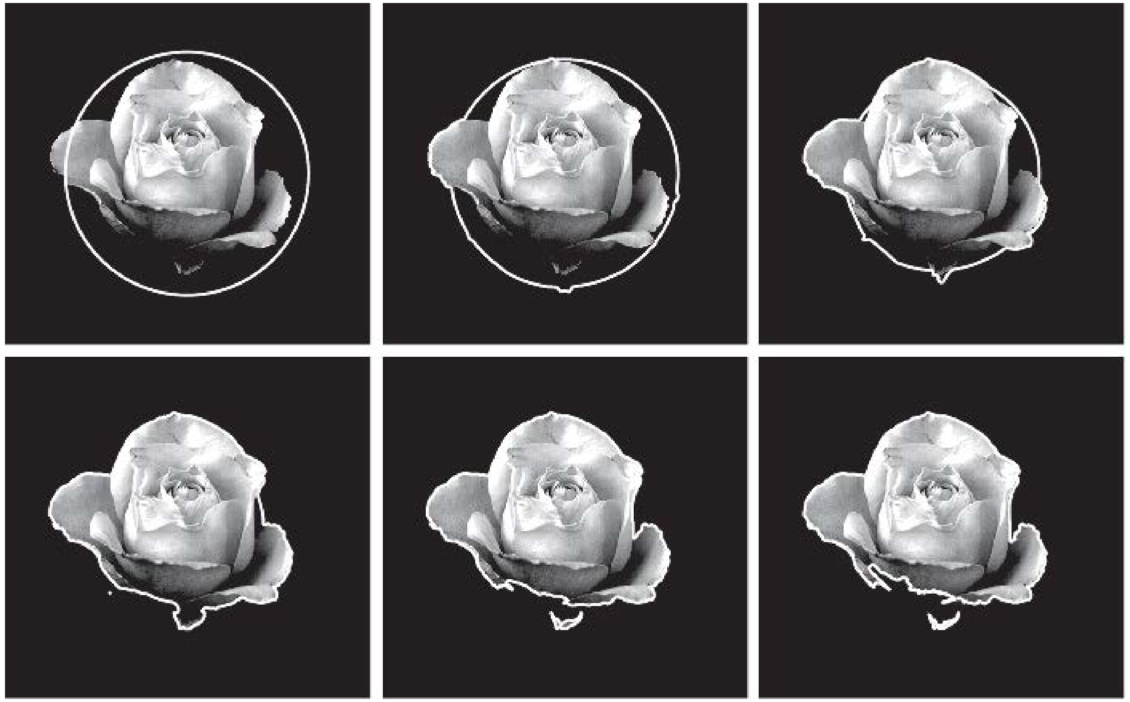
$$c_1 = \frac{\int_\Omega f(x,y) H(\phi(x,y)) dxdy}{\int_\Omega H(\phi(x,y)) dxdy}$$

$$c_2 = \frac{\int_\Omega f(x,y) [1 - H(\phi(x,y))] dxdy}{\int_\Omega [1 - H(\phi(x,y))] dxdy}$$

H: Heaviside function, $H(\phi) = \frac{1}{2}\left[1 + \frac{2}{\pi} \arctan\left(\frac{\phi}{\varepsilon}\right)\right]$

- **The force function:**

$$F = -\left[\mu \nabla \cdot \left(\frac{\nabla \phi}{\|\nabla \phi\|} \right) - \nu - \lambda_1 (f - c_1)^2 + \lambda_2 (f - c_2)^2 \right]$$



a b c
d e f

FIGURE 11.34 (a) Image and initial contour. Results after: (b) 100, (c) 300, (d) 500, (e) 700, and (f) 1100 iterations, respectively. We used $\mu = 0.5$ in all cases.

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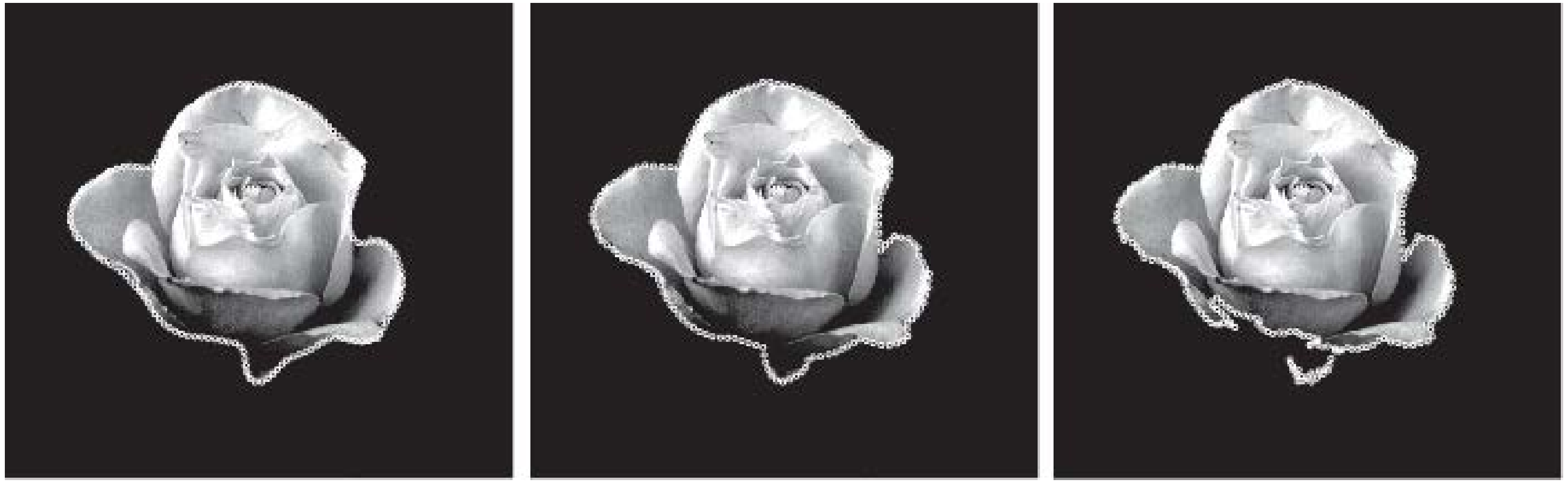
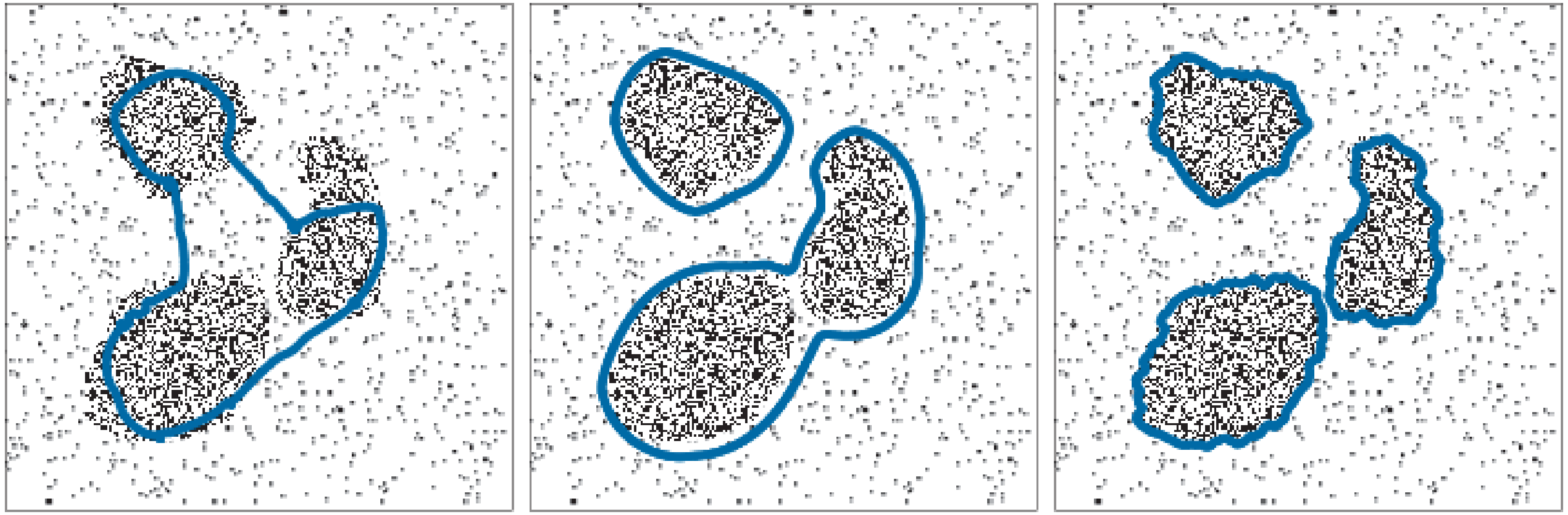
Comparison of Active Contours



a b c

Figure Segmentation comparisons: (a) Image segmented using snakes. (b) Result using edge-based level sets. (c) Result using region-based level sets



a b c

Figure Segmentation comparison between (a) snakes. (b) edge-based level sets. (c) region-based level sets

The use of motion in segmentation

Spatial Techniques

- A **difference** image of two images taken at time t_i and t_j

$$d_{ij}(x,y)=\begin{cases}1, & \text{if}\left|f(x,y,t_i)-f(x,y,t_j)\right|>T\\ 0, & \text{otherwise}\end{cases}$$

- **Accumulative difference image (ADI, 累积差值图像)**

Absolute ADI: $A_k(x,y)=\begin{cases}A_{k-1}(x,y)+1, & \text{if}\left|R(x,y)-f(x,y,k)\right|>T\\ A_{k-1}(x,y), & \text{otherwise}\end{cases}$

Positive ADI: $P_k(x,y)=\begin{cases}P_{k-1}(x,y)+1, & \text{if } R(x,y)-f(x,y,k)>T\\ P_{k-1}(x,y), & \text{otherwise}\end{cases}$

Negative ADI: $N_k(x,y)=\begin{cases}N_{k-1}(x,y)+1, & \text{if } R(x,y)-f(x,y,k)<-T\\ N_{k-1}(x,y), & \text{otherwise}\end{cases}$

where $R(x,y)$ is the reference image

- **Establish a reference image**

Obtaining a reference from a set of images containing one or more moving objects

a b c

FIGURE 10.65 ADIs of a rectangular object moving in a southeasterly direction. (a) Absolute ADI. (b) Positive ADI. (c) Negative ADI.

a b c

FIGURE 10.66 Building a static reference image. (a) and (b) Two frames in a sequence. (c) Eastbound automobile subtracted from (a), and the background restored from the corresponding area in (b). (Jain and Jain.)

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The use of motion in segmentation

FIGURE 10.67 LANDSAT frame. (Cowart, Snyder, and Ruedger.)

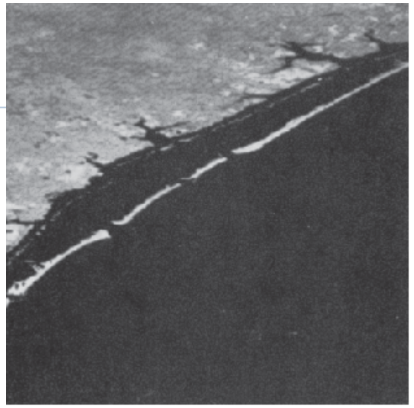
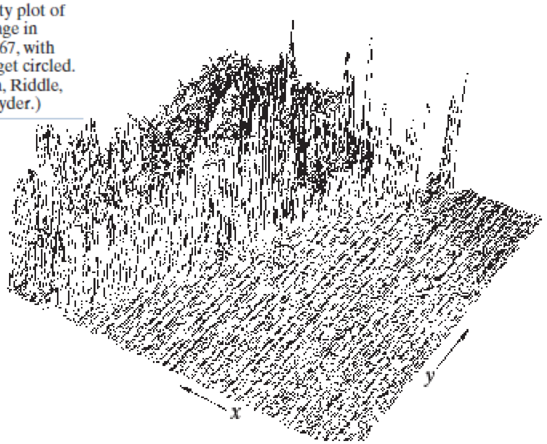
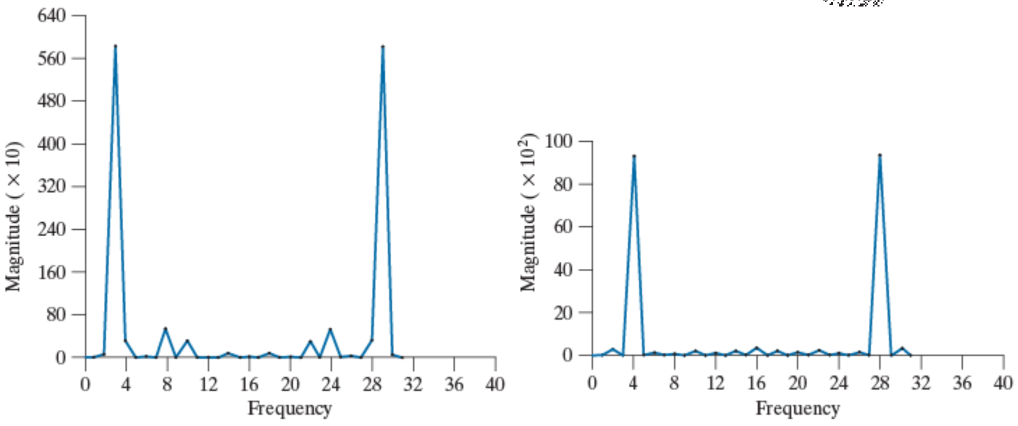


FIGURE 10.68 Intensity plot of the image in Fig. 10.67, with the target circled. (Rajala, Riddle, and Snyder.)





a b

FIGURE 10.69 (a) Spectrum of Eq. (10-121) showing a peak at $u_1 = 3$. (b) Spectrum of Eq. (10-122) showing a peak at $u_2 = 4$. (Rajala, Riddle, and Snyder.)

Frequency Domain Techniques

- Determine motion via a Fourier Transform
- For a sequence of K images of size $M \times N$ pixels, the 1D Fourier transform of the weighted projection onto the x and y axis respectively is

$$G_x(u_1, a_1) = \sum_{t=0}^{K-1} g_x(t, a_1) e^{-j2\pi u_1 t/K} \qquad u_1 = 0,1,\cdots,K-1$$

$$G_y(u_2, a_2) = \sum_{t=0}^{K-1} g_y(t, a_2) e^{-j2\pi u_2 t/K} \qquad u_2 = 0,1,\cdots,K-1$$

where the weighted projections are

$$g_x(t, a_1) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y,t) e^{j2\pi a_1 x \Delta t} \qquad t = 0,1,\cdots,K-1$$

$$g_y(t, a_2) = \sum_{y=0}^{N-1} \sum_{x=0}^{M-1} f(x,y,t) e^{j2\pi a_2 y \Delta t} \qquad t = 0,1,\cdots,K-1$$

The frequency-velocity relationship is

$$u_1 = a_1 V_1 \qquad\qquad u_2 = a_2 V_2$$

Image Segmentation

- **Traditional segmentation method**
  - Based on Discontinuity (Edge-based segmentation): point, line, edge detection
  - Based on Similarity
    - ✓ Thresholding: Global and Variable
    - ✓ Region-based segmentation: Region Growing, Region Splitting and Merging, Region Clustering, Superpixels, Graph Cuts
- **Morphological Image Processing**
  - Morphological operation
  - Morphological algorithms
  - Morphological Watersheds
- **Active Contours**
  - Snakes
  - Level Sets