Lecture 11 – Image Restoration and Noise （图像复原）

This lecture will cover:

- Model of Image Degradation Process（图像退化过程模型）
- Noise Reduction（噪声消除）
  - Noise Models（噪声模型）
  - Spatial Filtering （空间域滤波方法）
  - Frequency Domain Filtering（频率域滤波方法）

Model of Image Degradation （图像退化模型）

In Spatial domain: $g(x,y) = h\,(x,y) \star f(x,y) + \eta(x,y)$

where

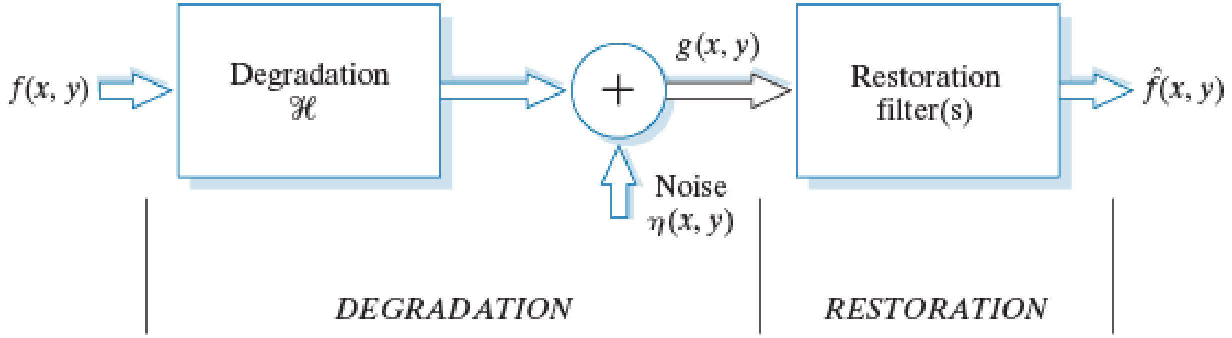
$g(x,y)$: a degraded image $\quad$ $f(x,y)$: input image,

$h(x,y)$: degradation function $\quad$ $\eta(x,y)$: additive noise term

In Frequency domain: $G(u,v) = H\,(u,v)F(u,v) + N(u,v)$

H **is a linear, position-invariant process**

FIGURE 5.1 A model of the image degradation/restoration process.



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Noise Models（噪声模型）

- Properties of Noise
- Noise Probability Density Function (PDF)(概率密度函数)
- Periodic Noise
- Estimation of Noise Parameter（噪声参数）

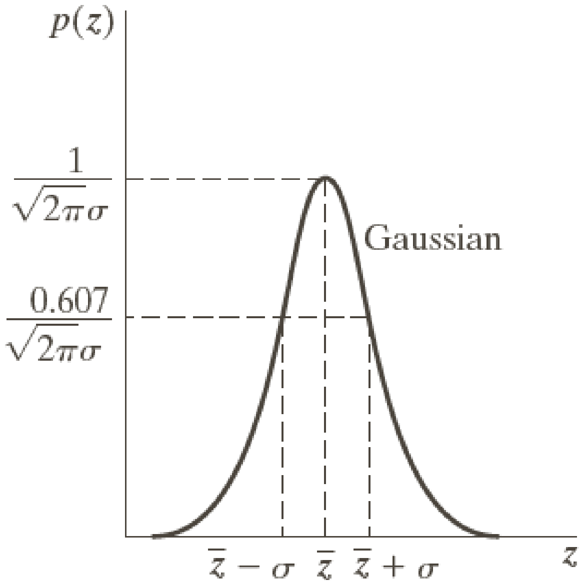
Properties of Noise

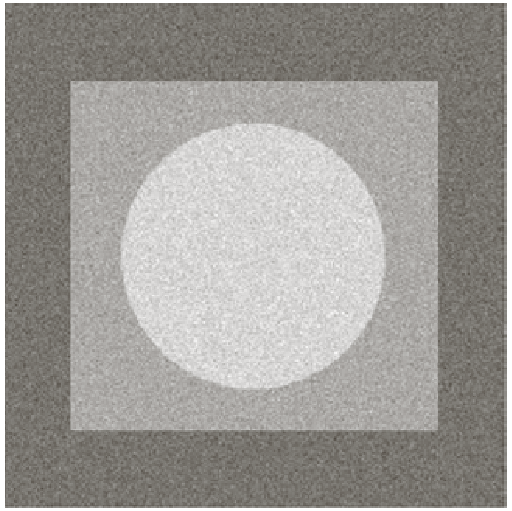
- **Spatial properties -**parameters that define spatial characteristics of noise
- **Frequency properties –** frequency content of noise
  - White noise
  - Periodic noise
- **Independent of spatial coordinates**
- **Uncorrelated with respect to the image itself**

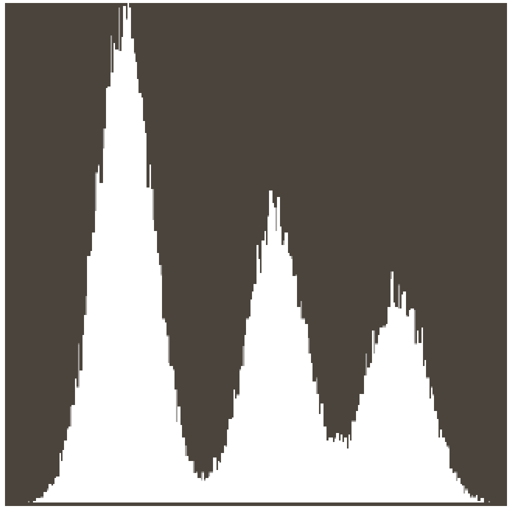
Gaussian Noise (高斯噪声)

Gaussian Noise(高斯噪声): $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$

$\bar{z}$: mean (average) σ: standard deviation σ^2: variance



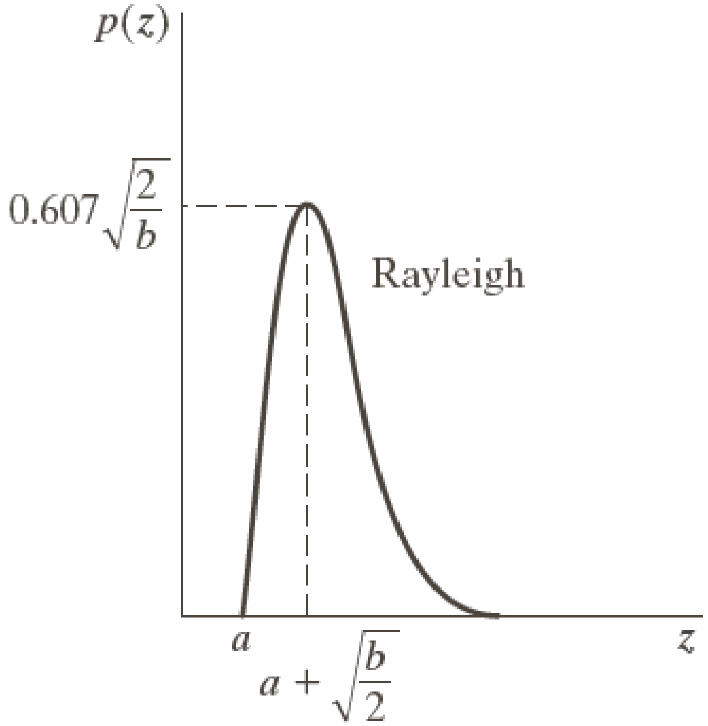


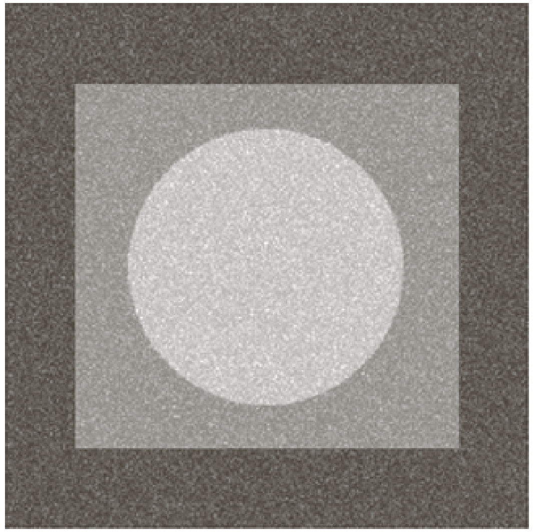


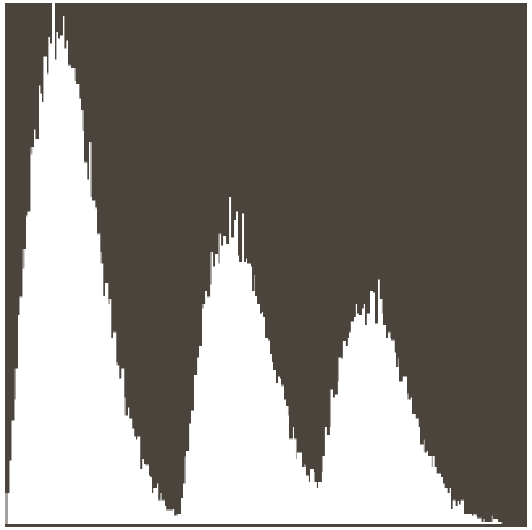
Rayleigh Noise（瑞利噪声）

Rayleigh Noise（瑞利噪声）：$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}}, & \text{for } z \geq a \\ 0, & \text{for } z < a \end{cases}$$

$$\bar{z} = a + \sqrt{\pi b/4} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$



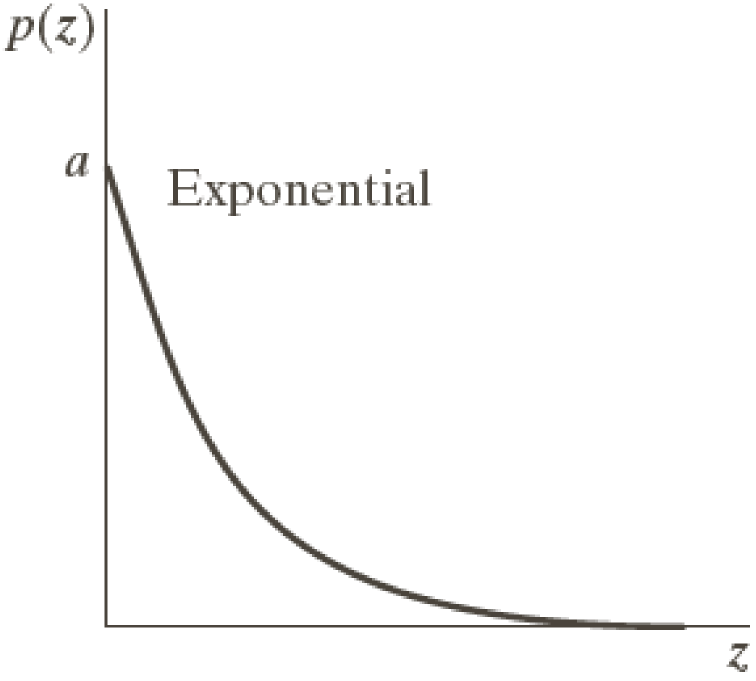


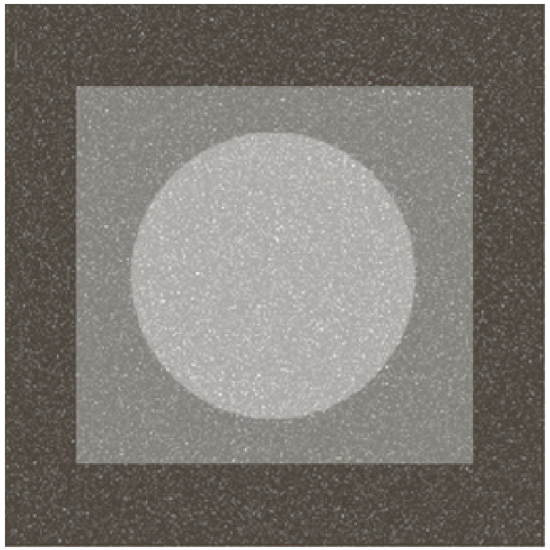


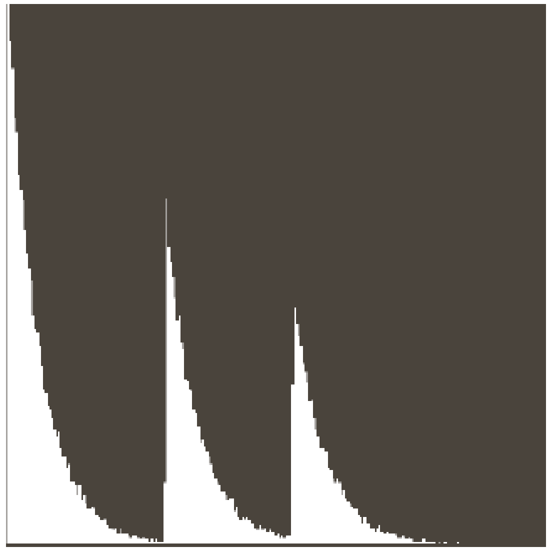
Exponential Noise（指数噪声）

Exponential Noise（指数噪声）： $p(z) = \begin{cases} ae^{-az}, \text{for } z \geq 0 \\ 0, \qquad\ \ \text{for } z < 0 \end{cases}$

$$\bar{z} = \frac{1}{a} \qquad\qquad \sigma^2 = \frac{1}{a^2}$$



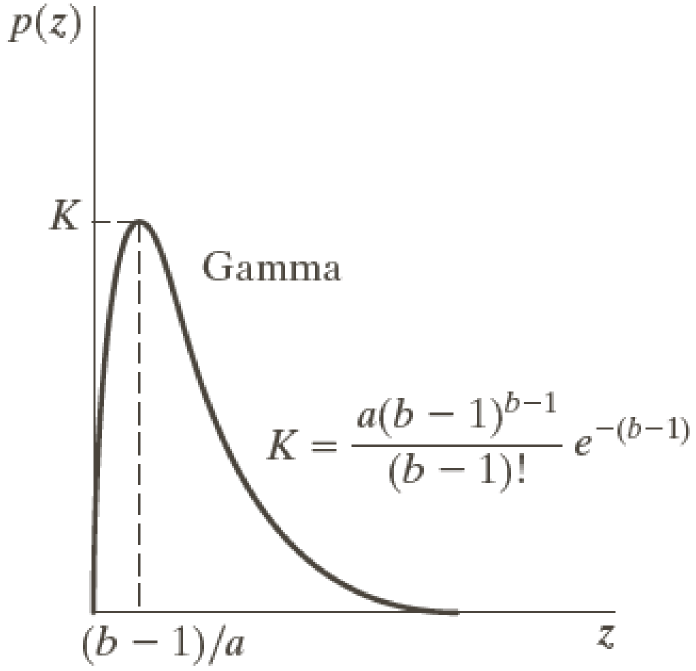


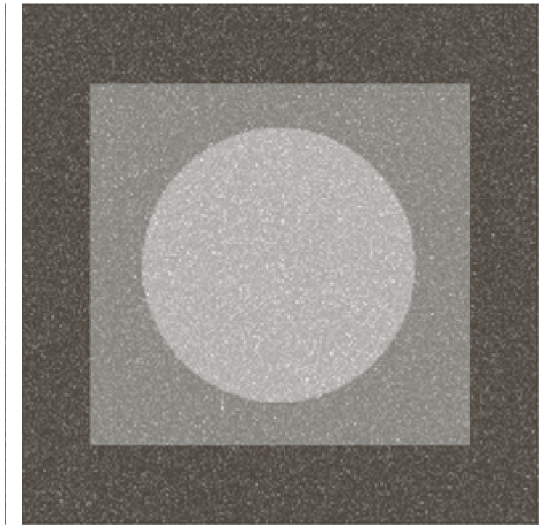


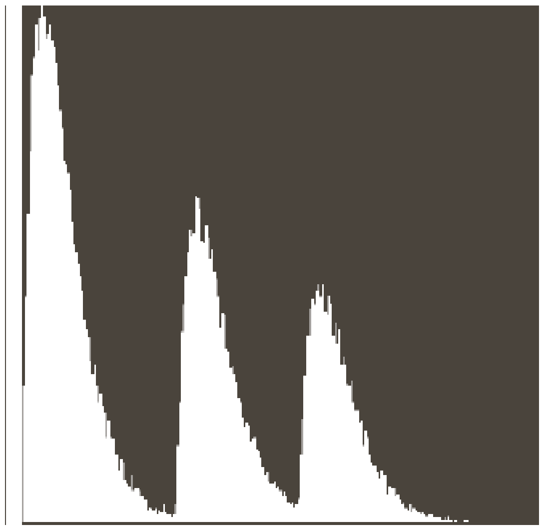
Erlang (gamma) Noise（爱尔兰/伽马噪声）

Erlang (gamma) Noise（爱尔兰/伽马噪声）：$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az}, & \text{for } z \ge 0 \\ 0, & \text{for } z < 0 \end{cases}$

$$\bar{z} = \frac{b}{a} \qquad \sigma^2 = \frac{b}{a^2}$$



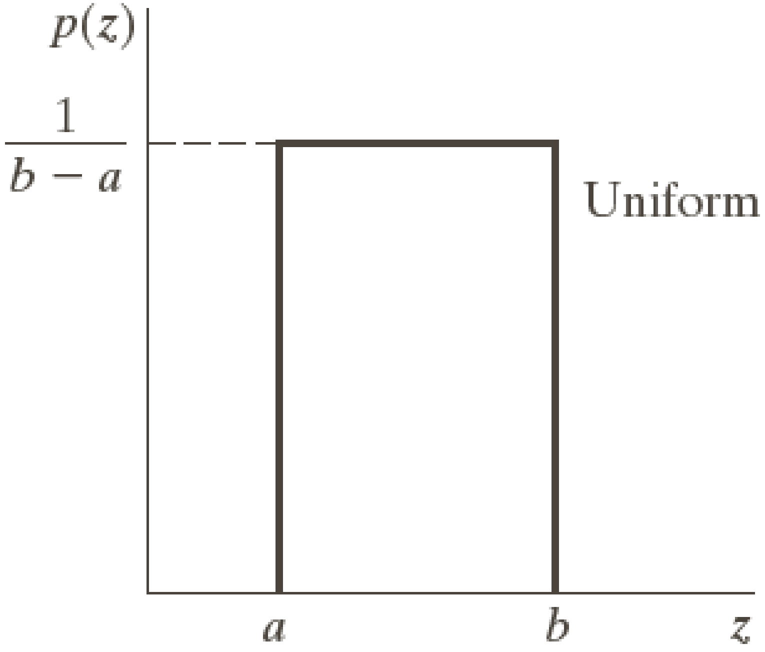


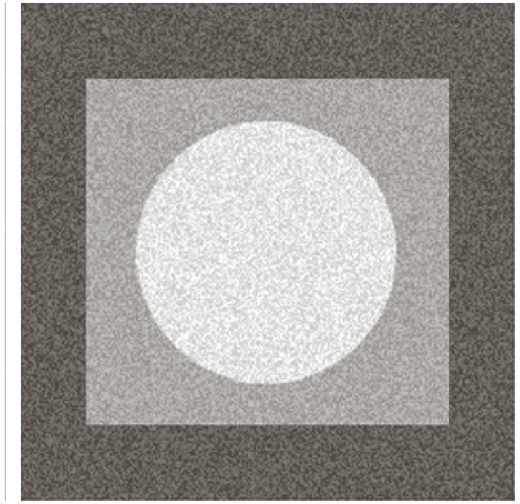


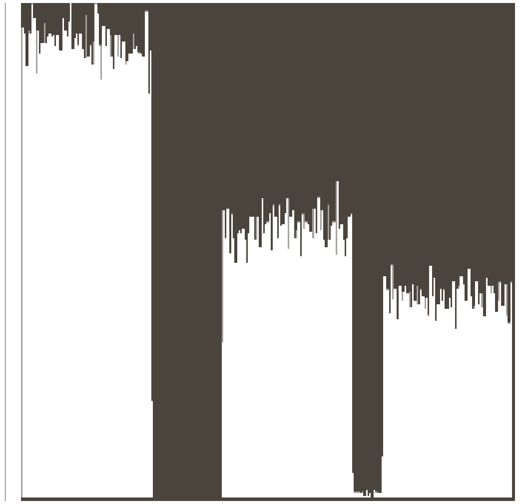
Uniform Noise（均匀噪声）

Uniform Noise（均匀噪声）：
$$p(z) = \begin{cases} \frac{1}{b-a}, & \text{for } a \le z \le b \\ 0, & \text{otherwise} \end{cases}$$

$$\bar{z} = \frac{a+b}{2} \qquad \sigma^2 = \frac{(b-a)^2}{12}$$



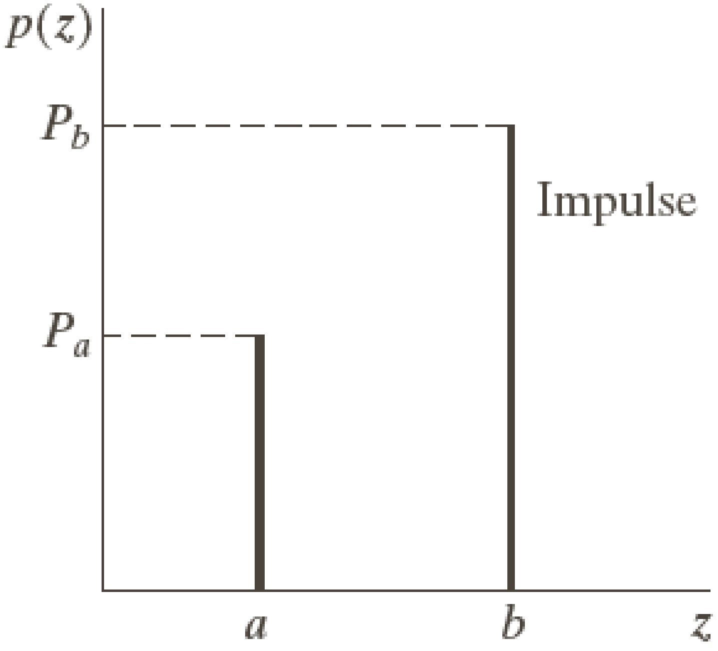


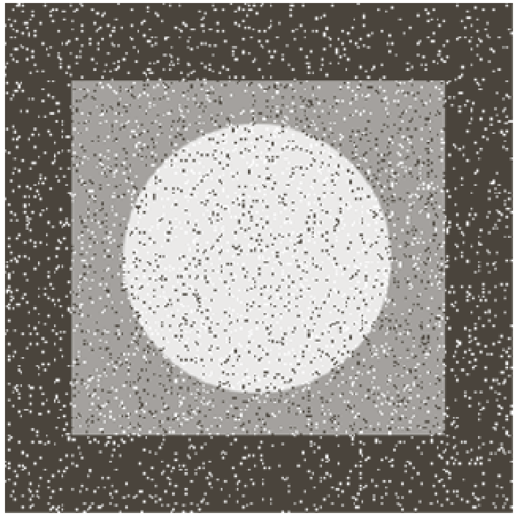


Impulse (salt-and-pepper) Noise（脉冲/椒盐噪声）

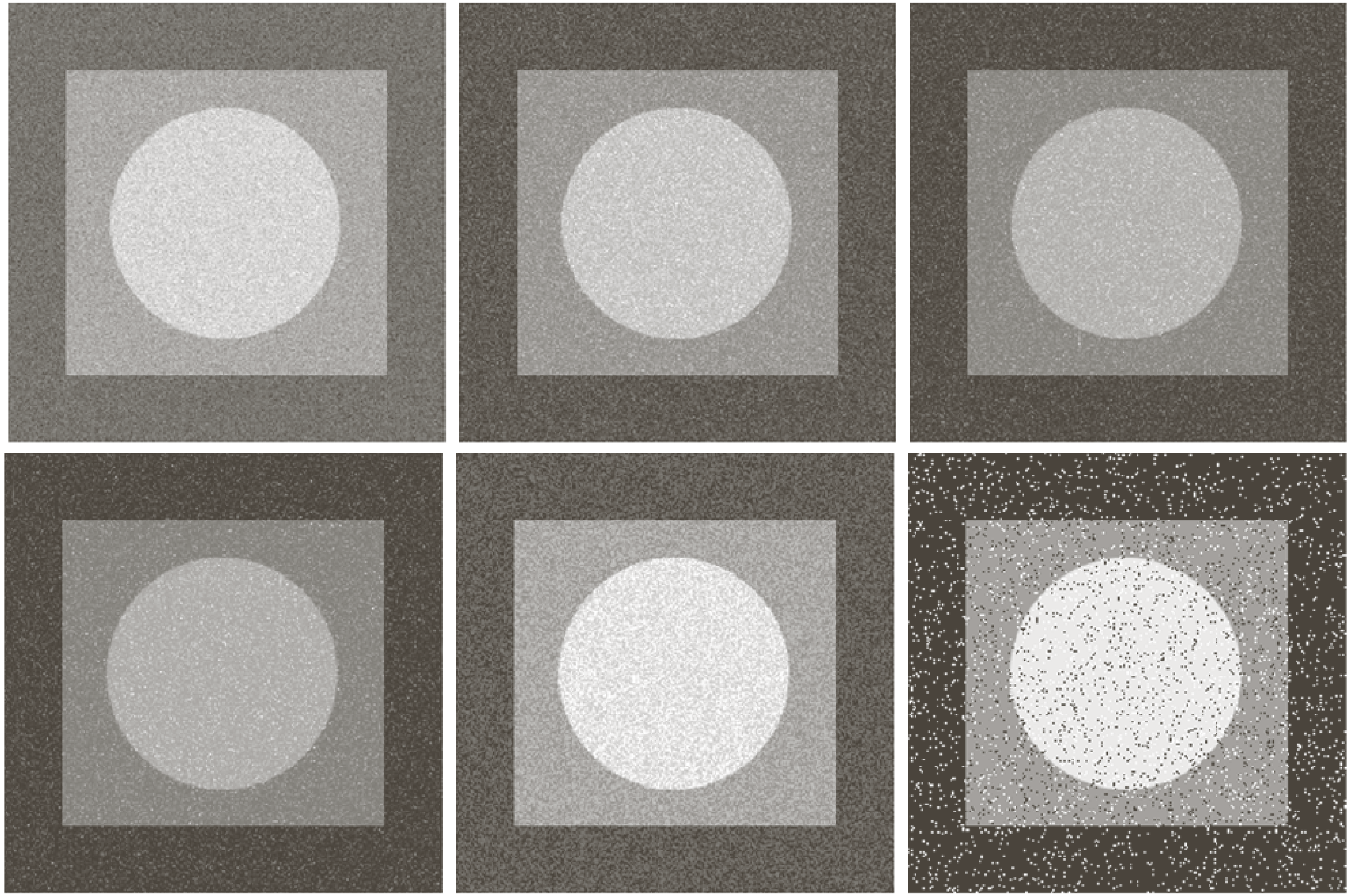
Impulse (salt-and-pepper) Noise（脉冲/椒盐噪声）:

$$p(z) = \begin{cases} P_a, & \text{for } z = a \\ P_b, & \text{for } z = b \\ 1 - P_a - P_b, & \text{otherwise} \end{cases}$$





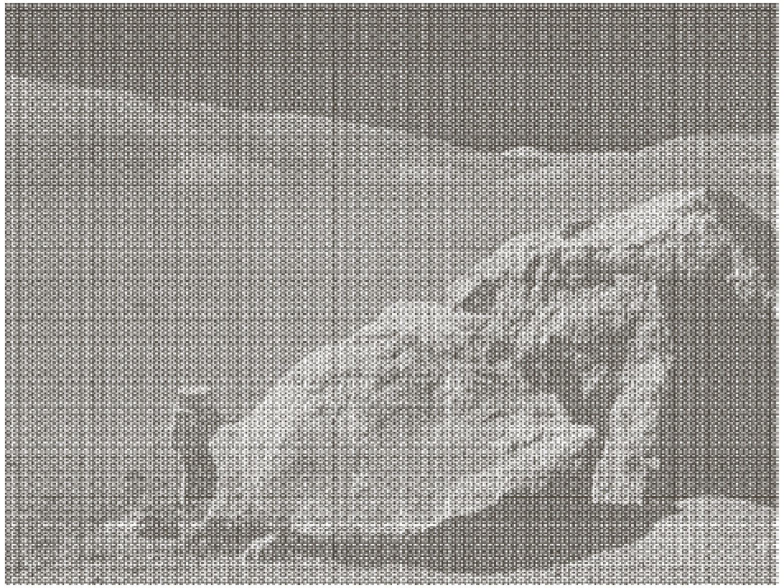
Noise

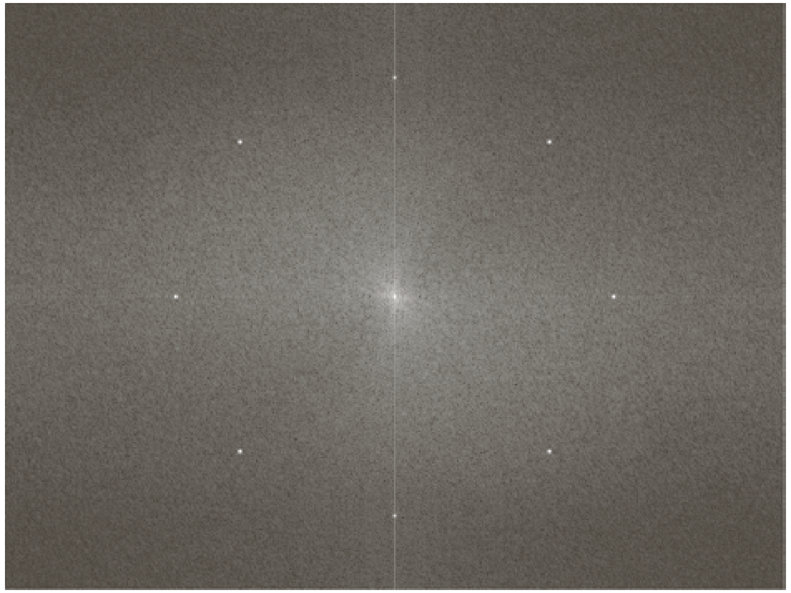


a b c
d e f

Images resulting from adding (a) Gaussian, (b) Rayleigh, (c) Gamma, (d) exponential, (e) uniform and (f) salt-and-pepper noise to the image

Periodic Noise （周期噪声）

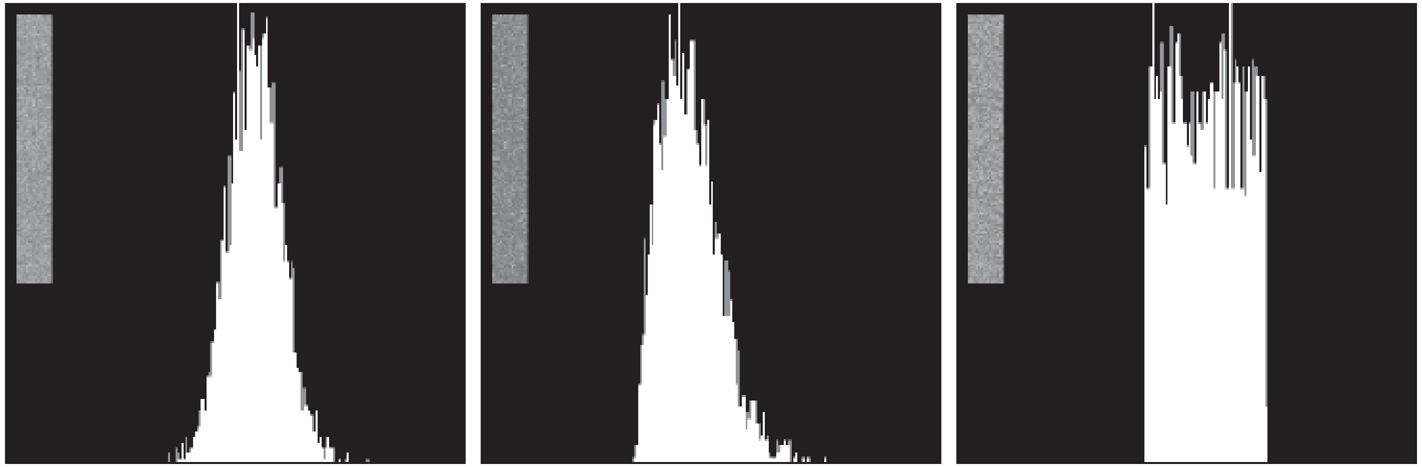




Estimation of Noise Parameter（噪声参数估计）

$$Mean: \quad \bar{z} = \sum_{i=0}^{L-1} z_i P_s(z_i)$$

$$Variance: \ \sigma^2 = \sum_{i=0}^{L-1} (\bar{z} - z_i)^2 P_s(z_i)$$



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Spatial Filtering（空间滤波）

Mean Filters (均值滤波器)

- Arithmetic mean filter (算术均值滤波器)
- Geometric mean filter (几何均值滤波器)
- Harmonic mean filter (谐波均值滤波器)
- Contraharmonic mean filter (逆谐波均值滤波器)

Order-statistic Filters (统计排序滤波器)

- Median filter (中值滤波器)
- Max and Min filter (最大值和最小值滤波器)
- Midpoint filter (中点滤波器)
- Alpha-trimmed mean filter (修正的阿尔法均值滤波器)

Adaptive Filters (自适应滤波器)

- Adaptive local noise reduction filter (自适应局部降噪滤波器)
- Adaptive median filter (自适应中值滤波器)

Mean Filters (均值滤波器)

- **Arithmetic mean filter (算术均值滤波器):**

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- **Geometric mean filter (几何均值滤波器):**

$$\hat{f}(x,y) = \left(\prod_{(s,t)\in S_{xy}} g(s,t) \right)^{\frac{1}{mn}}$$

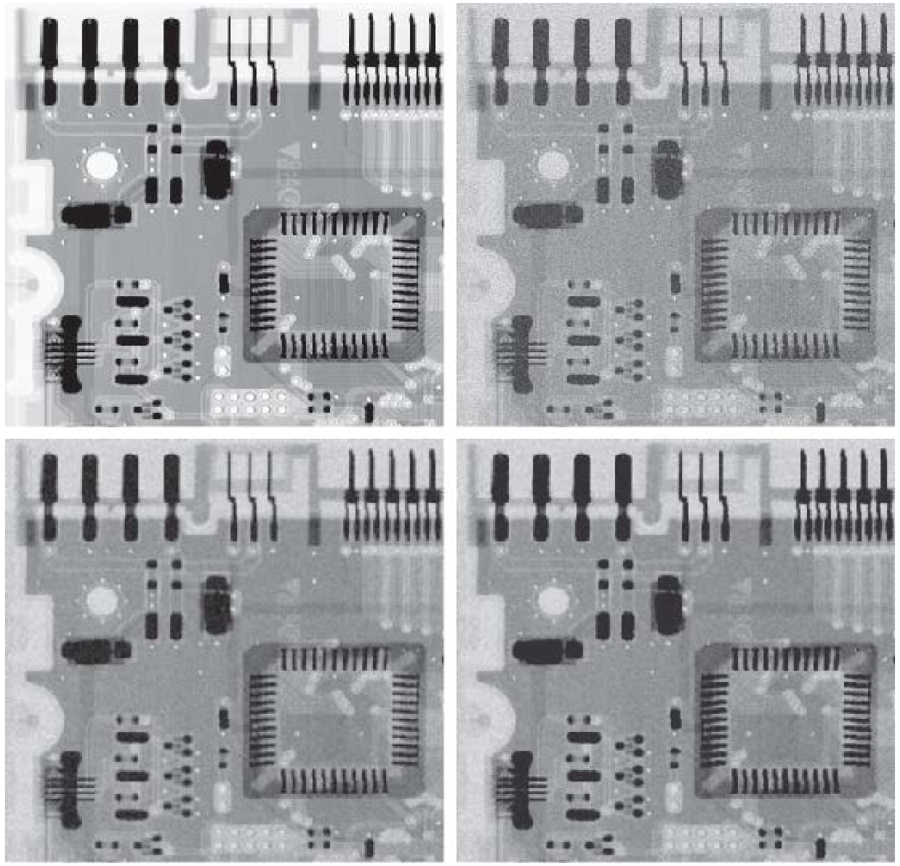


FIGURE 5.7 (a) X-ray image of circuit board. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

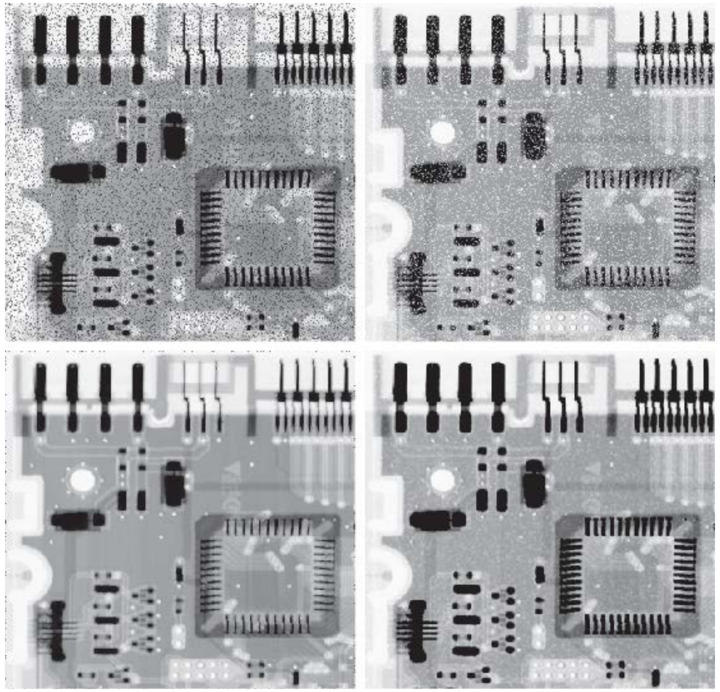
Mean Filters (均值滤波器)

- **Harmonic mean filter (谐波均值滤波器):**

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

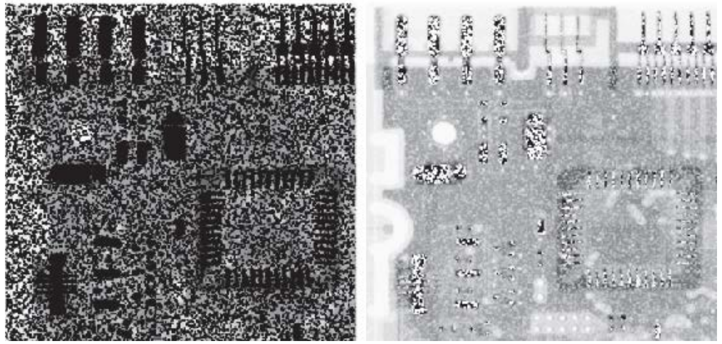
- **Contraharmonic mean filter (逆谐波均值滤波器):**

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter $Q = 1.5$. (d) Result of filtering (b) with $Q = -1.5$.



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering Fig. 5.8(b) using $Q = 1.5$.

Order-statistic Filters (统计排序滤波器)

- **Median filter (中值滤波器):**

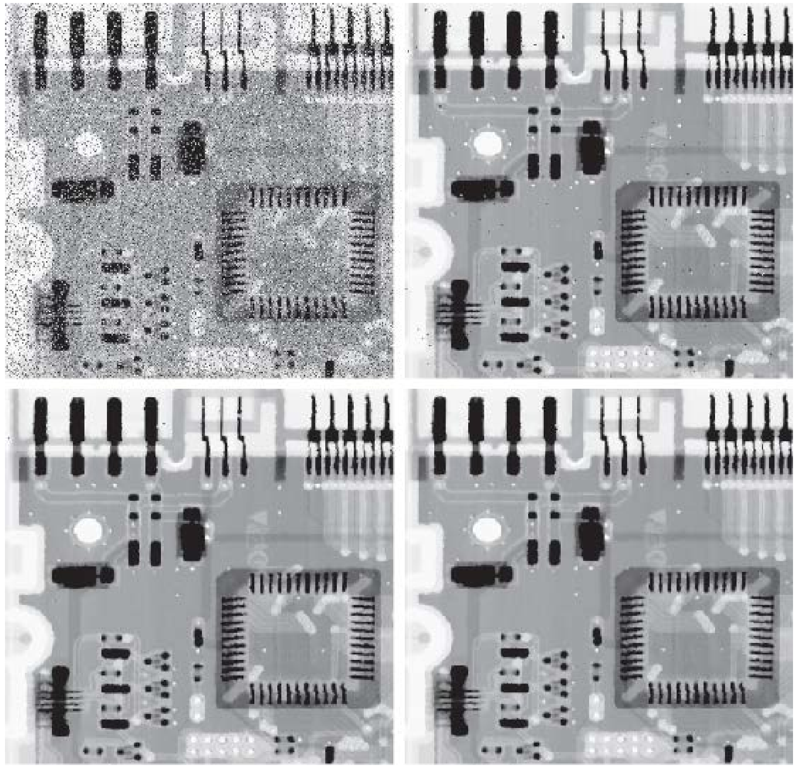
$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$$

- **Max filter (最大值滤波器)**

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

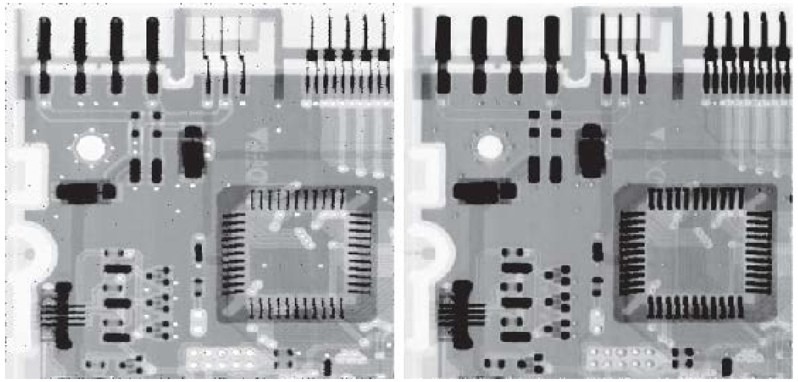
- **Min filter (最小值滤波器)**

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$



a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_s = P_p = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



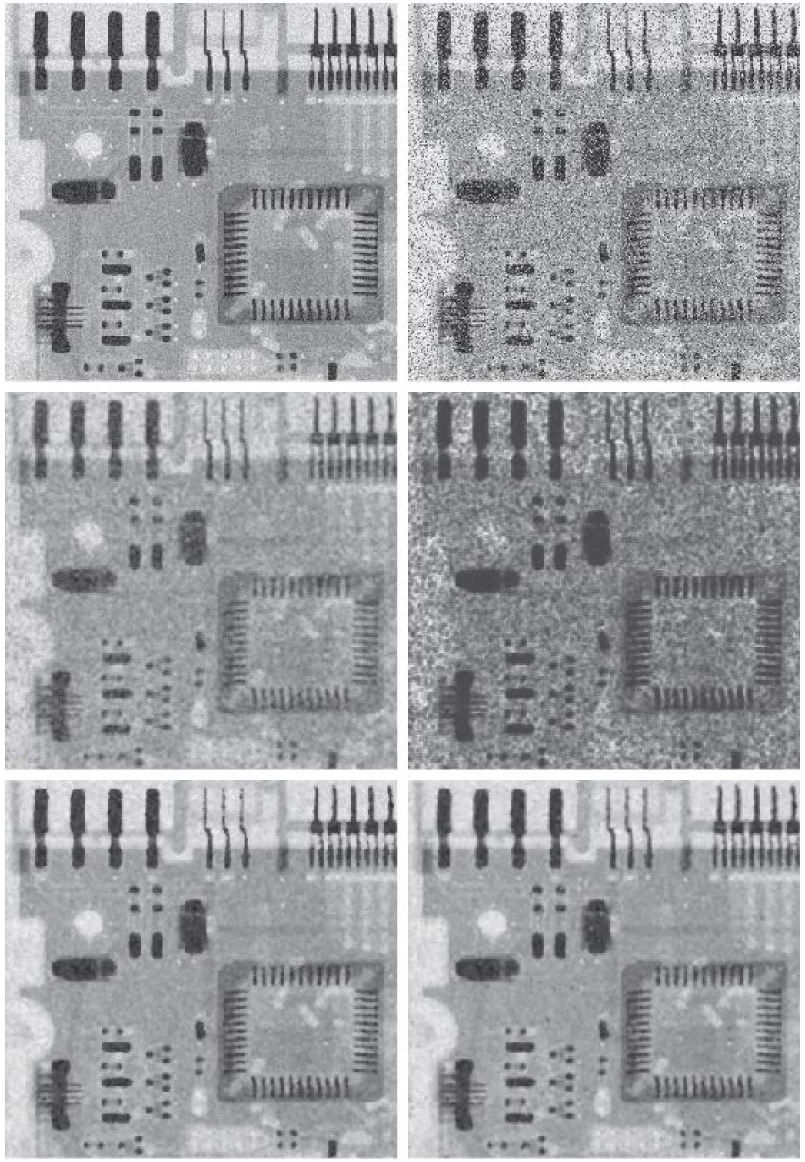
a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering Fig. 5.8(b) with a min filter of the same size.

Order-statistic Filters (统计排序滤波器)

a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. (c)-(f) Image (b) filtered with a 5×5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; (f) alpha-trimmed mean filter, with $d = 6$.



- **Midpoint filter (中点滤波器)**

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- **Alpha-trimmed mean filter (修正的阿尔法均值滤波器)**

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Adaptive Filters (自适应滤波器)

- **Adaptive local noise reduction filter (自适应局部降噪滤波器):**

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$

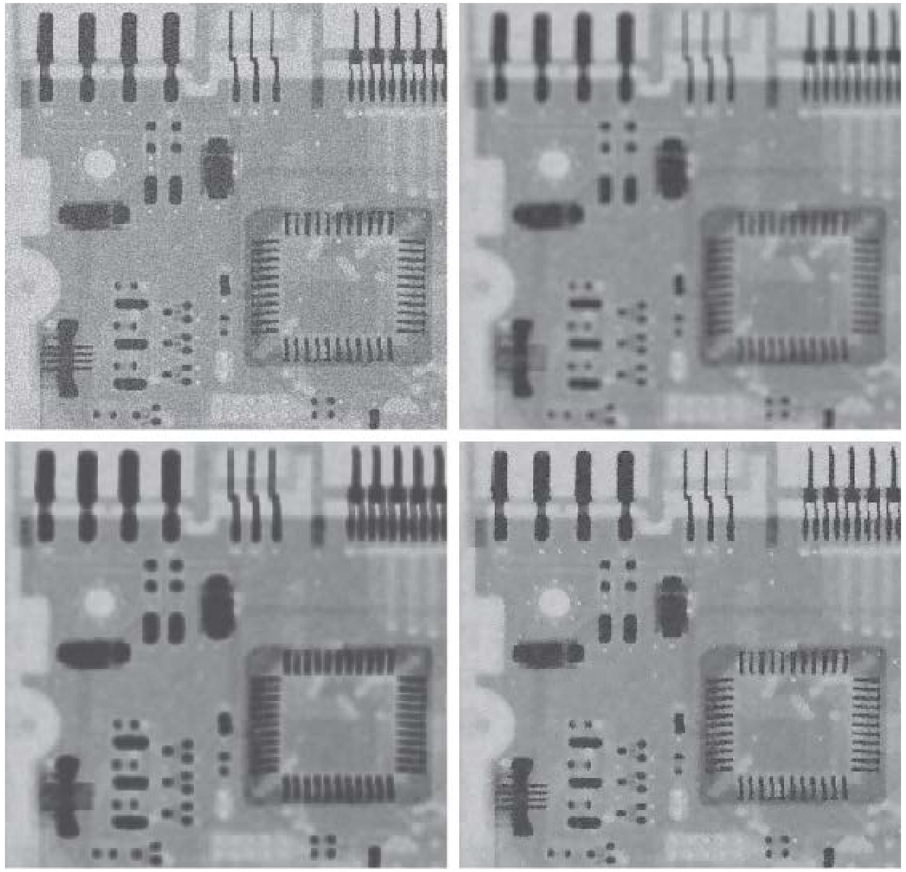
Where

$g(x,y)$: noisy image $\quad\quad$ σ_η^2 : the variance of noise

m_L: the local mean in S_{xy} $\quad\quad$ σ_L^2 : the local variance in S_{xy}

and

$$g(x,y) = f(x,y) + \eta(x,y)$$



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and a variance of 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise-reduction filtering. All filters used were of size 7 × 7.

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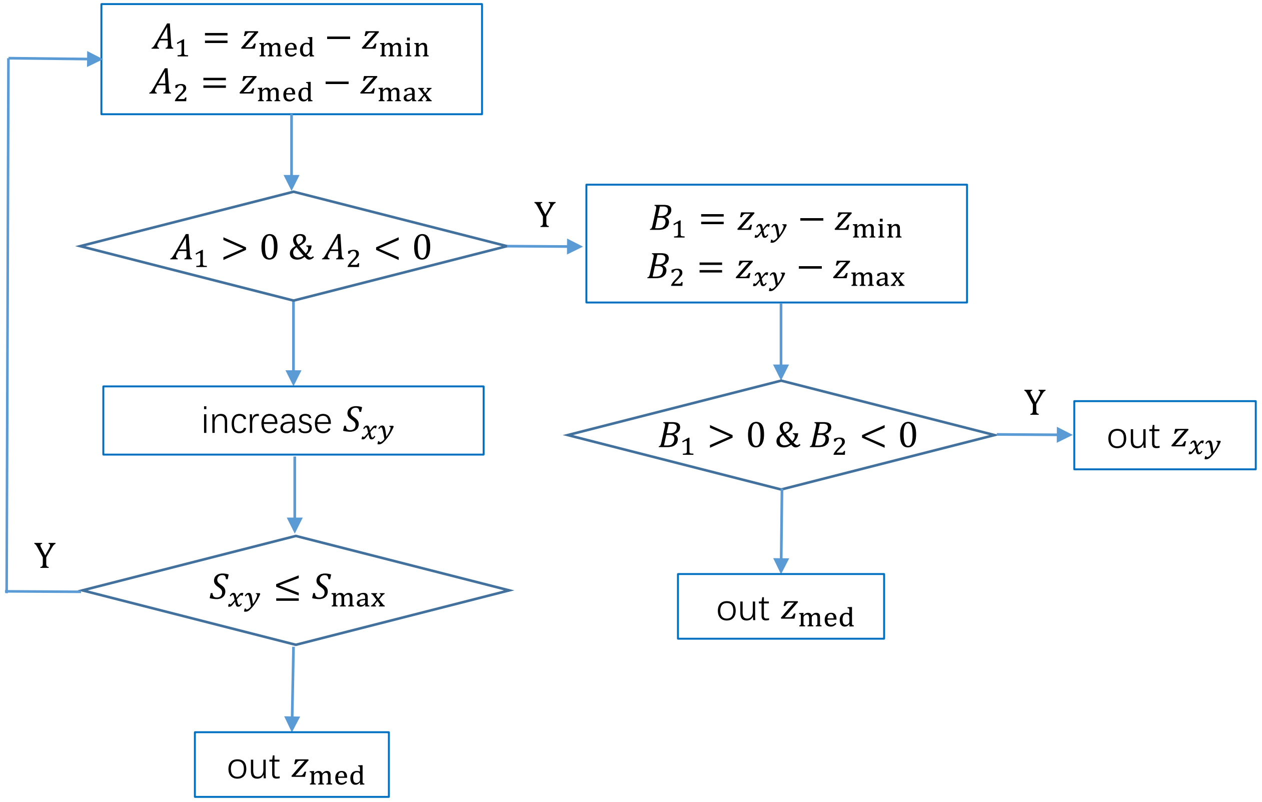
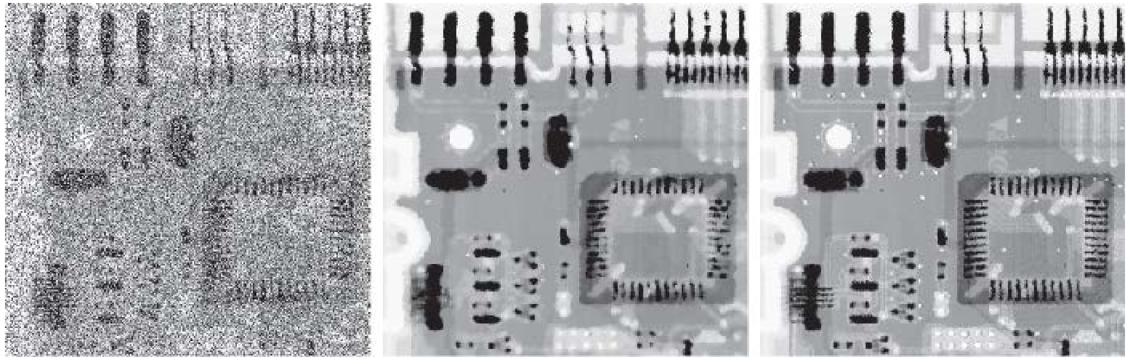
Adaptive Filters (自适应滤波器)

➢ **Adaptive Median filter (自适应中值滤波器):**





a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_s = P_p = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\text{max}} = 7$.

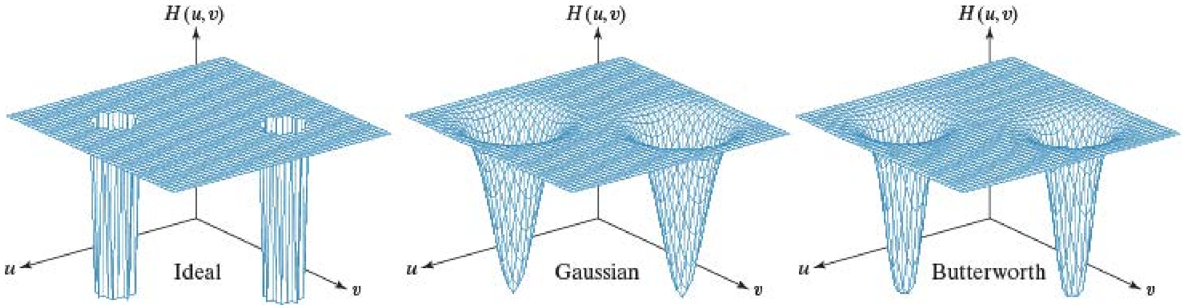
Frequency Domain Filtering（频率域滤波）

Mainly for periodic noise

- Bandreject Filters (带阻滤波器)
- Bandpass Filters (带通滤波器)
- Notch Filters (陷波滤波器)
- Optimum Notch Filters (最佳陷波滤波器)

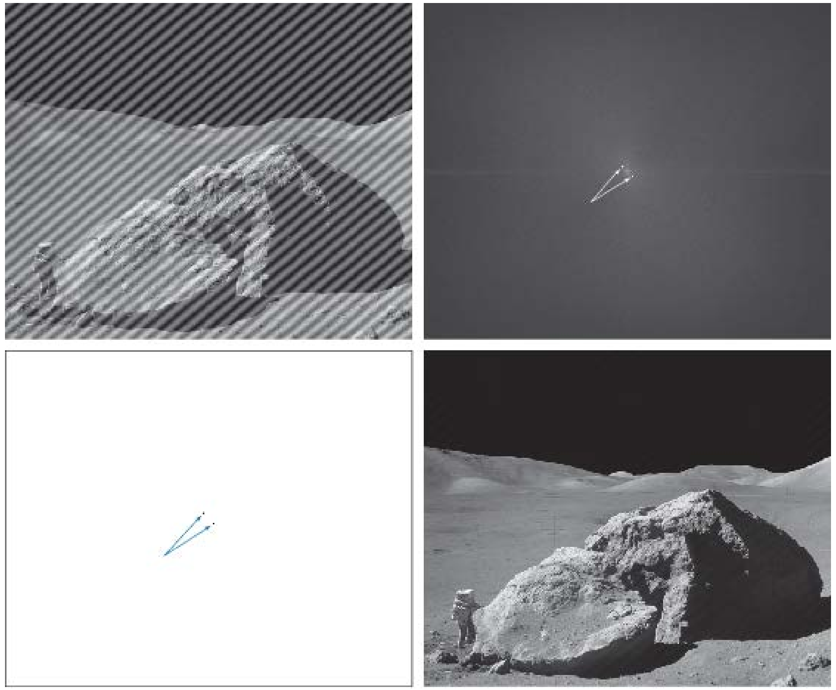
Bandreject Filters (带阻滤波器)

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$



a b c

FIGURE 5.15 Perspective plots of (a) ideal, (b) Gaussian, and (c) Butterworth notch reject filter transfer functions.



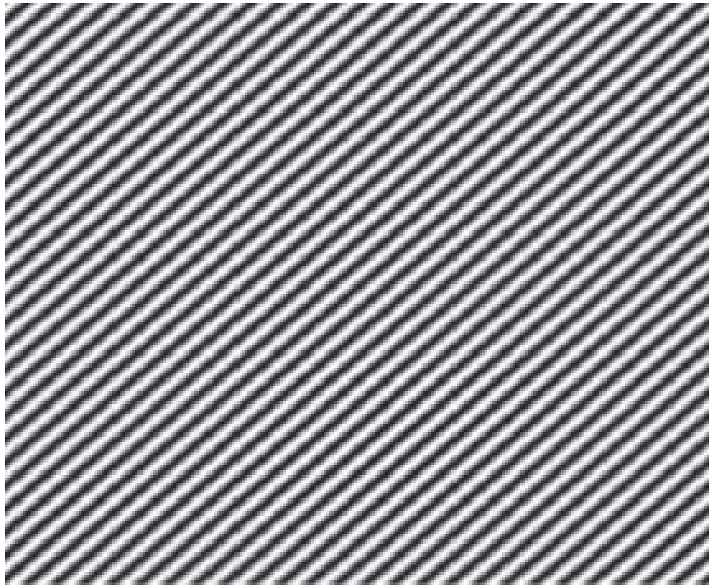
a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal interference. (b) Spectrum showing the bursts of energy caused by the interference. (The bursts were enlarged for display purposes.) (c) Notch filter (the radius of the circles is 2 pixels) used to eliminate the energy bursts. (The thin borders are not part of the data.) (d) Result of notch reject filtering. (Original image courtesy of NASA.)

Bandpass Filters (带通滤波器)

$$H_{\mathrm{BP}}(u,v) = 1 - H_{\mathrm{BR}}(u,v)$$

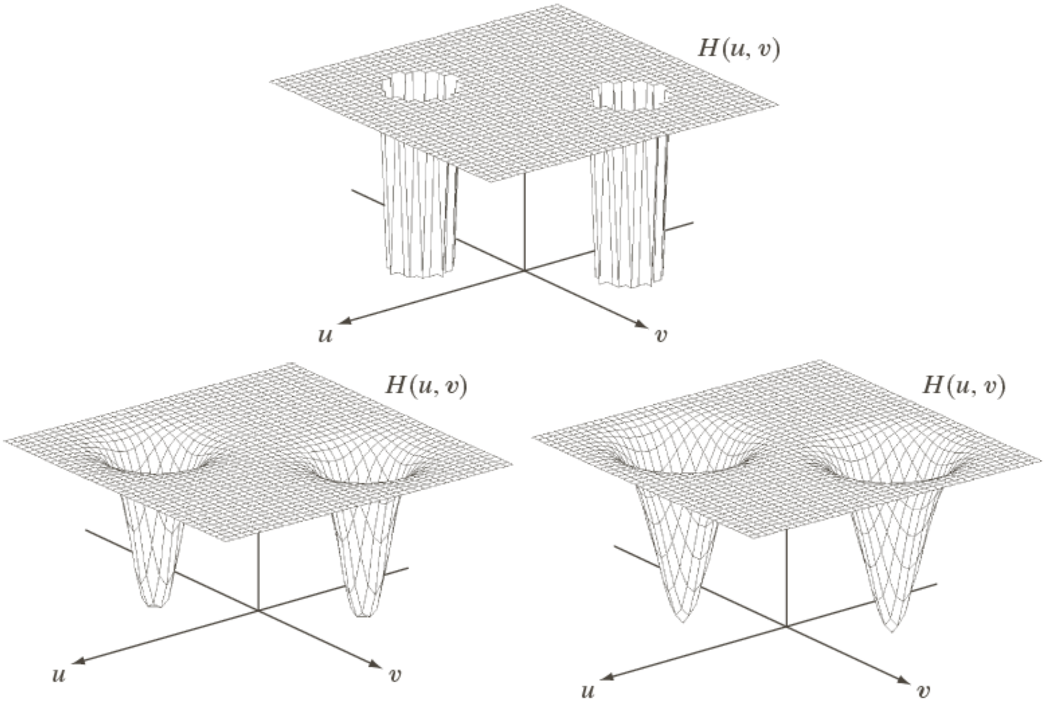
FIGURE 5.17 Sinusoidal pattern extracted from the DFT of Fig. 5.16(a) using a notch pass filter.



Notch Filters (陷波滤波器)

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v)H_{-k}(u,v)$$

$$H_{\mathrm{NP}}(u,v) = 1 - H_{\mathrm{NR}}(u,v)$$



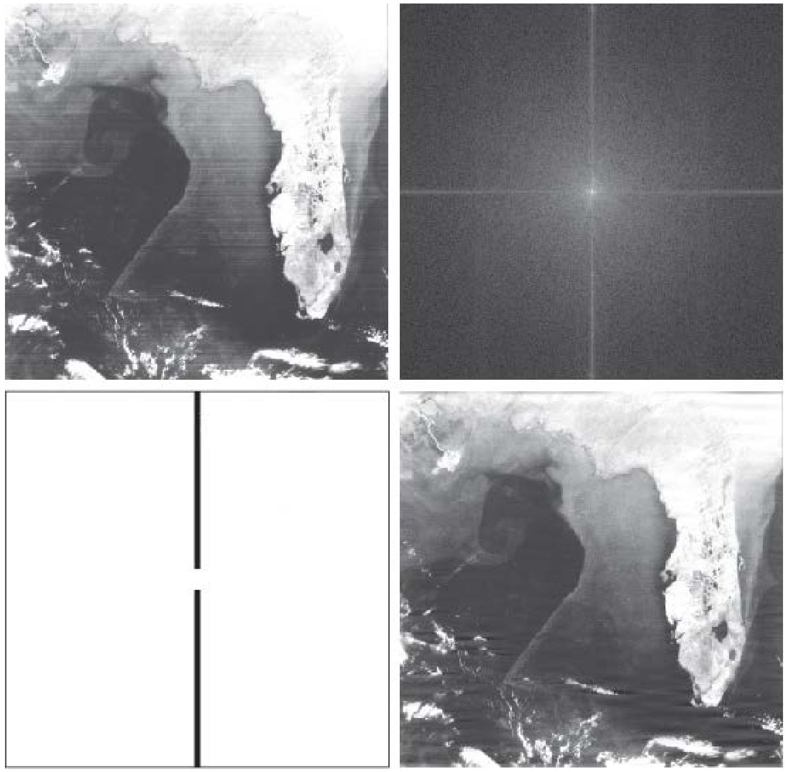


FIGURE 5.18 (a) Satellite image of Florida and the Gulf of Mexico. (Note horizontal sensor scan lines.) (b) Spectrum of (a). (c) Notch reject filter transfer function. (The thin black border is not part of the data.) (d) Filtered image. (Original image courtesy of NOAA.)

FIGURE 5.19 Noise pattern extracted from Fig. 5.18(a) by notch pass filtering.

Optimum Notch Filters (最佳陷波滤波器)

- **Noise pattern in spatial domain**

$$\eta(x,y) = \mathcal{F}^{-1}\{H_{\mathrm{NP}}(u,v)G(u,v)\}$$

- **Obtain estimate of $\boldsymbol{f(x,y)}$**

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y),$$ where $w(x,y)$ is weighed factor

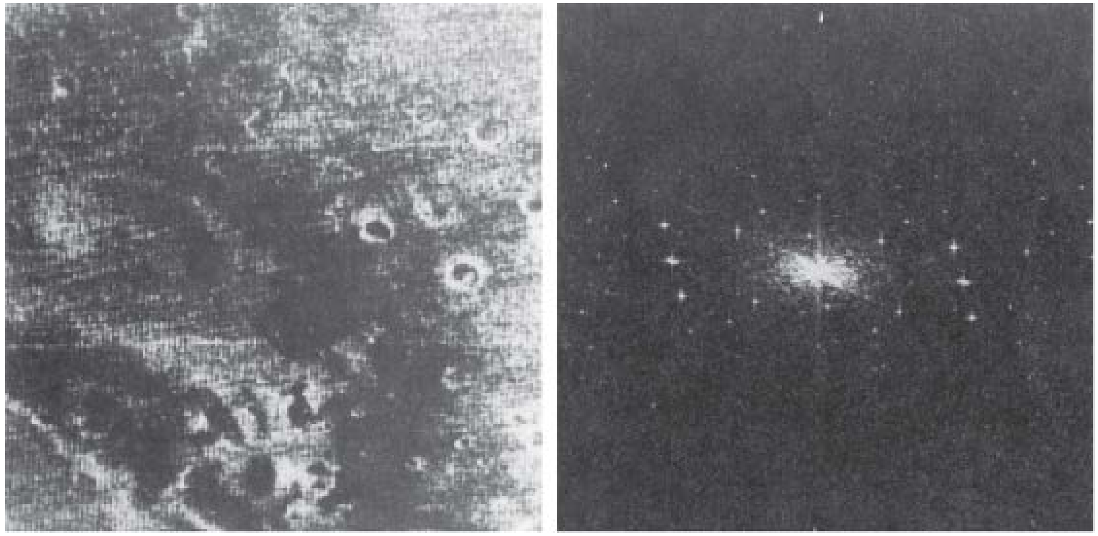
- **Estimate variance of $\boldsymbol{\hat{f}(x,y)}$**

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[\hat{f}(x+s,y+t) - \bar{\hat{f}}(x,y)\right]^2$$

- **Minimize $\sigma^2(x,y)$, and solve $w(x,y)$**

$$\frac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0 \quad \Rightarrow \quad w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

Optimum Notch Filters (最佳陷波滤波器)



a b

FIGURE 5.20 (a) Image of the Martian terrain taken by Mariner 6. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

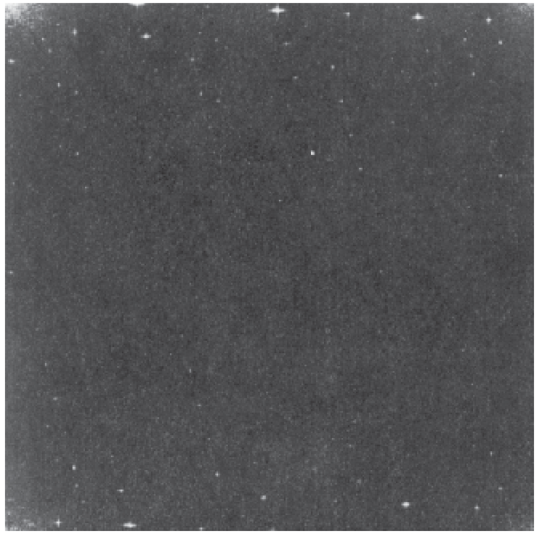
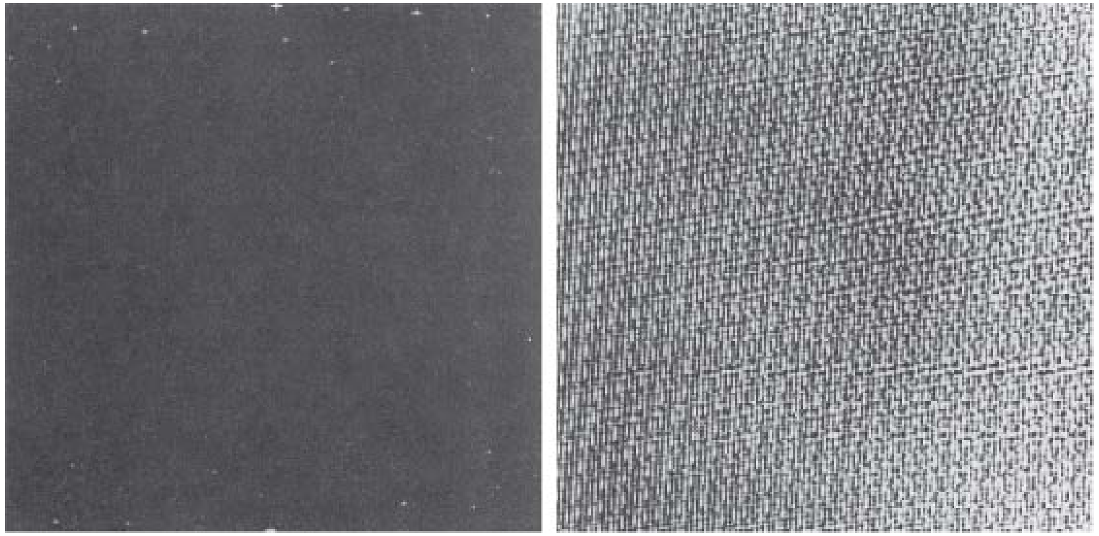


FIGURE 5.21 Uncentered Fourier spectrum of the image in Fig. 5.20(a). (Courtesy of NASA.)



a b

FIGURE 5.22 (a) Fourier spectrum of $N(u,v)$, and (b) corresponding spatial noise interference pattern, $\eta(x,y)$. (Courtesy of NASA.)

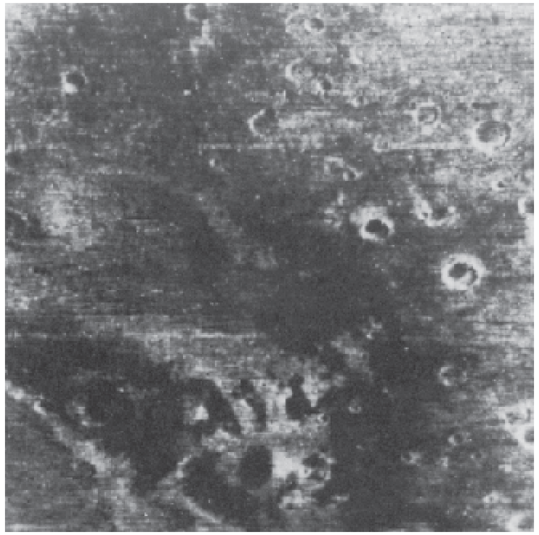


FIGURE 5.23 Restored image. (Courtesy of NASA.)

SNR (信噪比)

➢ **SNR (Signal-to-noise ratio):**

$$\mathrm{SNR} = \frac{\sum_{(x,y)} \hat{f}^2(x,y)}{\sum_{(x,y)}\left[g(x,y) - \hat{f}(x,y)\right]^2}$$

Where

$g(x,y)$: a degraded image

$\hat{f}(x,y)$: estimate of input image