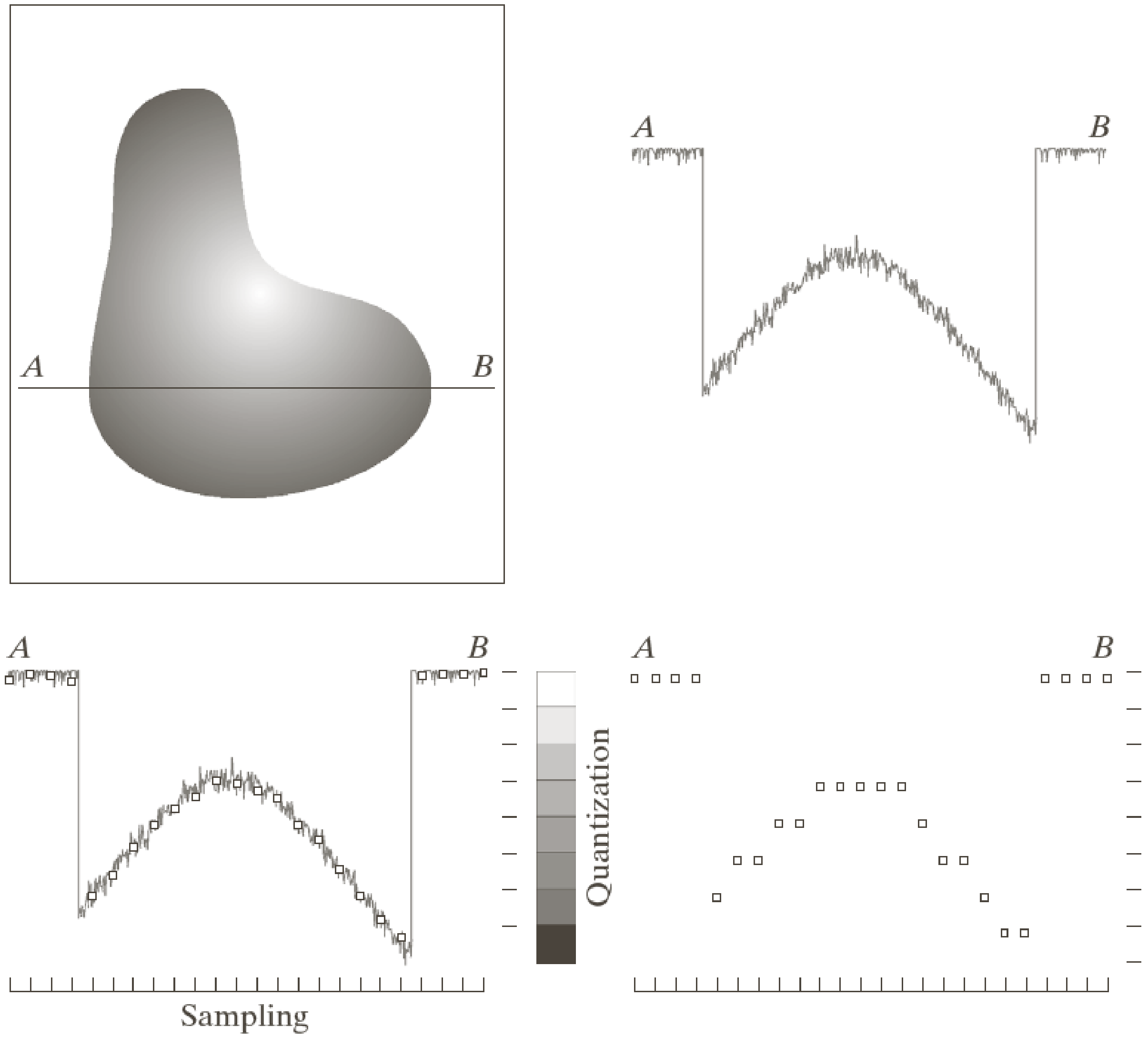
Lecture 2 - Image Fundamentals

This lecture will cover:

- **Sampling and Quantization**
- Pixels

Sampling and Quantization



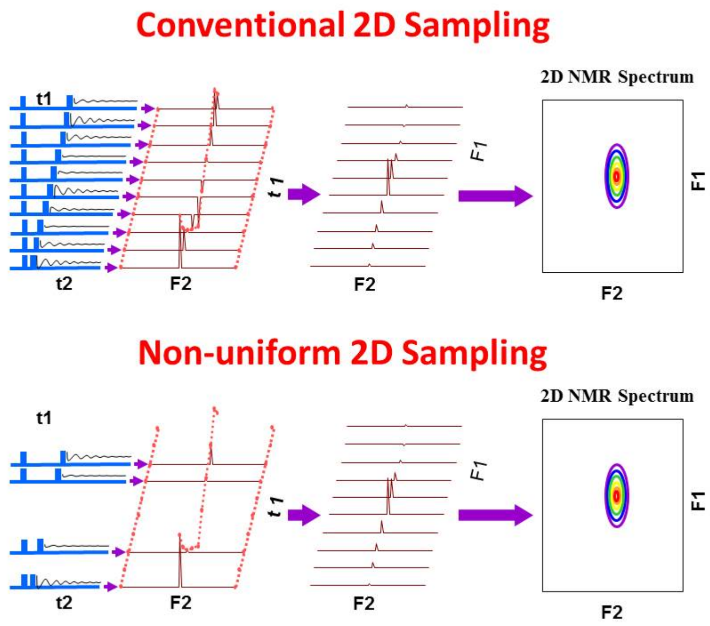
- ➢**Sampling (取样)**

  Digitize the coordinate values

- ➢**Quantization (量化)**

  Digitize the amplitude values

- ➢**Methods：** Uniform &Non-uniform



Matrix Representation

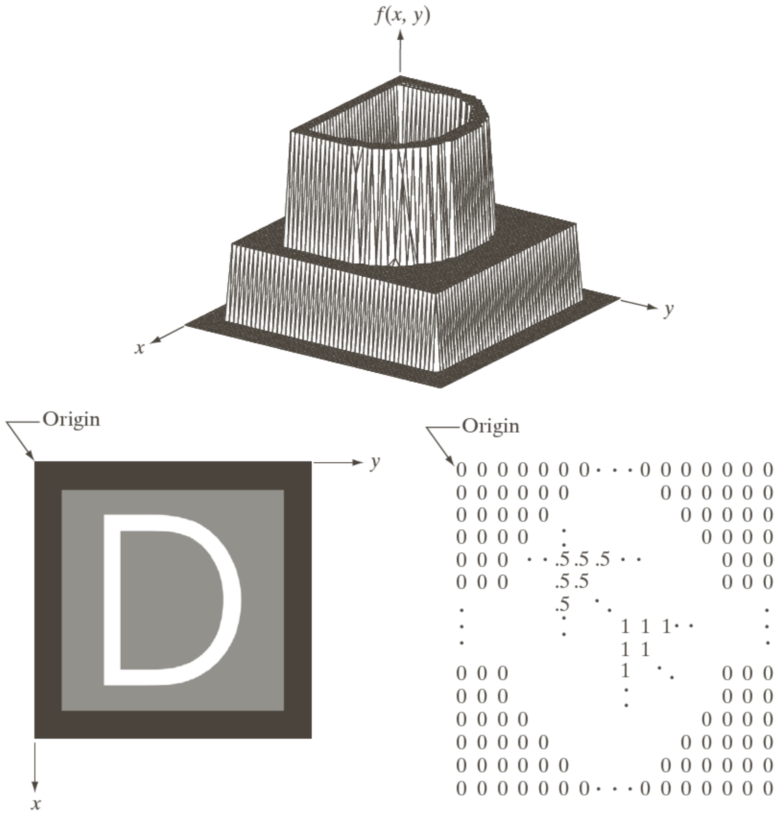
Three basic ways to represent *f*(*x*, *y*)

- Plot of function: *difficult to view and interpret*
- Visual intensity array: *for view*
- numerical array: *for processing and algorithm development*

$$[f(x,y)] = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \ddots & \cdots & \vdots \\ f(M-1,0) & f(M-1,1)\cdots & \cdots & f(M-1,N-1) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \ddots & \cdots & \vdots \\ a_{M-1,0} & a_{M-1,1}\cdots & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Intensity level $L = 2^k$, then $b = M \times N \times k$



Matrix Representation

Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial Resolution (空间分辨率)

Spatial Resolution: smallest discernible detail in an image

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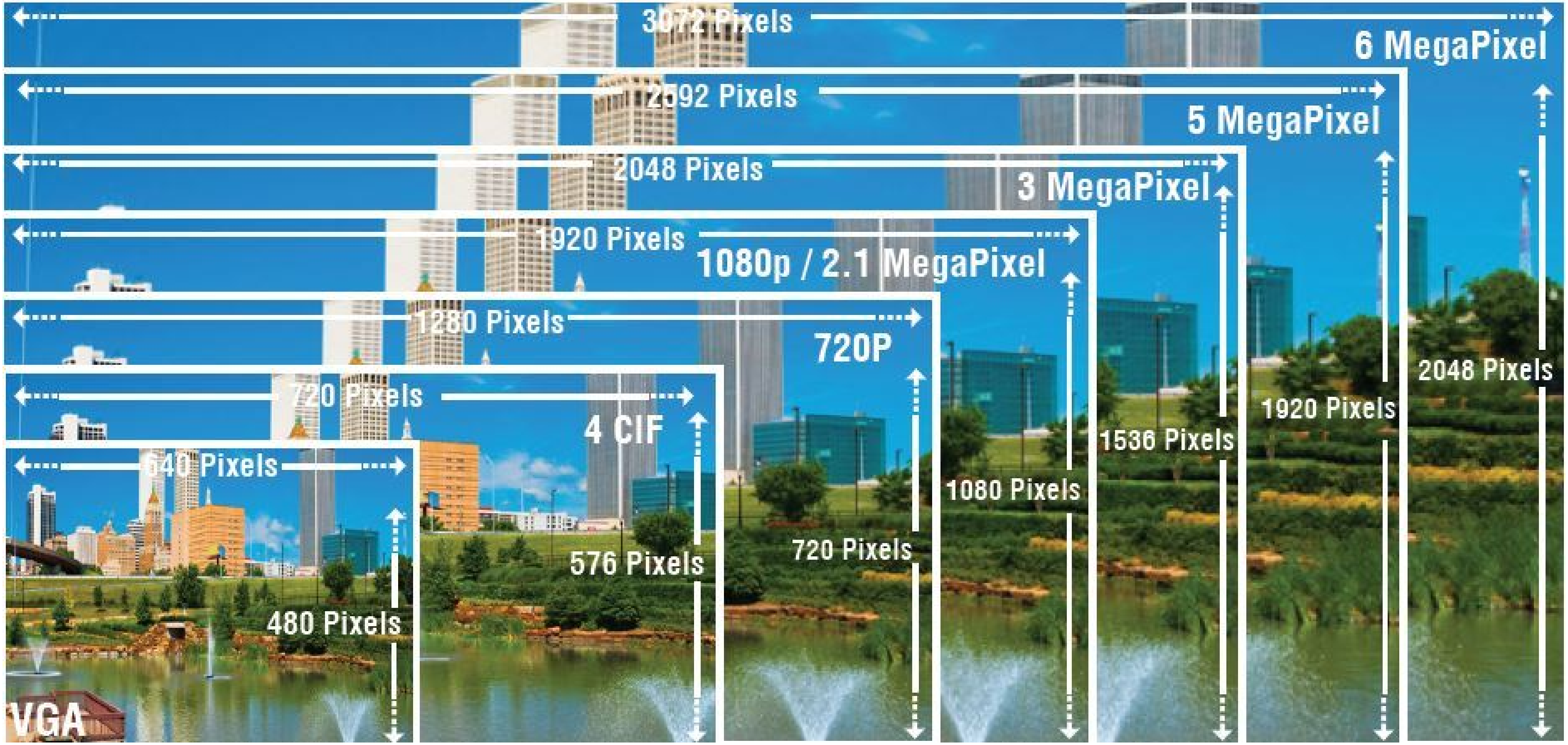
Spatial Resolution

Spatial resolution – Sampling (interval of pixels) vs Size (number of pixels)



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Spatial Resolution

Spatial resolution: dpi (Dots Per Inch)

1250 dpi

300 dpi

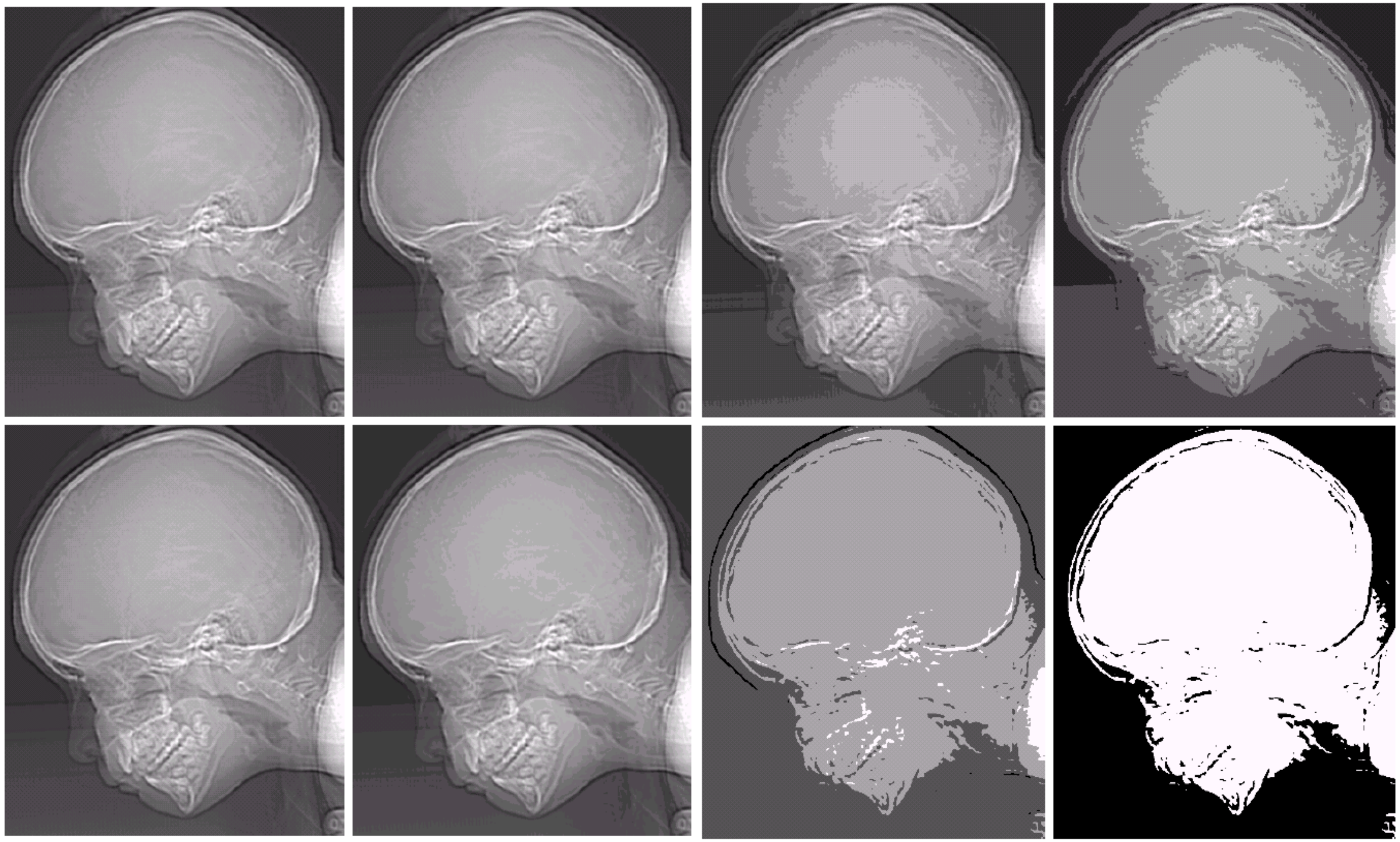
150 dpi

72 dpi

Intensity Resolution (灰度分辨率)

Intensity resolution: smallest discernible change in intensity level (灰度级)

8 bit	7 bit
6 bit	5 bit



4 bit	3 bit
2 bit	1 bit

Interpolation (插值)

- Use known data to estimate values at unknown locations
- Basic tool for geometric transformation
- A resampling (重取样) method

Question?

Optical zoom vs **Digital zoom**

Original | 10x Optical | 10x Digital

Lecture 2 - Image Fundamentals

This lecture will cover:

- Sampling and Quantization
- **Pixels**

Pixels

- **Neighbors of Pixel (像素邻域)**

- **Relationship between Pixels**
  - Adjacency (邻接性)
  - Connectivity (连通性)
  - Regions (区域)
  - Boundaries (边界)

- **Distance measures**
  - Euclidean distance (欧氏距离)
  - City-block distance (街区距离)
  - Chessboard distance (棋盘距离)

Neighbors of Pixel (像素邻域)

If a pixel p at coordinate (x, y)

- **$N_4(p)$ (4邻域)**

- **$N_D(p)$ (对角邻域)**

- **$N_8(p)$ (8邻域)**

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$ (4邻域)**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$ (对角邻域)**

- **$N_8(p)$ (8邻域)**

	q_1	
q_2	p	q_3
	q_4	

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$ (4邻域)**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$ (对角邻域)**

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

- **$N_8(p)$ (8邻域)**

r_1		r_2
	p	
r_3		r_4

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$ (4邻域)**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$ (对角邻域)**

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

- **$N_8(p)$ (8邻域) :** $N_4(p) \cup N_D(p)$

r_1	q_1	r_2
q_2	p	q_3
r_3	q_4	r_4

Pixels

- **Neighbors of Pixel (像素邻域)**
- **Relationship between Pixels**
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  - Connectivity (连通性)
  - Regions (区域)
  - Boundaries (边界)
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Adjacency (邻接性)

To define adjacency of pixels, we need identify

- **Type of Neighbor**

  $N_4(p)$, $N_D(p)$, $N_8(p)$

- **The set of intensity values V**
  - Binary image: $V = \{1\}$
  - Gray-scale image: $V = [L_{min}, L_{max}]$

q_1	p	q_2

0	1	1

Adjacency in a binary image

	q_1	
q_2	p	q_3

	39	
11	13	16

Adjacency in a gray-scale image

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Adjacency

Types of Adjacency:

- **4-adjacency (4邻接)**
- **8-adjacency (8邻接)**
- **M-adjacency (mixed adjacency)**

(M邻接, 混合邻接)

Adjacency

Types of Adjacency:

- **4-adjacency**
  - p, q $\in$ V
  - q $\in N_4(p)$
- 8-adjacency
- M-adjacency (mixed adjacency)

$$\begin{matrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{matrix}$$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Adjacency

Types of Adjacency:

- 4-adjacency
- **8-adjacency**
  - $p, q \in V$
  - $q \in N_8(p)$
- M-adjacency (mixed adjacency)

$$\begin{array}{ccc} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{array}$$

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

Adjacency

Types of Adjacency:

- 4-adjacency
- 8-adjacency
- **M-adjacency (mixed adjacency)**
  - p, q $\in V$
  - $q \in N_4(p)$ or
    $q \in N_D(p)$ and $N_4(p) \cap N_4(q) \notin V$

r_{11}	r_{12}	r_{13}
r_{21}	r_{22}	r_{23}
r_{31}	r_{32}	r_{33}

0	1	1
0	1	0
0	0	1

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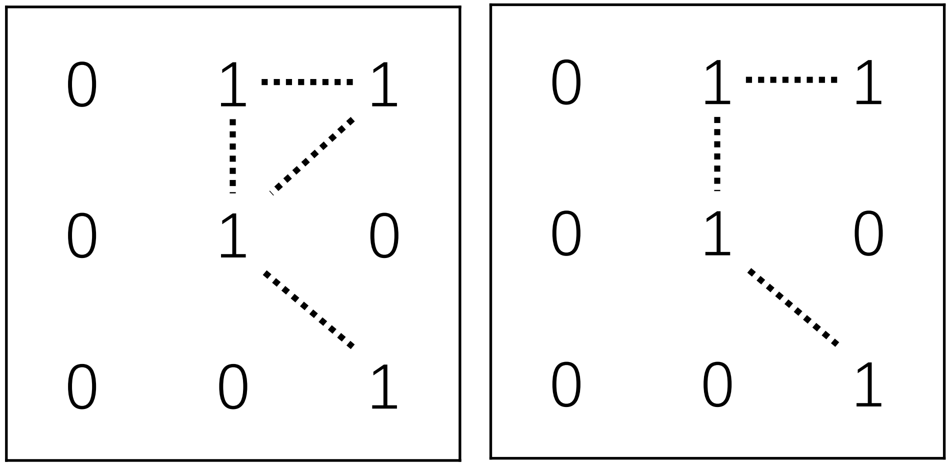
Connectivity (连通性)

Important concept used in establishing boundaries of objects and components of regions in an image

- *Path (通路)*
- *Connected (连通)*
- *Connected component (连通分量)*
- *Connected set (连通集)*

r_{11}	r_{12}	r_{13}
r_{21}	r_{22}	r_{23}
r_{31}	r_{32}	r_{33}



Region (区域)

R: a subset of an image which is also a connected set

- *Adjacent region (邻接区域)*
- *Disjoint region (不连接区域)*

$$
\begin{array}{l}
\left.\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right\} R_i \\
\left.\begin{array}{ccc} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array}\right\} R_j
\end{array}
$$

Boundary (边界)

A set of pixels that are adjacent to pixels in the complement of *R*.

- *Inner border and outer border*
- *Image border*
- *Edge*

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Boundary

A set of pixels that are adjacent to pixels in the complement of *R*.

- *Inner border and outer border*
- *Image border*
- ***Edge (边缘)***

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Pixels

- **Neighbors of Pixel (像素邻域)**
- **Relationship between Pixels**
  - Adjacency (邻接性)
  - Connectivity (连通性)
  - Regions (区域)
  - Boundaries (边界)
- **Distance measures**
  - Euclidean distance (欧氏距离)
  - City-block distance (街区距离)
  - Chessboard distance (棋盘距离)

Distance Measures

For pixels p, q and z, with coordinates (x, y), (s, t) and (v, w), D is a **distance function or metric** if

- $D(p, q) \geqq 0$ ($D(p, q) = 0$ only if $p = q$)
- $D(p, q) = D(q, p)$
- $D(p, z) \leq D(p, q) + D(q, z)$

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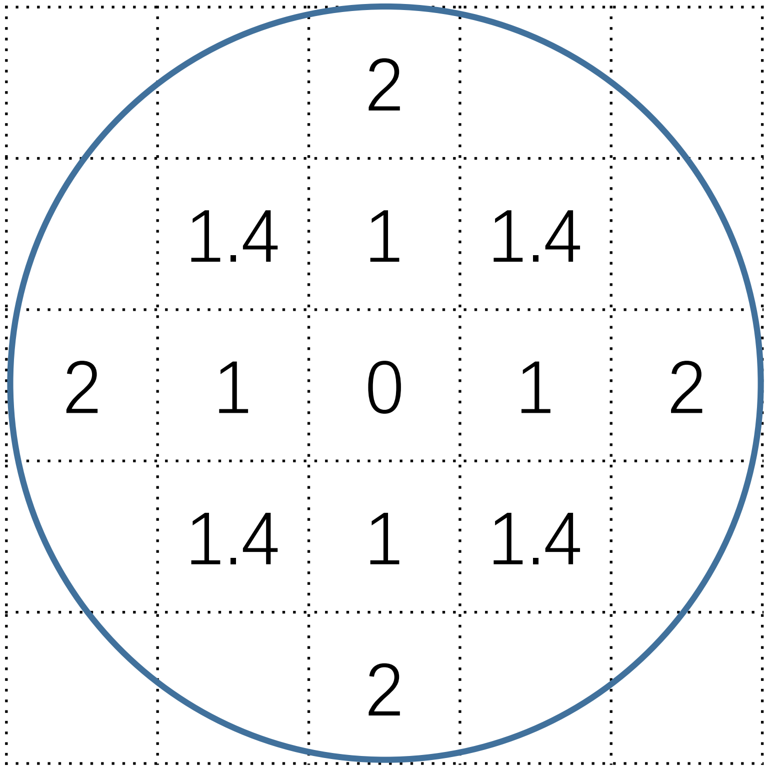
Distance Measures

- **Euclidean distance (欧氏距离):**

$$D_e(p, q) = [\,(x-s)^2+(y-t)^2]^{½}$$

- City-block distance:
- Chessboard distance



Distance Measures

- Euclidean distance：

  $$D_e(p, q) = [(x-s)^2+(y-t)^2]^{½}$$

- **City-block distance (街区距离):**

  $$D_4(p, q) = |x-s|+|y-t|$$

- Chessboard distance

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Distance Measures

- Euclidean distance：

  $$D_e(p, q) = [\ (x-s)^2+(y-t)^2]^{\frac{1}{2}}$$

- City-block distance:

  $$D_4(p, q) = |x-s|+|y-t|$$

- **Chessboard distance (棋盘距离)**

  $$D_8(p, q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

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Distance Measures

D_m distance is defined as the shortest m-path between the point

	r_{12}	1
r_{21}	r_{22}	
1		

D_m = ?

Distance Measures

No m-path between the point

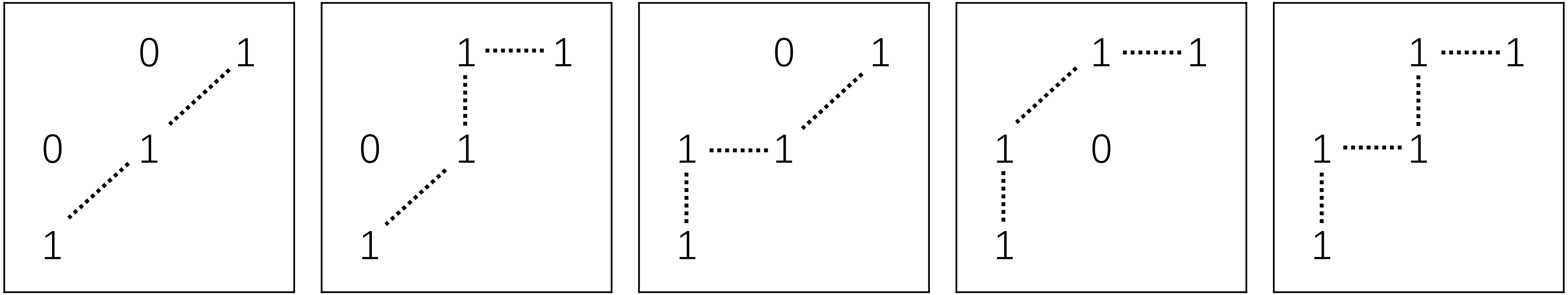
	0	1
0	0	
1		

	1	1
0	0	
1		

	0	1
1	0	
1		

Distance Measures

D_m **distance** is different by the values of r_{12}, r_{21} and r_{22}



$D_m = 2$ $D_m = 3$ $D_m = 3$ $D_m = 3$ $D_m = 4$