Lecture 12 – Image Restoration & Degeneration

This lecture will cover:

– Image Restoration（图像复原）

- Degradation Function（退化函数）
- Inverse Filtering (逆滤波)
- Wiener Filtering（维纳滤波）
- Constrained Least Squares Filtering（约束最小二乘方滤波）
- Geometric Mean Filtering（几何均值滤波）

Image Degradation（图像退化）

- For an operator H, let $\boldsymbol{g(x,y) = H[f(x,y)]}$, where
  - H is linear: $\boldsymbol{H[af_1(x,y) + bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]}$
  - H is position invariant: $\boldsymbol{H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)}$
- The impulse response: $\boldsymbol{h\,(x,\alpha,y,\beta) = H[\delta(x-\alpha, y-\beta)]}$, where $h\,(x,\alpha,y,\beta)$ is called point spread function (PSF,点扩散函数), and

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h\,(x,\alpha,y,\beta)\mathrm{d}\alpha\mathrm{d}\beta$$

- In presence of additive noise

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h\,(x,\alpha,y,\beta)\mathrm{d}\alpha\mathrm{d}\beta + \eta(x,y)$$

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Model of Image Degradation （图像退化模型）

In Spatial domain:

$$g(x,y) = h\,(x,y) \star f(x,y) + \eta(x,y)$$

Where

$g(x,y)$: a degraded image $\quad f(x,y)$: input image

$h(x,y)$: degradation function $\quad \eta(x,y)$: additive noise term

In Frequency domain:

$$G(u,v) = H\,(u,v)F(u,v) + N(u,v)$$

Image deconvolution(图像去卷积)---linear image restoration

Deconvolution filters(去卷积滤波器)--- filters used in the restoration process

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Degradation Function（退化函数）

To estimate degradation function

- Observation
- Experimentation
- Mathematical modeling

Blind Deconvolution(盲去卷积): restore an image by using a degradation function

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Observation

- Gather information from the image itself

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Where

$\hat{F}_s(u,v)$: the processed subimage

$G_s(u,v)$: observed subimage

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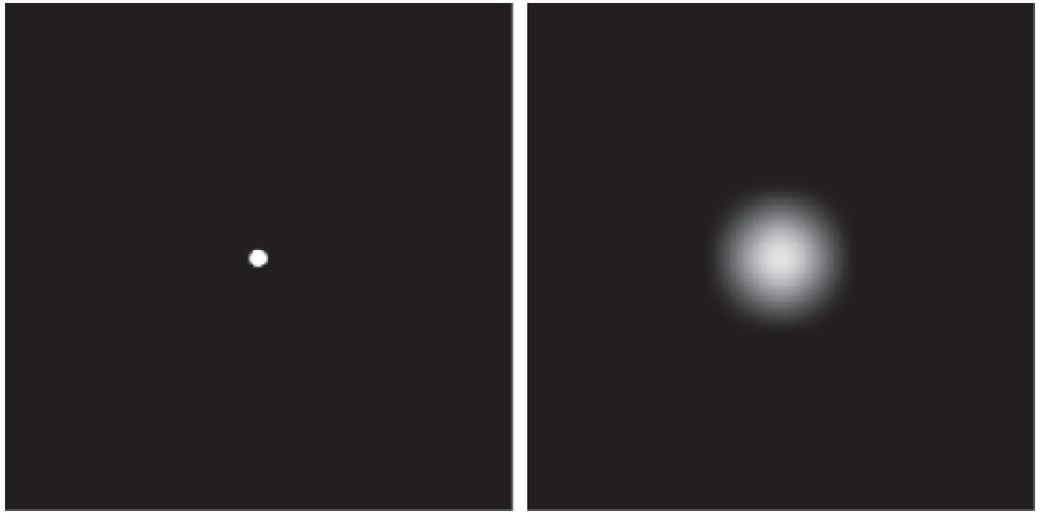
Experimentation

$$H(u,v) = \frac{G(u,v)}{A}$$

$G(u,v)$:the observed image $\quad$ A: a constant strength of the impulse

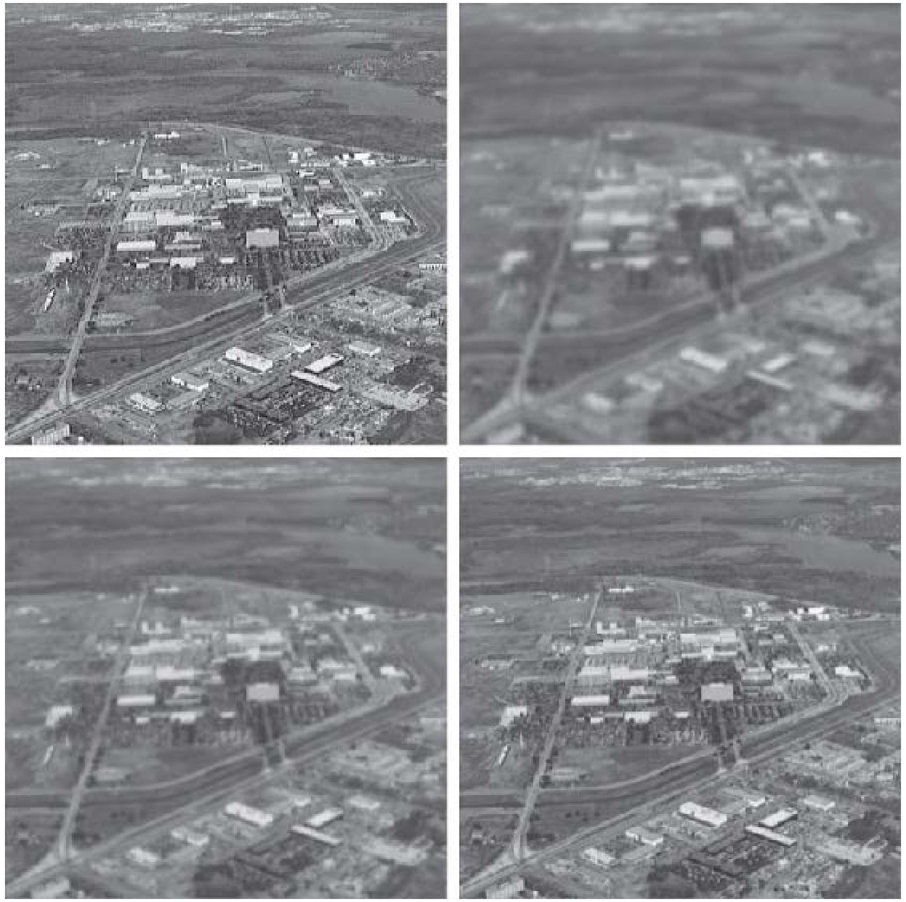
a b

FIGURE 5.24 Estimating a degradation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



Mathematical modeling

- Based on the physical characteristics: $H(u,v) = e^{-k(u^2+v^2)^{5/6}}$



a b
c d

FIGURE 5.25 Modeling turbulence. (a) No visible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. All images are of size 480×480 pixels. (Original image courtesy of NASA.)

Mathematical modeling

➢ By deriving a mathematical model from principles:

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin \pi(ua + vb) e^{-j\pi(ua+vb)}$$

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5-77) with $a = b = 0.1$ and $T = 1$.

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Discrete Degradation Function

- 1D discrete degradation model:

$$g(x) = \sum f(m)h(x-m)$$

where A and B are length of $f(x)$ and $h(x)$

- Consider extend $f(x)$ and $h(x)$ to a period $M \geq A + B - 1$,

$$f_e(x) = \begin{cases} f(x), & 0 \leq x \leq A-1 \\ 0, & A \leq x \leq M-1 \end{cases}$$

$$h_e(x) = \begin{cases} h(x), & 0 \leq x \leq B-1 \\ 0, & B \leq x \leq M-1 \end{cases}$$

then

$$g_e(x) = \sum_{m=0}^{M-1} f_e(m)h_e(x-m)$$

- 1D discrete degradation model in matrix:

$$g = Hf$$

where

$$g = [g_e(0) \quad g_e(1) \quad g_e(2) \cdots g_e(M-1)]^T$$

$$f = [f_e(0) \quad f_e(1) \quad f_e(2) \cdots f_e(M-1)]^T$$

$$H = \begin{bmatrix} h_e(0) & h_e(M-1) & h_e(M-2) & \cdots & h_e(1) \\ h_e(1) & h_e(0) & h_e(M-1) & \cdots & h_e(2) \\ h_e(2) & h_e(1) & h_e(0) & \cdots & h_e(3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_e(M-1) & h_e(M-2) & h_e(M-3) & \cdots & h_e(0) \end{bmatrix}$$

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2D Degradation Model

consider extend $f(x,y)$ and $h(x,y)$ to a period $M \geq A + C - 1$, $N \geq B + D - 1$

$$f_e(x,y) = \begin{cases} f(x,y), & 0 \leq x \leq A-1 \\ & 0 \leq y \leq B-1 \\ 0, & A \leq x \leq M-1 \\ & B \leq y \leq N-1 \end{cases}$$

and

$$h_e(x,y) = \begin{cases} h(x,y), & 0 \leq x \leq C-1 \\ & 0 \leq y \leq D-1 \\ 0, & C \leq x \leq M-1 \\ & D \leq y \leq N-1 \end{cases}$$

Then 2D discrete degradation model:

$$g_e(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_e(m,n) h_e(x-m, y-n)$$

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2D Degradation Model

2D discrete degradation model in matrix: $g = Hf$

Where $g = [g_e^j(x,y)]^T$ and $f = [f_e^j(x,y)]^T$

$$H = \begin{bmatrix} H^0 & H^{M-1} & H^{M-2} & \cdots & H^1 \\ H^1 & H^0 & H^{M-1} & \cdots & H^2 \\ H^2 & H^1 & H^0 & \cdots & H^3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H^{M-1} & H^{M-2} & H^{M-3} & \cdots & H^0 \end{bmatrix}$$

and

$$H^j = \begin{bmatrix} h_e(j,0) & h_e(j,N-1) & h_e(j,N-2) & \cdots & h_e(j,1) \\ h_e(j,1) & h_e(j,0) & h_e(j,N-1) & \cdots & h_e(j,2) \\ h_e(j,2) & h_e(j,1) & h_e(j,0) & \cdots & h_e(j,3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_e(j,N-1) & h_e(j,N-2) & h_e(j,N-3) & \cdots & h_e(j,0) \end{bmatrix}$$

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2D Degradation Model

2D discrete degradation model with noise:

$$g_e(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_e(m, n) h_e(x - m, y - n) + n_e(x, y)$$

2D discrete degradation model in matrix form:

$$g = Hf + n$$

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Algebraic Restoration Method（代数复原）

- Unconstrained restoration method

$$J(\hat{f}) = \|g - H\hat{f}\|^2 \quad \Longrightarrow \quad \frac{\partial J(\hat{f})}{\partial \hat{f}} = -2H^*(g - H\hat{f}) = 0$$

$$\hat{f} = H^{-1}(H^*)^{-1}H^*g$$

- Constrained restoration method

$$J(\hat{f}) = \|Q\hat{f}\|^2 + \lambda\left(\|g - H\hat{f}\|^2\right)$$

$$\Longrightarrow \quad \frac{\partial J(\hat{f})}{\partial \hat{f}} = 2Q^*Q\hat{f} - 2\lambda H^*(g - H\hat{f}) = 0$$

$$\hat{f} = (H^*H + \frac{1}{\lambda}Q^*Q)^{-1}H^*g$$

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Inverse Filtering (逆滤波)

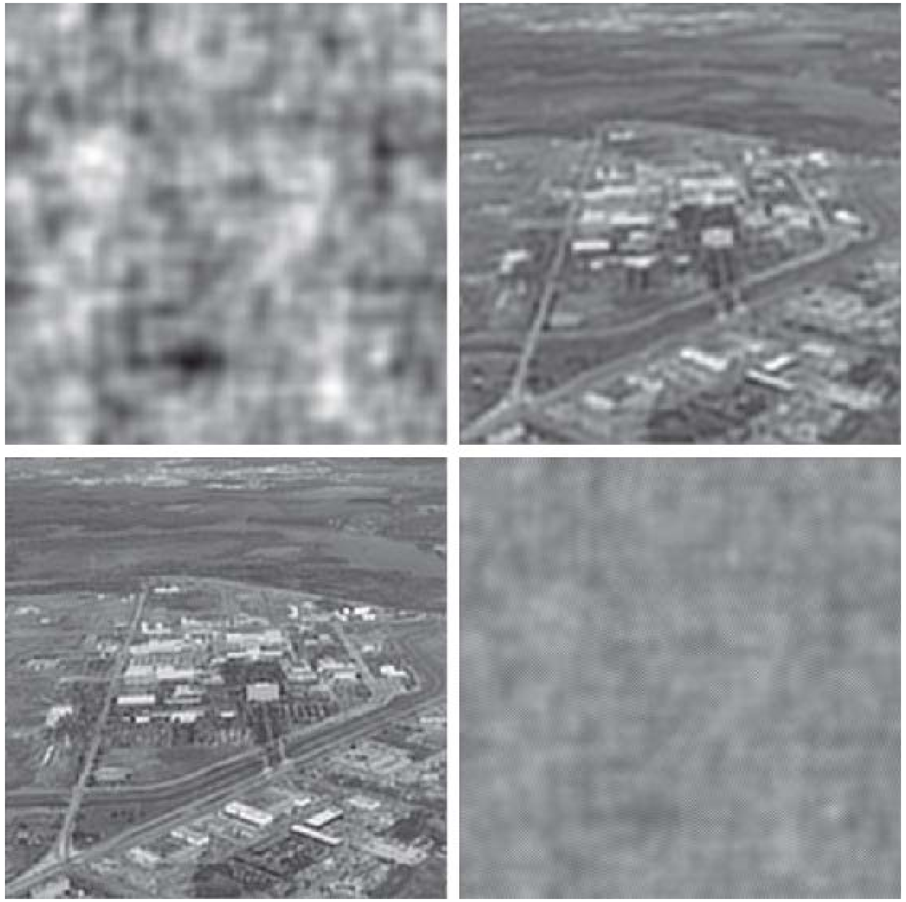
$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$= \frac{H(u,v)F(u,v) + N(u,v)}{H(u,v)}$$

$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) using Eq. (5-78). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40. (c) Result with H cut off outside a radius of 70. (d) Result with H cut off outside a radius of 85.



Wiener Filtering（维纳滤波）

- Expected value of mean square error

$$e^2 = E\left\{\left(f - \hat{f}\right)^2\right\}$$

The estimate of f in frequency domain

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \dfrac{S_n(u,v)}{S_f(u,v)}}\right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \dfrac{S_n(u,v)}{S_f(u,v)}}\right] G(u,v)$$

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SNR (信噪比)

- SNR (Signal-to-noise ratio) in Frequency domain:

$$SNR = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \frac{\left|\hat{F}(u,v\right|^2}{|N(u,v|^2}$$

- SNR (Signal-to-noise ratio) in spatial domain:

$$SNR = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \frac{\hat{f}(x,y)^2}{\left[f(x,y) - \hat{f}(x,y)\right]^2}$$

Wiener Filtering（维纳滤波）

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

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Wiener Filtering（维纳滤波）

a	b	c
d	e	f
g	h	i

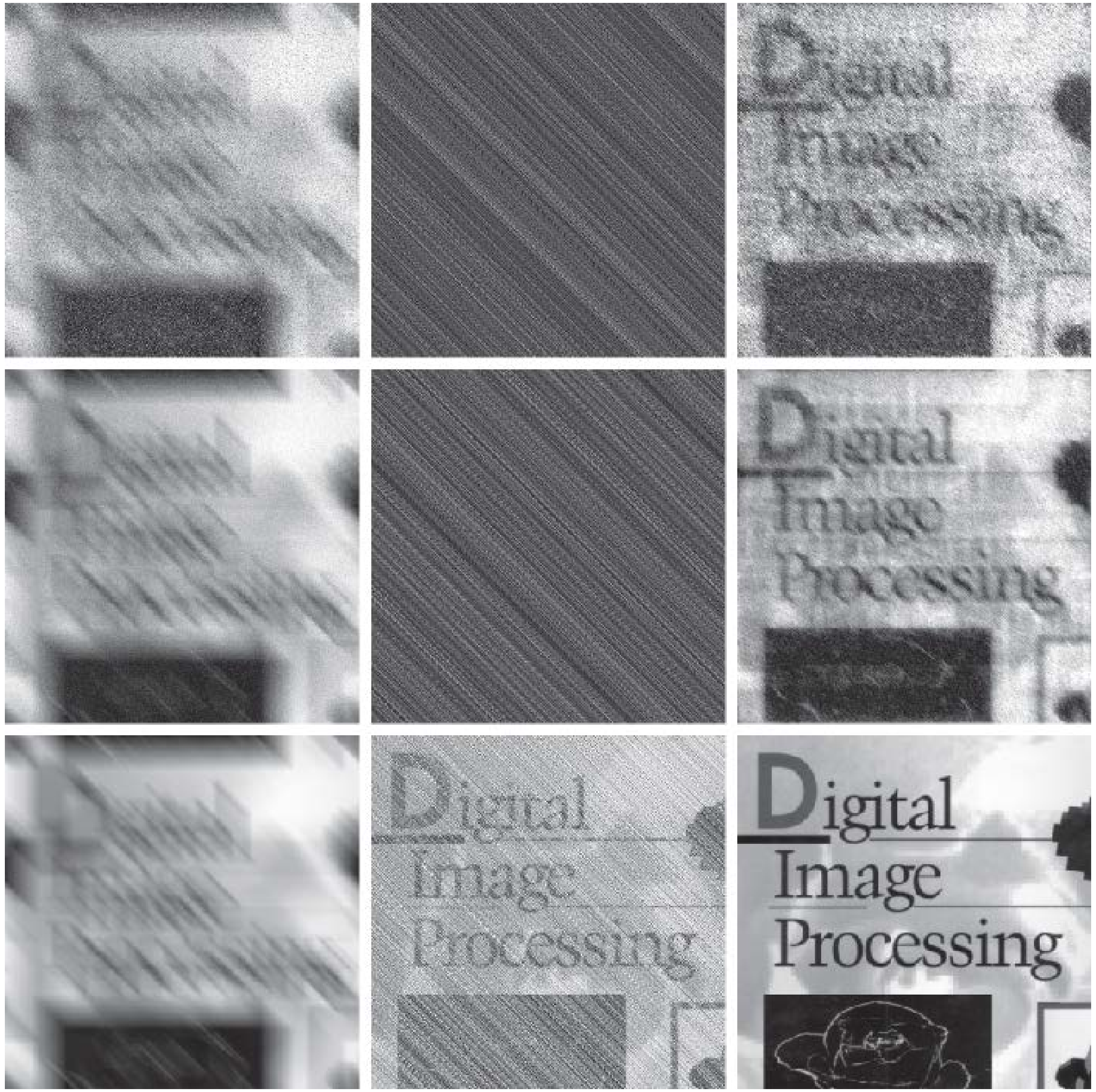

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a “curtain” of noise.

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Constrained Least Squares Filtering（约束最小二乘方滤波）

$$\hat{f} = (H^*H + \frac{1}{\lambda}Q^*Q)^{-1}H^*g \quad \Longrightarrow \hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v)$$

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

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Iterative Method

Let a residual (残差) $\boldsymbol{r} = \boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}} \quad \Longrightarrow \quad \|\boldsymbol{r}\|^2 = \boldsymbol{r}^*\boldsymbol{r}$

Adjust $\boldsymbol{\gamma}$ for $\|\boldsymbol{r}\|^2 = \|\boldsymbol{n}\|^2 \pm \boldsymbol{a}$, where a is an accuracy factor(精确度因子), then an iterative method follows:

1. **Specify an initial value of γ**
2. **Calculate $\|\boldsymbol{r}\|^2$**
3. **There will be three options**
    - Stop if $\|\boldsymbol{r}\|^2 \leq \|\boldsymbol{n}\|^2 \pm \boldsymbol{a}$
    - Increase $\boldsymbol{\gamma}$ if $\|\boldsymbol{r}\|^2 < \|\boldsymbol{n}\|^2 - \boldsymbol{a}$ and return to Step 2
    - Decrease $\boldsymbol{\gamma}$ if $\|\boldsymbol{r}\|^2 > \|\boldsymbol{n}\|^2 + \boldsymbol{a}$ and return to Step 2

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Estimation of Noise

The mean of noise:

$$\bar{m} = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} n(x,y)$$

The variance of noise:

$$\sigma_n^2 = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} [n(x,y) - \bar{m}]^2$$

The Power density of noise:

$$\|n\|^2 = MN[\sigma_n^2 + \bar{m}^2]$$

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Iterative Method

a b

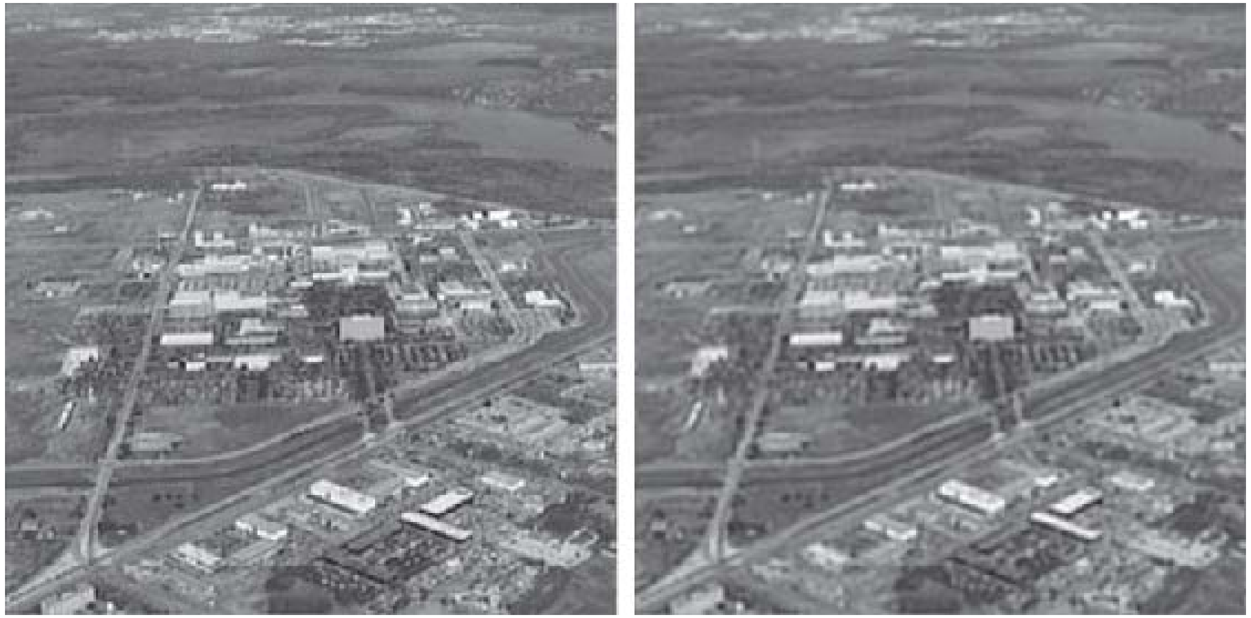

FIGURE 5.31
(a) Iteratively determined constrained least squares restoration of Fig. 5.25(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.

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Geometric Mean Filtering（几何均值滤波）

$$F(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_n(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha} G(u,v)$$

Where α, β: real positive

$\alpha = 1$: inverse filtering

$\alpha = 0$: Parametric Wiener filtering (参数维纳滤波器)

$\alpha = 0, \beta = 1$: Wiener filtering

$\alpha = 1/2, \beta = 1$: Spectrum equalization filter (谱均衡滤波器)

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