

# Lecture 19 – Feature Extraction(特征提取)

This lecture will cover:

- Image Features
- Feature Detection (Representation, 表示)
- Feature Description (描述)
  - Boundary feature descriptor
  - Region feature descriptor
  - Whole-image features

# Feature Extraction

## ➤ Image Feature

- A distinctive attribute or description of “something” to be labeled and differentiated;
- Can be individual image objects or entire images or sets of images
- Independent of location, rotation, scale, illumination

## ➤ Two principle aspects

- Feature Detection (Representation, 表示): *finding the features*
- Feature Description (描述): *assign quantitative (qualitative) attribute to the detected features.*

## ➤ Preprocess before feature extraction: to normalize input images

## ➤ Classification of features

- Invariant (the value remains unchanged) vs Covariant (produce the same result after transformation)
- Local (applied to a member of a set) vs Global (applied to the entire set)

## ➤ Feature Categories: *boundary, region and whole image*

# Feature Detection

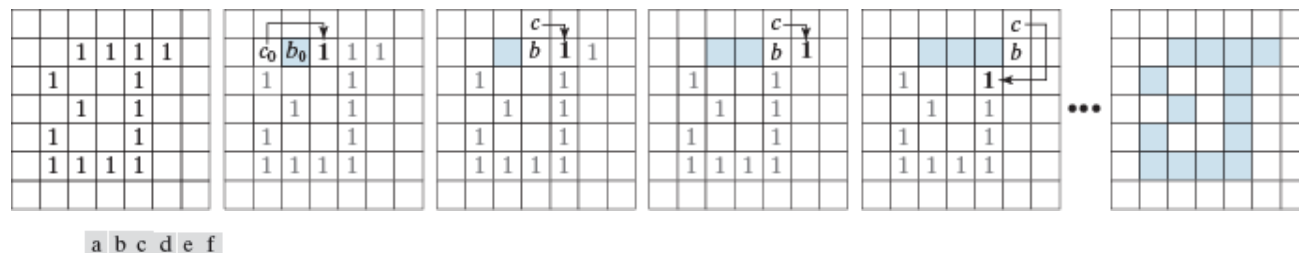
## Boundary preprocessing

- Boundary following (Tracing, 边界追踪)
- Chain codes (链码)
- Polygons (多边形)
- Signatures (标记图)
- Boundary segments (边界线段)
- Skeletons (骨架)

# Boundary following

## The following algorithm:

1. Let the starting point,  $b_0$ , be the *uppermost-leftmost* point in the image that is labeled 1. Denote by  $c_0$  the *west* neighbor of  $b_0$  [Fig.(b)]. Clearly,  $c_0$  is always a background point. Examine the 8-neighbors of  $b_0$ , starting at  $c_0$  and proceeding in a clockwise direction. Let  $b_1$  denote the *first* neighbor encountered whose value is 1, and let  $c_1$  be the (background) point immediately preceding  $b_1$  in the sequence. Store the locations of  $b_0$  for use in Step 5.
2. Let  $b = b_1$  and  $c = c_1$ .
3. Let the 8-neighbors of  $b$ , starting at  $c$  and proceeding in a clockwise direction, be denoted by  $n_1, n_2, \dots, n_8$ . Find the first neighbor labeled 1 and denote it by  $n_k$ .
4. Let  $b = n_k$  and  $c = n_{k-1}$ .
5. Repeat Steps 3 and 4 until  $b = b_0$ . The sequence of  $b$  points found when the algorithm stops is the set of ordered boundary points.



a b c d e f

**FIGURE 12.1** Illustration of the first few steps in the boundary-following algorithm. The point to be processed next is labeled in bold, black; the points yet to be processed are gray; and the points found by the algorithm are shaded. Squares without labels are considered background (0) values.

# Chain codes (链码)

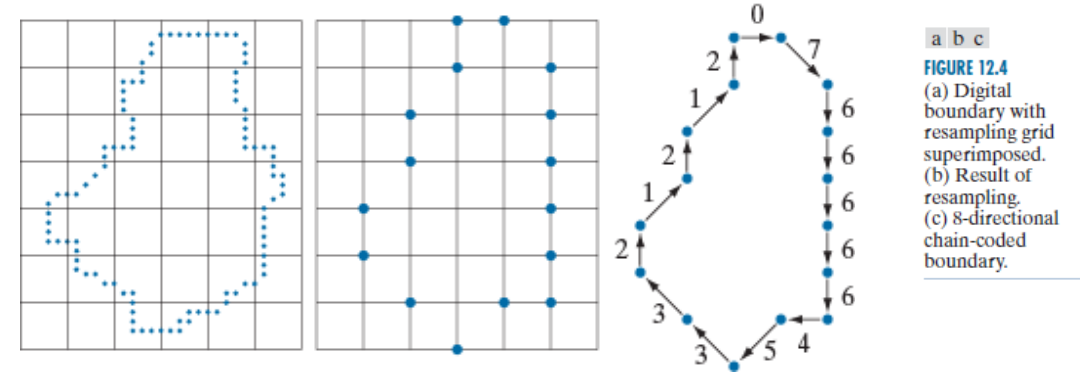
Representing a boundary by a connected sequence of straight-line segments of specified length and direction.

## ➤ Freeman chain code

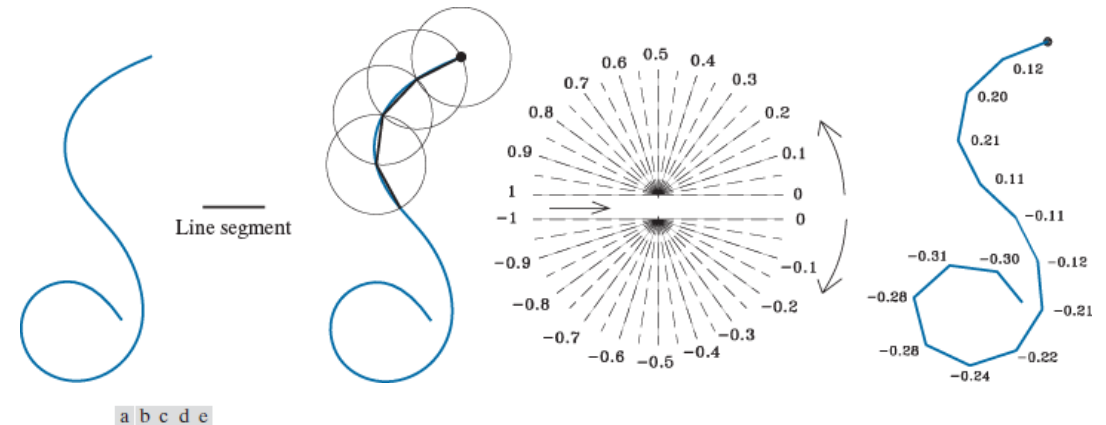
- Based on 4- or 8-connectivity of the segments
- The direction of each segment is coded by using a numbering scheme; and a boundary code is formed as a sequence of such directional numbers

## ➤ Slope chain code

- Placing straight-line segments of equal length around curve with the end points of segments touching the curve;
- Calculating the slope changes between contiguous line segments.

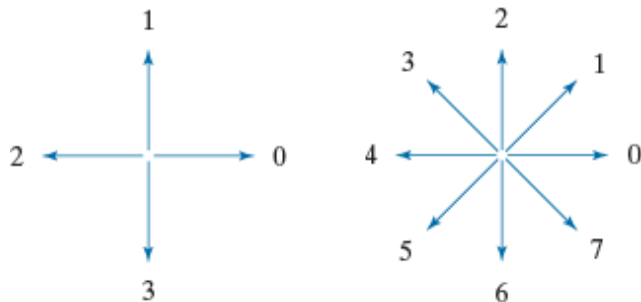


**FIGURE 12.4**  
(a) Digital boundary with resampling grid superimposed.  
(b) Result of resampling.  
(c) 8-directional chain-coded boundary.



**FIGURE 12.6** (a) An open curve. (b) A straight-line segment. (c) Traversing the curve using circumferences to determine slope changes; the dot is the origin (starting point). (d) Range of slope changes in the open interval  $(-1, 1)$  (the arrow in the center of the chart indicates direction of travel). There can be ten subintervals between the slope numbers shown. (e) Resulting coded curve showing its corresponding numerical sequence of slope changes. (Courtesy of Professor Ernesto Bribeas, IIMAS-UNAM, Mexico.)

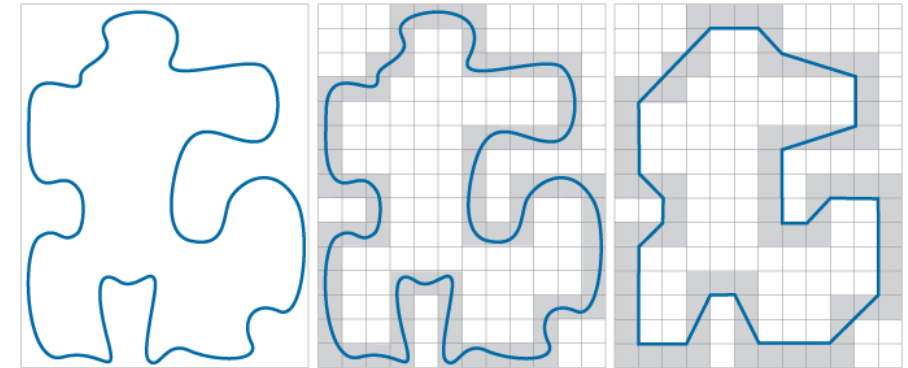
**FIGURE 12.3**  
Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.



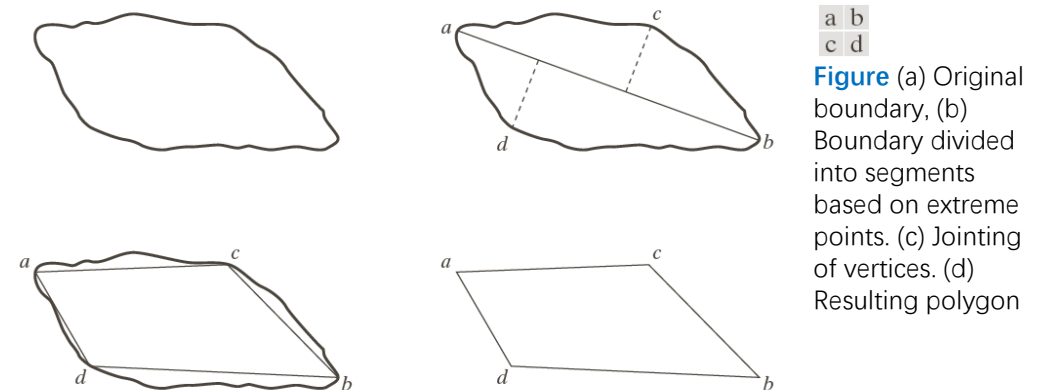
# Polygons (多边形)

The goal of a polygonal approximation is to capture the essence of the shape in a given boundary using the fewest possible number of segments.

- **Minimum-perimeter polygon (MPP, 最小周长多边形)**
  - To enclose a boundary by a set of concatenated cells;
  - The objective is to use the largest possible cell size acceptable in a given application, thus producing MPPs with the fewest number of vertices
- **Merging Techniques (聚合技术)**
  - To merge points along a boundary until the least square error fit of the points exceed a preset threshold
- **Splitting Techniques (分裂技术)**
  - To subdivide a segment successively into two parts until a specified criterion is satisfied;



**FIGURE 12.7** (a) An object boundary. (b) Boundary enclosed by cells (shaded). (c) Minimum-perimeter polygon obtained by allowing the boundary to shrink. The vertices of the polygon are created by the corners of the inner and outer walls of the gray region.

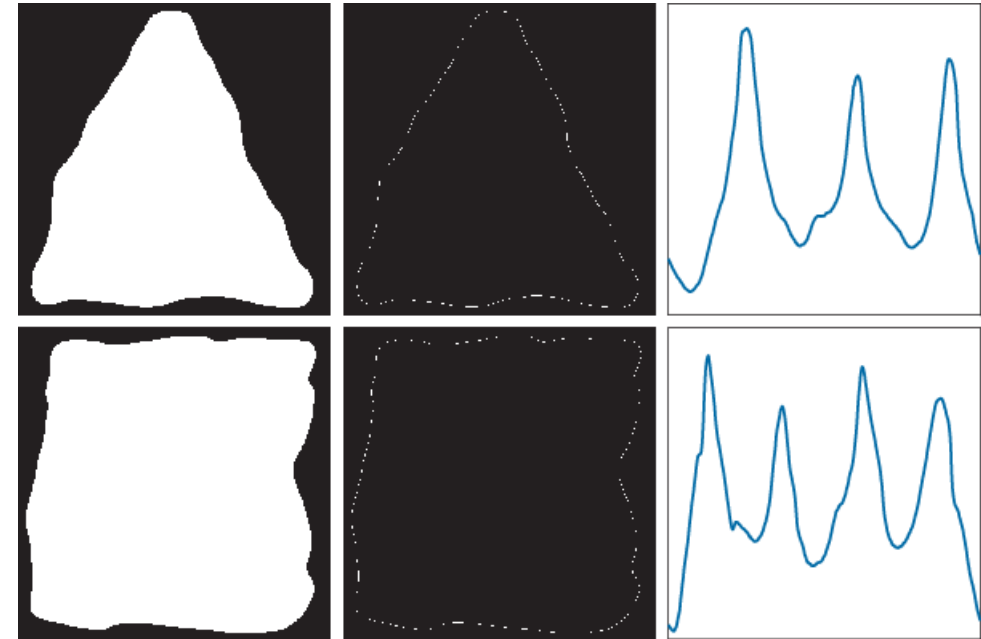


# Signatures (标记图)

A 1-D functional representation of a boundary

May be generated in various ways.

- To plot the distance from the centroid to the boundary as a function of angles.
- Slope density function
  - A histogram of tangent-angle values



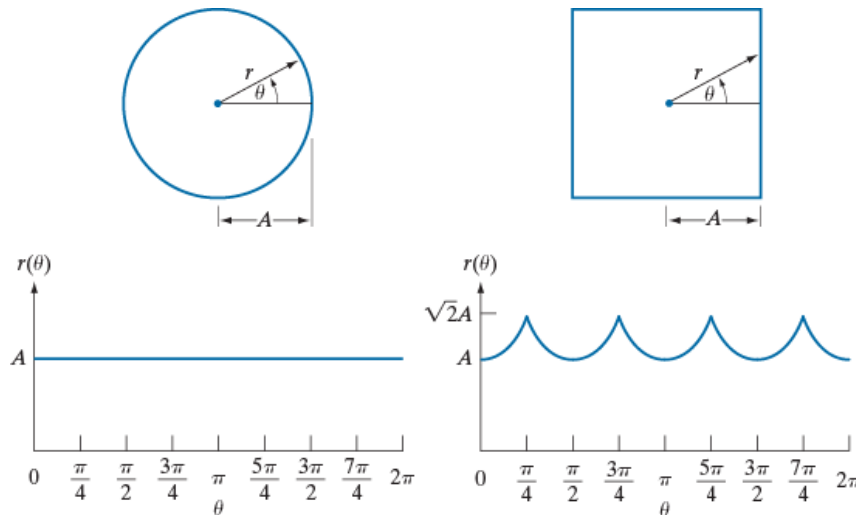
a b c  
d e f

**FIGURE 12.11**  
(a) and (d) Two binary regions, (b) and (e) their external boundaries, and (c) and (f) their corresponding  $r(\theta)$  signatures. The horizontal axes in (c) and (f) correspond to angles from  $0^\circ$  to  $360^\circ$ , in increments of  $1^\circ$ .

a b

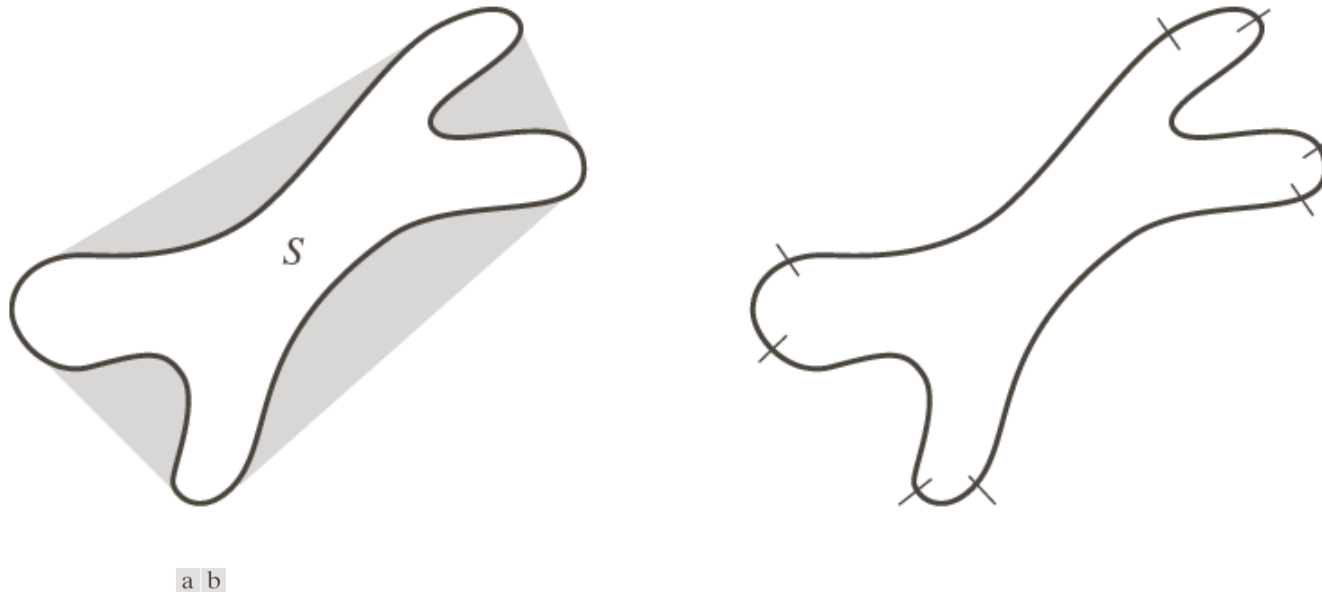
**FIGURE 12.10**

Distance-versus-angle signatures. In (a),  $r(\theta)$  is constant. In (b), the signature consists of repetitions of the pattern  $r(\theta) = A \sec \theta$  for  $0 \leq \theta \leq \pi/4$ , and  $r(\theta) = A \csc \theta$  for  $\pi/4 < \theta \leq \pi/2$ .



# Boundary segments (边界线段)

- Decomposing a boundary into segments
- Particularly attractive to the boundary containing one or more significant concavities.



a b

Figure (a) A region,  $S$ , and its convex deficiency (shaded). (b) Partitioned boundary.

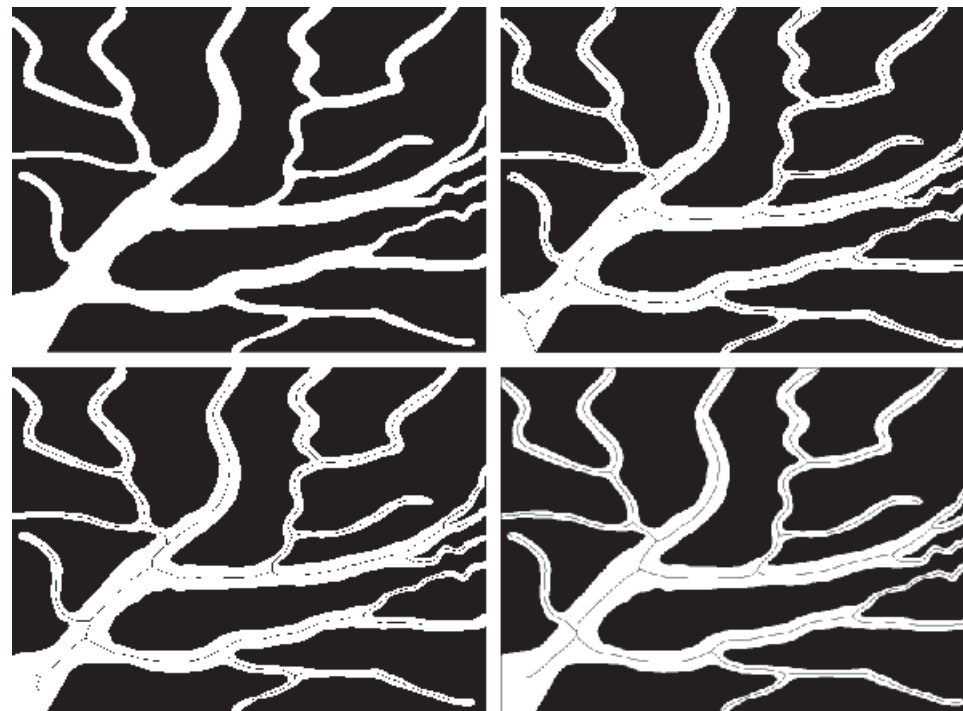
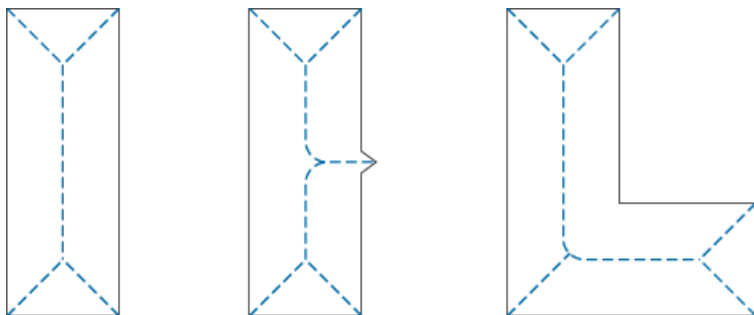


# Skeletons (骨架)

- Related to the shape of a region.
- To reduce a region to a tree or graph by computing its skeleton
- Obtained using two principal approaches:
  - By successively thinning the region (morphological erosion)
  - By computing the medial axis transform (MAT)

a b c

**FIGURE 12.12**  
Medial axes  
(dashed) of three  
simple regions.



a b  
c d

**FIGURE 12.14**  
(a) Thresholded  
image of blood  
vessels.  
(b) Skeleton  
obtained by  
thinning, shown  
superimposed  
on the image  
(note the spurs).  
(c) Result of 40  
passes of spur  
removal.  
(d) Skeleton  
obtained using the  
distance  
transform.

# Feature Description

- Boundary Descriptors (边界描绘子)
  - Basic Boundary Descriptors (基本边界描绘子)
  - Shape Numbers (形状数)
  - Fourier Descriptors (傅里叶描绘子)
  - Statistical Moments (统计矩)
- Regional Descriptors (区域描绘子)
  - Basic Regional Descriptors (基本区域描绘子)
  - Topological Descriptors (拓扑描绘子)
  - Texture (纹理)
  - Moment Invariants (不变矩)
- Whole-image features

# Basic Boundary Descriptors

➤ Length (长度)

➤ Diameter (直径) :

$$\text{Diam}(B) = \max_{i,j} [D(p_i, p_j)]$$

➤ Major Axis (长轴)

➤ Minor Axis (短轴)

➤ Basic rectangle (基本矩形)

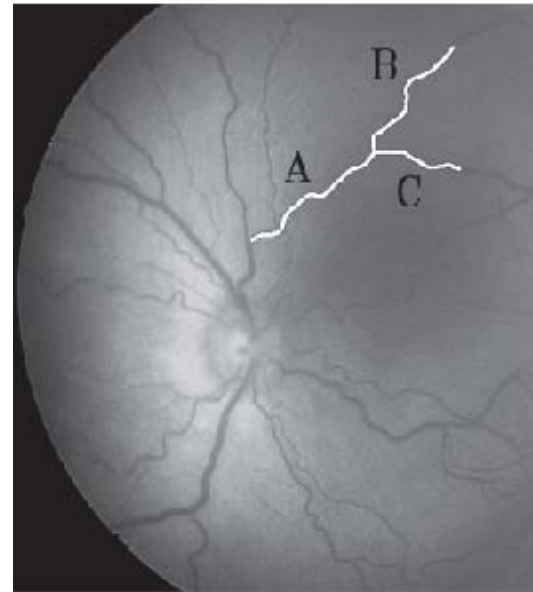
➤ Eccentricity (偏心率)

➤ Curvature (曲率)

➤ Tortuosity (弯曲度)

$$\tau = \sum_{i=1}^n |a_i|$$

where  $|a_i|$  are the values of slope changes



a b

FIGURE 12.15

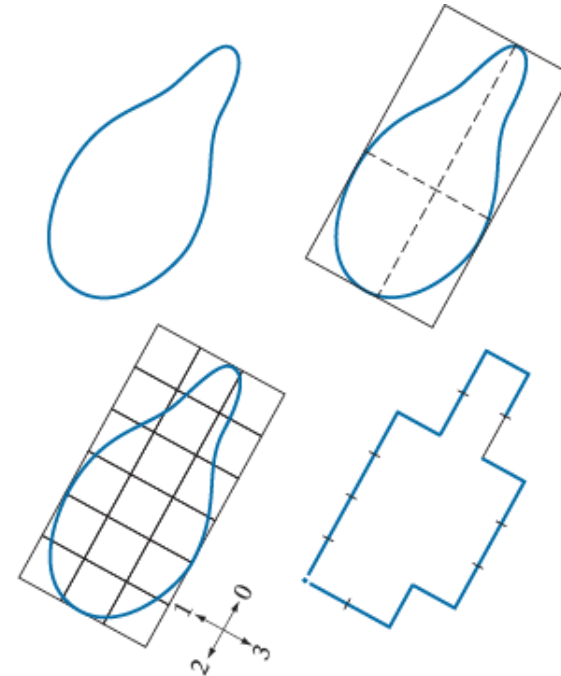
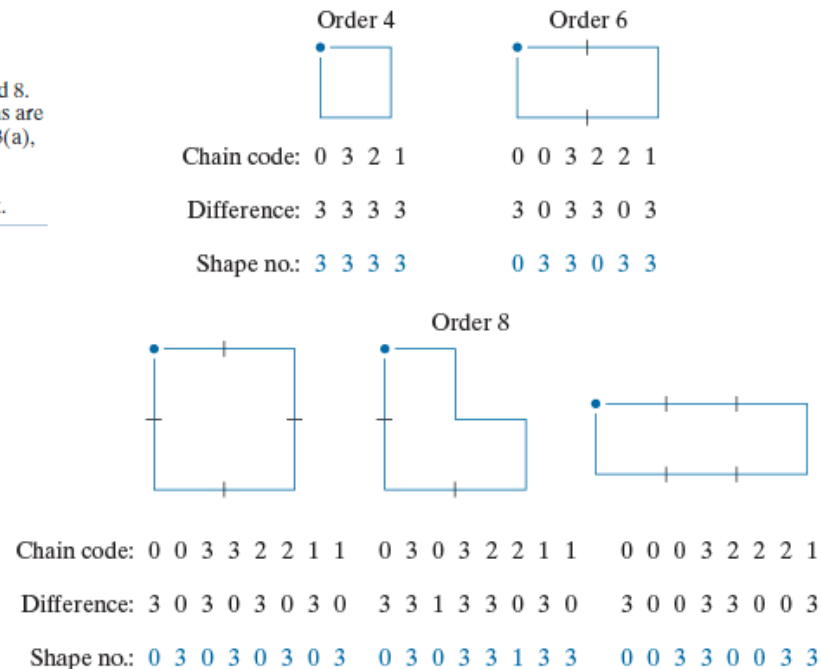
(a) Fundus image from a prematurely born baby with ROP.  
(b) Tortuosity of vessels A, B, and C.  
(Courtesy of Professor Ernesto Bribiesca, IIMAS-UNAM, Mexico.)

Curve	$n$	$\tau$
A	50	2.3770
B	50	2.5132
C	50	1.6285

# Shape Numbers (形状数)

- Based on Freeman Chain Code to describe the shape of boundary and defined as the first difference of smallest magnitude;
- The *order*  $n$ , of a shape number is the number of digits in its representation.

**FIGURE 12.16**  
All shapes of order 4, 6, and 8. The directions are from Fig. 12.3(a), and the dot indicates the starting point.



a b  
c d

**FIGURE 12.17**  
Steps in the generation of a shape number.

Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1  
 Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0  
 Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

# Fourier Descriptors (傅里叶描绘子)

- The sequence of coordinates

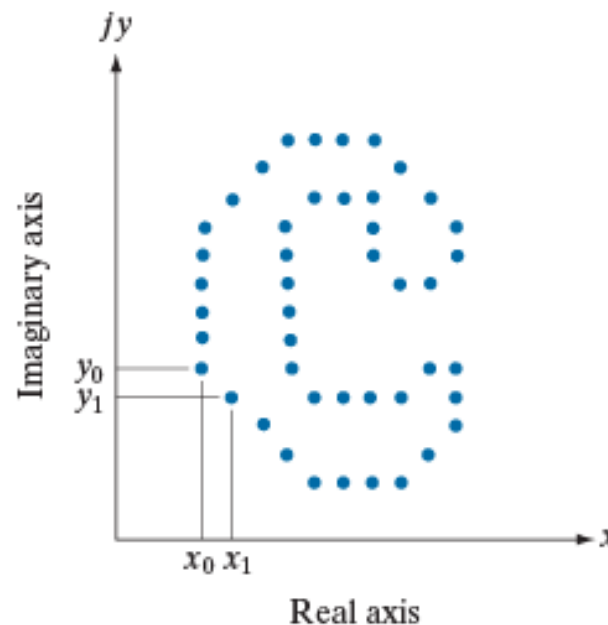
$$s(k) = x(k) + jy(k)$$

- The Fourier Descriptor

$$a(u) = \sum_{k=0}^{K-1} s(k) e^{-j2\pi uk/K}$$

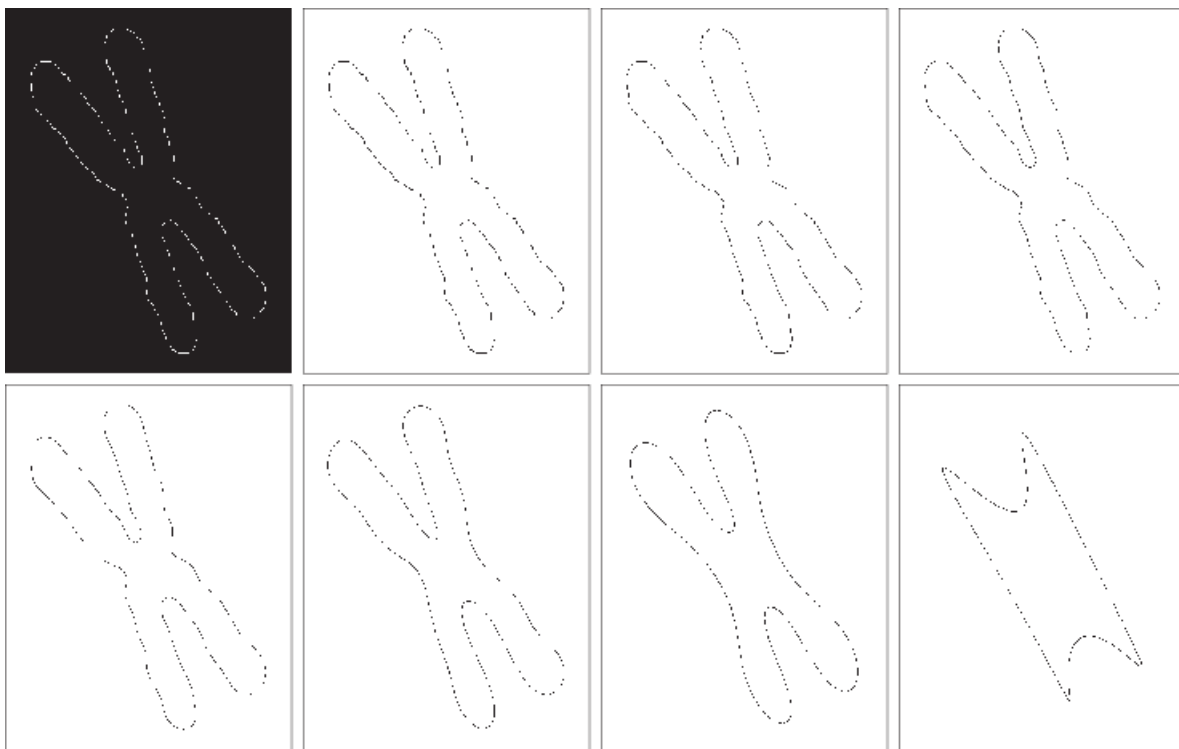
- The IDFT

$$s(k) = \frac{1}{K} \sum_{u=0}^{K-1} a(u) e^{j2\pi uk/K}$$



**FIGURE 12.18**  
A digital boundary and its representation as sequence of complex numbers. The points  $(x_0, y_0)$  and  $(x_1, y_1)$  are (arbitrarily) the first two points in the sequence.

# Fourier Descriptors (傅里叶描绘子)



a b c d  
e f g h

**FIGURE 12.19** (a) Boundary of a human chromosome (2868 points). (b)–(h) Boundaries reconstructed using 1434, 286, 144, 72, 36, 18, and 8 Fourier descriptors, respectively. These numbers are approximately 50%, 10%, 5%, 2.5%, 1.25%, 0.63%, and 0.28% of 2868, respectively. Images (b)–(h) are shown as negatives to make the boundaries easier to see.

**Table** Some basic properties of Fourier descriptors

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

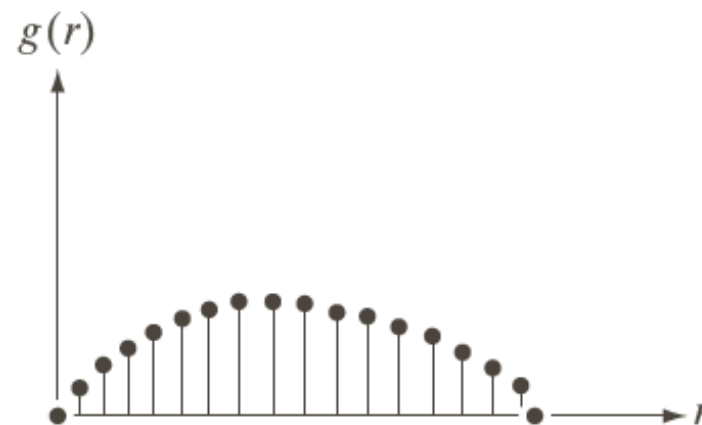
# Statistical Moments (统计矩)

➤ To quantitatively describe the shape of boundary segments (and of signature waveform) – for signature and boundary segments

- Mean:  $m = \sum_{i=0}^{K-1} r_i g(r_i)$
- The  $n$ th moment of  $r$  about its mean:  $\mu_n(r) = \sum_{i=0}^{K-1} (r_i - m)^n g(r_i)$

a b

**Figure** (a) Boundary segment, (b) representation as 1D function



# Basic Regional Descriptors

- Area (面积)
- Perimeter (周长)
- Compactness (致密性) :  $P^2/A$
- Circularity ratio (圆度率) :  $R_c = 4\pi A/P^2$
- Eccentricity (离心率) :

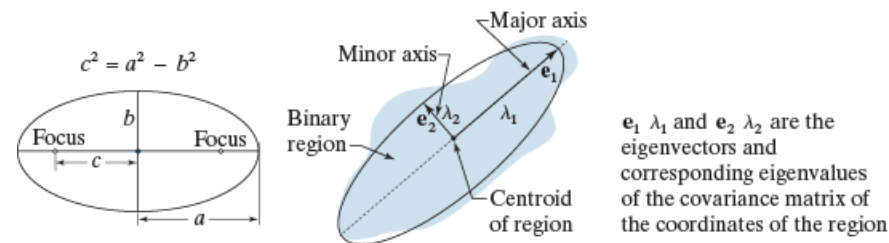
$$R_E = \sqrt{1 - (b/a)^2}$$

- Intensity related descriptors:
  - ✓ Mean and Median
  - ✓ Minimum and Maximum
  - ✓ The number of pixels with values above and below the mean

a b

FIGURE 12.21





(a) An ellipse in standard form.  
(b) An ellipse approximating a region in arbitrary orientation.



a b c d

FIGURE 12.22

Compactness, circularity, and eccentricity of some simple binary regions.

Descriptor				
Compactness	10.1701	42.2442	15.9836	13.2308
Circularity	1.2356	0.2975	0.7862	0.9478
Eccentricity	0.0411	0.0636	0	0.8117

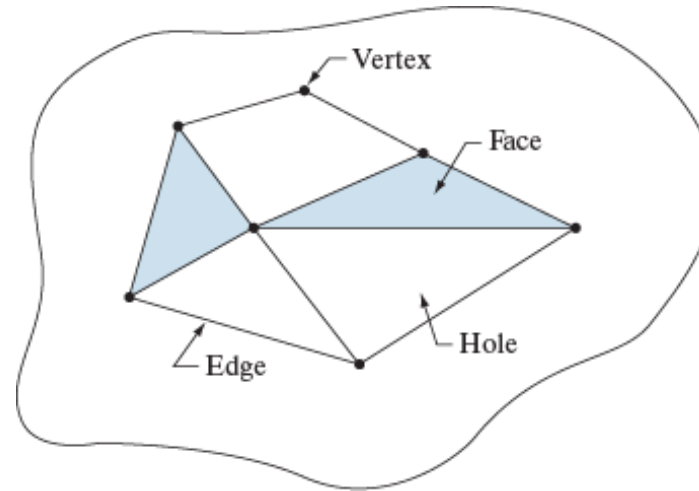
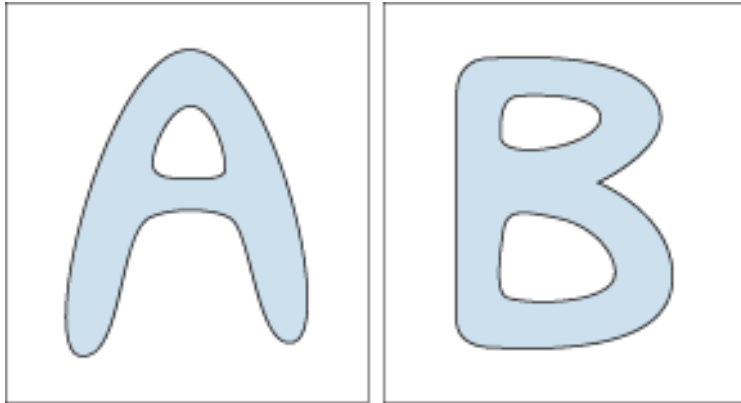


# Topological Descriptors (拓扑描绘子)

- The properties of a figure that are unaffected by any deformation.
- Euler Number (欧拉数):  $E = C - H$
  - Euler Formula (欧拉公式):  $E = C - H = V - Q + F$

a b

**FIGURE 12.26**  
Regions with Euler numbers equal to 0 and -1, respectively.



**FIGURE 12.27**  
A region containing a polygonal network.

# Texture (纹理)

- To qualify the texture content of region and provide measures of properties such as smoothness, coarseness and regularity.
- **Statistical approach:** characterization of textures as smooth, coarse, grainy, etc.

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

Where

$\mu_n(z)$ : the  $n$ th moment of  $z$  about the mean;

$p(z_i)$ : the normalized histogram

$m$ : average intensity (the mean value of  $z$ )

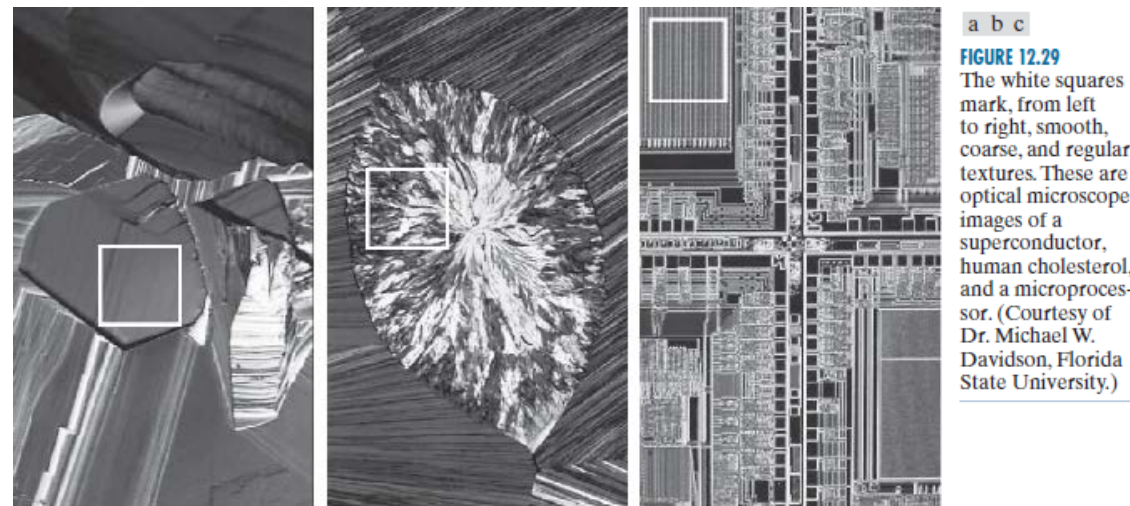
And

$$\mu_0(z) = 1, \quad \mu_1(z) = 0$$

$\mu_2(z)$ : the variance, a measure of intensity contrast; and  $R(z) = 1 - \frac{1}{1+\sigma^2(z)}$

$\mu_3(z)$ : a measure of skewness (偏斜度) of the histogram;

$\mu_4(z)$ : a measure of the relative flatness;



**FIGURE 12.29**  
The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

**TABLE 12.2**

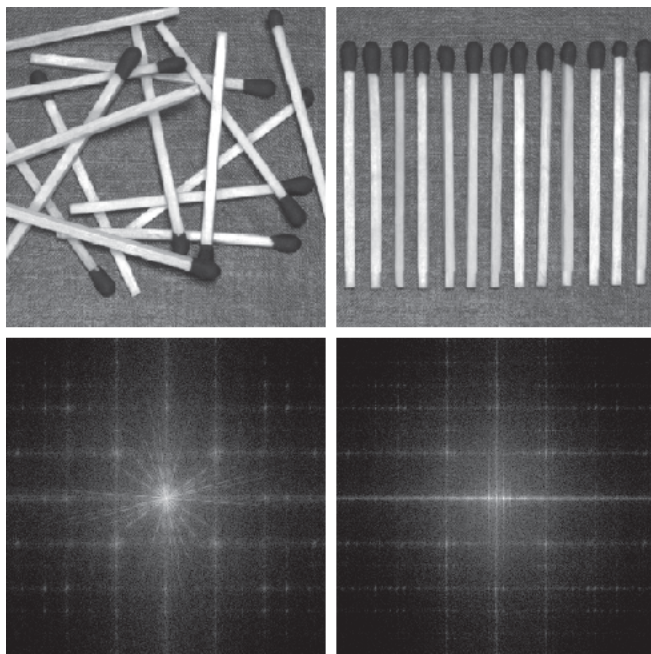
Statistical texture measures for the subimages in Fig. 12.29.

Texture	Mean	Standard deviation	$R$ (normalized)	3rd moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	-0.105	0.026	5.434
Coarse	143.56	74.63	0.079	-0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

# Texture (纹理)

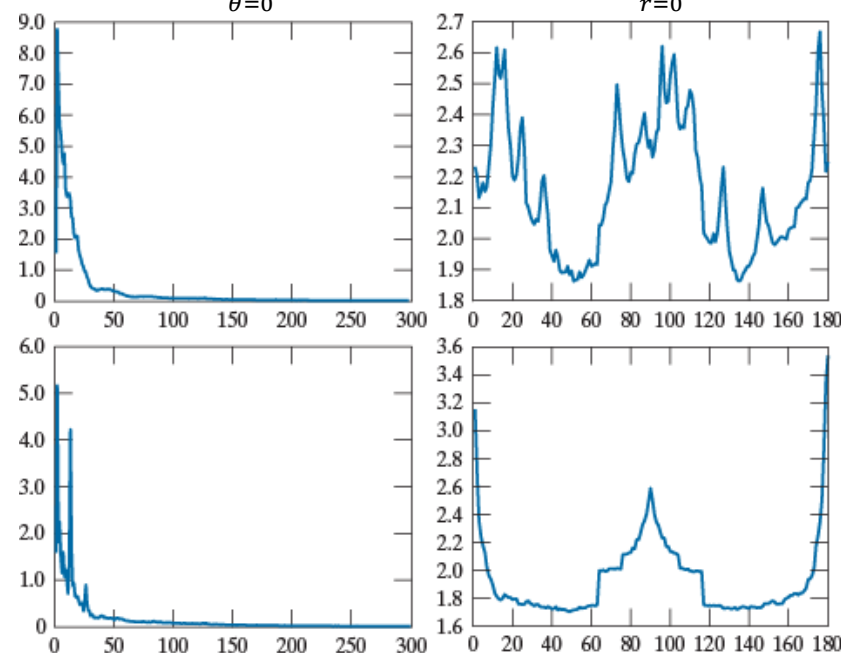
- **Spectral approach:**
  - ▣ based on properties of Fourier spectrum and to detect global periodicity in an image.
  - ▣ three useful features of the Fourier spectrum
    - ✓ Prominent peaks in the spectrum give the principal direction of the texture patterns
    - ✓ The location of the peaks in the frequency plane gives the fundamental spatial period of the pattern
    - ✓ Eliminating any periodic components via filtering leaves nonperiodic image elements which can be described by statistical techniques

a b  
c d  
**FIGURE 12.35**  
(a) and (b) Images of random and ordered objects. (c) and (d) Corresponding Fourier spectra. All images are of size  $600 \times 600$  pixels.



$$S(r) = \sum_{\theta=0}^{\pi} S_{\theta}(r)$$

$$S(\theta) = \sum_{r=0}^{R_0} S_r(\theta)$$



a b  
c d  
**FIGURE 12.36**  
(a) and (b) Plots of  $S(r)$  and  $S(\theta)$  for Fig. 12.35(a). (c) and (d) Plots of  $S(r)$  and  $S(\theta)$  for Fig. 12.35(b). All vertical axes are  $\times 10^5$ .

# Moment Invariants (不变矩)

- Central moment of order  $(p + q)$  :

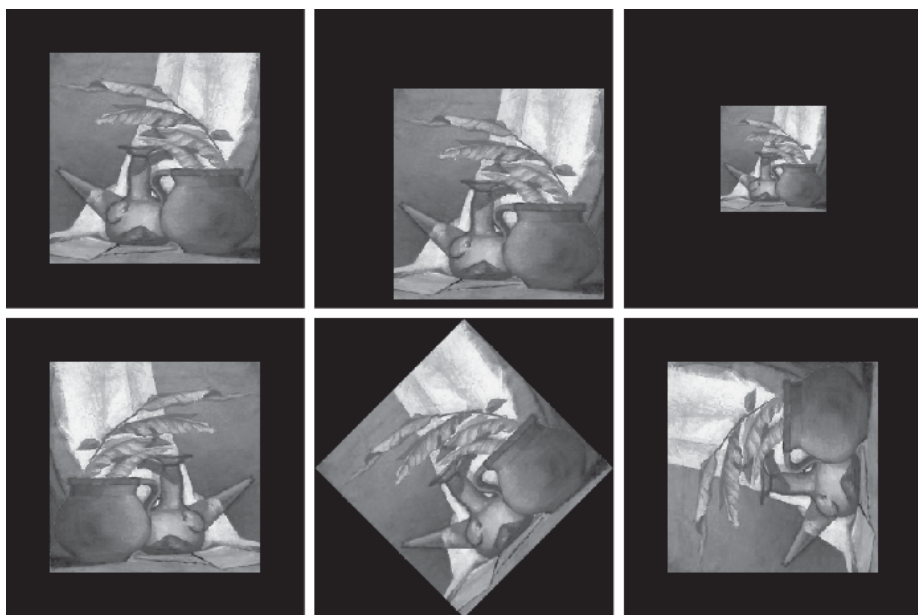
$$\mu_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - \bar{x})^p (y - \bar{y})^q f(x, y)$$

- The normalized central moment :

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^\gamma} \quad \text{where} \quad \gamma = \frac{p+q}{2} + 1$$

- 2D Moment Invariants:

$$\begin{aligned} \Phi_1 &= \eta_{02} + \eta_{20} & \Phi_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\ \Phi_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 & \Phi_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\ \Phi_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\ &\quad + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ \Phi_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\ \Phi_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\ &\quad + (3\eta_{12} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \end{aligned}$$



a b c  
d e f

FIGURE 12.37 (a) Original image. (b)–(f) Images translated, scaled by one-half, mirrored, rotated by 45°, and rotated by 90°, respectively.

TABLE 12.5

Moment invariants for the images in Fig. 12.37.

Moment Invariant	Original Image	Translated	Half Size	Mirrored	Rotated 45°	Rotated 90°
$\phi_1$	2.8662	2.8662	2.8664	2.8662	2.8661	2.8662
$\phi_2$	7.1265	7.1265	7.1257	7.1265	7.1266	7.1265
$\phi_3$	10.4109	10.4109	10.4047	10.4109	10.4115	10.4109
$\phi_4$	10.3742	10.3742	10.3719	10.3742	10.3742	10.3742
$\phi_5$	21.3674	21.3674	21.3924	21.3674	21.3663	21.3674
$\phi_6$	13.9417	13.9417	13.9383	13.9417	13.9417	13.9417
$\phi_7$	-20.7809	-20.7809	-20.7724	20.7809	-20.7813	-20.7809



# Whole-image features

- Feature descriptors applicable to entire images that are members of a large family of images.
- Two principal feature detection method for whole-image features
  - The Harris-Stephens corner detector:  
Sharp transitions of intensities
  - Maximally stable extremal regions (MSERs):  
“blob” oriented  
Establishing correspondence between two or more images
- Scale-invariant feature transform (SIFT)
  - For extracting invariant features from an image;
  - SIFT features (keypoints) are invariant to image scale and rotation, and are robust across a range of affine distortion, changes in 3D viewpoint, noise and changes of illumination.