# HW 5 Key

1. (15pt) A fisherman is restricted to catching at most two red grouper per day when fishing in the Gulf of Mexico. A field agent for the wildlife commission of inspects the day's catch for boats as they come to shore near his base. He has found that the number of red grouper caught has the following distribution

| Number of grouper | 0 | 1 | 2 |
|---|---|---|---|
| Probability | 0.2 | 0.7 | 0.1 |

Find the expectation, the variance, and the standard deviation for the individual daily catch of red grouper.

E(X)=(0)(0.2)+(1)(0.7)+(2)(0.1)=0.9
Var(X)=E(X2)-[E(X)]2={0(0.2)+(1)2(0.7)+(2)2(0.1)-(0.9)2 = 0.29
The standard deviation $\sigma = \sqrt{0.59} \approx 0.5385$

2. (20pt) For a laboratory assignment, if the equipment is working, the density function of the observed outcome X is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find the mean, variance, and standard deviation of X.

$$E(X) = 2\int_0^1 x(1-x)dx = \frac{1}{3}$$
$$E(X^2) = 2\int_0^1 x^2(1-x)dx = \frac{1}{6}$$
$$Var(X) = \frac{1}{6} - (\frac{1}{3})^2 = \frac{1}{18}$$
$$\sigma = \sqrt{\frac{1}{18}} = 0.2357$$

3. (45pt) From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find
(a) (5pt) the joint probability distribution of X and Y and (5pt) the joint probability table;

We can select x oranges from 3, y apples from 2, and 4-x-y bananas from 3 in $\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}$ ways. A random selection of 4 pieces of fruit can be made in $\binom{8}{4}$ ways.
Thus, $f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, x = 0,1,2,3; y = 0,1,2; 1 \le x + y \le 4.$

| f(x,y) | | x | | | | Row Totals |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | |
| y | 0 | 0 | 3/70 | 9/70 | 3/70 | 3/14 |
| | 1 | 1/35 | 9/35 | 9/35 | 1/35 | 4/7 |
| | 2 | 3/70 | 9/70 | 3/70 | 0 | 3/14 |
| Column Totals | | 1/14 | 3/7 | 3/7 | 1/14 | 1 |

(b) (20pt) the covariance between X and Y;

$$E(XY) = \sum_x \sum_y xyf(x,y) = \frac{9}{7}$$

$$\mu_X = \sum_x xg(x) = \frac{3}{2}$$

$$\mu_Y = \sum_y yh(y) = 1$$

$$\sigma_{XY} = \frac{9}{7} - \left(\frac{3}{2}\right)(1) = -\frac{3}{14}$$

(c) (15pt) the correlation coefficient of X and Y.

$$\sigma_X^2 = E(X^2) - \mu_X^2 = \sum_x \sum_y x^2 f(x,y) - \mu_X^2 = \sum_x x^2 g(x) - \mu_X^2 = \frac{15}{28}$$

$$\sigma_Y^2 = E(Y^2) - \mu_Y^2 = \sum_x \sum_y y^2 f(x,y) - \mu_Y^2 = \sum_y y^2 h(y) - \mu_Y^2 = \frac{3}{7}$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{-3/14}{\sqrt{15/28} \times \sqrt{3/7}} = -\frac{1}{\sqrt{5}} = -0.4472$$

4. (20pt) Let X represent the number that occurs when a green die is tossed and Y the number that occurs when a red die is tossed. Find the variance of the random variable

$$\mu_X = \mu_Y = \frac{1}{6}(1 + 2 + \cdots + 6) = 3.5$$

$$\sigma_X^2 = \sigma_Y^2 = \left(\frac{1}{6}\right)[1^2 + 2^2 + \cdots + 6^2] - 3.5^2 = \frac{35}{12}$$

(a) $2X - Y$;

$$\sigma_{2X-Y} = 4\sigma_X^2 + \sigma_Y^2 = \frac{175}{12}$$

(b) $X + 3Y - 5$.

$$\sigma_{X+3Y-5} = \sigma_X^2 + 9\sigma_Y^2 = \frac{175}{6}$$