

Chapter 16

The basis of special relativity

chapter 16

theory of relativity

Chapter 16: Foundation of Special relativity

- Einstein's hypothesis and the Lorentz transformation
- Relativity-theoretical space-time view
- Relativistic velocity transformation formula
- Momentum, mass, and energy in special relativity

By the end of the 19th century, classical physics was quite mature, "after adding only a few numbers after the decimal point of the known laws." But there were " two dark clouds in the seemingly clear sky of physics.

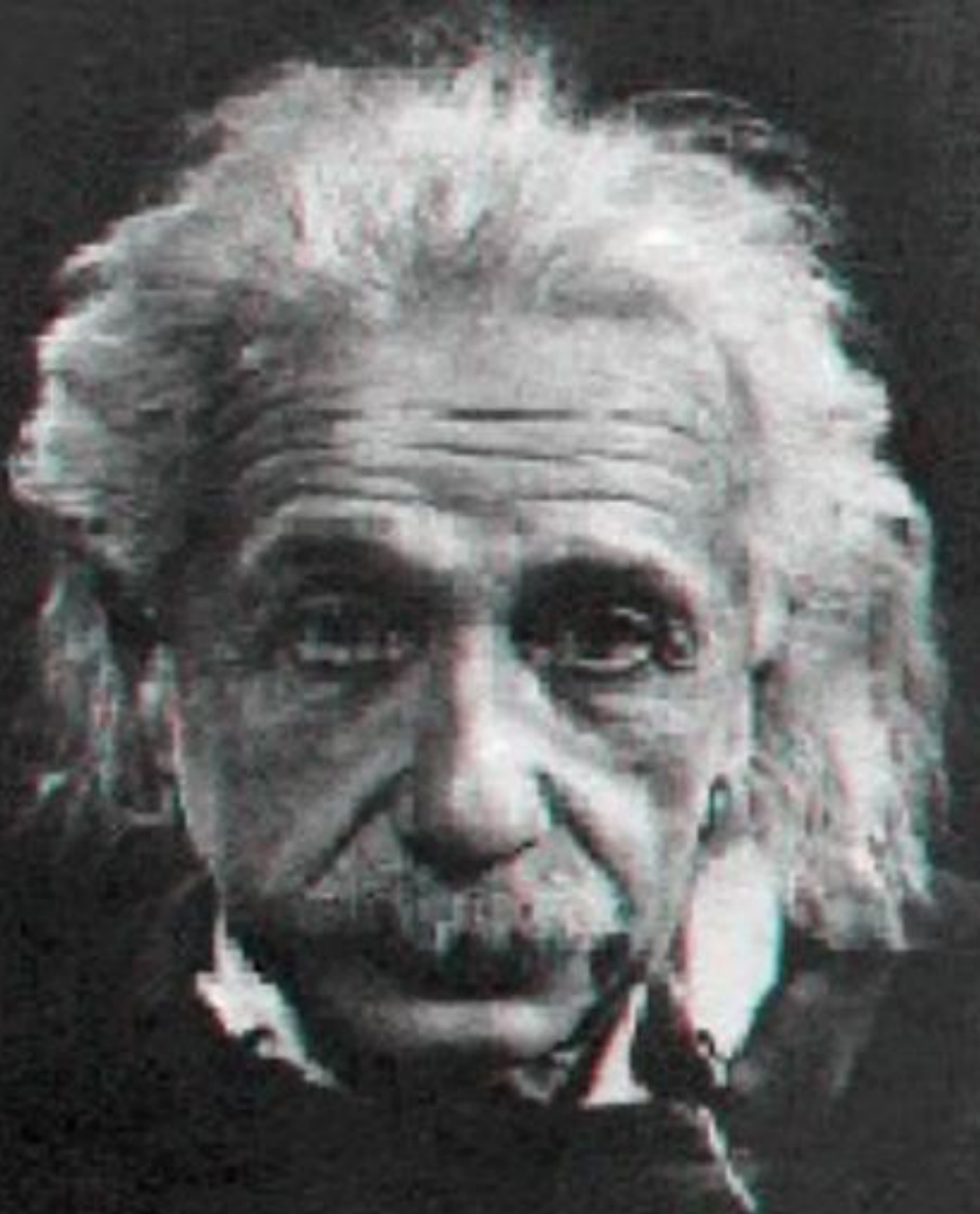
◆ The first cloud: The negative results of measuring the earth's relative "aether" movement contradict the existence of an absolutely static "aether" reference frame.

◆ The second cloud: the black body radiation law and the energy equal division theorem are in contradiction.

This leads to the birth of both relativity and quantum mechanics.

Relativity theory and quantum mechanics are the two basic theories of modern physics, which profoundly change people's understanding of the material world.

狭义相对论基础



Albert Einstein

1879 –1955

16.1 Einstein's hypothesis and the Lorentz transformation

The establishment of special relativity
space-time transformation

Absolute space-time view and the Galilean transformation

The underlying assumption of special relativity

Lorentz transformation



Time and space, or space-time for short. The motion of matter is closely related to the properties of space-time, so the study of the properties of space-time has always been a fundamental problem in physics.

The understanding of space-time in physics can be divided into three stages: Newtonian mechanics stage, special relativity stage and general relativity stage.

Special relativity is a theory of space-time in inertial systems that cannot deal with problems involving gravity. General relativity extends relativity to any frame of reference, which is a theory of spacetime and gravity.

space-time transformation

There is a train on the ground for a uniform linear motion, two clocks with identical structures and two identical meters. In the same frame of reference, the two clocks go equally fast, and the two meters are strictly equal in length.

Now put one bell and a ruler on the train (moving clock, moving ruler), leave the other bell and the other ruler on the ground (static clock, static ruler), and let both placed in the direction of the train movement.

Ask which one can go faster, the moving clock or the static bell? Which one is longer, the dynamic ruler or static ruler?

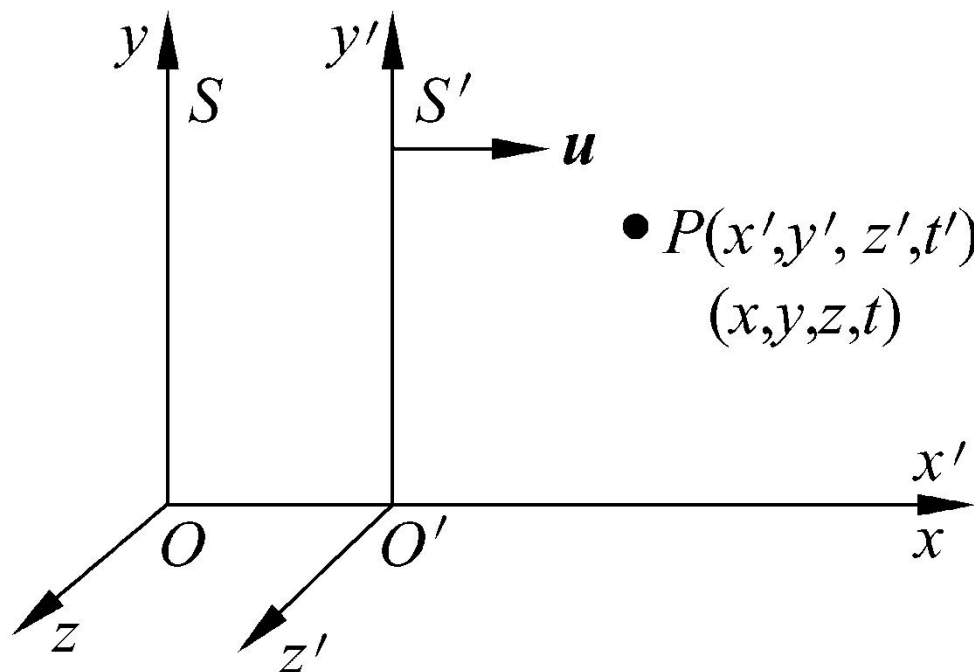
This is actually a transformation problem of time and space between the two inertial frames.

Event: Physical phenomenon with defined time and place of occurrence

The time and place of an event is called the space-time coordinate or space-time point of the event.

For example, a flash at a certain time t , arriving at a certain place (x, y, z) is an event, and its space-time coordinates are (x, y, z, t)

When discussing the nature of space-time, we are concerned with the space-time coordinates or space-time points, rather than being concerned with the events that introduce the concept of space-time.



How do you represent the time?

Definition of the time-zero point

$$t' = t = 0$$

Spatiotemporal transformation: the transformation relationship between the spatiotemporal coordinates (x', y', z', t') and (x, y, z, t) of the same event P in two inertial systems.

The spatiotemporal coordinates represent measurements of time and length, and then the spatiotemporal transformation reflects the relationship of time and space to the choice of reference frame.

16.1.1 Absolute space-time view and the Galilean transformation

Absolute space-time view: Newtonian mechanics believes that time and space are independent of each other, measured in two inertial frames of relative motion, the time interval and length are the same (the clock and the clock go as fast, the length is the same), that is, unrelated to the choice of the reference frame.

Galilean transformation:

$$x' = x - ut$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

The Galileo velocity transformation:

$$v'_x = v_x - u$$

$$v'_y = v_y$$

$$v'_z = v_z$$

16.1.2 Michelson-Mory experiment

1. Historical background

In 1864, Maxwell predicted that light was an electromagnetic wave. The rate of light propagation in a vacuum, according to the electromagnetism theory

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

among ϵ_0 And μ_0 The vacuum dielectric constant and the vacuum permeability, respectively, are independent of the reference frame.

Thus in any inertial frame, the propagation rate of light along all directions is equal to c.

Or, all inertial frames are equal for describing the propagation of electromagnetic waves.

The constant speed of light contradicts the principle of Galileo relativity: according to Galileo velocity, if the speed of light along all directions in an inertial system S is c , then the inertial system S is moving at speed u /medium, the speed of light along the direction of motion is $c - u$ and the opposite direction is $c + u$.

For describing the propagation of the electromagnetic waves, the S systems and the S' Inequality.

There are three possible options for resolving this contradiction:

(1) Modify the theory of electromagnetism and make it obey the Galileo relativity principle.

The correctness of electromagnetism theory has been verified experimentally, involving the high-speed motion such as the propagation of electromagnetic waves. Whether the Galileo transformation is applicable has not been verified experimentally. There is no reason to modify the theory of electromagnetism.

(2) Think that both the Galileo relativity principle and the electromagnetism theory are correct, but the law of electromagnetism is only held true in the inertial system of absolute rest.

This absolutely static inertial frame is the so-called "aether" reference frame.

According to the light ether hypothesis, light propagation in vacuum also requires a medium called the ether; the ether fills the entire space and is absolutely stationary; in the ether reference, the speed of light in all directions is c .

If the ether really exists, the earth will move relative to the ether, and the speed of light in different directions on the ground should be different.

Michelson, for the Morey Experiment

If there is Ether, u

Size must be related to

the direction of
Rotate the

interferometer

around the center

The interference

of two arms must

change

Both arms 90° position

orientation swap

difference changes

of 0.01 stripes

If the Ether view is

Expected movement amount

of 0.1 stripes

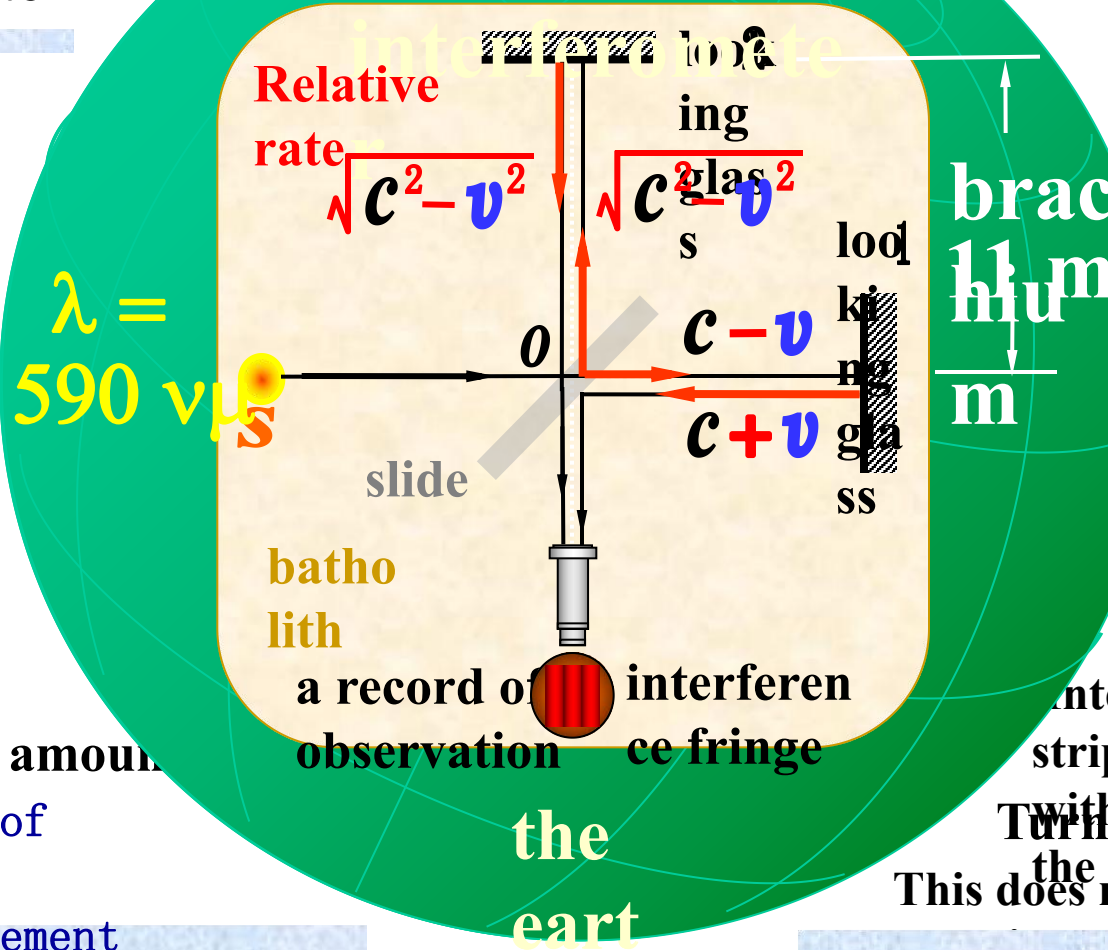
the instrument to

determine the movement

The of 0.01 After repeated

Look for an Ether
failed instance

Michelson



\vec{u} \vec{v} \vec{c}

Li T Li

g h g

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o ar to

n th t

ve h

rs e

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The

ar th h

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h A

interference

stripe movement

with the public

Turn the ether

the earth

This does not affect the

experimental principle.

After repeated careful observation of different seasons and different

times, no expected stripe movement was found. It has been known as the

"zero result" for searching for the ether

From 1881 to 1887, Michelson and Morey used interferometers invented by Michelson to try to measure the movement of the Earth's relative ether by measuring the difference in the speed of light in different directions, with a negative result: the speed of light in different directions on the ground was equal, so the ether did not exist.

(3) Einstein's practices:

The Galileo relativity principle is extended to make the electromagnetism theory obey the relativity principle after the extension. Einstein firmly believed that the principle of relativity is the expression of the universal law of nature, which revealed the relativity of simultaneity. In his 1905 paper entitled On Electrodynamics in motion, he put forward two basic hypotheses on which the special theory of relativity was based and created the special theory of relativity.

16.1.3 Einstein's hypothesis

1. Einstein's relativity principle

For describing the laws of physics (including the laws of mechanics), all inertial reference frames are equal, and there is no special absolute inertia frame.

2. The speed of light is a constant principle

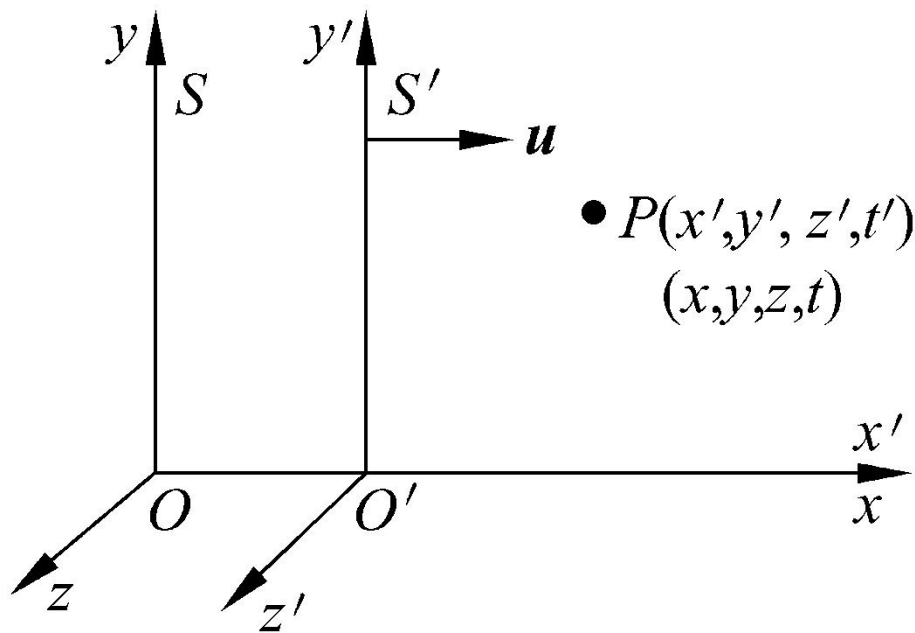
In all inertial frames, the propagation rate of light in the vacuum is equal to c . Or, the speed of light measured by the observer equals c , regardless of the movement of the light source and the observer.

For example, in 1964, the rays emitted by the muon (light source) with a velocity of $0.99975c$ were measured to verify that the speed of light is unchanged.

The basic clue that led Einstein to propose the principle of speed of light and to create the theory of special relativity was electromagnetism, while the negative result of Michael Sun Morley's experiment did not dominate.

But Einstein was still a student thinking about the relationship between the speed of light and the motion of objects, and spoke highly of the scientific significance of Michael Sun Murray's experiment.

16.1.4 Lorentz transformation



$$\begin{aligned}x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} \\y' &= y \\z' &= z\end{aligned}$$

1

$$t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1 - u^2/c^2}}$$

0

And $u / c \rightarrow 0$, back to the Galileo transform.

From the two basic principles proposed by Einstein, the Lorentz transformation is simply derived:

Galilean transformation:

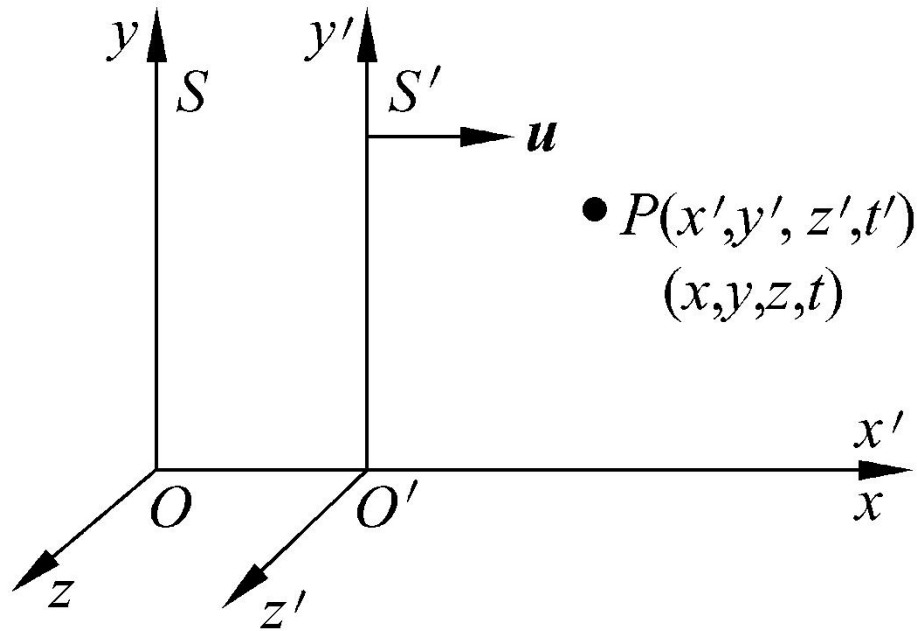
$$x' = x - ut \quad ?$$

$$y' = y \quad \checkmark$$

$$z' = z \quad \checkmark$$

$$t' = t \quad ?$$

Uniformity of space-time: any two space-time points in the same inertial system are equivalent. This requires that the new transformation can only be a linear transformation, including only one term of coordinates and time in the transformation relation.



**The new transformation can
be expressed as**

$$x' = \gamma'(x - ut)$$

$$x = \gamma(x' + ut')$$

**According to Einstein's relativity
principle**

$$\gamma' = \gamma$$

$$x' = \gamma(x - ut)$$

$$x = \gamma(x' + ut')$$

The parameters can be determined by the light speed constant principle:

Imagine when the origin is O'/When coincides with O, a flash from the origin. The speed of light is independent of the motion of the reference frame regardless in O'/Line or was observed in the O line

$$x' = ct', \quad x = ct$$

$$\left. \begin{array}{l} ct' = \gamma(c - u)t \\ ct = \gamma(c + u)t' \end{array} \right\} \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

**reducing the transforming relationship
between t and t' :**

$$x = \gamma(x' + ut') \rightarrow t' = \frac{1}{u} \left(\frac{x}{\gamma} - x' \right)$$

$$x' = \gamma(x - ut)$$

$$t' = \frac{1}{u} \left(\frac{x}{\gamma} - \gamma(x - ut) \right) = \frac{\gamma}{u} \left(\left(\frac{1}{\gamma^2} - 1 \right) x + ut \right)$$

$$1/\gamma^2 = (1 - u^2/c^2)$$

$$t' = \gamma \left(t - \frac{u}{c^2} x \right)$$

**Lorentz
transformation**

$u \rightarrow -u$

**inverse
transformation**

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$x = \frac{x' + ut'}{\sqrt{1 - u^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1 - u^2/c^2}}$$

$$t = \frac{t' + \frac{u}{c^2}x'}{\sqrt{1 - u^2/c^2}}$$

The transformation of the spatial interval and the time interval of the two events:

$$x'_2 - x'_1 = \frac{(x_2 - x_1) - u(t_2 - t_1)}{\sqrt{1 - u^2/c^2}}$$

$$y'_2 - y'_1 = y_2 - y_1$$

$$z'_2 - z'_1 = z_2 - z_1$$

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - \frac{u}{c^2}(x_2 - x_1)}{\sqrt{1 - u^2/c^2}}$$

$$x_2 - x_1 = \frac{(x'_2 - x'_1) + u(t'_2 - t'_1)}{\sqrt{1 - u^2/c^2}}$$

$$y_2 - y_1 = y'_2 - y'_1$$

$$z_2 - z_1 = z'_2 - z'_1$$

$$t_2 - t_1 = \frac{(t'_2 - t'_1) + \frac{u}{c^2}(x'_2 - x'_1)}{\sqrt{1 - u^2/c^2}}$$

[Example] A spacecraft flew at a speed of $0.8c$ relative to the ground, and the observer on the spacecraft measured the length of the spacecraft as 100m. A light pulse from the stern to the bow, ask the observer on the ground to measure, what is the space interval between the light pulse from the stern and reaching the bow?

The problem that only involves space-time transformation is called the kinematic problem, which is generally solved in the following steps:

(1) Set the reference frame

ship: S' System, ground: S system.

S' Line relative S line with $u = 0.8c$.

(2) Define the events and their space-time coordinates

Event 1: Light pulse emitted from the stern. In S'/spatiotemporal sitting in lines and S are marked as,,

$$(x'_1, t'_1) \quad (x_1, t_1)$$

Event 2: The light pulse reaches the bow. In S'/spatiotemporal sitting in lines and S are marked as,,

$$(x'_2, t'_2) \quad (x_2, t_2)$$

(3) Find the spatial interval between these two events in the S system by the Lorentz transformation.

known number $x'_2 - x'_1 = 100\text{m}, \quad t'_2 - t'_1 = (x'_2 - x'_1)/c$

$$x_2 - x_1 = \frac{(x'_2 - x'_1) + u(t'_2 - t'_1)}{\sqrt{1 - u^2/c^2}} = \frac{100 + 0.8 \times 100}{\sqrt{1 - 0.8^2}} \text{m} = 300\text{m}$$

(4) Discussion

The spatial interval of the two events by Galileo transformation

$$u = 0.8c: \quad x_2 - x_1 = 100 + 0.8c \times \frac{100}{c} = 180\text{m}$$

Lorentz transformation:

300m

It's a far cry

$$u = 0.08c: \quad x_2 - x_1 = 100 + 0.08c \times \frac{100}{c} = 108\text{m}$$

The Lorentz transformation:

108.3m

The result is close

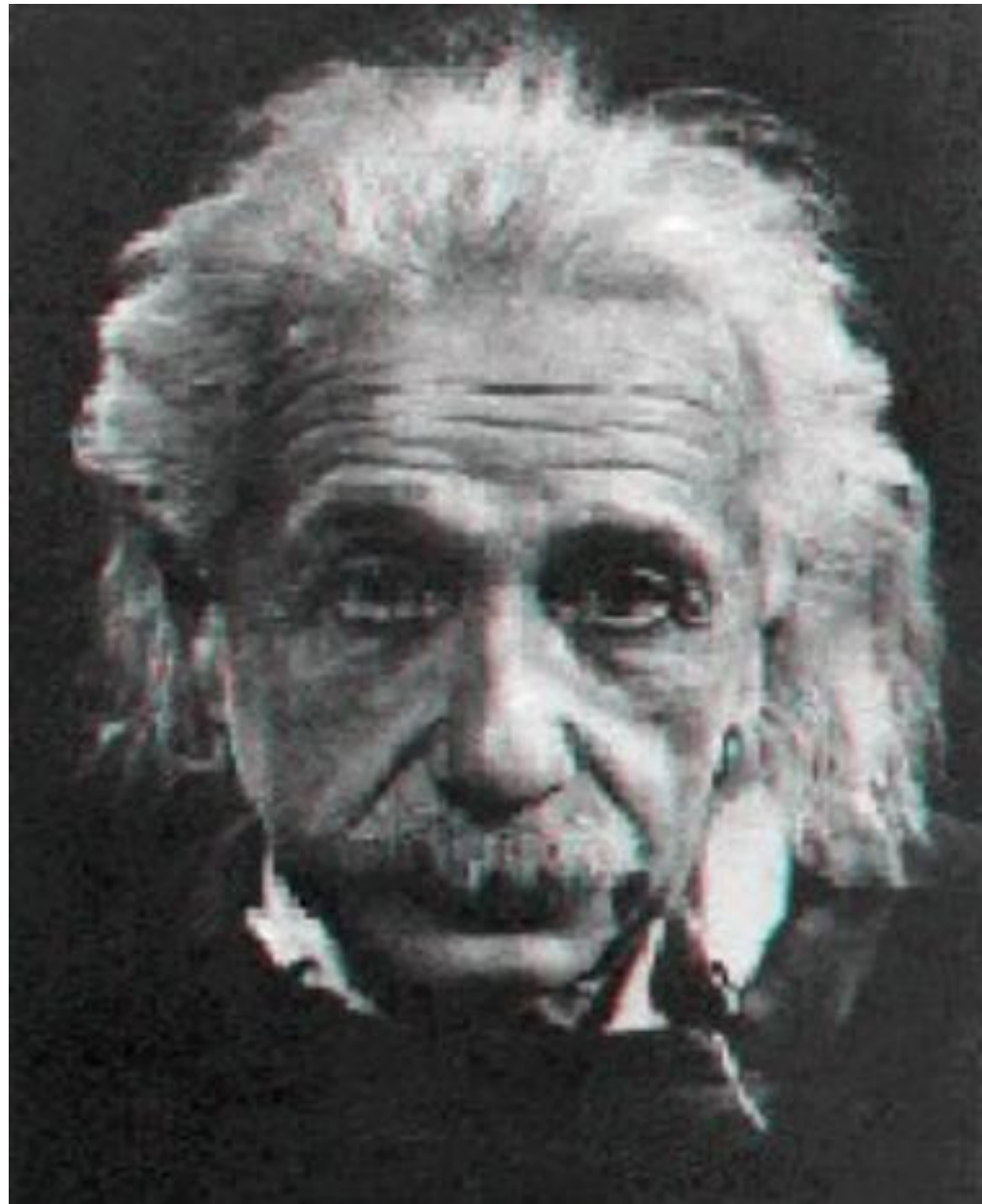
$u \ll c$: The Lorentz transformation returns to the Galileo transformation

16.2 The Space-temporal view of special relativity

relativity of simultaneity

time dilation

length contraction



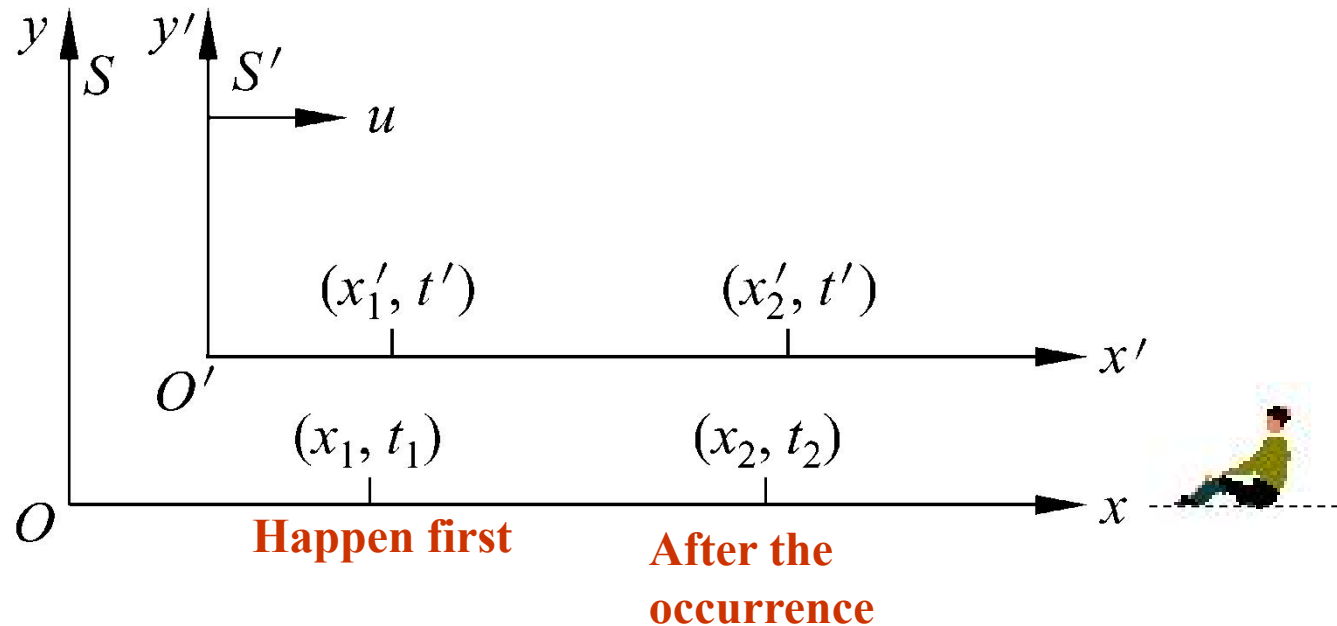
16.2.1 Relativity of simultaneity

Einstein showed the close relationship between the measurement of time and the simultaneity: all our judgments that time works in it are always judgments about simultaneous events. For example, if I say, "The train arrives here at 7 o'clock," this probably means, "The short needle of my watch points to the arrival here at the same time."

Absolute space-time view: If two events occur simultaneously in one inertial system, and they are observed in any other inertial system, which must occur at the same time

-The absoluteness of synchronization

According to the Lorentz transformation, synchronization is relative: two simultaneous events occurring simultaneously in one inertial system do not necessarily occur simultaneously in other inertial systems of relative motion.



$$t_2 - t_1 = \frac{(t'_2 - t'_1) + \frac{u}{c^2} (x'_2 - x'_1)}{\sqrt{1 - u^2/c^2}} = \frac{\frac{u}{c^2} (x'_2 - x'_1)}{\sqrt{1 - u^2/c^2}} > 0$$

This follows from the relativity of simultaneity: if two events occur in the direction of the relative motion of two inertial systems, if they occur simultaneously in one inertial system, they are observed in the other inertial system, and the event behind the motion of the previous inertial system always occurs first.

Lorentz first derived the Lorentz transformation, and the principle of relativity was first proposed by Poincare, but they did not grasp the key and revolutionary idea of simultaneous relativity. Both Lorenz and Poincare approached relativity, but failed to create it. Only 26 years old Einstein dared to question people's primitive concept of time, adhere that the simultaneous is relative, to complete this historical task.

16.2.2, the relativity of length —— length contraction

Measurement of the length of the stationary objects:

Measure the coordinates of both ends of the object, the difference x It is the length of the object, there is no requirement for the order of measurement, and the coordinates of the two ends of the object can not be measured at the same time, t_1 Can not be equal to the t_2 。

Measurement of the length of the moving objects:

Only the simultaneous determination of the coordinates of both ends of the object, $t_1=t_2$, The difference value of x is the length of the object.

Following the Lorentz transformation

$$\Delta x' = \frac{\Delta x - u(t_2 - t_1)}{\sqrt{1 - u^2/c^2}} = \frac{\Delta x}{\sqrt{1 - u^2/c^2}} \rightarrow \Delta x < \Delta x'$$

length contraction

Length shrinkage effect: observed in the inertial system, the length of the moving object in its direction of motion should be shortened.(Shorter than shorter)

Size: the spatial interval of two events occurring simultaneously along the direction of motion in a reference frame, which is called length measurement and denoted as l .

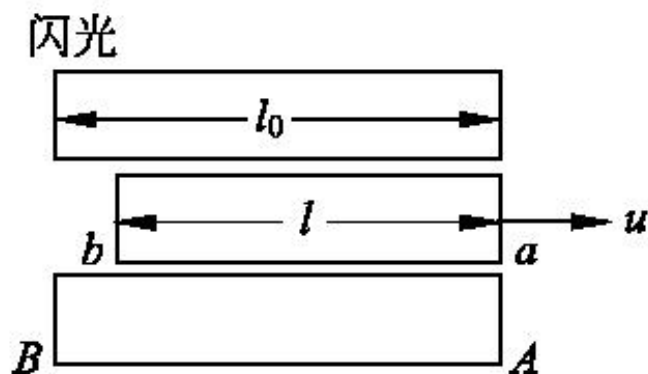
According to the relativity of simultaneity, in any other motion reference frame, these two events must not occur simultaneously, and their spatial interval is called the original length (intrinsic length), denoted as l_0 .

$$\Delta l = \Delta l' \sqrt{1 - u^2 / c^2}$$

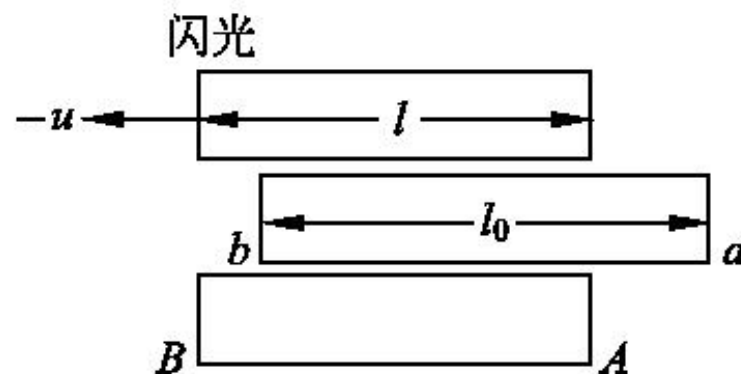
Length shrinkage effect: measure the length is longer than the original length

The length contraction effect occurs only in the direction of the object motion and does not contract in the vertical direction. This is often said to be a longitudinal contraction, not a lateral contraction.

The length contraction effect is purely the property of space-time, which is completely different from the real contraction and expansion occurring in the phenomenon of thermal expansion and cold contraction.



(a) 在地面上看



(b) 在火车上看

16.2.3, Relativity of time — time inflation (delay)

With its relativity, the time interval between the same two events occurring along the relative velocity direction is different for the different reference frames, i. e., the measure of time is relative.

located at S/The same site x in the lineage/At, first, after two events and, time interval.

$$(x', t'_1) \quad (x', t'_2) \quad \Delta t' = t'_2 - t'_1 > 0$$

Time interval between these two events in lineage S

$$\Delta t = \frac{\Delta t' + \frac{u^2}{c^2} (x' - x')}{\sqrt{1 - u^2/c^2}} = \frac{\Delta t'}{\sqrt{1 - u^2/c^2}} > \Delta t' \quad \text{time dilatation}$$

Time expansion effect: observed in one inertial system, the time interval between two events occurring in the same place in another inertial system with a uniform linear motion becomes larger.(time dilatation)

Original time (inherent time): the time interval between two events occurring at the same place in a reference frame.

By representing the original time

$$\Delta t = \frac{\Delta \tau}{\sqrt{1 - u^2 / c^2}}$$

Time expansion effect: the shortest original time

[Example] There are many calibrated static synchronous clocks (static clocks) that take them 1s time to walk through a grid. If one of the clocks is allowed to move relative to the static observer at a rate of $u = 0.8c$, how long does it take the pointer of the moving clock (moving clock) to move on a grid?

Solution Event 1: The pointer of the clock just starts to turn into a case

Event 2: The pointer turns through a case

In the reference frame, events 1 and 2 occur together and the time interval 1s is the original time.

In the view of the stationary observer, the time interval

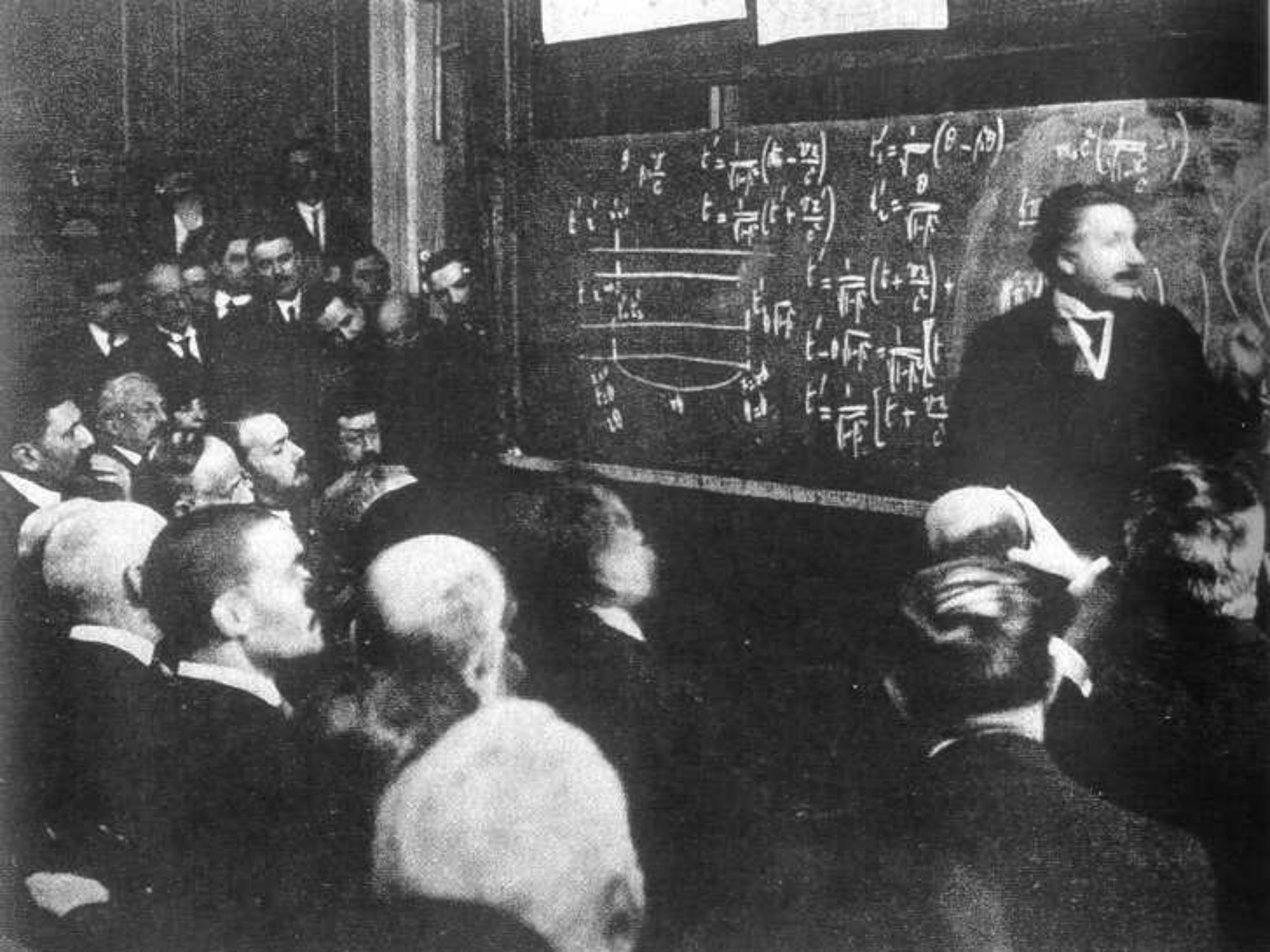
$$\Delta t = 1\text{s} / \sqrt{1 - 0.8^2} = 1.67\text{s}$$

In the observer's view, the time taken by the clock pointer to a lattice is 0.67s longer than that taken by the static clock pointer to a grid in this reference frame. Or, the moving clock is slower than a static clock.

This is purely the nature of space-time, not a change in the structure of the clock. The moving and static bells are exactly the same structure, and they walk as fast when put together.

Observing in one inertial system slows down the pace of any process (including physical, chemical, and life processes) that occurs at the same place in another moving inertial system.

Twin feint, effect, Ground level μ sub is detectable



$$t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \left(t - \frac{vx}{c^2} \right) \quad t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} (t - \beta x)$$
$$x' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} (x - vt)$$
$$t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \left(t' + \frac{vx'}{c^2} \right) \quad t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \left(t' + \beta x' \right)$$
$$x = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} (x' + vt')$$
$$t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \left(t + \frac{vx}{c^2} \right) + \dots$$
$$t - \frac{vx}{c^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \left[t' + \frac{vx'}{c^2} \right]$$
$$t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \left[t + \frac{vx}{c^2} \right]$$

Diagram: A spacetime diagram showing two horizontal lines representing worldlines. A curved line represents a path of light or a specific event. The diagram is labeled with t , x , t' , and x' .

16.3 Relativistic speed transformation formula

Definition of speed:

$$v'_x = \frac{dx'}{dt'}, \quad v'_y = \frac{dy'}{dt'}, \quad v'_z = \frac{dz'}{dt'}$$
$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

From the Lorentz transformation, we get $v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$

$$v'_y = \frac{v_y}{1 - \frac{uv_x}{c^2}} \sqrt{1 - u^2/c^2}$$

[Thinking] The lateral length does not shrink, but why does the transverse speed change?

$$v'_z = \frac{v_z}{1 - \frac{uv_x}{c^2}} \sqrt{1 - u^2/c^2}$$

If the transverse velocity of the object in the S system is zero and the velocity along the x-axis is v , then in S' Therefore, the transverse velocity of the object is also zero, while the velocity is oriented along the x-axis

$$v' = \frac{v - u}{1 - \frac{uv}{c^2}}$$

The inverse transformation is

$$v = \frac{v' + u}{1 + \frac{uv'}{c^2}}$$

[Example] Imagine a "light pursuit experiment", in which a train moving at speed u pursues a forward flash. Look out on the train, how fast is the flash?

separate The train is S' System, and the ground is the S line

Observation on the train, the speed of the flash

$$v' = \frac{v - u}{1 - \frac{uv}{c^2}} = \frac{c - u}{1 - \frac{uc}{c^2}} = c$$

Still equal to the light speed c , independent of the motion of the reference frame.