1 The rules of Fitch style natural deduction

Conjunction introduction (∧I)

m	A	
$\vdots$	$\vdots$	
n	B	
$\vdots$	$\vdots$	
p	$A \wedge B$	$\wedge$I, m, n

m	B	
$\vdots$	$\vdots$	
n	A	
$\vdots$	$\vdots$	
p	$A \wedge B$	$\wedge$I, m, n

Conjunction elimination (∧E)

m	$A \wedge B$	
$\vdots$	$\vdots$	
n	A	$\wedge$E, m

m	$A \wedge B$	
$\vdots$	$\vdots$	
n	B	$\wedge$E, m

Disjunction Introduction (∨I)

m	A	
$\vdots$	$\vdots$	
n	$A \vee B$	$\vee$I, m

m	B	
$\vdots$	$\vdots$	
n	$A \vee B$	$\vee$I, m

Disjunction Elimination (∨E)

m	$A \vee B$	
$\vdots$	$\vdots$	
n	$\quad \underline{A}$	
$\vdots$	$\quad \vdots$	
p	$\quad \varphi$	
q	$\quad \underline{B}$	
$\vdots$	$\quad \vdots$	
r	$\quad \varphi$	
s	φ	$\vee$E, m, n–p, q–r

Implication Introduction (→I)

m	$\quad \underline{A}$	
$\vdots$	$\quad \vdots$	
n	$\quad B$	
$n+1$	$A \to B$	$\to$I, m–n

Implication Elimination (→E)

m	A	
$\vdots$	$\vdots$	
n	$A \to B$	
$\vdots$	$\vdots$	
p	B	$\to$E, m, n

m	$A \to B$	
$\vdots$	$\vdots$	
n	A	
$\vdots$	$\vdots$	
p	B	$\to$E, m, n

Negation Introduction (¬I)

m	$\quad \underline{A}$	
$\vdots$	$\quad \vdots$	
n	$\quad \bot$	
$n+1$	$\neg A$	$\neg$I, m–n

Negation Elimination (¬E)

m	A	
$\vdots$	$\vdots$	
n	$\neg A$	
$\vdots$	$\vdots$	
p	$\bot$	$\neg$E, m, n

m	$\neg A$	
$\vdots$	$\vdots$	
n	A	
$\vdots$	$\vdots$	
p	$\bot$	$\neg$E, m, n

Contradiction Elimination (⊥E)

m	$\bot$	
$\vdots$	$\vdots$	
n	C	$\bot$E, m

Proof by Contradiction (C)

m	$\quad \underline{\neg A}$	
$\vdots$	$\quad \vdots$	
n	$\quad \bot$	
$n+1$	A	C, m–n

Repetition (R)

m	A	
$\vdots$	$\vdots \quad \vdots$	
$\vdots$	$\cdots \quad \underline{}$	
n	$\quad A$	R, m

Forall-introduction (∀I)

m	$u \quad \vdots$	
$\vdots$	$\quad \vdots$	
n	$\quad A[u]$	
$n+1$	$\forall x\, A$	$\forall$I, m–n

Forall-elimination (∀E)

m	$\forall x\, A$	
$\vdots$	$\vdots$	
n	$A[t]$	$\forall$E, m

Exists-Introduction (∃I)

m	$A[t]$	
$\vdots$	$\vdots$	
n	$\exists x\, A$	$\exists$I, m

Exists-Elimination (∃E)

p	$\exists x\, A$	
$\vdots$	$\vdots$	
m	$u \quad \underline{A[u]}$	
$\vdots$	$\quad \vdots$	
n	$\quad \varphi$	
$n+1$	φ	$\exists$E, p, m–n

2 Remarks

The biconditional ($\leftrightarrow$)

To simplify our formal proof system, we do not introduce any special rules for the connective $\leftrightarrow$. Instead, we simply regard the formula $A \leftrightarrow B$ as an *abbreviation* for $(A \to B) \wedge (B \to A)$.

Falsity ($\perp$)

The symbol $\perp$ stands for "contradiction" or "falsity". The formula $\perp$ is always false, and it is used in the rules for negation and contradiction above.

Negation ($\neg$)

As we have done before, it is possible to regard negation $\neg A$ as an abbreviation for $A \to \perp$. In this case, the negation introduction and elimination rules are simply instances of the implication introduction and elimination rules.

Repetition (R)

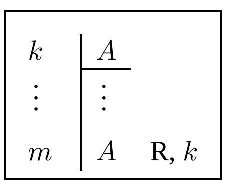
Let A be a formula written at line k (either as a hypothesis, or as a formula already proven). Then one can repeat A at line m if:

(1) $k < m$, and

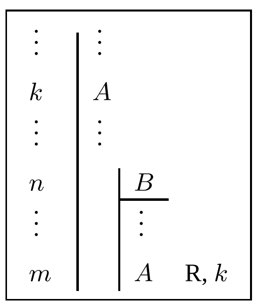
(2) every vertical from line k continues without interruption to line m.

Examples of repetition:

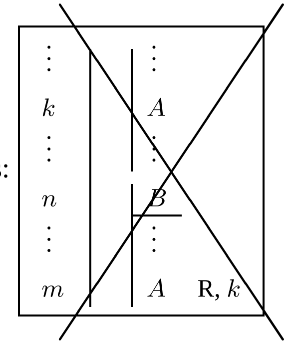
This is ok:



This too:



But not this:



Quantifiers

In the rules for quantifiers:

- in $\forall$E and $\exists$I, t is any term.
- in $\forall$I and $\exists$E, u is a *fresh variable*. Here "fresh" means that this variable does not occur anywhere else in the derivation. It may only occur in the subderivation from lines m–n. The "u" that is written between the vertical lines on line m is called a *guard* — it serves as a reminder that u must be fresh in this subderivation. In particular, this means that no formula containing u can be imported (repeated) into lines m–n from outside lines m–n. Also, this means that u cannot occur in the formula φ in lines n and $n + 1$ of $\exists$E.