§5.4

电　势

5.4.1 静电场力做功 静电场的环路定理

一、静电场力所做的功

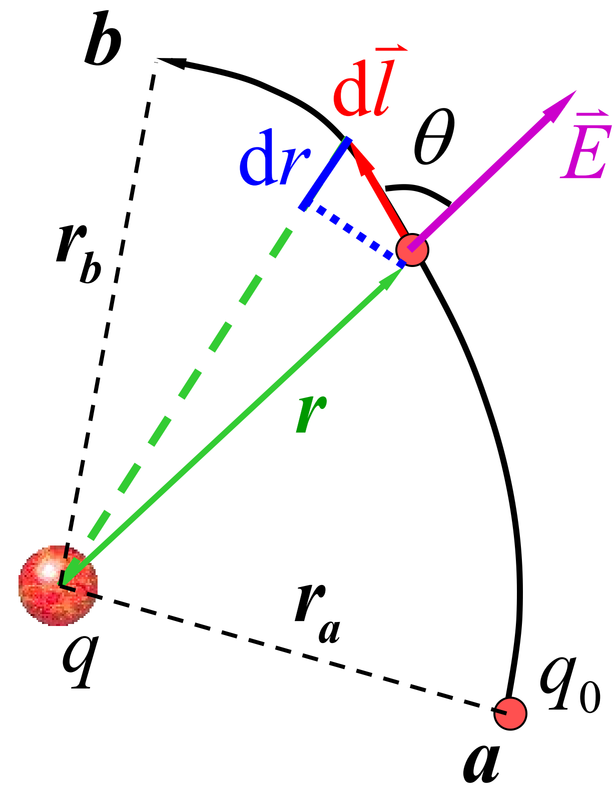
1. 点电荷的电场

$$\mathrm{d}A = q_0\vec{E}\cdot\mathrm{d}\vec{l} = \frac{qq_0}{4\pi\varepsilon_0 r^3}\vec{r}\cdot\mathrm{d}\vec{l} = \frac{qq_0}{4\pi\varepsilon_0 r^2}\mathrm{d}r$$

$$A = q_0\int_L \vec{E}\cdot\mathrm{d}\vec{l} = \frac{qq_0}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{\mathrm{d}r}{r^2}$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$

结果: W 仅与q_0的**始末**位置**有关**，与路径无关。



$$\vec{r}\cdot\mathrm{d}\vec{l} = r\mathrm{d}l\cos\theta = r\mathrm{d}r$$

2. 任意电荷的电场（视为点电荷系）

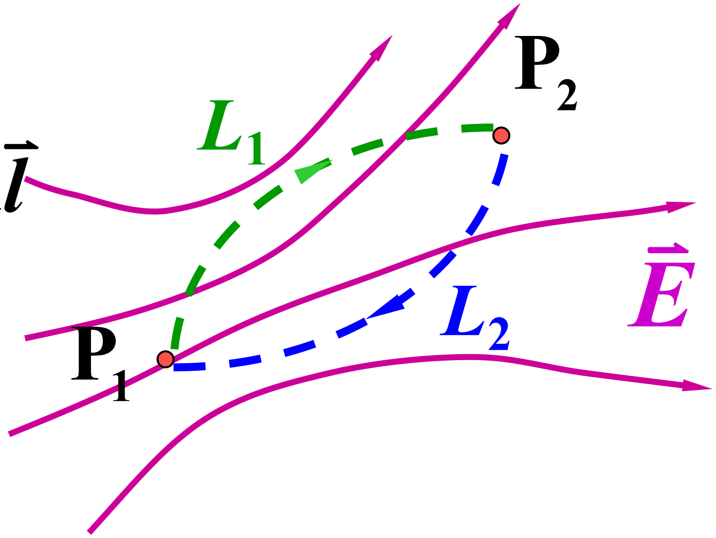
$$\vec{E}=\sum_i \vec{E}_i \Rightarrow A=q_0\int_l \vec{E}\cdot d\vec{l}=\sum_i q_0\int_l \vec{E}_i\cdot d\vec{l}$$

结论：静电场力做功与路径无关——**保守力**

二、静电场的环路定理

$$\oint_L \vec{E}\cdot d\vec{l}=\int_{P_1(L_1)}^{P_2}\vec{E}\cdot d\vec{l}+\int_{P_2(L_2)}^{P_1}\vec{E}\cdot d\vec{l}$$

$$=\int_{P_1(L_1)}^{P_2}\vec{E}\cdot d\vec{l}-\int_{P_1(L_2)}^{P_2}\vec{E}\cdot d\vec{l}$$



$$\boxed{\oint_L \vec{E}\cdot d\vec{l}=0}$$

$$\int_{P_1(L_1)}^{P_2}\vec{E}\cdot d\vec{l}=\int_{P_1(L_2)}^{P_2}\vec{E}\cdot d\vec{l}$$

5.4.2 电势差和电势

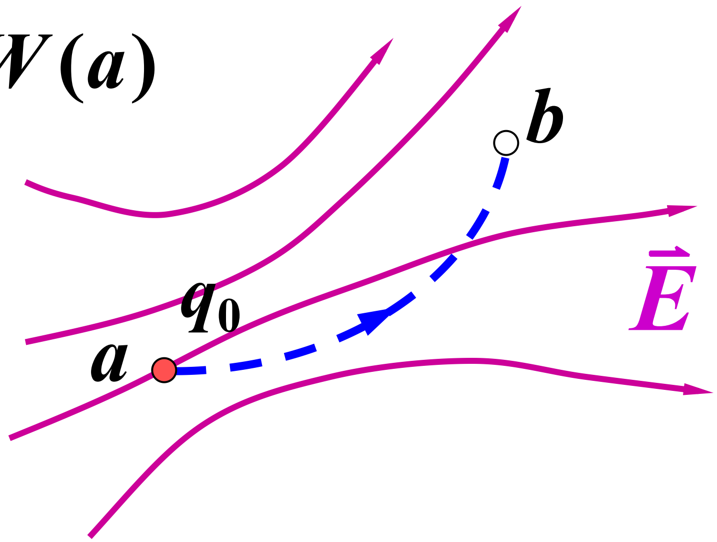
一、电势能

静电场是保守场，静电场力是**保守力**。静电场力所做的功就等于电荷**电势能的减少量**（增量的负值）。

$$A_{ab}=\int_a^b q_0\vec{E}\cdot\mathrm{d}\vec{l}=W(a)-W(b)=-\Delta W$$

$$A_{ab}\begin{cases}>0, & W(b)<W(a)\\ <0, & W(b)>W(a)\end{cases}$$

二、电势



$$\int_a^b \vec{E}\cdot\mathrm{d}\vec{l}=\frac{W(a)-W(b)}{q_0}$$

$$=\varphi_a-\varphi_b$$ **电势之差**

$$\varphi_a = \int_a^b \vec{E} \cdot d\vec{l} + \varphi_b$$

令 $\varphi_b = 0$

$$\varphi_a = \int_a^{\text{电势零点}} \vec{E} \cdot d\vec{l}$$

♦ **物理意义：** 把**单位正试验电荷**从点 a 移到电势零点时，静电场力所作的功。

电势零点的选择方法

(1) 电荷分布在**有限空间** 令 $\varphi_\infty = 0$ 则 $\varphi_a = \int_a^\infty \vec{E} \cdot d\vec{l}$

(2) 电荷为**无限大、长**分布 令 $\varphi_b = 0$ 则 $\varphi_a = \int_a^b \vec{E} \cdot d\vec{l}$

(3) 实际问题中常选择**地球电势为零**。

注意

电势差是绝对的，与电势零点的选择无关；

电势大小是相对的，与电势零点的选择有关。

♦ **静电场力的功** $A_{ab} = q_0\left(\varphi_a - \varphi_b\right)$

♦ **单位：** 伏特 (V)

原子物理中能量单位： $1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J}$

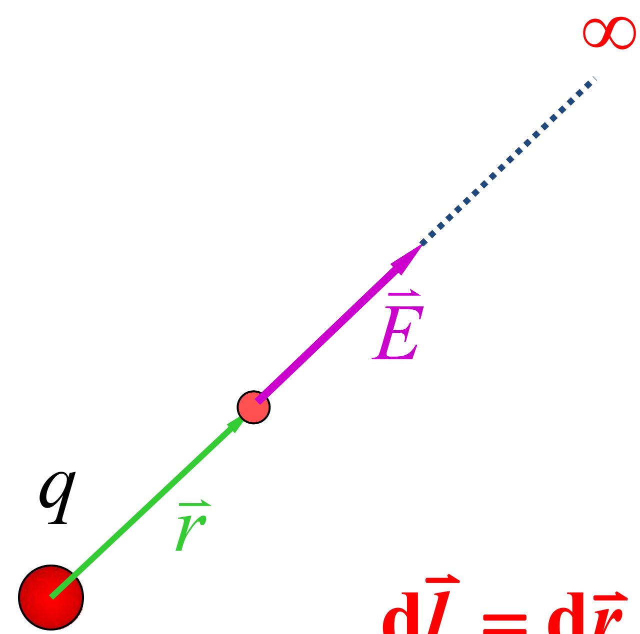
5.4.3 电势的计算

一、点电荷的电势

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r \qquad 令\ \varphi_\infty = 0$$

$$\varphi = \int_r^\infty \vec{E}\cdot \mathrm{d}\vec{l} = \int_r^\infty E\mathrm{d}r$$

$$= \int_r^\infty \frac{q}{4\pi\varepsilon_0 r^2}\mathrm{d}r$$



$$\mathrm{d}\vec{l} = \mathrm{d}\vec{r}$$

$$\boxed{\varphi = \frac{q}{4\pi\varepsilon_0 r}} \qquad \begin{cases} q > 0, & \varphi > 0 \\ q < 0, & \varphi < 0 \end{cases}$$

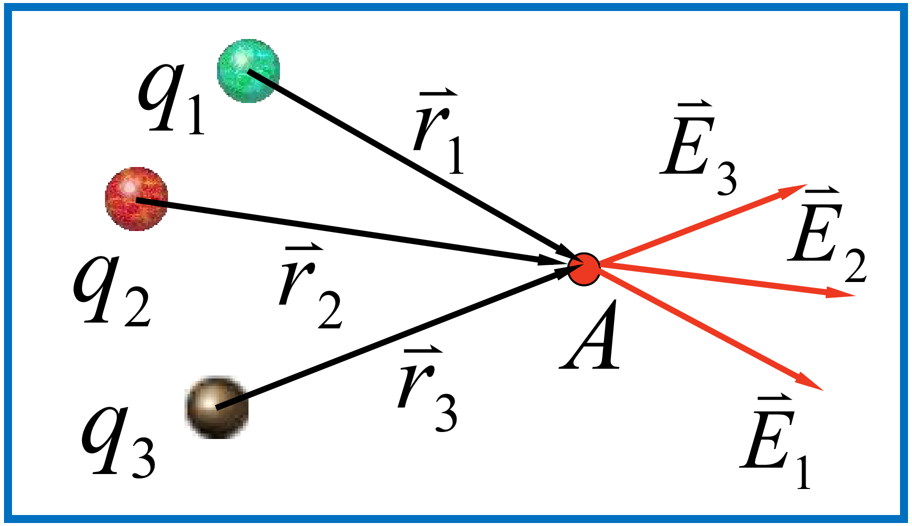
二、点电荷系的电势

$$\vec{E}=\sum_i \vec{E}_i$$

$$\varphi=\int_P^{\infty}\vec{E}\cdot d\vec{l}=\sum_i\int_P^{\infty}\vec{E}_i\cdot d\vec{l}$$

$$\boxed{\varphi=\sum_i \varphi_i=\sum_i\frac{q_i}{4\pi\varepsilon_0 r_i}}$$

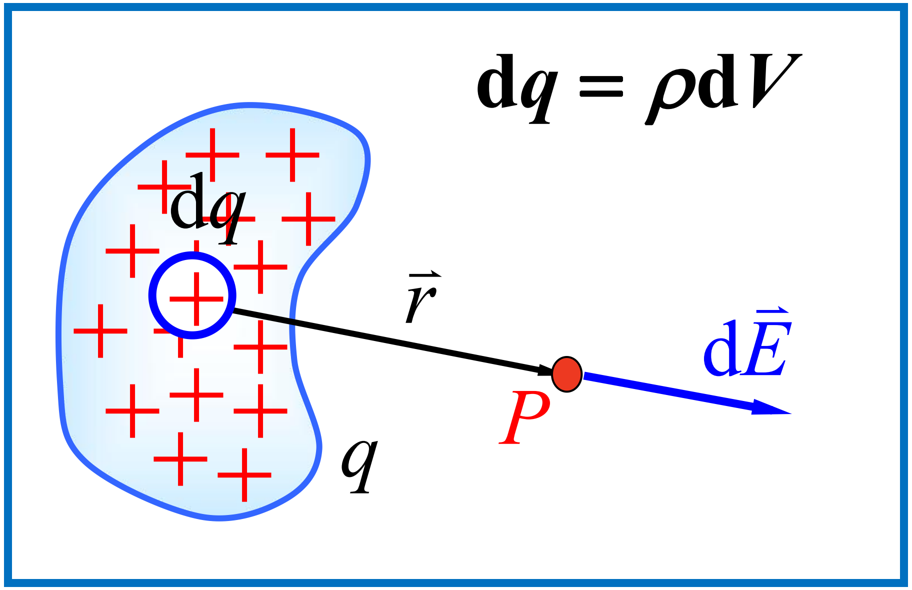
电势的叠加原理



三、连续带电体的电势

$$d\varphi=\frac{dq}{4\pi\varepsilon_0 r}$$

$$\boxed{\varphi=\int\frac{dq}{4\pi\varepsilon_0 r}}$$



四、电势的计算

1) 若已知在积分路径上电场 E 的分布函数，

由定义：$$\varphi = \int_P^{\infty} \vec{E} \cdot \mathrm{d}\vec{l}$$

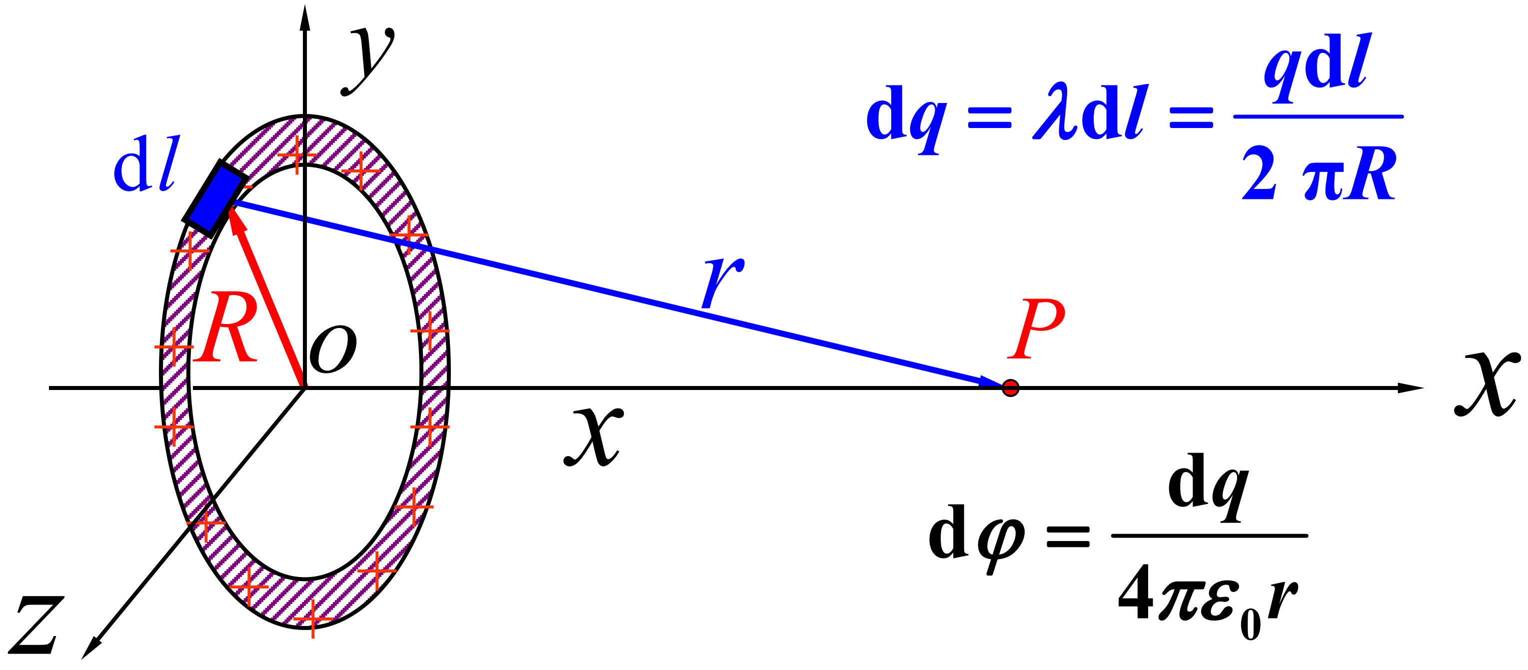
范围：能用高斯定理求场强的场。

2) 利用点电荷电势 $$\varphi = \frac{q}{4\pi\varepsilon_0 r}$$

及电势叠加原理 $$\varphi = \int \frac{\mathrm{d}q}{4\pi\varepsilon_0 r}、\varphi = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}$$

条件：有限大带电体，选无限远处电势为零。

例1 正电荷 q 均匀分布在半径为R 的细圆环上。求**圆环轴线上距环心为** x 处点 P 的电势。

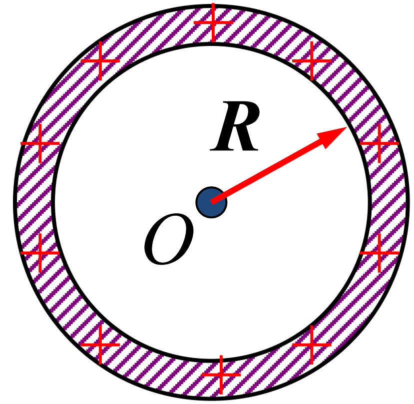


$$\mathrm{d}q = \lambda \mathrm{d}l = \frac{q\mathrm{d}l}{2\pi R}$$

$$\mathrm{d}\varphi = \frac{\mathrm{d}q}{4\pi\varepsilon_0 r}$$

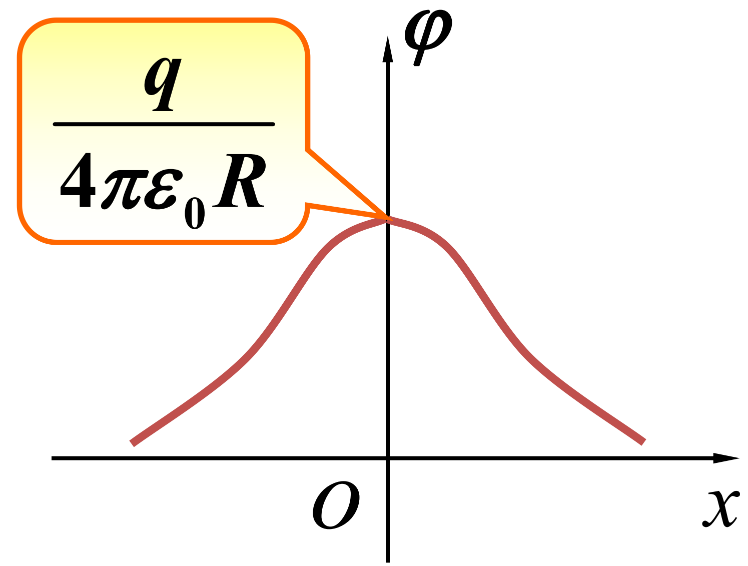
$$\varphi = \frac{1}{4\pi\varepsilon_0 r}\int \mathrm{d}q = \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

$$\boxed{\varphi = \frac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}}$$

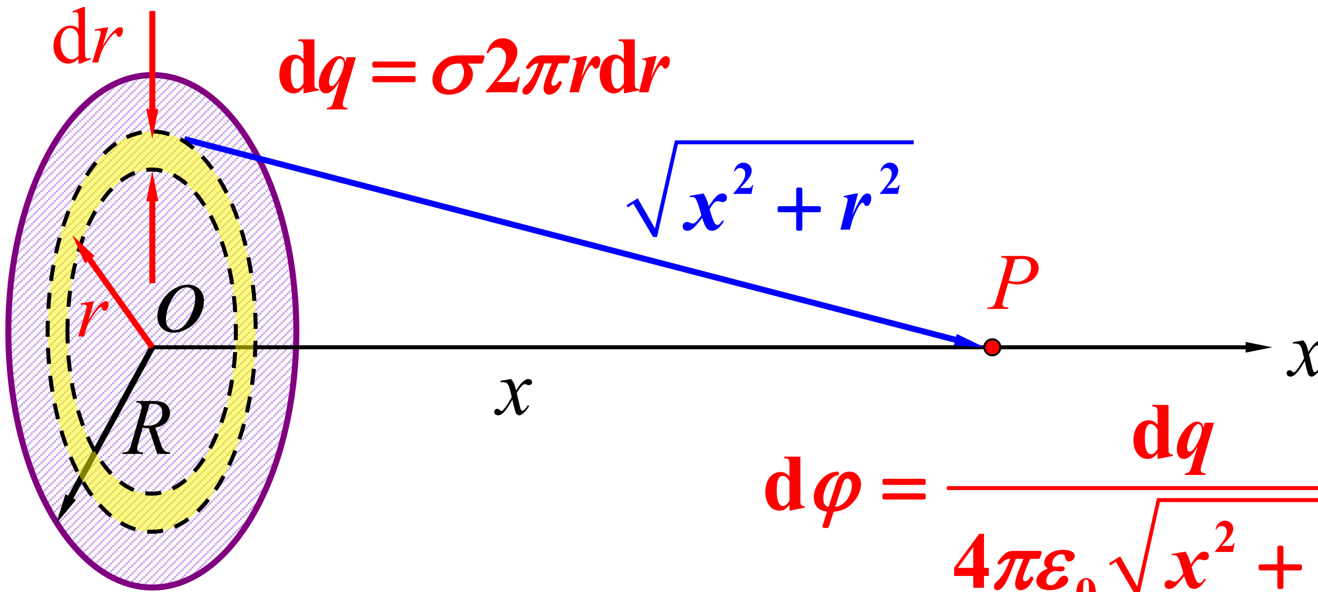


讨 论

$$\left\{\begin{array}{l} x = 0, \quad \boxed{\varphi_0 = \dfrac{q}{4\pi\varepsilon_0 R}} \\ \\ x >> R, \quad \varphi = \dfrac{q}{4\pi\varepsilon_0 x} \end{array}\right.$$



例2 均匀带电薄圆盘轴线上的电势



$$\mathrm{d}q=\sigma 2\pi r\mathrm{d}r$$

$$\mathrm{d}\varphi=\frac{\mathrm{d}q}{4\pi\varepsilon_0\sqrt{x^2+r^2}}$$

$$\varphi=\frac{1}{4\pi\varepsilon_0}\int_0^R\frac{\sigma 2\pi r\mathrm{d}r}{\sqrt{x^2+r^2}}=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R^2}-x\right)$$

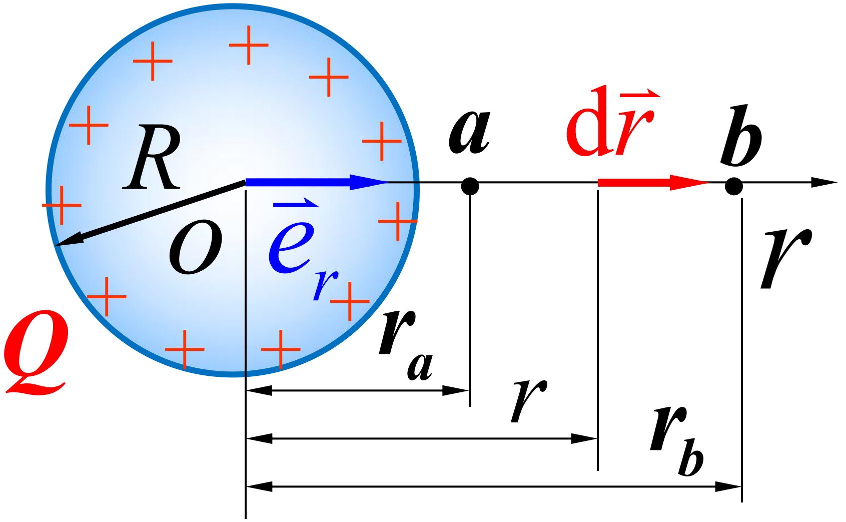
（点电荷电势）

$$x>>R \Rightarrow \sqrt{x^2+R^2}\approx x+\frac{R^2}{2x} \Rightarrow \varphi\approx Q/4\pi\varepsilon_0 x$$

例3 均匀带电球壳的电势

真空中，有一带电为Q，半径为R的带电球壳.

试求（1）球壳外两点间的电势差；（2）球壳内两点间的电势差；（3）球壳外任意点的电势；（4）球壳内任意点的电势.



解

$$\begin{cases} r < R, & \vec{E}_1 = 0 \\ r > R, & \vec{E}_2 = \dfrac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r \end{cases}$$

（1）$$\varphi_a - \varphi_b = \int_{r_a}^{r_b} \vec{E}_2 \cdot \mathrm{d}\vec{r}$$

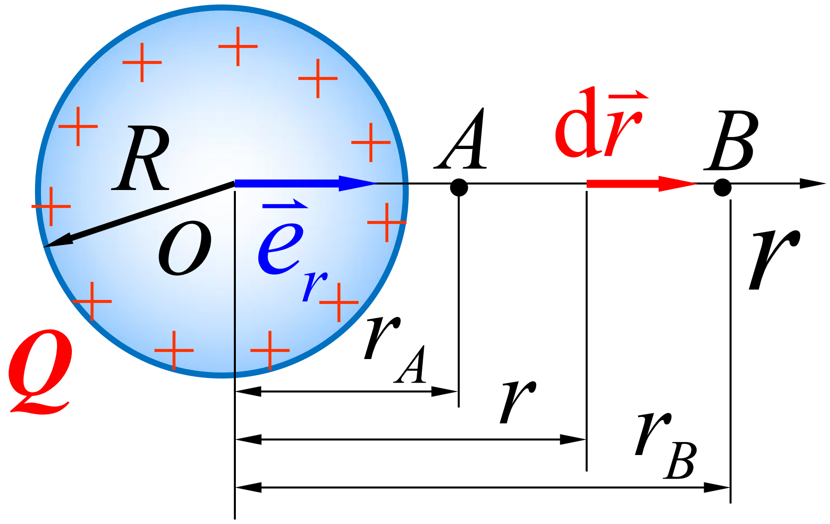
$$= \frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{\mathrm{d}r}{r^2}\vec{e}_r \cdot \vec{e}_r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(2) $r < R$

$$\varphi_a - \varphi_b = \int_{r_a}^{r_b} \vec{E}_1 \cdot \mathrm{d}\vec{r} = 0$$

(3) $r > R$

令 $r_b \to \infty, \quad \varphi_\infty = 0$



由 $\varphi_a - \varphi_b = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$ 可得 $\varphi_2(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$

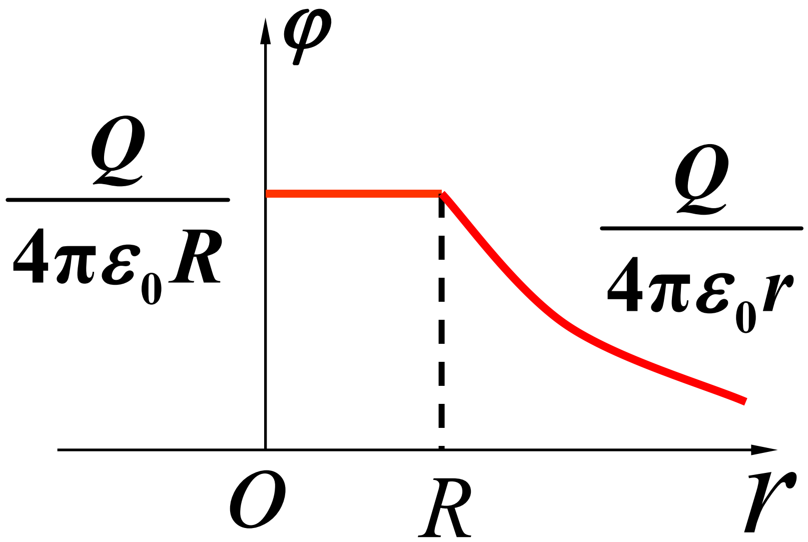
或 $\varphi_2(r) = \int_r^\infty \vec{E}_2 \cdot \mathrm{d}\vec{r} = \int_r^\infty \dfrac{Q}{4\pi\varepsilon_0 r^2}\mathrm{d}r = \dfrac{Q}{4\pi\varepsilon_0 r}$

(4) $r < R$

由 $\varphi_2(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$ 可得 $\varphi(R) = \dfrac{Q}{4\pi\varepsilon_0 R} = \varphi_1$

或 $\varphi_1(r) = \int_r^R \vec{E}_1 \cdot \mathrm{d}\vec{r} + \int_R^\infty \vec{E}_2 \cdot \mathrm{d}\vec{r} = \dfrac{Q}{4\pi\varepsilon_0 R}$

$$\begin{cases} \varphi_2(r) = \dfrac{Q}{4\pi\varepsilon_0 r} \\ \varphi_1(r) = \dfrac{Q}{4\pi\varepsilon_0 R} \end{cases}$$



例4 “无限长”带电直导线的电势

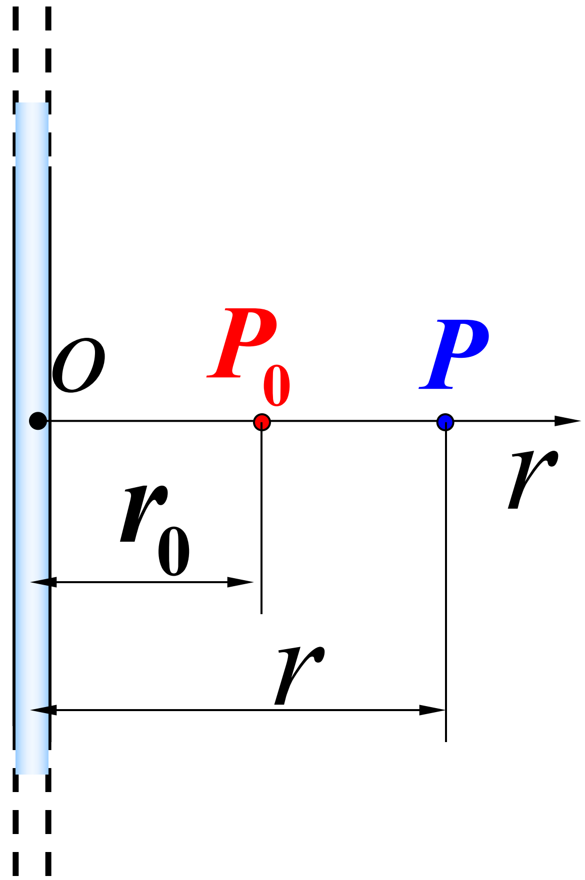
解 $$\varphi_P=\int_P^{P_0}\vec{E}\cdot\mathrm{d}\vec{l}+\varphi_{P_0}$$

令 $\varphi_{P_0}=0$

$$\varphi_P=\int_r^{r_0}\vec{E}\cdot\mathrm{d}\vec{r}$$

$$=\int_r^{r_0}\frac{\lambda}{2\pi\varepsilon_0 r}\vec{e}_r\cdot\mathrm{d}\vec{r}$$

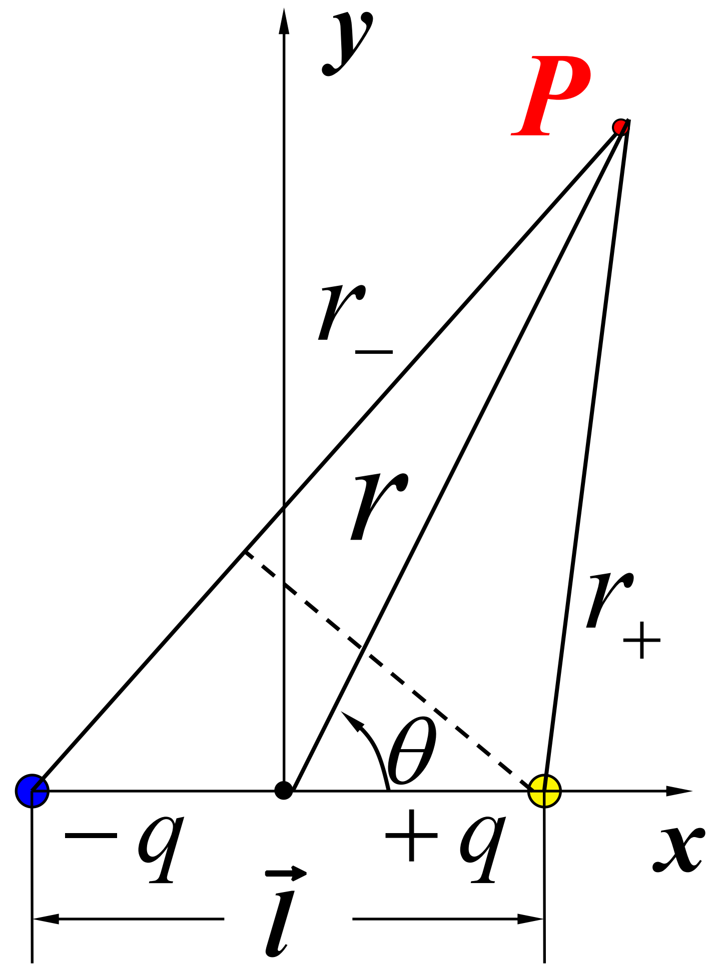
$$=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r}$$



能否选 $\varphi_\infty=0$ ？

例5 求电偶极子 $p=ql$ 电场的电势分布.

解 $\begin{cases} \varphi_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+} \\ \varphi_- = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_-} \end{cases}$



$$\varphi = \varphi_+ + \varphi_- = \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_+ r_-}$$

$\because l << r$

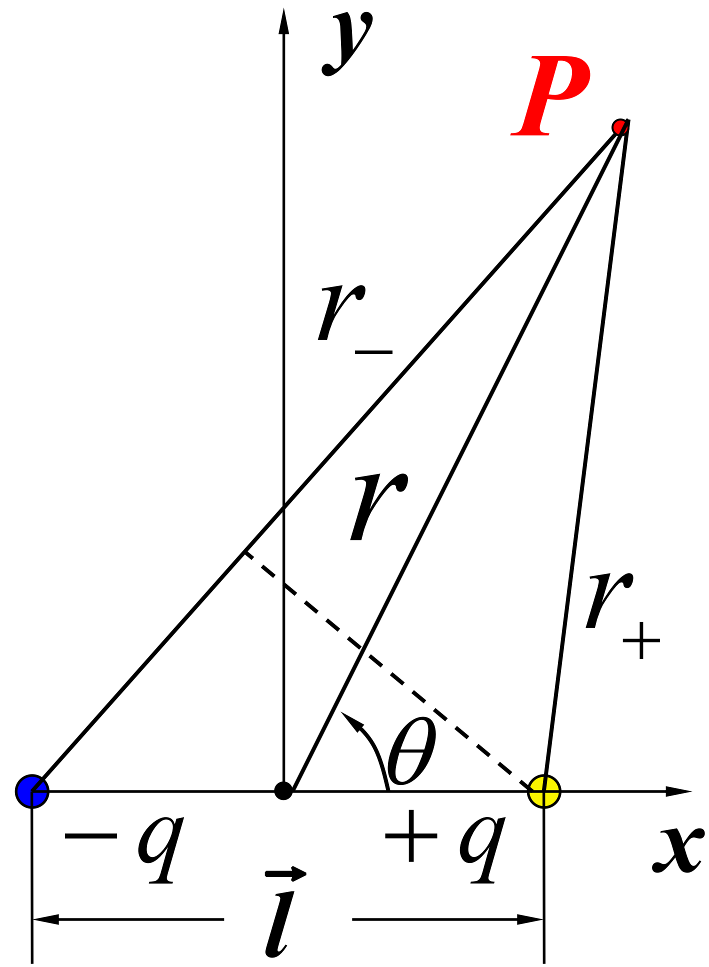
$\therefore r_- - r_+ \approx l\cos\theta$

$r_- r_+ \approx r^2$

$$\varphi=\varphi_{+}+\varphi_{-}=\frac{q}{4\pi\varepsilon_0}\frac{r_{-}-r_{+}}{r_{+}r_{-}}$$

$$\approx\frac{q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}=\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$



$$\begin{cases}\theta=0 & \varphi\approx\dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2}\\[2ex] \theta=\pi & \varphi\approx-\dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2}\\[2ex] \theta=\pi/2 & \varphi=0\end{cases}$$

5.4.4 等势面 电势梯度

一、等势面（电势图示法）

空间电势相等的点连接起来所形成的面称为等势面.

规定任意两相邻等势面间的电势差相等.

1. 在静电场中，电荷沿等势面移动时，电场力做功

$$A_{ab}=q_0(\varphi_a-\phi_b)=\int_a^b q_0\vec{E}\cdot\mathrm{d}\vec{l}=0$$

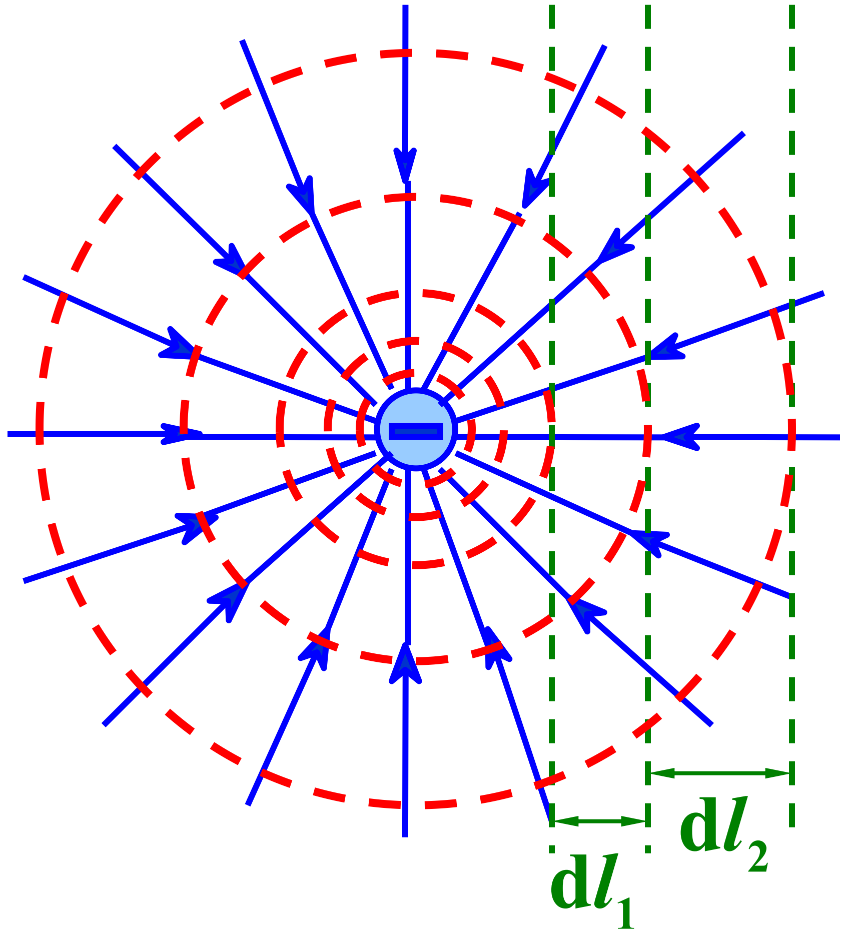
2. 在静电场中，电场强度 $\vec{E}$ 总是与等势面垂直的，即电场线是和等势面正交的曲线簇.

$$A_{ab}=\int_a^b q_0\vec{E}\cdot\mathrm{d}\vec{l}=0$$

$$q_0\neq 0 \quad \vec{E}\neq 0 \quad \mathrm{d}\vec{l}\neq 0 \qquad \boxed{\therefore\quad \vec{E}\perp\mathrm{d}\vec{l}}$$

♦ 按规定，电场中任意两相邻等势面之间的电势差相等，即等势面的**疏密程度**同样可以表示场强的大小。

点电荷的等势面

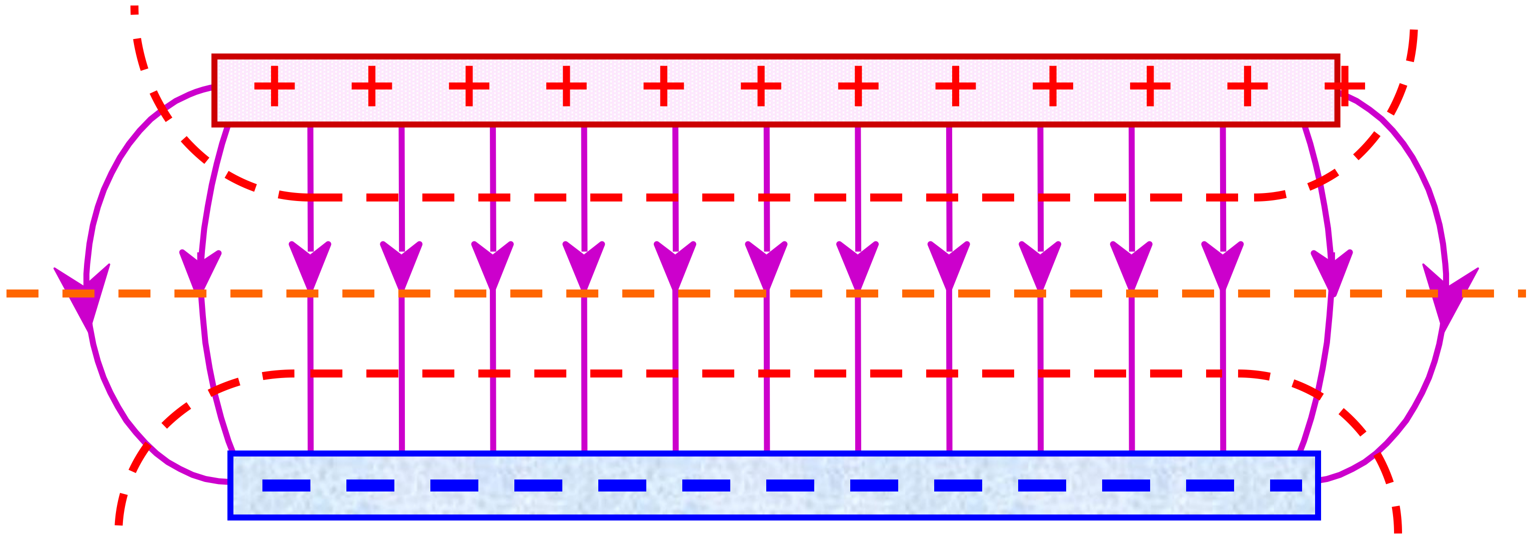


$$\varphi_a - \varphi_b = \int_a^b \vec{E} \cdot d\vec{l}$$

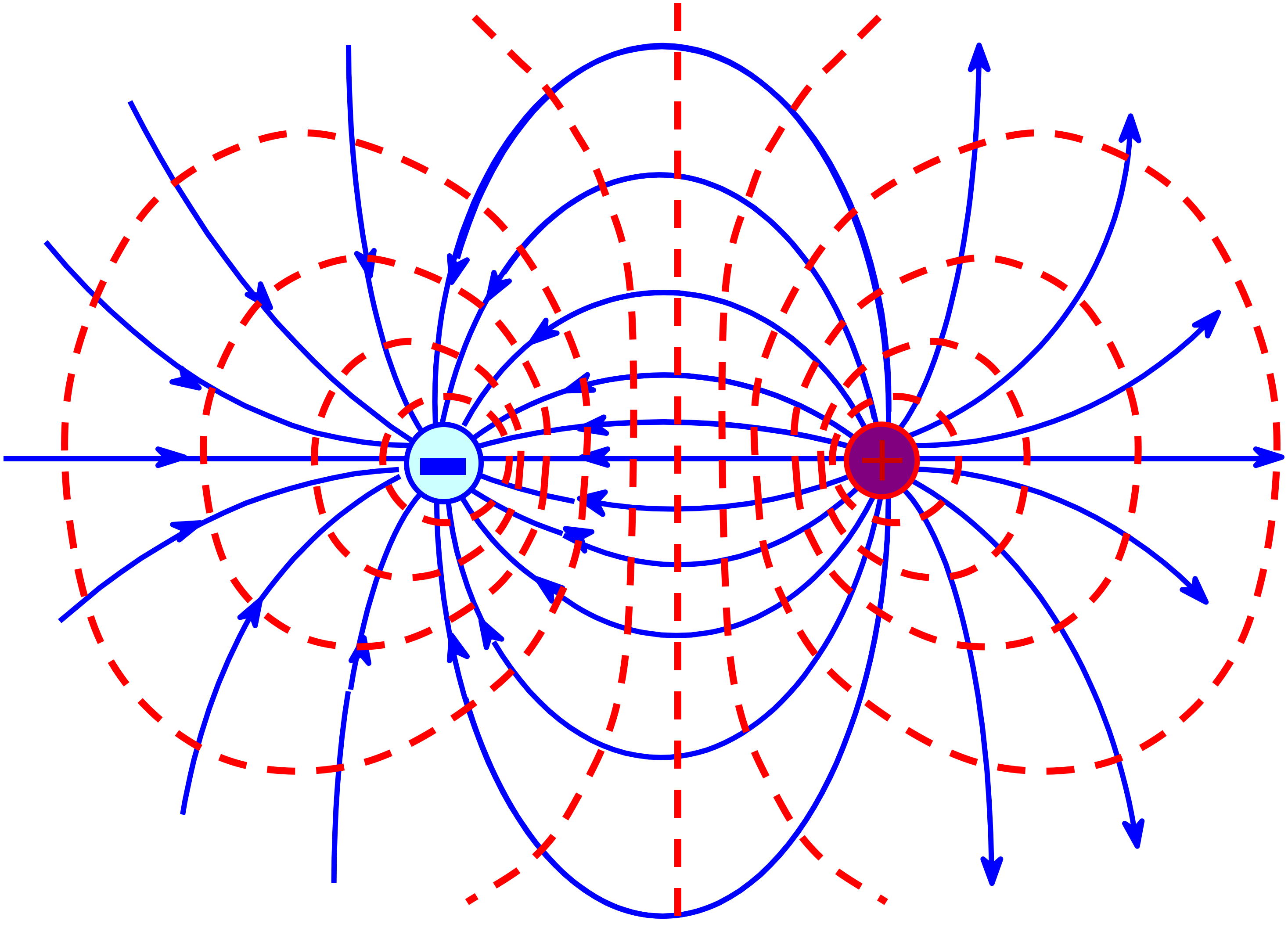
$$dl_2 > dl_1$$

$$\Rightarrow E_2 < E_1$$

两平行带电平板的电场线和等势面



一对等量异号点电荷的电场线和等势面



二、电场强度与电势梯度

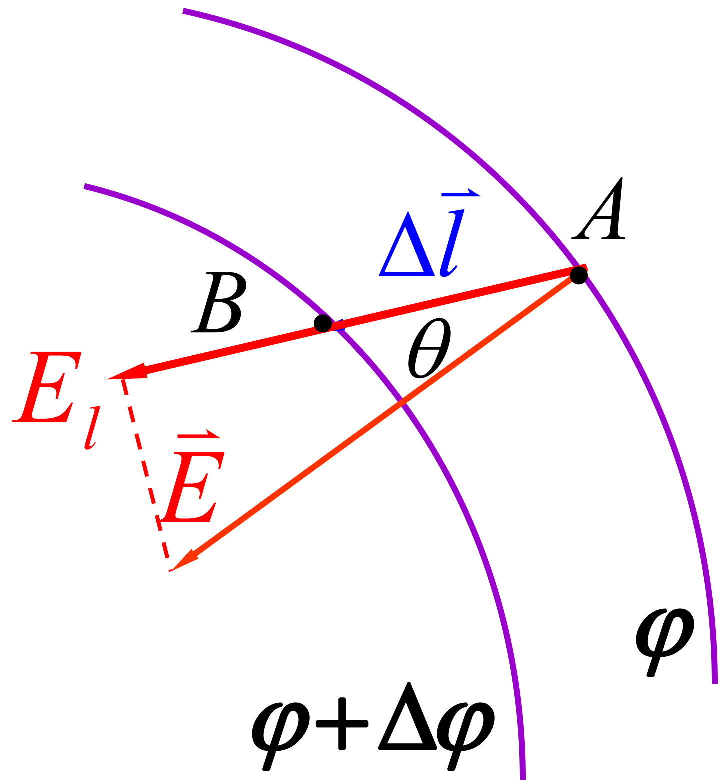
$$A = \vec{E} \cdot \Delta \vec{l} = E\cos\theta \Delta l$$

$$= \varphi - (\varphi + \Delta\varphi) = -\Delta\varphi$$

$$E\cos\theta = E_l$$

$$-\Delta\varphi = E_l \Delta l, \quad E_l = -\frac{\Delta\varphi}{\Delta l}$$

$$E_l = -\lim_{\Delta l \to 0} \frac{\Delta\varphi}{\Delta l} = -\frac{\mathrm{d}\varphi}{\mathrm{d}l}$$

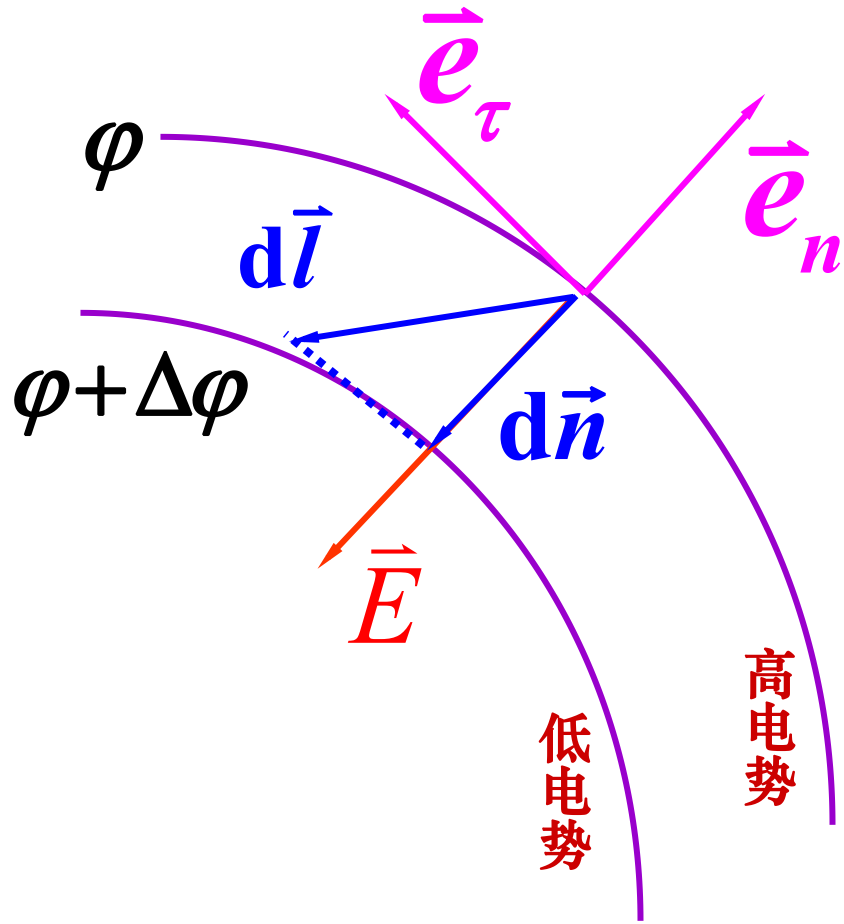


电场中某一点的电场强度沿某一方向的分量，等于这一点的电势沿该方向单位长度上电势变化率的负值.

$$E_l = -\frac{\mathrm{d}\varphi}{\mathrm{d}l} \qquad E_\mathrm{n} = -\frac{\mathrm{d}\varphi}{\mathrm{d}n}$$

$$\because \mathrm{d}l > \mathrm{d}n \qquad \therefore E_\mathrm{n} > E_l$$

$$\vec{E} = -\frac{\mathrm{d}\varphi}{\mathrm{d}n}\vec{e}_\mathrm{n}$$



大小 $\left|\vec{E}\right| = \left|\frac{\mathrm{d}\varphi}{\mathrm{d}n}\right|$

方向 与$\vec{e}_\mathrm{n}$ 相反，由**高**电势处指向**低**电势处

物理意义

(1) 空间某点电场强度的大小取决于该点领域内电势的空间变化率.

(2) 电场强度的方向恒指向电势降落的方向.

讨论

◆ 直角坐标系中

$$\vec{E}=-(\frac{\partial\varphi}{\partial x}\vec{i}+\frac{\partial\varphi}{\partial y}\vec{j}+\frac{\partial\varphi}{\partial z}\vec{k})=-\mathrm{grad}\varphi$$

$$\vec{E}=-\nabla\varphi$$ （电势梯度）

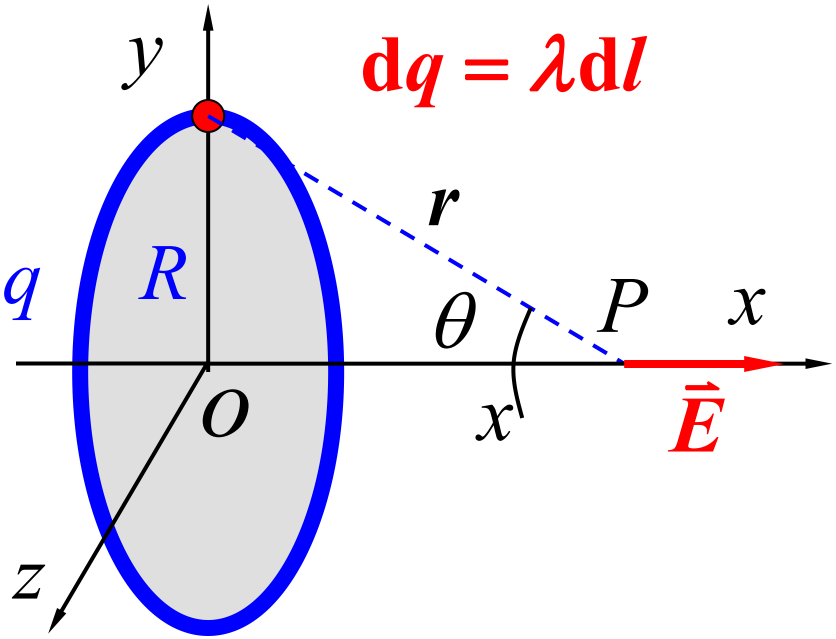
◆ 为求电场强度 提供了一种新的途径

求$\vec{E}$的三种方法：

- 利用电场强度叠加原理
- 利用高斯定理
- 利用电势与电场强度的关系*

例1* 求一均匀带电细圆环轴线上任一点的电场强度。

解： $\vec{E} = -\nabla\varphi$



$$\varphi = \frac{q}{4\pi\varepsilon_0(x^2+R^2)^{1/2}}$$

$$E = E_x = -\frac{\partial\varphi}{\partial x}$$

$$= -\frac{\partial}{\partial x}\left[\frac{q}{4\pi\varepsilon_0(x^2+R^2)^{1/2}}\right]$$

$$= \frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

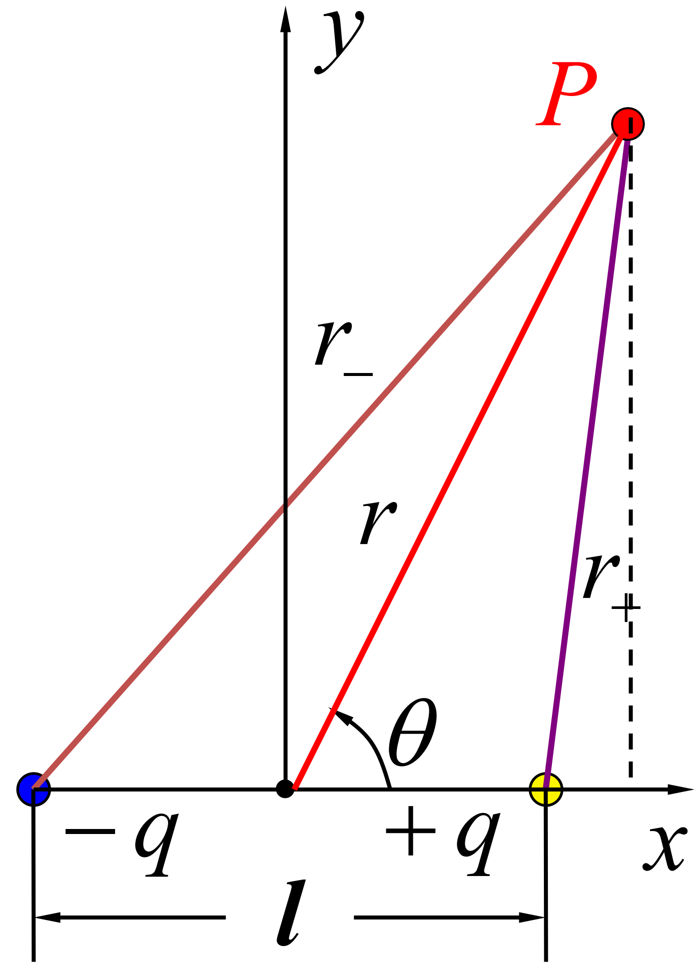
例2* 用场强与电势梯度的关系，计算电偶极子的电场强度分布。

$$\varphi = \frac{px}{4\pi\varepsilon_0 (x^2 + y^2)^{3/2}}$$

$$E_x = -\frac{\partial \varphi}{\partial x}$$

$$= -\frac{p}{4\pi\varepsilon_0}\frac{y^2 - 2x^2}{(x^2 + y^2)^{5/2}}$$

$$E_y = -\frac{\partial \varphi}{\partial y} = \frac{p}{4\pi\varepsilon_0}\frac{3xy}{(x^2 + y^2)^{5/2}}$$



$$\begin{cases} E_x = -\dfrac{p}{4\pi\varepsilon_0}\dfrac{y^2-2x^2}{(x^2+y^2)^{5/2}} \\ E_y = \dfrac{p}{4\pi\varepsilon_0}\dfrac{3xy}{(x^2+y^2)^{5/2}} \end{cases}$$

$$E = \sqrt{E_x^2 + E_y^2}$$

$$= \frac{p}{4\,\pi\varepsilon_0}\frac{(4x^2+y^2)^{1/2}}{(x^2+y^2)^2}$$

$$\begin{cases} y = 0 \qquad E = \dfrac{2p}{4\pi\varepsilon_0}\dfrac{1}{x^3} \\ x = 0 \qquad E = \dfrac{p}{4\pi\varepsilon_0}\dfrac{1}{y^3} \end{cases}$$