CS0441

Discrete Structures Section 2

KP. Wang **Assignment #1** **Solutions**

1. Let p, q, and r be the propositions $p:$ You have the flu. $q:$ You miss the final examination. r :You pass the course. Express each of these propositions as an English sentence.

   a) $p \rightarrow q$

   b) $\neg q \leftrightarrow r$

   c) $q \rightarrow \neg r$

   d) $p \vee q \vee r$

   e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

   f) $(p \wedge q) \vee (\neg q \wedge r)$

   Solution:

   a) If you have the flu, then you will miss the final examination.

   b) If you don't miss the final examination, then you will pass the course.

   c) If you miss the final examination,then you will not pass the course.

   d) You have the flu, miss the final examination or pass the course.

   e) You have the flu or miss the final examination only if you will not pass the course.

   f) You have the flu and miss the final examination or you don't miss the final examination and you pass the course.

2. Let p, q, and r be the propositions $p:$ You get an A on the final exam. $q:$ You do every exercise in this book. $r:$ You get an A in this class. Write these propositions using p, q, and r and logical connectives (including negations).

   a) You get an A in this class, but you do not do every exercise in this book.

   b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

   c) To get an A in this class, it is necessary for you to get an A on the final.

   d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

   e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

   f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

   Solution:

   a) $r \wedge \neg q$,

b) $p \wedge q \wedge r$,

c) $p \to r$,

d) $(p \wedge \neg q) \to r$,

e) $(p \wedge q) \to r$,

f) $r \leftrightarrow (q \vee p)$.

3. Construct following truth tables.

a) $p \to (\neg q \vee r)$.

p	q	r	$\neg q$	$\neg q \vee r$	$p \to (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

b) $\neg p \to (q \to r)$.

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

c) & d) Parts (c) and (d) we can combine into a single table.

p	q	r	$p \to q$	$\neg p$	$\neg p \to r$	$(p \to q) \vee (\neg p \to r)$	$(p \to q) \wedge (\neg p \to r)$
T	T	T	T	F	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	F	T	T	F
T	F	F	F	F	T	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	F

e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$.

p	q	r	$p \leftrightarrow q$	$\neg q$	$\neg q \leftrightarrow r$	$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$.

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

4. a) bitwise $OR = 1111111$; bitwise $AND = 0000000$; bitwise $XOR = 1111111$.

   b) bitwise $OR = 11111010$; bitwise $AND = 10100000$; bitwise $XOR = 01011010$.

   c) bitwise $OR = 100111\ 1001$; bitwise $AND = 0001000000$; bitwise $XOR = 1000111001$.

   d) bitwise $OR = 1111111111$; bitwise $AND = 0000000000$; bitwise $XOR = 1111111111$.

5. You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of g : "You can graduate," m : "You owe money to the university," r : "You have completed the requirements of your major," and b : "You have an overdue library book."

   Solution: Recall that p only if q means $p \to q$. In this case, if you can graduate then you must have fulfilled the three listed requirements. Therefore the statement is $g \to (r \wedge (\neg m) \wedge (\neg b))$. Notice that in everyday life one might actually say "You can graduate if you do these things," but logically that is not what the rules really say.

6. To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service. Express your answer in terms of w : "You can use the wireless network in the airport," d : "You pay the daily fee," and s :"You are a subscriber to the service."

   Solution: Recall that q unless p implies $\neg p \to q$. Hence, we can express as $\neg s \to (d \to w)$.

7. You are eligible to be President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country. Express your answer in terms of e : "You are eligible to be President of the U.S.A.," a: "You are at least 35 years old," b : "You were born in the U.S.A," p : "At the time of your birth, both of your parents where citizens," and r : "You have lived at least 14 years in the U.S.A."

   Solution: If you are eligible to be President, then you must satisfy the requirements: $e \to (a \wedge (b \vee p) \wedge r)$. Notice that it is only the requirement of being native-born that can be overridden by having parents who were citizens, so $b \vee p$ is grouped as one of the three conditions.

8. *Solution:* De Morgan's laws tell us that to negate a conjunction we form the disjunction of the negations, and to negate a disjunction we form the conjunction of the negations.

   a) This is the conjunction "Jan is rich, and Jan is happy." So the negation is "Jan is not rich, or Jan is not happy."

   b) This is the disjunction "Carlos will bicycle tomorrow, or Carlos will run tomorrow." So the negation is "Carlos will not bicycle tomorrow, and Carlos will not run tomorrow." We could also render this as "Carlos will neither bicycle nor run tomorrow."

   c) This is the disjunction "Mei walks to class, or Mei takes the bus to class." So the negation is "Mei does not walk to class, and Mei does not take the bus to class." (Maybe she gets a ride with a friend.) We could also render this as "Mei neither walks nor takes the bus to class."

   d) This is the conjunction "Ibrahim is smart, and Ibrahim is hard working." So the negation is "Ibrahim is not smart, or Ibrahim is not hard working."

9. *Solution:* The proposition $\neg(p \leftrightarrow q)$ is true when p and q do not have the same truth values, which means that p and q have different truth values (either p is true and q is false, or vice versa). These are exactly the cases in which $p \leftrightarrow \neg q$ is true. Therefore these two expressions are true in exactly the same instances, and therefore are logically equivalent.

10. *Solution:* We'll determine exactly which rows of the truth table will have F as their entries. In order for $(p \to r) \vee (q \to r)$ to be false, we must have both of the two conditional statements false, which happens exactly when r is false and both p and q are true. But this is precisely the case in which $p \wedge q$ is true and r is false, which is precisely when $(p \wedge q) \to r$ is false. Since the two propositions are false in exactly the same situations, they are logically equivalent.

11. Show that $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.

    Solution: We can prove using the derivation as follows

$$
\begin{aligned}
(p \vee q) \wedge (\neg p \vee r) \to (q \vee r) &\equiv (p \wedge \neg p) \vee (q \wedge \neg p) \vee (q \wedge r) \vee (p \wedge r) \to q \vee r, \text{ Distributive Law} \\
&\equiv \neg((q \wedge \neg p) \vee (q \wedge r) \vee (p \wedge r)) \vee q \vee r, \text{ Implication Identity} \\
&\equiv ((\neg q \vee p) \wedge (\neg q \vee \neg r) \wedge (\neg p \vee \neg r)) \vee q \vee r, \text{ De Morgan's Law} \\
&\equiv (\neg q \vee p \vee q \vee r) \wedge (\neg q \vee \neg r \vee q \vee r) \wedge (\neg p \vee \neg r \vee q \vee r), \text{ Distributive Law} \\
&\equiv T \wedge T \wedge T \equiv T.
\end{aligned}
$$

12. *Solution:* To show that these are not logically equivalent, we need only find one assignment of truth values to p, q, r, and s for which the truth values of $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ differ. Let us try to make the first one false. That means we have to make $r \to s$ false, so we want r to be true and s to be false. If we let p and q be false, then each of the other three simple conditional statements ($p \to q, p \to r$, and $q \to s$) will be true. Then $(p \to q) \to (r \to s)$ will be $\mathrm{T} \to \mathrm{F}$, which is false; but $(p \to r) \to (q \to s)$ will be $\mathrm{T} \to \mathrm{T}$, which is true.

13. Show that if p, q, and r are compound propositions such that p and q are logically equivalent and q and r are logically equivalent, then p and r are logically equivalent.

    Solution: The question can be expressed as $[(\mathrm{p} \leftrightarrow \mathrm{q}) \wedge (\mathrm{q} \leftrightarrow \mathrm{r})] \to (\mathrm{p} \leftrightarrow \mathrm{r})$. To say that p and q are logically equivalent is to say that the truth tables for p and q are identical; similarly, to say that q and r are logically equivalent is to say that the truth tables for q and r are identical. Clearly if the truth tables for p and q are identical, and the truth tables for q and r are identical, then the truth tables for p and r are identical. Therefore p and r are logically equivalent.

14. *Solution:* a) With a little trial and error we discover that setting $p = \mathrm{F}$ and $q = \mathrm{F}$ produces $(\mathrm{F} \vee \mathrm{T}) \wedge (\mathrm{T} \vee \mathrm{F}) \wedge (\mathrm{T} \vee \mathrm{T})$, which has the value T. So this compound proposition is satisfiable. (Note that this is the only satisfying truth assignment.)

    b) We claim that there is no satisfying truth assignment here. No matter what the truth values of p and q might be, the four implications become $\mathrm{T} \to \mathrm{T}, \mathrm{T} \to \mathrm{F}, \mathrm{F} \to \mathrm{T}$, and $\mathrm{F} \to \mathrm{F}$, in some order. Exactly one of these is false, so their conjunction is false.

    c) This compound proposition is not satisfiable. In order for the first clause, $p \leftrightarrow q$, to be true, p and q must have the same truth value. In order for the second clause, $(\neg p) \leftrightarrow q$, to be true, p and q must have opposite truth values. These two conditions are incompatible, so there is no satisfying truth assignment.