Chapter 7

constant current and constant magnetic field

Primary coverage

1. Constant electric current

2. Magnetic induction strength of the magnetic field

3 The Bio-Saval law

4 Gaussian theorem and ampere loop theorem for the magnetic field

5 Ampere force with the Lorentz force

6 Magnetic media

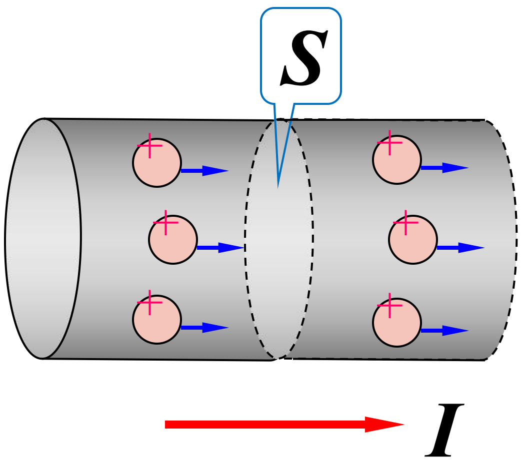
§ 7.1

Constant current

7.1.1 Continuity equation of the current current

1. Current current density current field

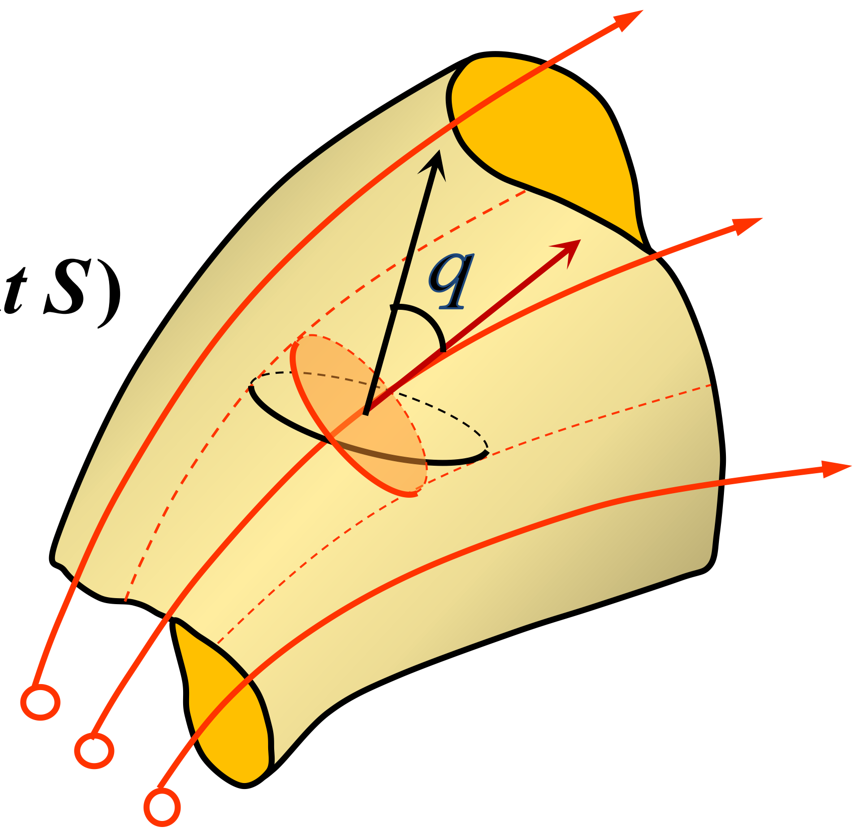
The current is defined as the rate of charge through the section S



$$I = \frac{dq}{dt}$$

$$dQ = nq(udt\,S)$$

$$\boxed{I = nquS}$$



u For the electron, the drift velocity size

$$dQ = nq(udt\underline{dS\cos\theta})$$

$$\mathrm{d}I = \frac{\mathrm{dQ}}{\mathrm{d}t} = \frac{nq(u\mathrm{d}t\mathrm{d}S\cos\theta)}{\mathrm{d}t} = nqu\mathrm{d}S\cos\theta$$

$$= nq\vec{u}\cdot\mathrm{d}\vec{S}$$

Introducing the element vector

$$\mathrm{d}\vec{S} = \mathrm{d}S\,\vec{e}_{\mathrm{n}}$$

If there are several charge carriers

$$\vec{j} = \sum_i n_i q_i \vec{u}_i$$

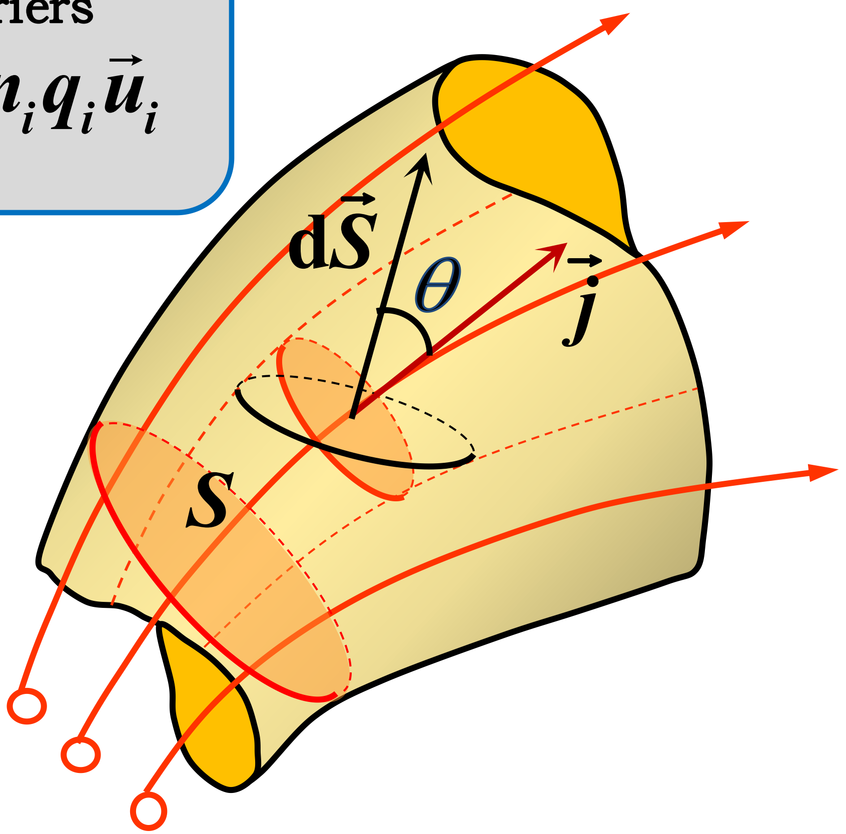
Current density vector

$$\vec{j} = nq\vec{u}$$

$$\mathrm{d}I = \vec{j}\cdot\mathrm{d}\vec{S}$$

Current through any surface S

$$I = \int_S \vec{j}\cdot\mathrm{d}\vec{S} = \int_S j\cos\theta\,\mathrm{d}S$$



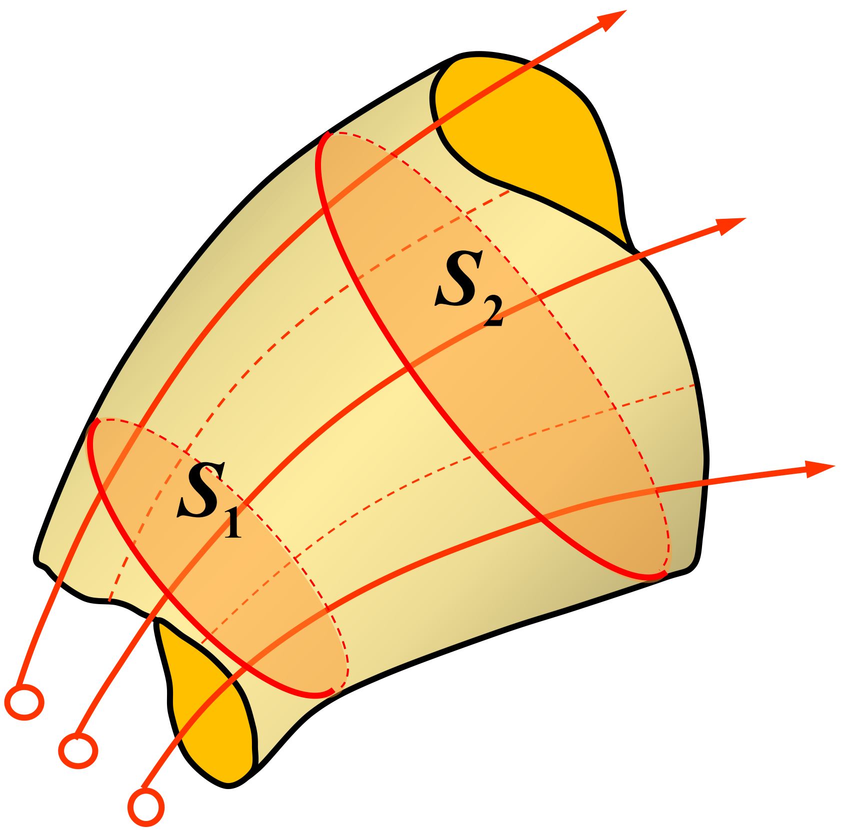
The current density reflects the distribution of the current at each point of the space

$$\vec{j}_1 \neq \vec{j}_2$$

In the bulk conductors, the current distribution is complex

$\vec{j}(x, y, z)$ To form a vector field —— current field

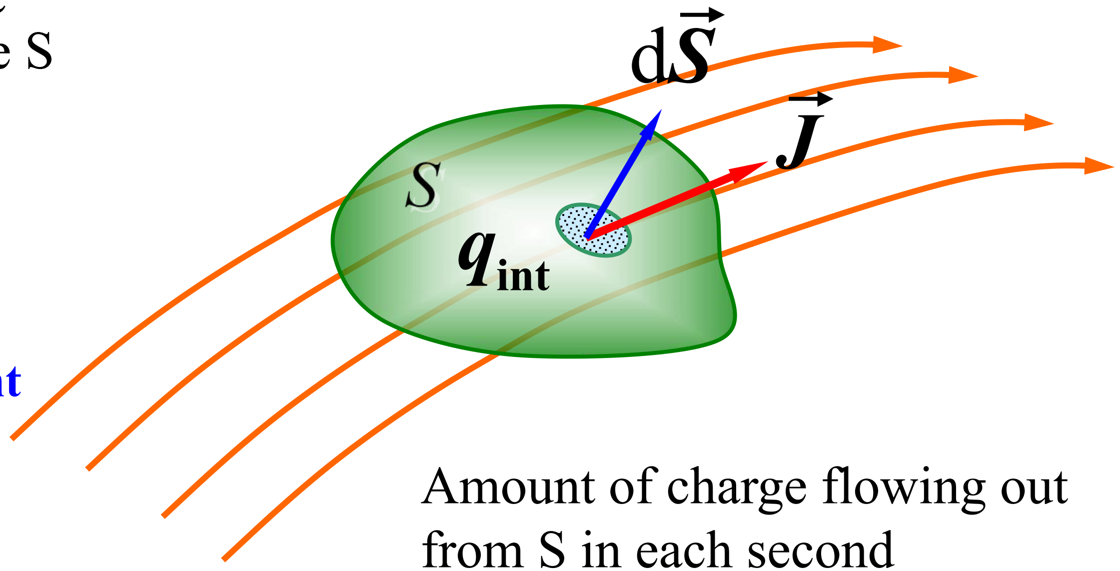
Current lines are introduced to describe the distribution of the current field



The tangent direction at each point on the current line is the same direction as the current density at that point, and the degree of density of the curve represents the magnitude of the current density

2. Continuity equation of the current The constant condition of the current

Through a current, closed surface S



$$I = \oint_S \vec{j} \cdot \mathrm{d}\vec{S}$$

It represents the number of current lines from the closed surface, that is, the current through the closed surface

Amount of charge flowing out from S in each second

$$\frac{\mathrm{d}q}{\mathrm{d}t} = I = \oint_S \vec{j} \cdot \mathrm{d}\vec{S}$$

The continuity equation of the electric current

$$\oint_S \vec{j} \cdot \mathrm{d}\vec{S} = -\frac{\mathrm{d}q_{\text{int}}}{\mathrm{d}t}$$

Conserved by the electric charge:

$$\frac{\mathrm{d}q}{\mathrm{d}t} = -\frac{\mathrm{d}q_{\text{int}}}{\mathrm{d}t}$$

Current lines have a head and a tail

◆ steady current

The direction and size of the current density $\vec{j}$ of each point in the conductor do not change with time, and the current is called constant current (also known as steady constant current)

The continuity equation of the electric current

$$\oint_S \vec{j}\cdot \mathrm{d}\vec{S} = -\frac{\mathrm{d}q_{\text{int}}}{\mathrm{d}t}$$

For a constant current, the number of current lines entering any closed surface at any time is equal to the number of current lines that penetrate the closed surface

$$\oint_S \vec{j}\cdot \mathrm{d}\vec{S} = 0$$

This is the constant-current condition

The constant current line is a closed curve without a head and a tail

◆ constant electric field

If the conductor carries a constant current, the charge distribution in the conductor does not change with time, and the power in any closed surface does not change with time

$$\frac{\mathrm{d}q_{\text{int}}}{\mathrm{d}t}=0$$ (dynamic balance)

A constant electric field is an electric field present inside and outside the conductor through which a constant current passes.

The spatial distribution of the constant electric field does not change with time

- The spatial distribution of the macroscopic charges
  electric-field distribution

Don't change over time

- satisfied $$\oint_S \vec{E}\cdot\mathrm{d}\vec{S}=\sum q_{\text{int}}/\varepsilon_0$$
- satisfied $$\oint_l \vec{E}\cdot\mathrm{d}\vec{l}=0$$

The concept of the electric potential can be introduced

7.1.2 Ohm's law

1. Differential form of Ohm's law

Theoretically, it can be proved that when the temperature of the metal is kept constant, the current density $\vec{j}$ in the metal is proportional to the electric field strength $\vec{E}$

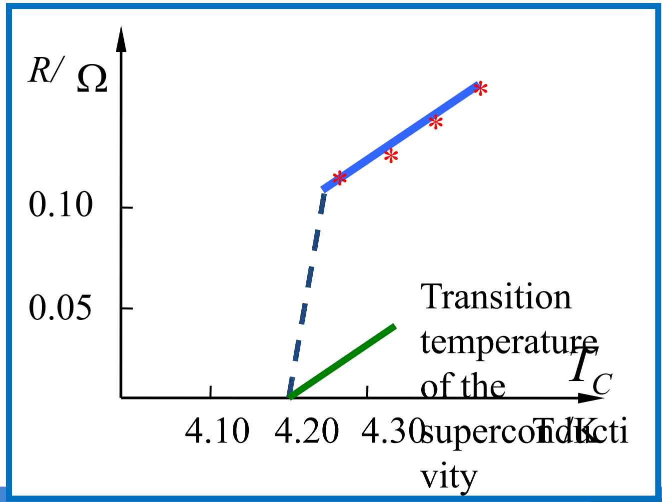
$$\vec{j} = \gamma \vec{E}$$

conductivity

Its reciprocal is called the resistivity $\rho = \dfrac{1}{\gamma}$

discuss

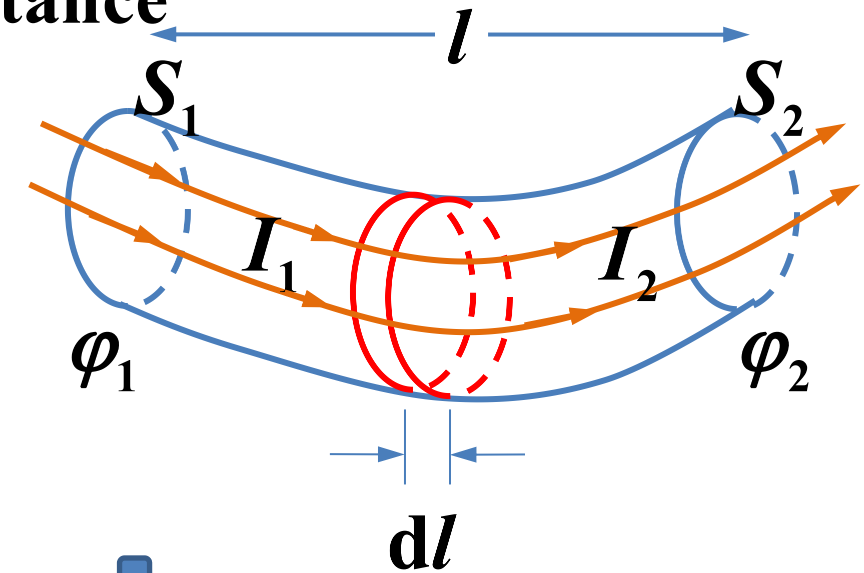
Some metals and compounds suddenly reduce their resistivity to zero at near absolute zero, a phenomenon called superconductivity.



Mercury was measured at 4.2K

The nearby resistance suddenly drops to zero

2. Ohm's law of electrical resistance



Judging from the constant current condition

$$I_1 = I_2$$

Therefore, the electric potential difference in the dl segment is

$$\mathrm{d}\varphi = \vec{E} \cdot \mathrm{d}\vec{l}$$

section S_1、S_2The difference between the electric potential is

$$\varphi_1 - \varphi_2 = \int \vec{E} \cdot \mathrm{d}\vec{l} = \int \rho \vec{j} \cdot \mathrm{d}\vec{l}$$

$$= \int \rho j \mathrm{d}l = I \boxed{\int \frac{\rho \mathrm{d}l}{S}} \quad R$$

Conductor with a uniform cross-section:

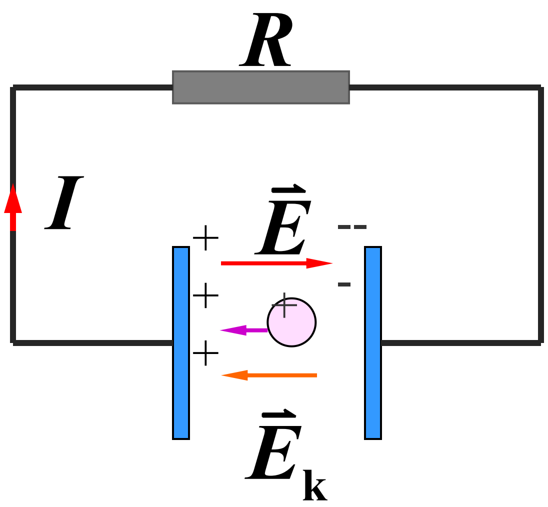
$$R = \rho \frac{l}{S}$$

$$U = IR$$

ohm's law

7.1.3 Power supply electromotive force and full circuit Ohm law

1. Power supply electromotive force



♦ Non-static power: can continuously separate the positive and negative charges to make the positive charge reverse electrostatic field force direction movement.

♦ Power supply: devices that provide non-static power.

♦ Non-electrostatic field strength: $\vec{E}_{\mathrm{k}}$ non-static power per unit of positive charge.

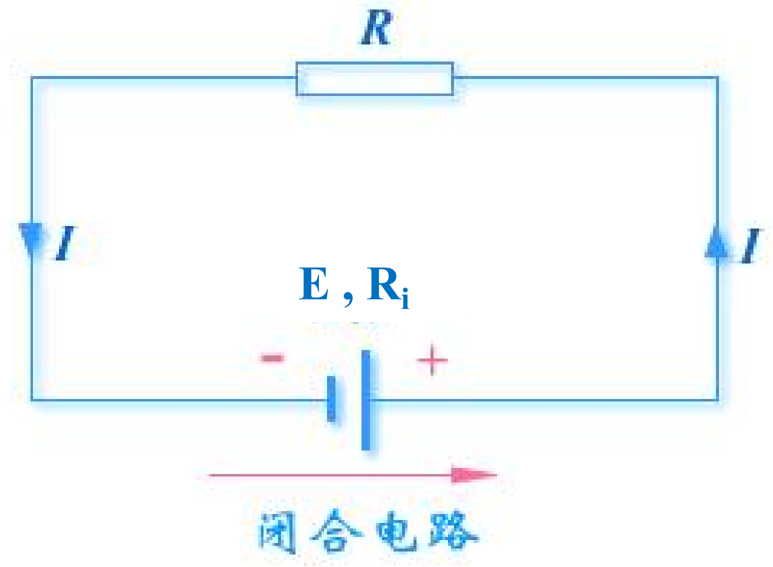
♦ Definition of electromotive force: inside the power supply, the unit positive charge circle from the negative electrode to the positive electrode, the work done by non-static power.

$$\mathrm{E} = \frac{A}{q} = \int_{-}^{+} \vec{E}_{\mathrm{k}} \cdot \mathrm{d}\vec{l}$$

Non-static power exists throughout the circuit

$$\mathrm{E} = \oint_{L} \vec{E}_{\mathrm{k}} \cdot \mathrm{d}\vec{l}$$

2. Full-circuit Ohm's law



$$\boxed{\vec{j}=\frac{1}{\rho}(\vec{E}_{\mathrm{e}}+\vec{E}_{\mathrm{k}})=\gamma(\vec{E}_{\mathrm{e}}+\vec{E}_{\mathrm{k}})}$$

$$\oint_L(\vec{E}_{\mathrm{k}}+\vec{E})\cdot\mathrm{d}\vec{l}$$

$$=\oint_L\vec{E}_{\mathrm{k}}\cdot\mathrm{d}\vec{l}=\mathsf{E}$$

$$\oint_L(E+E_{\mathrm{k}})\cdot\mathrm{d}\vec{l}=\oint_L\frac{\vec{j}\cdot\mathrm{d}\vec{l}}{\gamma}$$

$$=\oint_L\frac{j\,\mathrm{d}l}{\gamma}$$

For the uniform circuit

$$j=I/S$$

$$\mathrm{E}=\oint_L\frac{\vec{j}\cdot\mathrm{d}\vec{l}}{\gamma}$$

$$=\int_{\mathrm{out}}\frac{j\,\mathrm{d}l}{\gamma}+\int_{\mathrm{in}}\frac{j\,\mathrm{d}l}{\gamma}$$

$$=I\left(\int_{\mathrm{out}}\frac{\mathrm{d}l}{\gamma S}+\int_{\mathrm{in}}\frac{j\,\mathrm{d}l}{\gamma S}\right)$$

$$\mathrm{E} = I\Big(\underbrace{\int_{\text{out}} \frac{\mathrm{d}l}{\gamma S}}_{R} + \underbrace{\int_{\text{in}} \frac{\mathrm{d}l}{\gamma S}}_{R_{\mathrm{i}}}\Big)$$

R: Resistor of the external circuit

R_{i}: Internal resistance of power supply

Full-circuit Ohm's law

$$\mathrm{E} = I(R + R_{\mathrm{i}})$$

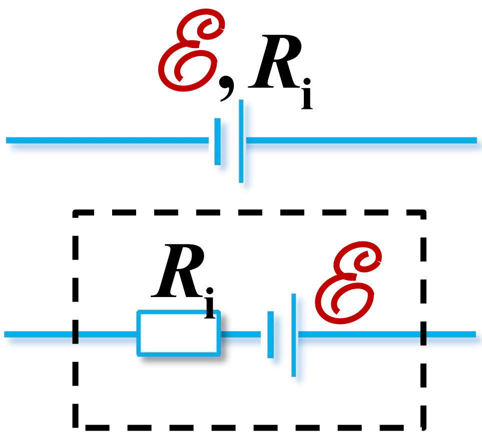
3. An Ohm's law of source-containing circuits

stipulate

♦ The power supply is an ideal power E supply with electromotive force and a resistance in series R_{i}

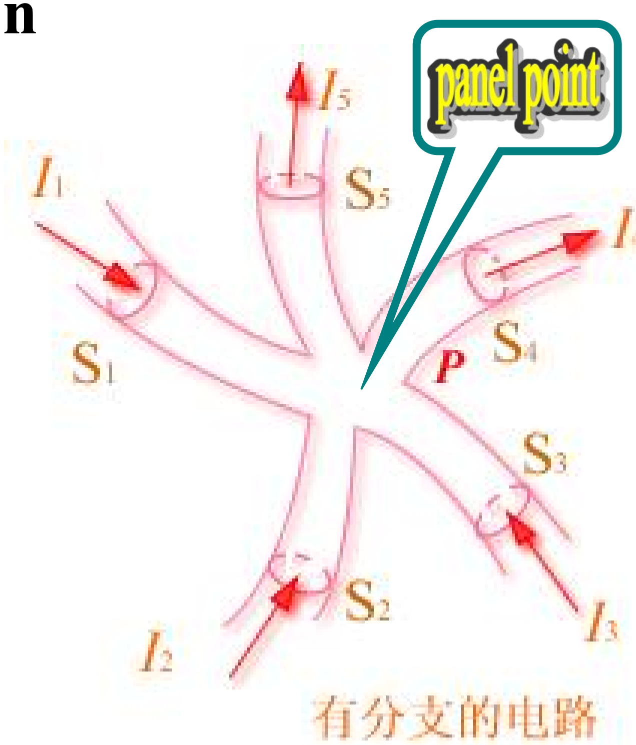
♦ The conductor resistance is zero, no electrical potential landing

♦ Following the current direction, the current flows through the resistance and the potential decreases; the current flows through the power supply and the potential increases



4. The Kirchhoff equation and its application

Dealing with complex circuit problems



1. Kirchhoff's first equation

In branched circuits, judging from the constant current condition:

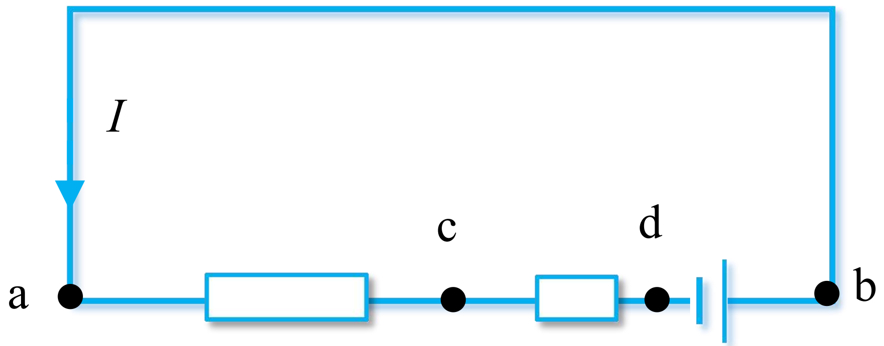
$$\sum I_i = 0$$

The node current equation

That is, the algebraic sum of the current of the outflow node is zero.

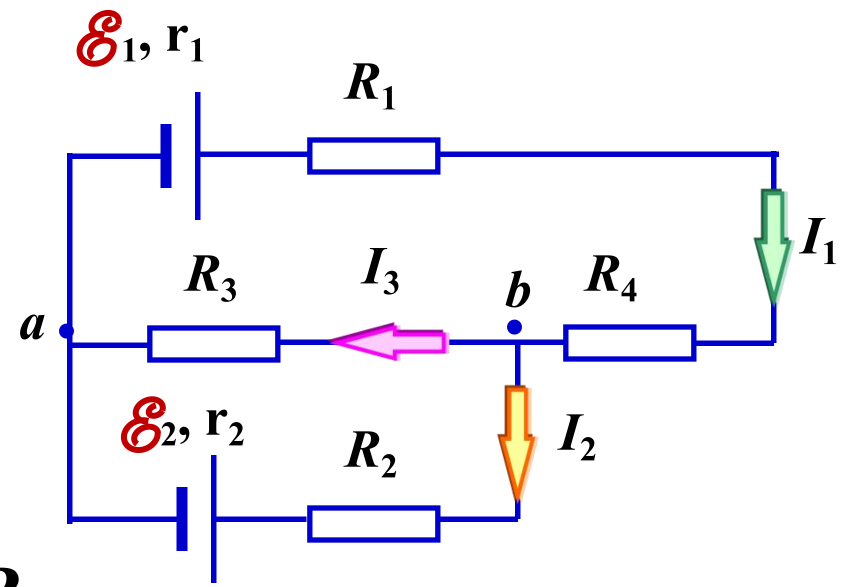
2. Kirchhoff's second equation

$$\left(\sum \pm IR\right) - \left(\sum \pm \mathscr{E}\right) = 0$$



That is, the weekly potential decrease and potential increase in the circuit are equal.

Example 2 is shown in Fig, $\mathcal{E}_1 = 3.0\text{V}$, $\mathcal{E}_2 = 1.0\text{V}$, $R_{i1} = 0.5\text{W}$, $R_{i2} = 1.0\text{W}$, $R_1 = 4.5\text{W}$, $R_2 = 19.0\text{W}$, $R_3 = 10.0\text{W}$, $R_4 = 5.0\text{W}$ 。Find the current distribution in the circuit.



Solution: List of the Kirchhoff equation

For the node b:

$$-I_1 + I_3 + I_2 = 0$$

A R on the loop₁ bR_3a :

$$\mathcal{E}_1 = I_1(r_1 + R_1 + R_4) + I_3 R_3$$

A R on the loop₃ bR_2a :

$$-\mathcal{E}_2 = I_2(r_2 + R_2) - I_3 R_3$$

Substitution of data: $I_1 = 0.16\text{A},\quad I_2 = 0.02\text{A},\quad I_3 = 0.14\text{A}$

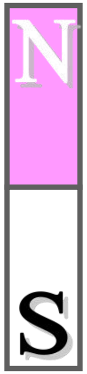
§ 7.2

Magnetic field, magnetic induction strength

7.2.1 Magnetic phenomena and magnetic field

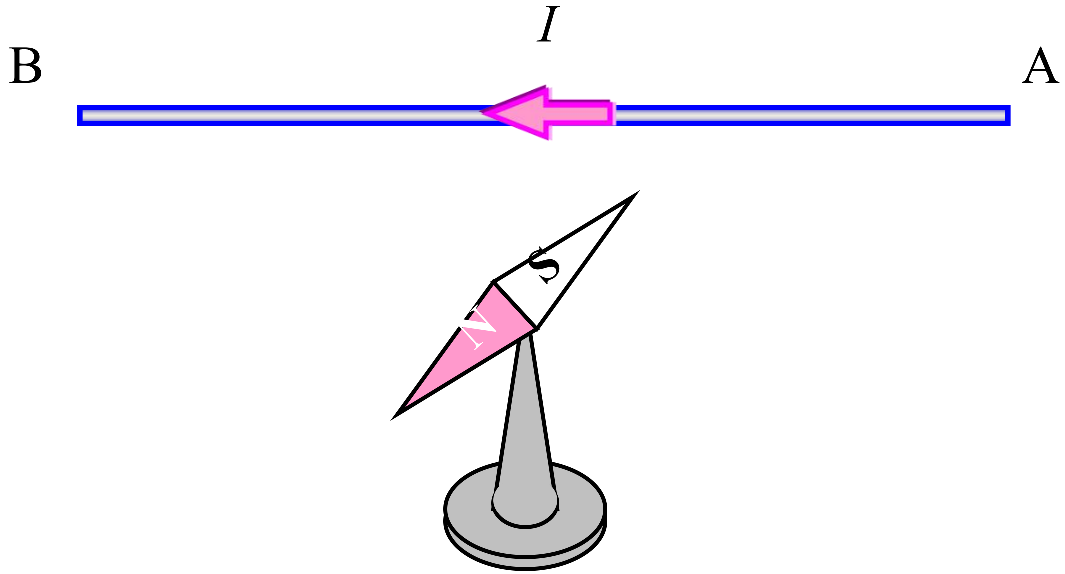
- ♦ **Interactions between the magnets:**

The same pole repel, the different pole phase suction



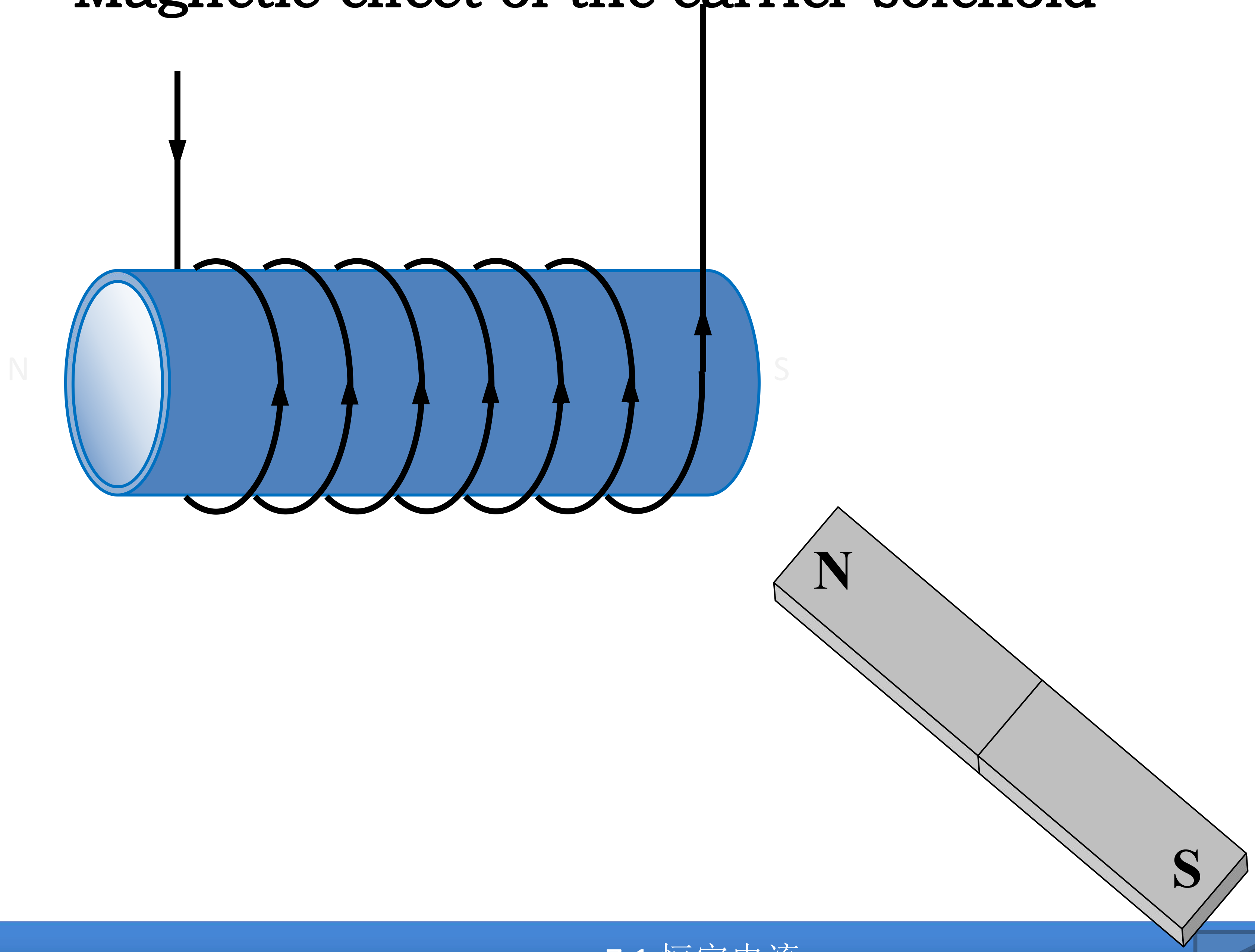
magnet

- ♦ **magnetic effect of current**



Danish physicist-Oster

Magnetic effect of the carrier solenoid

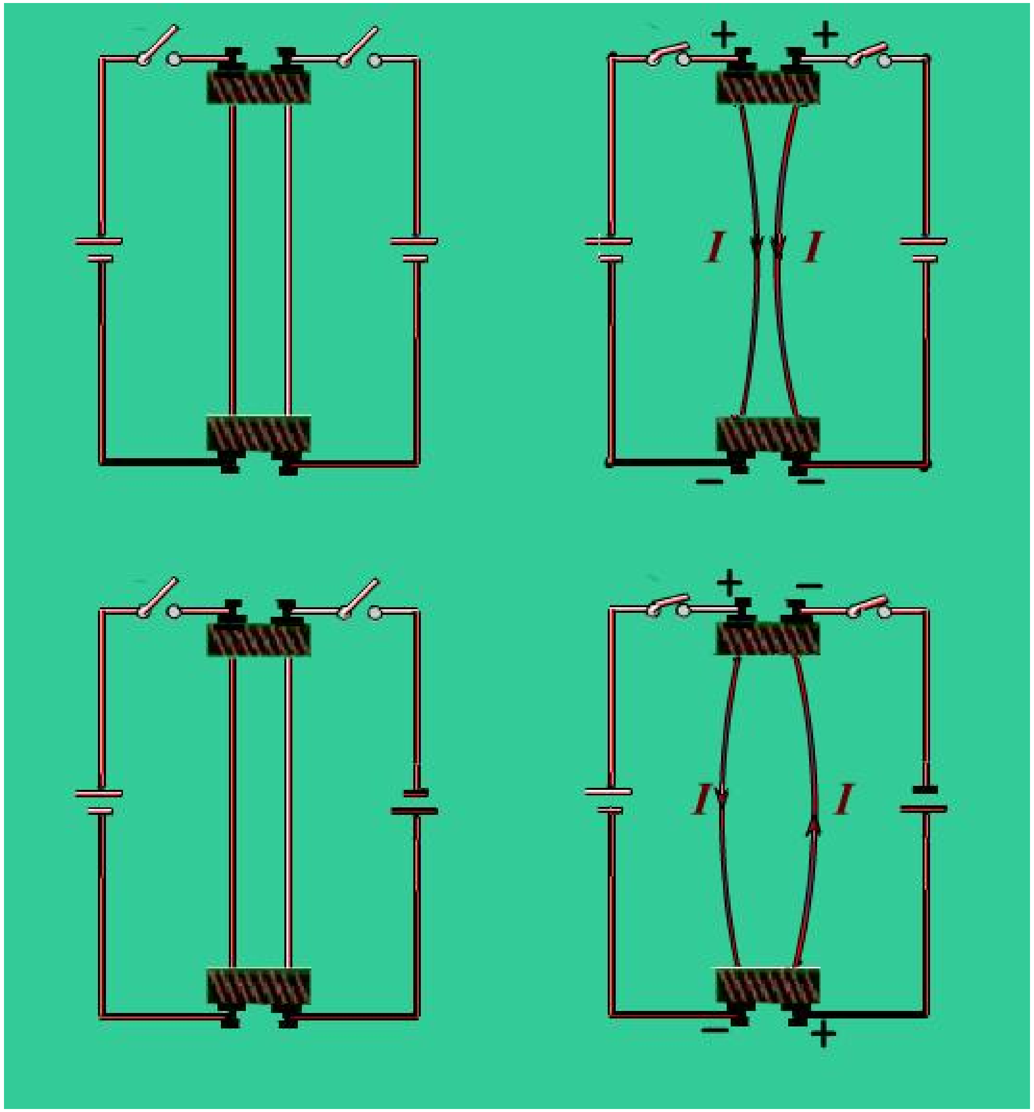


The interaction between the electric current and the electric current

phenomenon:

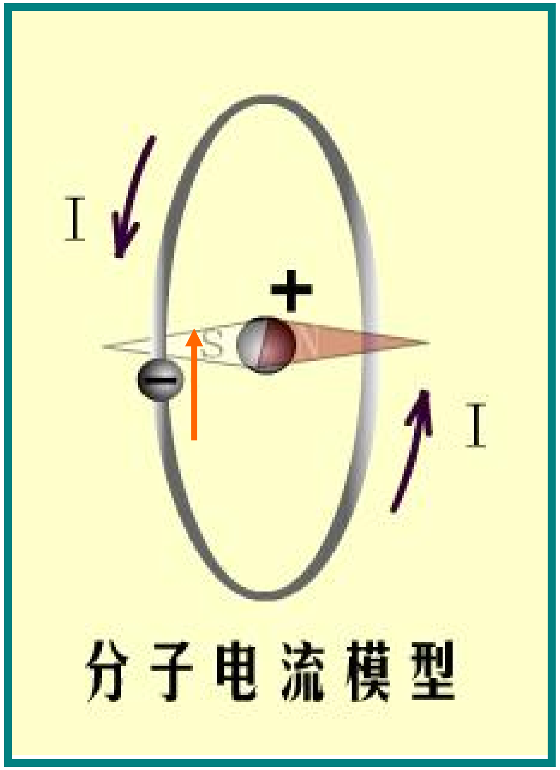
The directional current attracts each other;

The reverse currents are mutually exclusive.



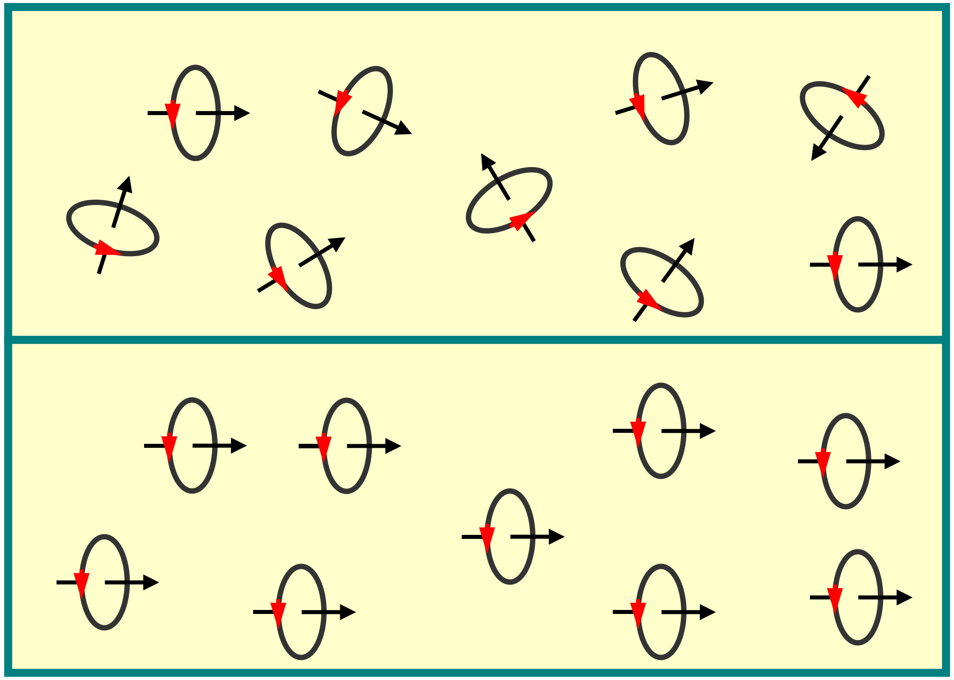
The molecular current hypothesis of the ♦ Ampere

Magnetic origin



Inside the atoms, the movement of the electrons (rotating around the nucleus and rotating inside it) forms a small electric current

—— molecular current



The direction of the internal molecular current is neatly arranged in a certain manner

—— magnetic body

Summary: Magnetic magnetic phenomenon between magnets, between magnets and current, and between current and current, or all magnetic phenomena can be attributed to the magnetic effect of current.

There is only one source of a current and a magnet: the movement of the charge

Conclusion: From the microscopic point of view, the magnetic force is the manifestation of the interaction between the moving charges.

♦ **magnetic field**

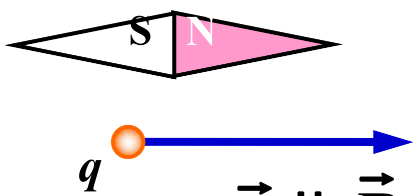
There is a magnetic field in the space around the moving charge, and the magnetic force on the other moving charge is actually the magnetic field acting on it.

Motion charge ⇔ **magne tic field** ⇔ **Motion charge**

7.2.2, Magnetic induction intensity

The Lorentz force of the moving point charge in a magnetic field is analyzed

1.

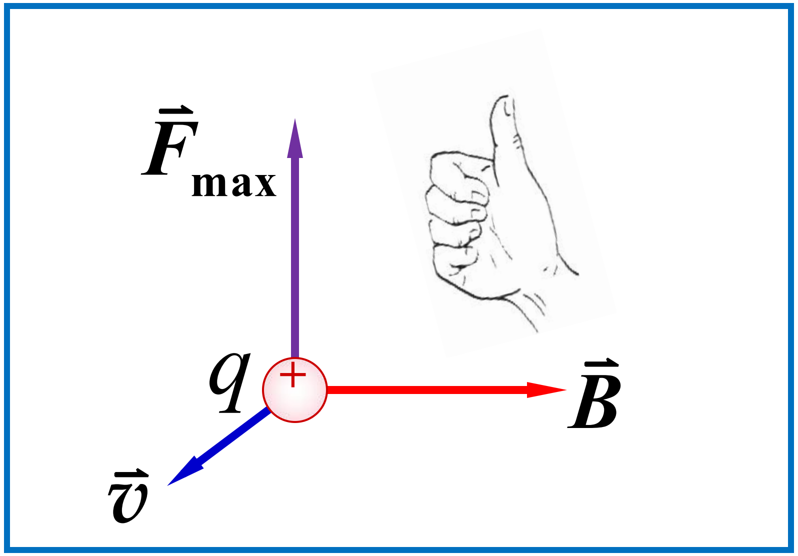


$$\vec{v} \parallel \vec{B}$$

$$F = 0$$

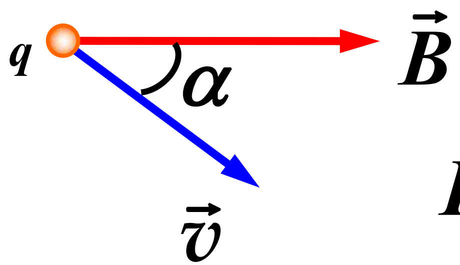
Magnetic induction intensity size

$$B = \frac{F_{\max}}{qv}$$



$$\vec{F}_m = q\vec{v} \times \vec{B}$$

2.



$$F = qvB\sin\alpha$$

$$\vec{v} \perp \vec{B}$$

$$\alpha = 90^\circ$$

$$F_{\max} = qvB$$

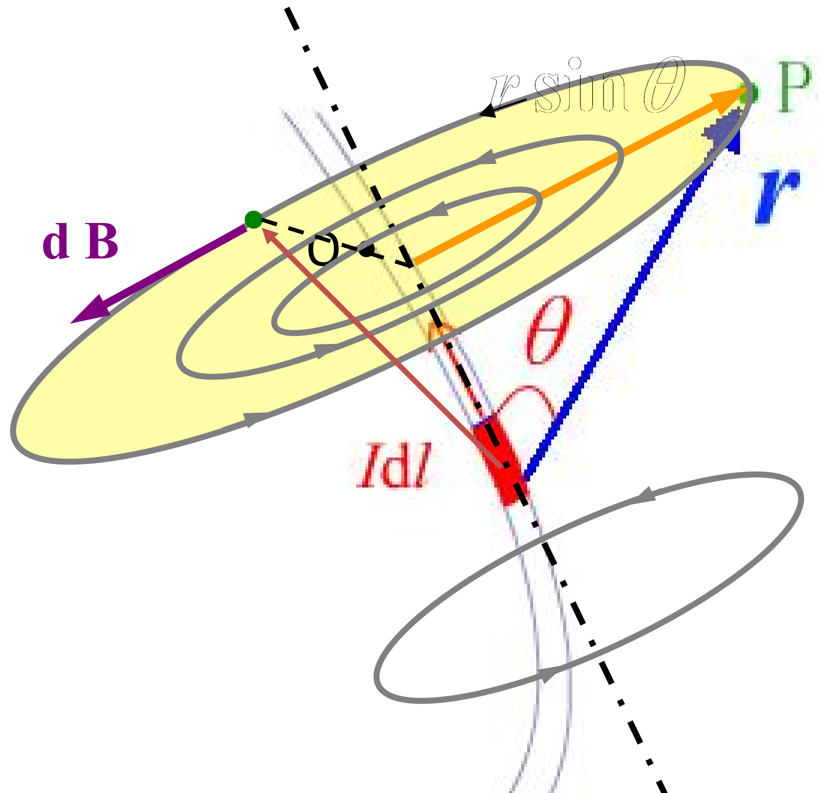
§ 7.3

Biot-Saval law

7.3.1 Biot-Saval Law

Biot-Saval's law is the relationship between a current element $I\mathrm{d}\vec{l}$ and the magnetic induction strength $\mathrm{d}\vec{B}$ it stimulates

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l} \times \vec{e}_r}{r^2}$$



permeability of vacuum

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N}\cdot\mathrm{A}^{-2}$$

big or small:

$$\mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}l\sin\theta}{r^2}$$

Note: The magnetic sensor line of the current element is the concentric circle of the circle on the axis of the current element.

The winding and current satisfy the right-hand rule

The magnetic field of any current (current carrier wire) can be regarded as the superposition of the magnetic field generated by infinite multiple current elements, i. e

$$\vec{B} = \int \mathrm{d}\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\mathrm{d}\vec{l} \times \vec{e}_r}{r^2}$$

The magnetic field satisfies the superposition principle

7.3.2 Application of the Biot-Saval Law

basic step

(1) The current is decomposed into countless current elements $I\mathrm{d}\vec{l}$

(2) by the current element $\mathrm{d}\vec{B}$ (according to the law)

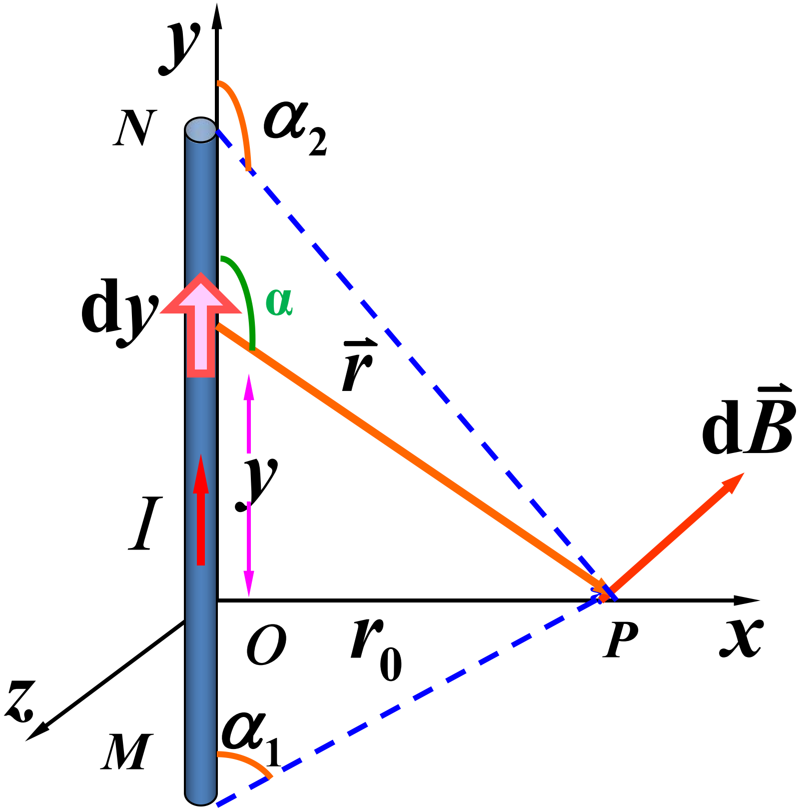
(3) The $\mathrm{d}\vec{B}$ is decomposed in the coordinate system, and the symmetry analysis is done by the principle of magnetic field superposition to simplify the calculation steps

(4) integrate over $\mathrm{d}\vec{B}$ for $\vec{B} = \mathrm{d}\vec{B}$

$$B_x = \int_L \mathrm{d}B_x,\quad B_y = \int_L \mathrm{d}B_y,\quad B_z = \int_L \mathrm{d}B_z$$

Vector synthesis: $\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$

Magnetic field generated by the carrier straight wire



$\mathrm{d}\vec{B}$ Both in the negative direction along the z-axis

separate: $$\mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}y\sin\alpha}{r^2}$$

$$\boxed{y = -r_0\cot\alpha,\ r = r_0/\sin\alpha}$$

$$\mathrm{d}y = r_0\mathrm{d}\alpha/\sin^2\alpha$$

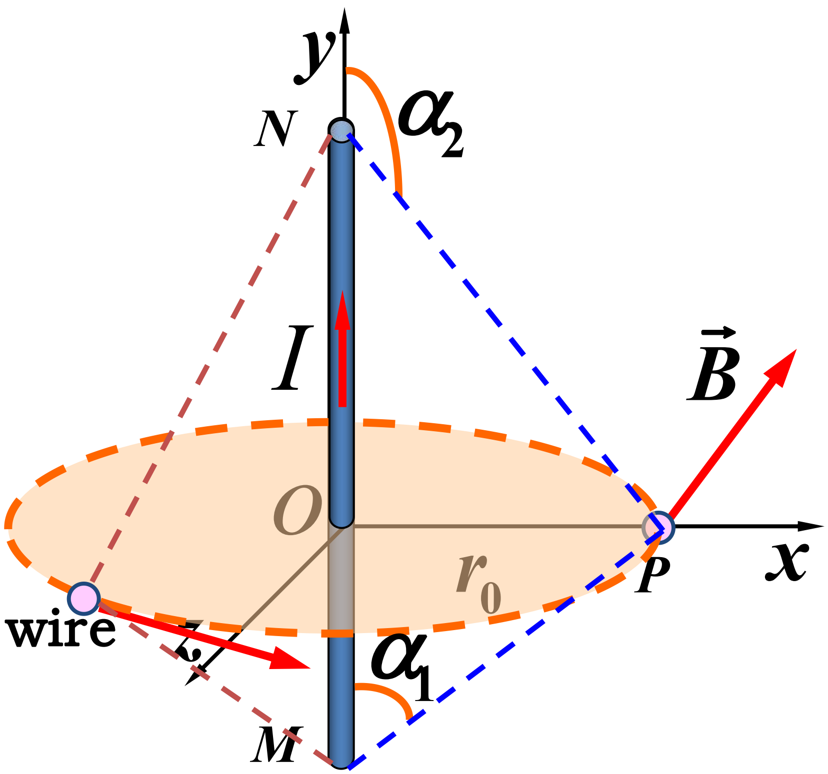
$$\mathrm{d}B = \frac{\mu_0 I}{4\pi r_0}\sin\alpha\,\mathrm{d}\alpha$$

$$B = \int \mathrm{d}B = \frac{\mu_0 I}{4\pi r_0}\int_{\alpha_1}^{\alpha_2}\sin\alpha\,\mathrm{d}\alpha$$

$$= \frac{\mu_0 I}{4\pi r_0}(\cos\alpha_1 - \cos\alpha_2)$$

$$\boxed{B = \frac{\mu_0 I}{4\pi r_0}(\cos\alpha_1 - \cos\alpha_2)}$$

Direction: Current and magnetic induction intensity into the right-handed spiral rule



◆ **The magnetic field of infinite long load and long straight wire**

$$\theta_1 \to 0 \qquad \theta_2 \to \pi \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

◆ One end of the half-infinite long load wire

$$\theta_1 \to \frac{\pi}{2} \qquad \theta_2 \to \pi \quad \Rightarrow \quad B = \frac{\mu_0 I}{4\pi r}$$

- The magnetic field generated by the current element

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l} \times \vec{e}_r}{r^2}$$

- Magnetic field integration of arbitrary current (symmetry)

■ Reflux straight wire

$$\boxed{B = \frac{\mu_0 I}{4\pi r_0}(\cos\alpha_1 - \cos\alpha_2)}$$

Infinite long, half infinite long

■ Circular current (special position (center), N turn, arc current)

■ solenoid

§ 7.4

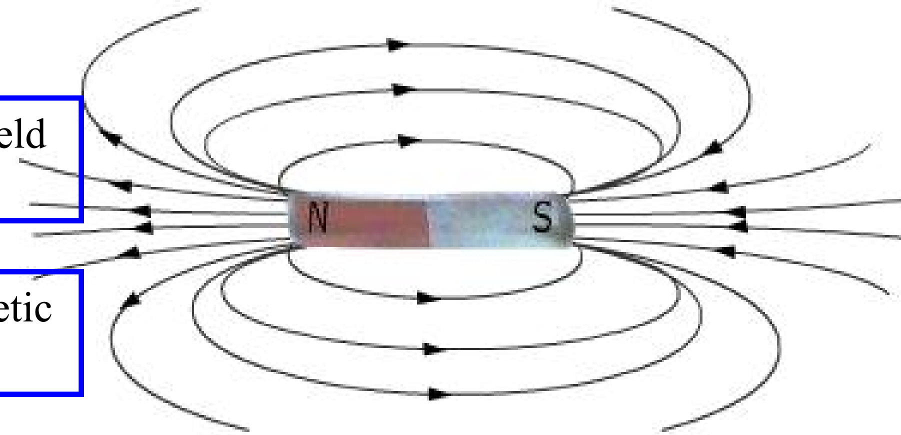
Gaussian theorem sum of magnetic fields Ampere circuital theorem

7.4.1 The Gaussian theorem for the magnetic field

1. Magnetic line, magnetic flux

◆ Magnetic induction line: the tangent direction of one point of the line is consistent with the direction of the magnetic field of the point.

Magnetic induction lines are dense	→	Magnetic field is strong
Magnetic induction lines are sparse	→	Weak magnetic field

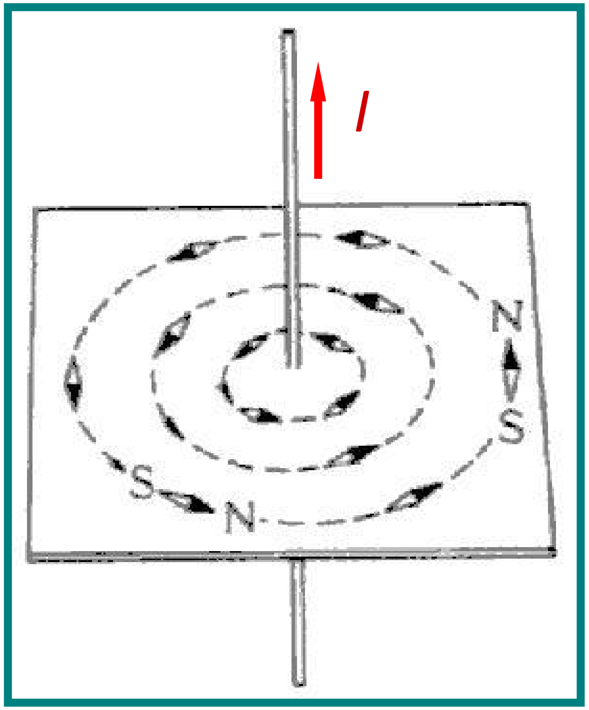


Magnetic induction line diagram of the permanent magnet

◆ Properties of the magnetic induction wires

1. The magnetic sensor line is a closed curve

2. The magnetic sensor line is connected to the current sleeve into a right-hand spiral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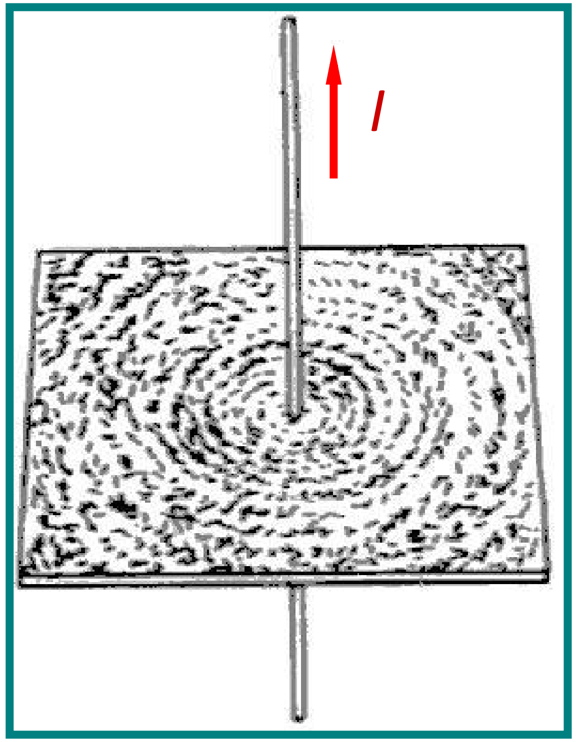
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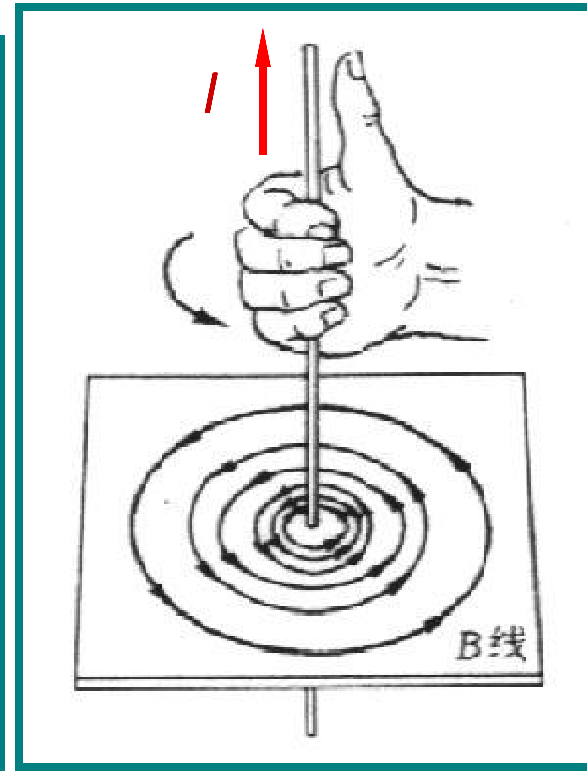
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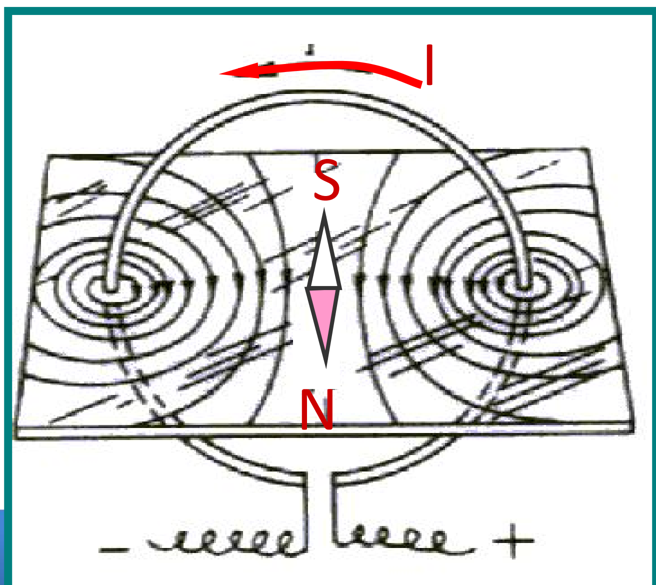
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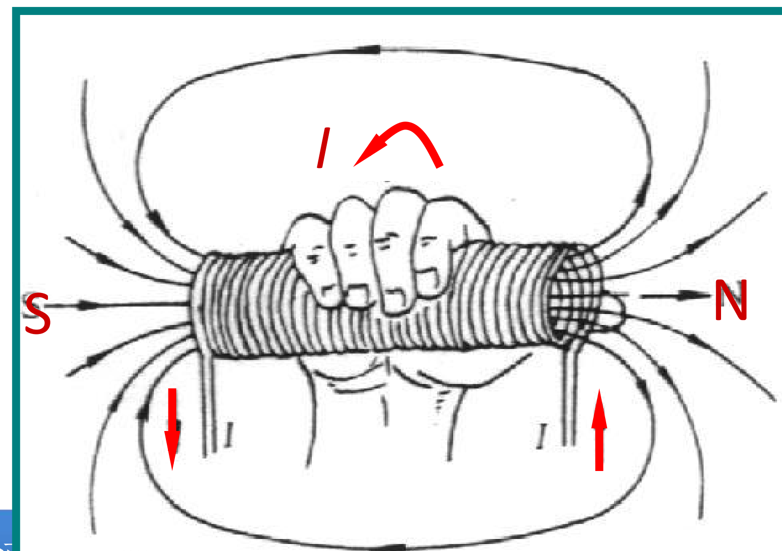
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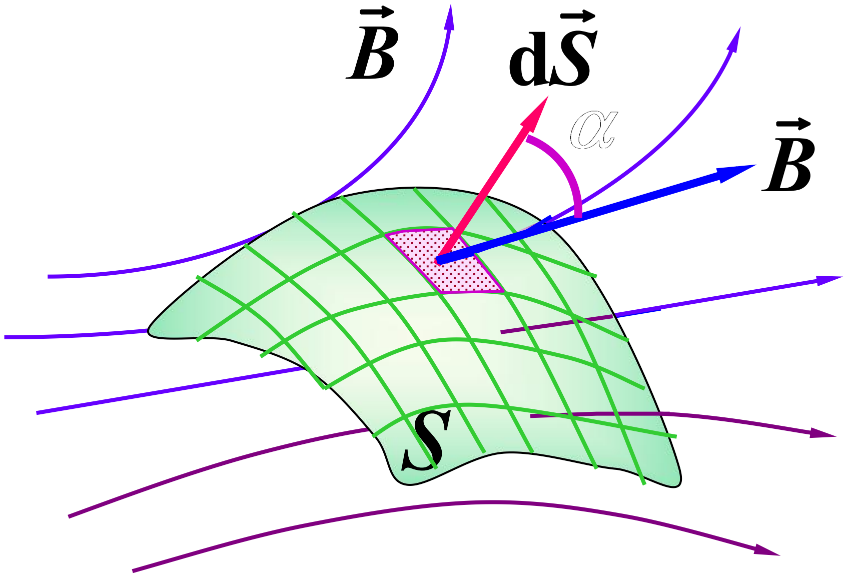
◆ magnetic flux

The magnetic flux through a finite surface with area S in a magnetic field is the number of magnetic lines passing through that area

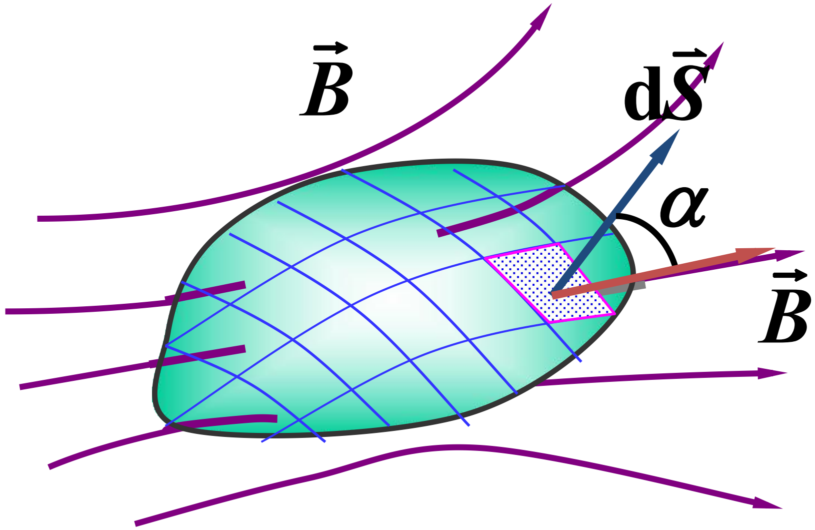
$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$

Unit: Wei [Bo], Wb



For a closed surface

$$\Phi = \oint_S \vec{B} \cdot d\vec{S}$$



2. The Gaussian theorem for the magnetic field

The magnetic sensor line of the current element is closed

According to the principle of magnetic field superposition, the magnetic sensor lines are all closed curves

Gaussian law of magnetic field (magnetic flux continuity theorem): the magnetic flux in any magnetic field passing through any closed surface is always equal to zero.

$$\oint_S \vec{B} \cdot \mathrm{d}\vec{S} = 0$$

The magnetic field is a passive field

The magnetic monopole (magnetic charge) does not exist

$$\Phi_{\mathrm{e}} = \oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i \qquad \oint_S \vec{D} \cdot \mathrm{d}\vec{S} = \sum_i q_{0i}$$

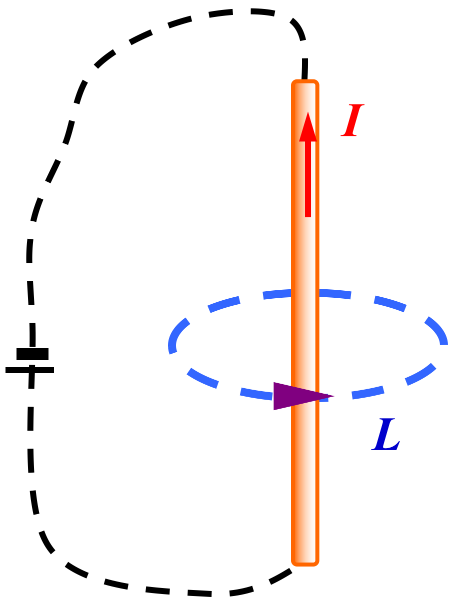
7.4.2 Ampere loop theorem of magnetic field

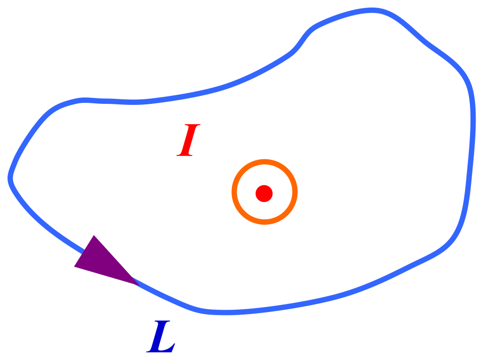
Take the magnetic field generated by the infinite long load straight wire for example

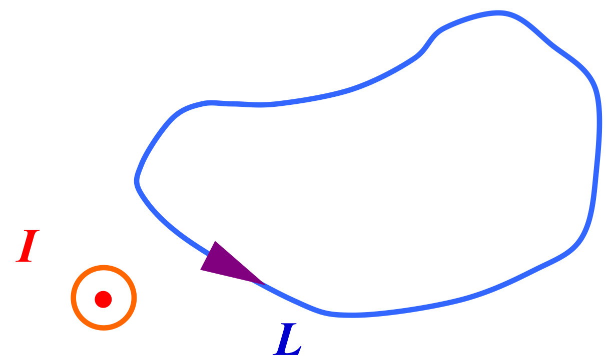
(1) Take the symmetrical loop surrounding the current

(2) Take any loop surrounding the current

(3) Take any loop that does not surround the current







Note: The loop is in the plane perpendicular to the wire

(1) The closed loop L shall be a circular loop, and the long straight conductor is located in its center

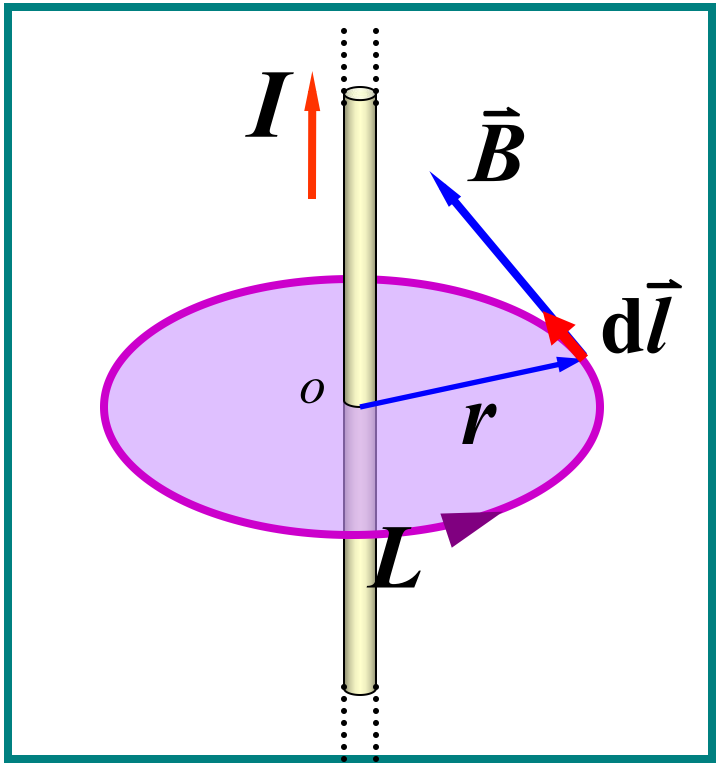
$$B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B}\cdot \mathrm{d}\vec{l} = B\mathrm{d}l\cos 0 = \frac{\mu_0 I}{2\pi r}\mathrm{d}l$$

$$\oint_L \vec{B}\cdot \mathrm{d}\vec{l} = \frac{\mu_0 I}{2\pi r}\oint_L \mathrm{d}l$$

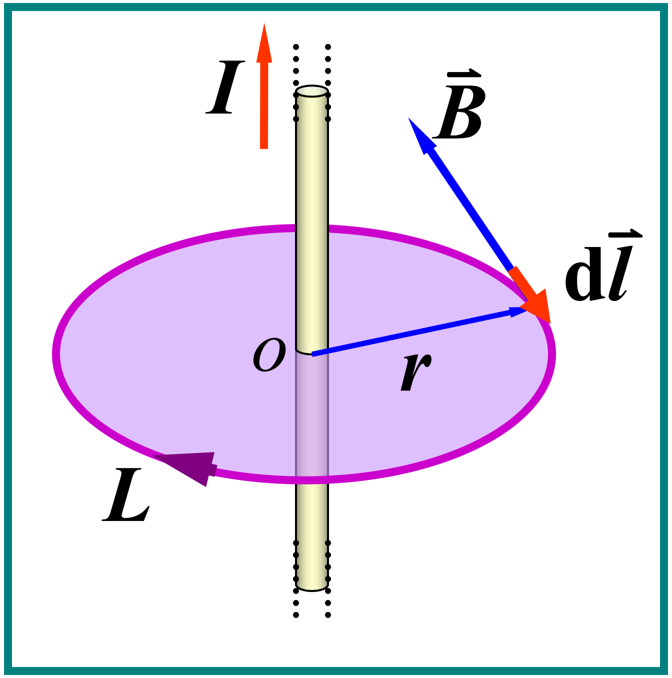
$$\oint_L \mathrm{d}l = 2\pi r$$

$$\Rightarrow \oint_L \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 I$$



Let L it form the right helix I

Note: B The circulation along this circular loop is related only related to the current I surrounded by the closed loop, not to the size of the loop.

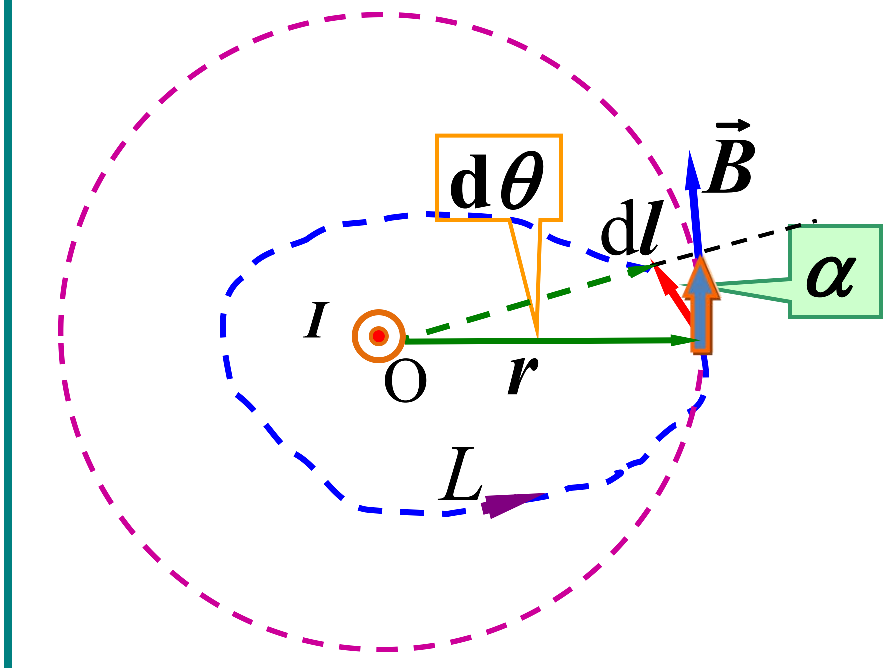


If the loop direction becomes counterclockwise, then

$$\oint_L \vec{B}\cdot\mathrm{d}\vec{l} = -\frac{\mu_0 I}{2\pi}\int_0^{2\pi}\mathrm{d}\varphi = -\mu_0 I$$

$$= \mu_0(-I)$$

(2) The loop L of any shape of the surrounding current



L forms a right helix with I

$$\vec{B}\cdot d\vec{l} = B\,dl\cos\alpha$$

$$dl\cos\alpha = r\,d\theta$$

$$\vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi r} r\,d\theta = \frac{\mu_0 I}{2\pi} d\theta$$

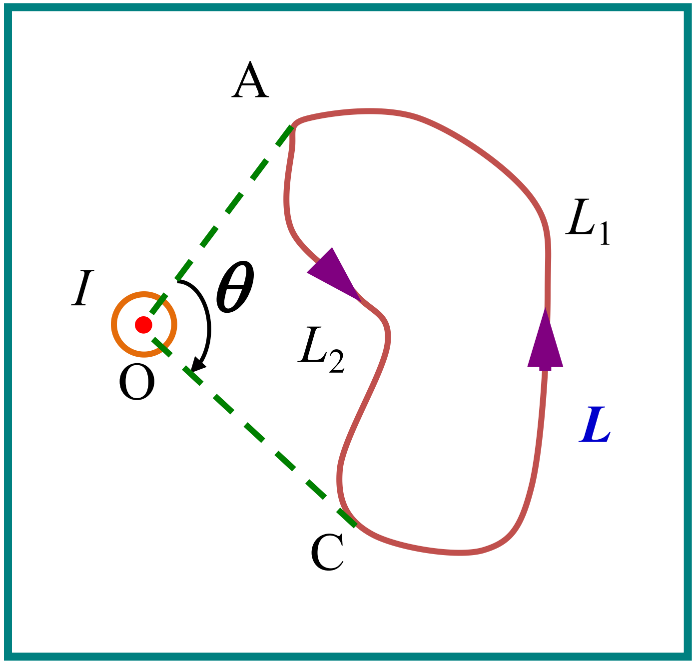
$$\boxed{\oint_L \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\oint_L d\theta}$$

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Note: The circulation value of B is independent of the size and shape of the loop.

(3) Take any loop that does not surround the current



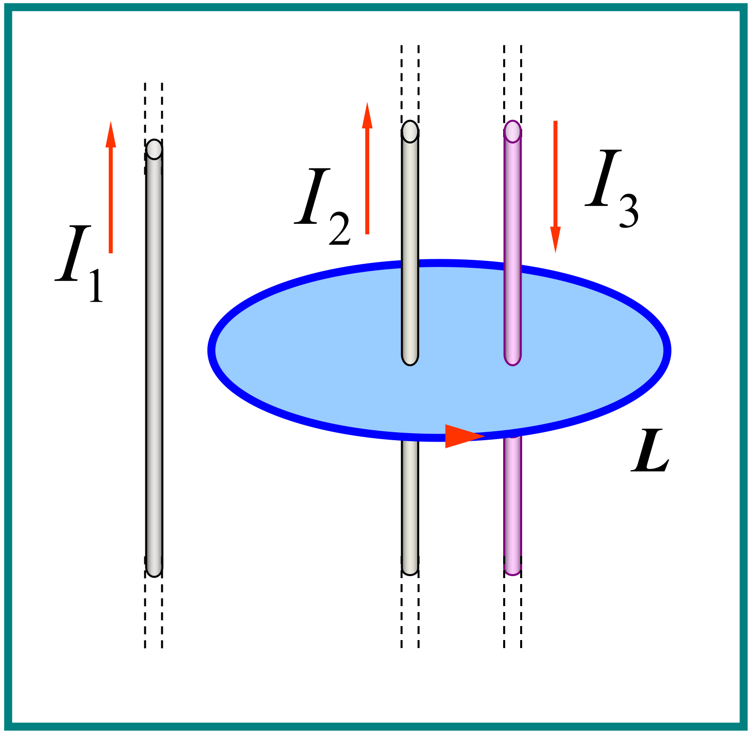
$$\oint_L \vec{B}\cdot d\vec{l} = \int_{L_1} \vec{B}\cdot d\vec{l} + \int_{L_2} \vec{B}\cdot d\vec{l}$$

$$= \frac{\mu_0 I}{2\pi}\left(\int_{L_1} d\theta + \int_{L_2} d\theta\right)$$

$$= \frac{\mu_0 I}{2\pi}[\theta - \theta] = 0$$

Note: When the closed path L does not surround the current, this current does not contribute to the B loop along this closed path.

(4) Multiple current situation



$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\oint_L \vec{B} \cdot \mathrm{d}\vec{l} = \oint_L \vec{B}_1 \cdot \mathrm{d}\vec{l} + \oint_L \vec{B}_2 \cdot \mathrm{d}\vec{l} + \oint_L \vec{B}_3 \cdot \mathrm{d}\vec{l}$$

$$= \mu_0 (I_2 - I_3)$$

Results for any shape of closed current (extended current) of any loop

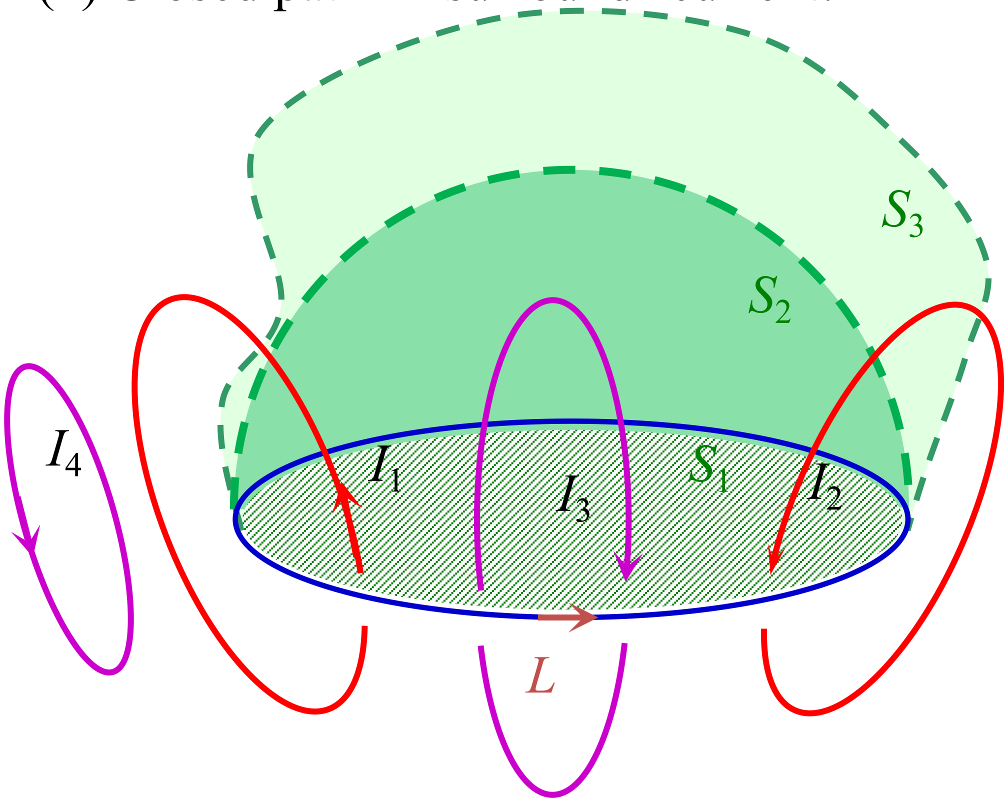
Ampere circuital theorem $\boxed{\oint_L \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \sum I_{\mathrm{int}}}$ $\boxed{\oint_L \vec{E} \cdot \mathrm{d}\vec{l} = 0}$

A few attention

$$\oint_L \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \sum I_{\text{int}}$$

(1) The current should be a closed constant current.

(2) Closed path L "surround" current:



Only the current hinged with L is counted as the current surrounded by L.

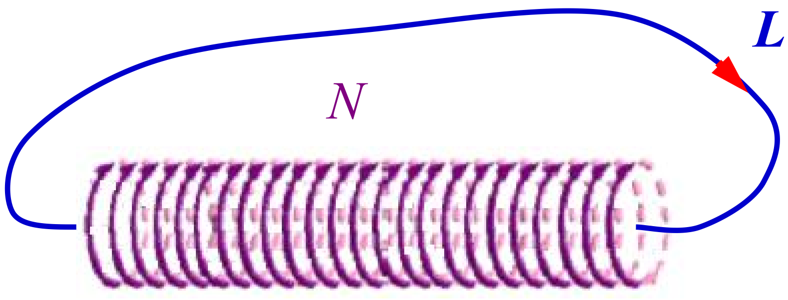
only I_1、I_2Surrounded by a loop L, and I_1For positive, I_2For negative.

A few attention

(3) The current I is positive and negative: when I and L form the right helix, I is positive; otherwise I is negative.

(4) If the current loop is helical, and the integral loop L and N turn current hinge, then

$$\boxed{\oint_L \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 NI}$$

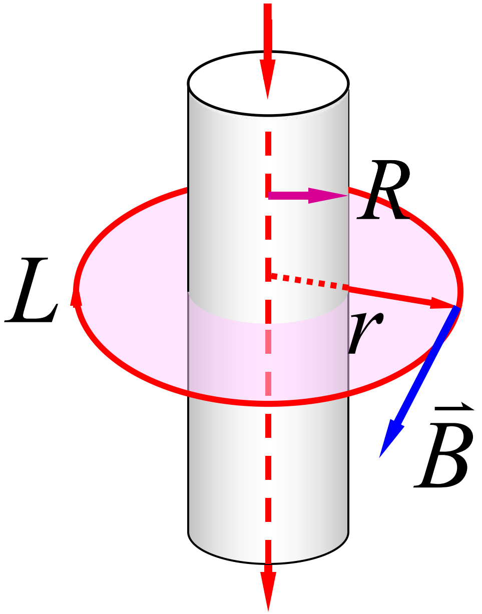


(5) The ampere loop theorem shows

A constant magnetic field is a rotating field, that is, the current stimulates the magnetic field in the way of vortex, where there is a current, there must be a closed magnetic sensor line around it.

7.4.3 Application of the ampere loop theorem

1. The magnetic field inside and outside the cylinder



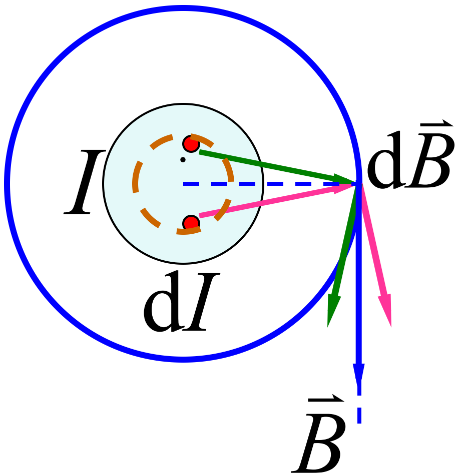
Solution: (1) symmetry analysis (2) Select the circuit

$$r > R \qquad \oint_L \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

$$2\pi r B = \mu_0 I \qquad \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

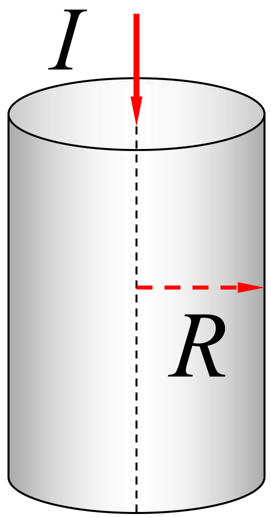
$$0 < r < R \qquad \oint_L \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \frac{\pi r^2}{\pi R^2} I$$

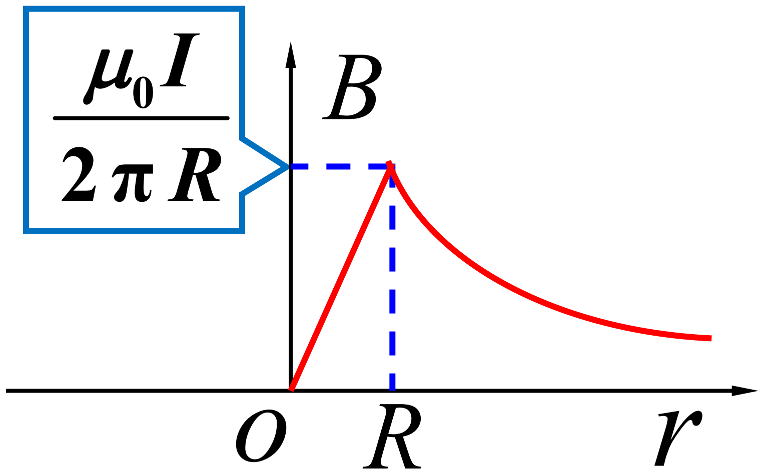
$$2\pi r B = \frac{\mu_0 r^2}{R^2} I \qquad \boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$



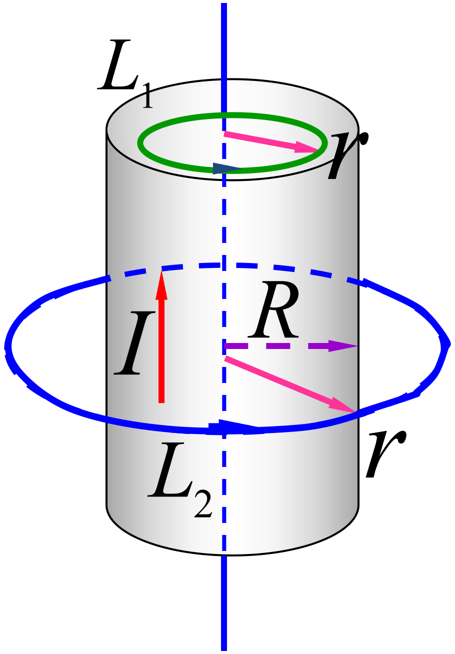
$$\begin{cases} 0 < r < R, & B = \dfrac{\mu_0 I r}{2\pi R^2} \\ r > R, & B = \dfrac{\mu_0 I}{2\pi r} \end{cases}$$

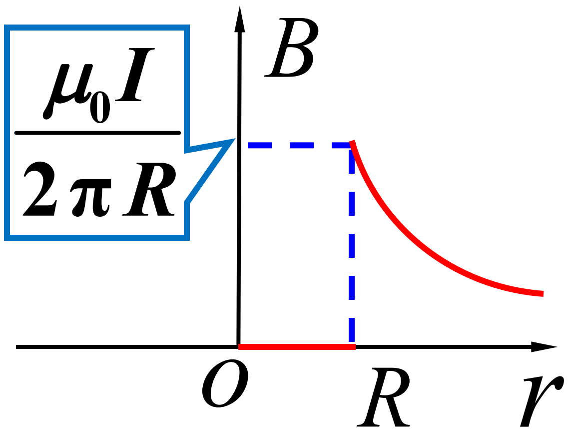
Of the direction $\vec{B}$ of the same with I , into the right helix





Thinking: the magnetic field of the infinite long carrier flow cylinder surface



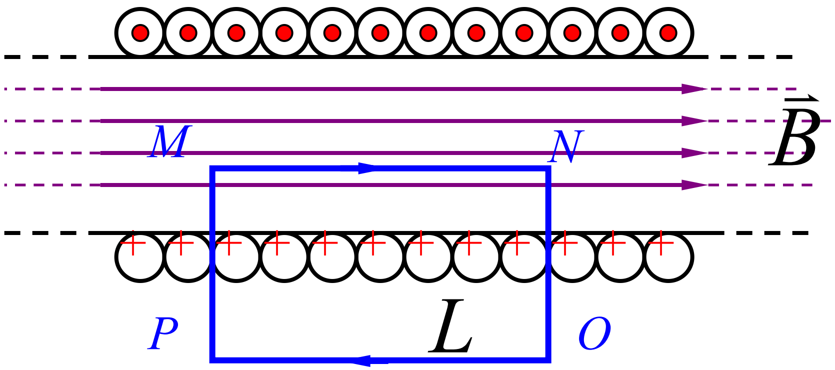


separate: $0 < r < R, \quad \oint_{L_1} \vec{B} \cdot \mathrm{d}\vec{l} = 0 \qquad B = 0$

$r > R, \quad \oint_{L_2} \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I \qquad B = \frac{\mu_0 I}{2\pi r}$

2. The magnetic field inside the infinite length of the straight solenoid

Solution: (1) symmetry analysis in the spiral tube is a uniform field, the direction along the axis, the external magnetic sensor strength tends to zero, that is.



$$B \cong 0$$

(2). Select the circuit. L

The direction of the magnetic field $\vec{B}$ is in the right spiral with the current. I

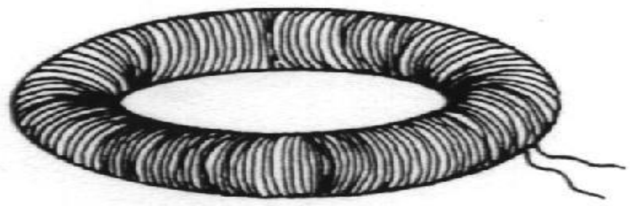
$$\oint_l \vec{B} \cdot \mathrm{d}\vec{l} = \int_{MN} \vec{B} \cdot \mathrm{d}\vec{l} + \cancel{\int_{NO} \vec{B} \cdot \mathrm{d}\vec{l}} + \cancel{\int_{OP} \vec{B} \cdot \mathrm{d}\vec{l}} + \cancel{\int_{PM} \vec{B} \cdot \mathrm{d}\vec{l}}$$

$$B \cdot \overline{MN} = \mu_0 n \overline{MN} I \qquad \boxed{B = \mu_0 n I}$$

Infinite long load solenoid internal magnetic field is equal everywhere, external, magnetic field is zero.

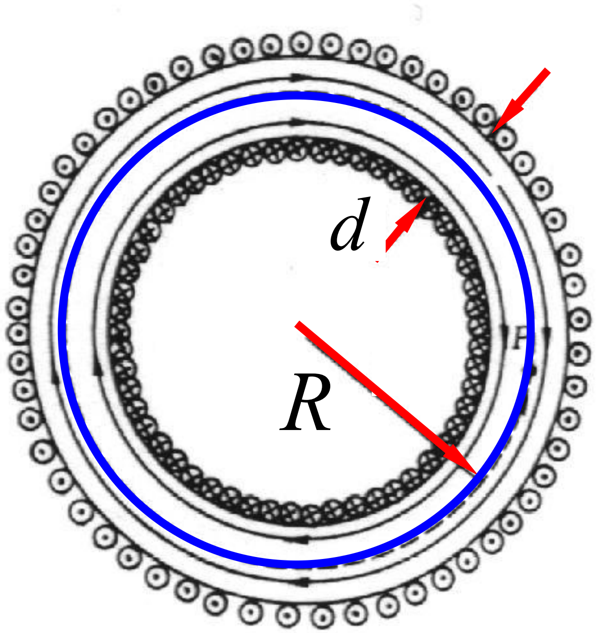
3. The magnetic field inside the loop of the coiled screw

Solution: (1) Symmetry analysis; inside the ring, the lines are concentric circles $\vec{B}$ and zero outside the ring. $\vec{B}$



(2). Select the circuit.

$$\oint_L \vec{B}\cdot \mathrm{d}\vec{l} = 2\pi RB = \mu_0 NI$$



$$B = \frac{\mu_0 NI}{2\pi R}$$

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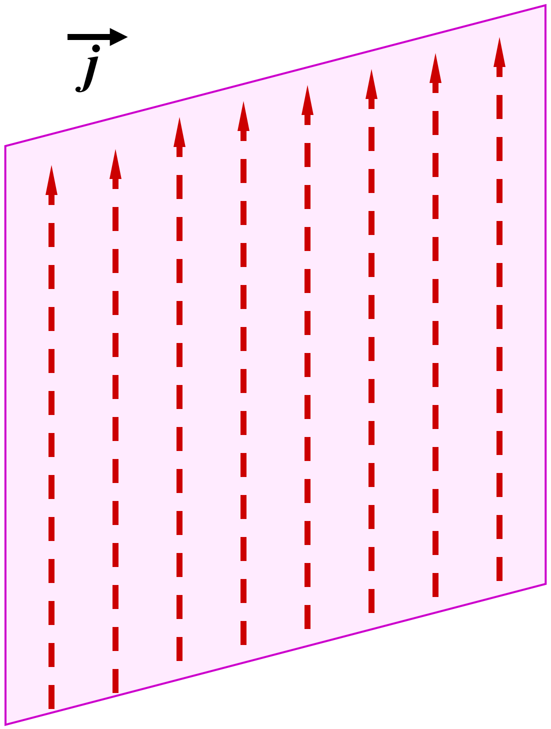
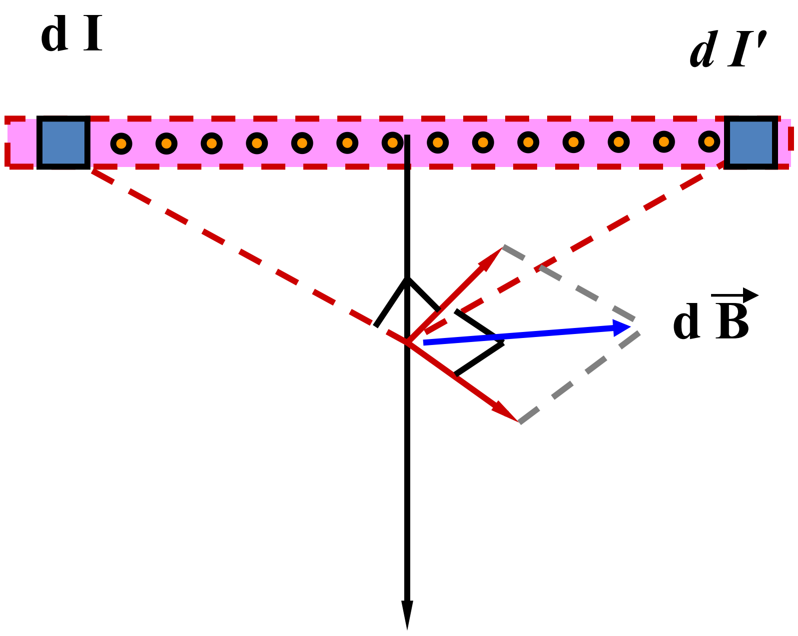
$$L = 2\pi R \qquad B = \mu_0 NI/L$$

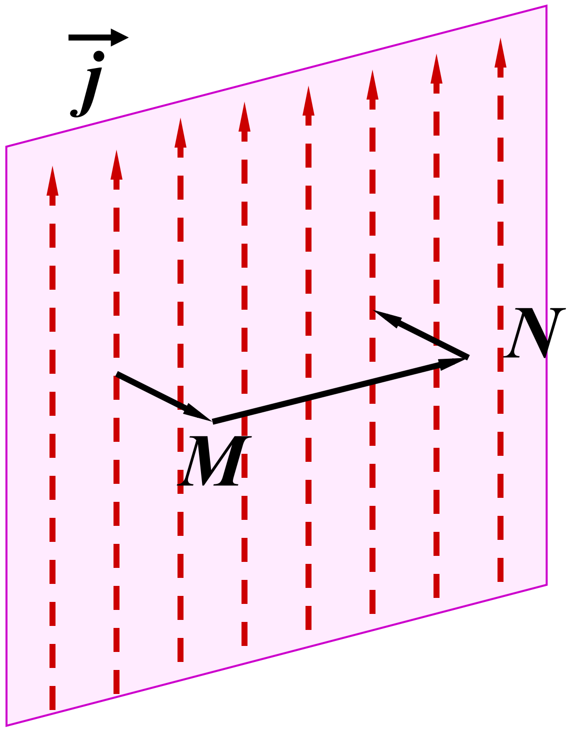
At that time, $2R >> d$ the screw ring can be regarded as a uniform field.

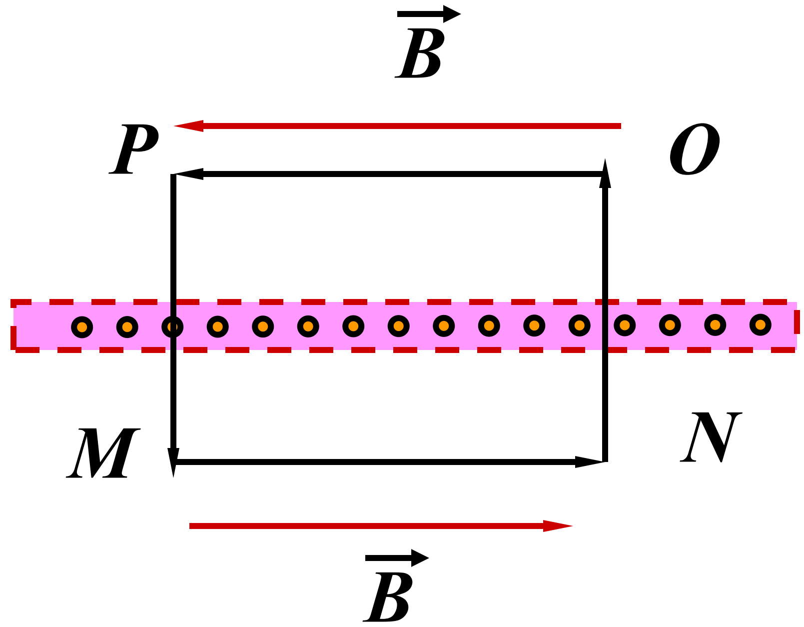
4. Magnetic field in the plane of an infinite flow load





separate:





$$\oint_L \vec{B}\cdot d\vec{l} = \int_{MN}\vec{B}\cdot d\vec{l} + \int_{NO}\cancel{\vec{B}\cdot d\vec{l}} + \int_{OP}\vec{B}\cdot d\vec{l} + \int_{PM}\cancel{\vec{B}\cdot d\vec{l}}$$

$$2Bl = \mu_0 jl$$

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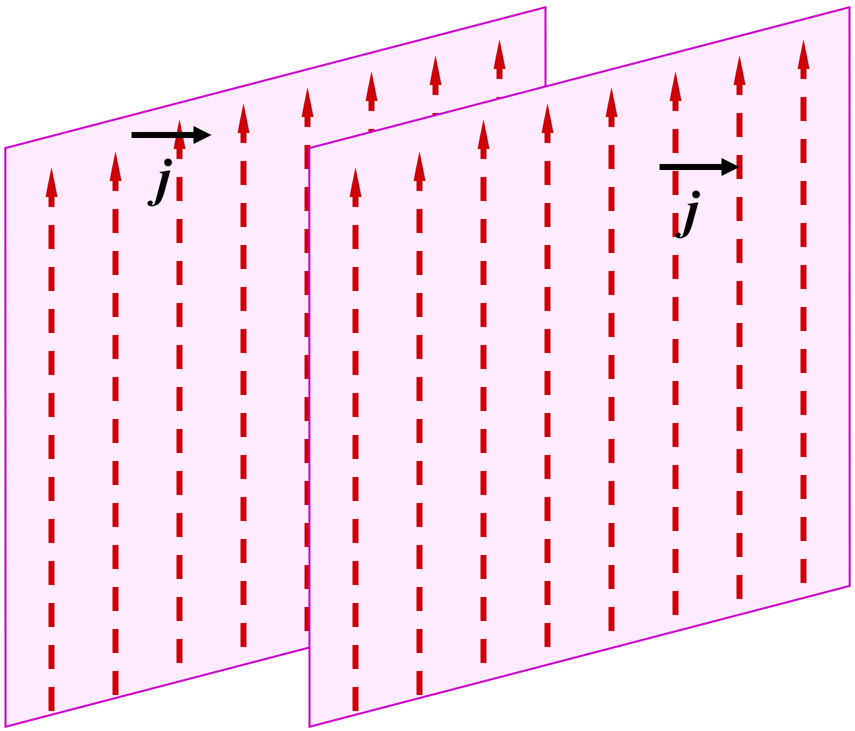
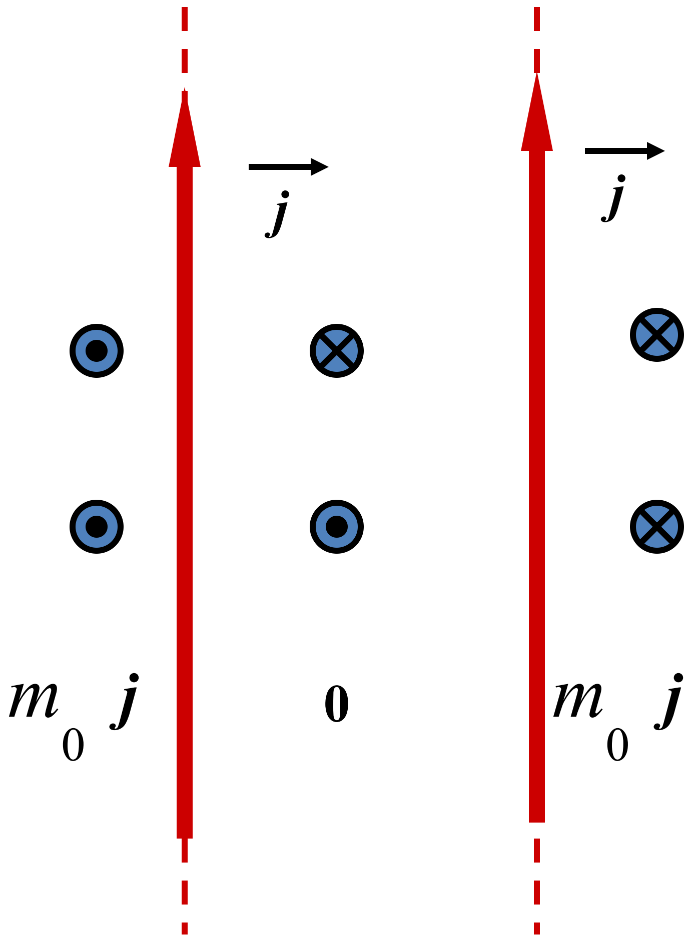
$$\boxed{B = \frac{1}{2}\mu_0 j}$$

Thinking: Magmagnetic induction intensity on both sides of two infinite conductor plates

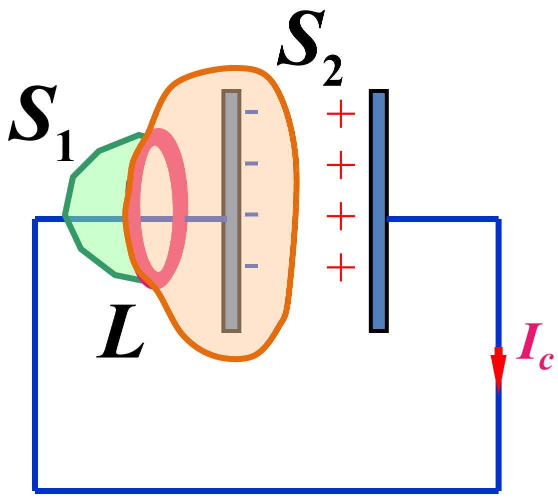




7.4.4 Shift current and full current

In a constant magnetic field, the ampere loop theorem

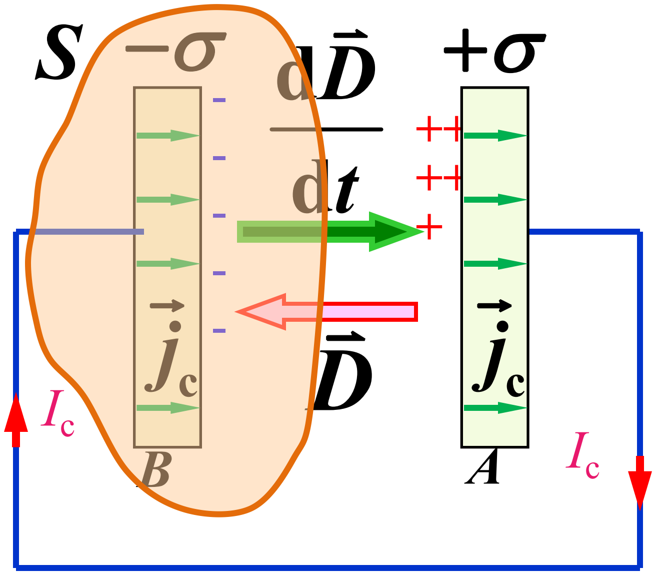
$$\oint_L \vec{H}\cdot d\vec{l} = \sum I = \int_S \vec{j}\cdot d\vec{S}$$



(Make any surface S with L as the edge)

$$\oint_L \vec{H}\cdot d\vec{l} = \int_{S_1} \vec{j}_c\cdot d\vec{S} = I_c$$

$$\oint_L \vec{H}\cdot d\vec{l} = \int_{S_2} \vec{j}_c\cdot d\vec{S} = 0$$



Maxwell hypothesized that a changing electric field is a current, called a displacement current, which "continues" the current of the whole circuit.

♦ **Displacement current density**

$$\vec{j}_d = \frac{\partial \vec{D}}{\partial t}$$

$$I_d = \int_S \vec{j}_d \cdot d\vec{S} = \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

The current continuity equation

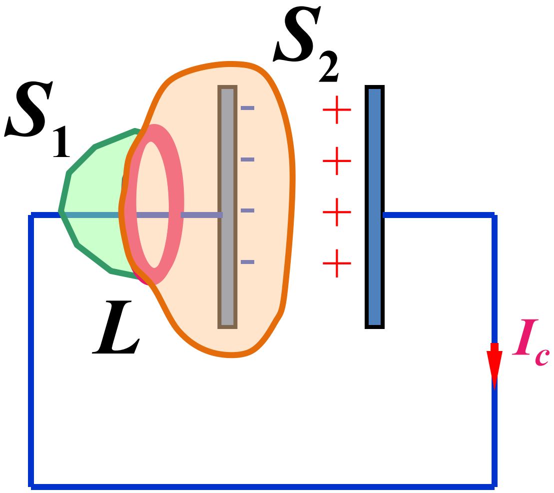
The Gaussian theorem of D

$$\frac{d}{dt}\oint_S \vec{D} \cdot d\vec{S} + \oint_S \vec{j}_c \cdot d\vec{S} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\oint_S \vec{D}\cdot \mathrm{d}\vec{S} + \oint_S \vec{j}_\mathrm{c}\cdot \mathrm{d}\vec{S} = 0$$

$$\oint_S \left(\frac{\partial \vec{D}}{\partial t} + \vec{j}_\mathrm{c}\right)\cdot \mathrm{d}\vec{S} = 0$$

$$\oint_S (\vec{j}_\mathrm{d} + \vec{j}_\mathrm{c})\cdot \mathrm{d}\vec{S} = 0$$



$$S_1 + S_2 = S$$

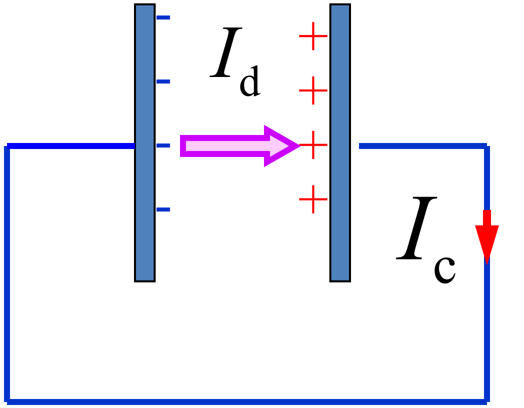
event

$$\int_{S_1} (\vec{j}_\mathrm{d} + \vec{j}_\mathrm{c})\cdot \mathrm{d}\vec{S}$$

$$= \int_{S_2} (\vec{j}_\mathrm{d} + \vec{j}_\mathrm{c})\cdot \mathrm{d}\vec{S} = I_\mathrm{d} + I_\mathrm{c}$$

That is, it is continuous

$$\int (\vec{j}_\mathrm{d} + \vec{j}_\mathrm{c})\cdot \mathrm{d}\vec{S}$$



♦ **total current** $\boldsymbol{I_t = I_c + I_d}$

1) Full current is continuous;

2) The displacement current stimulates the magnetic field just like the conduction current;

3) The conduction current produces joule heat, and the displacement current does not produce joule heat.

The universal ampere loop theorem

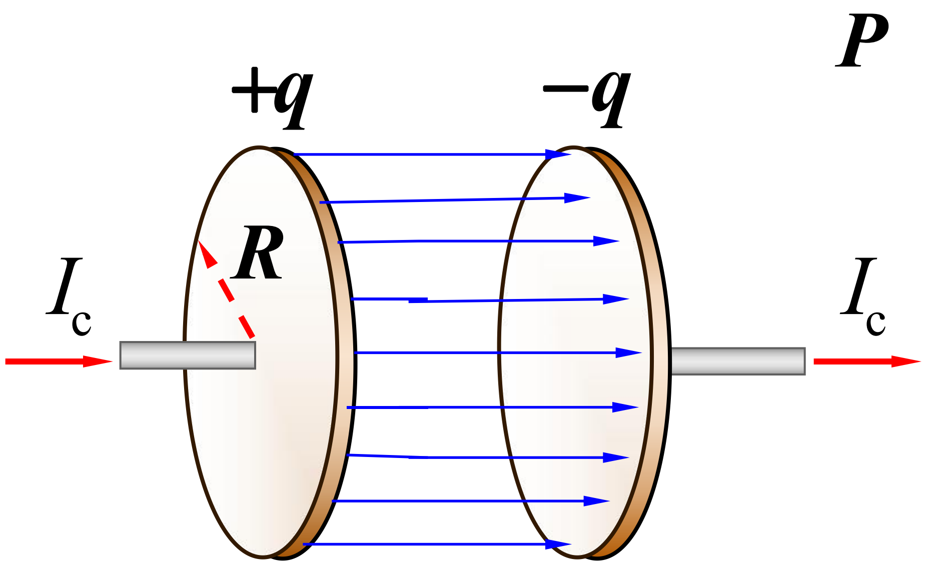
$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I_t = \mu_0 \int_s (\vec{j}_c + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{S}$$

Example 1 has a circular parallel plate capacitor, $R = 0.3\,\mathrm{m}$ Now charge it, so that the conduction current on the circuit, $I_c = \mathrm{d}Q/\mathrm{d}t = 5\,\mathrm{A}$ if omit the edge effect, seek: the distance between the two plates axis $r_1 = 0.2\,\mathrm{m}$ and $r_2 = 0.4\,\mathrm{m}$ the magnetic induction intensity of the two.

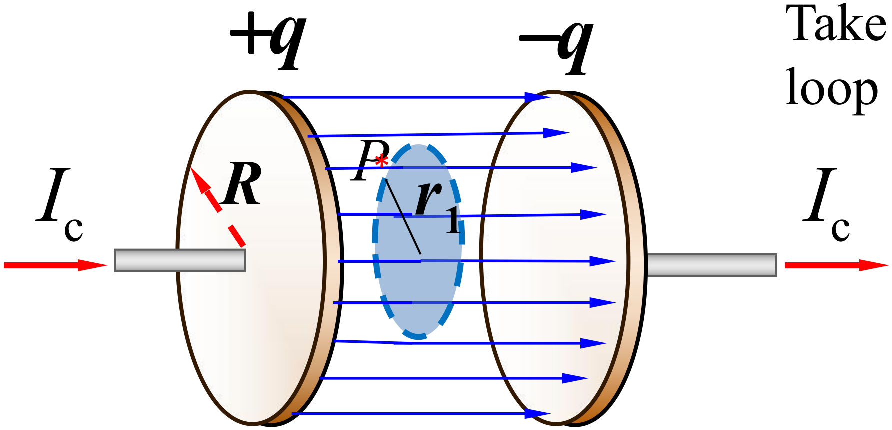
Solution: The size of the electric displacement vector between the two plates is

$$D = \varepsilon_0 E = \sigma = \frac{q}{\pi R^2}$$

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{1}{\pi R^2}\frac{\mathrm{d}q}{\mathrm{d}t} = \frac{I_c}{\pi R^2}$$



The electric field is axisymmetric, so the magnetic field is also axisymmetrically distributed.



Take the radius of $r_1 = 0.2\,\mathrm{m}$ the circular loop L_1

$$\oint_{L_1} \vec{B}_1 \cdot \mathrm{d}\vec{l} = 2\pi r_1 B_1$$

but

$$\int_{S_1} \frac{\mathrm{d}\vec{D}}{\mathrm{d}t} \cdot \mathrm{d}\vec{S} = \int_{S_1} \frac{\mathrm{d}D}{\mathrm{d}t} \mathrm{d}S = \pi r_1^2 \frac{\mathrm{d}D}{\mathrm{d}t} = \frac{r_1^2 I_c}{R^2}$$

By the universal ampere loop theorem

$$2\pi r_1 B_1 = \mu_0 \frac{r_1^2 I_c}{R^2}$$

event $$B_1 = \mu_0 \frac{r_1 I_c}{2\pi R^2}$$ **Regeneration of data** $$B_1 = 2.2 \times 10^{-6}\ \mathrm{T}$$

Take the radius of $r_2 = 0.4\,\mathrm{m}$the circular loop L2

$$\int_{S_2} \frac{\mathrm{d}\vec{D}}{\mathrm{d}t} \cdot \mathrm{d}\vec{S} = \int_{S_2} \frac{\mathrm{d}D}{\mathrm{d}t} \mathrm{d}S = \pi R^2 \frac{\mathrm{d}D}{\mathrm{d}t} = I_\mathrm{c}$$

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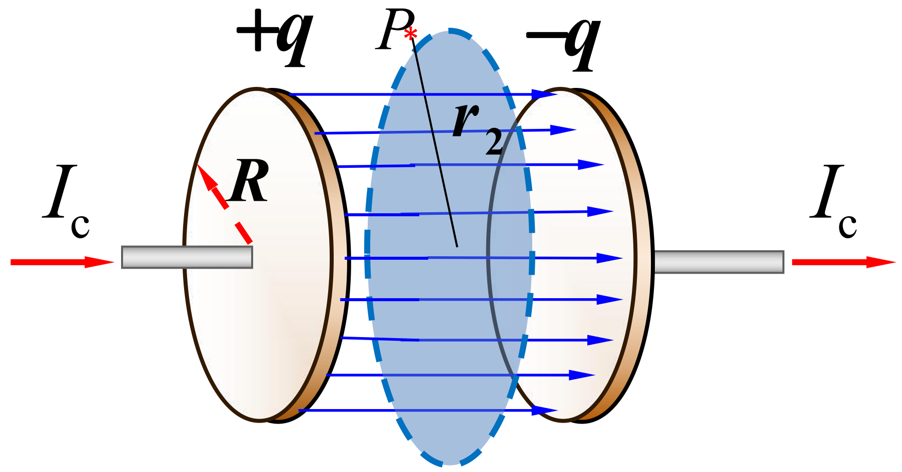
$$2\pi r_2 B_2 = \mu_0 I_\mathrm{c}$$

$$B_2 = \mu_0 \frac{I_\mathrm{c}}{2\pi r_2} = 2.5 \times 10^{-6}\ \mathrm{T}$$



§ 7.5

Ampere force and Lorentz force

7.5.1 Amperometric force

1. Ampere's law

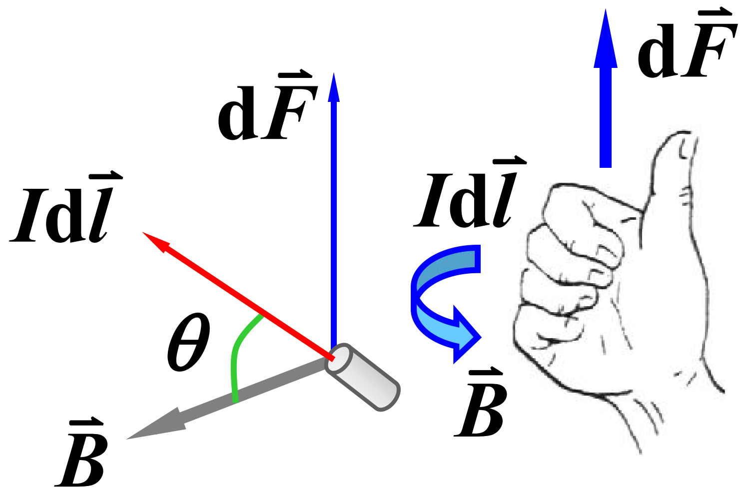
The force of the magnetic field on the carrier wire —— ampere force

The amperometric force of a current $I\mathrm{d}\vec{l}$ element is

$$\mathrm{d}F = I\mathrm{d}\vec{l} \times \vec{B}$$ **Ampere's law**

The amperometric force of any shaped carrier conductor in any magnetic field is

$$\vec{F} = \int_L I\mathrm{d}\vec{l} \times \vec{B}$$



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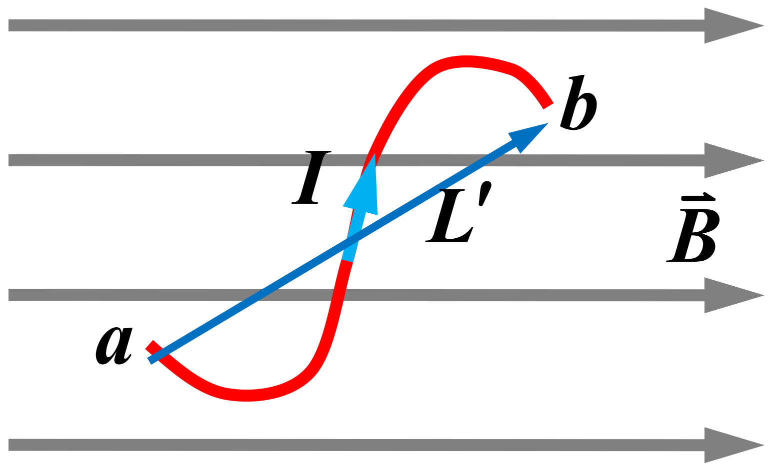
1. A current carrier conductor of any shape in a uniform magnetic field

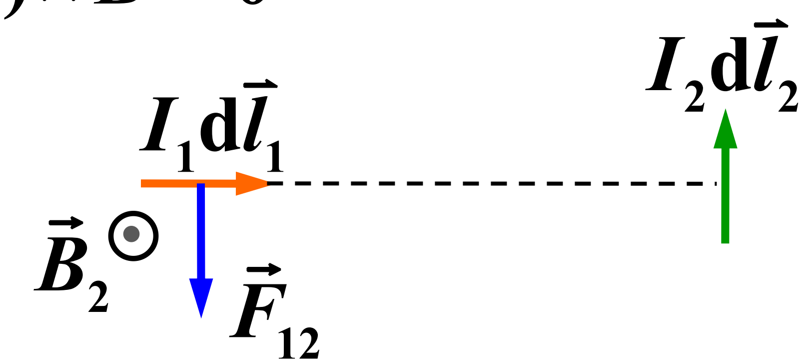


$$\vec{F} = \int_a^b I\mathrm{d}\vec{l} \times \vec{B} = I\left(\int_a^b \mathrm{d}\vec{l}\right) \times \vec{B}$$
$$= I\vec{L}' \times \vec{B}$$

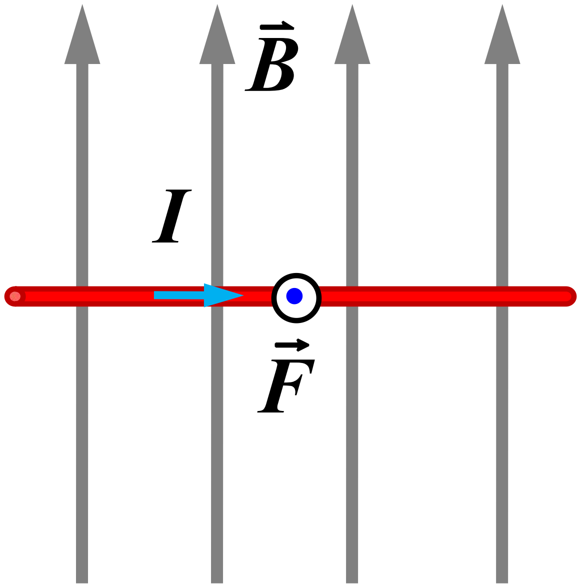
2. A closed flow carrier coil of any shape in a uniform magnetic field

$$\vec{F} = \oint_L I\mathrm{d}\vec{l} \times \vec{B} = I\left(\oint_L \mathrm{d}\vec{l}\right) \times \vec{B} = 0$$

3. The amperometric force between two isolated current cells does not satisfy Newton's third law



3. Uniform magnetic field and straight wire are vertical to each other



$$F = ILB$$