Section 1.1

12. **a)** If you have the flu, then you miss the final exam.

b) You do not miss the final exam if and only if you pass the course.

c) If you miss the final exam, then you do not pass the course.

d) You have the flu, or miss the final exam, or pass the course.

e) It is either the case that if you have the flu then you do not pass the course or the case that if you miss the final exam then you do not pass the course (or both, it is understood).

f) Either you have the flu and miss the final exam, or you do not miss the final exam and do pass the course.

14. **a)** $r \wedge \neg q$ **b)** $p \wedge q \wedge r$ **c)** $r \rightarrow p$ **d)** $p \wedge \neg q \wedge r$ **e)** $(p \wedge q) \rightarrow r$ **f)** $r \leftrightarrow (q \vee p)$

37. The techniques are the same as in Exercises 31–36, except that there are now three variables and therefore eight rows. For part **(a)**, we have

p	q	r	$\neg q$	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

For part **(b)**, we have

p	q	r	$\neg p$	$q \rightarrow r$	$\neg p \rightarrow (q \rightarrow r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Parts **(c)** and **(d)** we can combine into a single table.

p	q	r	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow r$	$(p \rightarrow q) \vee (\neg p \rightarrow r)$	$(p \rightarrow q) \wedge (\neg p \rightarrow r)$
T	T	T	T	F	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	F	T	T	F
T	F	F	F	F	T	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	F

For part **(e)** we have

p	q	r	$p \leftrightarrow q$	$\neg q$	$\neg q \leftrightarrow r$	$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Finally, for part **(f)** we have

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

Section 1.2

32. **a)** We look at the three possibilities of who the innocent men might be. If Smith and Jones are innocent (and therefore telling the truth), then we get an immediate contradiction, since Smith said that Jones was a friend of Cooper, but Jones said that he did not even know Cooper. If Jones and Williams are the innocent truth-tellers, then we again get a contradiction, since Jones says that he did not know Cooper and was out of town, but Williams says he saw Jones with Cooper (presumably in town, and presumably if we was with him, then he knew him). Therefore it must be the case that Smith and Williams are telling the truth. Their statements do not contradict each other. Based on Williams' statement, we know that Jones is lying, since he said that he did not know Cooper when in fact he was with him. Therefore Jones is the murderer.

b) This is just like part **(a)**, except that we are not told ahead of time that one of the men is guilty. Can none of them be guilty? If so, then they are all telling the truth, but this is impossible, because as we just saw, some of the statements are contradictory. Can more than one of them be guilty? If, for example, they are all guilty, then their statements give us no information. So that is certainly possible.

Section 1.3

7. De Morgan's laws tell us that to negate a conjunction we form the disjunction of the negations, and to negate a disjunction we form the conjunction of the negations.

a) This is the conjunction "Jan is rich, and Jan is happy." So the negation is "Jan is not rich, or Jan is not happy."

b) This is the disjunction "Carlos will bicycle tomorrow, or Carlos will run tomorrow." So the negation is "Carlos will not bicycle tomorrow, and Carlos will not run tomorrow." We could also render this as "Carlos will neither bicycle nor run tomorrow."

c) This is the disjunction "Mei walks to class, or Mei takes the bus to class." So the negation is "Mei does not walk to class, and Mei does not take the bus to class." (Maybe she gets a ride with a friend.) We could also render this as "Mei neither walks nor takes the bus to class."

d) This is the conjunction "Ibrahim is smart, and Ibrahim is hard working." So the negation is "Ibrahim is not smart, or Ibrahim is not hard working."

17. The proposition $\neg(p \leftrightarrow q)$ is true when p and q do not have the same truth values, which means that p and q have different truth values (either p is true and q is false, or vice versa). These are exactly the cases in which $p \leftrightarrow \neg q$ is true. Therefore these two expressions are true in exactly the same instances, and therefore are logically equivalent.

$$
\begin{aligned}
\neg(p \leftrightarrow q) &= \neg((p \to q) \land (q \to p)) \\
&= (\neg(p \to q)) \lor (\neg(q \to p)) \\
&= (\neg(\neg p \lor q)) \lor (\neg(\neg q \lor p)) \\
&= (p \land \neg q) \lor (q \land \neg p) \\
&= (\neg q \lor (q \land \neg p)) \land (p \lor (q \land \neg p)) \\
&\because \neg q \lor q \equiv \neg p \lor p \equiv T \\
&= (\neg q \lor \neg p) \land (p \lor q) \\
&= (p \to \neg q) \land (\neg q \to p) \\
&= (p \leftrightarrow \neg q)
\end{aligned}
$$

25. We'll determine exactly which rows of the truth table will have F as their entries. In order for $(p \to r) \lor (q \to r)$ to be false, we must have both of the two conditional statements false, which happens exactly when r is false and both p and q are true. But this is precisely the case in which $p \land q$ is true and r is false, which is precisely when $(p \land q) \to r$ is false. Since the two propositions are false in exactly the same situations, they are logically equivalent.

30. The conclusion $q \vee r$ will be true in every case except when q and r are both false. But if q and r are both false, then one of $p \vee q$ or $\neg p \vee r$ is false, because one of p or $\neg p$ is false. Thus in this case the hypothesis $(p \vee q) \wedge (\neg p \vee r)$ is false. An conditional statement in which the conclusion is true or the hypothesis is false is true, and that completes the argument.

$$
\begin{aligned}
& (p \vee q) \wedge (\neg p \vee r) \to (q \vee r) \\
& \equiv \sim\big((p \vee q) \wedge (\sim p \vee r)\big) \vee (q \vee r) \\
& \equiv \big(\sim(p \vee q) \vee (\sim(\sim(p \vee r)))\big) \vee (q \vee r) \\
& \equiv \big((\sim p \wedge \sim q) \vee (p \wedge \sim r)\big) \vee (q \vee r) \\
& \equiv \big((\sim p \wedge \sim q) \vee p\big) \wedge \big((\sim p \wedge \sim q)\ \sim r\big) \vee (q \vee r) \\
& \equiv \big(\underbrace{(\sim p \vee p)}_{T} \wedge (\sim q \vee p)\big) \wedge \big((\sim p \wedge \sim q) \vee \sim r\big) \vee (q \vee r) \\
& \equiv (\sim q \vee p) \wedge \big((\sim p \wedge \sim q) \vee (\sim r \vee r) \vee q\big) \\
& \equiv (\sim q \vee p) \wedge \big((\sim p \wedge \sim q) \vee \underbrace{(T \vee q)}_{T}\big) \\
& \equiv \big[(\sim q \wedge (\sim p \wedge \sim q)) \vee T\big] \vee \big[(p \wedge (\sim p \wedge \sim q)) \vee T\big] \\
& \equiv T
\end{aligned}
$$

33. To show that these are *not* logically equivalent, we need only find one assignment of truth values to p, q, r, and s for which the truth values of $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ differ. Let us try to make the first one false. That means we have to make $r \to s$ false, so we want r to be true and s to be false. If we let p and q be false, then each of the other three simple conditional statements ($p \to q$, $p \to r$, and $q \to s$) will be true. Then $(p \to q) \to (r \to s)$ will be $T \to F$, which is false; but $(p \to r) \to (q \to s)$ will be $T \to T$, which is true.