HW 3 Key

1. (20pt) Daily sales records for a car dealership show that it will sell 0, 1, 2, or 3 cars, with probabilities as listed

Number of Sales	0	1	2	3
Probability	0.5	0.3	0.15	0.05

a) (15pt) Find the probability distribution for the number of sales in a 2 day period, assuming that the sales as independent from day to day.

Let us use (a, b) indicate a cars sold the first day and b cars sold the second day. Then the computation for two days sales as:

P(X=0) = {(0, 0)} = (0.5)(0.5) = 0.25,

P(X=1) = {(0, 1)+(1, 0)} = (0.3)(0.5)+(0.5)(0.3) = 0.3

P(X=2) = {(2, 0)+(1, 1)+(0, 2)} = (0.15)(0.5)+(0.3)(0.3)+(0.5)(0.15) = 0.24

P(X=3) = {(3, 0)+(2, 1)+(1, 2)+(0, 3)} = (0.05)(0.5)+(0.15)(0.3)+(0.3)(0.15)+(0.5)(0.05) = 0.14

P(X=4) = {(1, 3)+(2, 2)+(3, 1)} = (0.3)(0.05)+(0.15)(0.15)+(0.05)(0.3) = 0.0525

P(X=5) = {(2, 3)+(3, 2)} = (0.15)(0.05) + (0.05)(0.15) = 0.015

P(X=6) = {(3, 3)} = (0.05)(0.05) = 0.0025

A tabular form:

Number of Sales	0	1	2	3	4	5	6
Probability	0.25	0.3	0.24	0.14	0.0525	0.015	0.0025

b) (5pt) Find the probability that two or more sales are made in the next two days.

P(X≥2) = 1 – P(X<2) = 1 – P(X=0) – P(X=1) = 1 – 0.25 – 0.3 = 0.45

2. (20pt) An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & elsewhere \end{cases}$$

(a) (10pt) Verify that this is a valid density function.

$f(x) \geq 0$

$$\int_1^{\infty} 3x^{-4}dx = -x^{-3}\Big|_1^{\infty} = 1$$

(b) (5pt) Evaluate F(x).

For x⩾1，$F(x) = \int_1^x 3t^{-4}dt = 1 - x^{-3}$

So, $F(x) = \begin{cases} 0, & x < 1 \\ 1 - x^{-3}, & x \geq 1 \end{cases}$

(c) (5pt) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

$P(X > 4) = 1 - F(4) = 4^{-3} = 0.0156$

3. (20pt) A coin is tossed twice. Let Z denote the number of heads on the first toss and W the total number of heads on the 2 tosses. If the coin is unbalanced and a head has a 40% chance of occurring, find

(a) (5pt) the joint probability distribution of W and Z;

P(H) = 0.4, P(T) = 0.6, and S = {HH, HT, TH, TT}. Let (W, Z) represents a typical outcome of the experiment. The particular outcome (1, 0) indicates a total of 1 head and no heads on the first toss corresponds to the event TH. Therefore, f(1, 0) = P(W=1, Z=0) = P(TH) = P(T)P(H) = (0.6)(0.4) = 0.24. Similar calculations for the outcomes (0, 0), (1, 1), and (2, 1) lead to the following joint probability distribution:

		w		
	$f(w,z)$	0	1	2
z	0	0.36	0.24	
	1		0.24	0.16

(b) (5pt) the marginal distribution of W;

Summing the columns, the marginal distribution of W is

w	0	1	2
$g(w)$	0.36	0.48	0.16

(c) (5pt) the marginal distribution of Z;

Summing the rows, the marginal distribution of Z is

z	0	1
$h(z)$	0.60	0.40

(d) (5pt) the probability that at least 1 head occurs.

P(W⩾1) = f(1, 0)+f(0, 1)+f(2,1) = 0.24+0.24+0.16 = 0.64

4. (20pt) The joint density function of the random variables X and Y is

$$f(x,y) = \begin{cases} 6x, & 0 < x < 1, 0 < y < 1 - x \\ 0, & elsewhere \end{cases}$$

(a) (10pt) Determine whether X and Y are independent or not.

$h(y) = 6\int_0^{1-y} x dx = 3(1-y)^2, for\ 0 < y < 1$. Since $f(x|y) = \frac{f(x,y)}{h(y)} = \frac{2x}{(1-y)^2}, for\ 0 < x < 1 - y$, involves the variable y, X and Y are not independent.

(b) (10pt) Find P(X > 0.3 | Y = 0.5).

$$P(X > 0.3|Y = 0.5) = 8\int_{0.3}^{0.5} xdx = 0.64$$

5. (20pt) Let X, Y, and Z have the joint probability density function

$$f(x,y,z) = \begin{cases} kxy^2z, & 0 < x,y < 1, 0 < z < 2 \\ 0, & elsewhere \end{cases}$$

(a) (10pt) Find k.

$k\int_0^2\int_0^1\int_0^1 kxy^2zdxdydz = \frac{k}{3} = 1$. So, k=3.

(b) (10pt) Find P(X < 0.25, Y > 0.5, 1 < Z < 2).

$$P(X < 0.25, Y > 0.5, 1 < Z < 2) = 3\int_1^2\int_{0.5}^1\int_0^{0.25} xy^2zdxdydz = \frac{21}{512}$$