§ 5.2

Electrostatic field
Electric field strength

5.2.1 Electric field and electric field strength

1. electric field

The charge excite electric field around it, which has a force on any charge in it.

electric charge ⟺ **electric field** ⟺ **electric charge**

The electric field is a form of matter that exists and also has energy, momentum, velocity

Electric field excited by the stationary charge ⟹ **electrostatic field**

stable distribution
Independent of the presence of other charges

2. Definition of the electric field strength

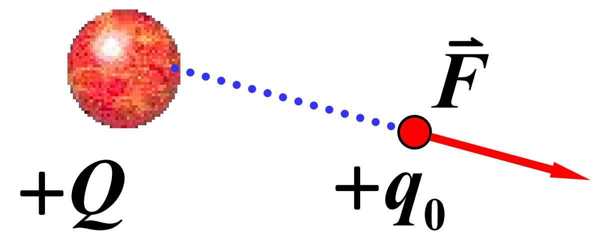
$$\vec{E} = \frac{\vec{F}}{q_0}$$

The strength of the electric field is equal to the force of the unit test charge located at the same point.

unit $\quad \mathrm{N \cdot C^{-1}} \quad \mathrm{V \cdot m^{-1}}$

♠ The charge q is forced in the electric field

$$\vec{F} = q\vec{E}$$



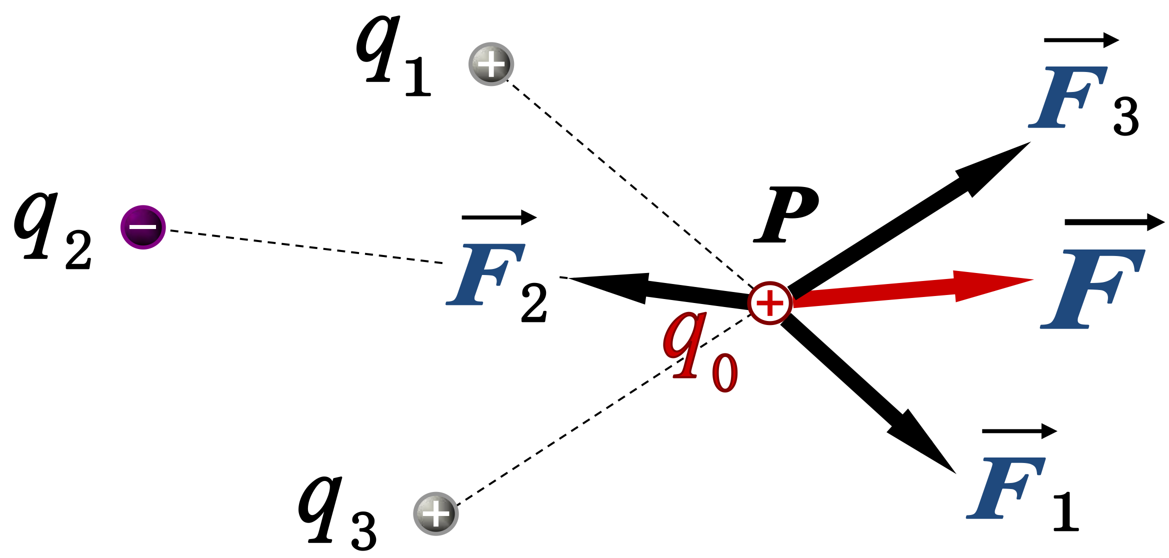
$+Q$: Field source charge

$+q_0$: test charge

Test charge: charge is small enough, dimension is small enough. Therefore, it has almost no impact on the original electric field, and can reflect the situation of spatial points.

5.2.2 Superposition principle of electric field strength

Total Coulomb`s force $\vec{F} = \sum_i \vec{F}_i$



$$\vec{E}_1 = \frac{\vec{F}_1}{q_0},$$

$$\vec{E}_2 = \frac{\vec{F}_2}{q_0},$$

$$\cdots$$

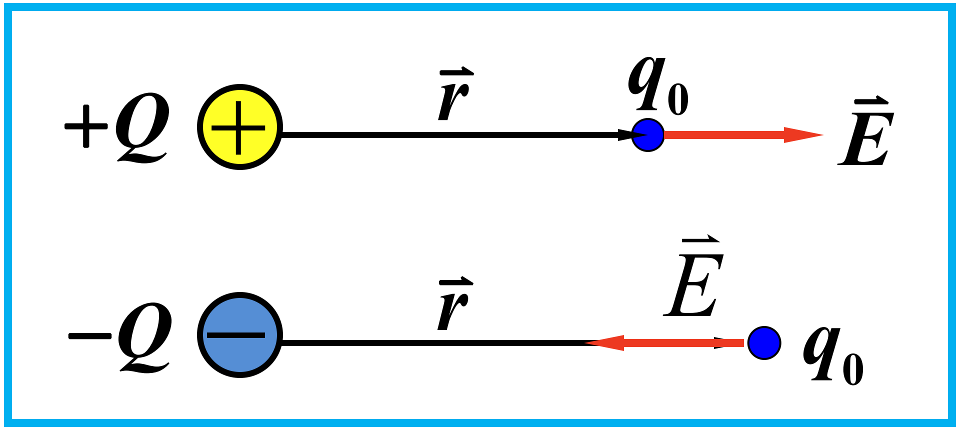
$$\vec{E}_n = \frac{\vec{F}_n}{q_0}$$

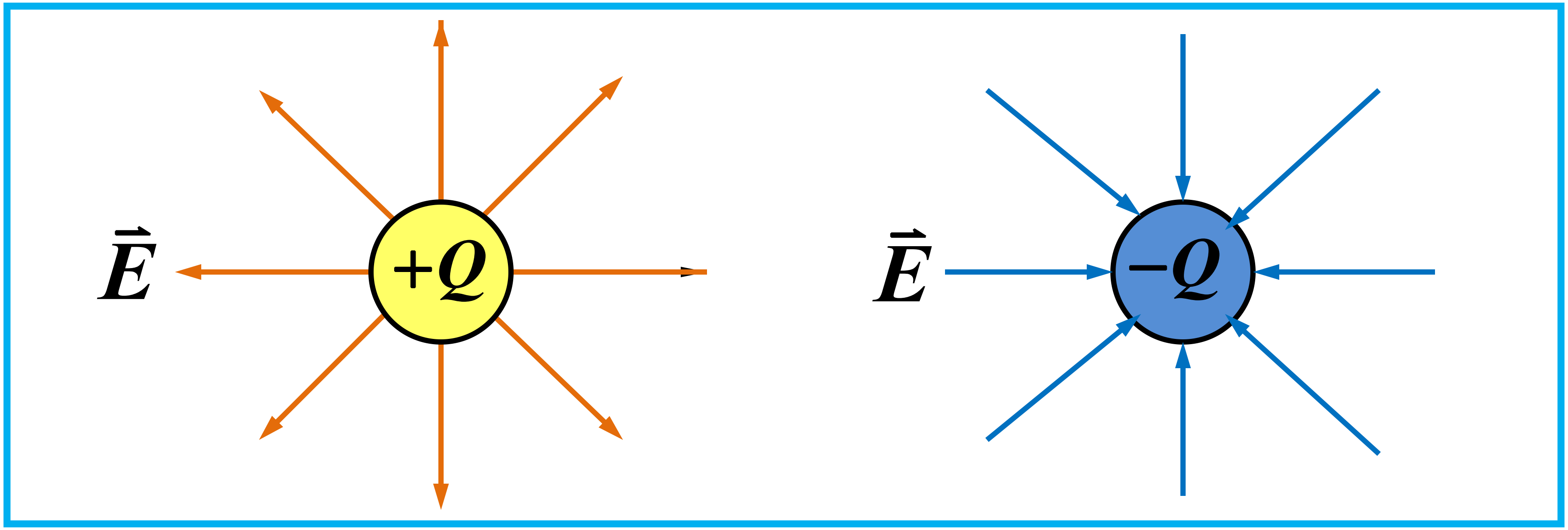
Total Electric field strength $\vec{E} = \sum_i \vec{E}_i$

5.2.3 Calculation of the electric field intensity

1. Electric field of the point charge

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{e}_r$$





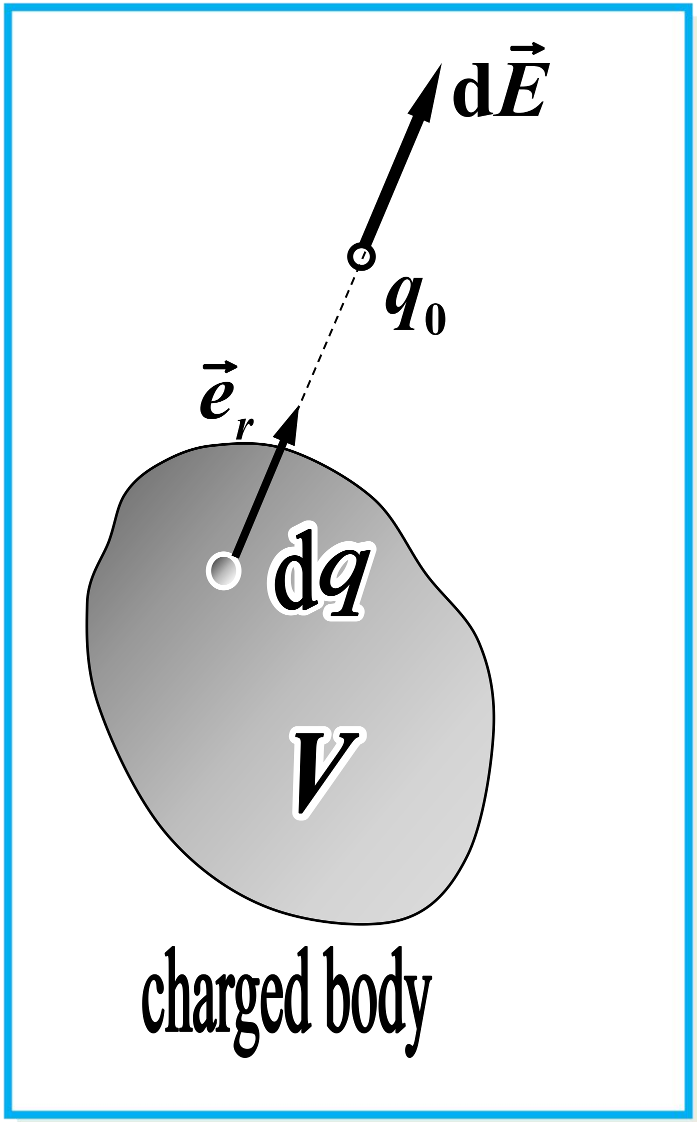
$$r \to 0 \quad E \to \infty ?$$

2. Electric field of the point-charge system

$$\vec{E} = \sum_i \vec{E}_i = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\vec{e}_{ri}$$

3. Electric field of a continuous charged body

$$\vec{E} = \int \mathrm{d}E = \int_q \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{d}q}{r^2}\vec{e}_r$$



volume charge density:

$$\rho = \frac{q}{V}$$

$$\mathrm{d}q = \rho \mathrm{d}V$$

charge density

volume charge density:

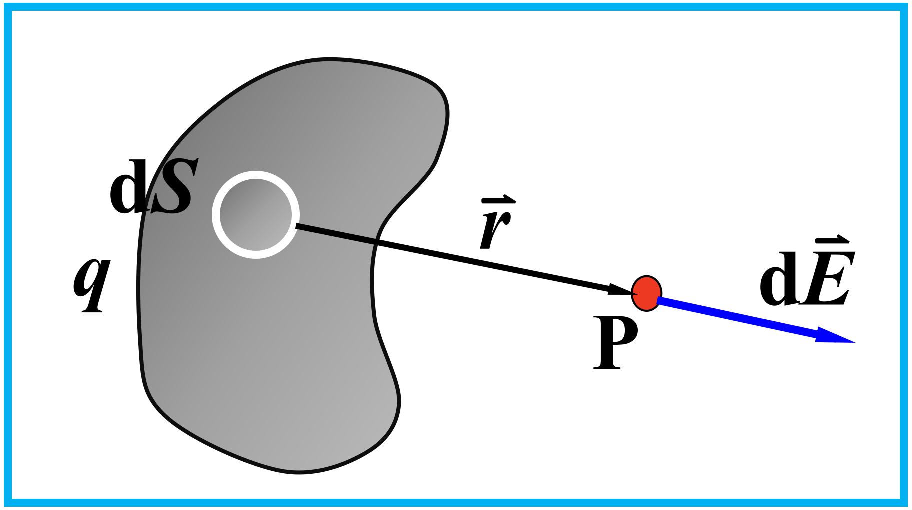
$$\rho = \frac{q}{V} \qquad dq = \rho dV$$

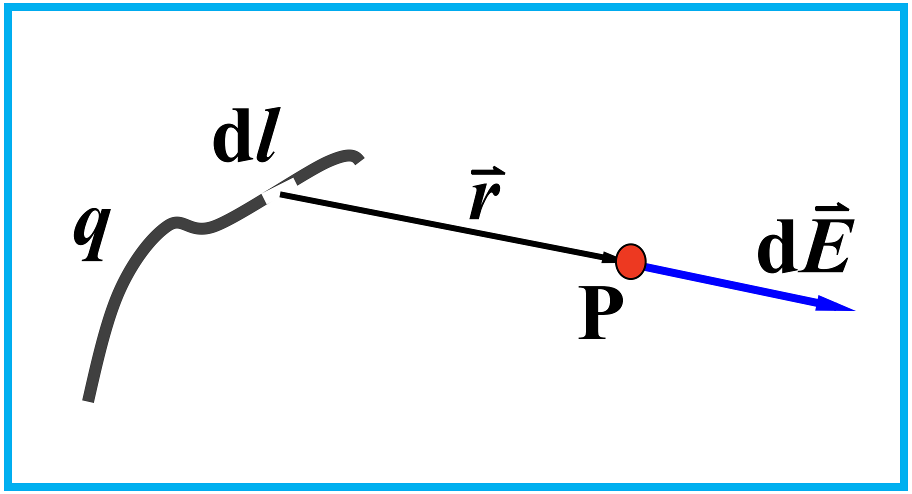
density of surface charge:

$$\sigma = \frac{q}{S} \qquad dq = \sigma dS$$

linear charge density:

$$\lambda = \frac{q}{L} \qquad dq = \lambda dl$$

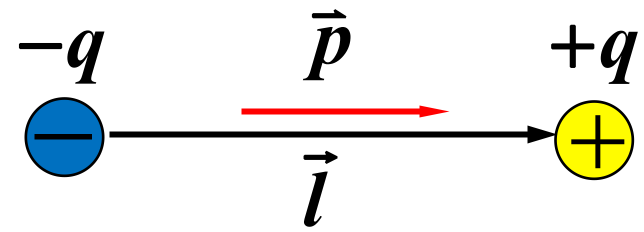




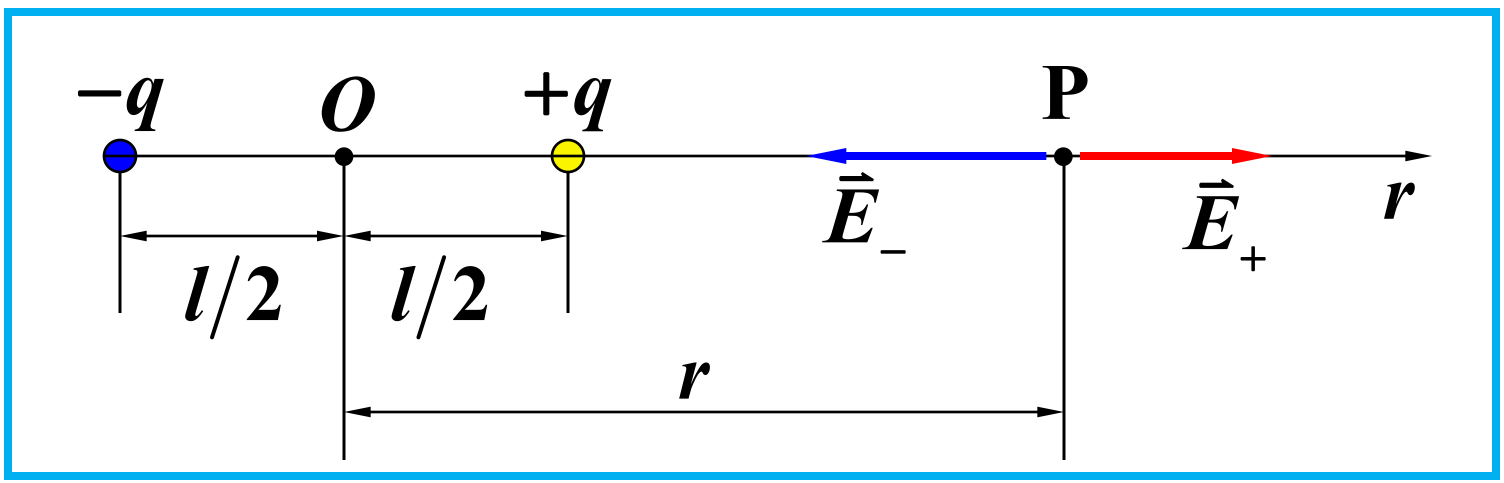
4. Electric fields of several typical charged systems

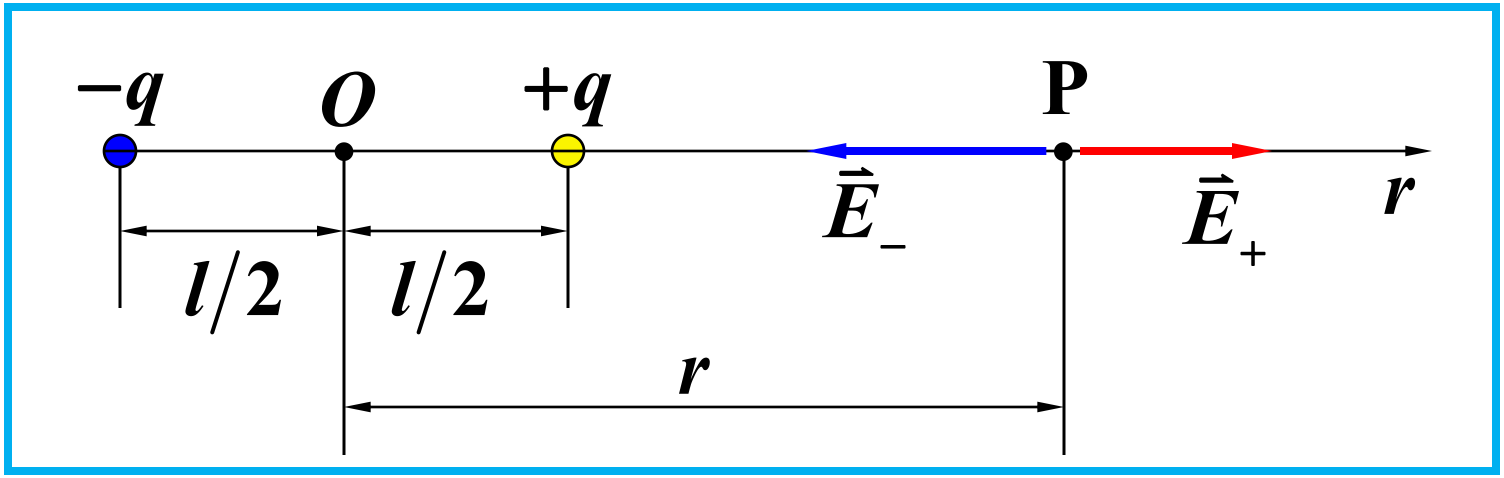
1. The electric field strength of the electric dipole

Electric dipole moment (electrical moment) $\vec{p} = q\vec{l}$



(1) Electric field strength at a point on the extension line of the electric dipole axis





$$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l/2)^2} \qquad E_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l/2)^2}$$

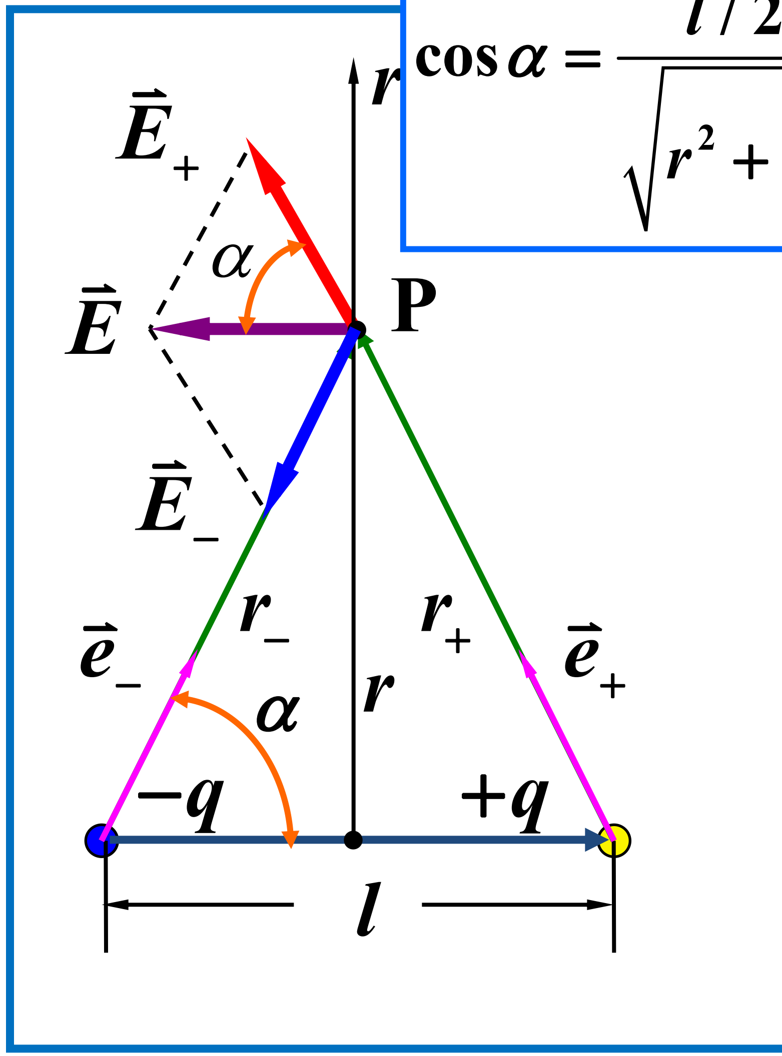
$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{q}{4\pi\varepsilon_0}\left[\frac{2rl}{(r^2-l^2/4)^2}\right]$$

$$r >> l \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{2ql}{r^3} \qquad \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}}$$

(2) The electric field intensity at a point in the medium vertical line of the electric dipole axis

$$\begin{cases} \vec{E}_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+^2}\vec{e}_+ \\[2ex] \vec{E}_- = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_-^2}\vec{e}_- \end{cases}$$

$$\cos\alpha = \frac{l/2}{\sqrt{r^2 + \dfrac{l^2}{4}}}$$



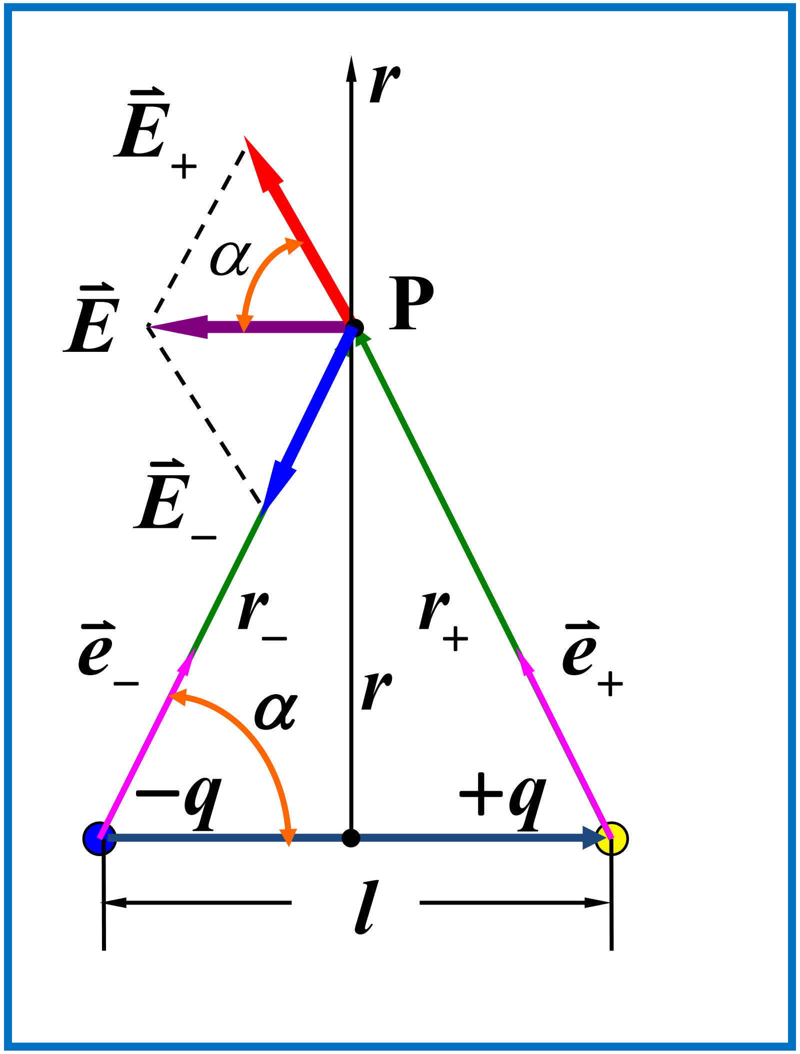
$$r_+ = r_- = \sqrt{r^2 + \left(\frac{l}{2}\right)^2}$$

$$E_+ = E_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2 + \dfrac{l^2}{4}\right)}$$

$$\boxed{\begin{aligned} \vec{E} &= \vec{E}_{+x} + \vec{E}_{-x} \\ E &= 2E_+\cos\alpha \end{aligned}}$$

$$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2+\frac{l^2}{4}\right)}$$

$$\cos\alpha = \frac{l/2}{\sqrt{r^2+\frac{l^2}{4}}}$$



$$E = 2E_+\cos\alpha$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{ql}{(r^2+\frac{l^2}{4})^{3/2}}$$

$$r >> l \quad \Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{ql}{r^3}$$

$$\boxed{\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}}$$

(3) The electric field intensity at any point in the space

$$p_{\parallel} = p\cos\theta$$

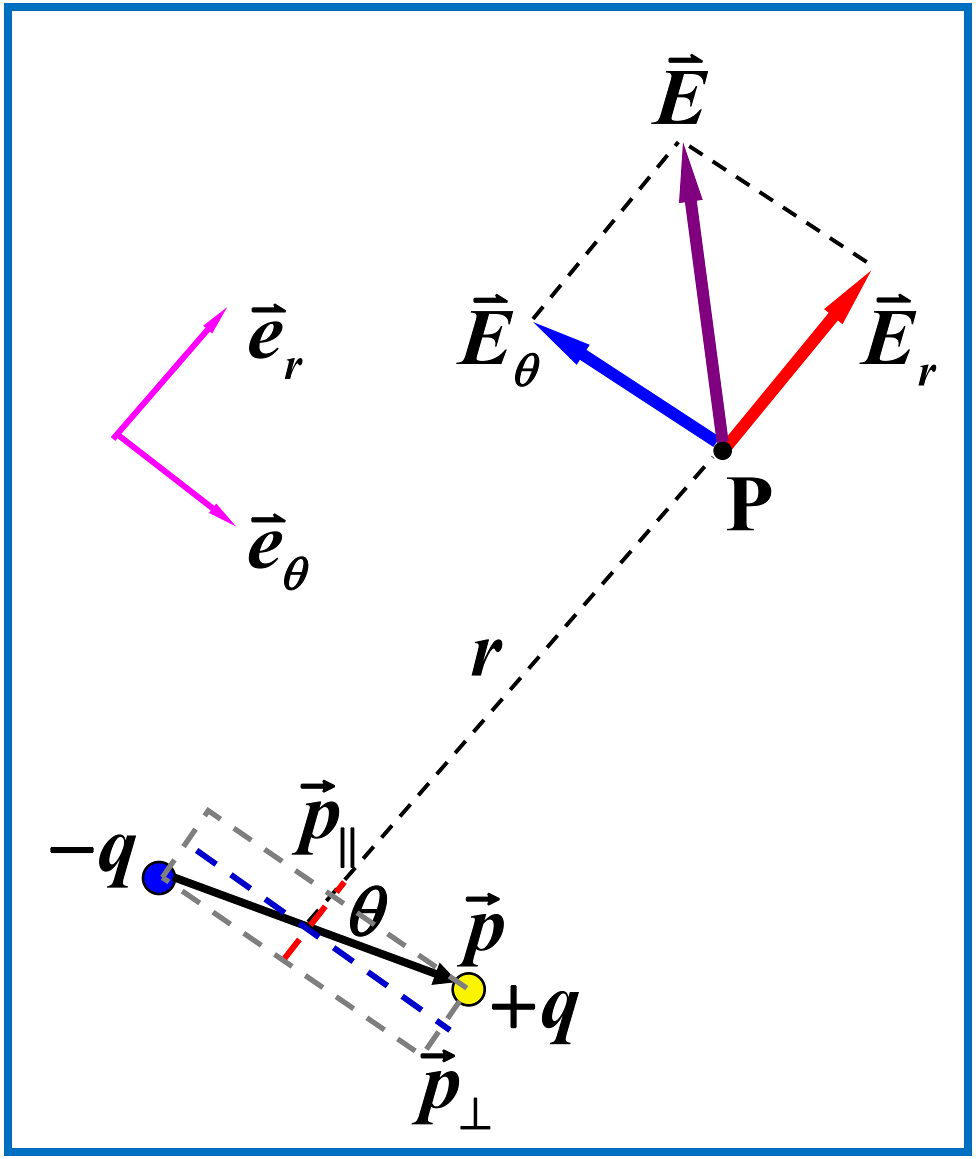
$$p_{\perp} = p\sin\theta$$

$$\vec{E} = \vec{E}_r + \vec{E}_\theta$$

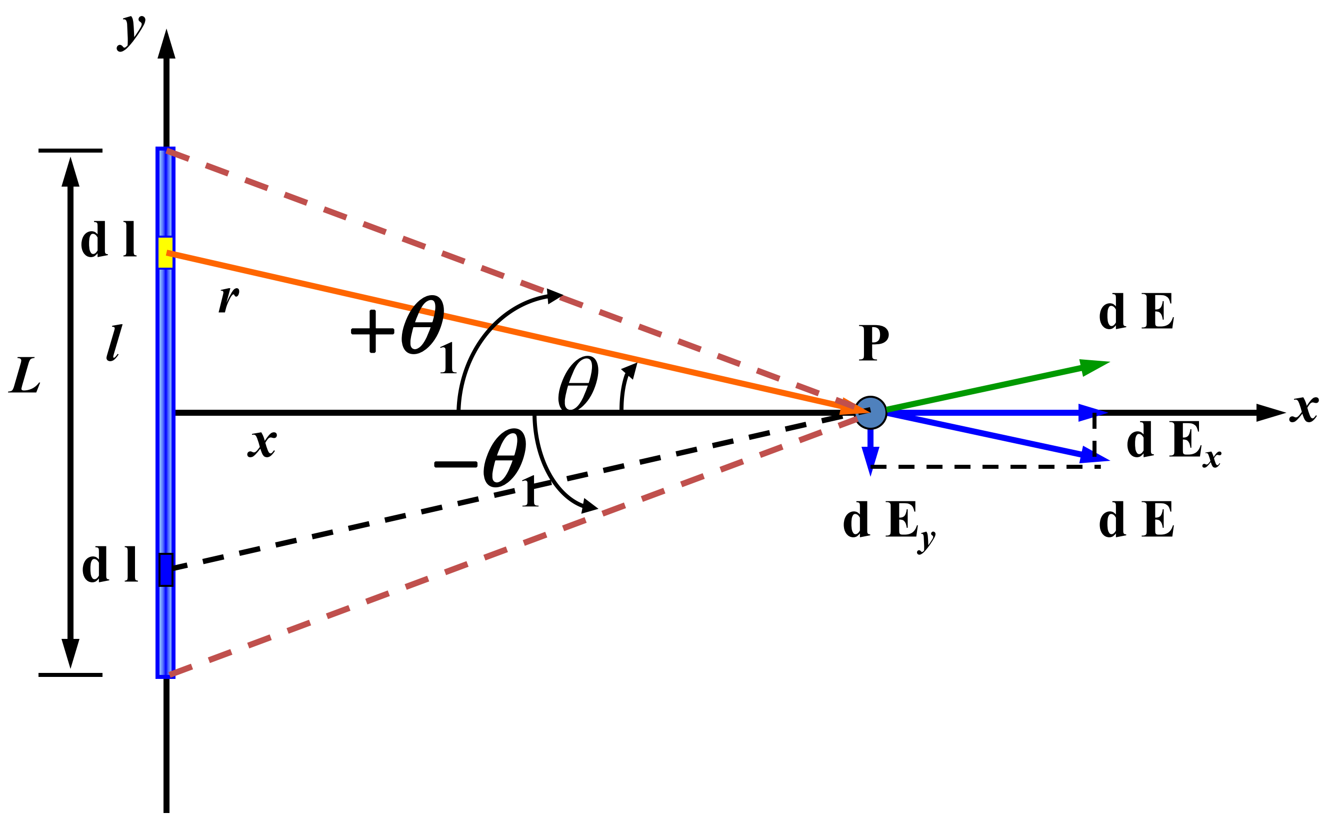
among:

$$\vec{E}_r = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\vec{e}_r$$

$$\vec{E}_\theta = -\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\vec{e}_\theta$$



Example 1-4-1 Find the field strength on the vertical line in a uniform charged line. With a uniform charged line, the length is L and the line charge density is $\lambda\ (\lambda > 0)$, find the field strength of a point on the vertical line in the line.



Field strength on the vertical line in a uniformly charged line

Solution: take the length of the line element dl, its power is

$$\mathrm{d}q = \lambda \mathrm{d}l$$

The symmetry analysis, the total field strength E direction of point P should be along the x-axis, i. e

$$E = \int \mathrm{d}E_x$$

but $\mathrm{d}E_x = \mathrm{d}E\cos\theta = \dfrac{\lambda \mathrm{d}l}{4\pi\varepsilon_0 r^2} \cdot \dfrac{x}{r} = \dfrac{\lambda x \mathrm{d}l}{4\pi\varepsilon_0 r^3}$

owing to $l = x\tan\theta \Rightarrow \mathrm{d}l = \dfrac{x}{\cos^2\theta}\mathrm{d}\theta$

$r = x / \cos\theta$

$$\mathrm{d}E_x = \frac{\lambda \mathrm{d}l x}{4\pi\varepsilon_0 r^3} = \frac{\lambda\cos\theta}{4\pi\varepsilon_0 x}\mathrm{d}\theta$$

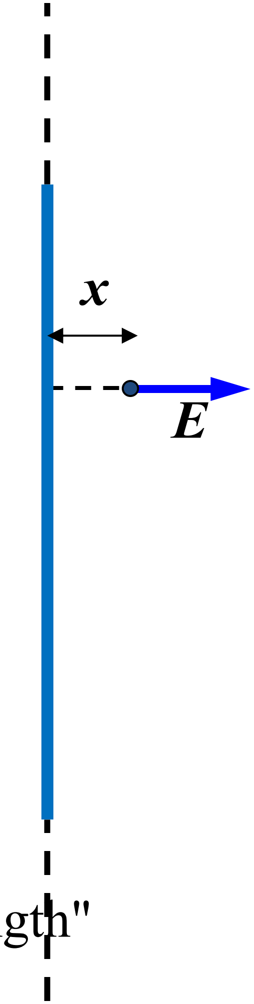
$$E = \int \mathrm{d}E_x = \int_{-\theta_1}^{+\theta_1} \frac{\lambda\cos\theta}{4\pi\varepsilon_0 x}\mathrm{d}\theta = \frac{\lambda\sin\theta_1}{2\pi\varepsilon_0 x}$$

$$\sin\theta_1 = \frac{L/2}{\sqrt{(L/2)^2 + x^2}}$$ To substitute:

$$E = \frac{\lambda\sin\theta_1}{2\pi\varepsilon_0 x} = \frac{\lambda L}{4\pi\varepsilon_0 x(x^2 + L^2/4)^{1/2}}$$

The direction is perpendicular to the charged line and points to the side far away from the line

$$E = \frac{\lambda \sin\theta_1}{2\pi\varepsilon_0 x} = \frac{\lambda L}{4\pi\varepsilon_0 x(x^2 + L^2/4)^{1/2}}$$



Discuss:

(1) When $x \ll L \quad (x^2 + L^2/4)^{1/2} \approx L/2$

$$\Rightarrow \quad E \approx \frac{\lambda}{2\pi\varepsilon_0 x}$$

At this point, the charged straight line can be regarded as "infinite length" relative to x.

Note: The field strength of any point around an infinite long charged line is inversely proportional to the distance from that point to the charged line.

$$E = \frac{\lambda \sin\theta_1}{2\pi\varepsilon_0 x} = \frac{\lambda L}{4\pi\varepsilon_0 x(x^2 + L^2/4)^{1/2}}$$

(2) When x>> L, $(x^2 + L^2/4)^{1/2} \approx x$

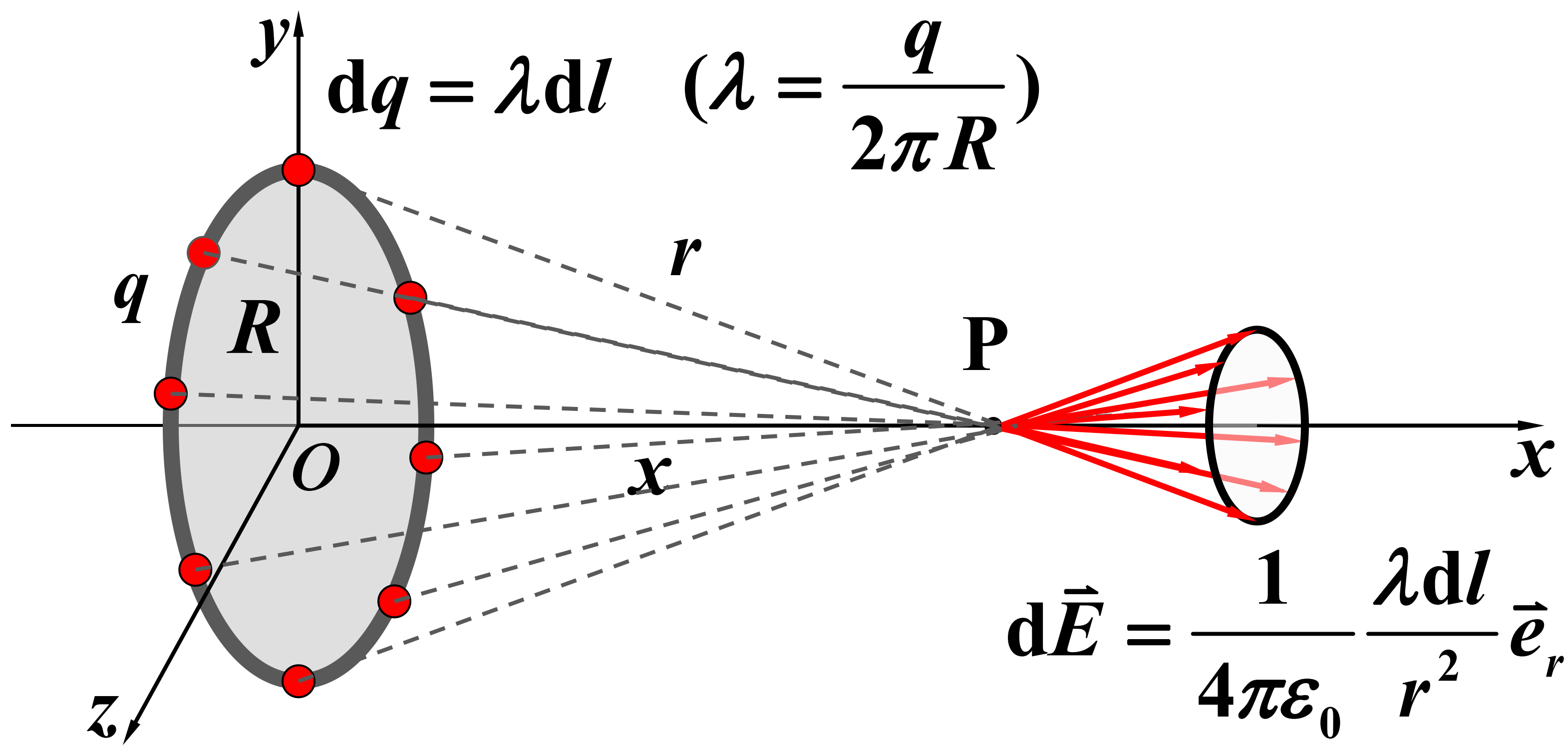
$$\Rightarrow E \approx \frac{\lambda L}{4\pi\varepsilon_0 x^2} = \frac{q}{4\pi\varepsilon_0 x^2}$$

Note: Far away from the charged line, the electric field of the charged line is equivalent to an electric field of a point charge q.

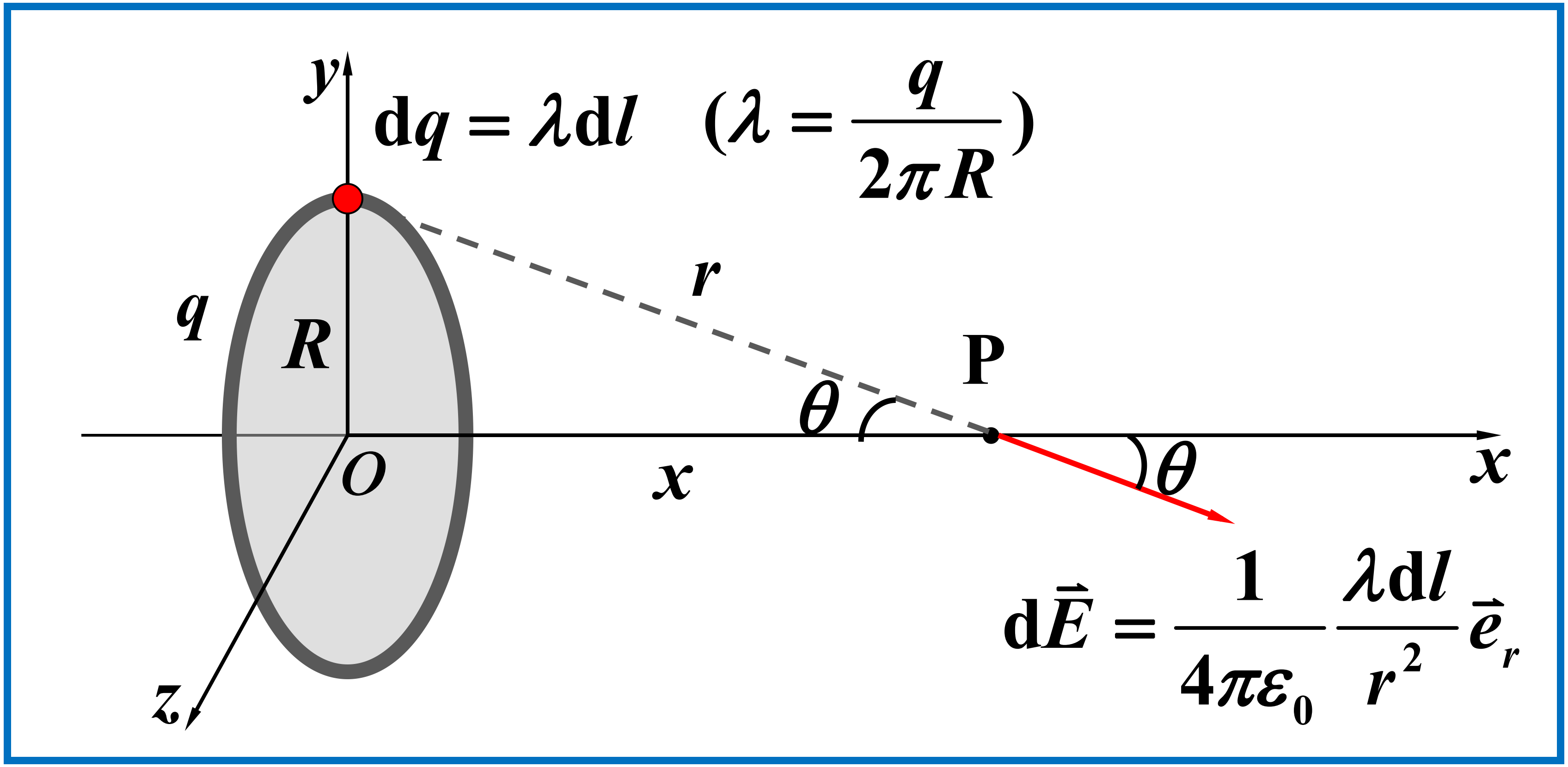
Example 2 of the field strength on the axis perpendicular to the uniformly charged ring. A uniform charged fine ring, radius of R, the total power of q, find the field strength of one point of the axis of the circle.

separate: $\vec{E} = \int \mathrm{d}\vec{E}$ There is a symmetry $\vec{E} = E_x \vec{i}$

$$\mathrm{d}q = \lambda \mathrm{d}l \quad \left(\lambda = \frac{q}{2\pi R}\right)$$



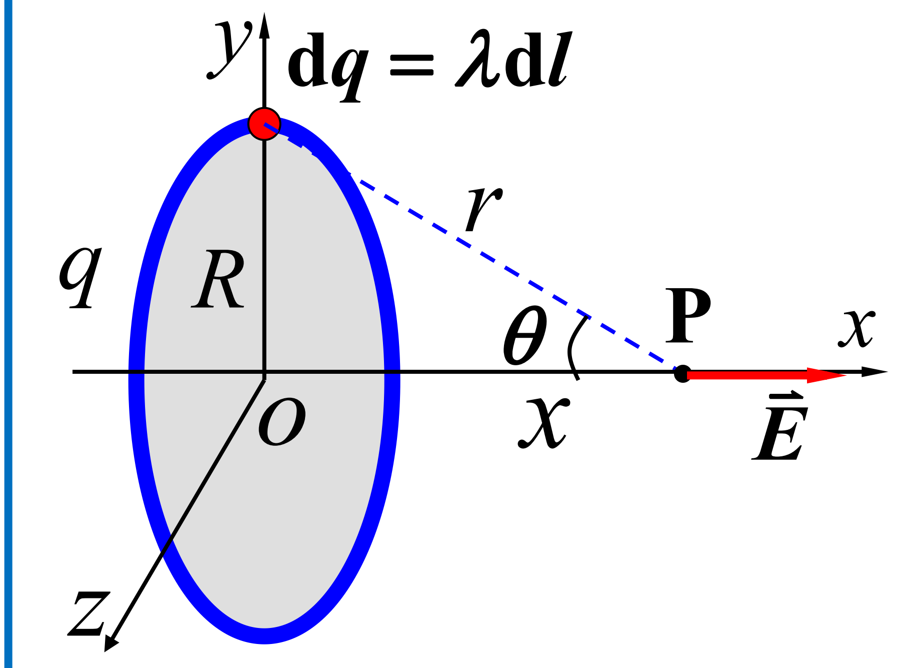
$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \mathrm{d}l}{r^2}\vec{e}_r$$



$$E=\int_l \mathrm{d}E_x=\int_l \mathrm{d}E\cos\theta=\int\frac{\lambda\mathrm{d}l}{4\pi\varepsilon_0 r^2}\cdot\frac{x}{r}$$

$$=\int_0^{2\pi R}\frac{x\lambda\mathrm{d}l}{4\pi\varepsilon_0 r^3}=\frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

$$E=\frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$



discuss:

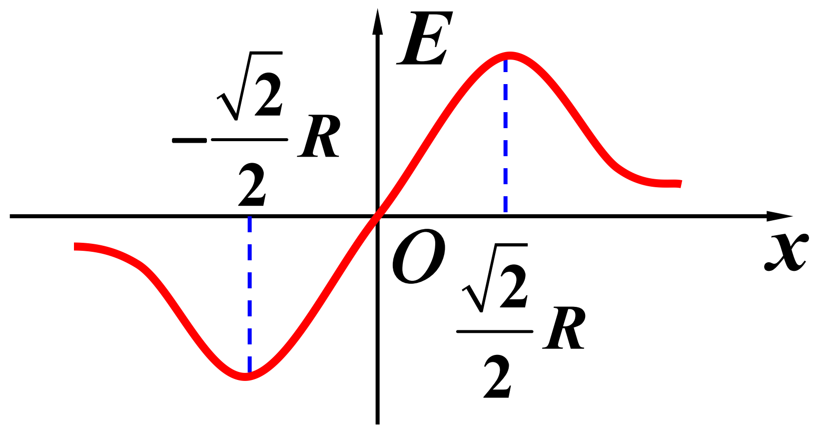
(1) $x>>R$

$$E\approx\frac{q}{4\pi\varepsilon_0 x^2}$$

(Point-charge electric field strength)

(2) $x=0,\quad E_0=0$

(3) $\frac{\mathrm{d}E}{\mathrm{d}x}=0,\quad x=\pm\frac{\sqrt{2}}{2}R$



Example 3 of the field strength on the axis perpendicular to the uniformly charged disk.

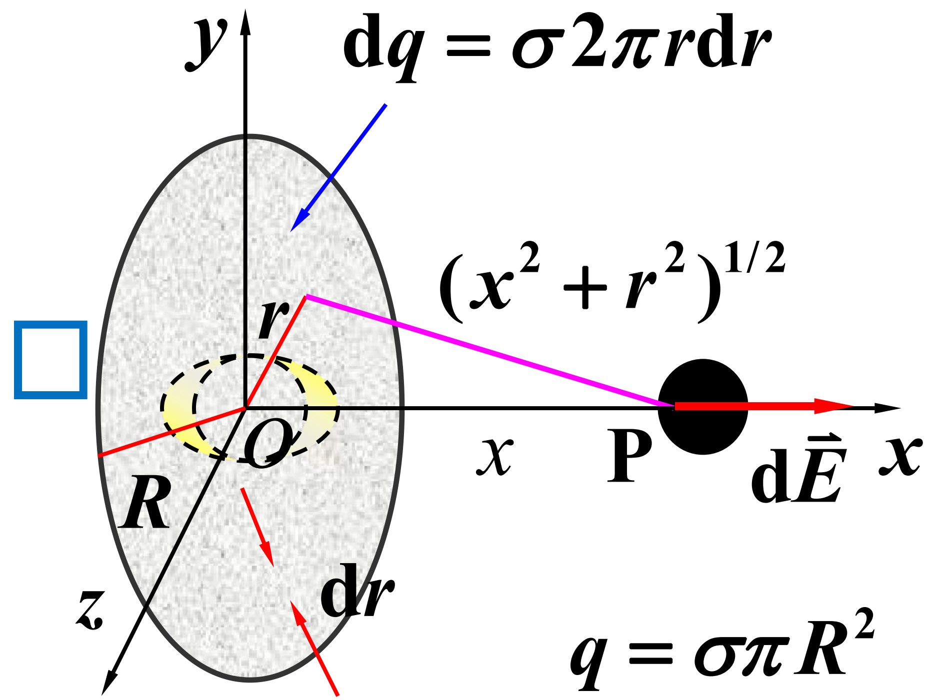
There is a thin disk with a radius R of and a uniform charge distribution, with a charge surface density of. Find the electric field strength at any point on the axis of the disk center and the vertical disk surface.

Solution: by the above example

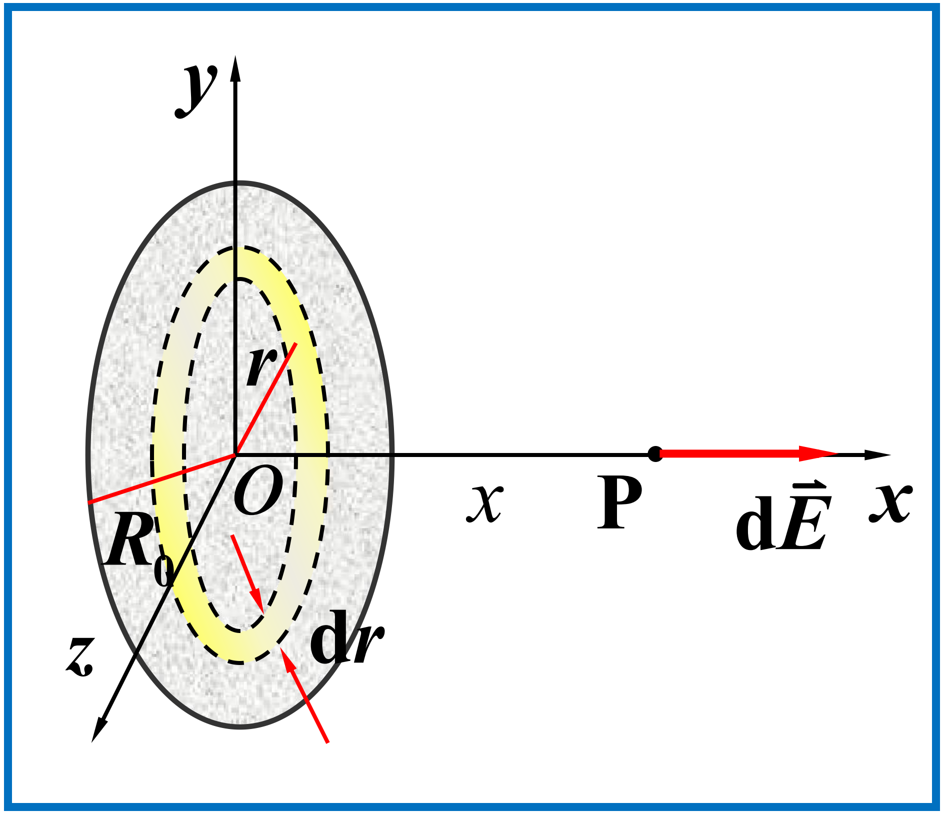
$$E = \frac{\boxed{q}\, x}{4\pi\varepsilon_0 (x^2 + R^2)^{3/2}}$$

$$\mathrm{d}E_x = \frac{\boxed{\mathrm{d}q} \cdot x}{4\pi\varepsilon_0 (x^2 + r^2)^{3/2}}$$

$$= \frac{\sigma}{2\varepsilon_0} \frac{x r \mathrm{d}r}{(x^2 + r^2)^{3/2}}$$



$$\mathrm{d}E_x = \frac{\sigma}{2\varepsilon_0}\frac{x r \mathrm{d}r}{(x^2+r^2)^{3/2}}$$



$$E = \int \mathrm{d}E_x$$

$$= \frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{r\mathrm{d}r}{(x^2+r^2)^{3/2}}$$

$$E = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{(x^2+R^2)^{1/2}}\right]$$

$$\boxed{E=\frac{\sigma}{2\varepsilon_0}[1-\frac{x}{(x^2+R^2)^{1/2}}]}$$

discuss:

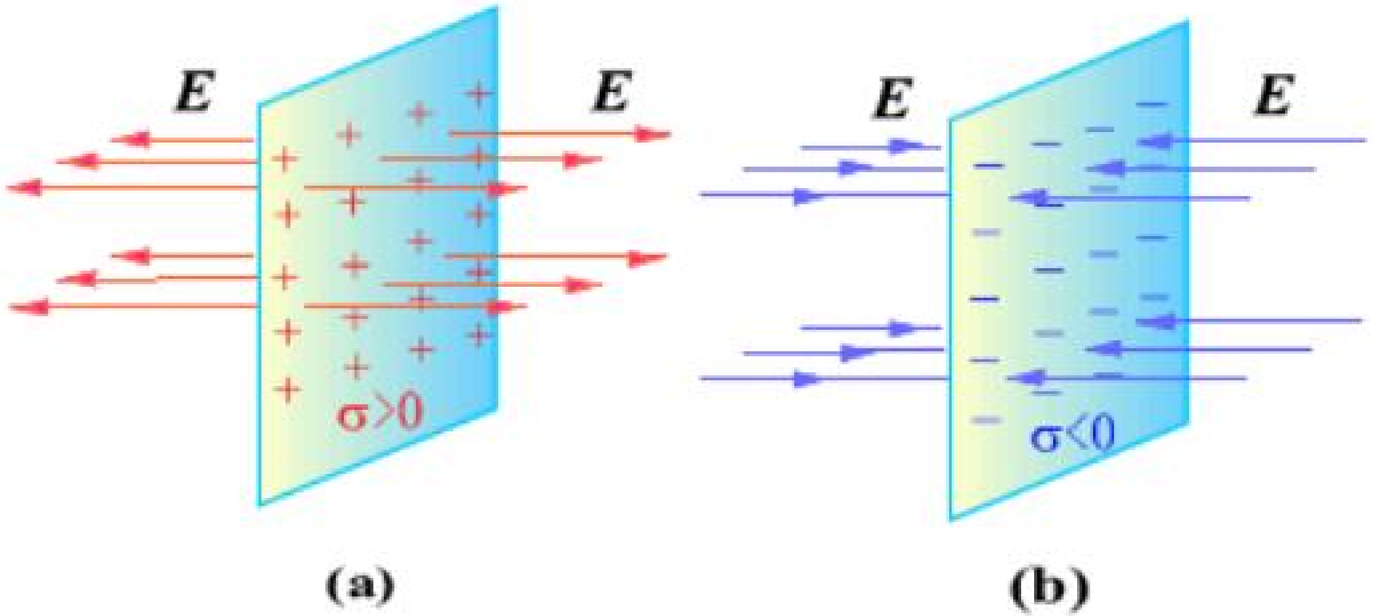
$x >> R$ $\qquad \boxed{E \approx \frac{q}{4\pi\varepsilon_0 x^2}}$ (Point-charge electric field strength)

$x << R$ $\qquad \boxed{E \approx \frac{\sigma}{2\varepsilon_0}}$ (The electric field strength of an infinitely large, uniformly charged plane)

$$(x^2+R^2)^{-1/2}=\frac{1}{x}\left(1-\frac{1}{2}\cdot\frac{R^2}{x^2}+\cdots\right)\approx\frac{1}{x}\left(1-\frac{1}{2}\cdot\frac{R^2}{x^2}\right)$$

discuss

$$E \approx \frac{\sigma}{2\varepsilon_0}$$



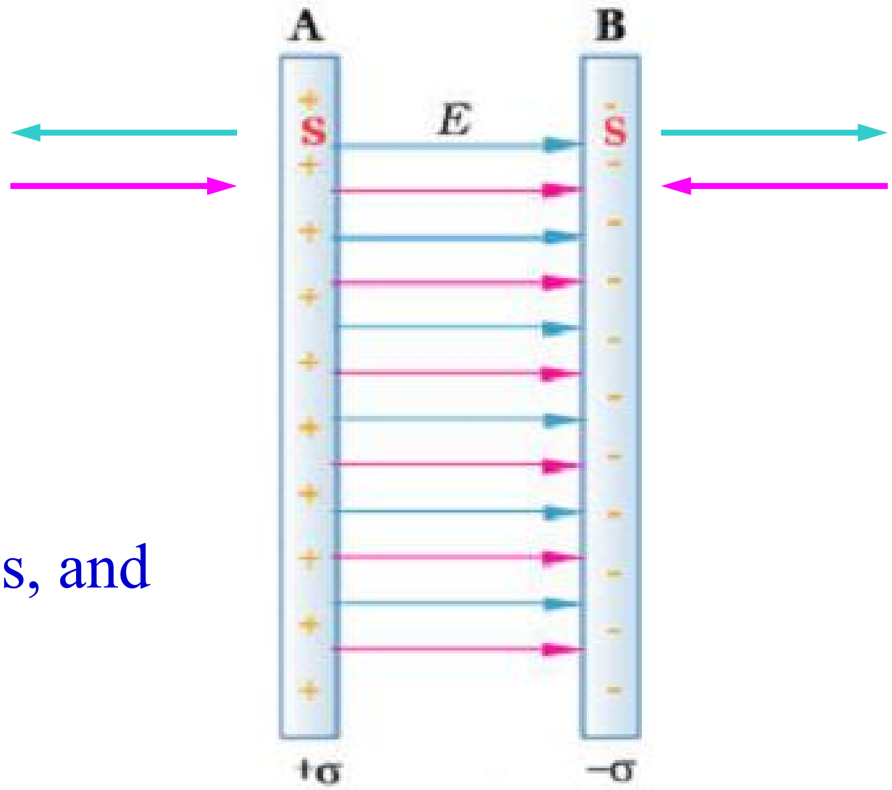
Electric field in an "infinitely large" uniformly charged plane

Conclusion: Around an infinite uniform charged plane, the electric field is a uniform field, with all directions perpendicular to the plane and parallel to each other.

Thinking: Knowing two uniform parallel planes with equal positive and negative charges (that is, the surface charge density is the same), we find the electric field distribution of this charged system.

Using the electric-field superposition principle

$$E = \frac{\sigma}{\varepsilon_0}$$



Conclusion: The electric field is all concentrated between the two planes, and it is a uniform electric field.

The electric field confined to the above region is called the "infinite large" electric field in a uniformly charged parallel plane.

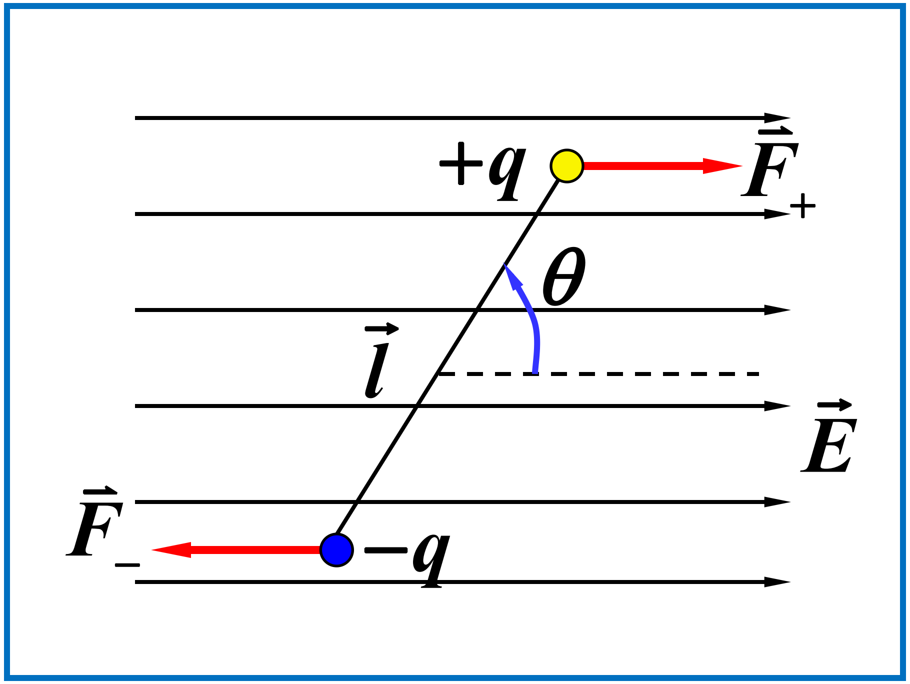
5.2.4 Effect of the uniform strong electric field on the electric dipole

$$\vec{F} = \vec{F}_+ + \vec{F}_-$$

$$= q\vec{E} - q\vec{E} = 0$$

$$M = qlE\sin\theta$$

$$= pE\sin\theta$$



$$\vec{M} = \vec{p}\times\vec{E} \quad \begin{cases} \theta = 0 \\ \theta = \pi \end{cases} \quad \vec{M} = 0$$

stable equilibrium

Unstable balance

If in the nonuniform strong electric field $\vec{F} = \vec{F}_+ + \vec{F}_- = q\vec{E}_+ - q\vec{E}_- \neq 0$