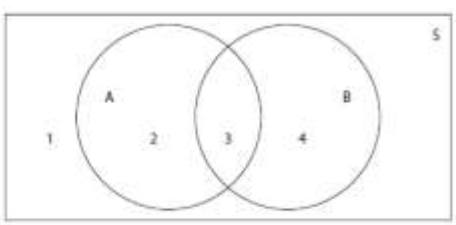
STAT 1151 Fall 2023

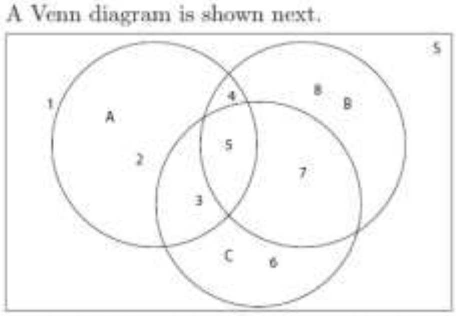
HW 1

1. (30pt) Five applicants (Jim, David, Mary, Sue, and Nancy) are available for two identical jobs. A supervisor selects two applicants to fill these jobs.
   a. (5pt) Give the sample space associated with this experiment.
   Suppose that we use the first letter to represent the applicant, the sample space of selecting two people is S = {JD, JM, JS, JN, DM, DS, DN, MS, MN, SN}
   b. (5pt) Let A denote the event that at least one male is selected. How many outcomes are in A?
   A = {JD, JM, JS, JN, DM, DS, DN}. n(A)=7.
   c. (5pt) Let B denote the event that exactly one male is selected. How many outcomes are in B?
   B = {JM, JS, JN, DM, DS, DN}. n(B)=6.
   d. (5pt) Write the event symbolically that two males are selected in terms of A and B.
   $A \cap \bar{B}$ or $A\bar{B}$.
   e. (10pt) List the outcomes in $\bar{A}, AB, A \cup B$, and $\overline{AB}$
   $\bar{A}$ = {MS, MN, SN};
   AB = {JM, JS, JN, DM, DS, DN};
   $A \cup B$ = {JD, JM, JS, JN, DM, DS, DN};
   $\overline{AB}$ ={JD, MS, MN, SN}.

2. (10pt) Let A, B, and C be events relative to the sample space S. Using Venn diagrams, shade the areas representing the following events:
   a. (3pt) (A ∩ B)’
   b. (3pt) (A ∪ B)’
   c. (4pt) (A ∩ C) ∪ B.



(a) From the above Venn diagram, $(A \cap B)'$ contains the regions of 1, 2 and 4.
(b) $(A \cup B)'$ contains region 1.
(c) A Venn diagram is shown next.



$(A \cap C) \cup B$ contains the regions of 3, 4, 5, 7 and 8.

3. (20pt) In a class of 60 students, 13 could not roll their tongue, 17 had attached earlobes, and 10 could roll their tongue and had attached earlobes. Let T denote the event that the selected student can roll his or her tongue and E denote the event that the student has attached earlobes. Suppose a student is randomly selected from the class. For each of the following events, 1) represent the event symbolically and then 2) find its probability.
   a. (5pt) The student can roll his or her tongue.
   Symbolic representation is T. Since total number of students is 60 and 13 of them cannot roll tongue, n(T) = 60 – 13 = 47.
   $$P(T) = \frac{47}{60} = 0.78$$
   b. (5pt) The student can neither roll tongue nor has attached earlobes.
   Symbolic representation is $\bar{T} \cap \bar{E}$.
   $$(\bar{T} \cap \bar{E}) = n(\overline{T \cup E}) = n(S) - n(T \cup E) = 60 - 54 = 6.$$
   $$P(\bar{T} \cap \bar{E}) = \frac{6}{60} = 0.1$$
   c. (5pt) The student has attached earlobes but cannot roll tongue.
   Symbolic representation is $E \cap \bar{T}$. $n(E \cap \bar{T}) = n(E) - n(ET) = 17 - 10 = 7.$
   $$P(E \cap \bar{T}) = \frac{7}{60} = 0.12$$
   d. (5pt) The student can roll tongue or has attached earlobes but not both.
   Symbolic representation is $T\bar{E} \cup \bar{T}E$. $n(T\bar{E} \cup \bar{T}E) = 37 + 7 = 44.$
   $$P(T\bar{E} \cup \bar{T}E) = \frac{44}{60} = 0.73$$

4. (15pt) Suppose a class of 8 boys and 7 girls are attend a theatrical performance, and the teacher obtains 15 tickets (one for each student) in a row. How many ways are there to order the students under the following conditions:
   a. (5pt) The children are randomly assigned seats.
   There are 15! ways.
   b. (5pt) Boys and girls are alternated so that boys sit by girls and girls sit by boys
   Since there is one boy more than girl, the boy must sit in the first and the last seats. Otherwise, two boys would sit together at the end. Thus, we have 8!7! ways to arrange.
   c. (5pt) All boys sit together, and all girls sit together.
   We can sit all boys first then girls which is 8!7! ways. Or we can sit all girls first then boys which is 7!8!. Add them together: 2(8!7!) ways.

5. (15pt) A California study concluded that following 7 simple health rules can extend a man's life by 11 years on the average and a woman's life by 7 years. These 7 rules are as follows: no smoking, get regular exercise, use alcohol only in moderation, get 7 to 8 hours of sleep, maintain proper weight, eat breakfast, and do not eat between meals. In how many ways can a person adopt 5 of these rules to follow
   a. (5pt) if the person presently violates all 7 rules?
   There are $\binom{7}{5} = 21$ ways.

b. (5pt) if the person never drinks and always eats breakfast?

There are $\binom{5}{3} = 10$ ways.

c. (5pt) if the person has no smoking or drinking habits, has swimming as daily routine, and has a great weight control?

There are $\binom{3}{1} = 3$ ways.

6. (10pt) Four couples go to dinner together. The waiter seats the men randomly on one side of the table and the women randomly on the other side of the table. What is the probability that all four couples are seated across from one another?

The possible ways to seat men on one side is 4!, and women on the other side is 4!. Thus, total possible ways to seat men and women on opposite sides are 2(4!4!). 2 indicates there are two sides. The event for couple sit across each other will be 2(4!). Therefore, the probability is: $\frac{2\cdot 4!}{2\cdot 4!\cdot 4!} = \frac{1}{24}$.