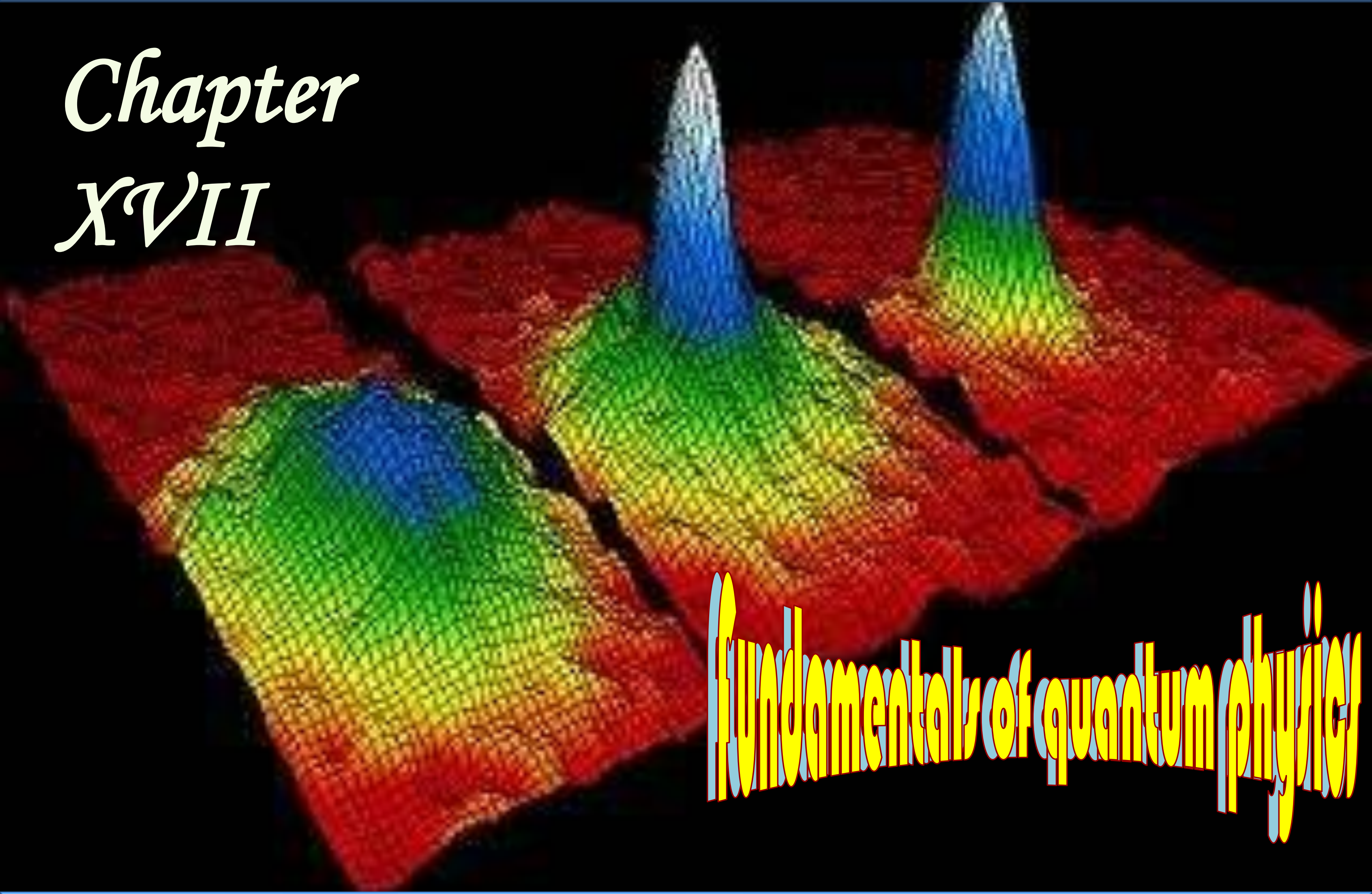
Chapter XVII

Fundamentals of quantum physics



Primary coverage

1 Introduction of the quantum concept

2 Bohr model

3 Mass wave particle duality

§ 17.1

The proposal of the quantum concept

17.1.1 Planck

1. black-body radiation

1. Thermal radiation: the experiment proves that objects can emit different electromagnetic waves at different temperatures, and the electromagnetic radiation with the distribution of wavelength (frequency) and temperature is called thermal radiation.

2. Monochromatic irradiance (radiation ability): the energy of electromagnetic waves emitted from the unit surface area of the object per unit of time nearby.

Monochromatic emission degree unit: $M_{\lambda}(T)\ \mathrm{W/m^3}$

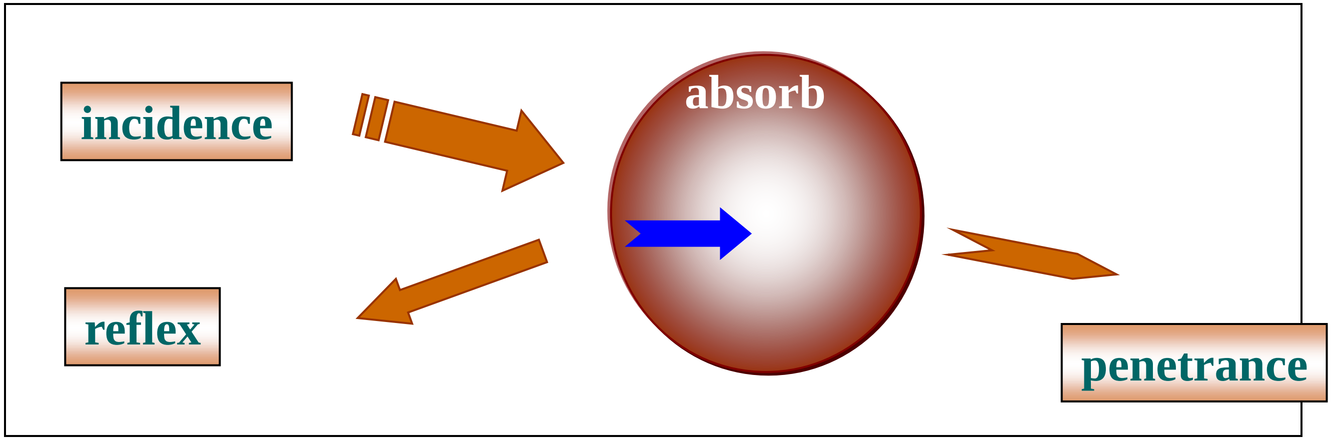
3. Irradiation (total radiation ability)

$$M(T) = \int_0^{\infty} M_{\lambda}(T)\,\mathrm{d}\lambda$$

Monochromatic absorption ratio and monochromatic reflection ratio

➢**Monochromatic absorption ratio $_{\lambda}$(T) ：**

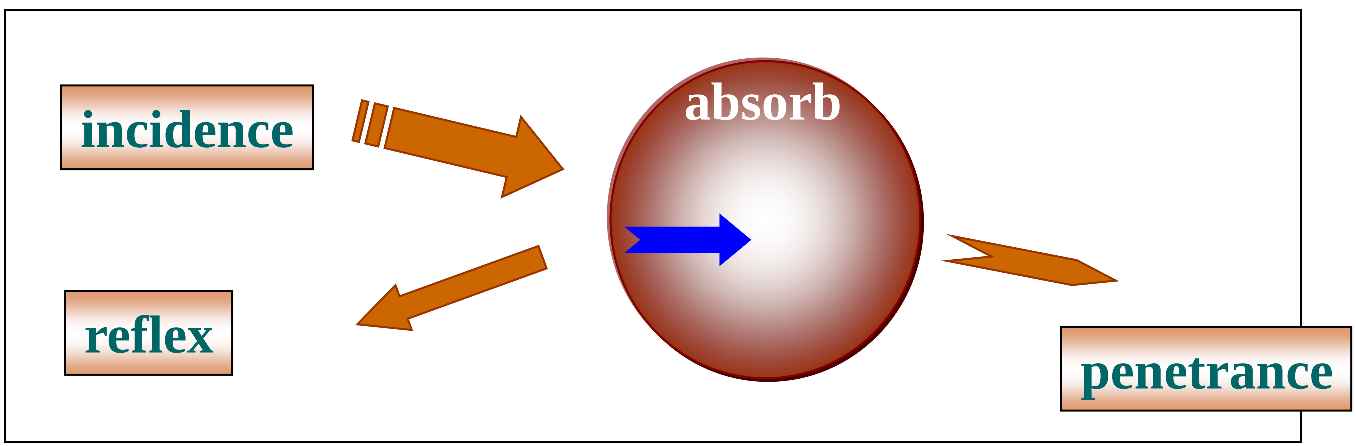
The ratio of the absorbed energy to the incident energy in the wavelength range λ **to** $\lambda + \mathrm{d}\lambda$.



➢**Monochromatic reflection ratio, $r_\lambda(T)$：**

The ratio of the absorbed energy to the incident energy in the wavelength range λ to $\lambda + \mathrm{d}\lambda$

For the opaque objects$_\lambda(T) + r_\lambda(T)=1$



Kirchhoff's law

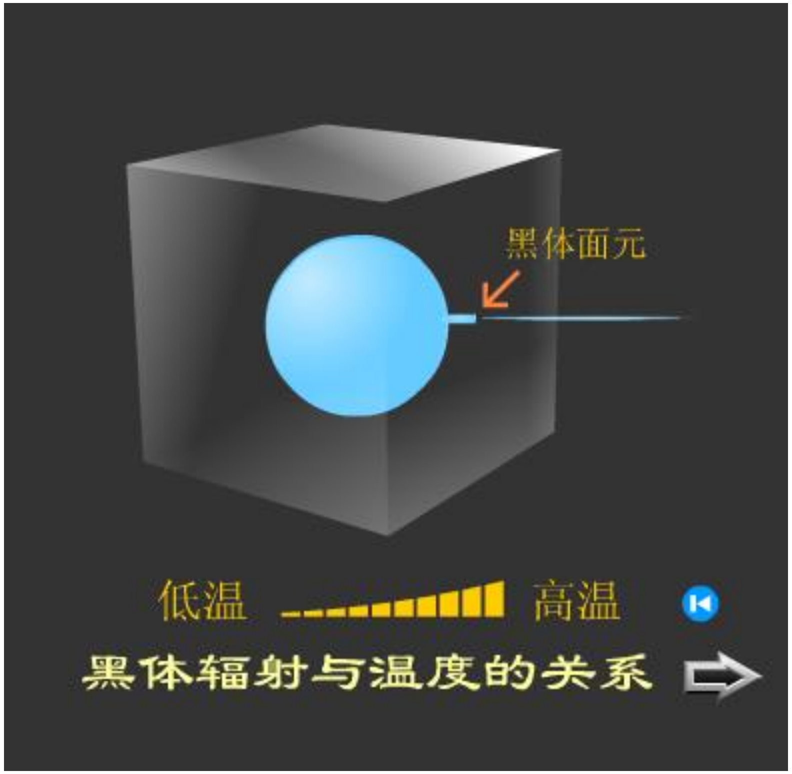
Monochromatic irradiance of any object M$_\lambda$(T) and the monochromatic absorption ratio$_\lambda$(T), equal to the absolute bolbody at the same temperature T M at the same wavelength$_B$（λ，T），i．e

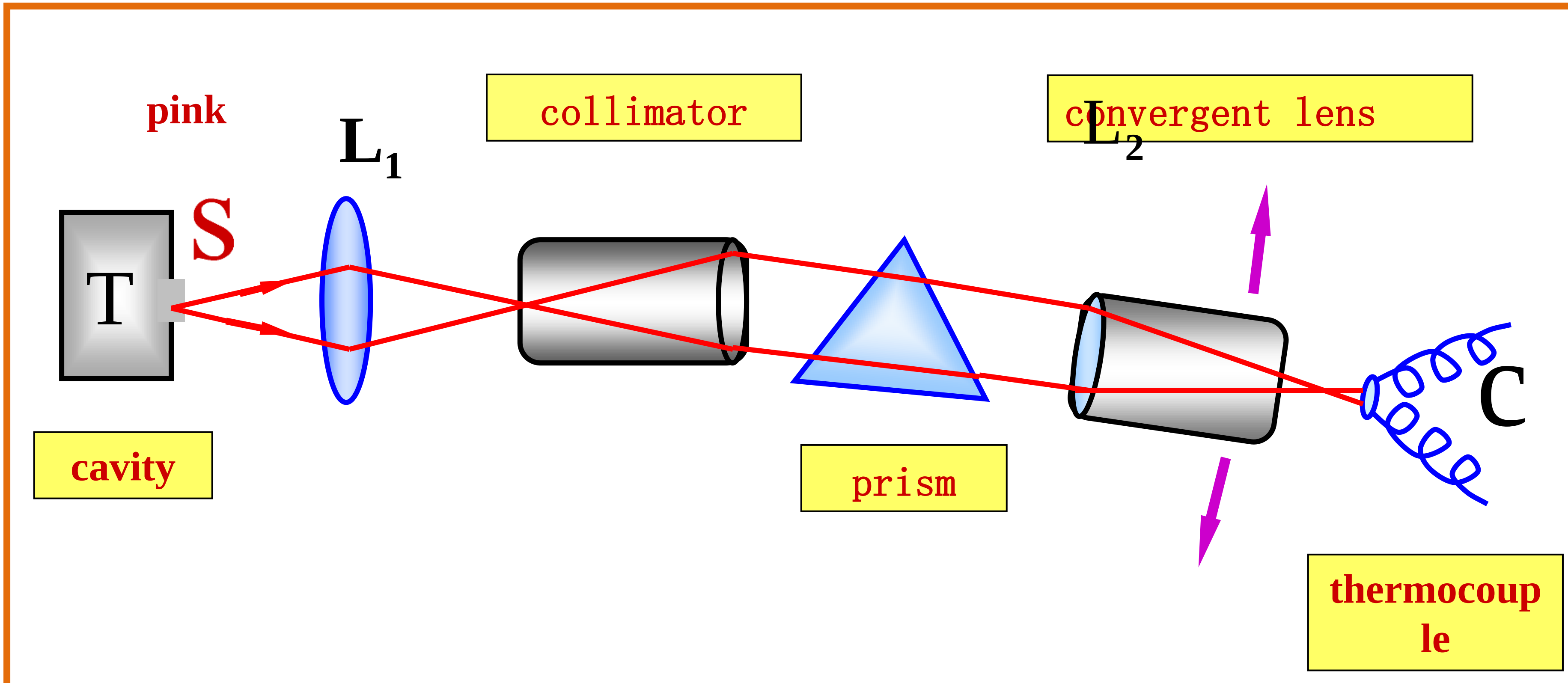
$$\frac{M_\lambda(T)}{\alpha_\lambda(T)} = M_{\mathrm{B}}(\lambda, T)$$

Generally speaking, a good absorber is a good radiation body.

The stronger the radiation capacity, the stronger the absorption capacity.

4. Black body: an object that can completely absorb the electromagnetic radiation of various frequencies irradiating on it is called black body.(Boldface is the ideal model.)





Experimental device for measuring the dradiation radiation

5. The basic law of blackbody thermal radiation

(1) Stetefan- -Boltzmann's law

$$M(T)=\int_0^{\infty} M_\lambda(T)\mathrm{d}\lambda=\sigma T^4$$

Stetfan- -Boltzmann constant

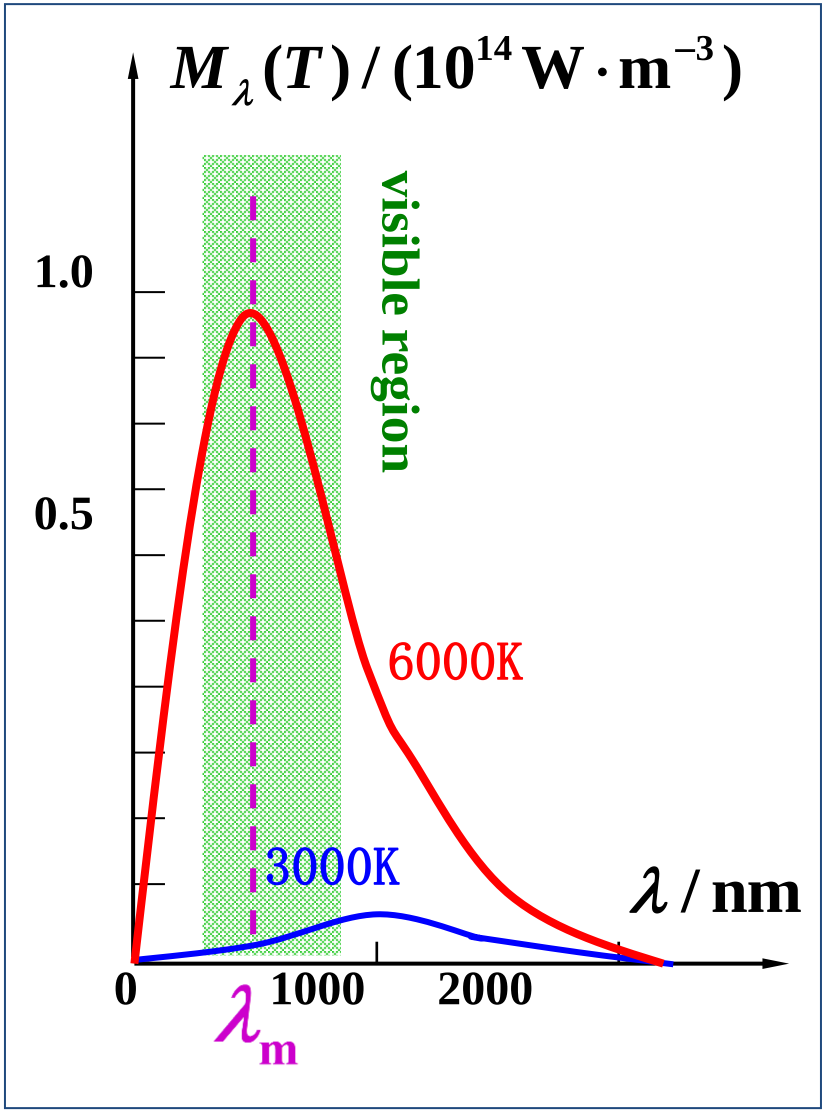
$$\sigma=5.670\times10^{-8}\,\mathrm{W\cdot m^{-2}\cdot K^{-4}}$$

(2) Wien's law of displacement

$$\lambda_{\mathrm{m}} T=b$$

peak wavelength

constant $b=2.898\times10^{-3}\,\mathrm{m\cdot K}$



Example 1 (1) What is the wavelength $(20^{\circ}\mathrm{C})$ corresponding to the peak of the monochromatic irradiance degree?(2) If the wavelength corresponding to the peak of a black body monochrome irradiance is within the range of the red spectral line, what temperature should be?(3) What is the ratio of the above two irradiation degrees?

separate: **(1) By Venn's displacement law**

$$\lambda_{\mathrm{m}} = \frac{b}{T} = \frac{2.898\times10^{-3}}{293}\,\mathrm{nm} = 9890\ \mathrm{nm}$$

(2) Take

$$\lambda_{\mathrm{m}} = 650\ \mathrm{nm}$$

$$T' = \frac{b}{\lambda_{\mathrm{m}}} = \frac{2.898\times10^{-3}}{6.5\times10^{-7}}\,\mathrm{K} = 4.46\times10^{3}\,\mathrm{K}$$

(3) By Stetfan- -Boltzmann's law

$$M(T')/M(T) = (T'/T)^{4} = 5.37\times10^{4}$$

Example 2 Solar peak wavelength $\lambda_{\mathrm{m}} = 483\ \mathrm{nm}$ of monochromatic irradiance,

Try to estimate the temperature of the solar surface.

Solution: by Venn's displacement law

$$T = \frac{b}{\lambda_{\mathrm{m}}} = \frac{2.897 \times 10^{-3}}{500 \times 10^{-9}}\mathrm{K} \approx 6000\mathrm{K}$$

The surface temperature of other luminous stars in the universe can also be speculated in this way.

2. The Planck energy subhypothesis

从经典物理理论出发推导M(λ, T)函数表达式

1. Rayleigh —— kins formula

$$M_o(\lambda, T) = CT\lambda^{-4}$$

The long wave fits with the experimental curve

Short wave is very different —— ultraviolet disaster

2. The Wien formula

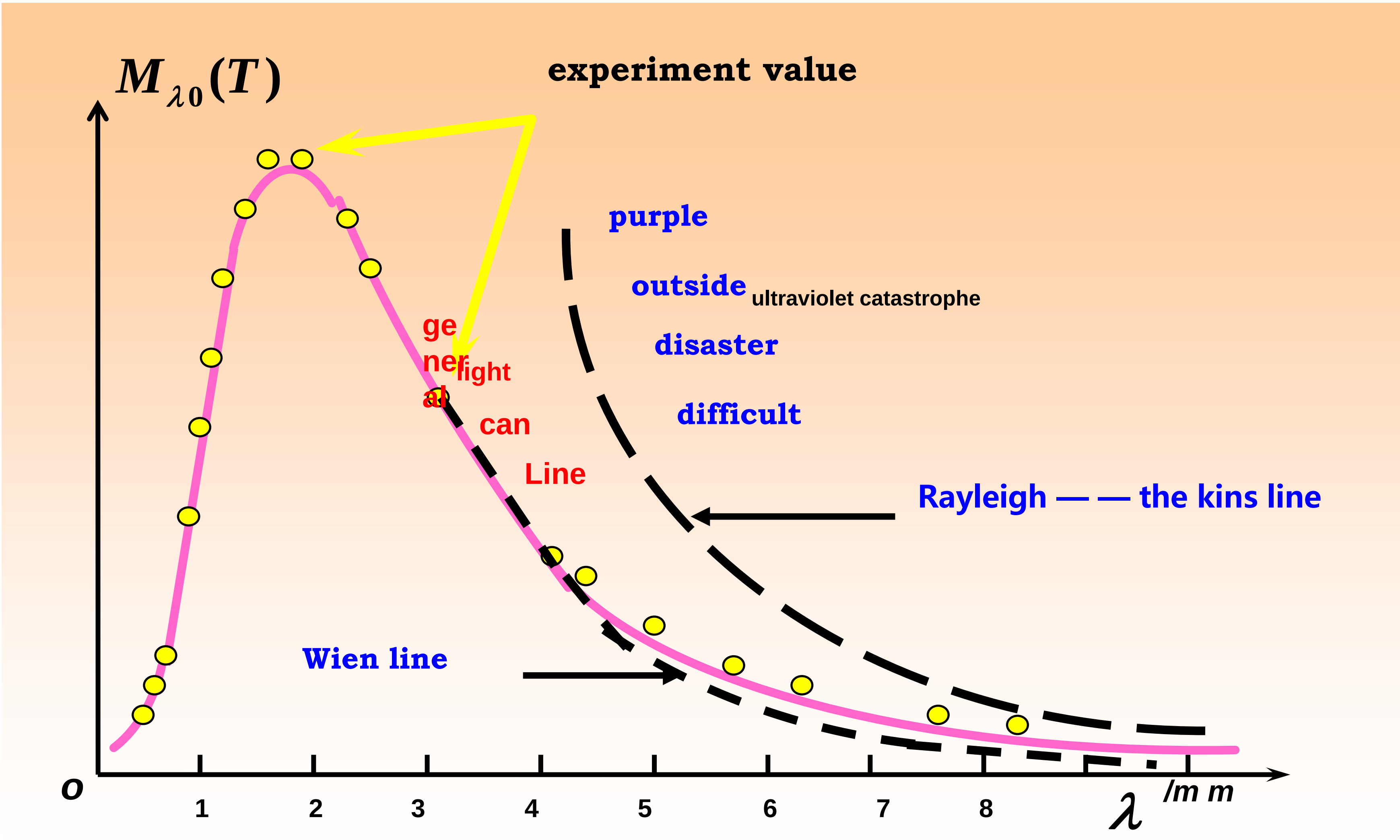
$$M_o(\lambda, T) = C_1\lambda^{-5}e^{-\frac{C_2}{\lambda T}}$$

The short wave is close to the experimental curve

The long wave differs greatly

Judging from the classical theory

$M_o(\lambda, T)$公式的努力均遭失败


$M_{\lambda 0}(T)$
experiment value
purple
outside
ultraviolet catastrophe
disaster
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Rayleigh — — the kins line
Wien line
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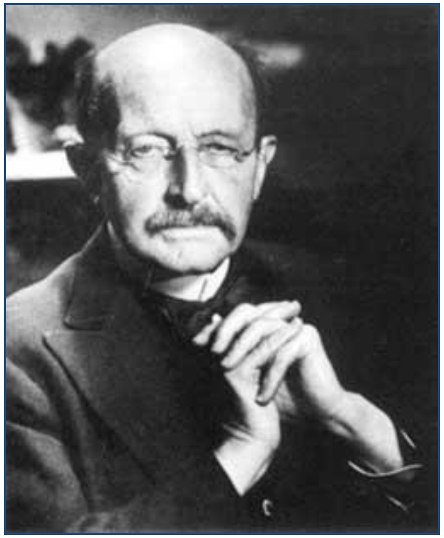
λ
/m m

Planck (1858- -1947)

A German theoretical physicist, quantum theory founder. In 1900 he was in German physics
At the meeting, read out on the normal spectrum
The theory of the distribution of energy in China thesis.
Lauer called this one
Heaven is a " quantum theory of
Birth day ".the quantum theory
And relativity constitute the study of modern physics foundation.

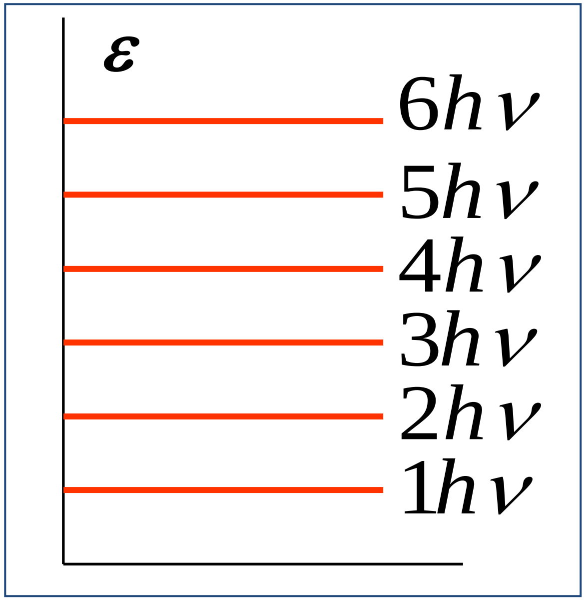


Planck believes that the vibration of electrons in the wall of a metal cavity can be regarded as a one-dimensional harmonic oscillator. When it absorbs or emits electromagnetic radiation energy, it can not continuously absorb or emit energy as classical physics thought in the past, but is proportional to the frequency of the oscillator

The energy ons are units to absorb or emit energy.

$$\varepsilon = h\nu$$

The absorption or emission energy of the charged harmonic oscillator on the cavity wall shall be



$$\varepsilon = nh\nu \quad (n = 1, 2, 3, \cdots)$$

Planck constant

$$h = 6.6260755 \times 10^{-34}\ \mathrm{J \cdot s}$$

The Planck black-body radiation formula

$$M_{\lambda 0}(T) = \frac{2\pi hc^2\lambda^{-5}}{\mathrm{e}^{hc/kT\lambda} - 1}$$

Example 3 has the mass of a fork tip is 0.050kg, adjust its frequency to, amplitude. ask

$$\nu = 480\ \mathrm{Hz} \qquad A = 1.0\mathrm{mm}$$

(1) the quantum number of the tip vibration;

(2) What is the amplitude change when the quantum number increases from n **to?** $n+1$

Solution (1) $$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m(2\pi\nu)^2 A^2 = 0.227\mathrm{J}$$

$$E = nh\nu \qquad n = \frac{E}{h\nu} = 7.13\times10^{29}$$

motif energy $$h\nu = 3.18\times10^{-31}\ \mathrm{J}$$

（2）

$$E = nh\nu$$

$$A^2 = \frac{E}{2\pi^2 m\nu^2} = \frac{nh}{2\pi^2 m\nu}$$

$$2A\mathrm{d}A = \frac{h}{2\pi^2 m\nu}\mathrm{d}n$$

$$\Delta A = \frac{\Delta n}{n}\frac{A}{2} \qquad \Delta n = 1$$

$$\Delta A = 7.01\times10^{-34}\,\mathrm{m}$$

It shows that in the macroscopic range, the effect of energy quantization is extremely obvious, that is, the energy of macroscopic objects can be regarded as completely continuous.

17.1.2 Photoelectric effect

1. The Einstein Light quantum hypothesis

The energy of the photon is $E = h\nu$

The static mass of the photon is $m_0 = 0$

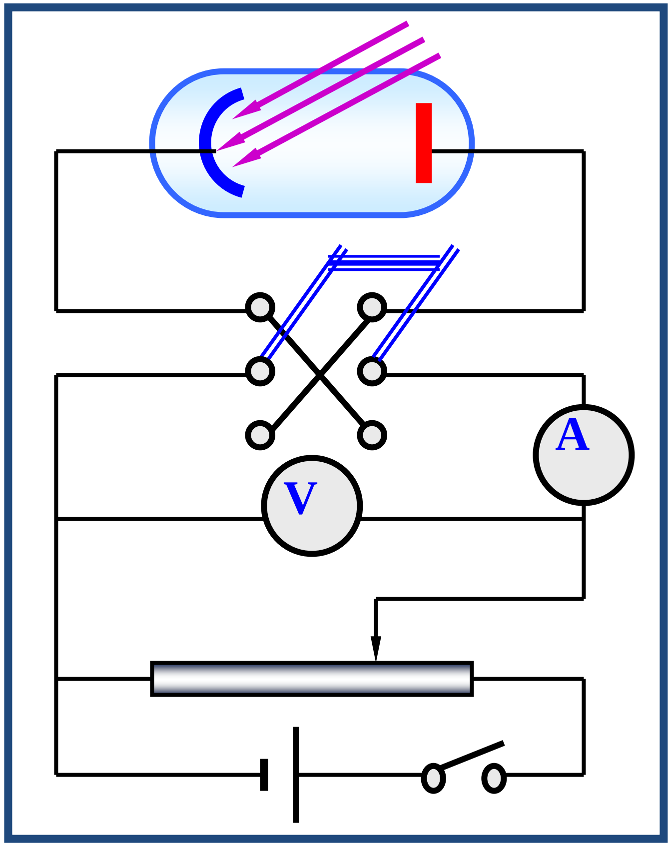
The relation of the relativistic energy and the momentum is $E^2 = p^2c^2 + m_0^2c^4$

The momentum of the photon is $p = \dfrac{E}{c} = \dfrac{h}{\lambda}$

The energy-flow density of the monochromatic light is $S = Nh\nu$

2. The law of the photoelectric effect experiment

(1) Experimental device



Light irradiation to the metal surface, electrons escape from the metal surface, called the photoelectron.

(2) Experimental rules

◈Cutoff frequency (red limit) ν_0

$$\nu > \nu_0$$

Only if the photoelectric effect occurs, the cutoff frequency and material related and light intensity independent.

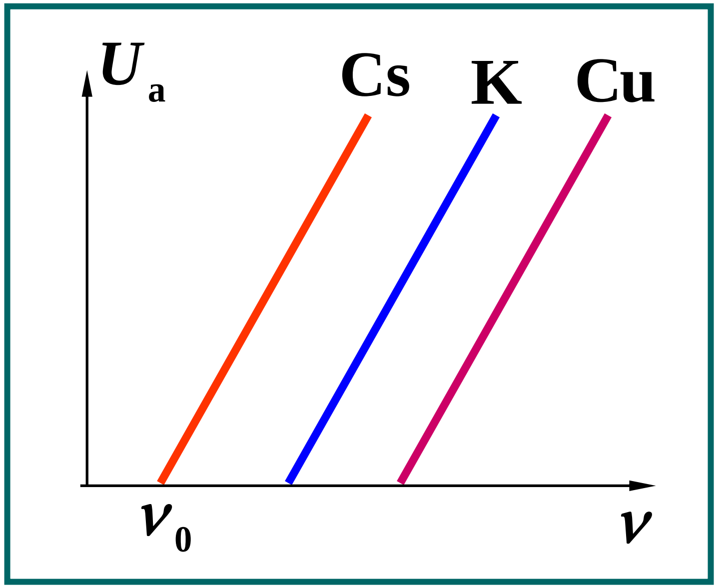
Cut-off frequencies for several metals

metal	Cesium, sodium, zinc, silver, and platinum				
cut-off frequency $\nu_0 / 10^{14}$ Hz	4.69	5.53	8.06	11.55	15.28

◈Stop voltage U_a

$$eU_a = E_{k,max} = \frac{1}{2} m v_m^2$$

The check potential difference is linear with the incident light frequency.



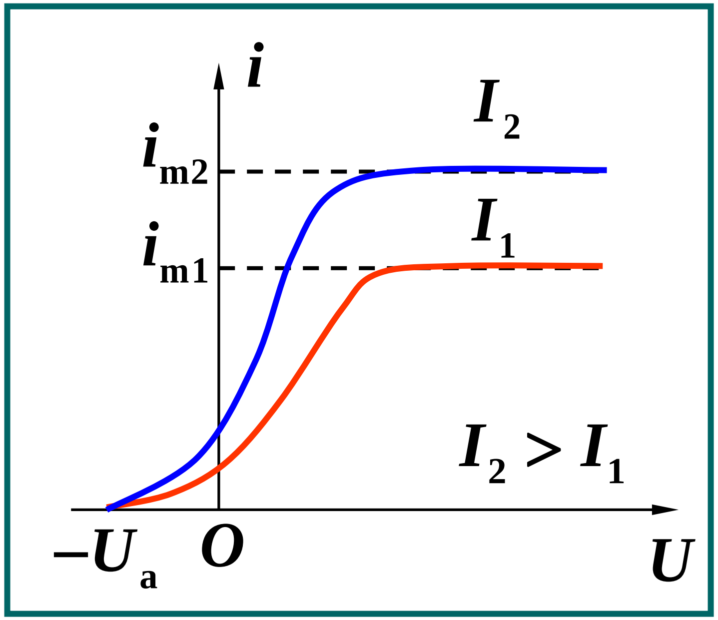
◈instantaneity

When the light hits the metal surface, the photoelectrons escape almost immediately

◈Current saturation i_m value

$$i_m \propto I \text{ (light intensity)}$$

The stop voltage U_a is independent of the light intensity



3. Difficulties encountered by the classical theory

◈Red limit problem

According to the classical theory, no matter what the frequency of the incident light, as long as its intensity

Large enough to allow the electron to have enough energy to escape the metal. With the real

The test result is inconsistent.

◈Transient problem

According to the classical theory, the energy needed for electrons to escape a metal is needed

A certain amount of time to accumulate, until enough for electrons to escape the metal

So far on the surface. Not consistent with the experimental results.

4. Einstein equation

$$h\nu = \frac{1}{2}mv_{\mathrm{m}}^{2} + A_0$$

Escape work is related to the material

◈**For the same metal, A_0 certainly, $E_{\mathrm{k}} \propto \nu$, is independent of the light intensity**

The escape work of several metals

metal	Sodium, zinc, aluminum, copper, silver, and platinum
A_0 / eV	2.29 3.34 3.74 4.47 4.78 6.33

Einstein equation $$h\nu = \frac{1}{2} m v_{\mathrm{m}}^2 + A_0$$

◈work function $A_0 = h\nu_0$

Generate the photoelectric effect conditions $\nu > \nu_0 = A_0 / h$ (cut-off frequency)

◈The greater the light intensity, the more the number of photons, that is, per unit time

The more children, the larger the photocurrent. $\nu > \nu_0$

◈Photons shoot to the metal surface, one photon carrying energy $h\nu$ to one

Subicity is absorbed by an electron, if $\nu > \nu_0$ the electron immediately escapes,

No time accumulation (instantaneous).

Example 1 Monochromatic light with a wavelength of 450nm shines onto the surface of pure sodium.

Find (1) the photon energy and momentum of this light;

(2) the kinetic energy of the sodium surface;

(3) If the energy of the photon is 2.40 eV, what is its wavelength?

Solution (1) $$E = h\nu = \frac{hc}{\lambda} = 4.42\times10^{-19}\,\mathrm{J} = 2.76\mathrm{eV}$$

$$p = \frac{h}{\lambda} = \frac{E}{c} = 1.47\times10^{-27}\,\mathrm{kg\cdot m\cdot s^{-1}} = 2.76\,\mathrm{eV}/c$$

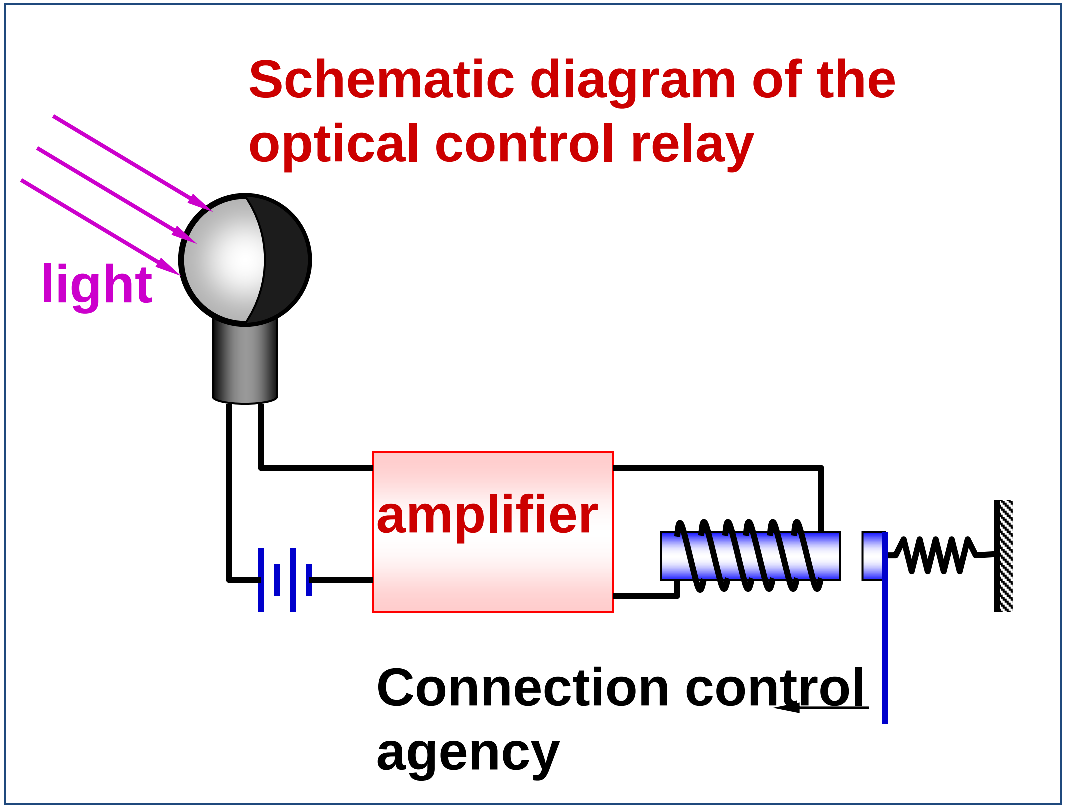
（2） $$E_\mathrm{k} = E - A_0 = (2.76 - 2.29)\mathrm{eV} = 0.47\,\mathrm{eV}$$

（3） $$\lambda = \frac{hc}{E} = 5.18\times10^{-7}\,\mathrm{m} = 518\,\mathrm{nm}$$

3. The application of photoelectric effect in modern technology

Optical control relay, automatic control,

Automatic counting, automatic alarm, etc.



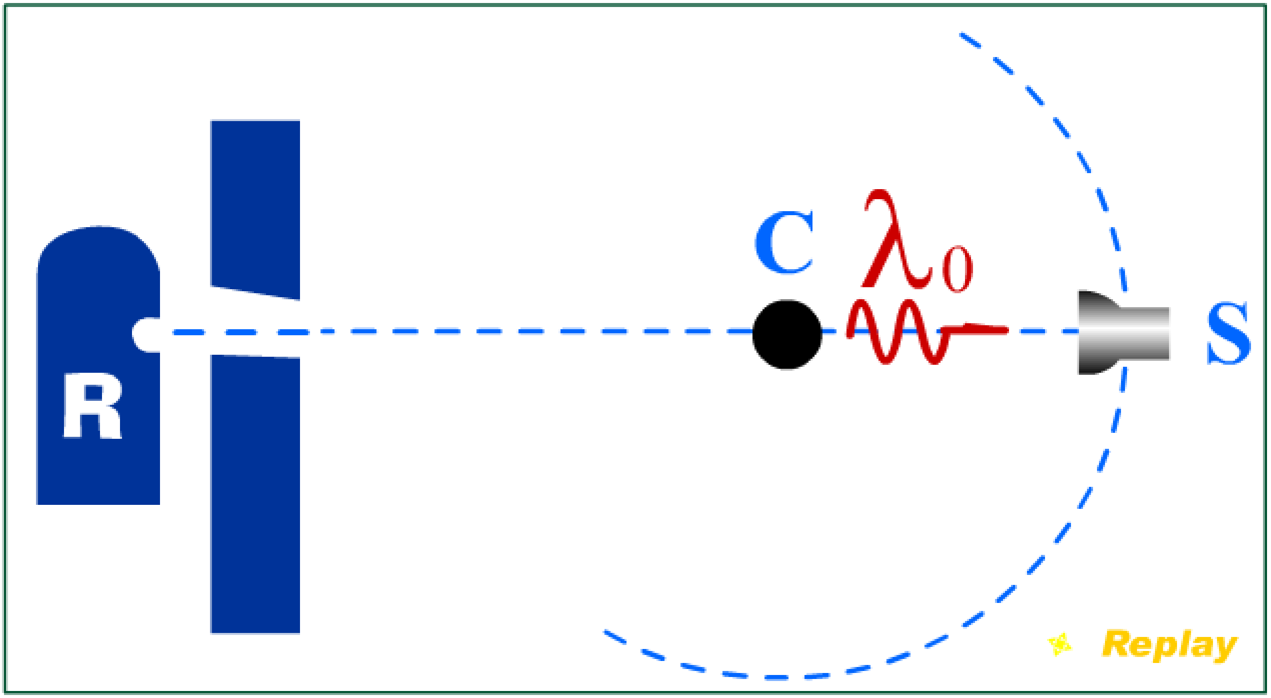
photomultiplier

17.1.3 The Compton effect

In 1920, the American physicist Compton observed the X-ray scattering by matter, and found that the scattering rays contained a changed wavelength of the components.

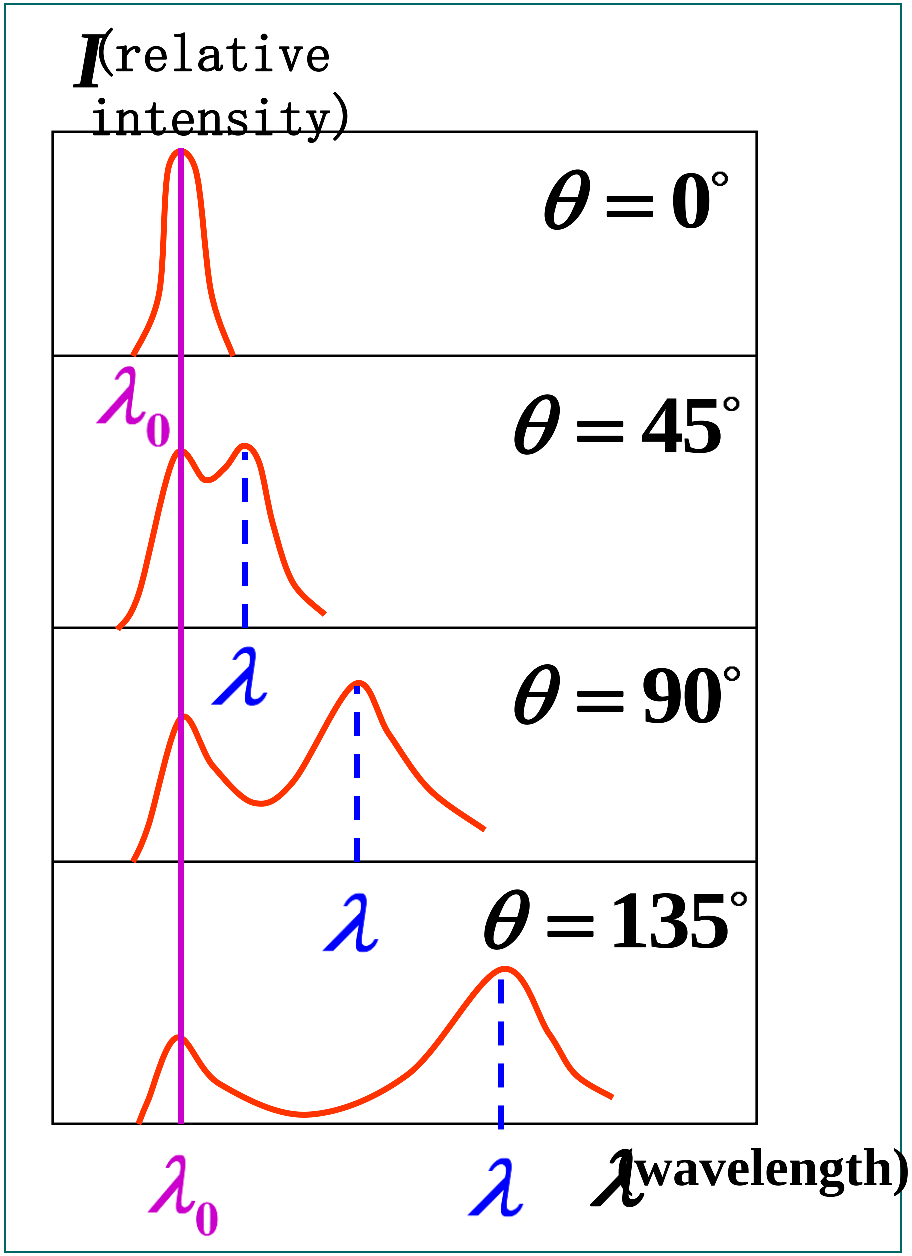
1. Experimental regularities of the Compton effect

1. experimental installation



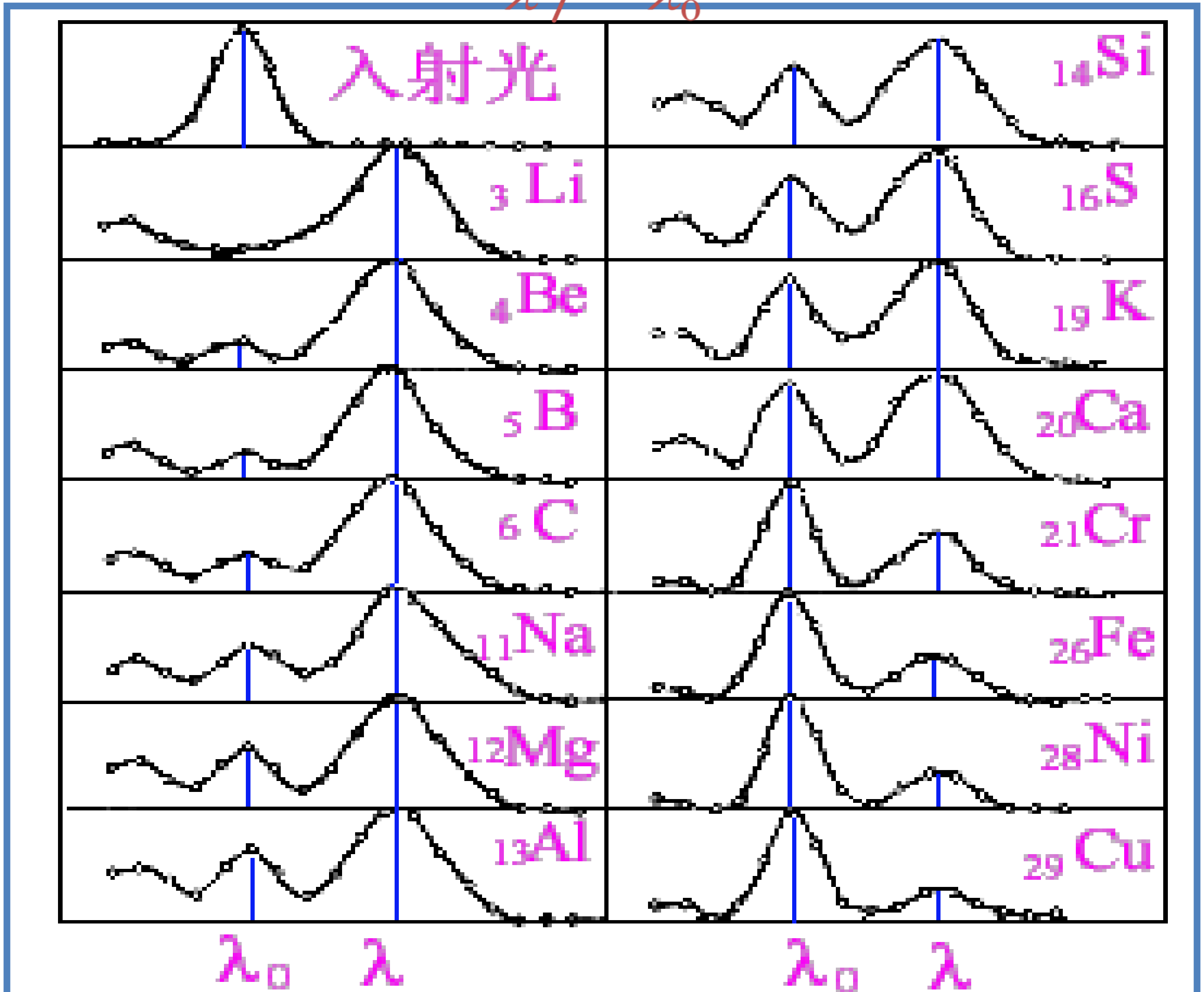
2. Experimental results

In the scattered X-rays in addition to the incident wavelength, there is the wavelength is longer than the incident wavelength.



With the scattering matter at the same scattering angle

$I_{\lambda}/I_{\lambda_0}$



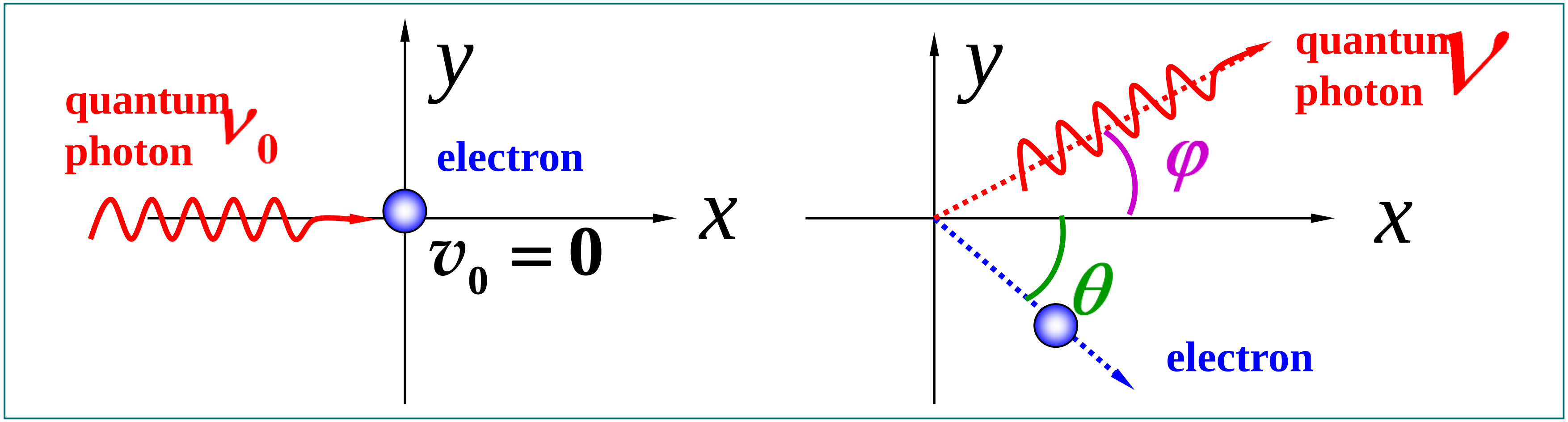
Difficulties of the classical theory

According to the classical electromagnetic theory, the charged particle is forced by the action of the incident electromagnetic wave, thus radiating the electromagnetic wave in all directions. The frequency of the scattering beam should be the same as the incident beam frequency, and the charged particle only plays the role of energy transmission.

So, classical theory cannot explain the wavelength of scattering.

3. Photon theory explains the Compton effect

1. Physical model



◈ The incident photon (X-ray or ray γ) energy is large.

$$E = h\nu$$ The scope is: $10^4 \sim 10^5 \mathrm{eV}$

◈Solid surface electron binding can be regarded as near free electron.

◈The electron thermal motion energy, which can be approximated as a stationary electron. $\ll h\nu$

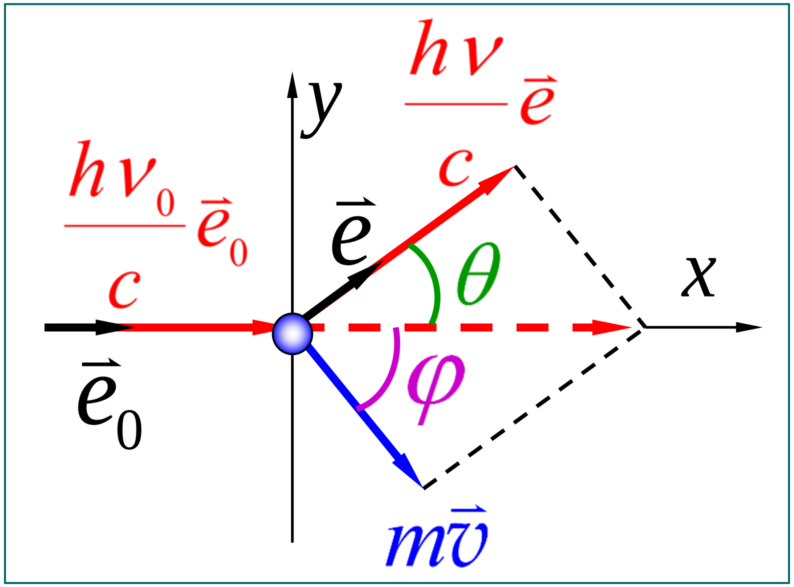
◈The electron recoil velocity is very large and needs to be treated with relativistic mechanics.

2 Qualitative analysis

(1) When the incoming photon collides with the weakly bound electron in the scattering matter, part of the energy is transmitted to the electrons, the energy of the scattered photon decreases, the frequency drops and the wavelength becomes larger.

(2) When a photon collides with a tightly bound electron in an atom, however, the energy will not decrease significantly, so the same wavelength as the incident light appears in the scattering beam.

3 for the quantitative calculation



conservation of energy

$$h\nu_0 + m_0c^2 = h\nu + mc^2$$

conservation of momentum

$$\frac{h\nu_0}{c}\vec{e}_0 = \frac{h\nu}{c}\vec{e} + m\vec{v}$$

$$m^2v^2 = \frac{h^2\nu_0^2}{c^2} + \frac{h^2\nu^2}{c^2} - 2\frac{h^2\nu_0\nu}{c^2}\cos\theta$$

$$m^2 v^2 = \frac{h^2 \nu_0^2}{c^2} + \frac{h^2 \nu^2}{c^2} - 2\frac{h^2 \nu_0 \nu}{c^2}\cos\theta$$

$$m^2 c^4 (1 - \frac{v^2}{c^2}) = m_0^2 c^4 - 2h^2 \nu_0 \nu (1 - \cos\theta) + 2m_0 c^2 h(\nu_0 - \nu)$$

$$m = m_0 (1 - v^2 / c^2)^{-1/2}$$

$$\frac{c}{\nu} - \frac{c}{\nu_0} = \frac{h}{m_0 c}(1 - \cos\theta) = \lambda - \lambda_0 = \Delta\lambda$$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) = \frac{2h}{m_0 c}\sin^2\frac{\theta}{2}$$

- **Compton wavelength**

$$\lambda_C = \frac{h}{m_0 c} = 2.43\times10^{-12}\ \mathrm{m}$$

- **Compton formula**

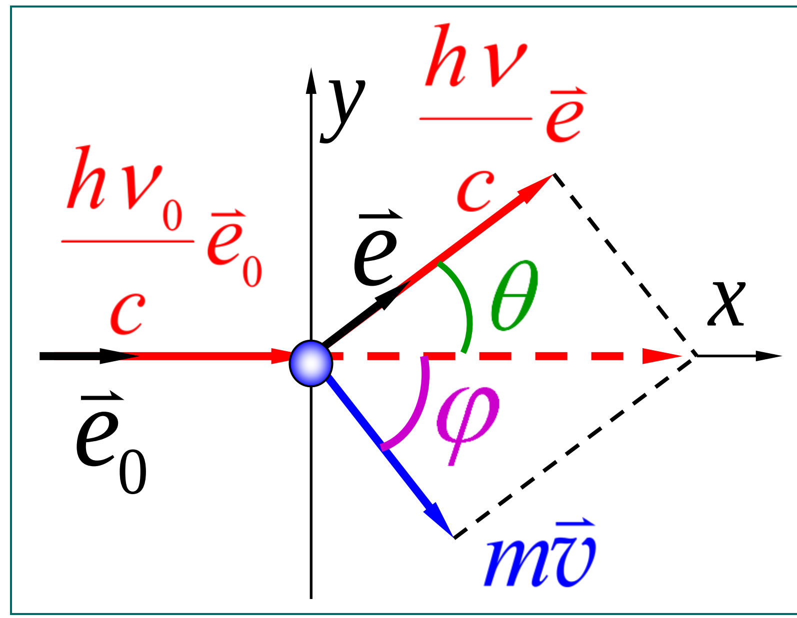
$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) = \lambda_C(1-\cos\theta)$$

4 Conclusion

◈The wavelength of scattered $\Delta\lambda$ wavelength is only related θ .

$$\theta = 0, \Delta\lambda = 0$$

$$\theta = \pi, \ \ (\Delta\lambda)_{\max} = 2\lambda_C$$



◈The scattered photon energy decreases

$$\lambda > \lambda_0, \nu < \nu_0$$

5 for the discussion

◆ **Light has a wave-particle duality**

In general, light is volatile and its particles.

◆**If,** $\lambda_0 >> \lambda_C$ **the Compton effect** $\lambda \approx \lambda_0$ **is not observed in visible light.**

- $\Delta\lambda$ θ **Independent of matter, it is the interaction between photons and near-free electrons.**

- **The scattered light** $\Delta\lambda = 0$ **is the action of photons and tightly bound electrons. The Compton effect is not obvious.**

6. Physical meaning

◈ **The correctness of the photon hypothesis, the correctness of the special relativity mechanics.**

◈ **The microscopic particle interactions also follow the laws of conservation of energy and momentum.**

A wavelength X-ray is elastically collides with a stationary free electron, observed in the direction angling the incident angle, q $\lambda_0 = 1.00\times10^{-10}\text{m}$ 90°

(1) What is the change of the scattering wavelength? $\Delta\lambda$

(2) How much kinetic energy does the recoil electron get?

(3) How much energy loss does the photon lose in a collision?

Solution (1) $\Delta\lambda = \lambda_C(1-\cos\varphi) = \lambda_C(1-\cos 90^\circ) = \lambda_C$

$$= 2.43\times10^{-12}\,\text{m}$$

(2) The kinetic energy of the recoil electrons

$$E_\text{k} = mc^2 - m_\text{e}c^2 = \frac{hc}{\lambda_0} - \frac{hc}{\lambda} = \frac{hc}{\lambda_0}\left(1-\frac{\lambda_0}{\lambda}\right) = 295\,\text{eV}$$

(3) Photon-lost energy = the kinetic energy of the recoil electrons

§ 17.2

Hydrogen atomic model of Bohr

17.2.1 Spectra of hydrogen atoms, the Rydber equation

1. The regularity of the hydrogen atomic spectrum

◈In 1885, the Swiss mathematician Bar discovered the visible light of hydrogen atoms

$$\sigma = \frac{1}{\lambda} = \frac{4}{B}\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad n = 3, 4, 5, \cdots$$

◈In 1890, the Swedish physicist Ridberg gave the formula for the hydrogen atomic spectrum

$$\boxed{\sigma = \frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

$$m = 1, 2, 3, \cdots, \quad n = m+1, m+2, m+3, \cdots$$

Rydberg constant $R = 1.097 \times 10^7\,\mathrm{m}^{-1}$

ultraviolet — Lyman series $$\sigma = \frac{1}{\lambda} = R(\frac{1}{1^2} - \frac{1}{n^2}), \quad n = 2, 3, \cdots$$

visible light — Balmer series $$\sigma = \frac{1}{\lambda} = R(\frac{1}{2^2} - \frac{1}{n^2}), \quad n = 3, 4, \cdots$$

infrared

Paschen series $$\sigma = \frac{1}{\lambda} = R(\frac{1}{3^2} - \frac{1}{n^2}), \quad n = 4, 5, \cdots$$

Brackett series $$\sigma = \frac{1}{\lambda} = R(\frac{1}{4^2} - \frac{1}{n^2}), \quad n = 5, 6, \cdots$$

Pfund series $$\sigma = \frac{1}{\lambda} = R(\frac{1}{5^2} - \frac{1}{n^2}), \quad n = 6, 7, \cdots$$

Humphrey department $$\sigma = \frac{1}{\lambda} = R(\frac{1}{6^2} - \frac{1}{n^2}), \quad n = 7, 8, \cdots$$

◈Reeds combination principle

$$\boxed{\sigma = T(m) - T(n)}$$

$T(m)$、$T(n)$ It is called a spectral term

Experiments show that the combination principle is also applicable to the atomic spectra of other elements (such as alkali metal elements), but the representation of the spectral terms is more complex.

17.2.2 Rutherford's atomic planet model

- **In 1897, the J.J. Tom Sun found the electronics.**

- **In 1903, Tom Sun proposed the atomic "Raisin cake model".**

The positive charge in the atoms and the mass of the atoms are immersed in them.

$$10^{-10}\,\mathrm{m}$$

Rutherford (E.Rufherford， 1871—1937)

A British physicist. In 1899, it was discovered that uranium salts emitted α and β rays, and the decay theory and law of natural radioactive elements were proposed.

According to the α particle scattering experiment, the atomic nuclear model is proposed, and the study of atomic structure is led to the correct orbit, so it is known as the father of atomic physics.

◈ **Rutherford's atomic nuclear model (planetary model)**

At the center of the atom is a positively charged nucleus, which concentrates almost all the entire mass of the atom. Electrons rotate around this nucleus, and the size of the nucleus is very small compared to the whole atom.

17.2.3 Bohr

1. Difficulties of planetary models

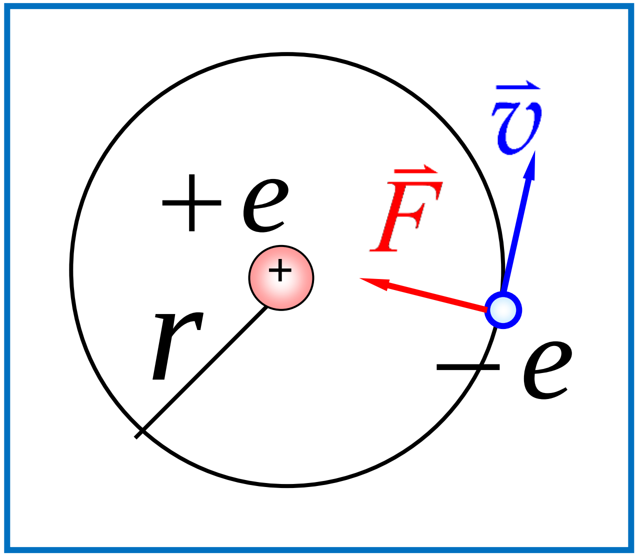
According to the classical electromagnetic theory, the electrons will move uniformly around the nucleus, and the accelerated electrons will constantly radiate the electromagnetic waves outward.

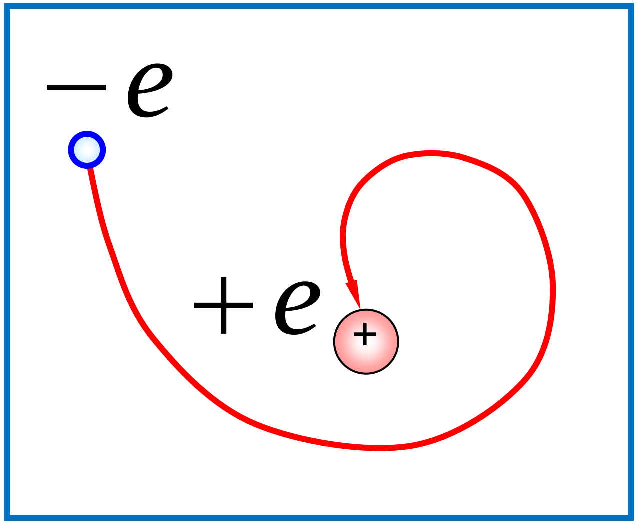
◈The atoms are constantly radiating energy outward,

The energy gradually decreases, the frequency of electron rotation around the core is gradually changed, the emission spectrum should be continuum;

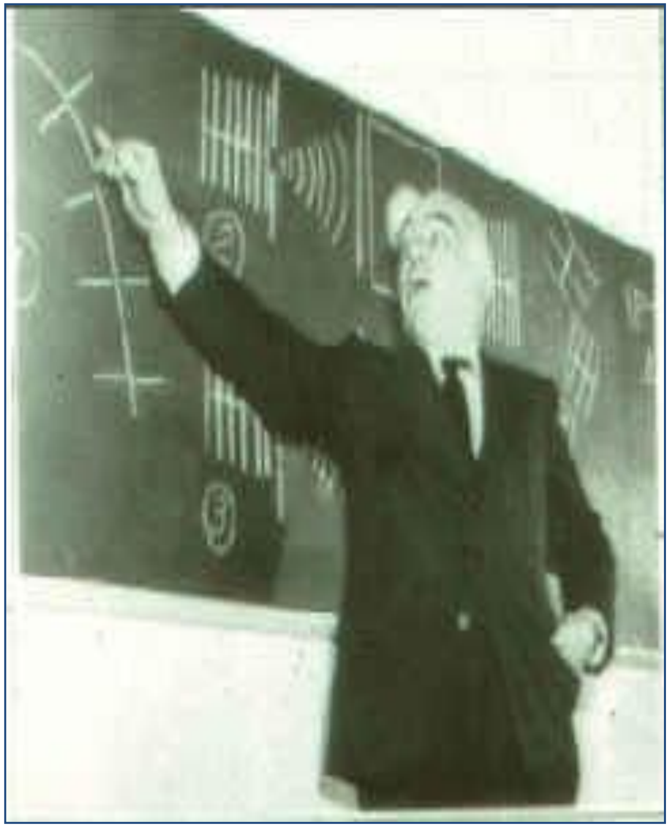
◈ By decreasing the total atomic energy, the electron

Will gradually approach the nucleus and then meet, and the atoms are unstable.





Bohr (Bohr.Niels 1885—1962)



A Danish theoretical physicist, one of the founders of modern physics.

Based on the Rutherford nuclear model, three assumptions about the theory of atomic stability and quantum transition can fully explain the laws of the hydrogen spectrum.

In 1922, Bohr won the Nobel Prize in physics.

2. Bohr's three assumptions

Suppose an electron in an atom can only move in a certain orbit without radiating electromagnetic waves, and then the atom is in a stable state (steady state) and has a certain amount of energy.

(Condition of atom)

Consider the angular momentum of the electron. v r

L $h/2\pi$

(Angular momentum quantization condition of the electron)

$$L = mvr_n = n\frac{h}{2\pi}$$

principal quantum number

$$n = 1, 2, 3, \cdots$$

Suppose three the atoms transition from high energy to low energy E_n

E_m To emit a photons with a frequency of. ν

$$h\nu = E_n - E_m$$

(Photon frequency condition)

◈**Hydrogen atom energy level formula**

By Newton's law
$$\frac{e^2}{4\pi\varepsilon_0 r_n^2} = m\frac{v_n^2}{r_n}$$

By the hypothesis 2-quantization condition
$$m v_n r_n = n\frac{h}{2\pi}$$

$$\boxed{r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2 = r_1 n^2 \qquad (n = 1, 2, 3, \cdots)}$$

$n = 1$, Bohr radius
$$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.529 \times 10^{-10}\,\mathrm{m} = a$$

Total energy of the first-orbital atom n
$$E_n = \frac{1}{2} m v_n^2 - \frac{e^2}{4\pi\varepsilon_0 r_n}$$

$$E_n = \frac{1}{2} m v_n^2 - \frac{e^2}{4\pi\varepsilon_0 r_n}$$

$$\boxed{E_n = -\frac{me^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2} = \frac{E_1}{n^2}}$$

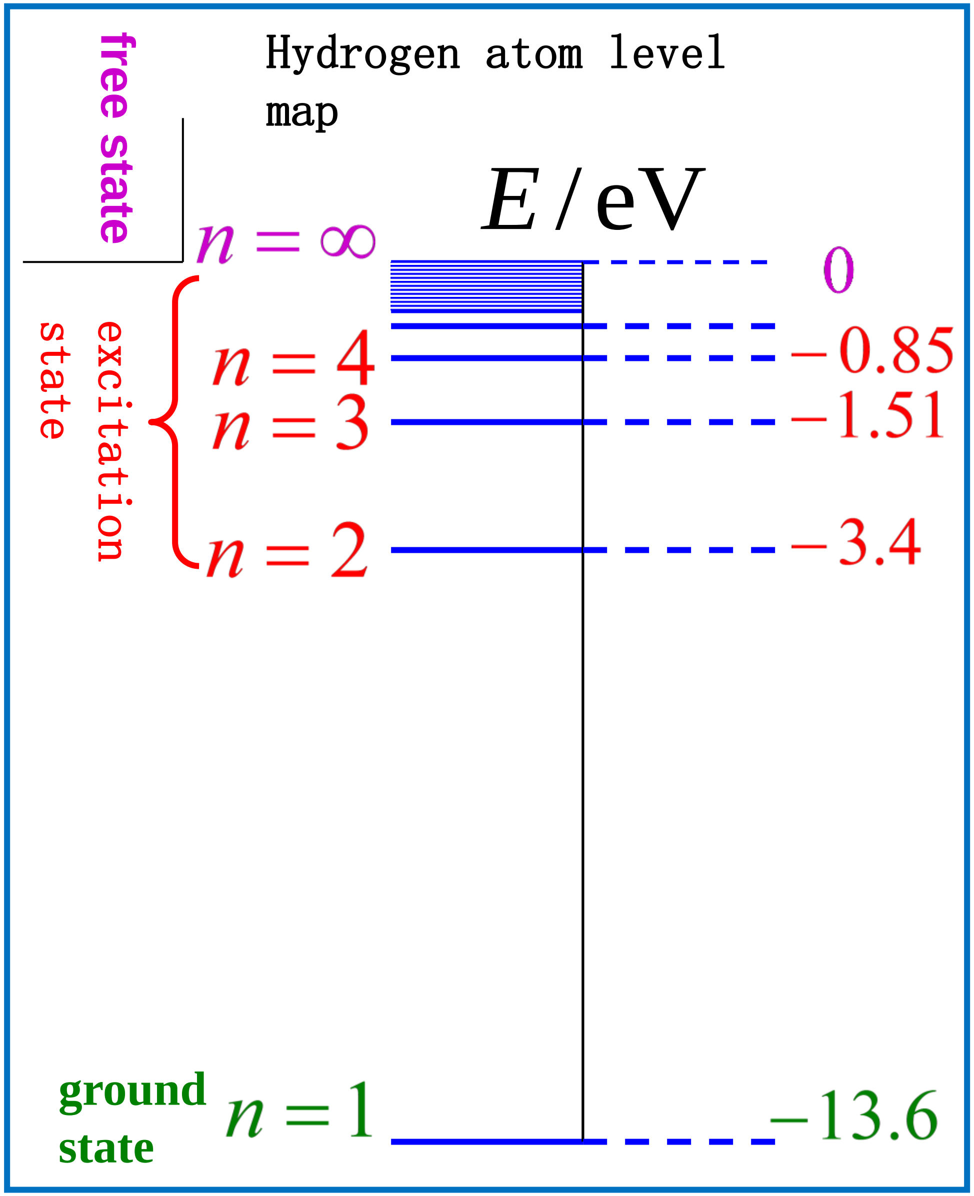
The ground state energy $(n=1)$

$$E_1 = -\frac{me^4}{8\varepsilon_0^2 h^2} = -13.6\text{eV}$$

(ionization energy)

excited state energy $(n>1)$

$$E_n = E_1 / n^2$$



◈**Interpretation of the spectra of hydrogen atoms by the Bohr theory**

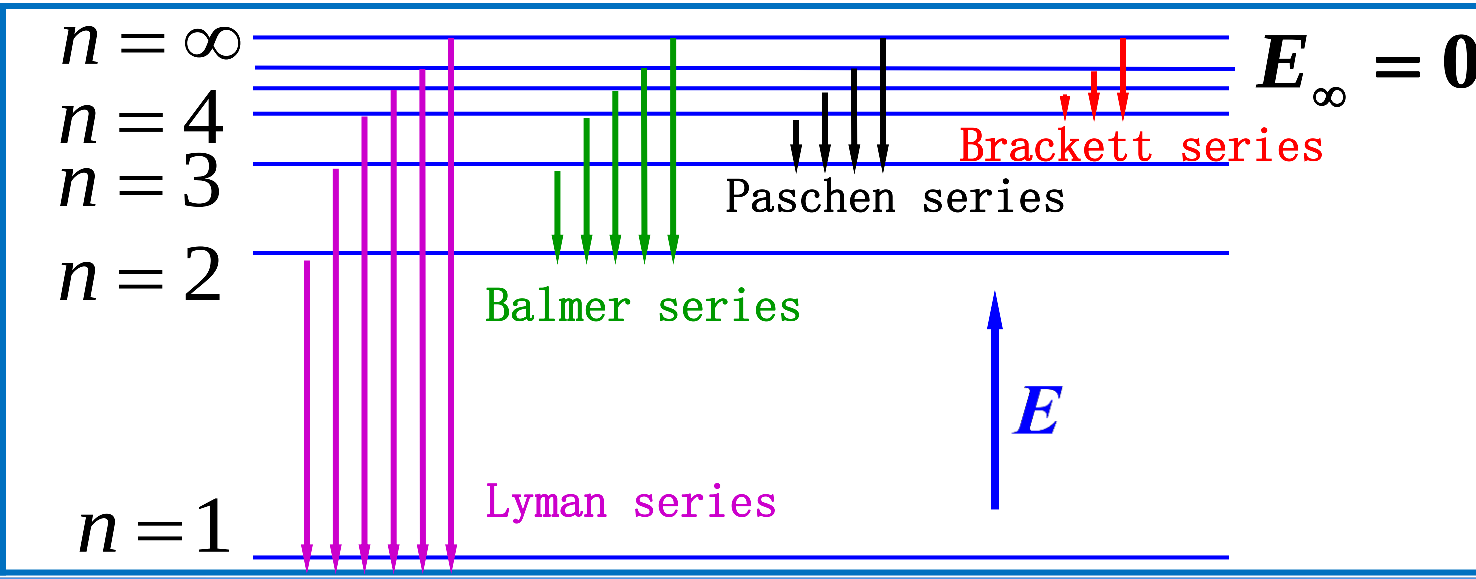
$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2}$$

$$\boxed{h\nu = E_n - E_m}$$

$$\sigma = \frac{1}{\lambda} = \frac{\nu}{c} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \quad n > m$$

$$\frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097\times 10^7\,\mathrm{m}^{-1} \approx R$$ (Rydberg constant)

With the spectral lineage Hydrogen atom energy level transitions



3. The significance and difficulty of the Bohr theory of hydrogen atoms

(1) Correctly point out the existence of the atomic energy level (atomic energy quantization);

(2) to correctly point out the concept of fixed state and angular momentum quantization;

(3) Correct explanation of hydrogen atoms and hydrogen-like ion spectrum;

(4) Unable to explain the more complex atoms than hydrogen atoms;

(5) It is incorrect to treat the motion of microscopic particles as a definite orbit;

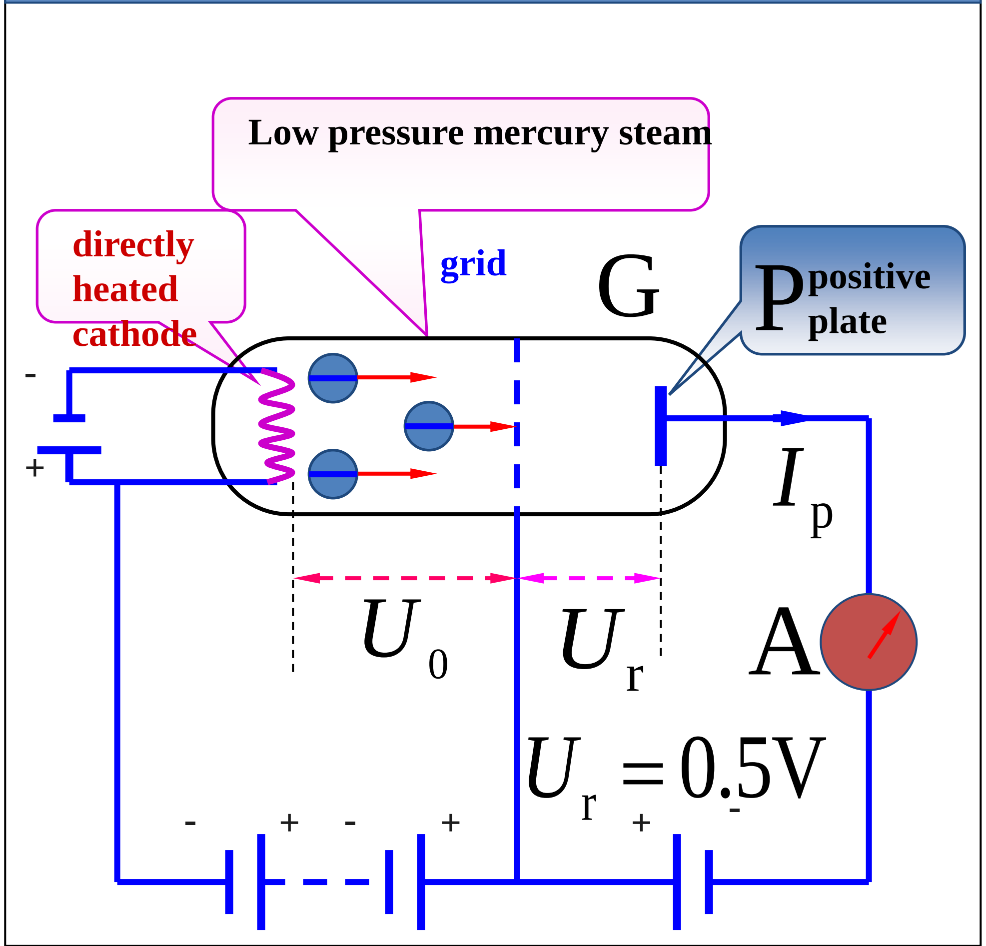
(6) It is a semiclassical and semi-quantum theory, which has a logical disadvantage, namely, the theory

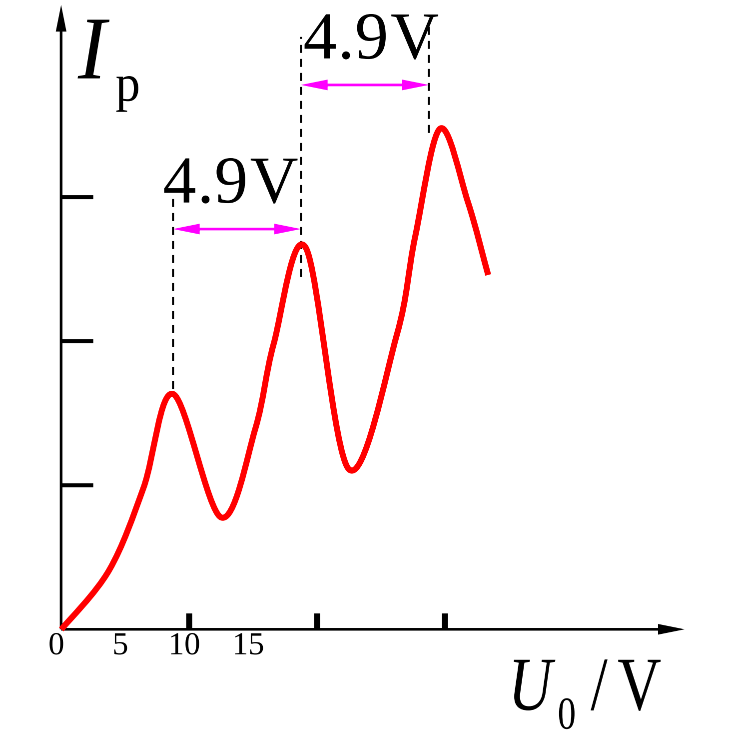
The microscopic particles are regarded as the particle of classical mechanics, and

Give them the characteristics of the quantization.

In 1914, Frank-Herz experimentally confirmed that atoms had separate energy levels, and in 1925 they won the Nobel Prize in physics.

The Frank-by-Hertz experimental setup





Relationship between the plate current and the accelerating voltage

§ 17.3

Material wave particle duality

17.3.1 Wave-particle duality of light

1. Volatility: the interference and diffraction of light

2. Particle property: $E = h\nu$ (photoelectric effect and Compton effect, etc.)

Describe the light corpuscular property

$$\boxed{\begin{gathered} E = h\nu \\ p = \frac{h}{\lambda} \end{gathered}}$$

Describe the light undulatory property

17.3.2 Material wave

French physicist de Broglie (Louis Victor de Broglie 1892 1987)

Nature is clearly symmetric in many ways, and he analogy the hypothesis of matter waves.

"For the whole century, the theory of radiation has been too neglected than the study of particles; is it wrong in physical theory? Do we think too much about the image of 'particles' and excessively ignore the image of waves?”

1. De Broipo (1924)

De Broglie assumes that physical particles have wave-particle duality.

$$\boxed{E = h\nu} \qquad \boxed{p = \frac{h}{\lambda}}$$

◈ **Debro's formula**

$$\nu = \frac{E}{h} = \frac{mc^2}{h} \qquad \lambda = \frac{h}{p} = \frac{h}{mv}$$

pay attention to

1) If then $v \ll c \qquad m = m_0$

If then $v \to c \qquad m = \gamma m_0$

2) The de Broglie wavelength of the macroscopic object is so small that it is difficult to measure by experiment, so the macroscopic object shows only particle properties.

In a beam of electrons, the kinetic energy of the electron is 200eV, how to find the de Broglie wavelength of the electron? λ

separate $\quad v << c, \quad E_k = \frac{1}{2} m_0 v^2 \qquad v = \sqrt{\frac{2E_k}{m_0}}$

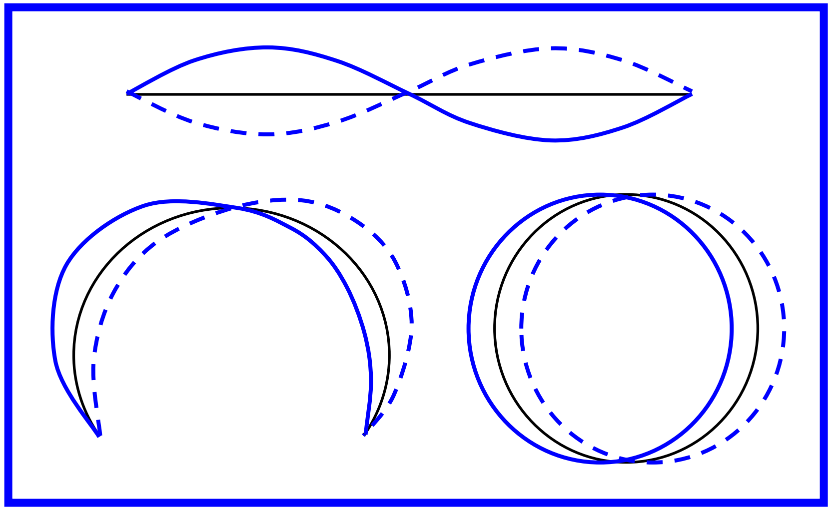
$$v = \sqrt{\frac{2 \times 200 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \mathrm{m \cdot s^{-1}} = 8.4 \times 10^{6} \mathrm{m \cdot s^{-1}}$$

$$\because v << c \qquad \therefore \lambda = \frac{h}{m_0 v} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 8.4 \times 10^{6}} \mathrm{nm}$$

$$\lambda = 8.67 \times 10^{-2} \mathrm{nm}$$

The order of magnitude of this wavelength is comparable to the order of magnitude of the X-ray wavelength.

Example 2 derived the angular momentum quantization condition in the hydrogen atomic Boer theory from the de Broglie wave.



Solving the string fixed at both ends can form a stable standing wave if its length is equal to the wavelength.

When bending the string into a circle

$$2\pi r = n\lambda, \quad n = 1, 2, 3, 4, \cdots$$

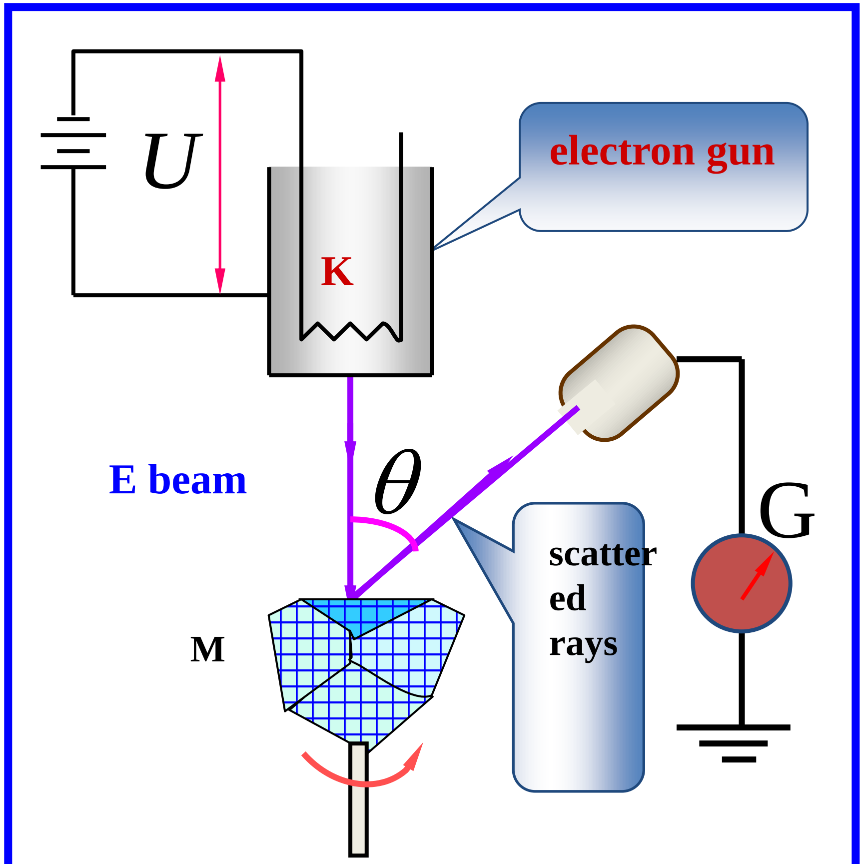
Electrons move around the core with a de Broglie wavelength of $\lambda = \dfrac{h}{mv}$

$$2\pi rmv = nh$$

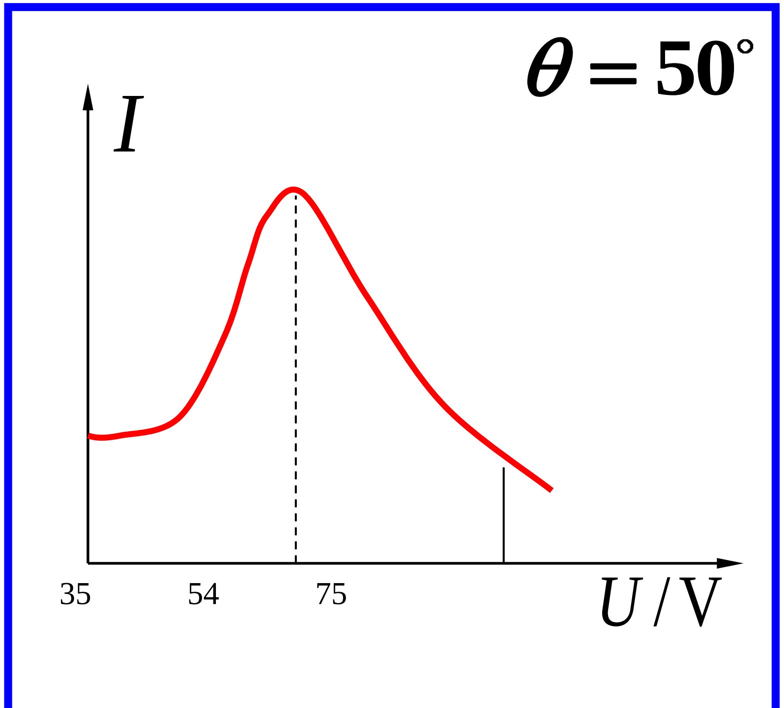
The angular-momentum quantization conditions $L = mvr = n\dfrac{h}{2\pi}$

2. Electron diffraction experiments

1. Davissun-GeMo electron diffraction experiment (1927)



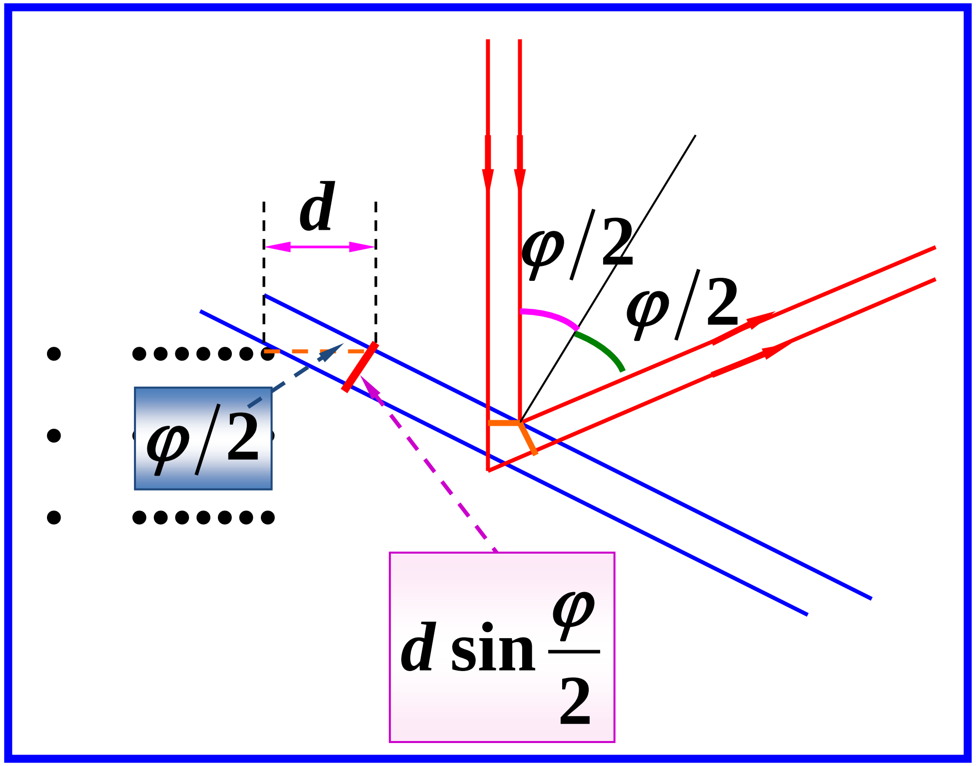
Electrons were quantified by the nickel crystal diffraction experiments



When the scattering angle $\theta = 50^{\circ}$

Current vs. accelerated voltage curves

Interbeam beam beam beam beam beams in two adjacent crystal surfaces



$$2d\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}=k\lambda$$

$$d\sin\varphi=k\lambda$$

$$k=1,\quad \varphi=50^\circ$$

Nickel crystal

$$d=0.215\,\text{nm}=0.125\times10^{-9}\,\text{m}$$

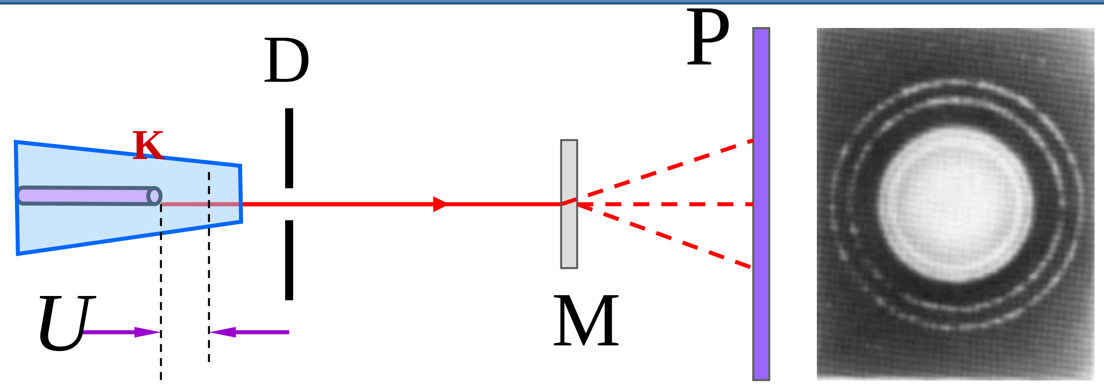
$$\lambda=d\sin\varphi=1.65\times10^{-10}\,\text{m}$$

The wavelength of the electron wave

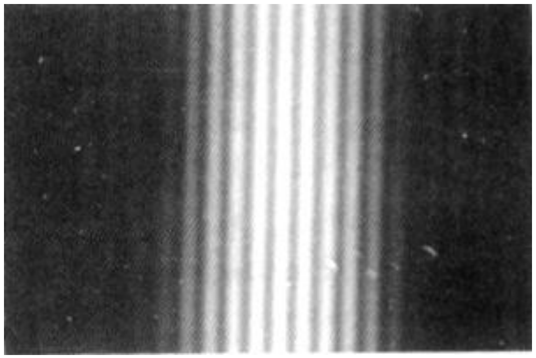
$$\lambda=\frac{h}{m_e v}=\frac{h}{\sqrt{2m_e E_k}}=1.67\times10^{-10}\,\text{m}$$

2. G .P . Tom Sun electron diffraction experiment (1927)

Diffraction of the electron beam through the polycrystalline aluminum foil



Double slit diffraction diagram



3. Application examples

In 1932 German Ruska successfully developed an electron microscope; in 1981 German Binich and Swiss Rorell made a scanning tunnel microscope.

Example 3 The trial calculates that the temperature is $25^{\circ}\mathrm{C}$ the de Broglie wavelength of the slow neutron.

When the solution is in thermal equilibrium, the average translational kinetic energy of the energy equal partition theorem can be expressed as

$$T = 298\mathrm{K}$$

$$\bar{\varepsilon} = \frac{3}{2}kT = 3.85\times10^{-2}\mathrm{eV} \qquad m_{\mathrm{n}} = 1.67\times10^{-27}\mathrm{kg}$$

$$p = \sqrt{2m_{\mathrm{n}}\bar{\varepsilon}} = 4.54\times10^{-24}\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-1}$$

The DeBroglie wavelength of the slow neutrons $\lambda = \dfrac{h}{p} = 0.146\mathrm{nm}$

17.3.3 Statistical interpretation of wave-particle duality
probability wave

1. Wave-particle duality of the particles

The classical particle —— is not divided by the whole, which has a definite position and a moving track;

The classical wave —— the spatial distribution of some actual physical quantity changes periodically, and the wave has coherent superposition.

Duality —— requires unifying waves and particles to the same object.

2. Statistical interpretation of de Brogbo

◈ In 1926, —— is a probability wave.

Statistical explanation: The strength of the DeBroglie wave somewhere is proportional to the probability of the particle appearing in the immediate vicinity.

The philosophical significance of the concept of probability: under a given condition, it is impossible to accurately predict the result, but only to predict the probability of some possible result.

3. Wave-function probability density of the matter wave

At a certain time t, the probability of particle occurrence within a unit volume near a location in space is $\left(x,y,z\right)$

$$P\left(x,y,z,t\right)=\left|\Psi\left(x,y,z,t\right)\right|^2=\Psi\Psi^*$$

The probability of finding a particle in full space is 1, so

normalizing condition

$$\int_{\text{total space}} \left|\Psi\left(x,y,z,t\right)\right|^2 \mathrm{d}V = 1$$

standard condition

The wave function Ψ must be single-valued, finite, and continuous

17.3.4 Uncertain relationship

◈ **Heisenberg proposed the principle of uncertainty in 1927**

The microscopic particles cannot be described both by definite position and definite momentum.

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

Physical significance

1. The coordinates and momentum of microscopic particles in the same direction cannot be measured accurately and accurately at the same time, and there is an ultimate insurmountable limit to their accuracy.

2. The source of uncertainty is wave-particle duality, which is the fundamental attribute of nature.

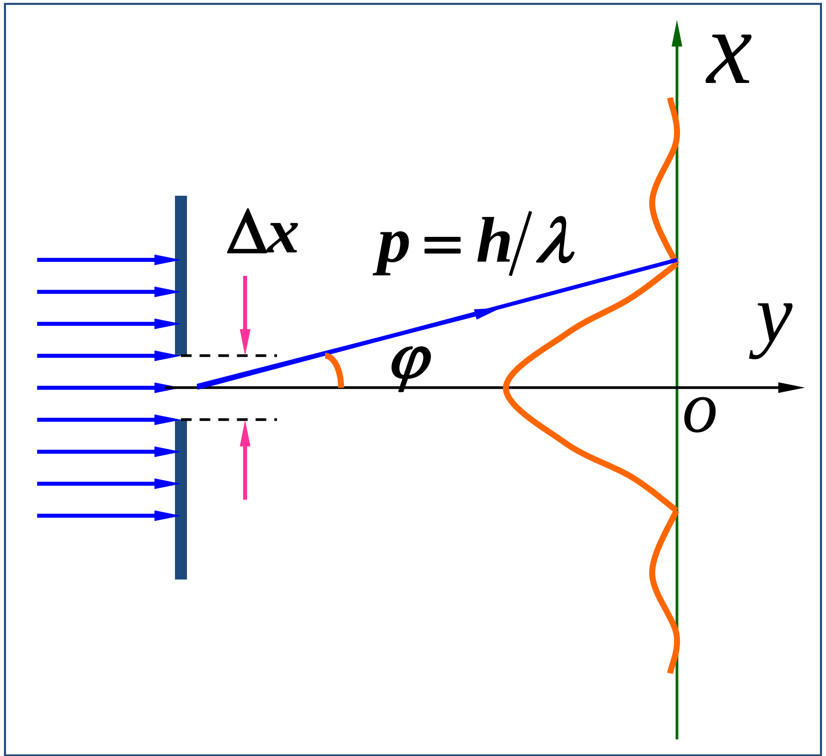
◈Undeterminuncertainty relationship by electron diffraction

The position of the electron through the seam is uncertain as the seam width.

$$\Delta x$$

Level I diffraction angle $\sin\varphi = \lambda/\Delta x$

The momentum in the x direction after the slit



Single-seam diffraction experiments of the electrons

$$\Delta p_x = p\sin\varphi = p\frac{\lambda}{\Delta x}$$

$$\lambda = \frac{h}{p} \qquad \Delta p_x = \frac{h}{\Delta x}$$

$$\Delta x \Delta p_x = h$$

Consider diffraction secondary have

$$\boxed{\Delta x \Delta p_x \geq h}$$

3. For macroscopic particles, they can h be regarded as accurate measurements of both position and momentum. $\Delta x \Delta p_x \to 0$

Example 1 A bullet with a mass of 10 g has a rate of. If the uncertain range of $200\,\mathrm{m \cdot s^{-1}}$ momentum is momentum (which is very accurate in the macroscopic range), what is the uncertain range of the bullet position? 0.01%

Momentum of the solution bullet $p = mv = 2\,\mathrm{kg \cdot m \cdot s^{-1}}$

The uncertain range of the momentum

$$\Delta p = 0.01\% \times p = 2\times10^{-4}\,\mathrm{kg \cdot m \cdot s^{-1}}$$

Range of uncertainty for the positions

$$\Delta x \geq \frac{h}{\Delta p} = \frac{6.63\times10^{-34}}{2\times10^{-4}}\,\mathrm{m} = 3.3\times10^{-30}\,\mathrm{m}$$

Example 2 If an electron has a $200\mathrm{m\cdot s^{-1}}$ rate of 0.01% of the momentum (which is also accurate enough), how large is the uncertain position range of the electron?

The momentum of the solved electron

$$p = mv = 9.1\times10^{-31}\times200\mathrm{kg\cdot m\cdot s^{-1}}$$

$$p = 1.8\times10^{-28}\mathrm{kg\cdot m\cdot s^{-1}}$$

The uncertain range of the momentum

$$\Delta p = 0.01\%\times p = 1.8\times10^{-32}\mathrm{kg\cdot m\cdot s^{-1}}$$

Range of uncertainty for the positions

$$\Delta x \geq \frac{h}{\Delta p} = \frac{6.63\times10^{-34}}{1.8\times10^{-32}}\mathrm{m} = 3.7\times10^{-2}\mathrm{m}$$