Q1

(a) $\nabla f(x,y) = (4\cos(4x+3y),\ 3\cos(4x+3y))$

$\nabla f(-6,4) = (4\cos(-12),\ 3\cos(-12))$

$D_u f(-6,4) = 2\sqrt{3}\cos(-12) - \frac{3}{2}\cos(-12)$

(b) $\nabla f(x,y) = (2xy^2,\ 2x^2y)$

$\nabla f(1,2) = (8,4)$

$D_u f(1,2) = \frac{2}{3}\cdot 8 - \frac{\sqrt{5}}{3}\cdot 4 = \frac{16}{3} - \frac{4\sqrt{5}}{3}$

(c) $\nabla f(x,y,z) = (2xyz - yz^3,\ x^2z - xz^3,\ x^2y - 3xyz^3)$

$\nabla f(2,-1,-1) = (3,-2,2)$

$D_u f(2,-1,-1) = -\frac{8}{5} + \frac{6}{5} = -\frac{2}{5}$

(d) $\nabla f(x,y,z) = (yze^{xyz},\ xze^{xyz},\ xye^{xyz})$

$\nabla f(0,-1,-1) = (1,0,0)$

$D_u f(0,-1,-1) = \frac{3}{13}$

Q2

(a) $\nabla f(x,y) = (y\cos(xy),\ x\cos(xy))$

$\nabla f(2,3) = (3\cos 6,\ 2\cos 6)$

$Max = |\nabla f(2,3)| = \sqrt{13} \cdot \cos 6$

Direction: $(3\cos 6, 2\cos 6)$.

(b) $\nabla f(s,t) = (e^{st} + (t+s)te^{st}, e^{st} + (t+s)se^{st})$

$\nabla f(0,2) = (5,1)$

$max = |\nabla f(0,2)| = \sqrt{26}$

Direction: $(5,1)$

(c) $\nabla f(x,y,z) = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\nabla f(2,3,-1) = \left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$

$Max = |\nabla f(2,3,1)| = 1$

Direction $\left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$

Q3

(a) $F(x,y,z) = 2(x-2)^2 + (y-1)^2 + (z-3)^2$

$F_x = 4x-8 \quad F_y = 2y-2 \quad F_z = 2z-6$

At $(3,3,5)$

$F_x = 4 \quad F_y = 2y-2, \quad F_z = 2z-6$

Tangent Plane: $4(x-3) + 4(y-3) + 4(z-5) = 0$

(b) $F(x,y,z) = xyz^2$

$F_x = yz^2, \quad F_y = xz^2, \quad F_z = 2xyz$

At $(3,2,1)$

$F_x = 2. \quad F_y = 3 \quad F_z = 12.$

Tangent Plane:

$$2(x-3) + 3(y-2) + 12(z-1) = 0$$

(c) $F(x,y,z) = x+y+z-e^{xyz}$

$F_x = 1 - yze^{xyz} \quad F_y = 1 - xe^{xyz} \quad F_z = 1 - xye^{xyz}$

Tangent plane: $x+y+(z-1) = 0$