16.5 Momentum, mass, and energy in special relativity

In relativistic dynamics, we will redefine physical quantities such as momentum, mass and energy based to new experimental facts, and determine their changing laws under interaction.

Because Newtonian mechanics is correct at low speeds, the newly defined physical quantities must tend to the corresponding quantity in Newtonian mechanics at $v \ll c$.

As a general principle, the changing laws of these physical quantities should also follow the laws of conservation of energy and momentum.

16.5.1&2 Momentum and mass in special relativity

Still define the momentum by the following below

$$\boldsymbol{p} = m\boldsymbol{v}$$

Still assume

$$\boldsymbol{f} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}$$

It is no different from Newtonian mechanics. But in Newtonian mechanics, the mass of an object is a constant quantity independent of the rate of motion. If this idea is retained, there will be

$$\boldsymbol{f} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = m_0 \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}$$

$$\boldsymbol{f} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = m_0 \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}$$

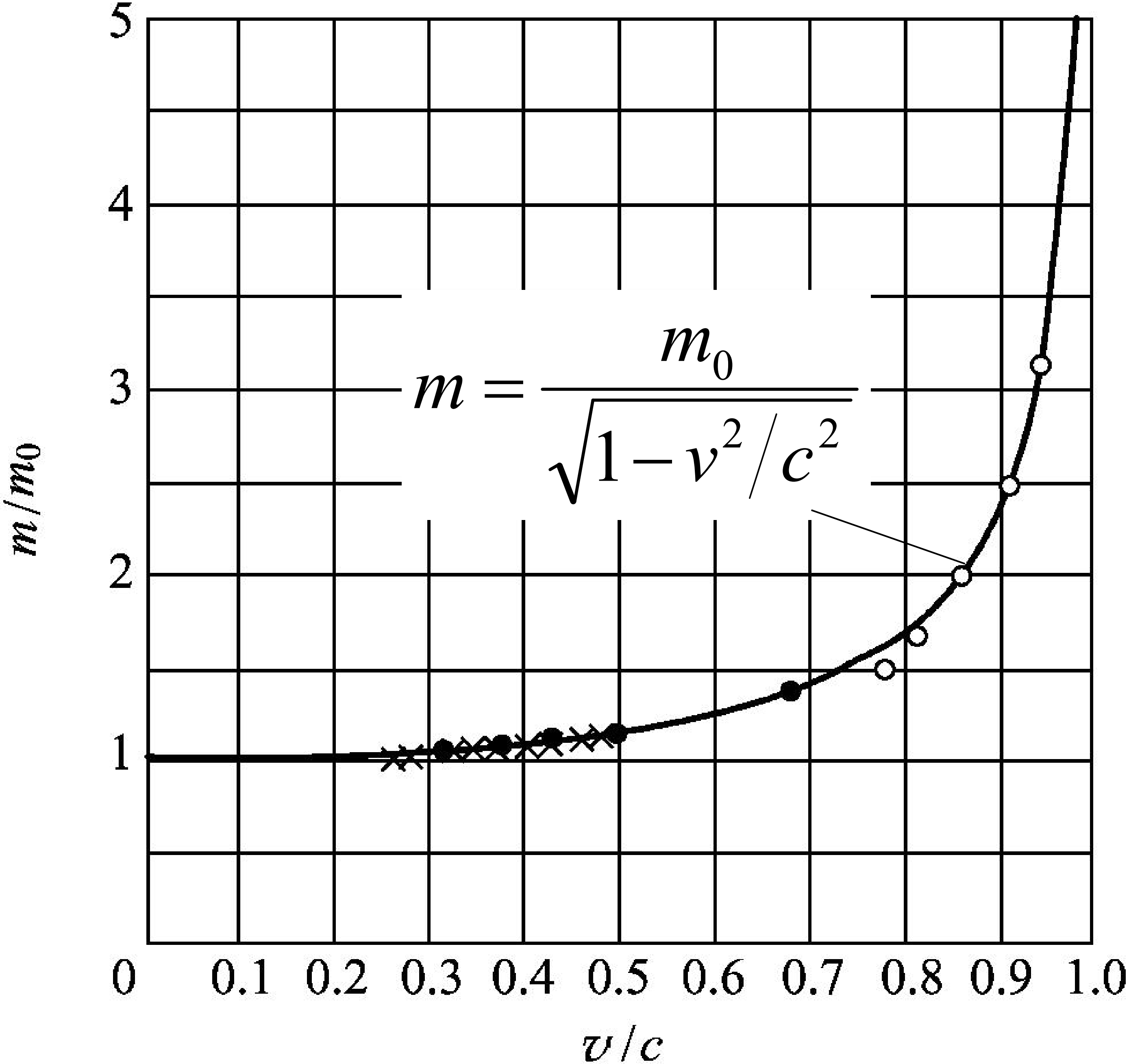
That is, the object is accelerated at a constant acceleration under a constant force. Long enough time can exceed the speed of light, which clearly goes against the basic principles of relativity.

Therefore, at high speeds, the mass can no longer be regarded as a constant quantity independent of the rate.

The experiments show that the relativistic mass of the object, and the motion rate of the object are satisfied

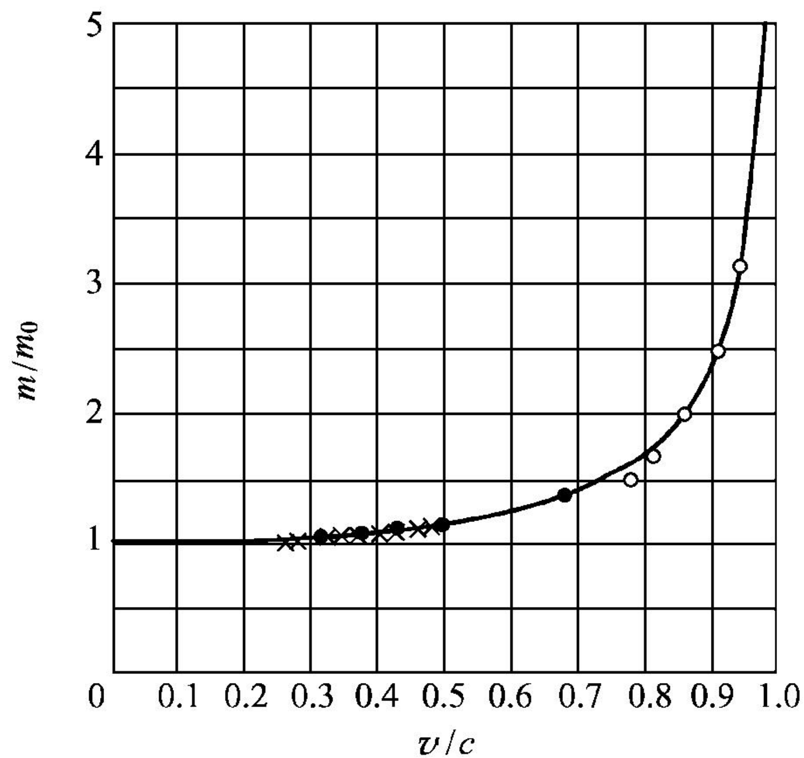
$$m = \gamma m_0 = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$$

(Quality and speed relationship)


5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
m/m_0
v/c
$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

Physical particles: static mass m_0 Particles of 0, such as electrons, protons, and neutrons.

When the rate is large, the mass increases dramatically, when accelerating the physical particles becomes very difficult, which must be considered in the design and operation of the accelerator.



$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$$

When v =c, the mass is meaningless, so that the speed of motion of the physical particle cannot reach the speed of light.

Momentum of particles:

$$\boldsymbol{p} = m\boldsymbol{v} = \frac{m_0 \boldsymbol{v}}{\sqrt{1 - v^2 / c^2}}$$

$$\boldsymbol{f} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \frac{\mathrm{d}m}{\mathrm{d}t}\boldsymbol{v} = m\boldsymbol{a} + \frac{\mathrm{d}m}{\mathrm{d}t}\boldsymbol{v}$$

In the high-speed case, the force exerted by the particle is no longer equal to its mass multiplied by the acceleration, and the change in mass must be considered. Only at dm / d t 0 does the direction of the acceleration tend to the direction of the force, consistent with the conclusion of Newton's law of motion.

16.5.3 Energy in Special Relativity

Relativity still retains the kinetic energy theorem in Newtonian mechanics: the work a force does to a particle is equal to the increment of the kinetic energy of the particle

$$\mathrm{d}E_{\mathrm{k}} = \boldsymbol{f} \cdot \mathrm{d}\boldsymbol{r}$$

To derive the kinetic energy formula of relativity: let the static mass be m_0The particles move from rest under the force f along the x-axis, and the kinetic energy of the particle when the velocity reaches v

$$E_{\mathrm{k}} = \int_{v=0}^{v} f \mathrm{d}x$$

$$E_{\mathrm{k}}=\int_{v=0}^{v}\frac{\mathrm{d}(mv)}{\mathrm{d}t}\mathrm{d}x=\int_{v=0}^{v}v\mathrm{d}(mv)=\int_{v=0}^{v}v\mathrm{d}\left(\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right)$$

$$=\left.\frac{m_0 v^2}{\sqrt{1-v^2/c^2}}\right|_0^v-\int_{v=0}^{v}\frac{m_0 v\mathrm{d}v}{\sqrt{1-v^2/c^2}}$$

$$=\frac{m_0 v^2}{\sqrt{1-v^2/c^2}}+\left.m_0 c^2\sqrt{1-v^2/c^2}\right|_0^v$$

$$=\frac{m_0 v^2}{\sqrt{1-v^2/c^2}}+m_0 c^2\sqrt{1-v^2/c^2}-m_0 c^2$$

$$=\frac{m_0 c^2}{\sqrt{1-v^2/c^2}}-m_0 c^2$$

The kinetic energy formula of relativity:

$$E_{\mathrm{k}} = (m - m_0)c^2 = mc^2 - m_0 c^2$$

The kinetic energy of the particle is proportional to the increment of the particle mass caused by the motion, with a proportional coefficient of c^2。

mass-energy relation:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

The energy of the particle is equal to its mass multiplied by c^2, While the kinetic energy of the particle is equal to its energy minus its static mass energy m_0c^2。

[Example] How fast is the movement rate of a particle, and its kinetic energy is equal to its static mass energy?

separate

$$E_{\mathrm{k}} = \left(\frac{m_0}{\sqrt{1 - v^2 / c^2}} - m_0 \right) c^2 = m_0 c^2$$

$$\sqrt{1 - v^2 / c^2} = \frac{1}{2}$$

$$v = \sqrt{\frac{3}{4}}\, c = 0.866c$$

When the rate reaches 0.866c, the kinetic energy of the particle equals its static matter energy.

During the nuclear reaction and the chemical reaction, the static mass of the matter should be reduced. The atic mass m reduced by the system during the reaction$_0$Called a mass loss.

According to the mass and energy relationship and energy conservation, the loss of the mass m$_0$Converted into an energy, m$_0c^2$Release out.

The energy released is only a very small fraction of the static mass energy. The energy released by nuclear reactions (nuclear fission, nuclear fusion) accounts for about one thousandth of the static mass energy of the nuclear fuel participating in the reaction. Chemical reactions, such as gasoline combustion, release a chemical energy of only 10 percent^{-10}the left and right sides.

The chemical reaction only involves the electromagnetic action, while the nuclear reaction is a strong interaction process, two orders of magnitude stronger than the electromagnetic action.

Heavy nuclear fission

$$X \to Y + Z$$

mass defect

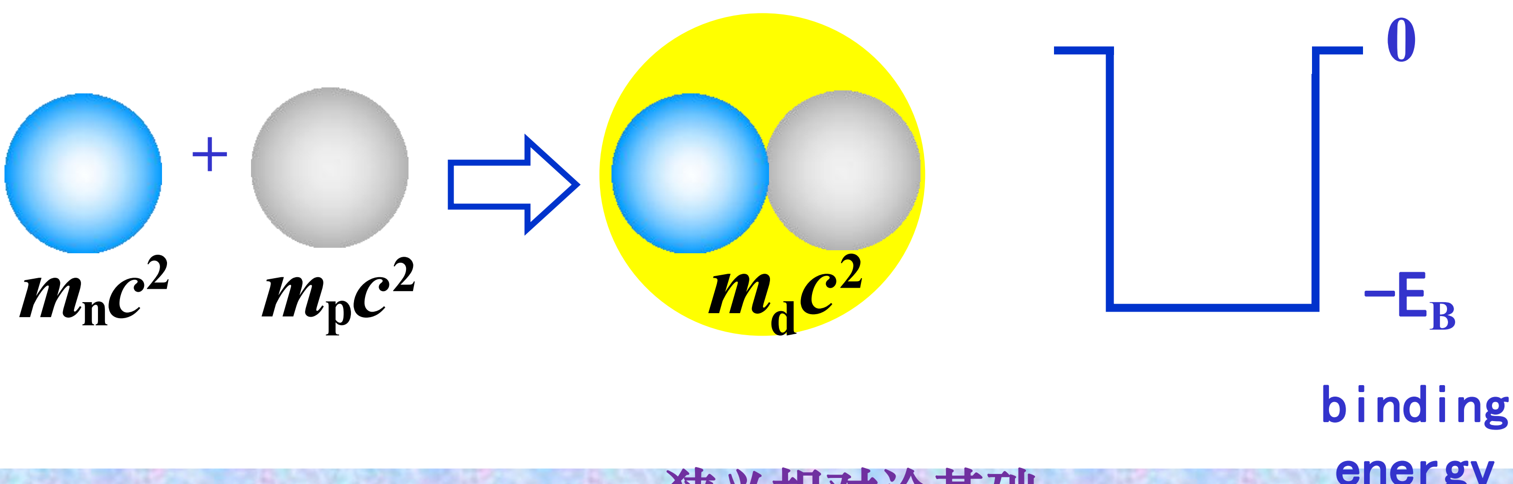
$$\Delta m_0 = m_{X0} - (m_{Y0} + m_{Z0})$$

disintegration energy

$$\Delta E = \Delta m_0 c^2$$

[Example] Calculate the proportion of the energy released by the combination of neutrons and protons into deuterons to the static mass energy of the deuterons, and compare it with the combination of electrons and protons into hydrogen atoms.

The interaction between the unnucleons (protons and neutrons) is called nuclear force. Protons and neutrons bind into deuterons through nuclear force attraction. Gravity does work, the system energy decreases, and the mass loss fusion energy occurs in the process of forming the deuteron



$$m_{\mathrm{n}} = 939.565\ 63\ \mathrm{Mev}\ / c^2$$

$$m_{\mathrm{p}} = 938.272\ 31\ \mathrm{Mev}\ / c^2$$

$$m_{\mathrm{d}} = 1875.613\ 39\ \mathrm{Mev}\ / c^2$$

mass defect: $\Delta m_0 = (m_{\mathrm{n}} + m_{\mathrm{p}}) - m_{\mathrm{d}}$

fusion energy: $\Delta E = \Delta m_0 c^2 = 2.23\,\mathrm{Mev}$

The portion of proton and neutron static energy participating in the reaction:

$$\frac{\Delta E}{m_{\mathrm{p}} + m_{\mathrm{n}}} = \frac{2.23}{938.272 + 939.565} \approx 1.2 \times 10^{-3}$$

Release of energy when electrons and protons combine to form hydrogen atoms

$$\Delta E = 13.6\,\mathrm{eV}$$

The portion of proton and electron static energy:

$$\frac{13.6}{938.272\times10^{6}} \approx 1.4\times10^{-8}$$

It's just a one-thousandth of the nuclear reaction.

cause:

Chemical reactions involve only electromagnetic interactions, while nuclear reactions are the process of nuclear force action. Nuclear forces are strong interactions with strengths two orders of magnitude greater than the strength of electromagnetic interactions.

16.5.4 Energy and momentum relations in special relativity

By and get

$$\boldsymbol{p} = \frac{m_0 \boldsymbol{v}}{\sqrt{1 - v^2/c^2}} \qquad E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

Energy and momentum relationship:

$$E^2 = p^2 c^2 + m_0^2 c^4$$

For the static mass of $m_0=0$ Particle

$$E = pc \rightarrow mc^2 = mvc \rightarrow v = c$$

Particles with zero static mass can only move at the speed of light.

The current experimental result: the upper limit of the photon static mass is 10^{-62}kg ,

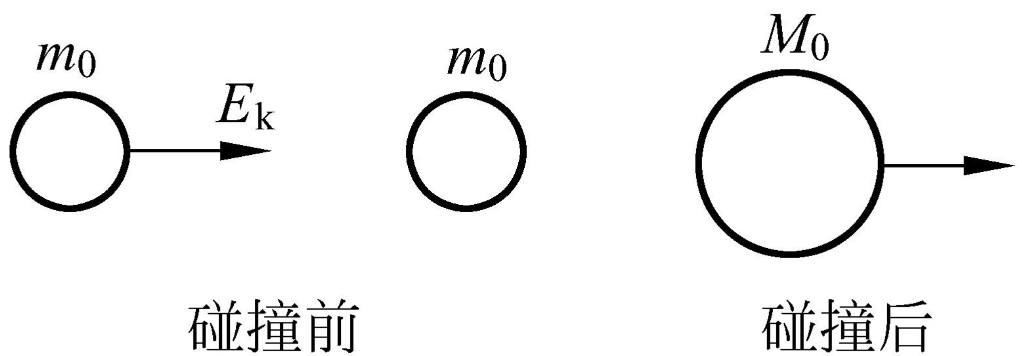
It can be considered that the photon static mass is zero and its motion velocity is c.

* Energy and momentum conservation

The subject of relativity is mainly those particle systems that are not affected by external influence. Momentum and energy conservation during the processes experienced by these systems. In relativity, the problem of striving for particle motion is not dominate.

For processes such as high-energy particle collision, fission and decay, the general measurement is the energy. However, the front and rear particles are far apart, so the interaction potential energy between particles is always ignored.

[Example] The static mass is m_0The kinetic energy is E_kThe energetic particles hit a static mass of m_0Of stationary target particles, and form composite particles. Seeking the static mass M of the composite particle$_0$。



separate

The collision particles interact strongly, can ignore external influences such as gravity, and the energy and momentum of the system

conservation of momentum: $$p'^2c^2 = p^2c^2$$

$$p^2c^2 = (E_k + m_0c^2)^2 - m_0^2c^4 = E_k^2 + 2m_0c^2E_k$$

conservation of energy: $E = E'$

$$E' = \sqrt{p'^2c^2 + M_0^2c^4} = \sqrt{p^2c^2 + M_0^2c^4} = \sqrt{E_k^2 + 2m_0c^2E_k + M_0^2c^4}$$

$$E_k + 2m_0c^2 = \sqrt{E_k^2 + 2m_0c^2E_k + M_0^2c^4}$$

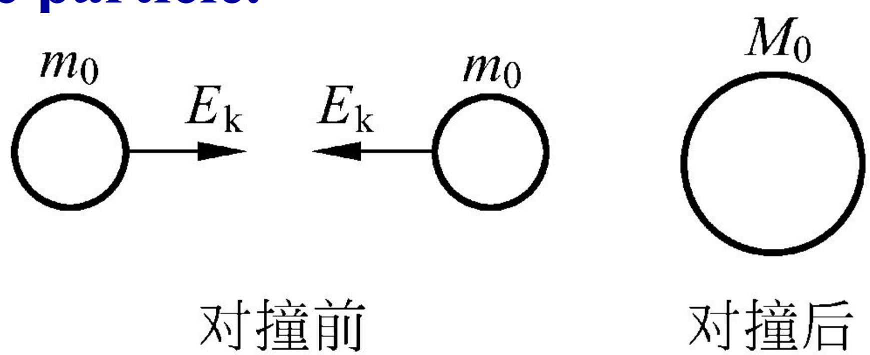
$$M_0 = \sqrt{4m_0^2 + \frac{2m_0E_k}{c^2}} = 2m_0\sqrt{1 + \frac{E_k}{2m_0c^2}}$$

Static mass M of the composite particle$_0$Greater than the static mass of 2m of the particles participating in the reaction$_0$, The increased static mass is called the mass excess, which is converted by the kinetic energy of the incident particle.

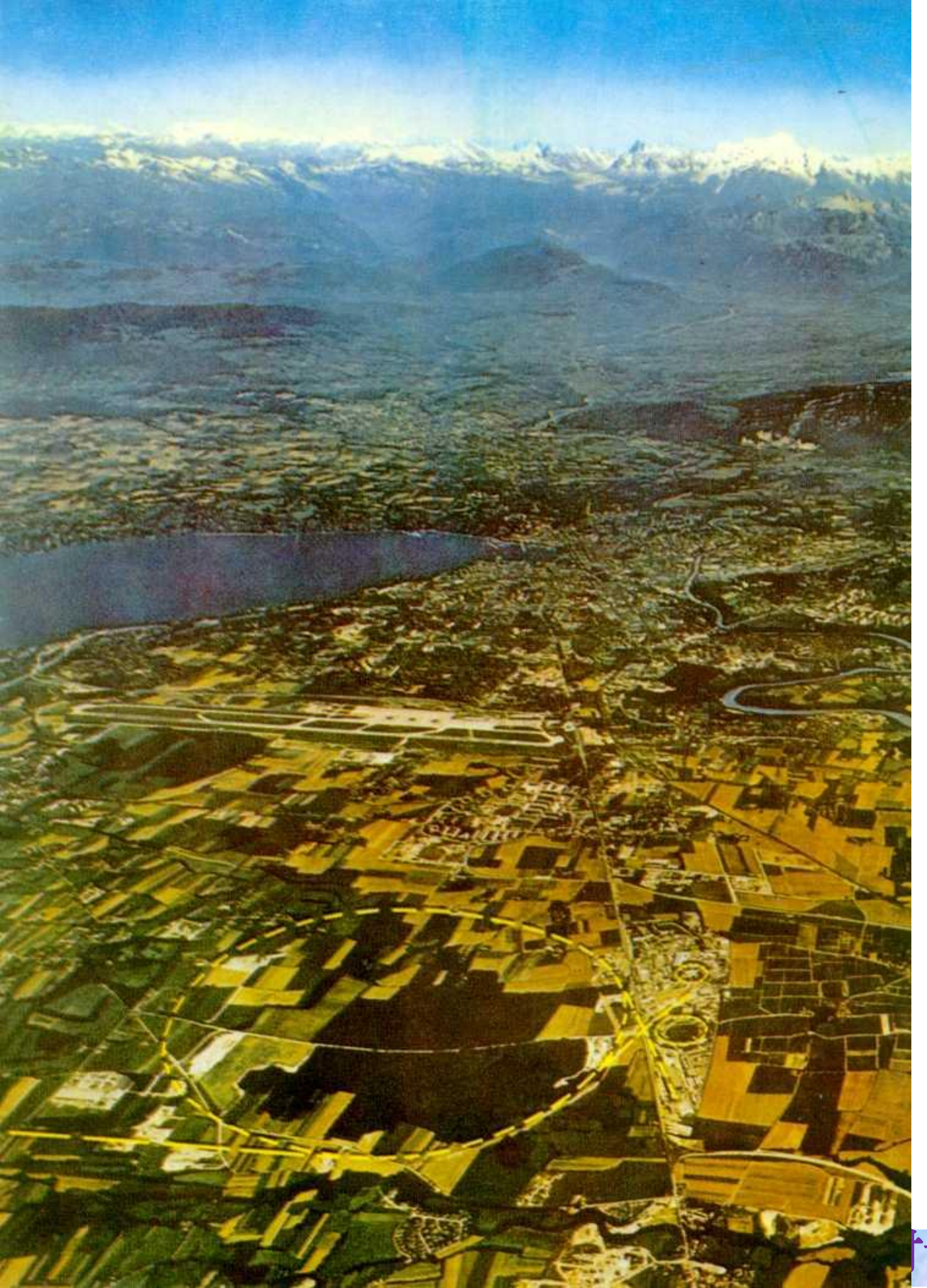
$$M_0 = 2m_0\sqrt{1+\frac{E_k}{2m_0c^2}}$$

In the stationary collision of the target particle, only one part of the kinetic energy of the incident particle is converted into the static mass energy of the composite particle, and the other part of the kinetic energy of the composite particle is "wasted".

[Example] Colliding can convert all the kinetic energy of the particles participating in the reaction into the static mass energy of the composite particle.



$$M_0 = 2m_0 + \frac{2E_k}{c^2}$$



EuroNuclear Centre CERN

C. Rubbia and S.van der Meer Found a W at the collider$_{\pm}$And Z_0Particles, confirming the unified theory of weak currents, won the 1984 Nobel Prize in Physics.