

## Chapter 14

# *diffraction of light*

(a)

(b)

# primary coverage

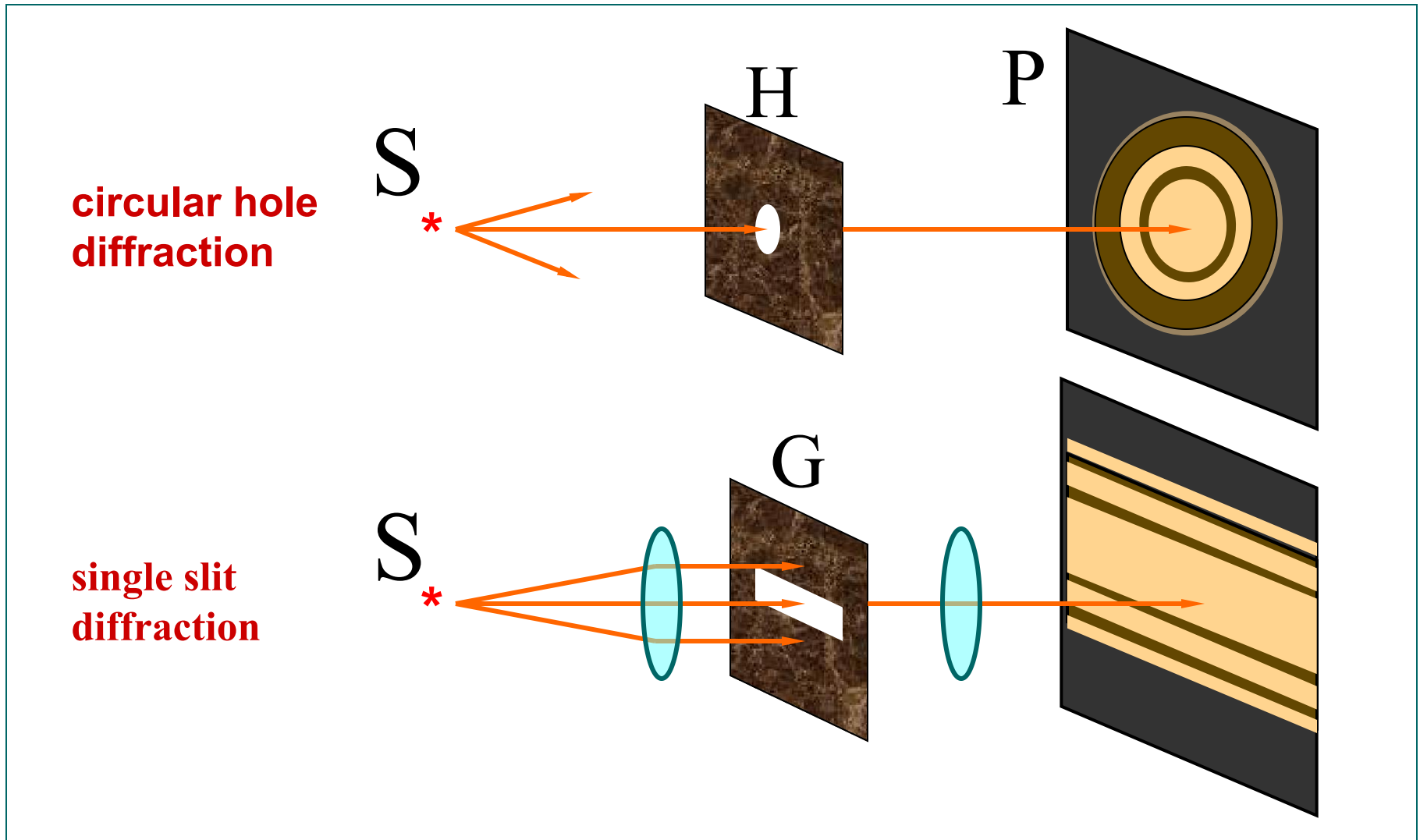
- 1 The diffraction phenomenon of the light
  - 2 The Huygens-Fresnel principle
  - 3 Single-slit Kluunhofer diffraction
  - 4 Fraunhofer round hole diffraction and optical instruments
- Resolution skills
- 5 The diffraction grating
  - 6 Grating spectra
  - 7 The X-ray diffraction



## § 14.1

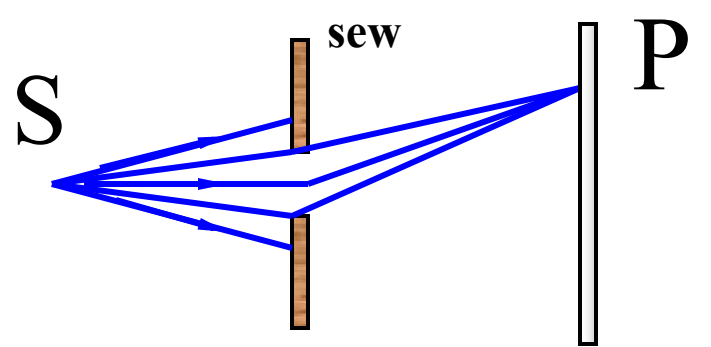
# Diffraction of light

# 1. Diffraction phenomenon of the light



2. Diffraction classification — according to the relative position of the light source, diffraction screen and receiving screen

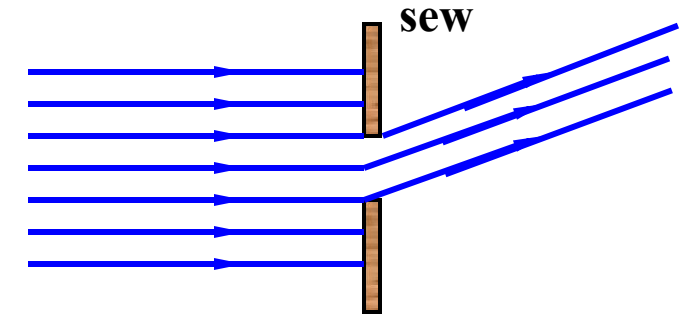
Fresnel diffraction



The diagram illustrates Fresnel diffraction. A point light source  $S$  on the left emits several blue rays that pass through a narrow slit (labeled 'sew') in a vertical barrier. The rays then travel to a vertical receiving screen  $P$  on the right. The distances from  $S$  to the slit and from the slit to  $P$  are finite, causing the rays to converge and diverge as they pass the slit.

The light source and (or) receiving screen are of limited distance from the diffraction screen

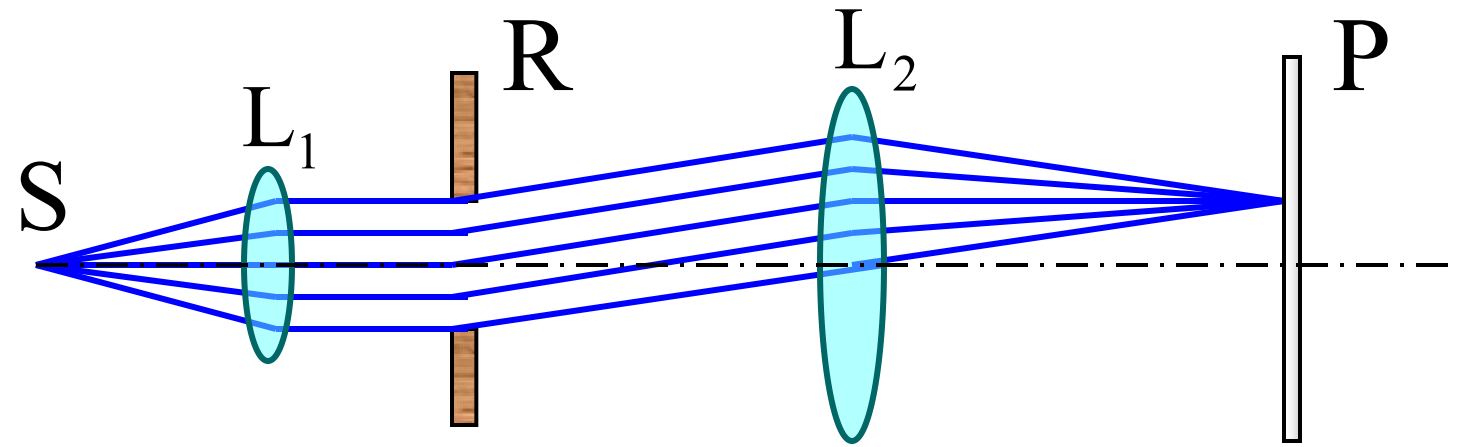
Fraunhofer diffraction



The diagram illustrates Fraunhofer diffraction. Parallel blue rays, representing light from a distant source, pass through a narrow slit (labeled 'sew') in a vertical barrier. After passing the slit, the rays diverge slightly. The distances from the source to the slit and from the slit to the observation point are effectively infinite.

The light source and the receiving screen distance from the diffraction screen are infinite distance

Fraunhofer diffraction implementation in experiments



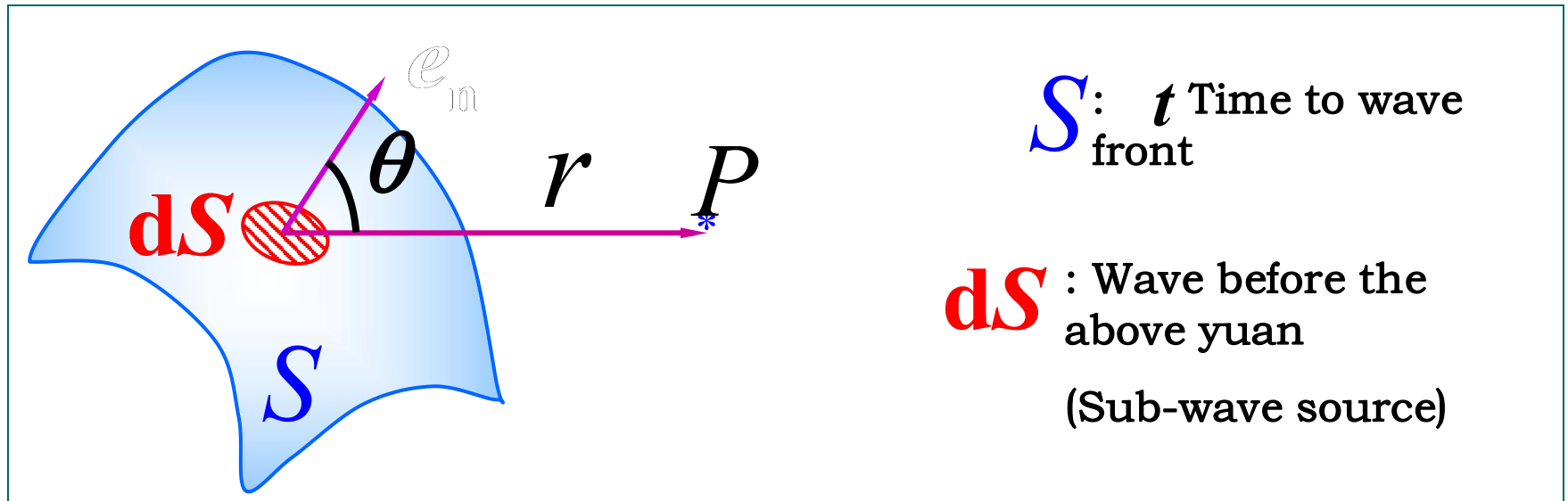
This diagram shows how Fraunhofer diffraction is implemented in a laboratory. A point source  $S$  emits rays that pass through a lens  $L_1$ . The lens  $L_1$  collimates the rays, making them parallel as they pass through a slit (labeled 'R'). These parallel rays then pass through a second lens  $L_2$ , which focuses them onto a vertical receiving screen  $P$ . A dashed horizontal line represents the optical axis.

## § 14.2

# The Huygens-Fresnel principle

## Huygens-Fresnel principle:

The perturbation of any point  $P$  in the diffraction wave field can be seen as a superposition of coherent vibrations at that point from the imaginary subwave source distributed continuously on the wavefront  $S$

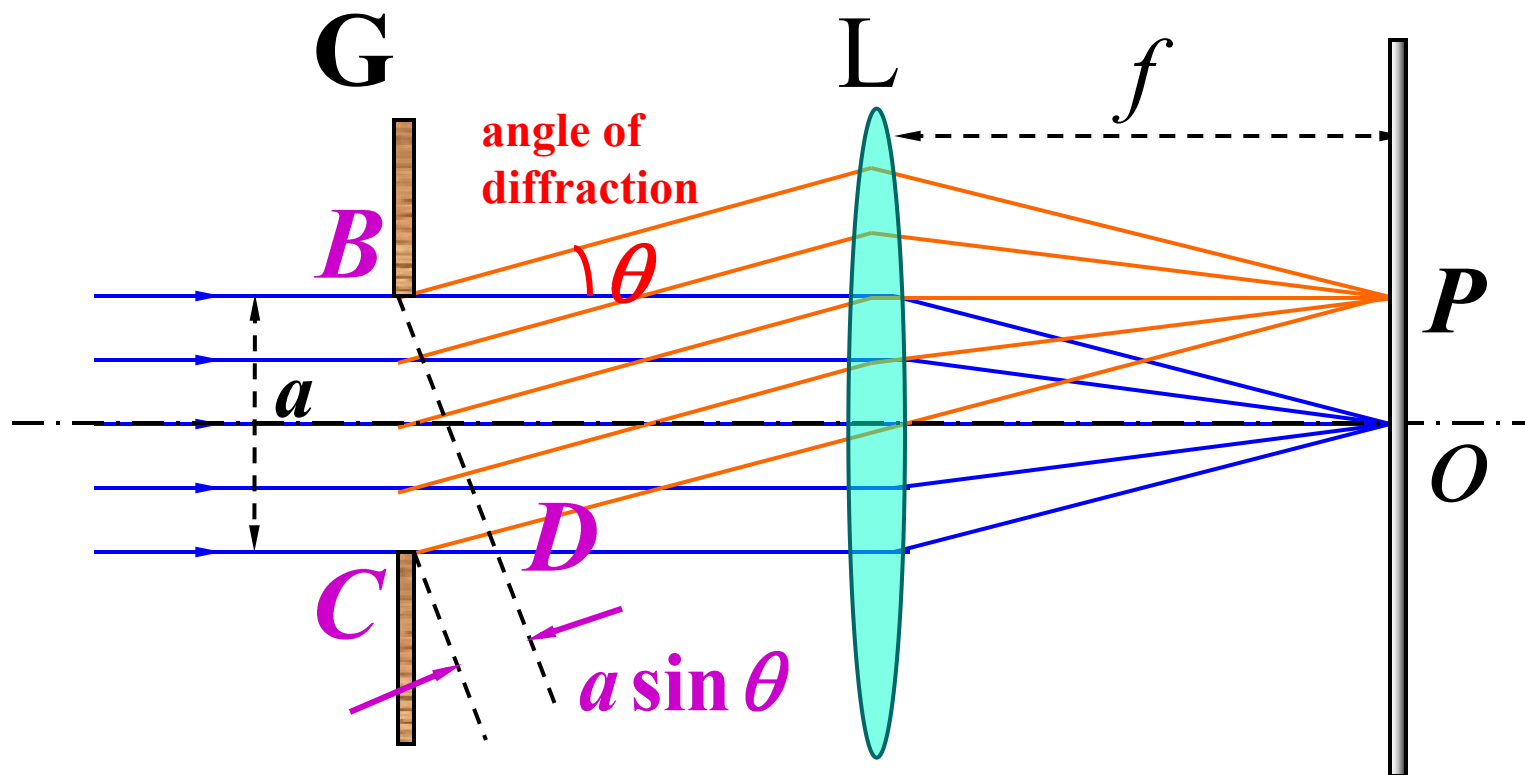


The vibration amplitude and related to  $\theta$  .  $\propto \frac{ds}{r}$  caused by the subwave at the point  $P$

## § 14.3

# Single-slit Kluunhofer diffraction



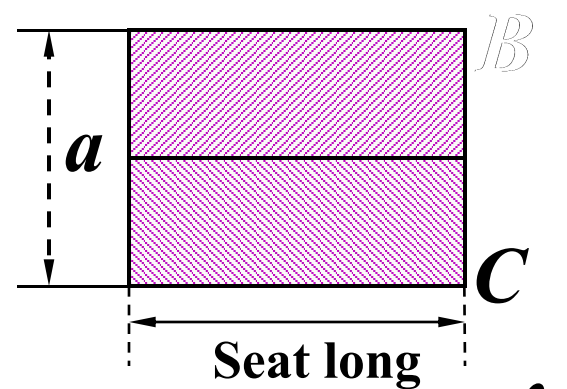


(Diffraction angle  $\theta$  : positive upward and negative downward. )

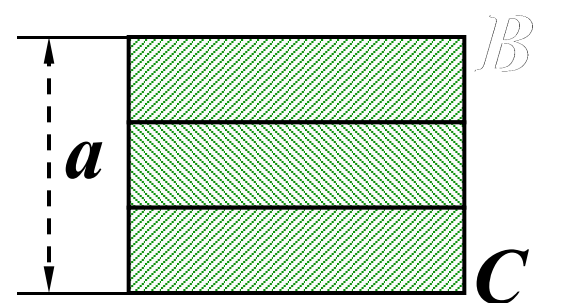
The Fresnel half-wave band method

$$CD = a \sin \theta = \pm k \frac{\lambda}{2} \quad (k = 1, 2, 3, \dots)$$

1. Half wave band method

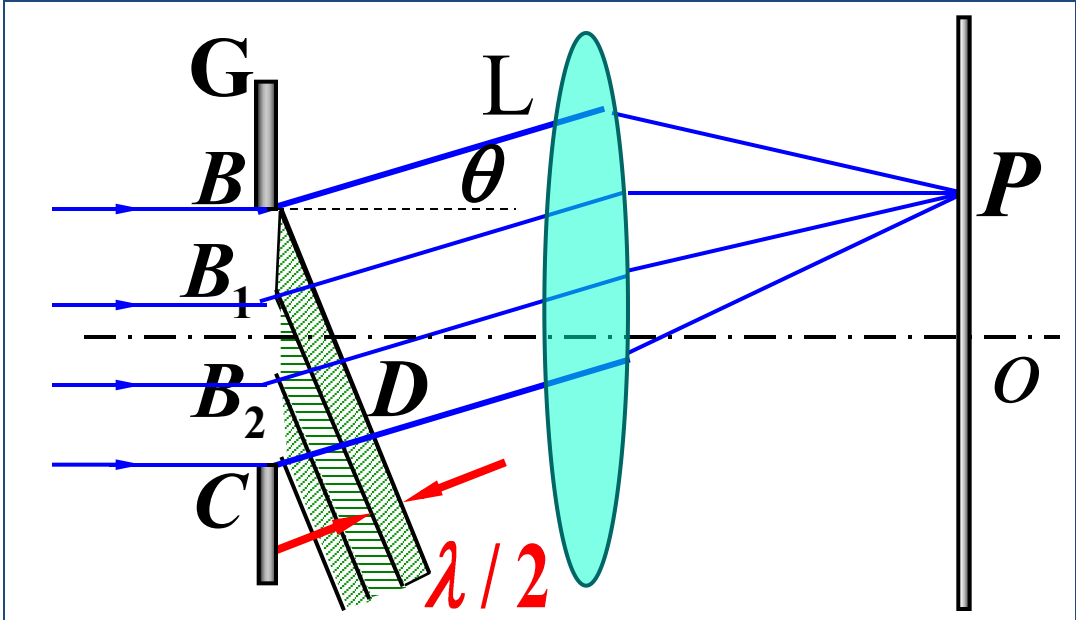
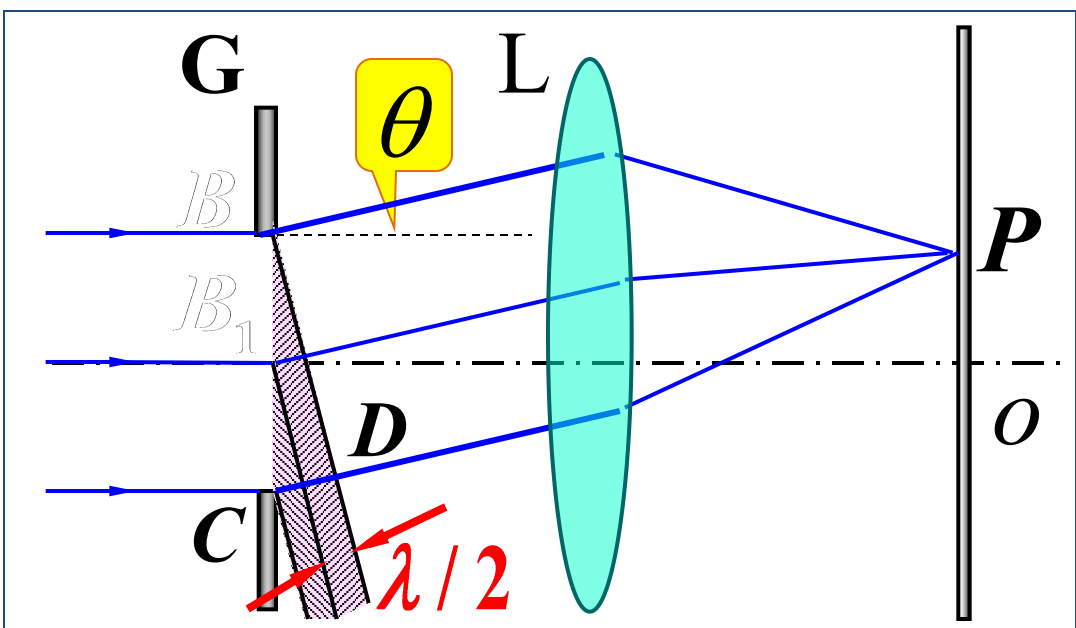


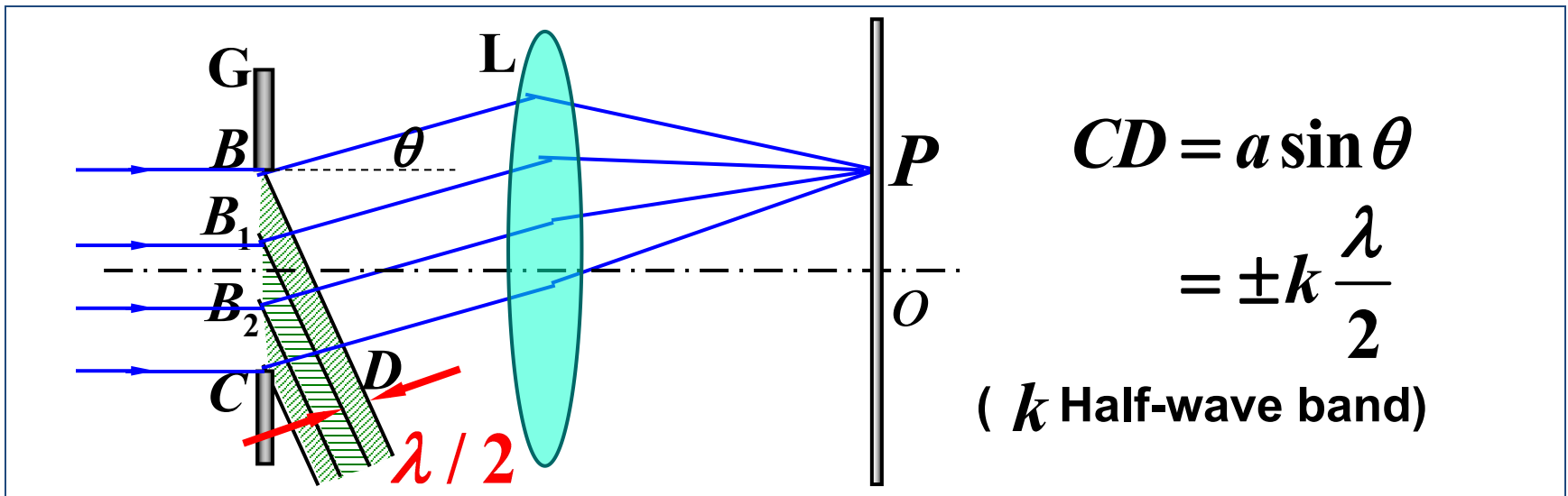
$$a \sin \theta = \pm(2k) \frac{\lambda}{2}$$



$$a \sin \theta = \pm(2k + 1) \frac{\lambda}{2}$$

$$k = 1, 2, 3, \dots$$





$$a \sin \theta = 0$$

Central bright grain  
center

$$a \sin \theta = \pm 2k \frac{\lambda}{2} = \pm k \lambda$$

Interference elimination (lines)  $2k$   
A half wave belt

$$a \sin \theta = \pm (2k + 1) \frac{\lambda}{2}$$

Interference strengthening  $2k + 1$   
A half wave belt

$$a \sin \theta \neq k \frac{\lambda}{2}$$

(Between light and shade)

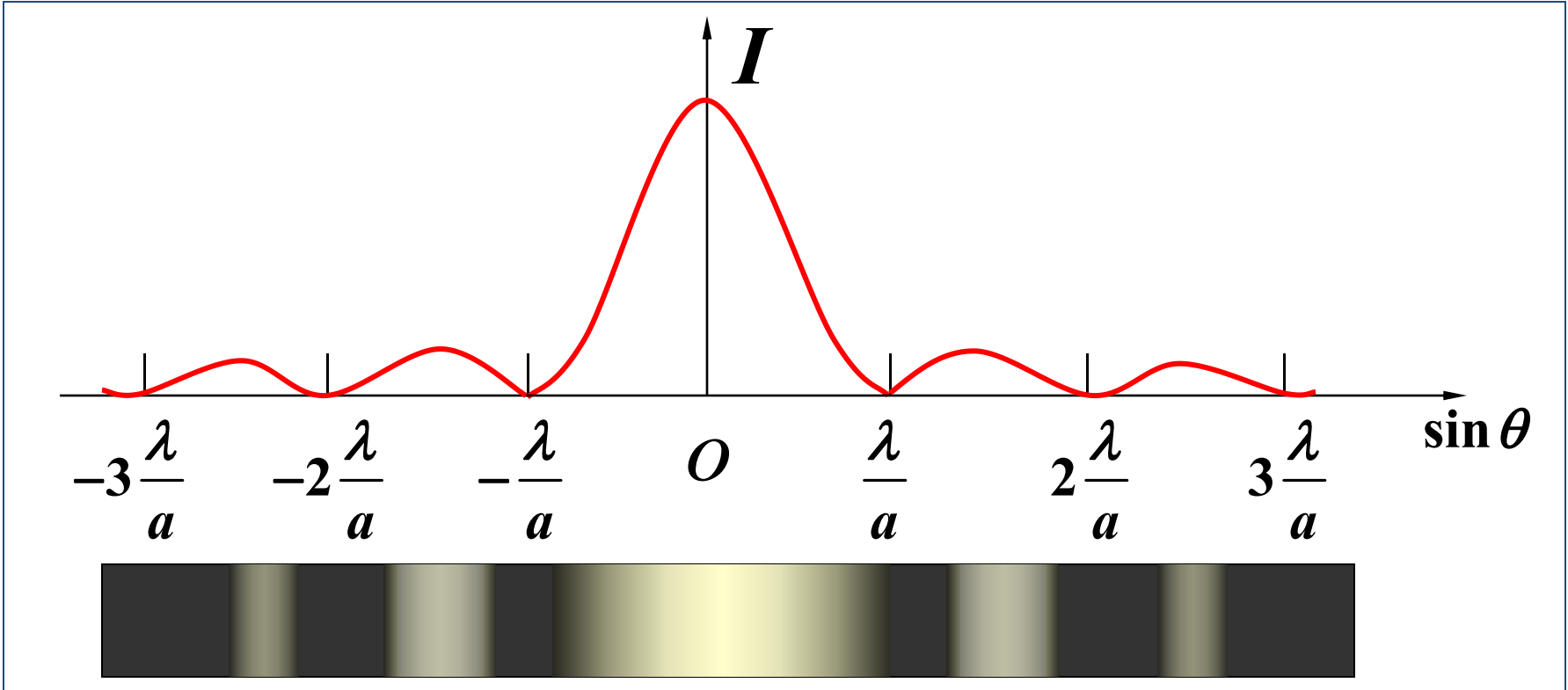
( $k = 1, 2, 3, \dots$ )

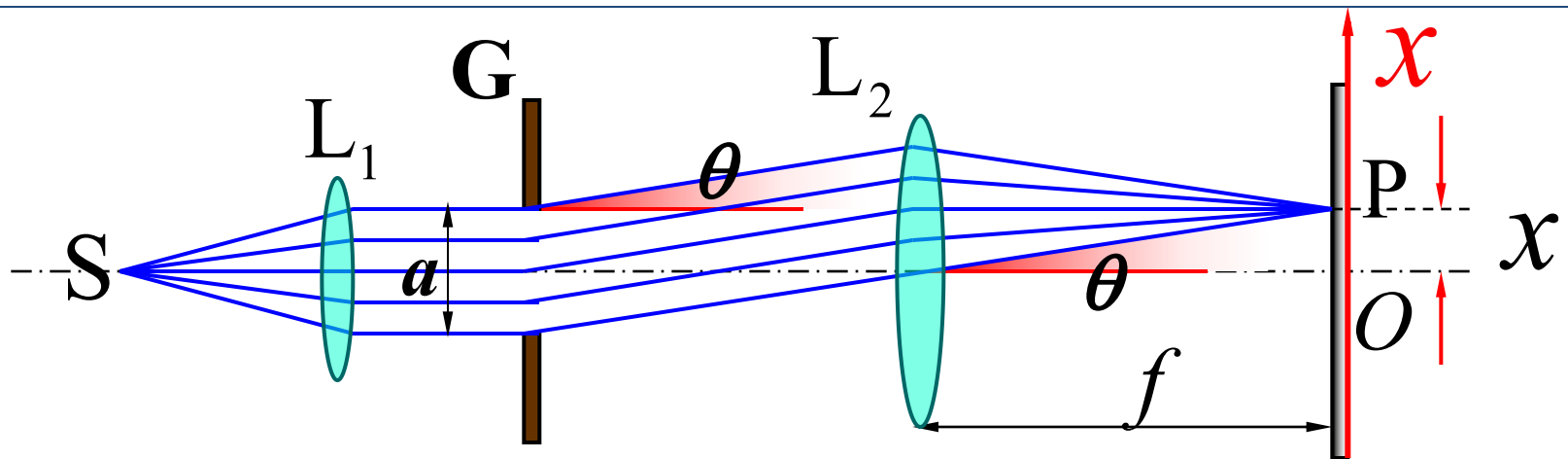
2. light distribution

$$\left\{ \begin{aligned} a \sin \theta &= \pm 2k \frac{\lambda}{2} = \pm k \lambda \\ a \sin \theta &= \pm (2k + 1) \frac{\lambda}{2} \end{aligned} \right.$$

Interference elimination  
(dark lines)

Interference strengthening  
(bright grain)



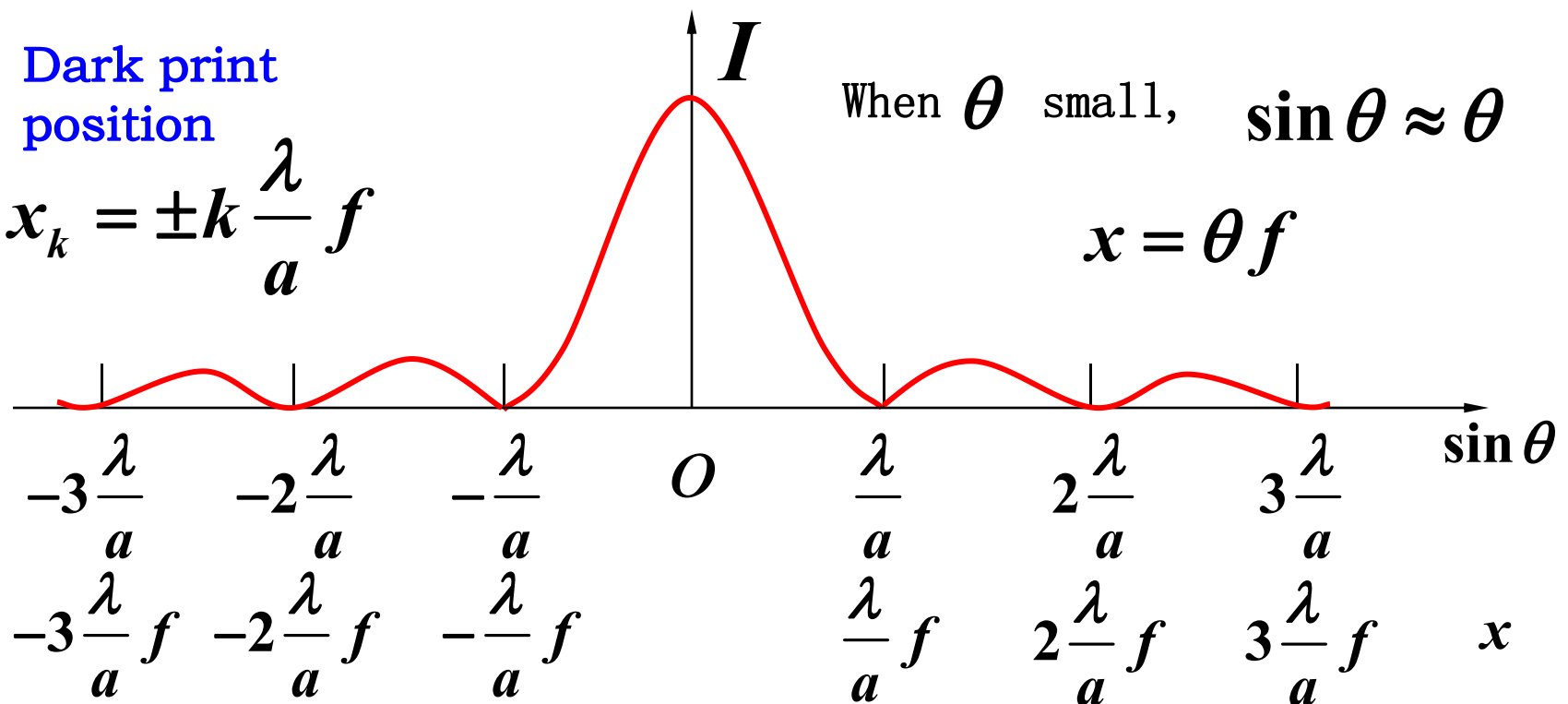


Dark print  
position

$$x_k = \pm k \frac{\lambda}{a} f$$

When  $\theta$  small,  $\sin \theta \approx \theta$

$$x = \theta f$$



discuss

$$\left\{ \begin{array}{ll} a \sin \theta = \pm 2k \frac{\lambda}{2} = \pm k \lambda & \text{Interference elimination} \\ & \text{(dark lines)} \\ a \sin \theta = \pm (2k + 1) \frac{\lambda}{2} & \text{Interference strengthening} \\ & \text{(bright grain)} \end{array} \right.$$

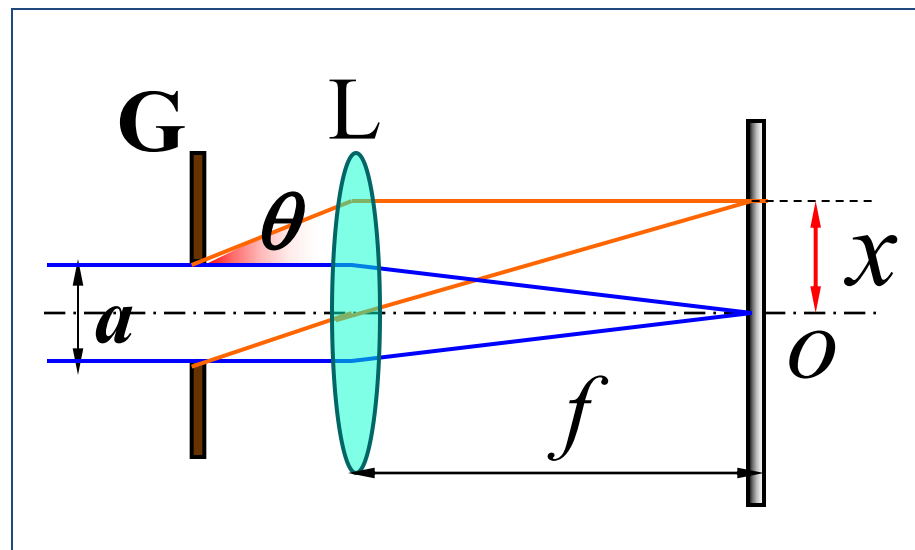
$$\sin \theta \approx \theta, \quad x_k = k \frac{\lambda}{a} f$$

1. Distance of the first dark grain from the center

$$x_1 = \frac{\lambda}{a} f$$

The diffraction angle of the first dark grain

$$\theta_1 = \arcsin \frac{\lambda}{a} \approx \frac{\lambda}{a}$$



The diffraction angle of the first dark grain  $\theta_1 \approx \frac{\lambda}{a}$

◆  $\lambda$  fixed

{	$a$ Increases, and $\theta_1$ decreases $\frac{\lambda}{a} \rightarrow 0, \theta_1 \rightarrow 0$	linear propagation of light
	$a$ Reduce, and $\theta_1$ increase $a \rightarrow \lambda, \theta_1 \rightarrow \frac{\pi}{2}$	Maximum diffraction

◆  $a$  fixed, the larger  $\lambda$ , the greater  $\theta_1$ , the more obvious the diffraction effect.

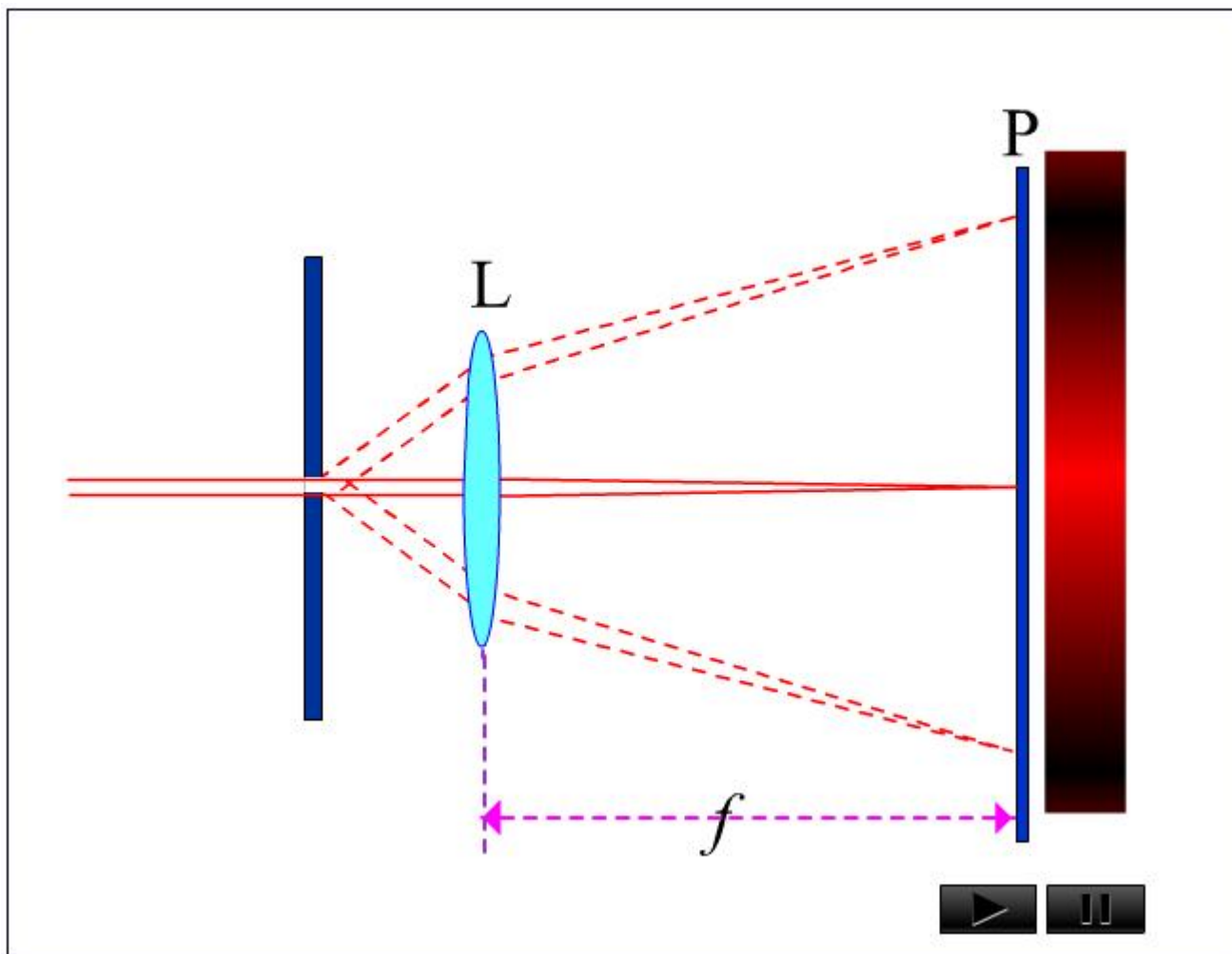
2. Central bright (  $k=1$  Between the two dark lines)  
pattern

angular region  $-\frac{\lambda}{a} < \sin \theta < \frac{\lambda}{a}$       Line range  $-\frac{\lambda}{a} f < x < \frac{\lambda}{a} f$

The angular width of the central bright pattern  $\Delta \theta_0 = 2\theta_1 = 2 \frac{\lambda}{a}$

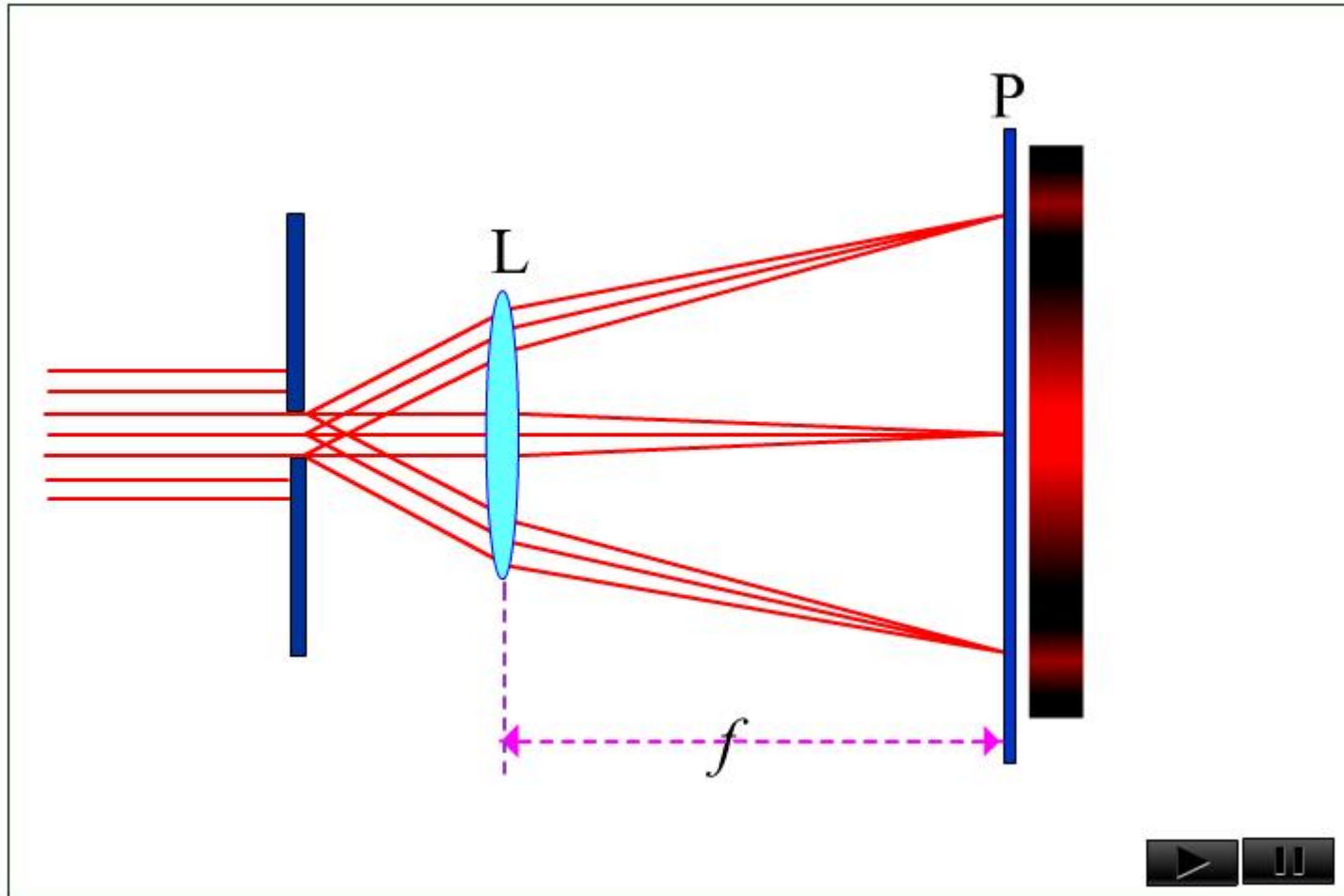


◆ The width of the single seam changes, how does the central bright grain width change?





◆ When the incident wavelength changes, how does the diffraction effect change?



$\lambda$  Larger,  $\theta_1$  greater diffraction effect.

### 3. Stripe width (adjacent stripe spacing)

$$\left\{ \begin{array}{l} a \sin \theta = \pm 2k \frac{\lambda}{2} = \pm k \lambda \\ a \sin \theta = \pm (2k + 1) \frac{\lambda}{2} \end{array} \right.$$

Interference elimination (dark lines)

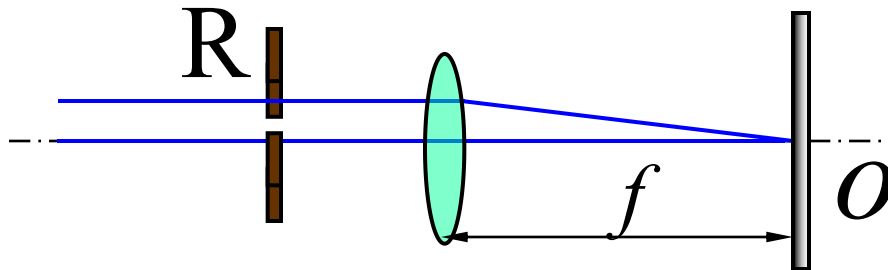
Interference strengthening (bright grain)

$$\Delta x = x_{k+1} - x_k = \frac{\lambda}{b} f$$

In addition to the central bright grain, the width of other bright and dark grain

### 4. Dynamic changes of single-seam diffraction

◆ The single slit moves up and down, and the diffraction pattern is unchanged according to the lens imaging principle.



The single seam moves up, and the zero-level bright pattern is still on the lens optical axis.

**Example 1:** a single seam with a width of  $b = 0.1 \text{ mm}$  and a poly lens with a focal length of  $50 \text{ cm}$  after the seam, and a parallel light of wavelength  $= 546.1 \text{ nm}$  to find the width of the central bright pattern on the screen at the focal plane and the distance between any two adjacent dark lines on the sides of the central bright pattern. If the single seam position is moved at a small distance, how will the diffraction stripe on the screen change?

**separate**

**Central bright grain width**

$$\Delta x_0 = \frac{2\lambda f}{b} = 5.46 \text{ mm}$$

**Other bright grain width**

$$\Delta x = \frac{\lambda f}{b} = 2.73 \text{ mm}$$



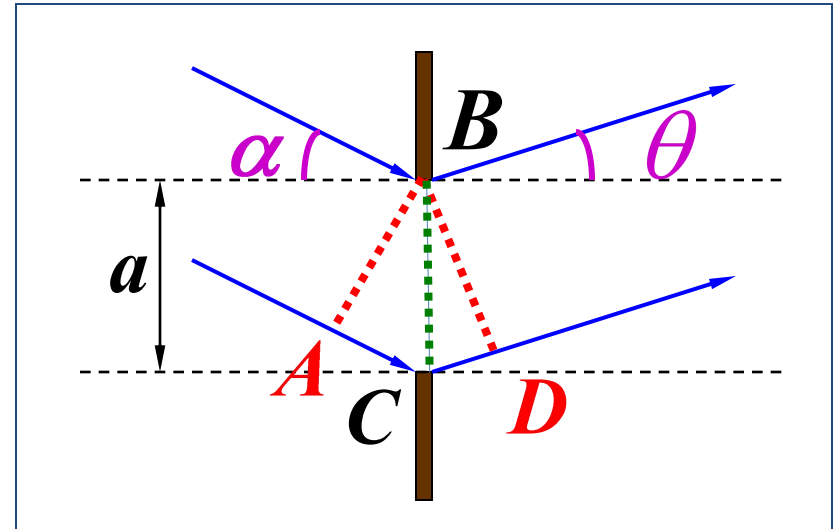
**If the single seam position is moved from a small distance, the diffraction stripe on the screen remains unchanged**



## 5. Calculation of optical path difference in non-vertical incident light

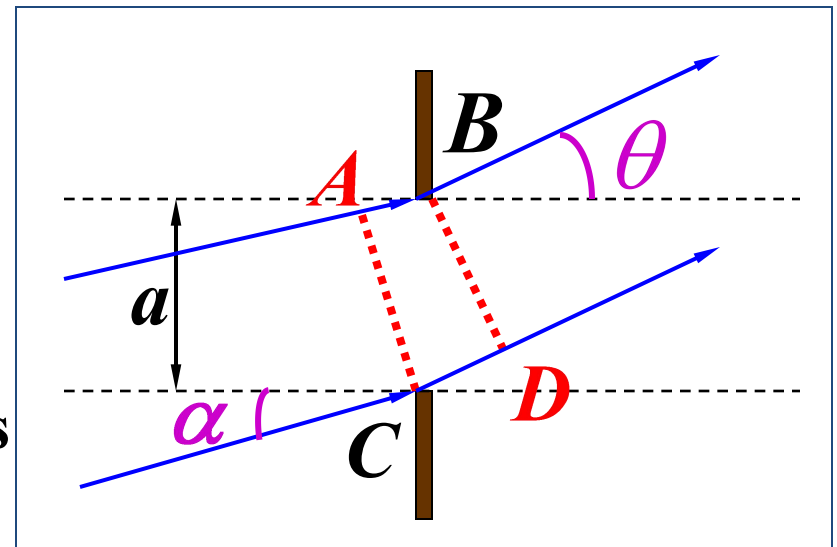
$$\Delta L = CD + AC$$
$$= a(\sin \theta + \sin \alpha)$$

(The central bright pattern moves down)



$$\Delta L = CD - BA$$
$$= a(\sin \theta - \sin \alpha)$$

(Central bright pattern moves upward)



Example 1 has a monochromatic plane wave transmitted oblique to a single slit of width  $A$ , as shown in the figure, seeking the diffraction angle of the dark lines.

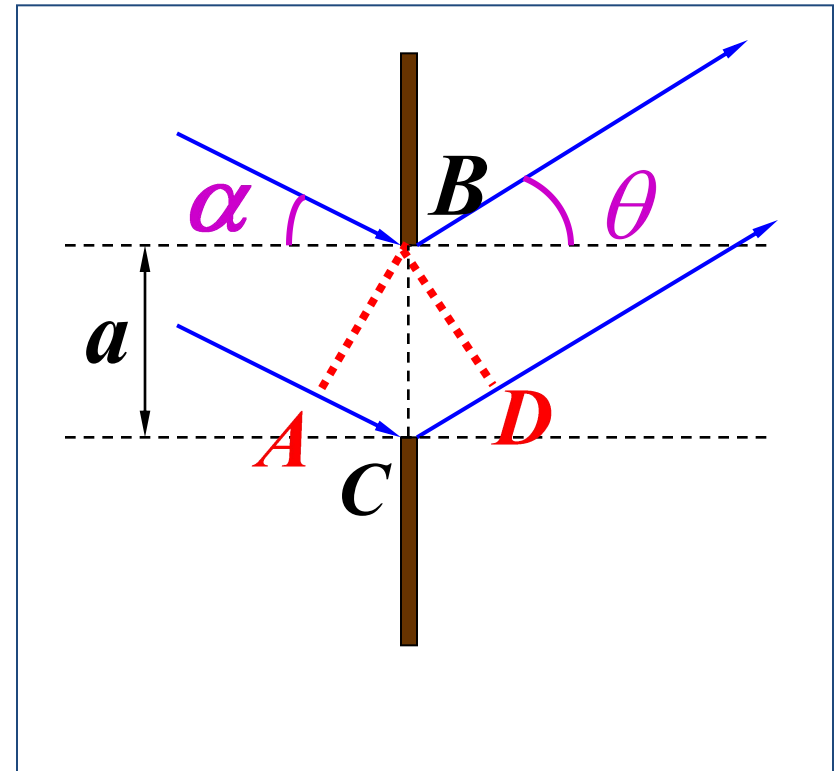
separate

$$\begin{aligned}\Delta L &= CD + AC \\ &= a(\sin \theta + \sin \alpha)\end{aligned}$$

By dark grain conditions

$$\begin{aligned}a(\sin \theta + \sin \alpha) &= \pm k\lambda \\ (k &= 1, 2, 3, \dots)\end{aligned}$$

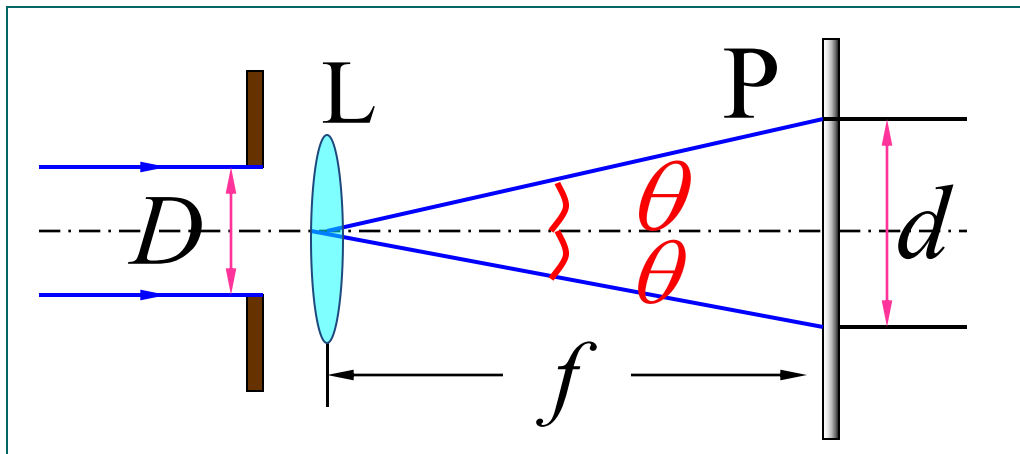
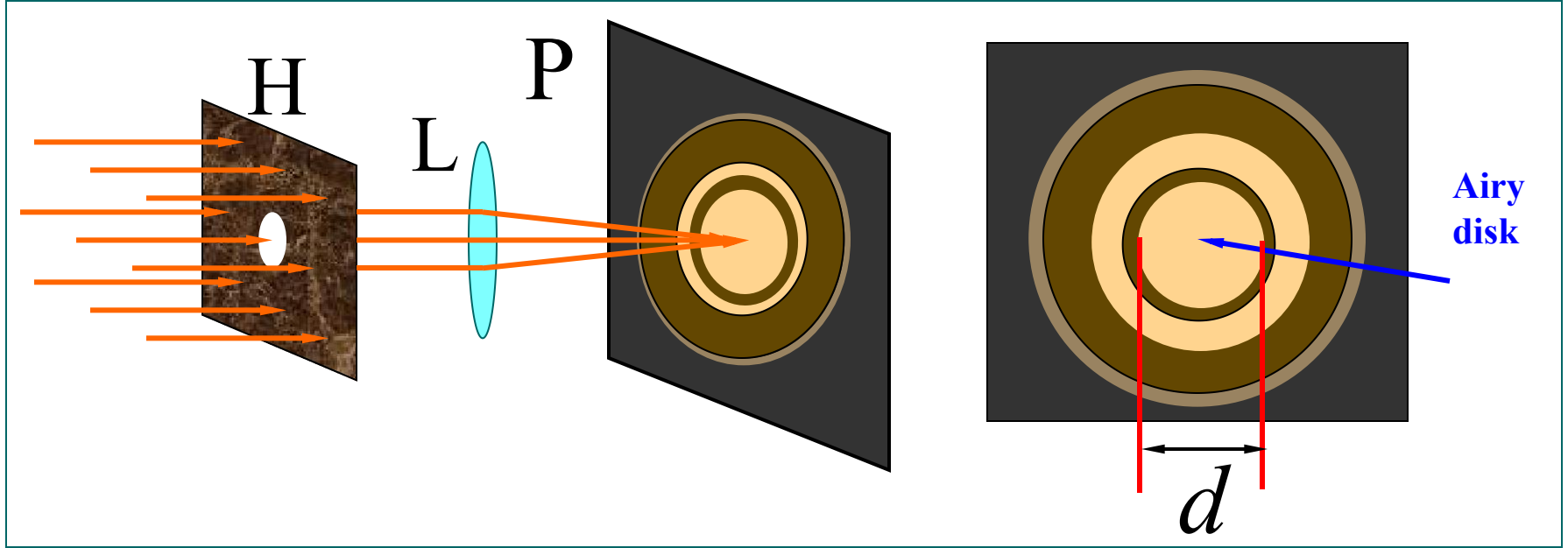
$$\theta = \arcsin\left(\frac{\pm k\lambda}{a} - \sin \alpha\right)$$



## § 14.4

**Furunhefer round pore diffraction  
And the resolution skills of the optical instruments**

## 14.4.1 Furunhefer round hole diffraction



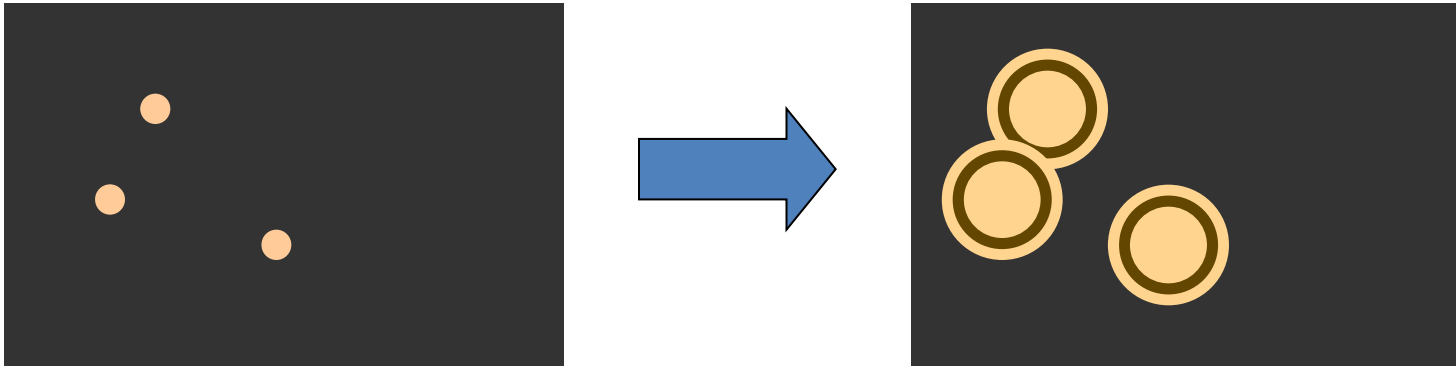
$d$  : Iris spot diameter

The half-angular width of the Airy spot

$$\Delta\theta = \frac{d/2}{f} = 1.22 \frac{\lambda}{D}$$



## 14.4.2 Resolution ability of optical instruments

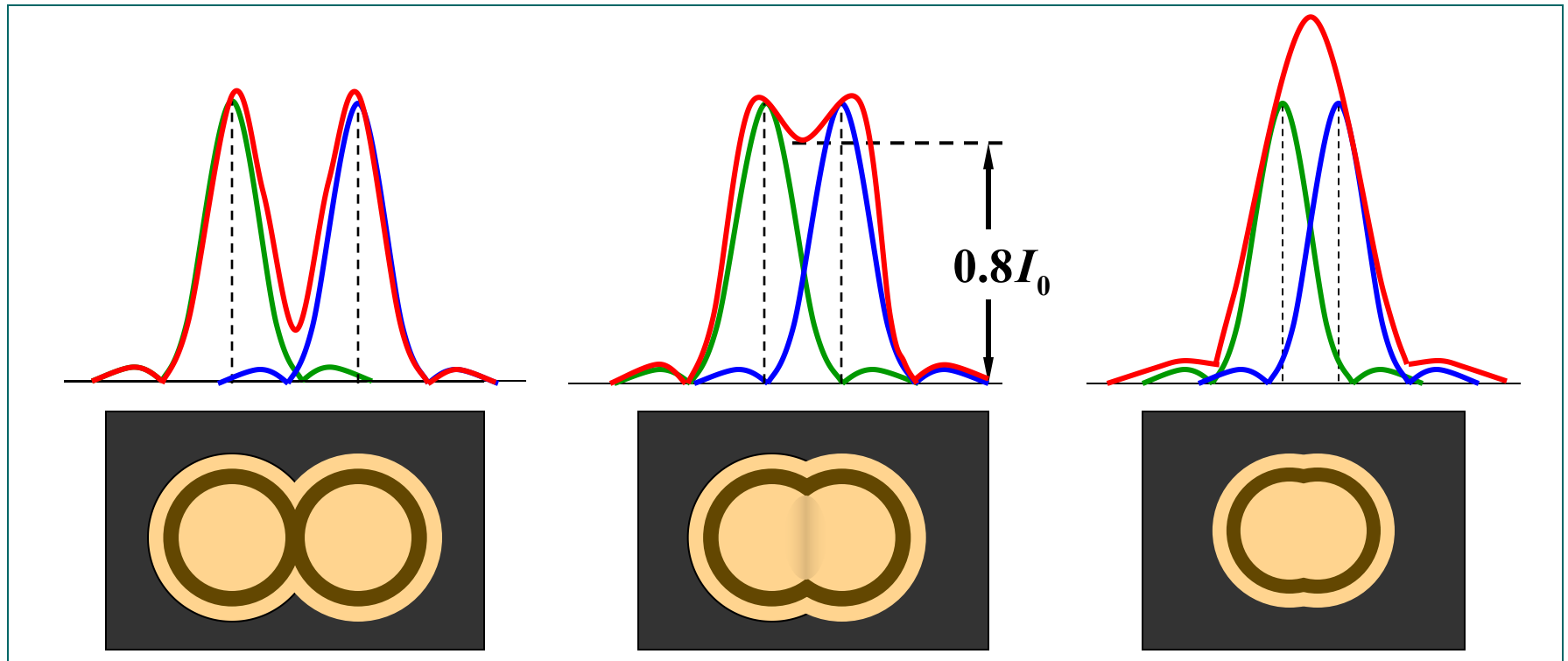


Due to the presence of diffraction, the ideal imaging optical instrument can not achieve the ideal situation of dot imaging.

The resolution ability of the optical instrument- -the ability of the instrument to distinguish the image of two adjacent objects.

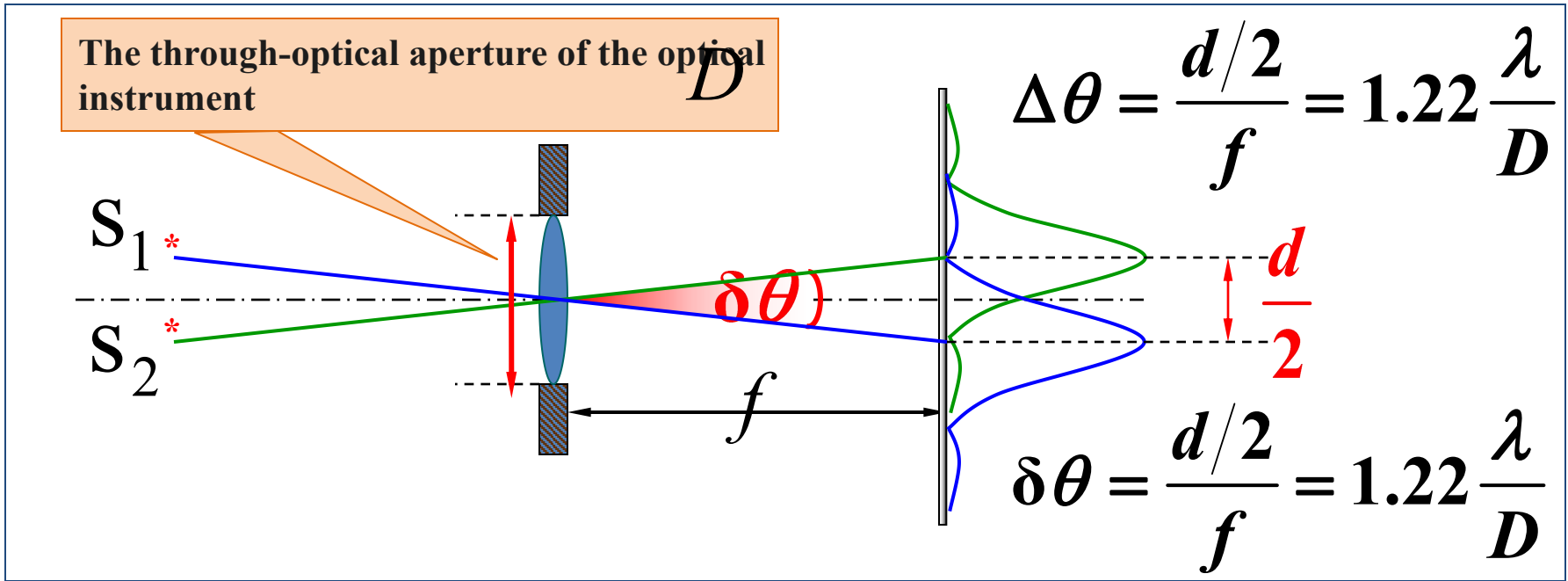
**Question: how far should the two points be at least apart, before the instrument or the human eye can distinguish between them?**

# 1. Rayleigh criterion



For two irrelevant point light sources (points) of equal intensity, the main maximum of the diffraction pattern of one point light source coincides with the first minimum of the diffraction pattern of the other point light source, when the image of the two point light sources (or points) can be distinguished.

2. The resolution ability of the optical instruments (Two dots just enough to distinguish)

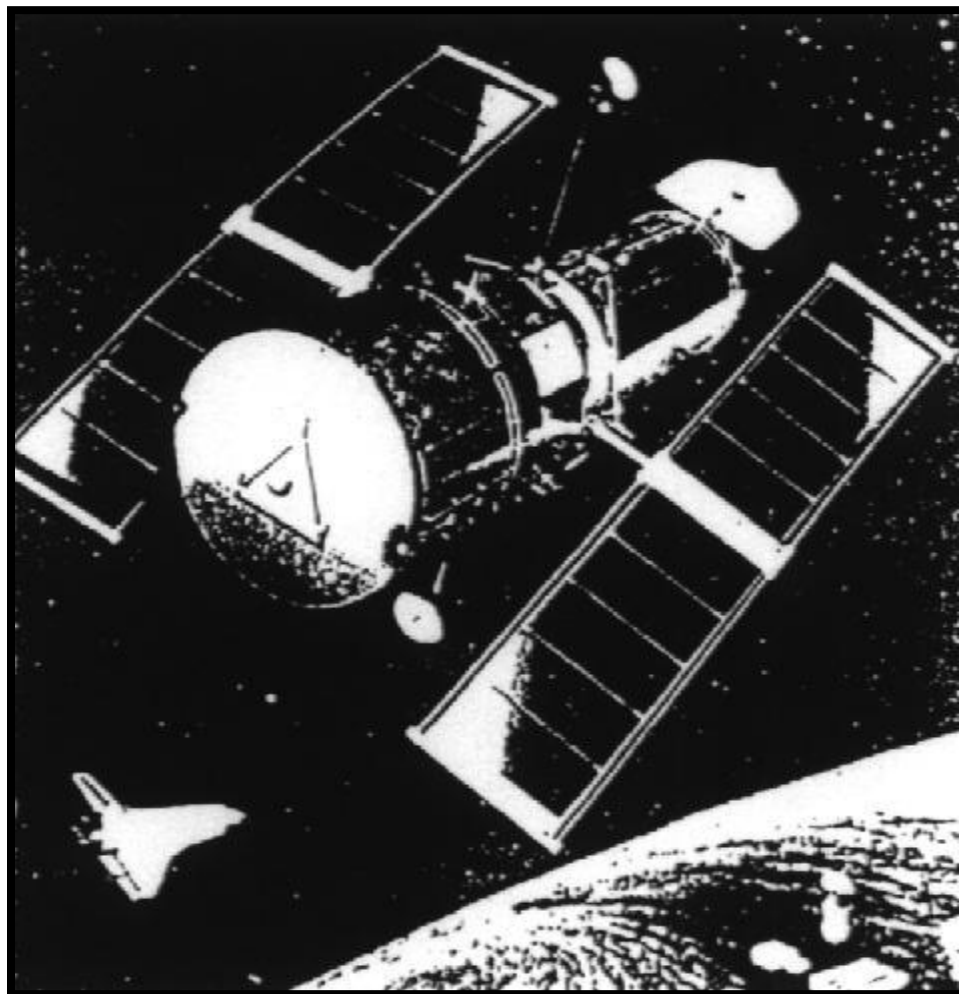


angle of minimum resolution  $\delta \theta = 1.22 \frac{\lambda}{D}$

Optical instrument resolution  $R = \frac{1}{\delta \theta} = \frac{D}{1.22 \lambda} \propto D, \frac{1}{\lambda}$



The Hubble Space Telescope, launched in 1990. The concave objective lens of the Hubble Space Telescope. The concave objective lens of the Hubble Space Telescope, launched in 1990, has a diameter of 2.4m and a minimum angle of discrimination of  $\delta\theta = 0.1''$ , orbiting the Earth at an altitude of 615km outside the atmosphere. orbiting the Earth at an altitude of 615km above the atmosphere. It can observe 13 billion light-years away in the depths of space. space, and has discovered 50 billion galaxies. galaxies.



**Example 1** Let the pupil diameter of the human eye under normal illumination be about 3mm, while in the visible light, the most sensitive wavelength of the human eye is 550nm, Q

**(1)** How big is the minimum resolution Angle of the human eye?

**(2)** If the object is placed 25cm away from the human eye (open visual distance), how long can the distance between the two objects be distinguished?

**Solution (1)**

$$\delta\theta = 1.22 \frac{\lambda}{D} = \frac{1.22 \times 5.5 \times 10^{-7} \text{ m}}{3 \times 10^{-3} \text{ m}}$$
$$= 2.2 \times 10^{-4} \text{ rad}$$

**(2)**

$$d = l \cdot \delta\theta = 25\text{cm} \times 2.2 \times 10^{-4}$$
$$= 0.0055\text{cm} = 0.055\text{mm}$$



## § 14.5

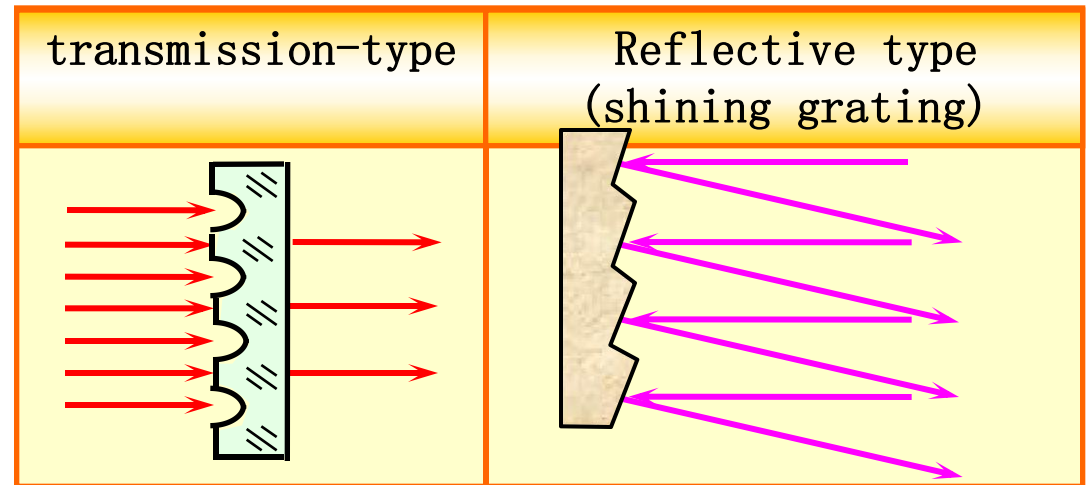
# diffraction grating

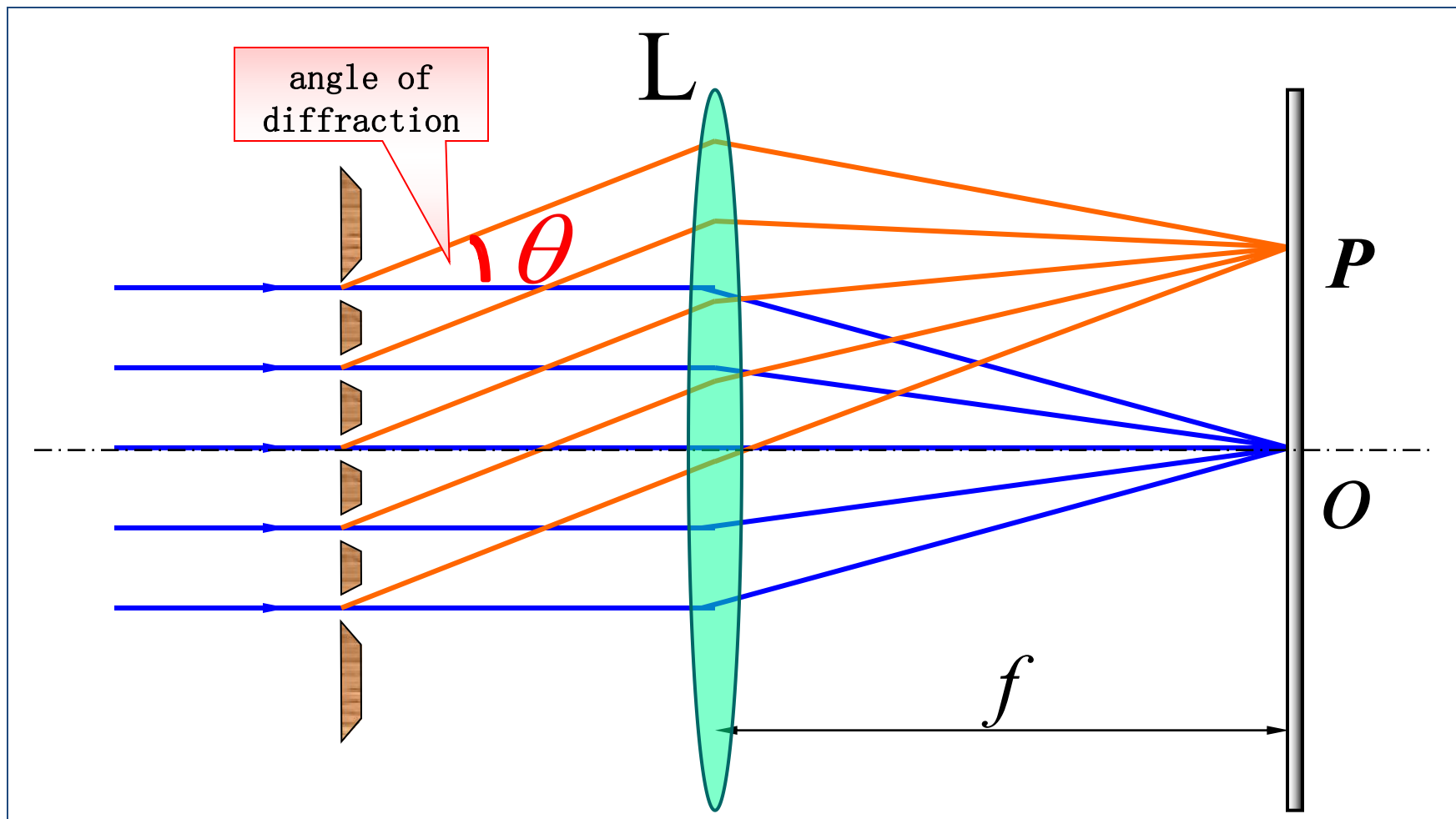
# 1. raster

Many equal width, slit, equal spacing arranged to form a periodic structure of the optical elements.

1. Transmission grating: cut a series of parallel equidistant scratches on the glass plate, the place is not light (diffuse reflection), the place is light (equivalent to the light slit).

2. Reflective grating: carve a series of equally spaced parallel fine grooves on the metal surface with high finish, which are serrated.







2. Formation of the grating diffraction stripes

The diffraction stripes of the grating are the total effect of single-seam diffraction and multi-seam interference.

Light path difference between two adjacent seams:

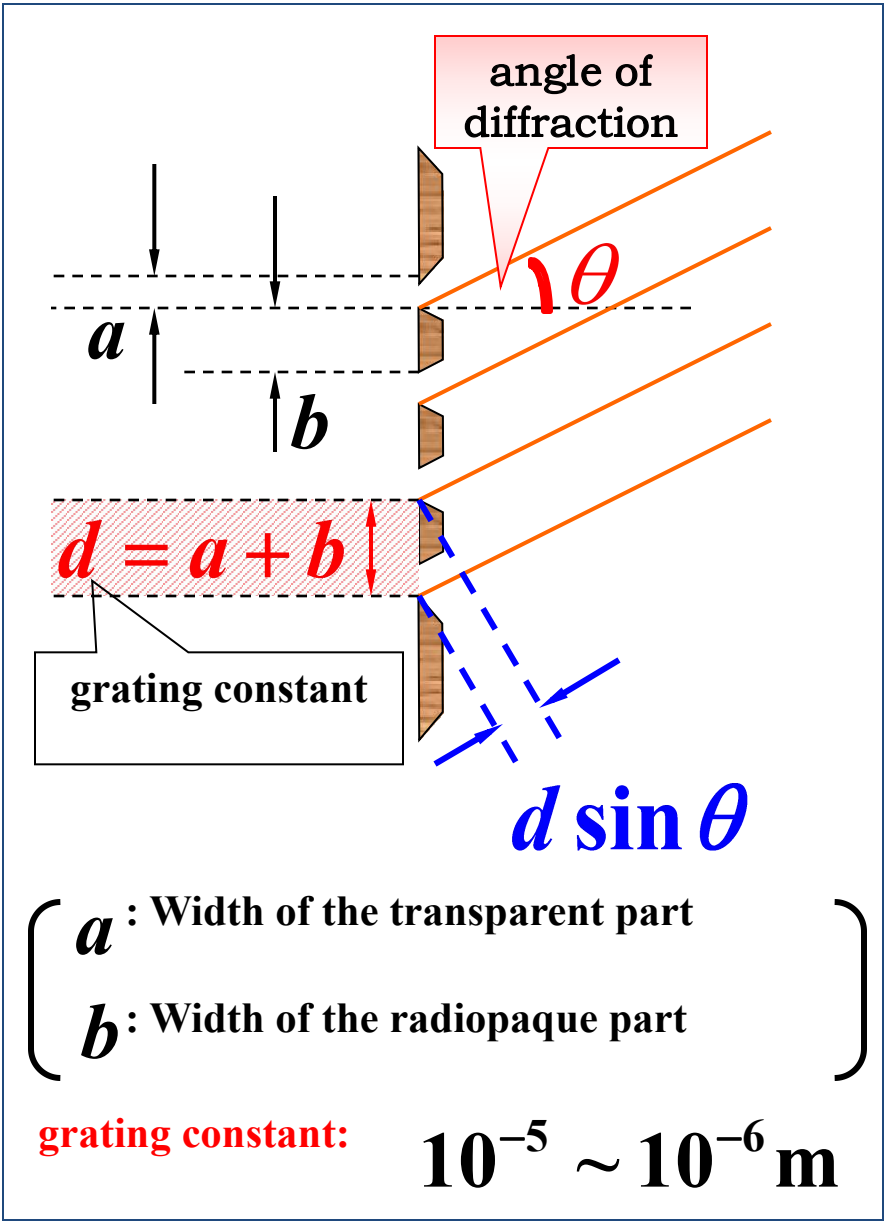
$$\Delta L = (a + b) \sin \theta = d \sin \theta$$

grating equation

Interference with bright pattern position

$$d \sin \theta = \pm k \lambda$$

$$(k = 0, 1, 2, \cdots)$$

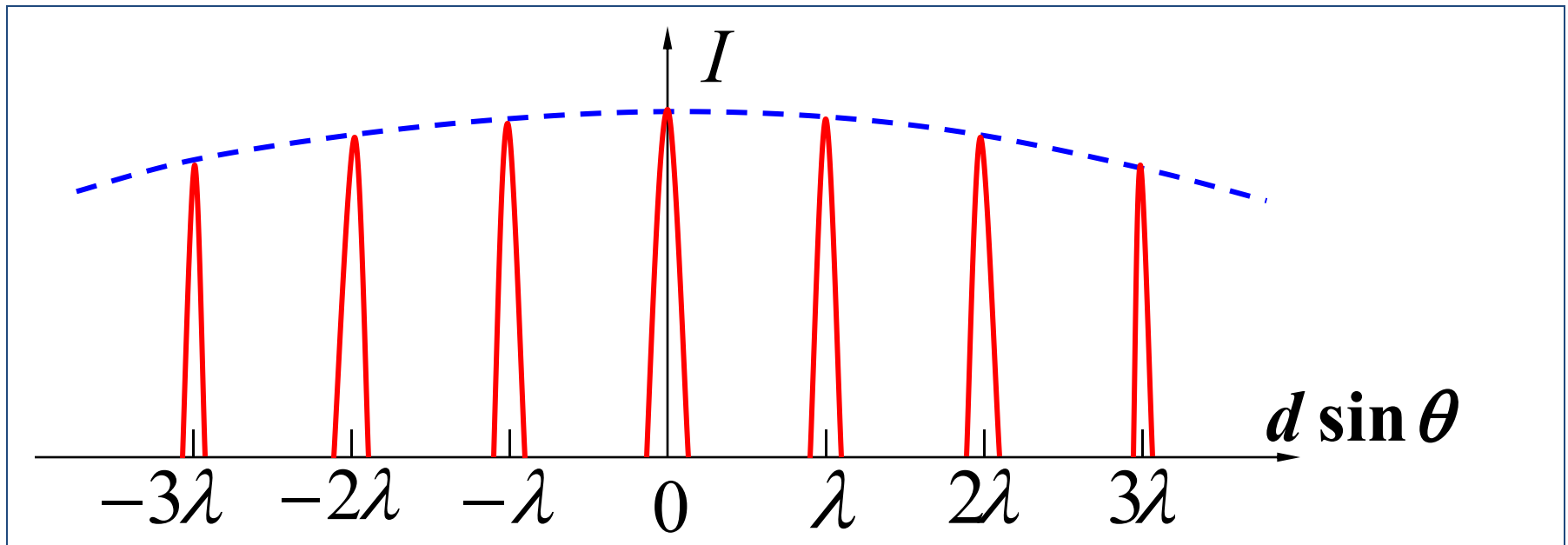


discuss

grating equation

$$d \sin \theta = \pm k \lambda \quad (k = 0, 1, 2, \dots)$$

◆ light distribution



◆ The highest series of stripes

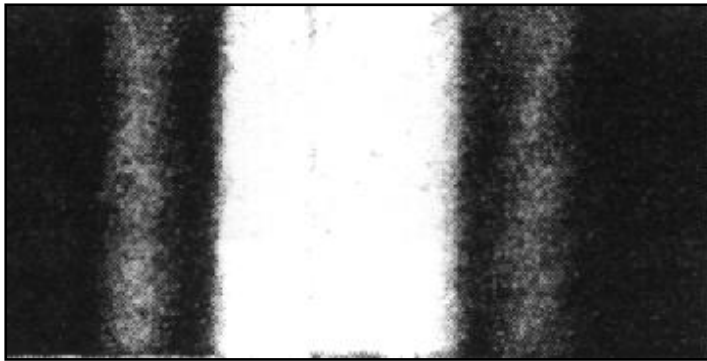
$$\sin \theta_k = \pm \frac{k \lambda}{d}$$

$$\theta = \pm \frac{\pi}{2},$$

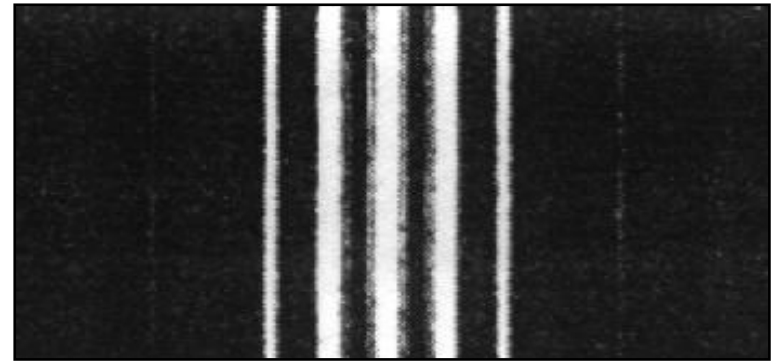
$$k_{\max} = \frac{d}{\lambda}$$

◆ The more the slit bars in the grating, the brighter the bright grain.

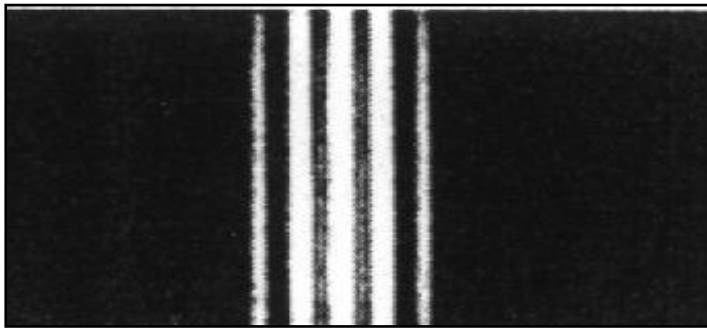
**Bright grain of light**  $I = N^2 I_0$  (N: Number of slits,  $I_0$  : Single slit light strength)



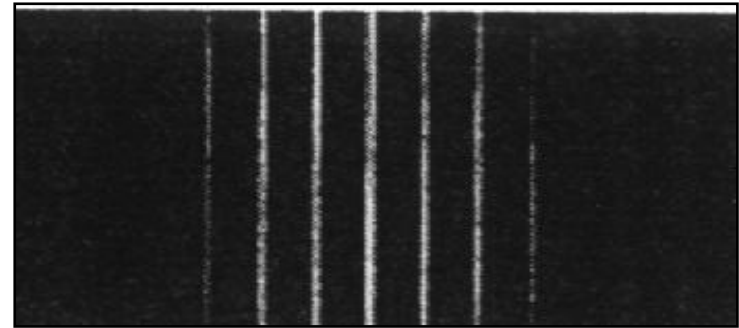
**A seam**



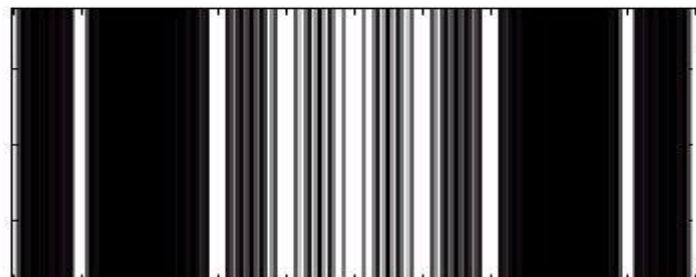
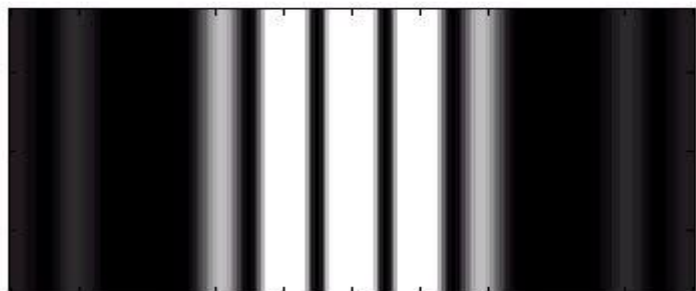
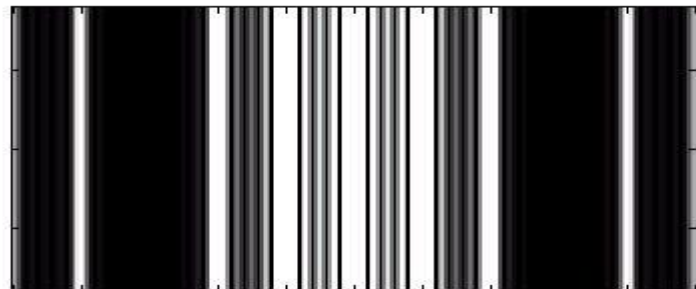
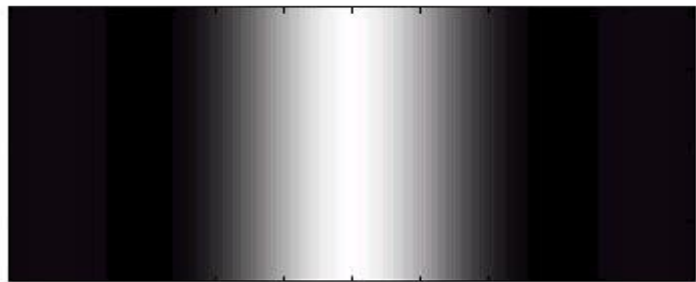
**Five joints**



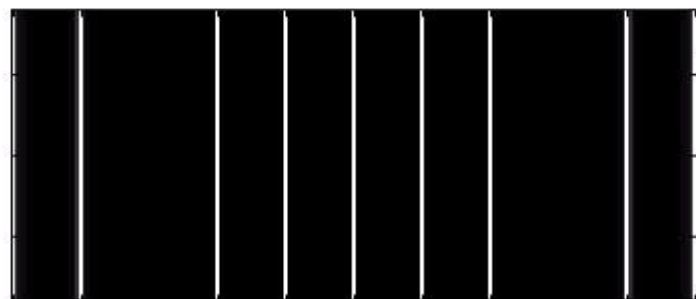
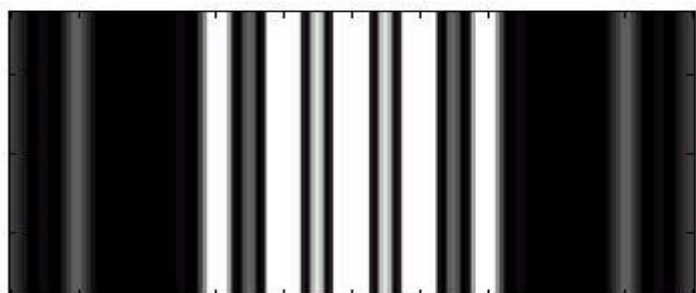
**Three joints**



**20 Seeds**



$\xi$



◆ Clear grain spacing

$$d \sin \theta = \pm k \lambda \quad (k = 0, 1, 2, \dots)$$

$$\Delta k = 1, \quad \sin \theta_{k+1} - \sin \theta_k = \frac{\lambda}{d}$$

(1) The smaller the grating constant, the narrower the bright grain and the farther the separation between the bright lines

$\lambda$  fixed,  $d$  decrease,  $\theta_{k+1} - \theta_k$  and increase.

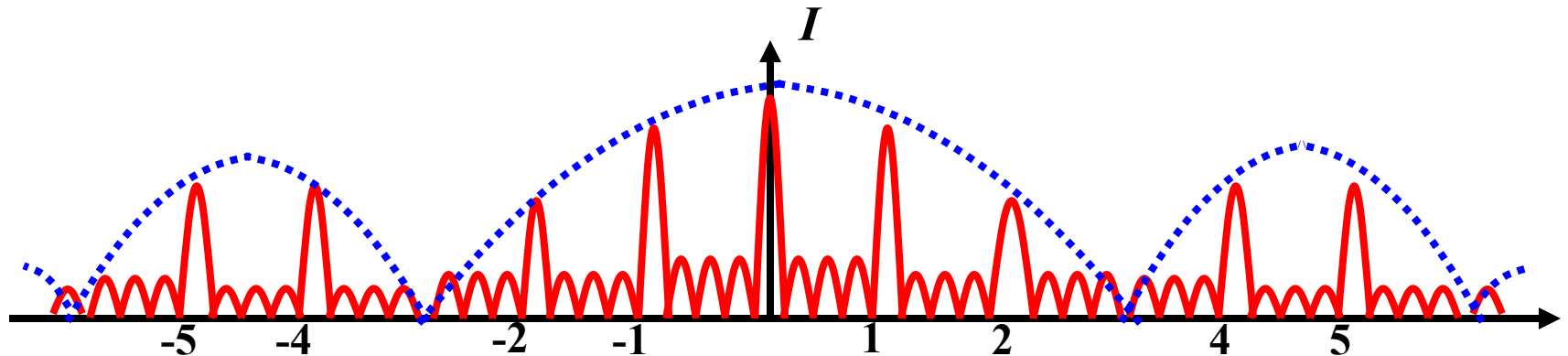
(2) The greater the wavelength of the incident light, the farther the separation between the bright lines

$d$  fixed,  $\lambda$  increase, and  $\theta_{k+1} - \theta_k$  increase.



## ◆ Dark lines and secondary bright lines

There is  $N - 1$  dark pattern between the two adjacent interference bright lines; there is a bright pattern between the two adjacent dark patterns. Because its brightness is much smaller than the interference bright pattern, it is called the secondary bright pattern, so the two adjacent interference bright patterns (also called the main bright pattern) will appear  $N - 2$  secondary bright pattern .



## ◆ Missing stage phenomenon

Multi-seam interference and  
bright pattern conditions

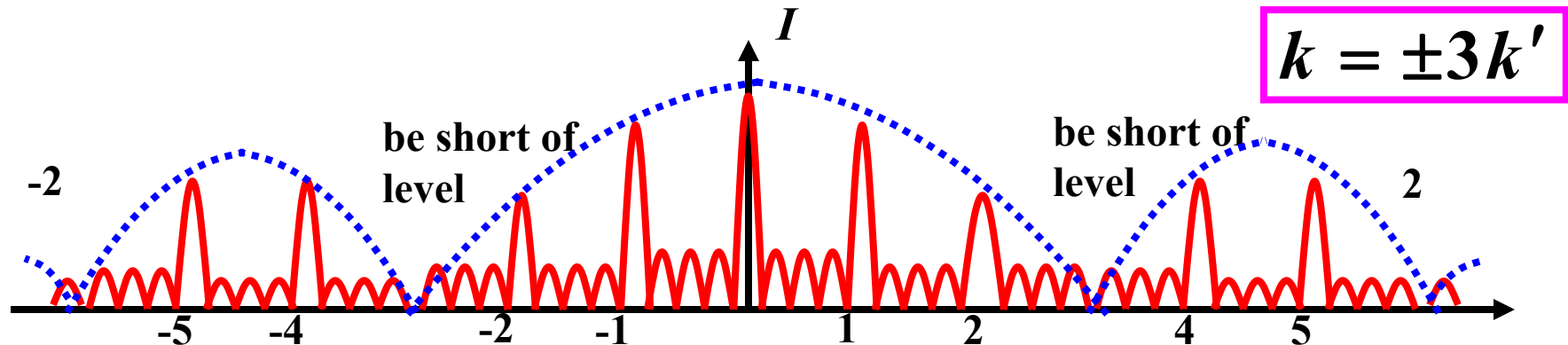
$$d \sin \theta = \pm k \lambda$$
$$(k = 0, 1, 2, \dots)$$

Single-seam diffraction dark  
pattern conditions

$$a \sin \theta = \pm k' \lambda$$
$$(k' = 1, 2, 3, \dots)$$

Main level of  
missing level

$$k = \pm \frac{d}{a} k' \quad (k' = 1, 2, 3, \dots)$$



Example 1 uses white light on a flat grating with 6500 nicks per centimeter to find the angle of the third level spectrum.

separate  $\lambda = 400 \sim 760 \text{ nm}$   $d = \frac{1}{6500} \text{ cm}$

purple light  $\sin \theta_1 = \frac{k \lambda_1}{d} = \frac{3 \times 4 \times 10^{-5} \text{ cm}}{1/6500 \text{ cm}} = 0.78$   $\theta_1 = 51.26^\circ$

glow  $\sin \theta_2 = \frac{k \lambda_2}{d} = \frac{3 \times 7.6 \times 10^{-5} \text{ cm}}{1/6500 \text{ cm}} = 1.48 > 1$  invisible

The angle of the third stage spectrum

$$\Delta\theta = 90.00^\circ - 51.26^\circ = 38.74^\circ$$

The maximum wavelength at which the third stage spectrum can occur

$$\lambda' = \frac{d \sin 90^\circ}{k} = \frac{d}{3} = 513 \text{ nm} \quad \text{Green Flash}$$





§ 14.6

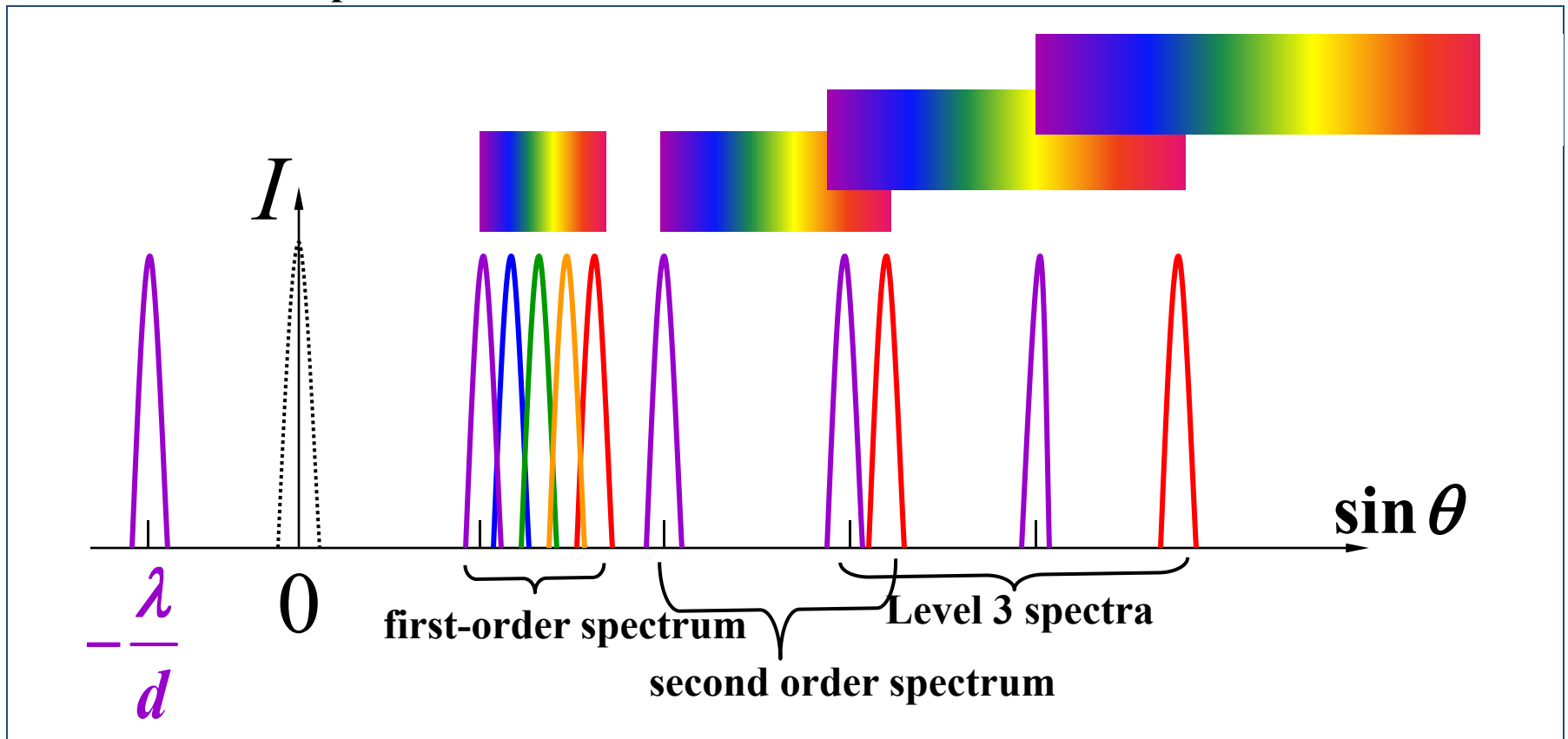
**grating spectrum**

# 14.6.1 The spectral principle of the grating

grating  
equation

$$d \sin \theta = \pm k \lambda \quad (k = 0, 1, 2, \dots)$$

When white light incident, different  $\lambda$ , different  $\theta_k$ , according to the wavelength order to form the spectrum.



**For example, the secondary spectrum overlaps the part of the spectral range**

$$\begin{cases} (b + b') \sin \theta = 3\lambda_{\text{紫}} \\ (b + b') \sin \theta = 2\lambda \end{cases}$$

$$\lambda = \frac{3}{2} \lambda_{\text{紫}} = 600 \text{ nm}$$

$$\lambda = 400 \sim 760 \text{ nm}$$

**Secondary spectral  
overlap part:**

$$600 \sim 760 \text{ nm}$$



## ◆ **Diffraction spectroscopy classification**

**Continuous spectrum: a hot object spectrum**

**Linear spectrum: Gas discharge in the discharge tube**

**Band spectrum: Molecular spectrum**



## ◆ spectrum analysis

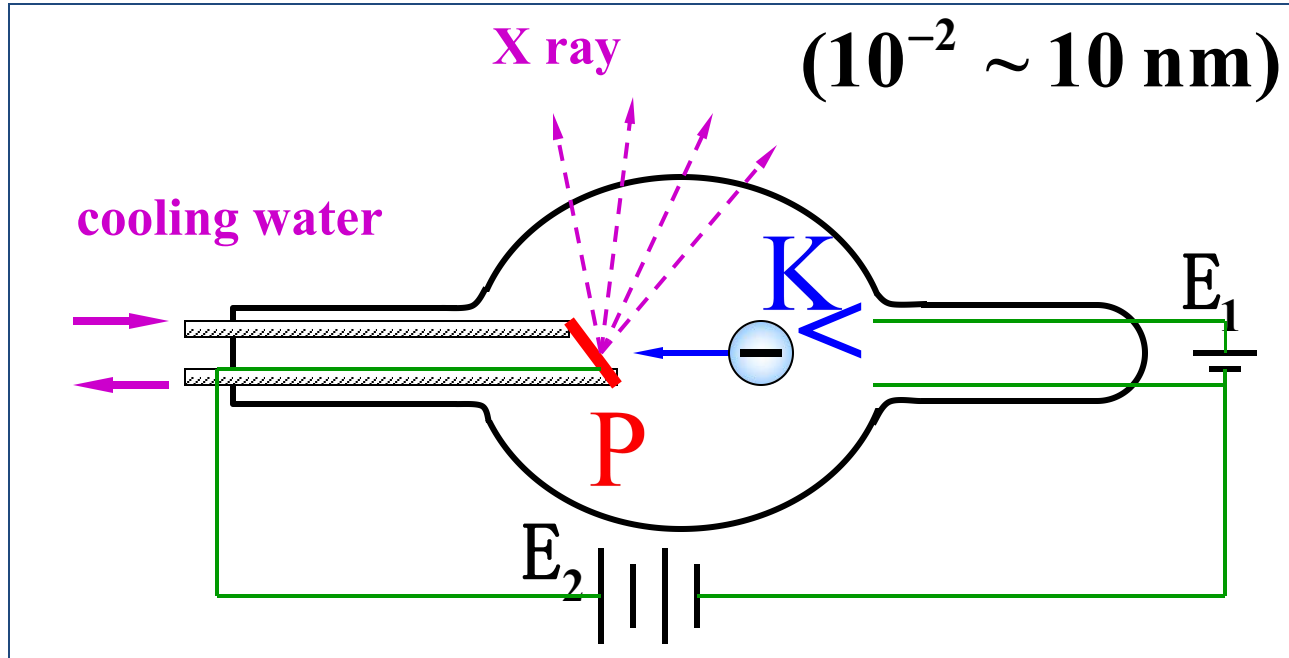
**Since different elements (or compounds) have their own specific spectra, the elements or compounds contained in the luminous substance can be analyzed from the composition of the line; the content of elements can be analyzed from the intensity of the line.**



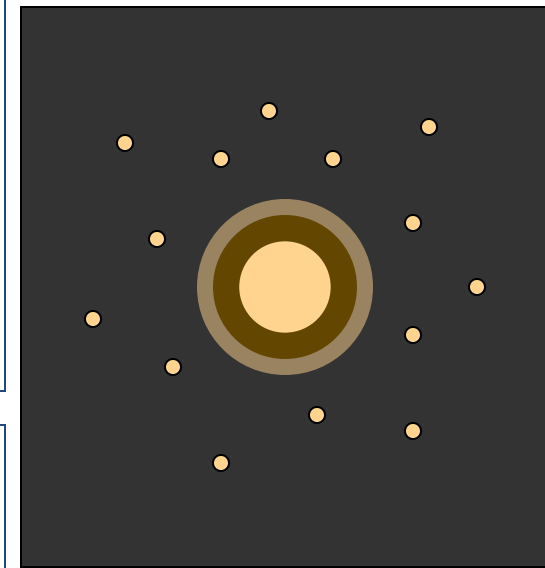
## § 14.7

# X x ray diffraction

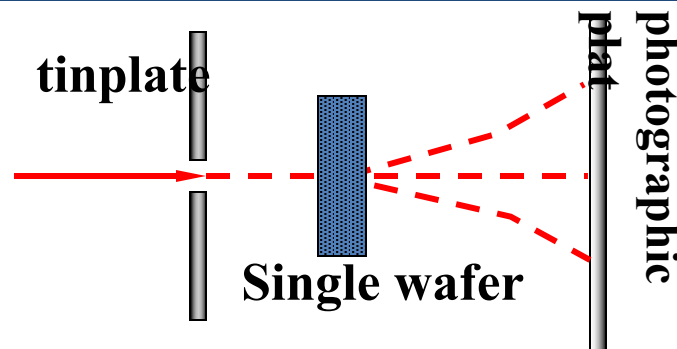
1. In 1885, Rentgen found that a metal hit by high-speed electrons fired a very penetrating ray, called an X-ray.



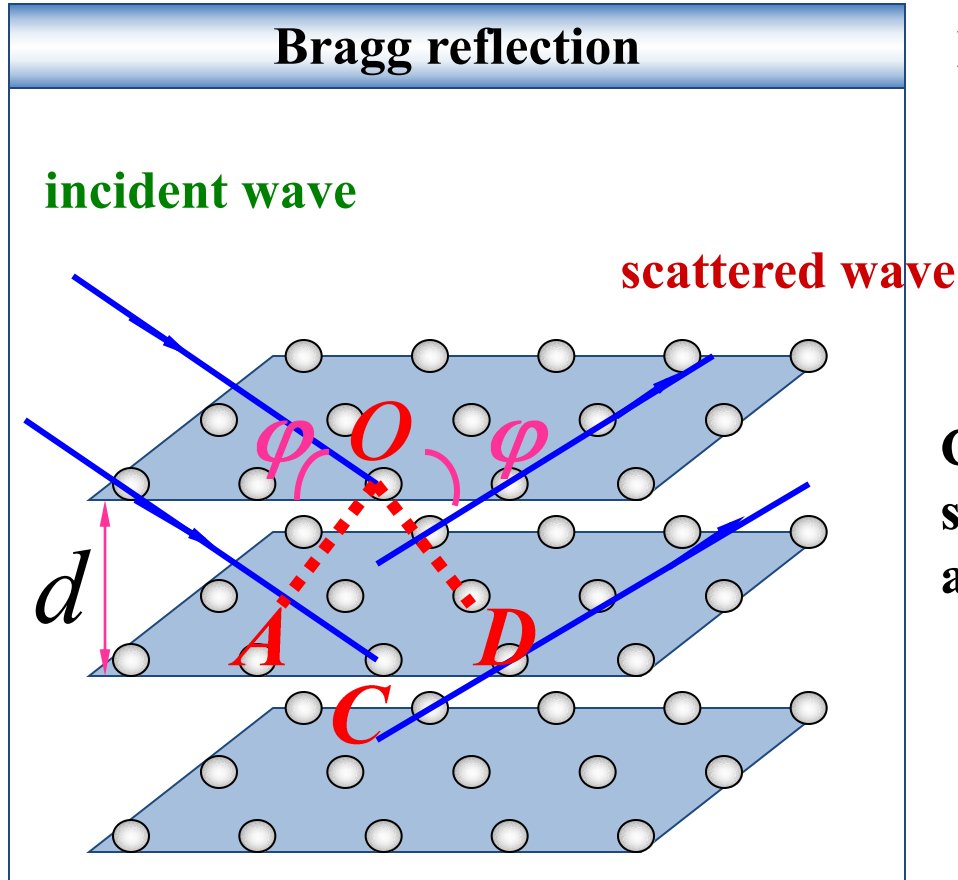
laue spot



**Diffraction of a single wafer**  
1912



2. In 1913, Prague and Son proposed a method to explain X-ray diffraction, and won the Nobel Prize in physics in 1915.



lattice constant  $d$       glancing angle  $\theta$

$$\Delta L = AC + CD$$
$$= 2d \sin \varphi$$

Conditions for two X-ray interference strengthening reflected by two adjacent crystal surfaces

Bragg's formula

$$2d \sin \varphi = k\lambda$$

$$k = 0, 1, 2, \dots$$



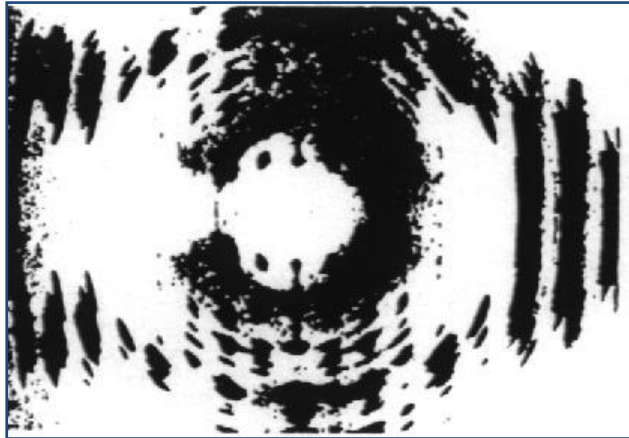


◆ Bragg's formula

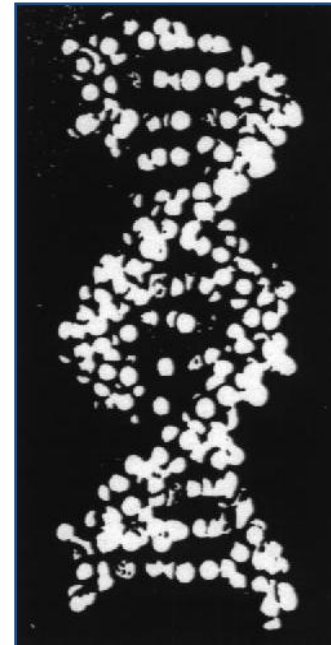
$$2d \sin \varphi = k\lambda \quad k = 0, 1, 2, \dots$$

use:

1. Measure the wavelength of rays to study the X-ray spectrum, and then study the atomic structure;
2. Study the structure of the crystal and further study the material properties.



**DNA, an X diffraction photo of the crystal**



**The double-helical structure of the DNA molecule**