§ 5.3

Gauss's law

5.3.1 Electric field line

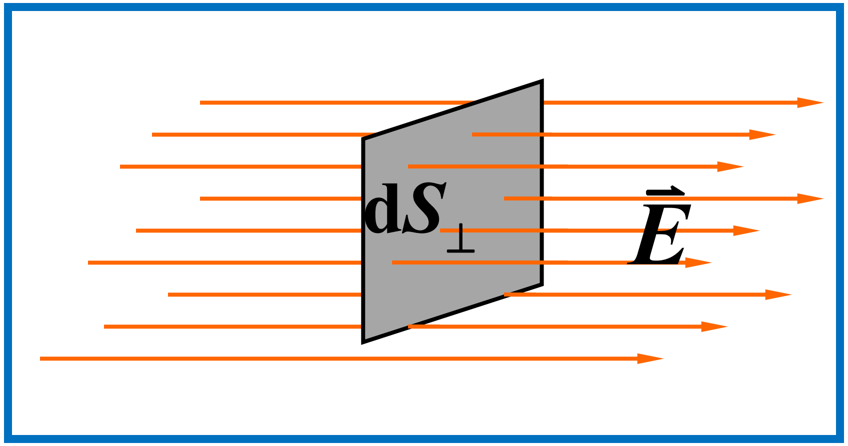
stipulate

1) The tangent direction of each point on the curve is the electric field direction of the point;

2) The number of electric field lines per unit area through the direction perpendicular to the electric field is the size of the electric field strength at the point.

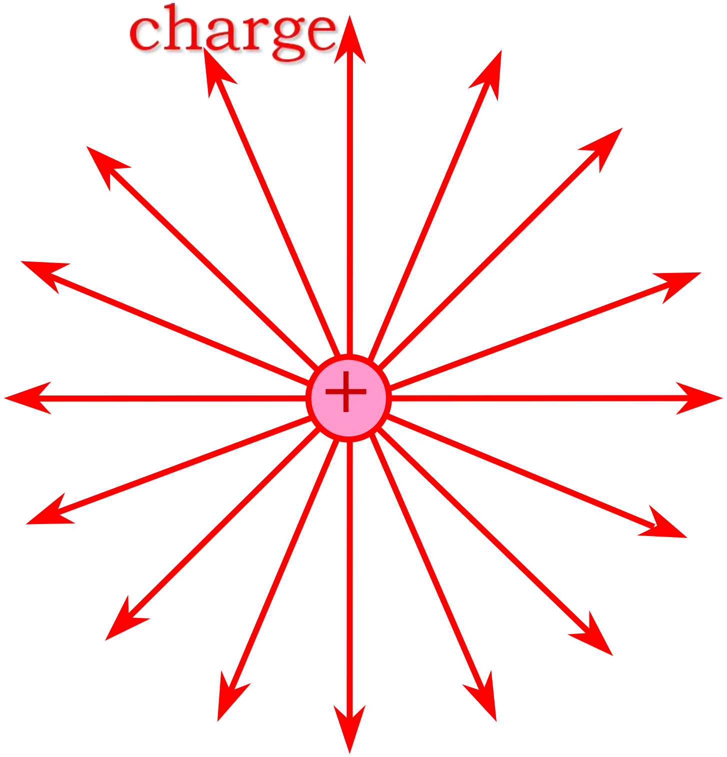
$$\left|\vec{E}\right| = E = \frac{\mathrm{d}N}{\mathrm{d}S_{\perp}}$$ Electric field line density

The density of the electric field line is used as the density of the field strength. $\vec{E}$

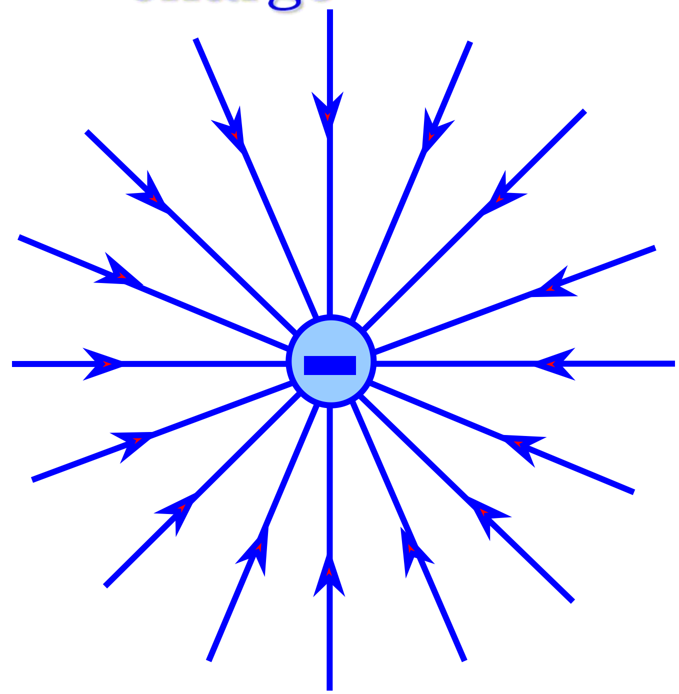


The electric field line of the point charge

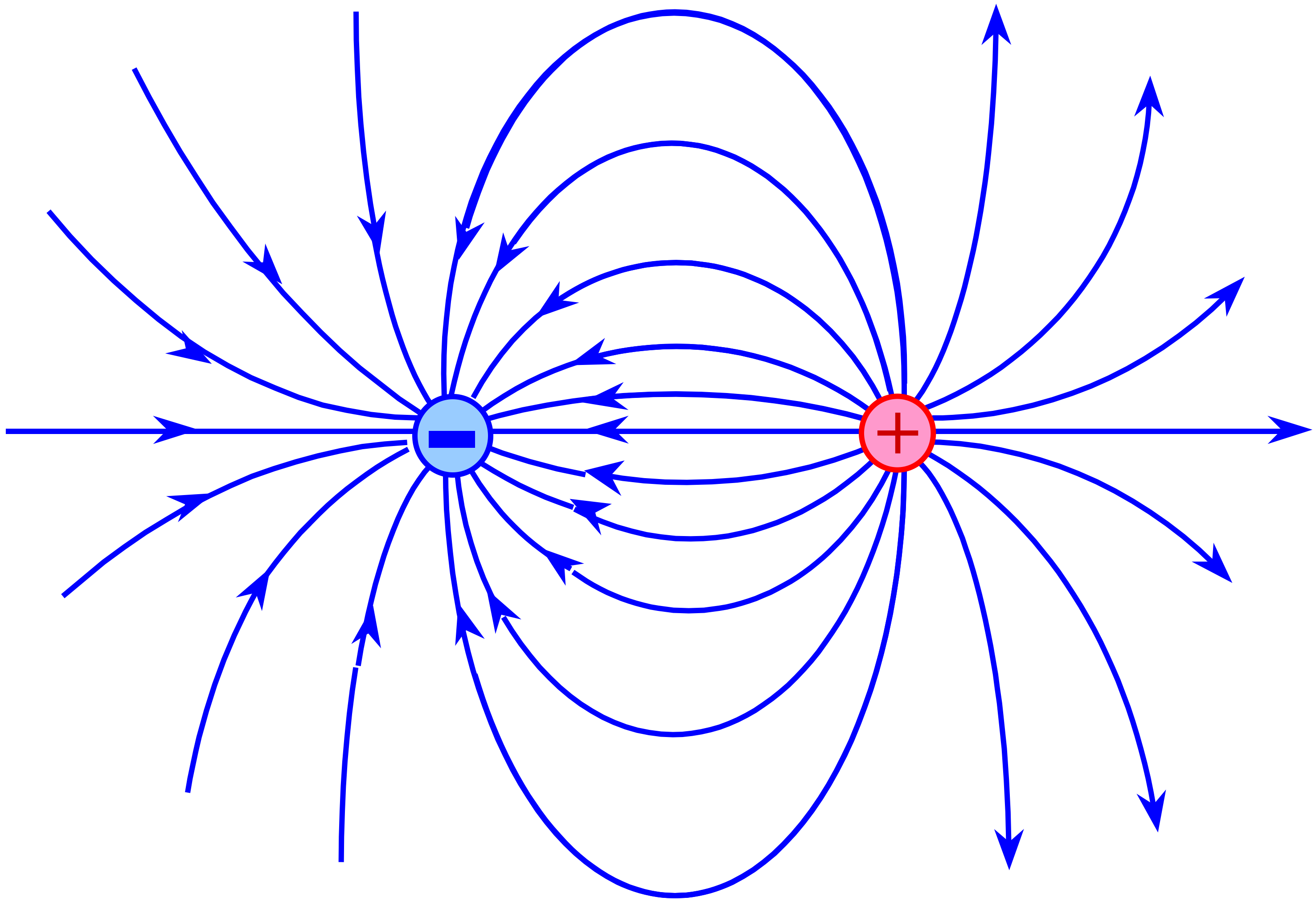
Positive point charge



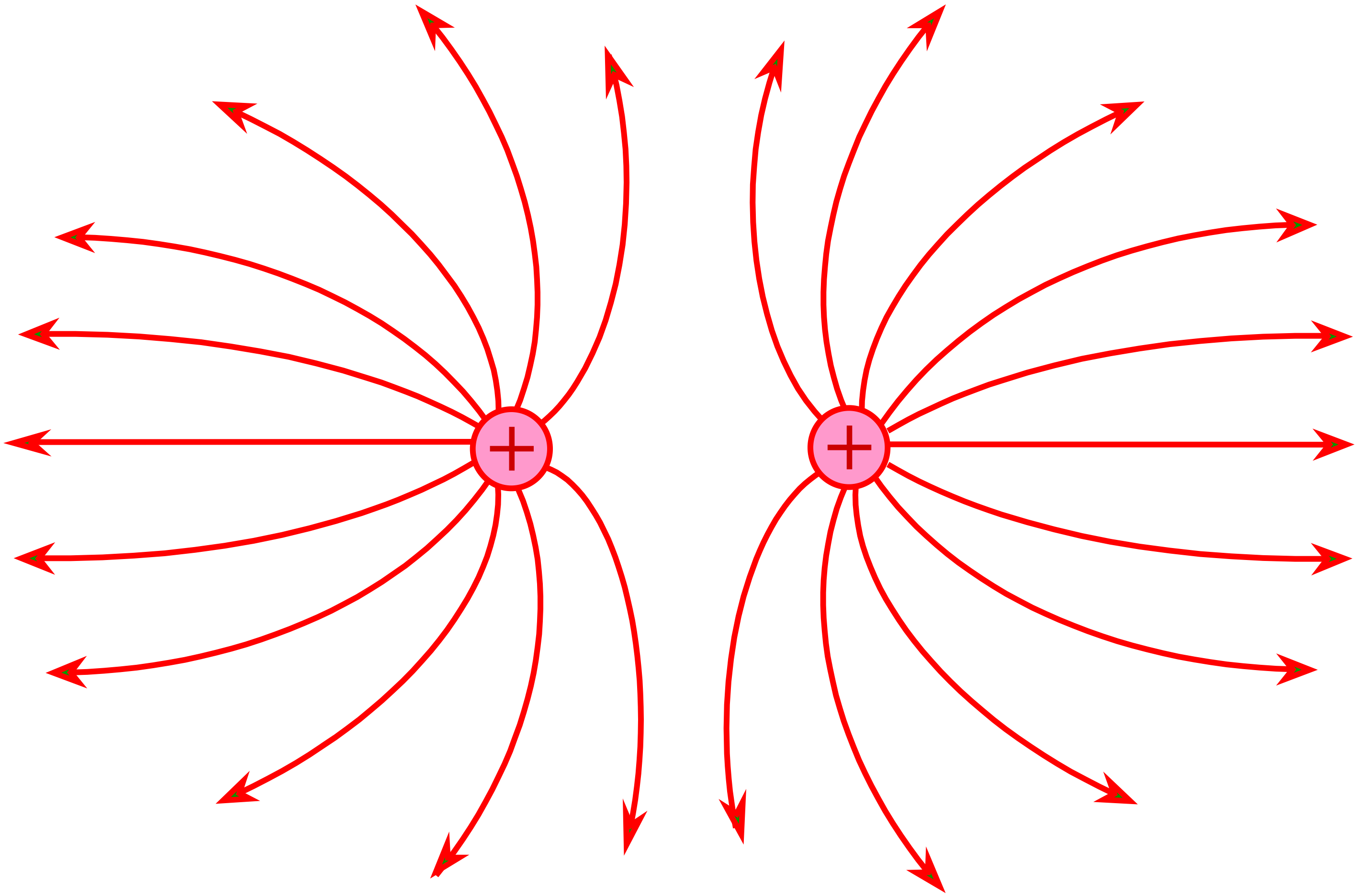
Negative point charge



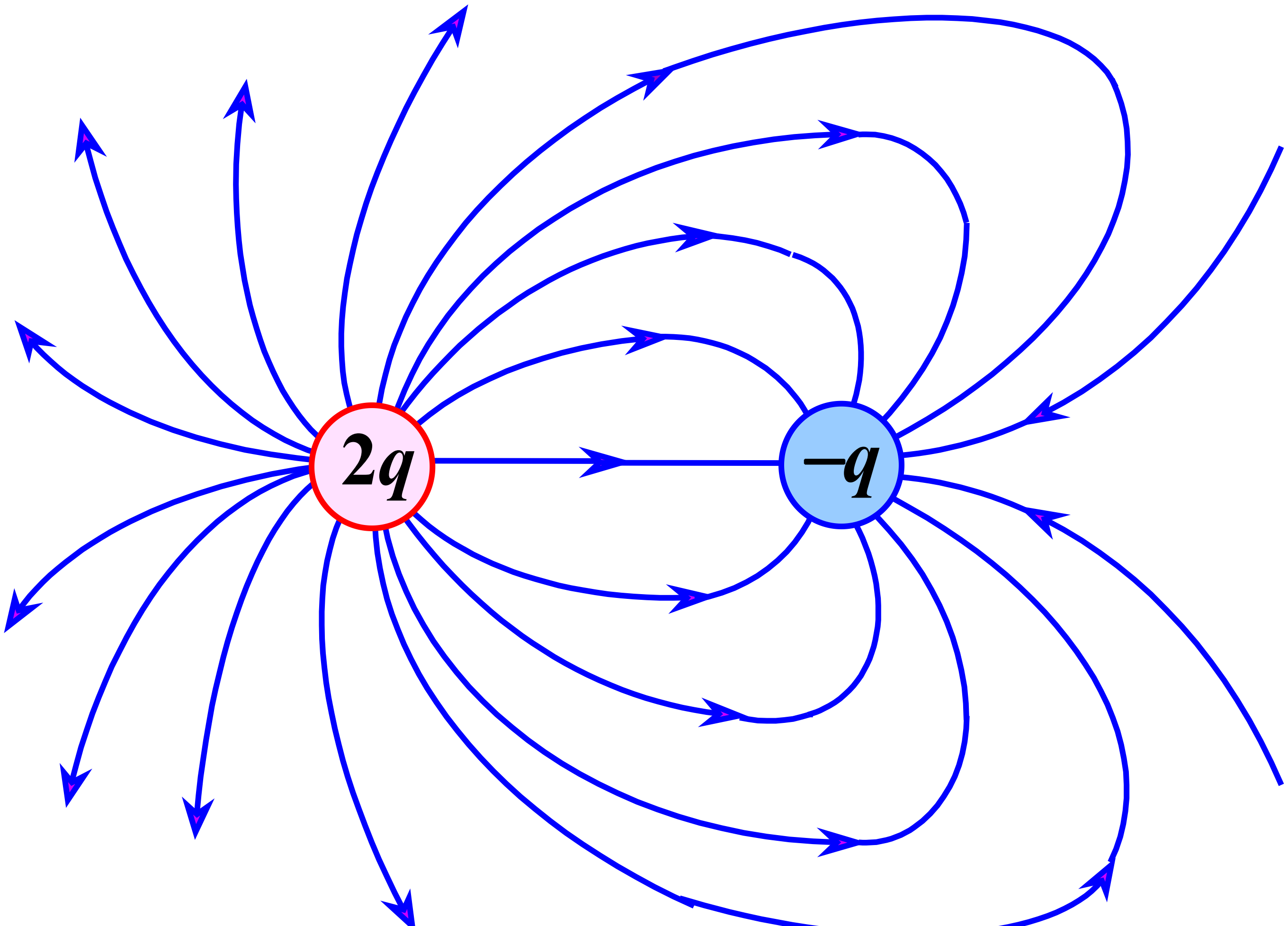
Electric field line with a pair of equal quantity and opposite point charges



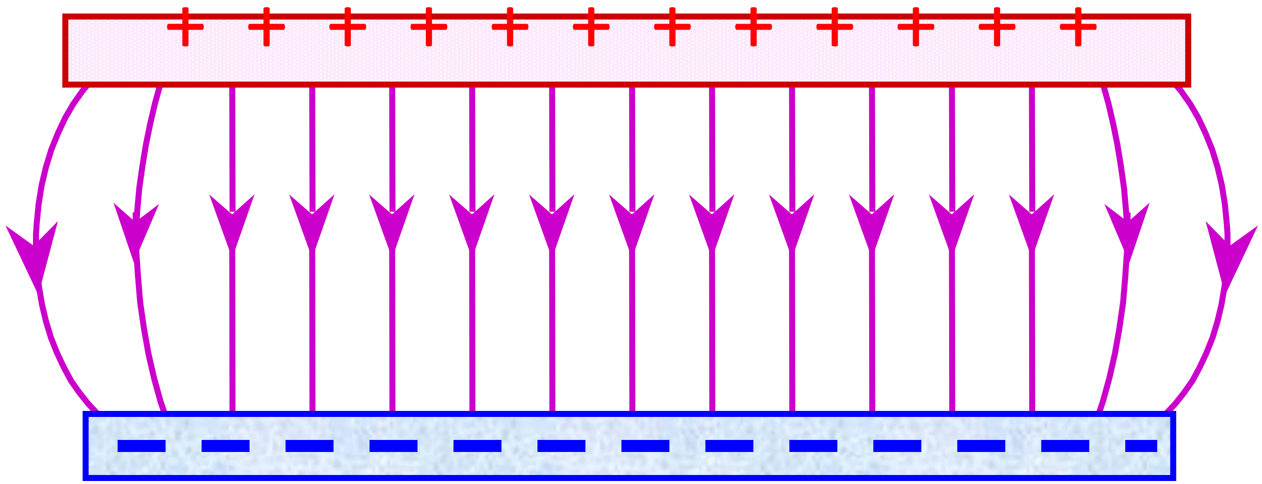
Electric field lines with an equal quantity and positive point charge



Electric field line with a pair of unequal quantity and opposite point charges



Electric field lines of charged parallel plate capacitors



Uniform electric field (uniform strong electric field): a set of electric field lines with parallel and uniform density.

Electric field line characteristics

1) Start with a positive charge, stop at a negative charge (or come from infinity, go

To infinity).

2) Electric field lines do not intersect.

3) Electric field line is not closed in electrostatic field.

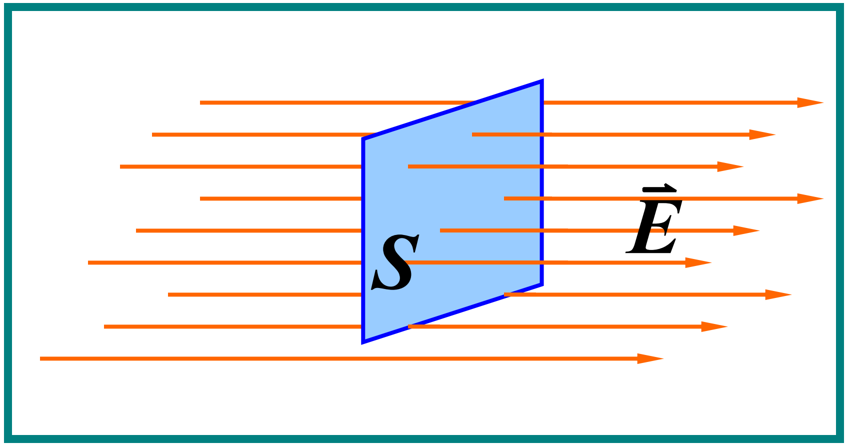
5.3.2 Electric field intensity flux

$$\left|\vec{E}\right| = E = \frac{\mathrm{d}N}{\mathrm{d}S_{\perp}}$$

The number of electric field lines passing through a certain surface in an electric field is called the electric field intensity flux Φ_e passing through this surface.

1. Uniform electric field $\vec{E}$, vertical plane

$$\boxed{\Phi_{\mathrm{e}} = ES}$$

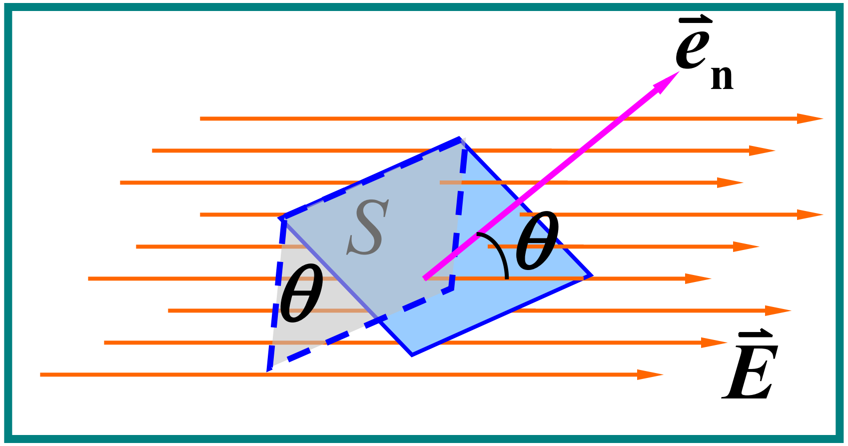


2. Uniform electric field $\vec{E}$, and clamp the Angle θ with the plane

$$\Phi_{\mathrm{e}} = ES_{\perp} = ES\cos\theta$$

$$\vec{S} = S\vec{e}_n$$

$$\boxed{\Phi_{\mathrm{e}} = \vec{E}\cdot\vec{S}}$$



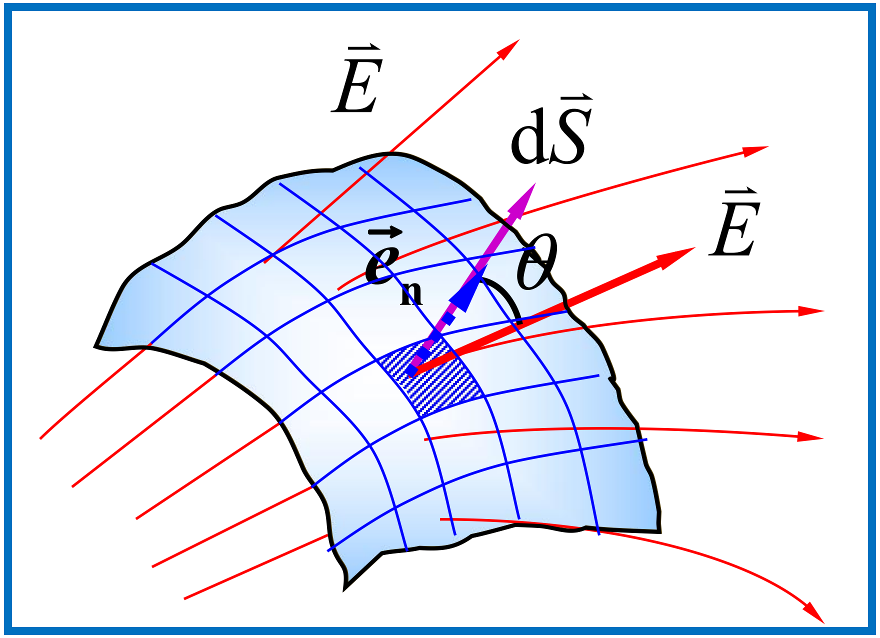
3. Nonuniform field, any curved surface

Small surface yuan $\mathrm{d}\vec{S} = \mathrm{d}S\vec{e}_n$

$$\mathrm{d}\Phi_{\mathrm{e}} = \vec{E}\cdot\mathrm{d}\vec{S} = E\mathrm{d}S\cos\theta$$

$$\Phi_{\mathrm{e}} = \int \mathrm{d}\Phi_{\mathrm{e}} = \int_S \vec{E}\cdot\mathrm{d}\vec{S}$$

surface integral

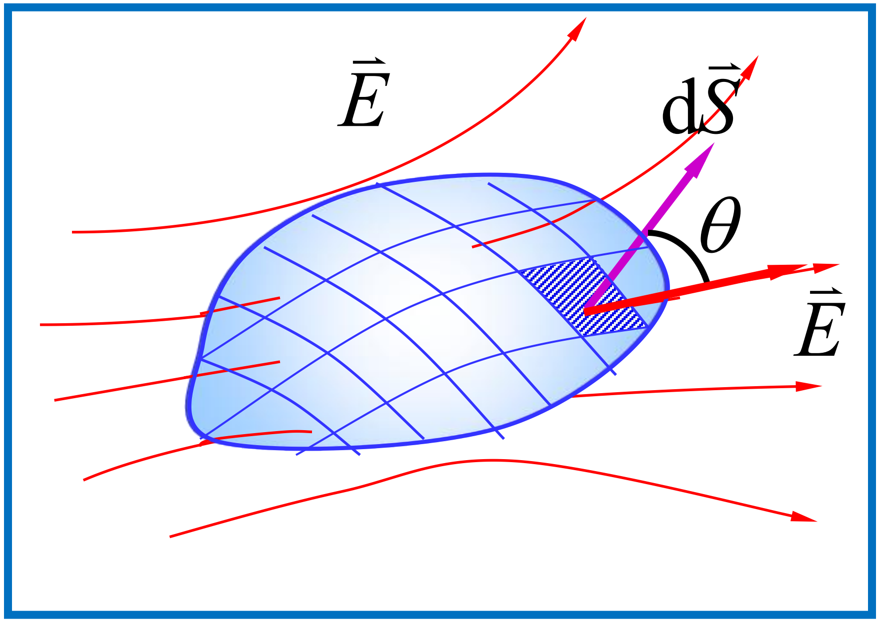


4. Any electric field, closed curved surface

$$\Phi_{\mathrm{e}} = \oint_S \vec{E}\cdot\mathrm{d}\vec{S}$$

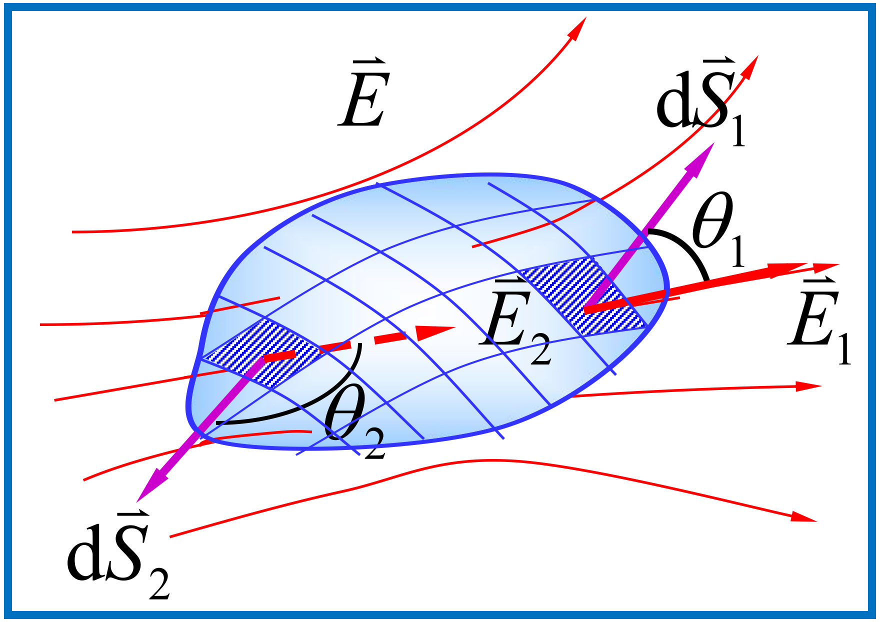
$$= \oint_S E\cos\theta\mathrm{d}S$$

Close area points



$\vec{e}_n$ External normal direction defined as closed surface

$$\mathrm{d}\Phi_e = E\mathrm{d}S\cos\theta$$



$$\theta_1 < \frac{\pi}{2}, \quad \mathrm{d}\Phi_{e1} > 0$$

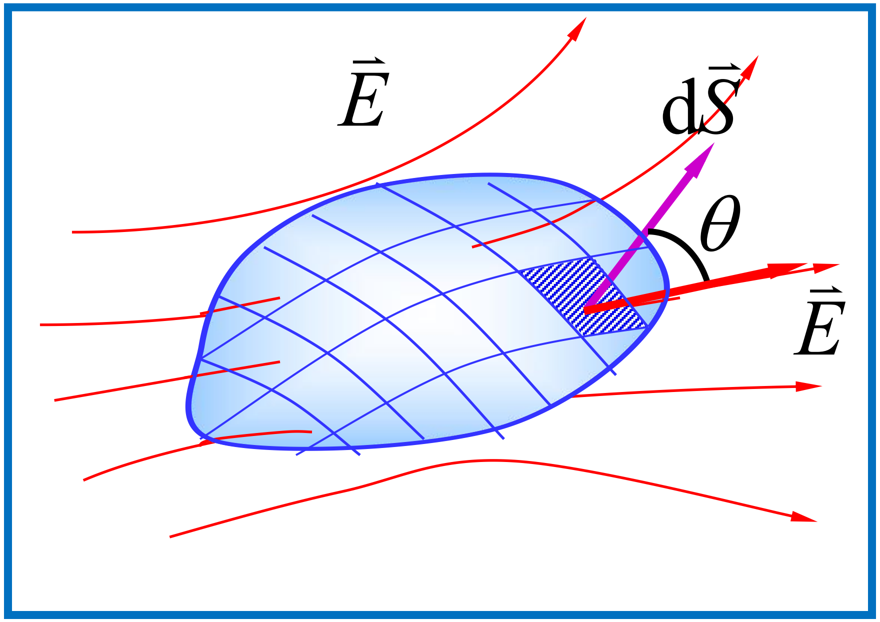
$$\theta_2 > \frac{\pi}{2}, \quad \mathrm{d}\Phi_{e2} < 0$$

$$\Phi_e = \oint_S \vec{E}\cdot\mathrm{d}\vec{S} = \oint_S E\cos\theta\mathrm{d}S$$

$$= \int_{S_{入}} E\cos\theta\mathrm{d}S + \int_{S_{出}} E\cos\theta\mathrm{d}S$$

Represents: The difference between the number of field lines that pass out and into the closed surface

$$\Phi_{\mathrm{e}} = \int_{S_{入}} E\cos\theta \mathrm{d}S + \int_{S_{出}} E\cos\theta \mathrm{d}S$$



Conclusion: For the closed surface

(1) If the F$_e$> 0, that is, if the electric field intensity flux is positive, there is a net electric field line from the inside of the surface;

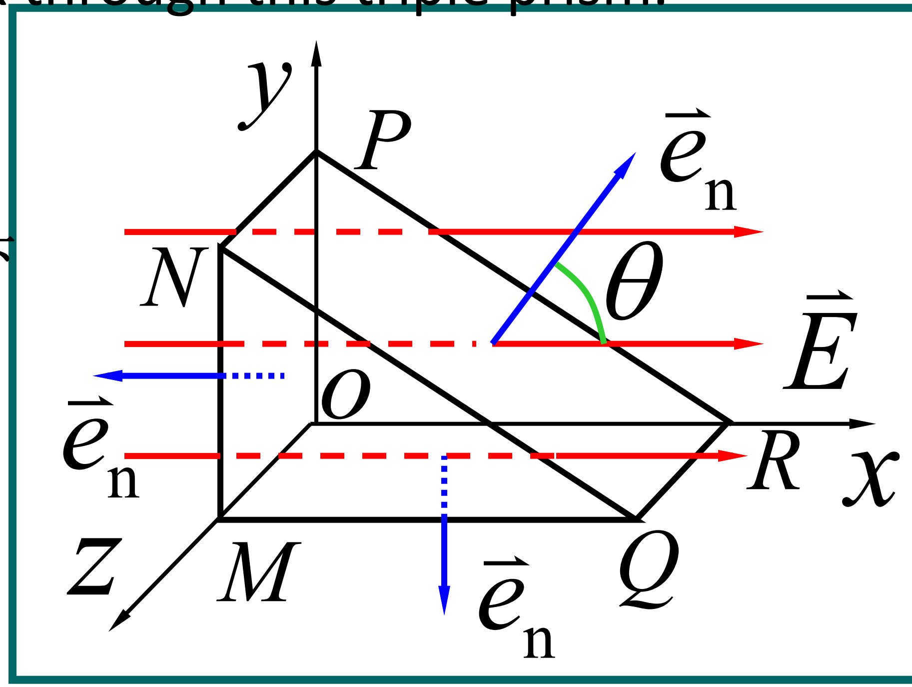
(2) If the F$_e$<0, that is, the electric field strength flux is negative, then there is a net electric field line from the outside into the surface.

Example 1 is shown in Fig, a triangular prism is placed in a uniformly strong electric field with the electric field intensity of $\vec{E}$. Find the electric field intensity flux through this triple prism.

$$\boldsymbol{\Phi}_{\mathrm{e}} = \boldsymbol{\Phi}_{\text{e前}} + \boldsymbol{\Phi}_{\text{e后}}$$

separate:

$$+ \boldsymbol{\Phi}_{\text{e左}} + \boldsymbol{\Phi}_{\text{e右}} + \boldsymbol{\Phi}_{\text{e下}}$$



$$\begin{cases} \Phi_{\text{e前}} = \Phi_{\text{e后}} = \Phi_{\text{e下}} \\ \qquad = \vec{E} \cdot \vec{S} = 0 \\ \Phi_{\text{e左}} = \vec{E} \cdot \vec{S}_{\text{左}} = ES_{\text{左}} \cos \pi = -ES_{\text{左}} \\ \Phi_{\text{e右}} = \vec{E} \cdot \vec{S}_{\text{右}} = ES_{\text{右}} \cos \theta = ES_{\text{左}} \end{cases}$$

$$\Rightarrow \quad \Phi_{\mathrm{e}} = \Phi_{\text{e前}} + \Phi_{\text{e后}} + \Phi_{\text{e左}} + \Phi_{\text{e右}} + \Phi_{\text{e下}} = 0$$

5.3.3 The Gaussian theorem of the electric field

Gaussian theorem

$$\boxed{\Phi_{\mathrm{e}} = \oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i}$$

In vacuum, the electric field strength flux through any closed surface is equal to the algebra sum of all charges enclosed by that surface divided by ε_0

(Independent of the out-of-plane charge, the closed surface is called a Gaussian surface)

Please think about: (1) what charges are related to the $\vec{E}$ on Gaussian surface?

(2) Which charges contribute to Φ_{e} of the closed surface S ?

Coulomb law

Superpositon principle of electric field strength

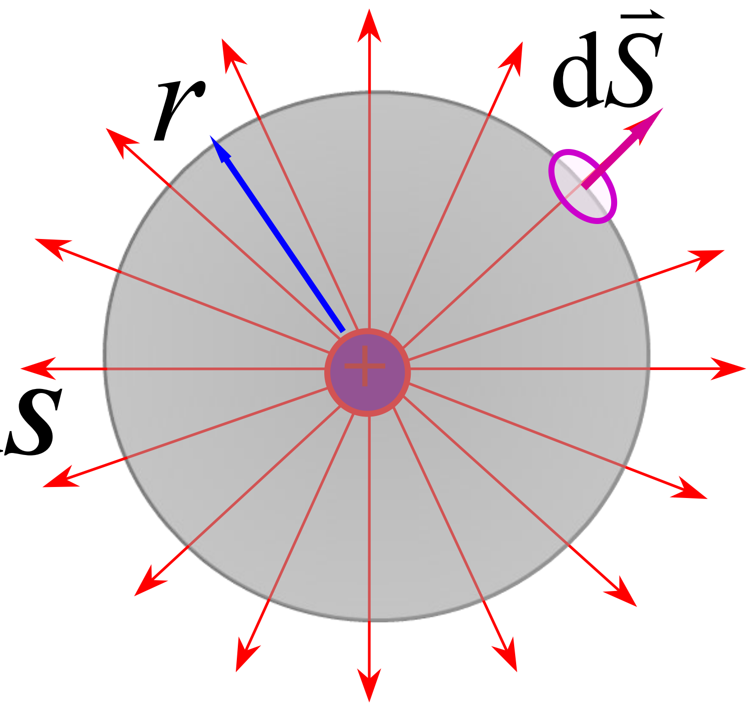
Gauss theorem

1. The point charge is located in the center of the sphere

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$\Phi_e = \oint_S \vec{E} \cdot d\vec{S} = \oint_S \frac{q}{4\pi\varepsilon_0 r^2} dS$$

$$\Phi_e = \frac{q}{\varepsilon_0}$$



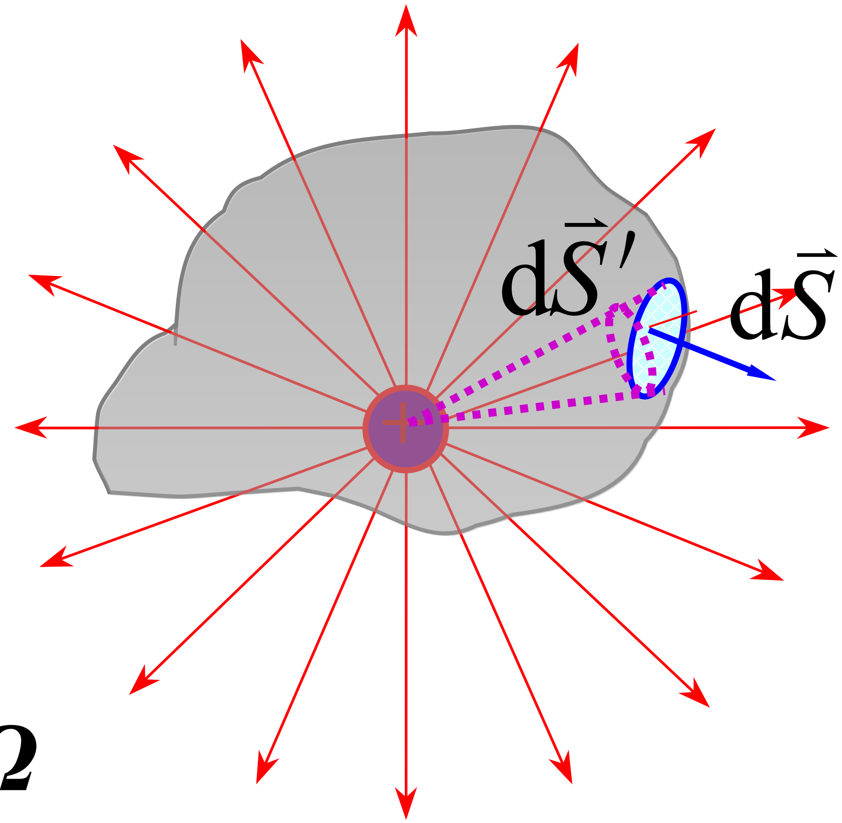
2. Point charge is in any closed surface

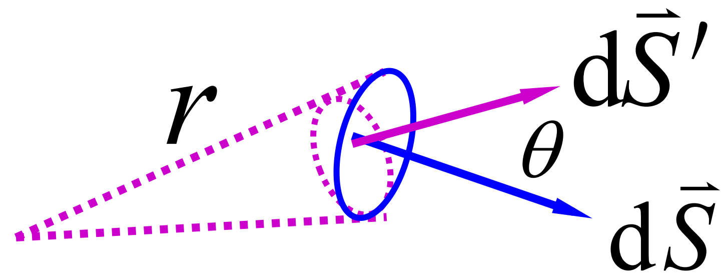
$$\mathrm{d}\Phi_{\mathrm{e}} = \frac{q}{4\pi\varepsilon_0 r^2}\mathrm{d}S\cos\theta$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{\mathrm{d}S'}{r^2}$$

Solid Angle $\quad \dfrac{\mathrm{d}S'}{r^2} = \mathrm{d}\Omega$

$$\Phi_{\mathrm{e}} = \frac{q}{4\pi\varepsilon_0}\oint \mathrm{d}\Omega = \frac{q}{\varepsilon_0}$$





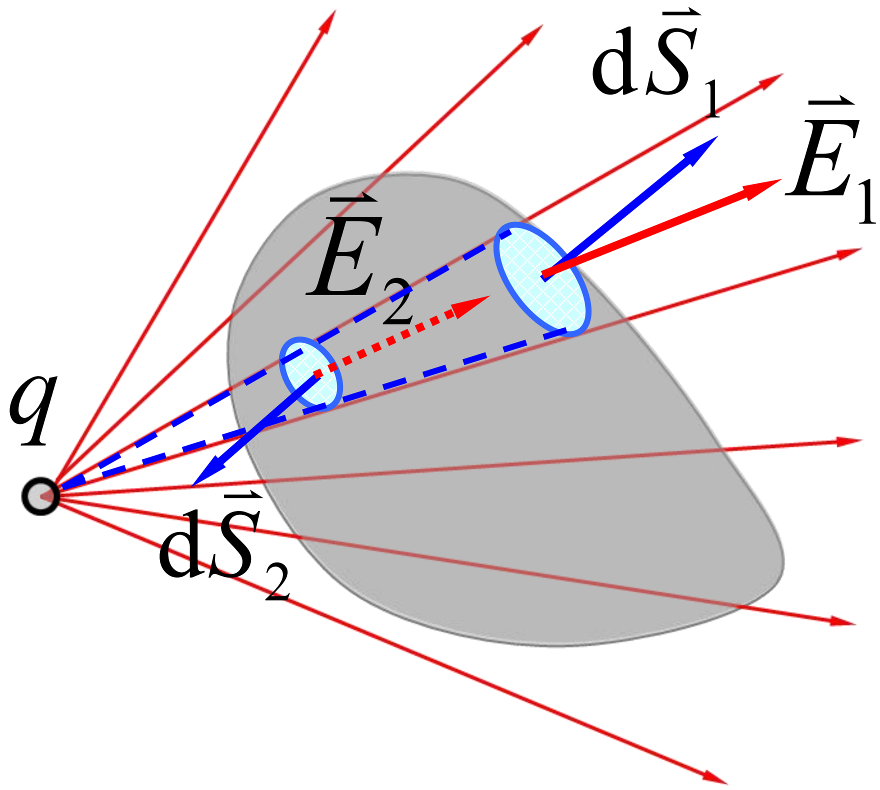
3. Point charge is outside the closed surface

$$\mathrm{d}\Phi_1 = \vec{E}_1 \cdot \mathrm{d}\vec{S}_1 > 0$$

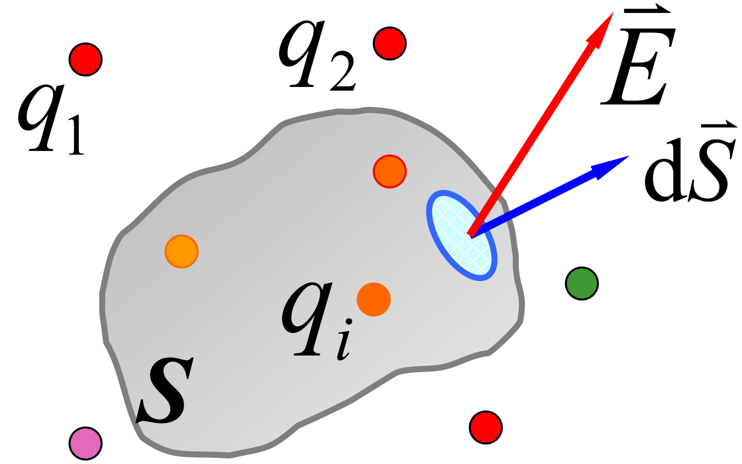
$$\mathrm{d}\Phi_2 = \vec{E}_2 \cdot \mathrm{d}\vec{S}_2 < 0$$

$$\mathrm{d}\Phi_1 + \mathrm{d}\Phi_2 = 0$$

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = 0$$



4. Electric field generated by multiple point charges



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$$

$$\Phi_{\mathrm{e}} = \oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \oint_S \sum_i \vec{E}_i \cdot \mathrm{d}\vec{S}$$

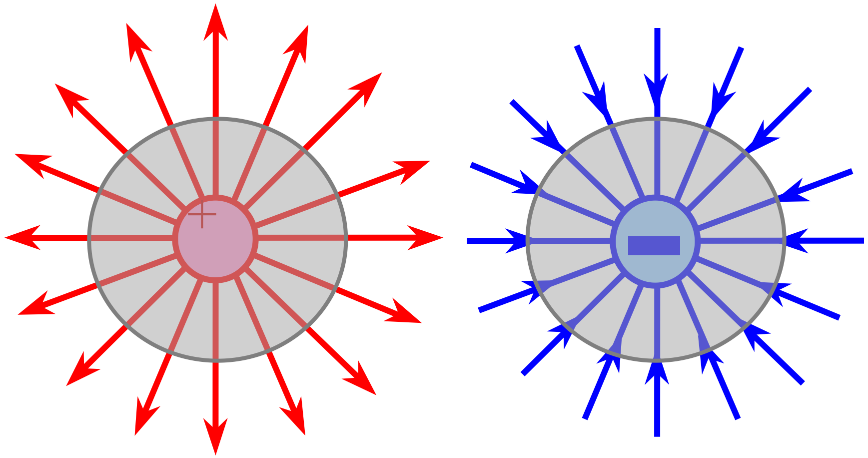
$$= \sum_{i(\mathrm{in})} \oint_S \vec{E}_i \cdot \mathrm{d}\vec{S} + \sum_{i(\mathrm{out})} \oint_S \vec{E}_i \cdot \mathrm{d}\vec{S}$$

$$\because \sum_{i(\mathrm{out})} \oint_S \vec{E}_i \cdot \mathrm{d}\vec{S} = 0$$

$$\therefore \Phi_{\mathrm{e}} = \sum_{i(\mathrm{in})} \oint_S \vec{E}_i \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0} \sum_{i(\mathrm{in})} q_i$$

Conclusion

$$\Phi_e = \oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i$$



1) The Gaussian surface is a closed surface (hypothetical surface).

2) The electric field strength of the Gaussian surface is the total electric field strength of all the internal and external charges.

3) Only the charge in the Gaussian surface contribute to the electric field intensity flux of the Gaussian surface.

5) The electric field intensity flux passing out the Gaussian surface is positive, and the one passing into is negative.

6) The static electricity field is an active field.

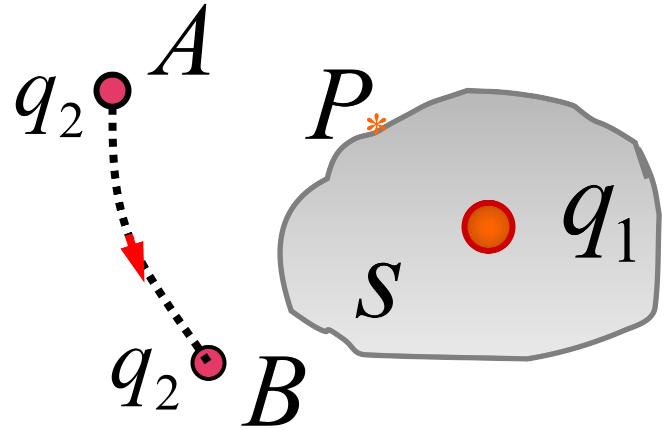
Discussion

1. Will the electric field strength change if q_2 moved from A to B? *yes*

Is there any change through the Gaussian face S?

Φ_{e}

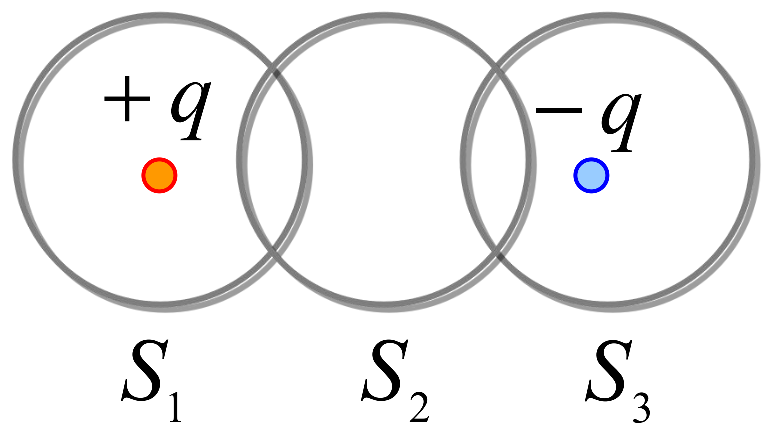
no



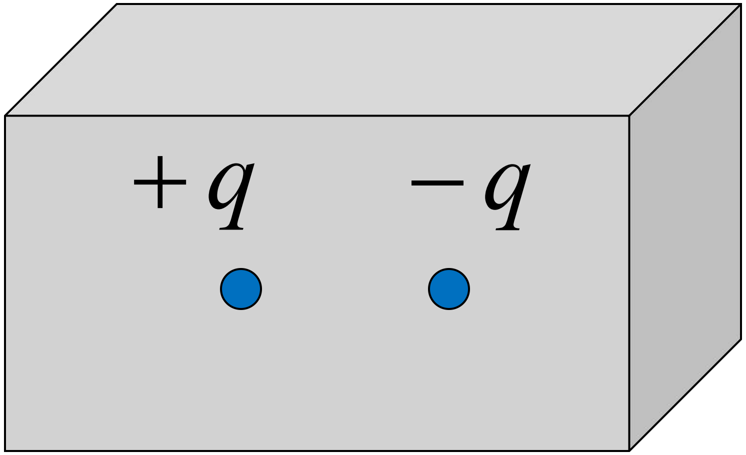
2. In the electrostatic field of the point charge **+q** and **-q**, make the following three closed surfaces S_1、S_2、S_3 to find the electric field intensity flux through each closed surface.

$$\Phi_{e1} = \oint_{S_1} \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

$$\Phi_{e2} = 0 \qquad \Phi_{e3} = \frac{-q}{\varepsilon_0}$$



Thinking: In the electrostatic field of the point charge +q and -q, do the following closed surface S in order to find the electric flux through the closed surface.



The electrical flux through the closed surface is equal to 0.

Thinking: Is the electric field strength of any point on the closed surface S 0?

5.3.4 Application of the Gauss Theorem

◆ **The electric field intensity is found by using the Gaussian theorem**

Principle: Gauss theorem $$\Phi_{\mathrm{e}} = \oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i$$

Range: In the charged body, the electrostatic field must have a high degree of symmetry.

step:

1. Make **symmetry analysis** according to the superposition principle of electric field strength;

2. Choose the appropriate **Gaussian surface** according to the symmetry;

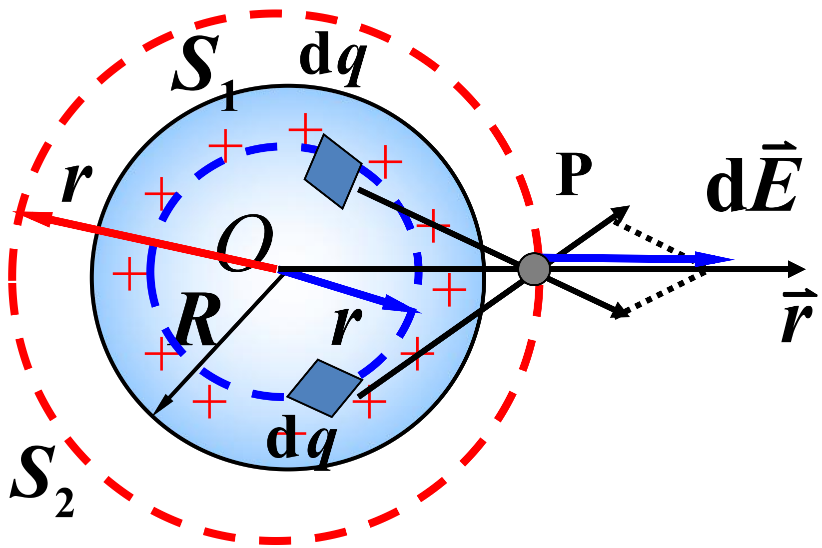
3. Apply the Gaussian theorem for **calculation**;

4. Write the partition function. $$\vec{E} = \vec{E}(\vec{r})$$

1. Electric field of a uniformly charged spherical shell

A thin spherical shell with $\boldsymbol{R}$ a radius of $\boldsymbol{q}$. Find the electric field strength at any point inside and outside the spherical shell.

Symmetry analysis:
Globerical symmetry



Choose the Gaussian surface as the concentric sphere.

Solution (1) $0 < r < R$

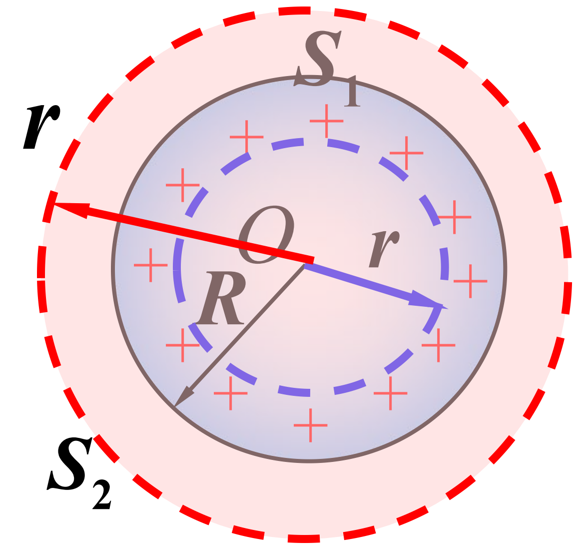
$$\oint_{S_1} \vec{E} \cdot d\vec{S} = 0$$

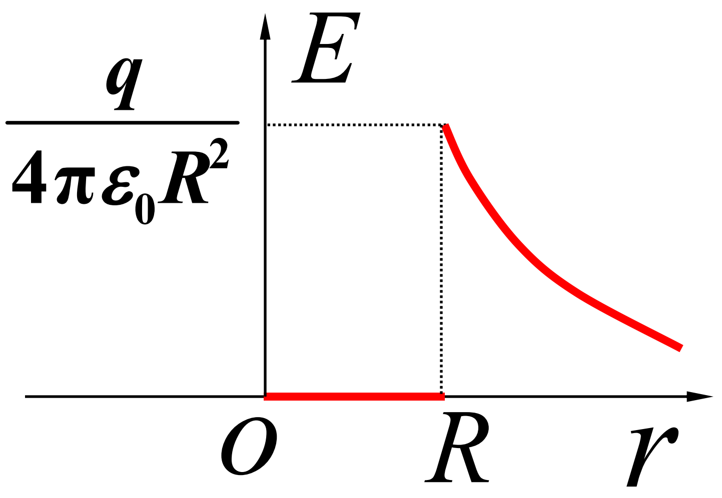
$$\boxed{\vec{E} = 0}$$

(2) $r > R$

$$\oint_{S_2} \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$4\pi r^2 E = \frac{q}{\varepsilon_0} \qquad \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$





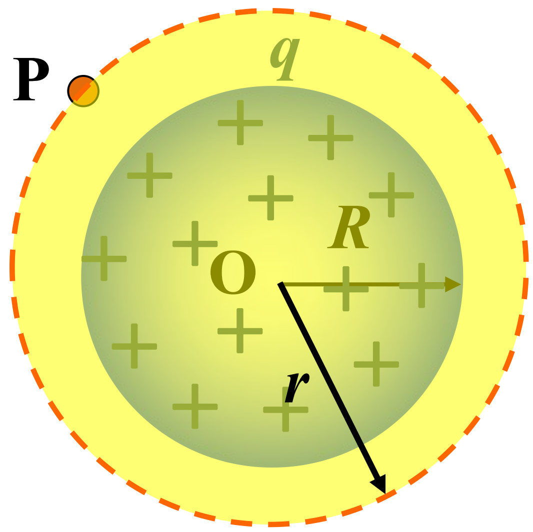
4. The electric field of a uniformly charged sphere

A uniformly charged sphere with radius R and charge of q.

Choose the Gaussian surface as the concentric sphere.

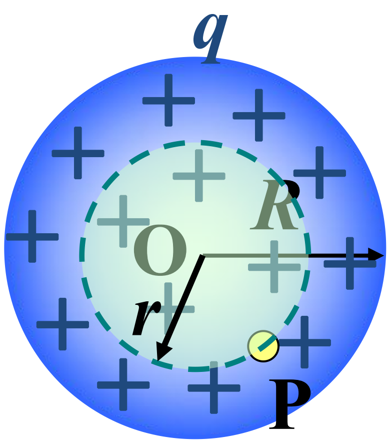
(1) For r > R, the charge in the Gaussian plane is q:

$$\Phi_e = \oint_S \vec{E} \cdot d\vec{S} = E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$



$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r}$$

(2) For r <R, the charge in the Gaussian plane is q ′:



$$q' = \rho V = \frac{q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 = \frac{r^3}{R^3} q$$

$$\Phi_e = \oint_S \vec{E} \cdot \mathrm{d}\vec{S} = E \cdot 4\pi r^2 = \frac{q'}{\varepsilon_0} = \frac{r^3}{R^3}\frac{q}{\varepsilon_0}$$

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}$$

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 R^3}\vec{r}}$$

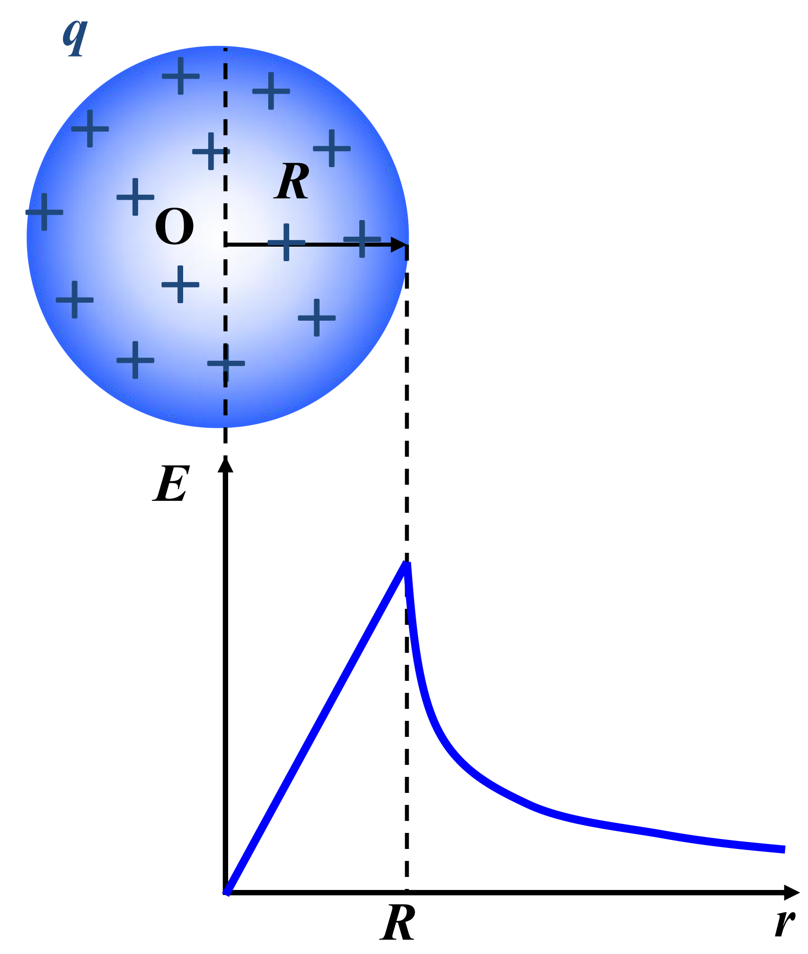
conclusion:

a. The field strength distribution outside the uniformly charged sphere is just like the point charge on the sphere when the charge is concentrated in the center.

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r}$$

b. The field strength within the sphere is proportional to the distance of the field point from the center of the sphere.

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 R^3}\vec{r}}$$

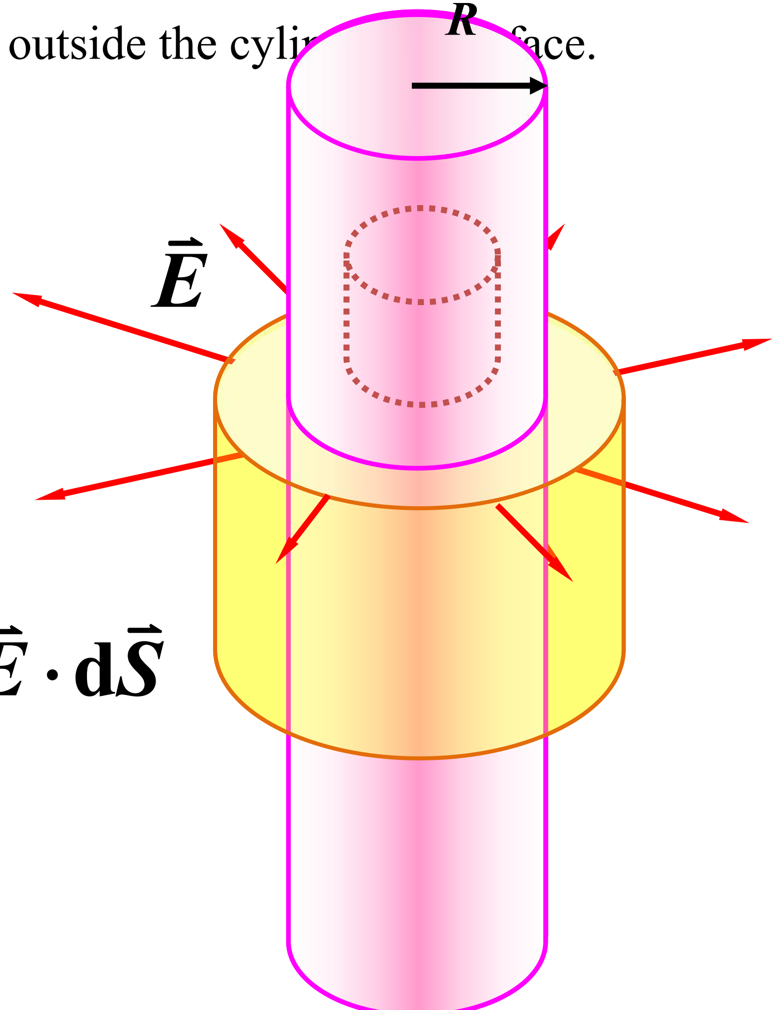


The electric field of a uniformly charged sphere

2. An electric field of an infinitely long, uniformly charged cylindrical surface

An infinitely long uniformly charged cylindrical surface with a radius of and a charge line density of

. Find the electric field strength of any point inside and outside the cyli… face.



Symmetry analysis: Axisymetry

The electric field intensity direction of each point is perpendicular to the axis

Select a closed cylindrical Gaussian surface

$$\oint_S \vec{E}\cdot \mathrm{d}\vec{S} =$$

$$\int_{s(柱面)} \vec{E}\cdot \mathrm{d}\vec{S} + \int_{s(上底)} \vec{E}\cdot \mathrm{d}\vec{S} + \int_{s(下底)} \vec{E}\cdot \mathrm{d}\vec{S}$$

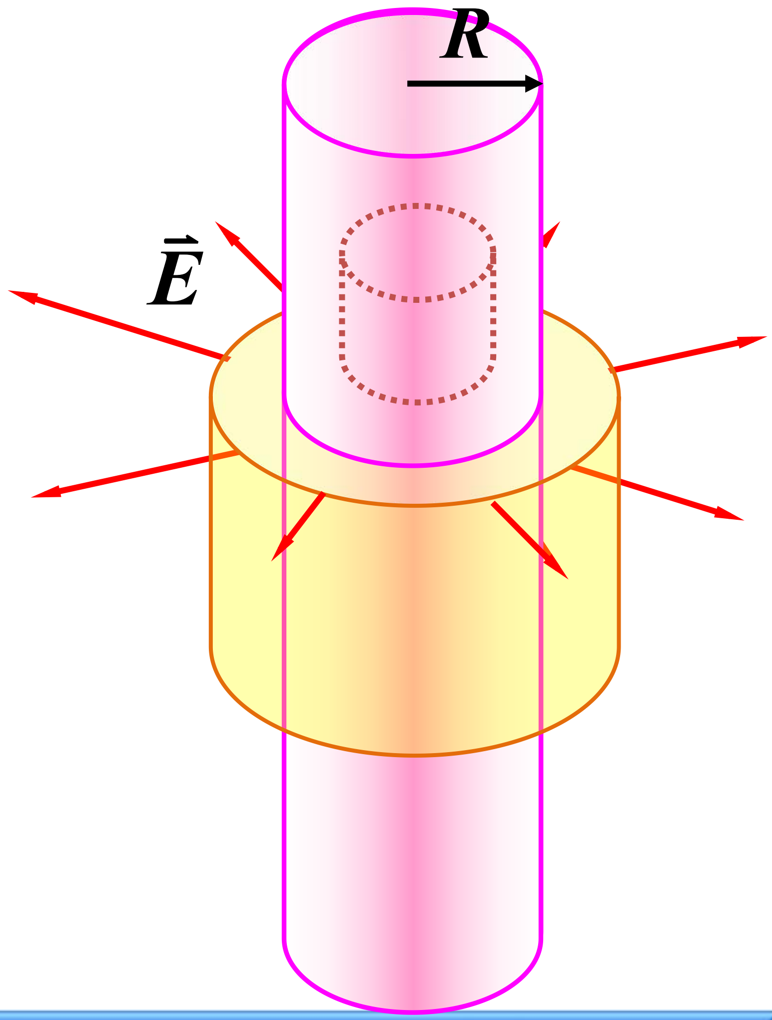
$$= \int_{s(柱面)} \vec{E}\cdot \mathrm{d}\vec{S}$$

(1) $r > R$

$$\oint_S \vec{E}\cdot \mathrm{d}\vec{S} = \int_{s(\text{柱面})} E\mathrm{d}S = E\int_{s(\text{柱面})} \mathrm{d}S = 2\pi r h E$$

$$2\pi r h E = \frac{\lambda h}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$



(2) $r \le R$

$$2\pi r h E = 0$$

$$\boxed{E = 0}$$

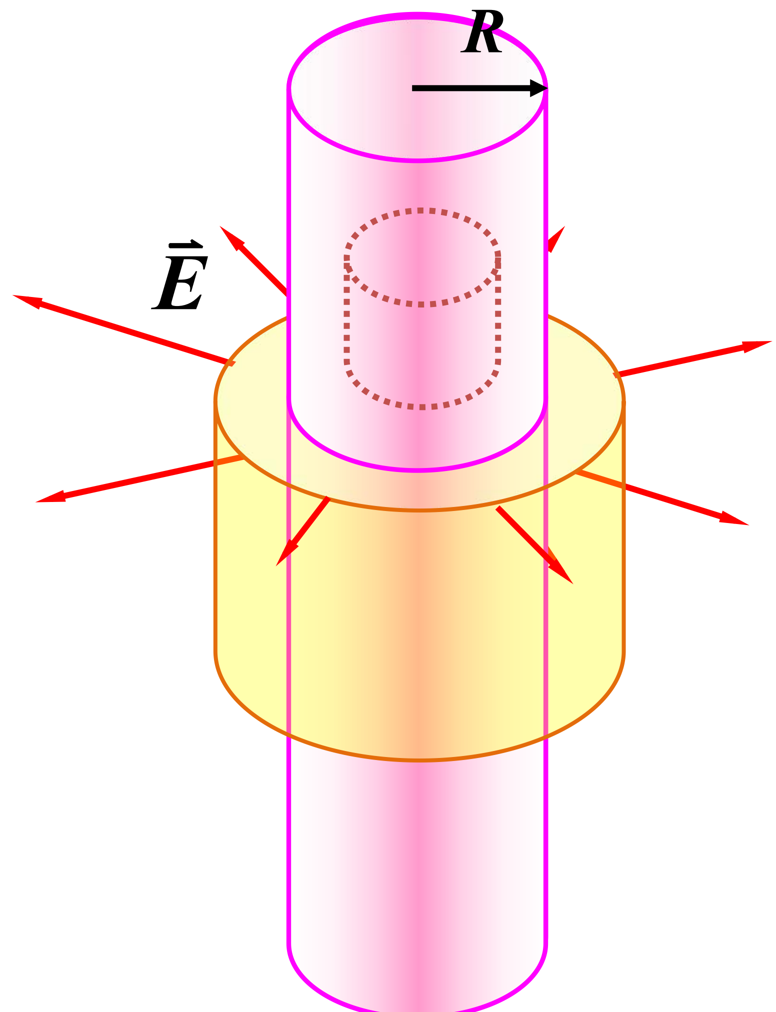
Thinking: If it is an infinite long uniform charged cylinder, what about the internal and external electric field?

$$r > R, \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$r \leq R \quad \oint_S \vec{E}\cdot \mathrm{d}\vec{S} = \int_{s(柱面)} E\mathrm{d}S$$

$$2\pi r h E = \frac{1}{\varepsilon_0}\lambda \frac{\pi r^2}{\pi R^2} h$$

$$\boxed{E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}}$$



3. An electric field in an infinitely large and uniformly charged plane

Infinite uniform charged plane, and set its surface charge density as σ.

Symmetry analysis: face weighing

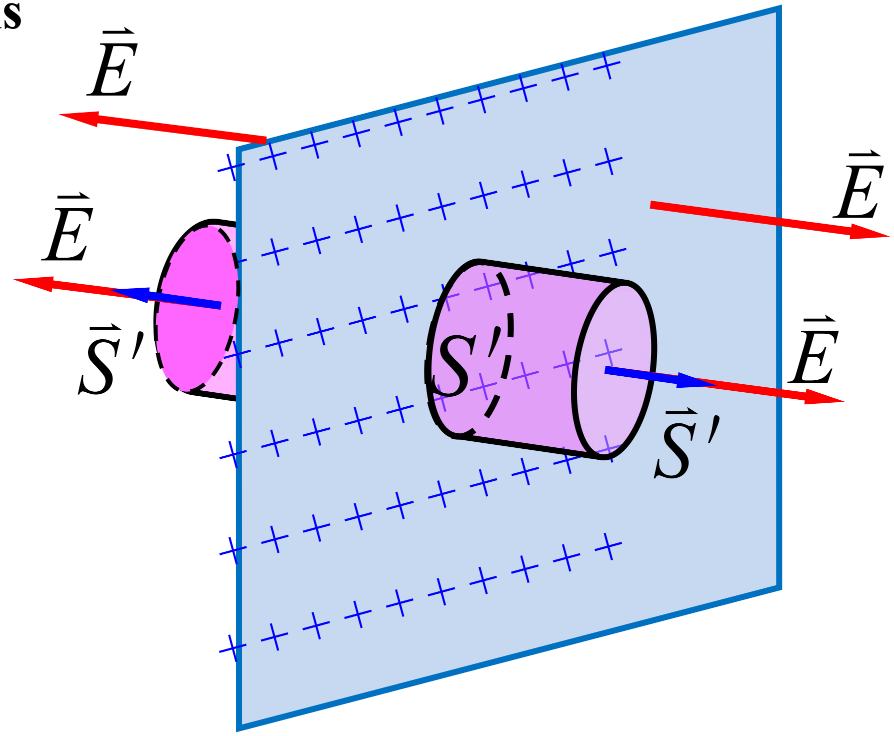
The electric field strength at each point is perpendicular to the plane

Select a closed cylindrical Gaussian surface

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{\sigma S'}{\varepsilon_0}$$

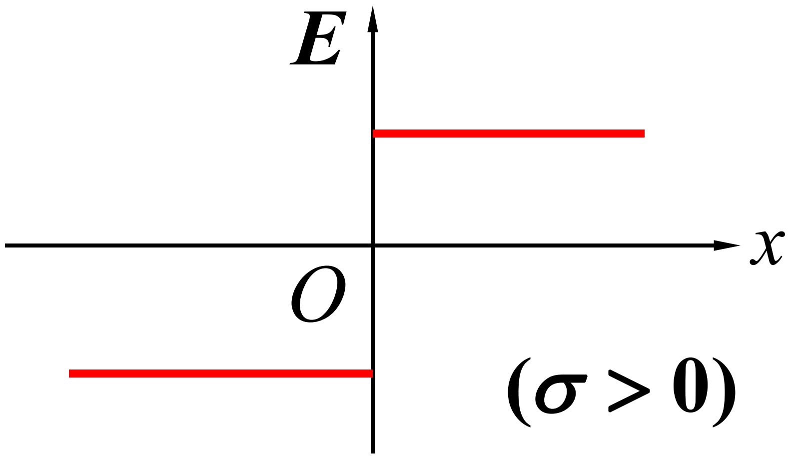
$$2S'E = \frac{\sigma S'}{\varepsilon_0}$$

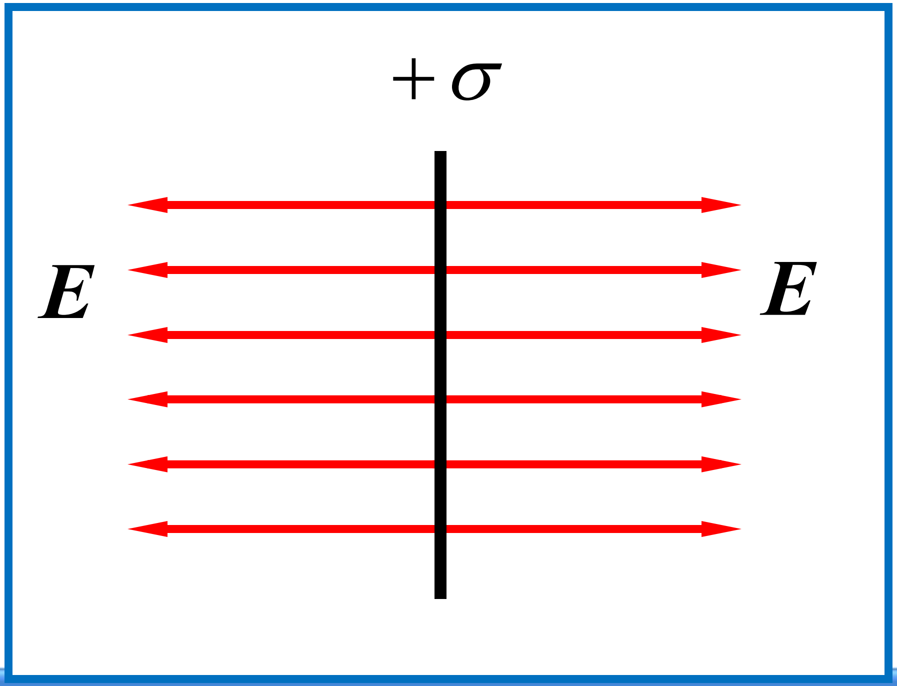
$$\boxed{E = \sigma / 2\varepsilon_0}$$

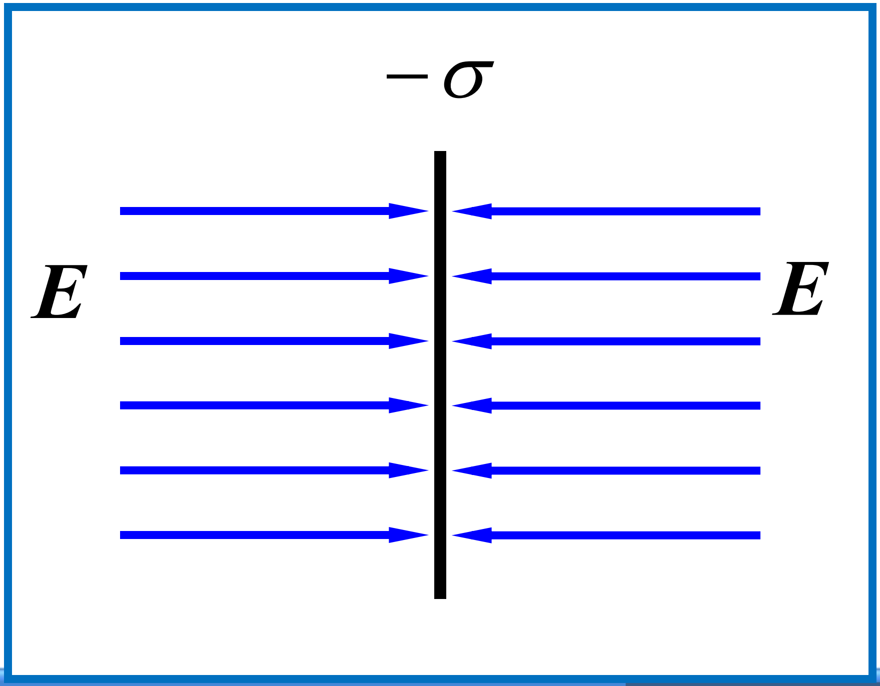


$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

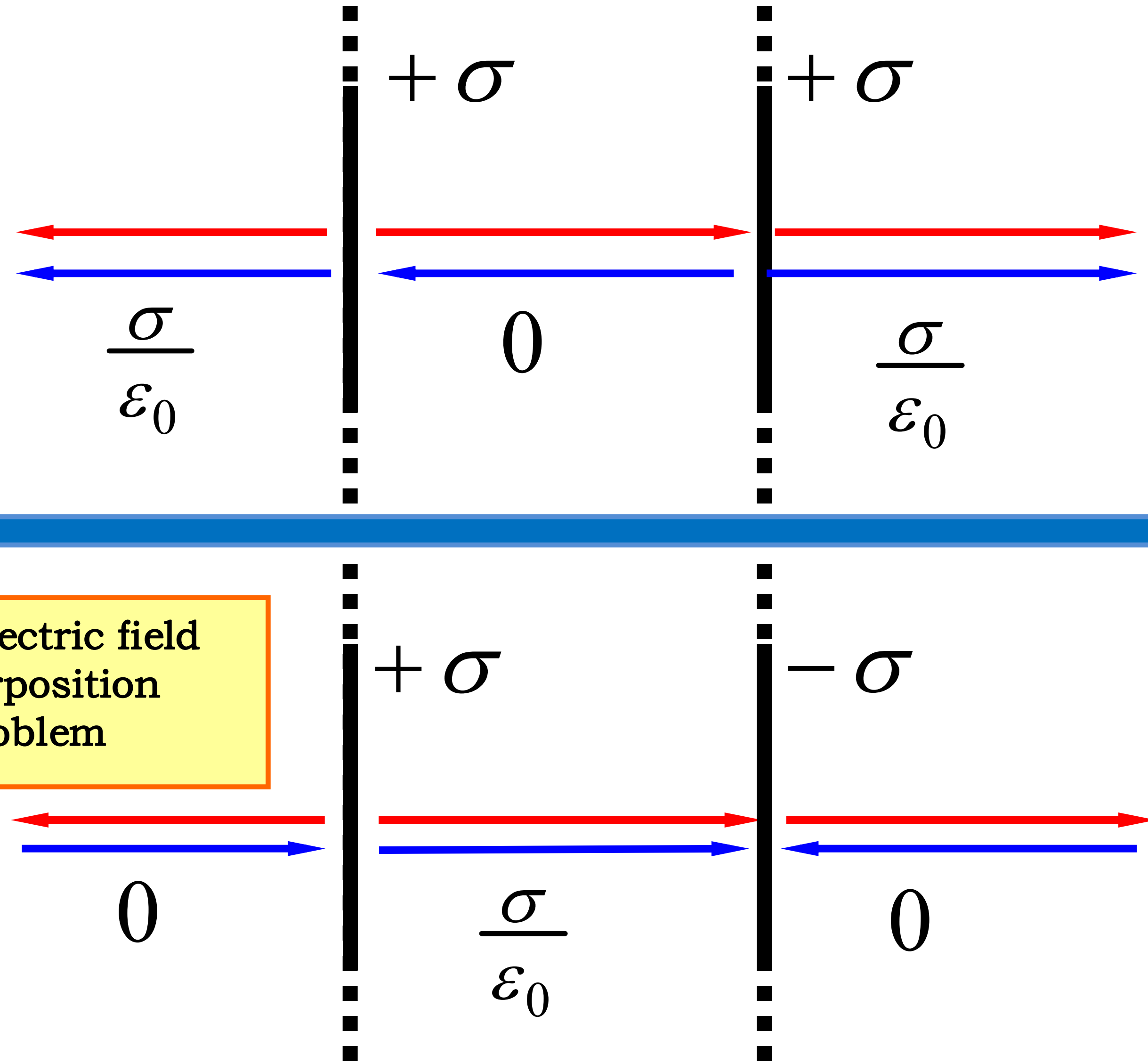
meanfield







discuss



Infinite large charged plane, The electric field superposition problem