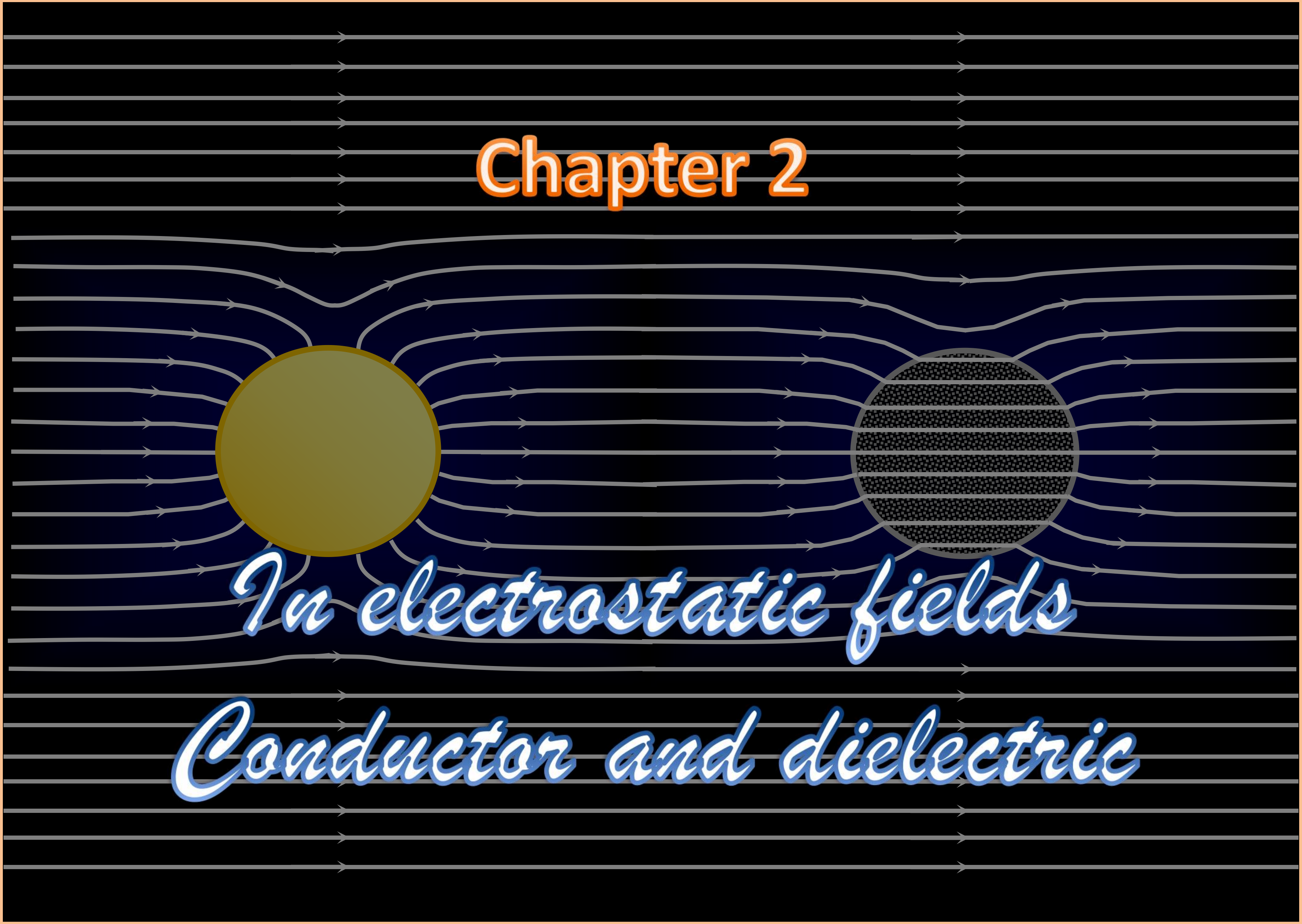
Chapter 2



In electrostatic fields Conductor and dielectric

primary coverage

1 The conductor in the electrostatic field

2 Capacitors and capacitors

3 Dielectric in the electrostatic field

4 Electrostatic energy of the charged systems

§ 6.1

Conductors in the electrostatic field

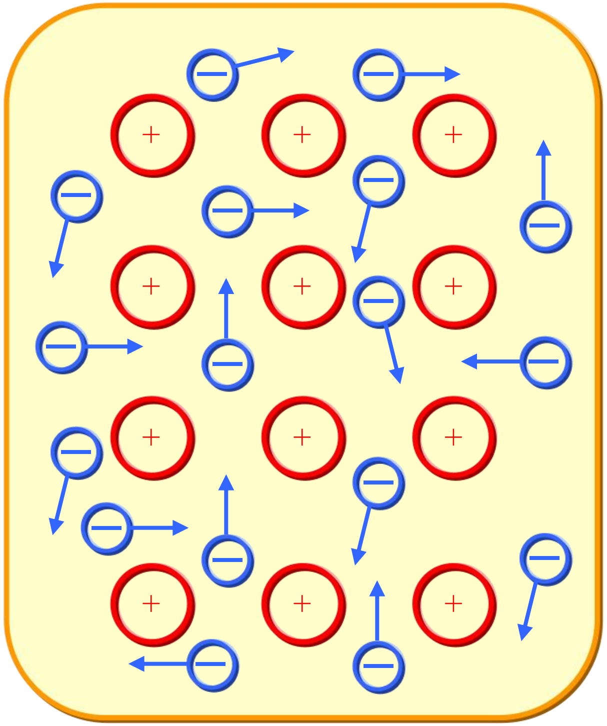
6.1.1 Electrostatic balance of the conductor

1. The electrical structure of the conductor

Electrical structural features of the metal conductors

1. The metal conductor is composed of negatively charged free electrons and a positively charged crystal lattice.

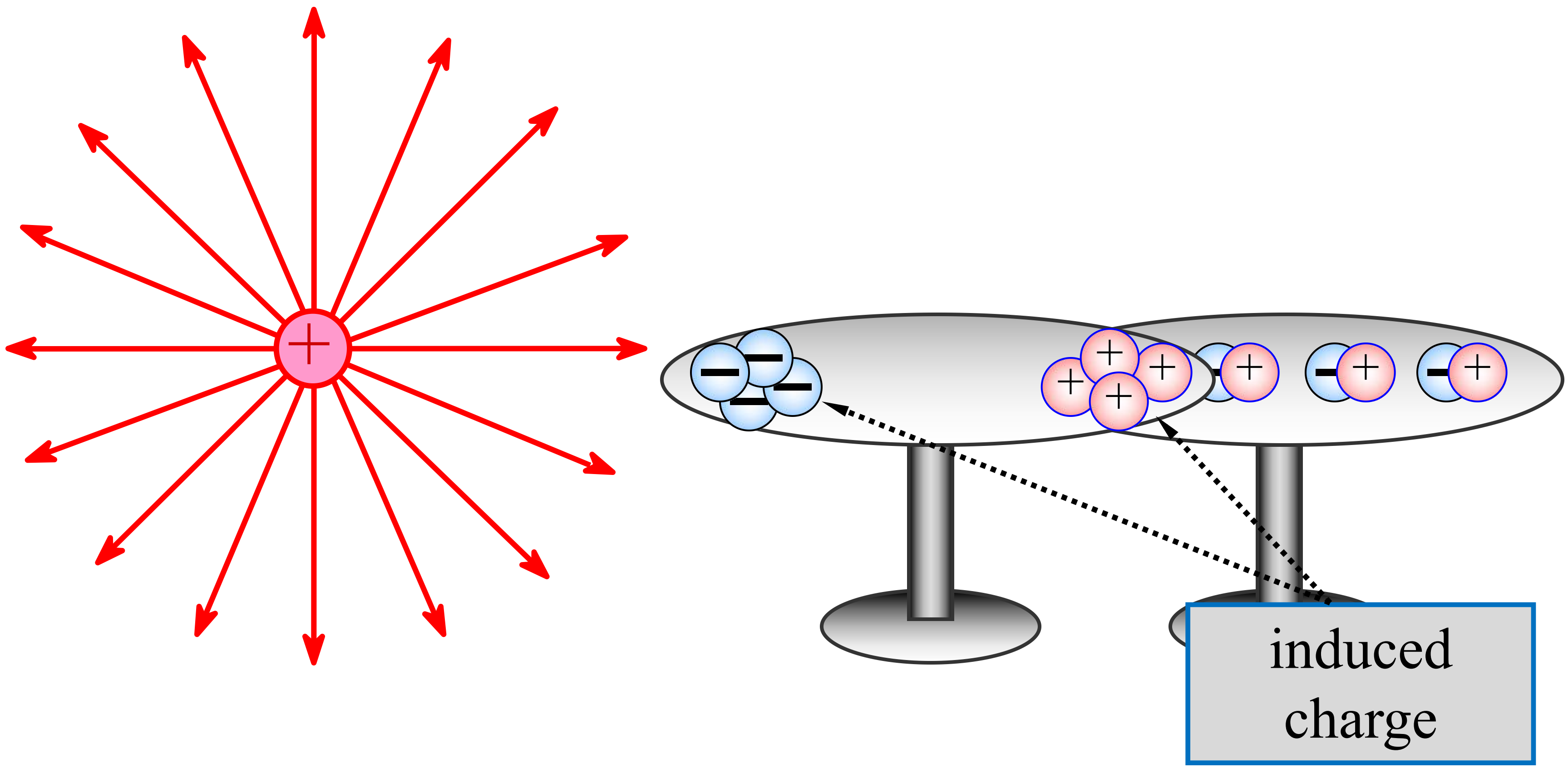
2. When the conductor is not charged nor affected by the external electric field, the two charges are evenly distributed within the conductor, with no macroscopic movement, and only the microscopic thermal motion exists.



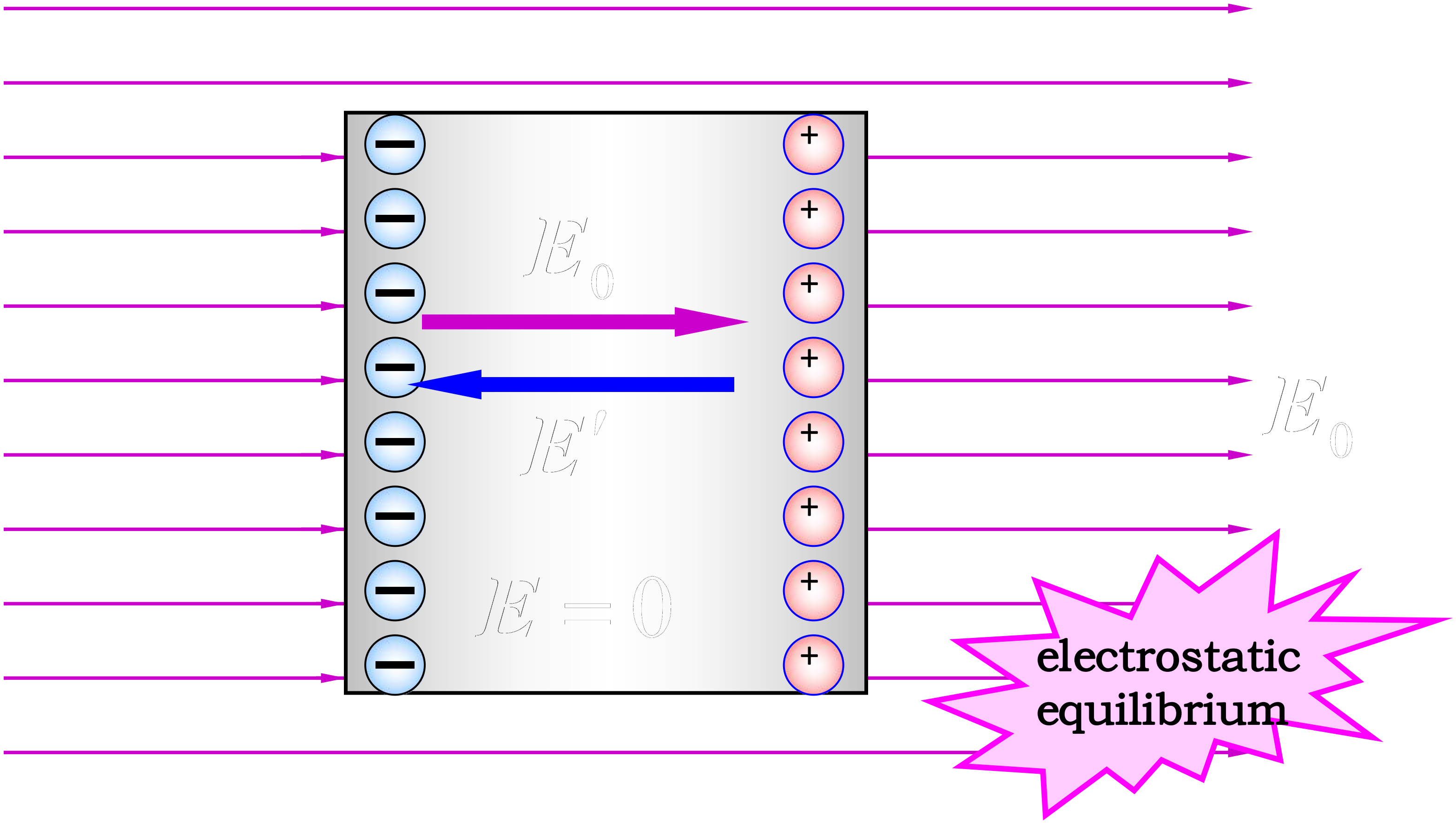
Schematic representation of the crystal lattice and the free electrons

electric neutrality

2. Electrostatic equilibrium of the conductor



The charge appearing on the conductor due to electrostatic induction is called the inductive charge



$$\boldsymbol{E}_{\text{int}} = \boldsymbol{E}_0 + \boldsymbol{E}'$$

Electric field strength within the conductor

External electric field strength

Inactive charge electric field strength

Electrostatic equilibrium state: There is no directed charge movement either inside the conductor or on the surface

Electrostatic balance conditions

(1) At any point inside the conductor, the electric field strength is zero;

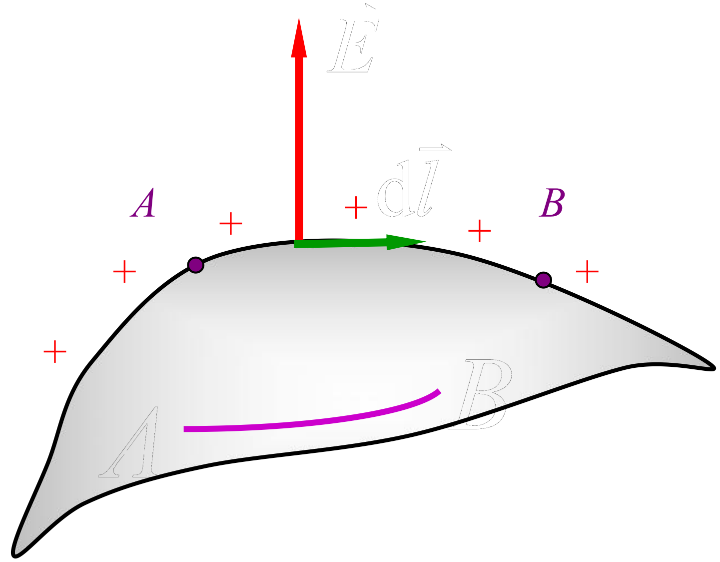
(2) The direction of the electric field strength at the surface of the conductor is perpendicular to the surface of the conductor.

The conductor is an isophore

- **The conductor surface is an equipotential surface**

$$\because \vec{E} \perp \mathrm{d}\vec{l}$$

$$\therefore U_{AB} = \int_A^B \vec{E} \cdot \mathrm{d}\vec{l} = 0$$



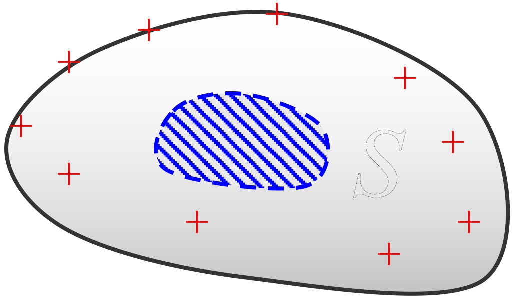
- **The internal electrical potential of the conductor is equal**

$$\because \vec{E}_{\text{int}} = 0 \qquad \therefore U_{AB} = \int_A^B \vec{E} \cdot \mathrm{d}\vec{l} = 0$$

6.1.2 The charge distribution on the electrostatic equilibrium conductor

1. Charge distribution in the interior and on the surface of the conductor

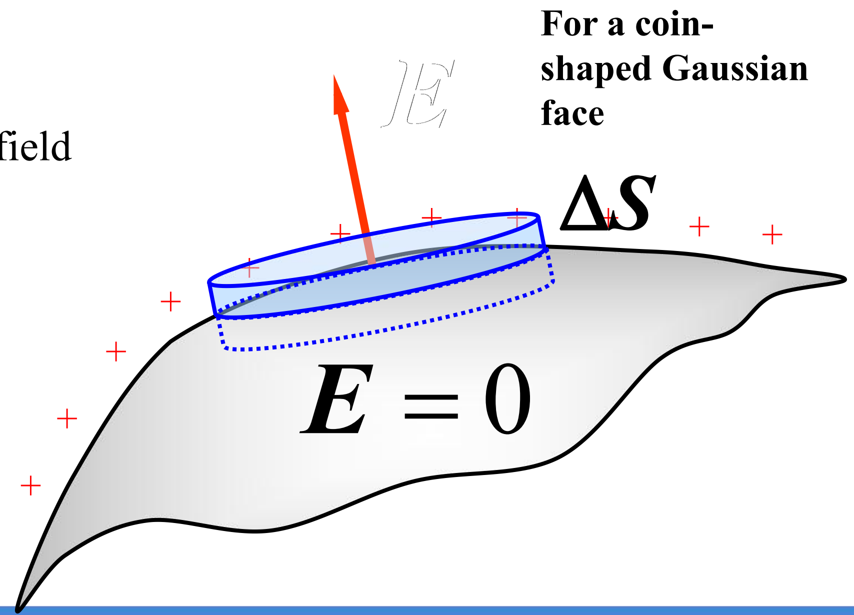
1. There is no net charge inside the conductor, and the charge can only be distributed on the conductor surface.



As Gaussian surface: $$\oint_S \vec{E}\cdot \mathrm{d}\vec{S} = 0 = \frac{q}{\varepsilon_0}$$

$$\because \vec{E} = 0 \qquad \therefore q = 0$$

2. The relationship between the conductor surface electric field strength and the charge surface density



$$\oint_S E\cdot \mathrm{d}S = \sigma\Delta S/\varepsilon_0$$

$$E\Delta S = \sigma\Delta S/\varepsilon_0$$

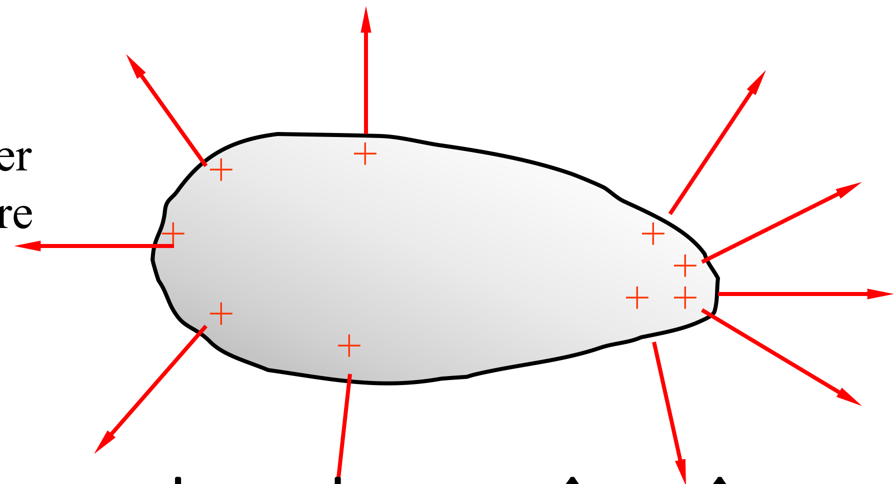
$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

2. Effect of the shape of the isolated conductors on the charge distribution

Cutting-edge discharge phenomenon

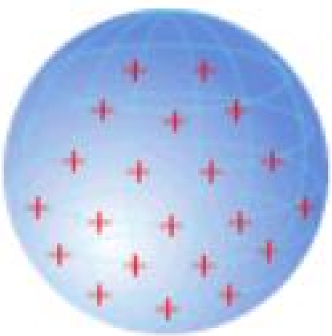
The conductor surface charge distribution is related to the conductor shape

1. Experiments show that when the isolated conductor is in electrostatic equilibrium, the greater the surface charge density and the surface curvature —— curvature, the greater the surface charge density.



$$\sigma\downarrow, E\downarrow; \quad \sigma\uparrow E\uparrow$$

2. For an isolated spherical charged conductor, the curvature of each part on the sphere is the same, so the charge is evenly distributed, that is, the charge density of the surface is the same everywhere on the sphere.



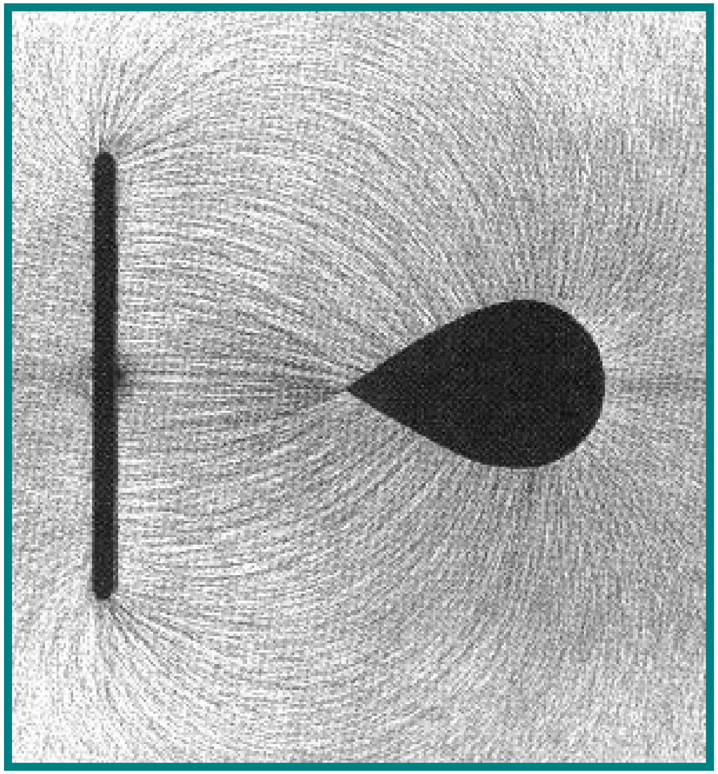
3. Tip discharge phenomenon

$$\sigma \uparrow \Rightarrow E \uparrow$$

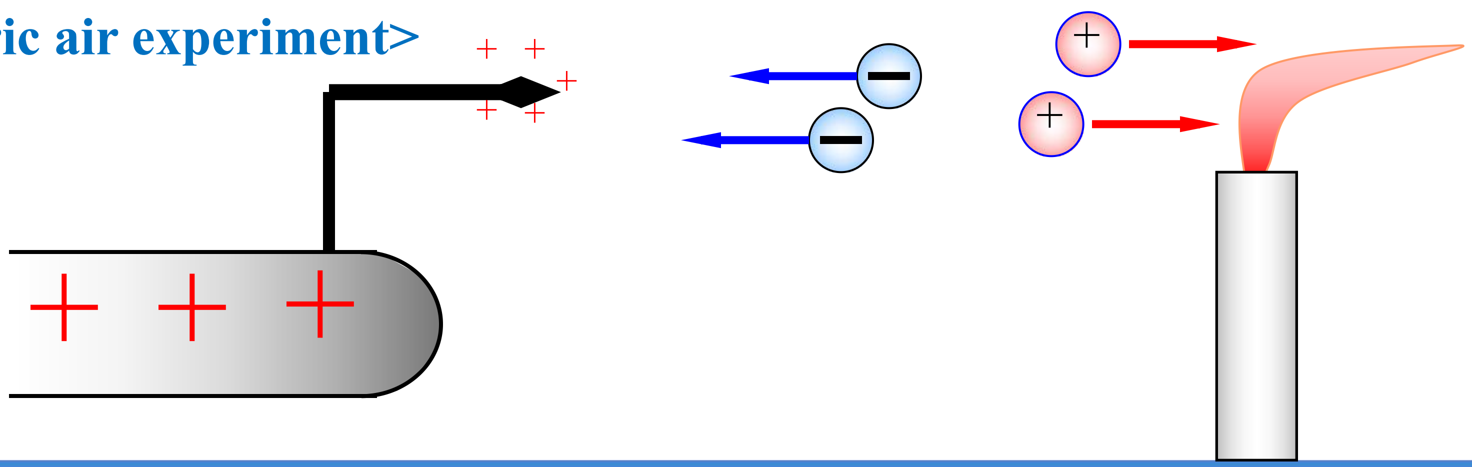
The strongest electric field near the tip of the charged conductor

The electric field near the tip of the charged conductor is particularly large, which can make the air near the tip ionize and become the discharge phenomenon of the conductor

—— **point discharge**

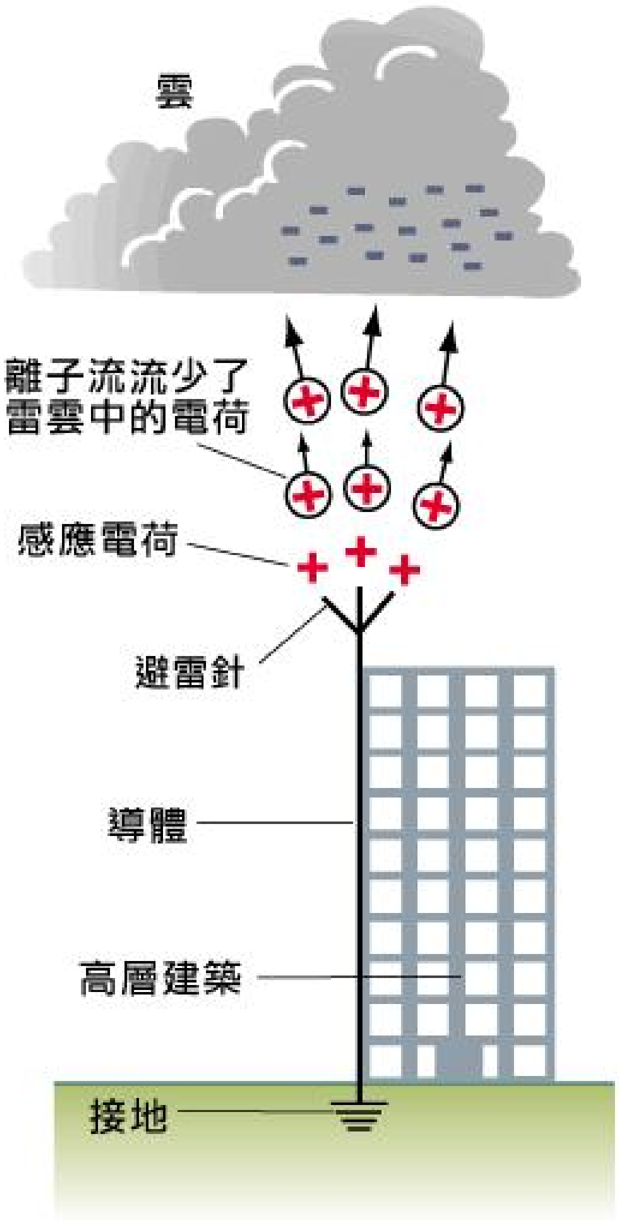


<Electric air experiment>



<lightning rod>

Utilization of the tip-discharge phenomena



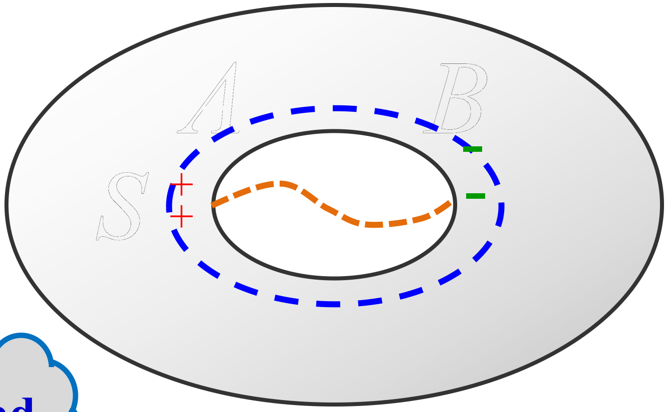
6.1.3 Electric field inside and outside the closed conductor cavity

electrostatic shield

1. The case of no charged body inside the conductor cavity

1. There is no charge on the surface of the cavity, and the charge can only be distributed on the outer surface

2. The cavity has no electric field, and the cavity is an equal potential area



Question: Is there any electric charge on the inner surface?

$$\oint_S \vec{E}\cdot \mathrm{d}\vec{S} = 0,\quad \sum q_i = 0$$

contradiction

If the inner surface is charged

$$\varphi_A - \varphi_B = \int_A^B \vec{E}\cdot \mathrm{d}\vec{l} \neq 0$$

The conductor is an isophore

2. Case of a charged body inside the conductor cavity

1. The inner surface of the cavity is charged, and the induced charge is equal to that of the charged body in the cavity

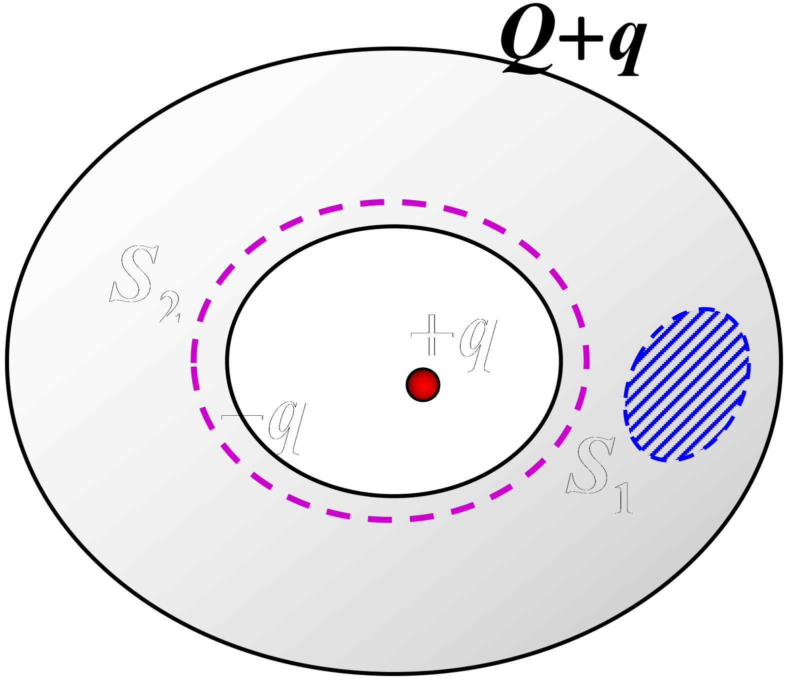
For Gaussian face S_2, Because the internal electric field is zero, so

$$\oint_{S_2} \vec{E} \cdot \mathrm{d}\vec{S} = 0$$

$$\sum q_i = 0$$

Charge charge in the cavity $+q$ add

The amount of electricity distributed on the inner surface of the conductor $-q$

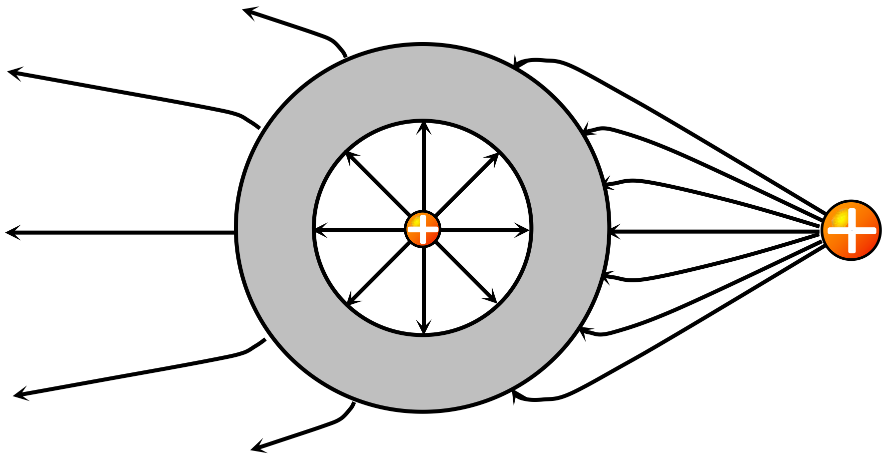


Because of the charge conservation in the conductors of this system

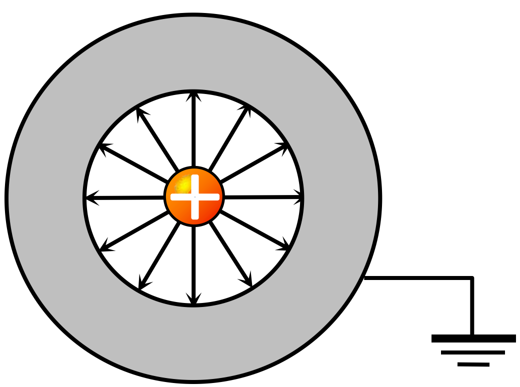
The amount of electricity distributed on the outer surface of the conductor is

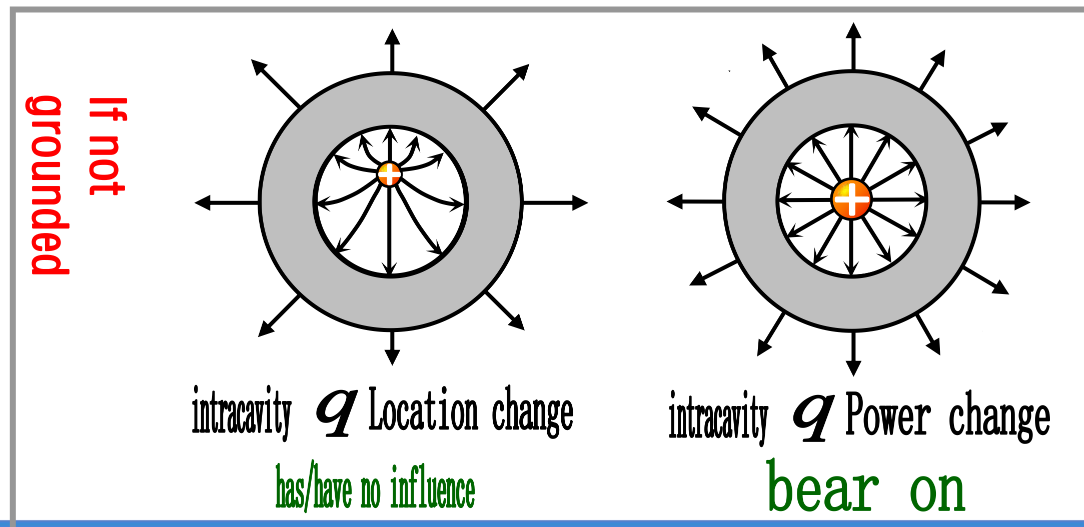
$$Q + q$$

2. The electric field in the cavity is only determined by the distribution of the charged bodies in the cavity and the surface induced charge in the cavity, and is independent of other charged bodies outside the conductor



3. Whether the external electric field of the cavity is affected by the charged body in the cavity is related to whether the cavity is grounded





3. electrostatic shield

Ground conductor cavity, the cavity and external electric field are independent, no interference with each other, called electrostatic shielding phenomenon.

4. Example of the calculation of the electrostatic field in the presence of conductors

Principles of electrostatic field field calculation in the presence of a conductor

1. Conditions of electrostatic balance

electric conductor $E_{\text{int}} = 0$ *or* $V = \text{const.}$

2. The basic property equation

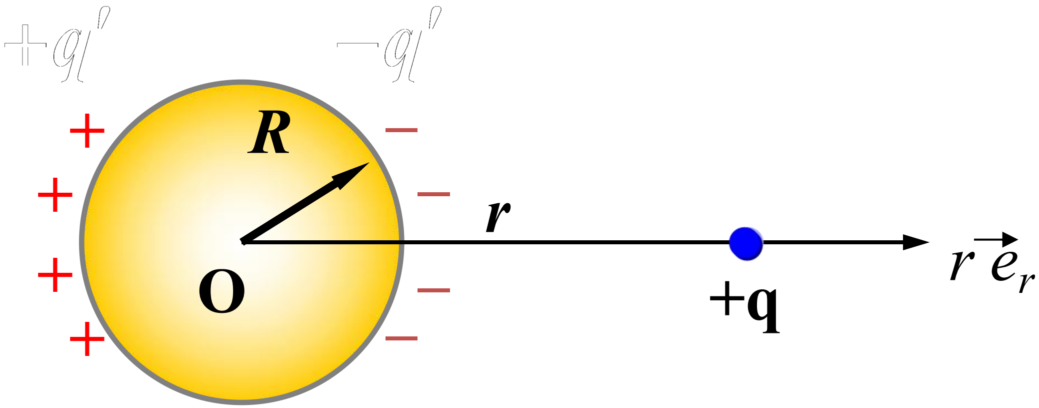
$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0} \sum_i q_i \qquad \oint_L \vec{E} \cdot \mathrm{d}\vec{l} = 0$$

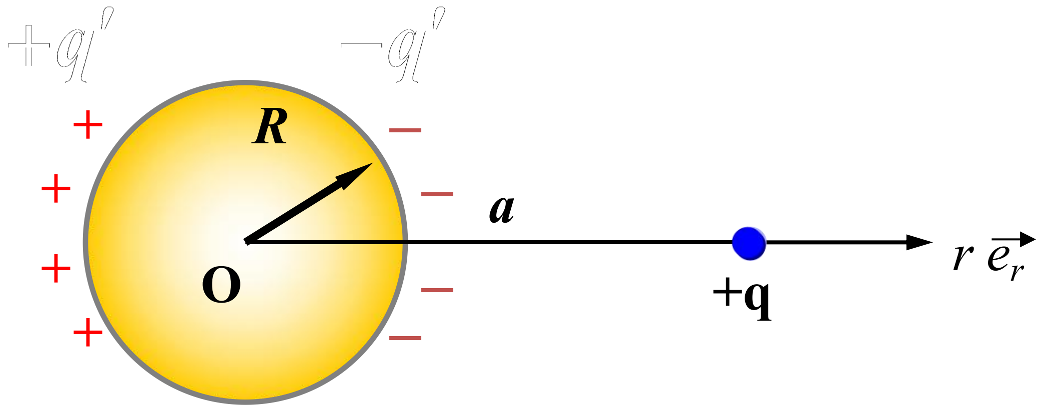
3. Law of conservation of charge $\sum Q_i = \text{const.}$

Example 1: Next to an uncharged metal ball, there is a little charge + q, the metal ball radius is R, try to find that:

(1) The electric field strength E generated at the center of the metal sphere and the potential of the center at this time.

(2) What is the net charge on the metal ball if it is grounded? The known center spacing between + q and the metal sphere is r.





separate:

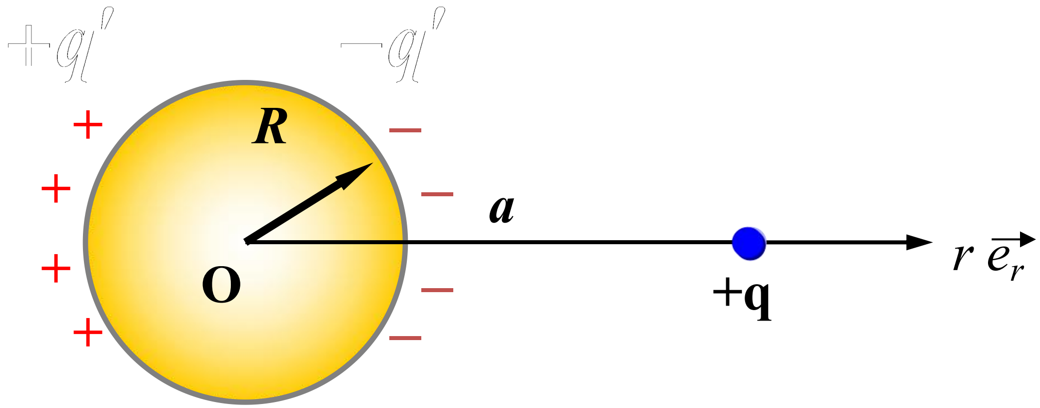
(1) The field strength of the spherical center O point is the superposition of the inductive charge electric field E and the electric field E'of the point charge q, namely:

$\pm q'$

$$\vec{E}_0 = \vec{E} + \vec{E}\,'$$

According to the electrostatic equilibrium condition, the internal field strength of the metal conductor sphere is zero everywhere, namely $E_0 = 0$, Then

$$\vec{E} = -\vec{E}' = -\frac{q}{4\pi\varepsilon_0 a^2}(-\vec{e}_r) = \frac{q}{4\pi\varepsilon_0 a^2}\vec{e}_r$$



The induced charge is distributed at the potential at the center of O:

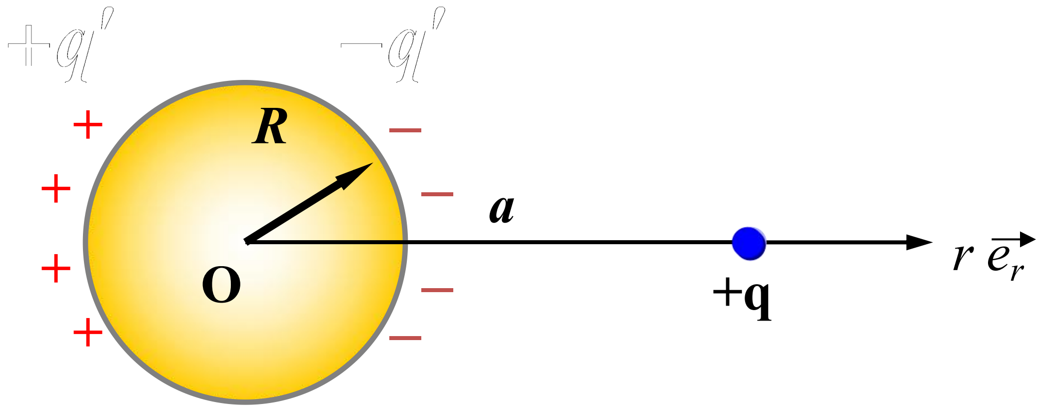
$$\varphi=\int_{\pm q'}\frac{\mathrm{d}q'}{4\pi\varepsilon_0 R}=\frac{1}{4\pi\varepsilon_0 R}\int_{\pm q'}\mathrm{d}q'=0$$

The electric potential of the point charge q at the spherical center O:

$$\varphi'=\frac{q}{4\pi\varepsilon_0 a}$$

O on the basis of the principle of potential superposition

$$\varphi_0=\varphi+\varphi'=\frac{q}{4\pi\varepsilon_0 a}$$



(2) If the metal ball is grounded, there is a net negative charge q '' on the ball, then the potential of the metal ball should be zero, as known from the superposition principle:

$$\varphi_0 = \frac{q}{4\pi\varepsilon_0 a} + \frac{q''}{4\pi\varepsilon_0 R} = 0$$

$$\Rightarrow \quad q'' = -\frac{R}{a}q \qquad \boxed{R < a} \qquad \Rightarrow \quad |q''| < q$$

§ 6.2

Capacitors and capacitors

6.2.1 Capacitance of the isolated conductor

$$C = \frac{Q}{\varphi}$$

unit

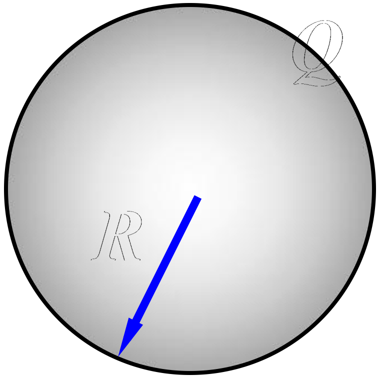
$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$

$$1\ \mu\mathrm{F} = 10^{-6}\ \mathrm{F}$$

$$1\ \mathrm{pF} = 10^{-12}\ \mathrm{F}$$

For example: the capacitance of an isolated conductor sphere

$$C = \frac{Q}{\varphi} = \frac{Q}{\dfrac{Q}{4\pi\varepsilon_0 R}} = 4\pi\varepsilon_0 R$$



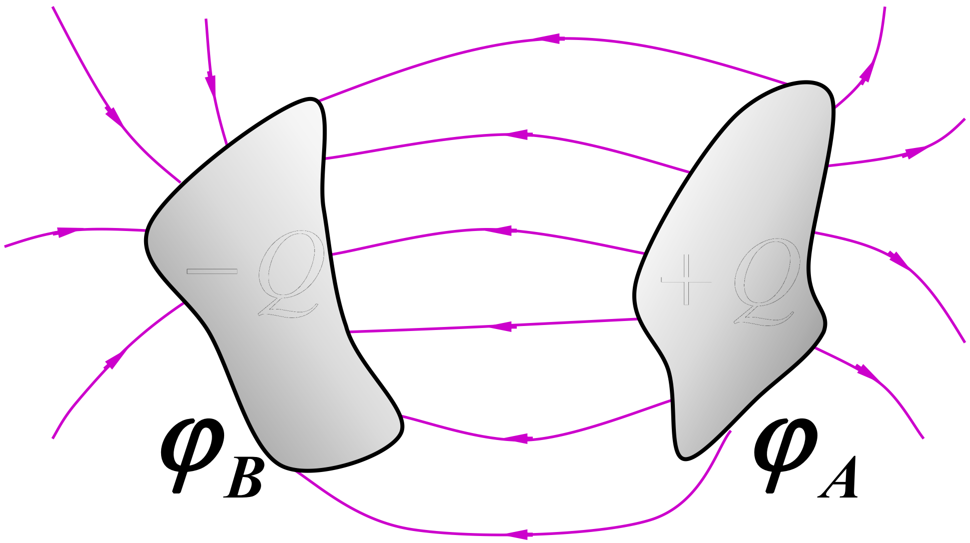
♦ the earth $R_{\mathrm{E}} = 6.4\times10^{6}\,\mathrm{m},\quad C_{\mathrm{E}} \approx 7\times10^{-4}\,\mathrm{F}$

6.2.2 Capacitors and their capacitors

The capacitance of the capacitors

$$C=\frac{Q}{\varphi_A-\varphi_B}=\frac{Q}{U}$$

Significance: The ability to store the charge



Note: the size of the capacitor is only related to the shape and relative position of the conductor, regardless of the amount of charge.

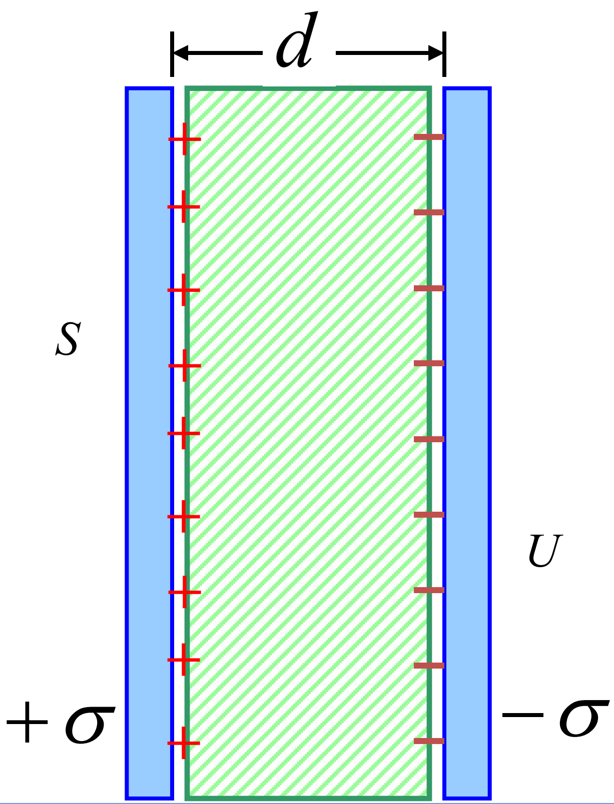
♦ Calculation of the capacitor capacitance

step

1. Two plates are charged respectively $\pm Q$ 2. Seek (Gauss theorem) $\vec{E}$

3. Please U 4. Please, $U_{AB} = \int_A^B \vec{E} \cdot \mathrm{d}\vec{l}$ C $C = Q / U$

1. Parallel plate capacitor



(1) Electric field strength between plates:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 S}$$

(2) Voltage between the two plates:

$$U = Ed = \frac{Qd}{\varepsilon_0 S}$$

(3) Parallel plate capacitor:

$$\boxed{C = \frac{Q}{U} = \frac{\varepsilon_0 S}{d}}$$

§ 6.3

Dielectric in the electrostatic field

6.3.1 Polarization of the dielectric medium

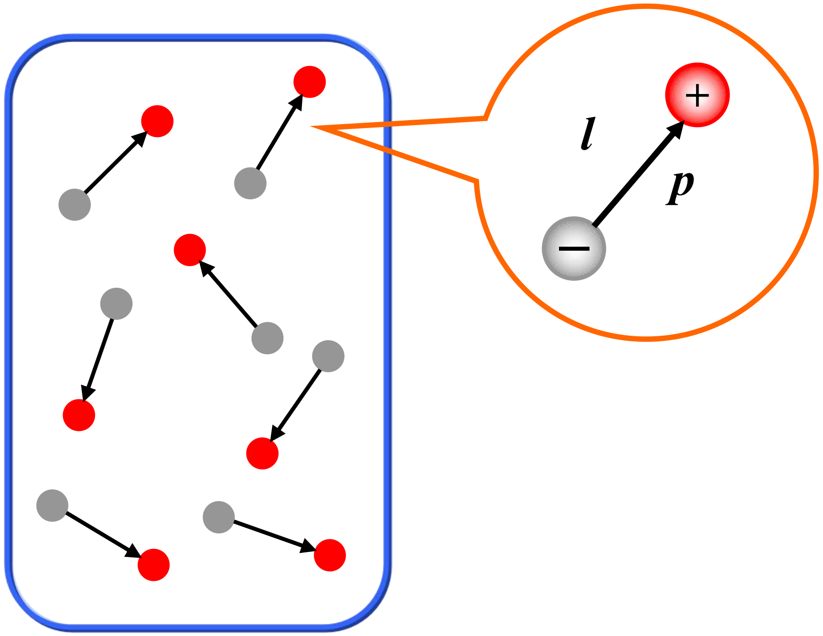
1. Classification of the dielectric medium

1. The center of gravity of the positive and negative charges in the molecules do not coincide, and there is a certain distance between them- -polar molecules (polar molecules)

Such as: water, epoxy resin, ceramics

The moment is: $\vec{p} = q\vec{l}$

The inherent moment

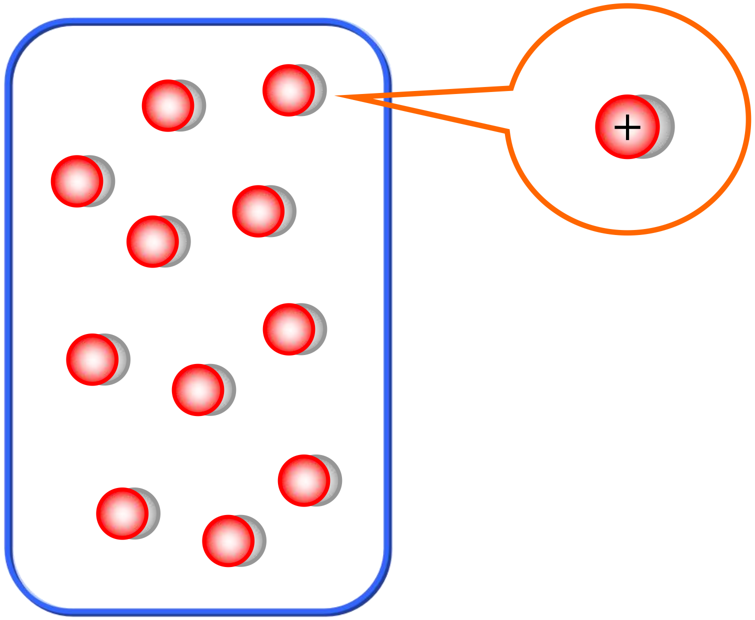


a.polar molecule

2. The center of gravity of positive and negative charges coincides, and such molecules are called infinite molecules (non-polar molecules)

Such as: methane, polypropylene ethylene, paraffin

electric moment: $\vec{p} = 0$



b.nonpolar molecule

2. The polarization of the dielectric medium

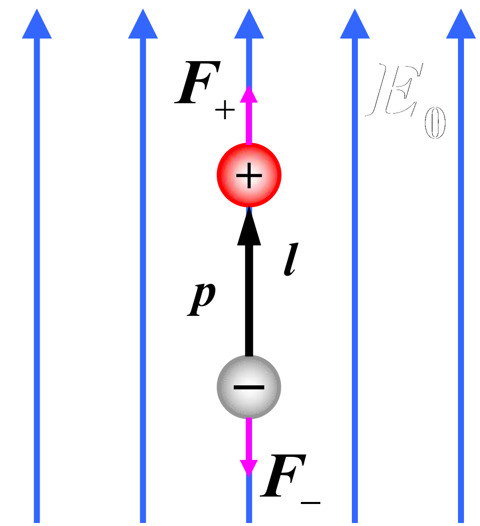
1. Displacement polarization of the infinite molecular dielectric

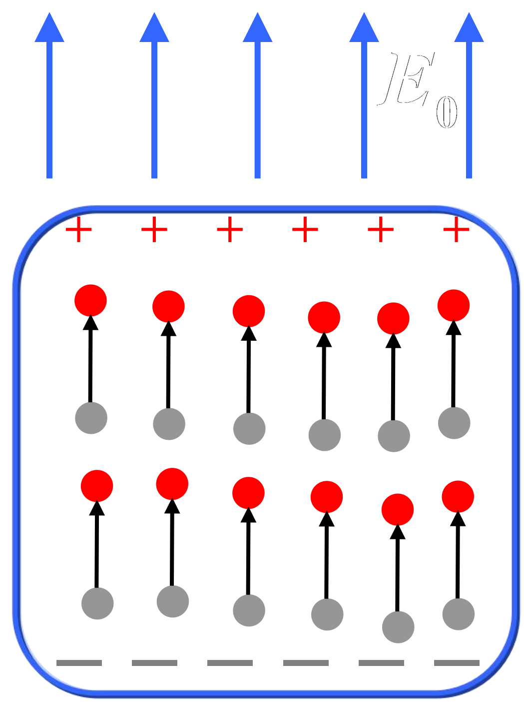
The external field causes the relative displacement of the positive and negative charge centers, equivalent to an electric dipole

Sense of electric moment $\vec{p} = q\vec{l}$

Both directions are along the external electric field

$$E_0 \uparrow \Rightarrow l \uparrow \Rightarrow p \uparrow$$



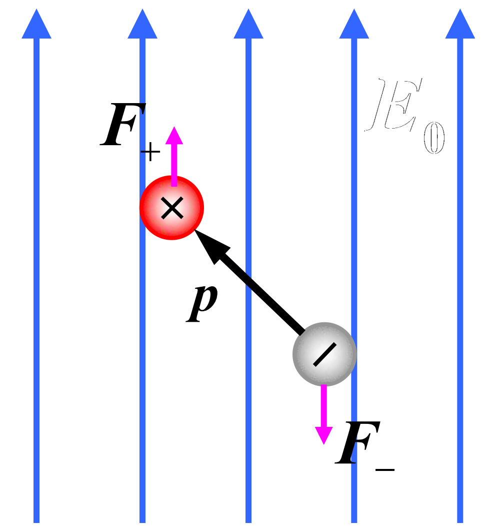


Positive and negative charge layers appear in the two end surfaces perpendicular to the external field

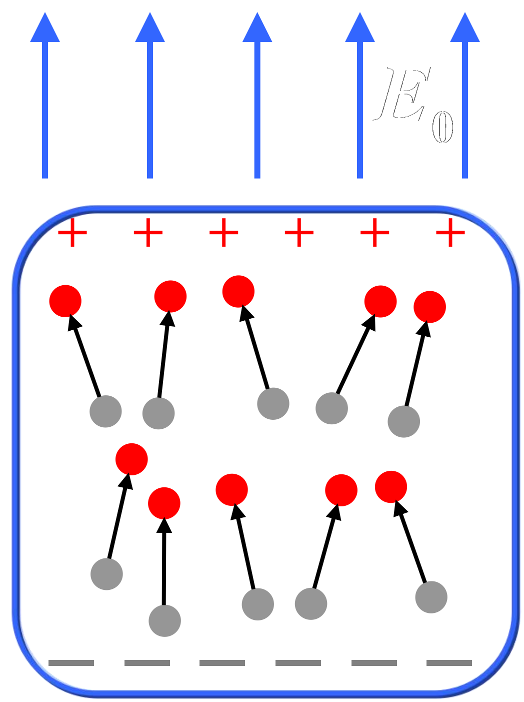
- -Polarized surface charge or the bound charge

2. Orientation polarization of the polar molecular dielectric

The external field generates a torque on the electric torque of the polar molecule



Note: Due to the interference of molecular thermal motion, the electric torque of each molecule cannot be arranged in the direction of the external electric field. The stronger the external electric field is, the more neat the arrangement of the molecular electric torque tends to be.



Because the electric torque tends to the direction of the external electric field, the positive and negative polarized surface charges appear in the two end surfaces of the medium perpendicular to the external field

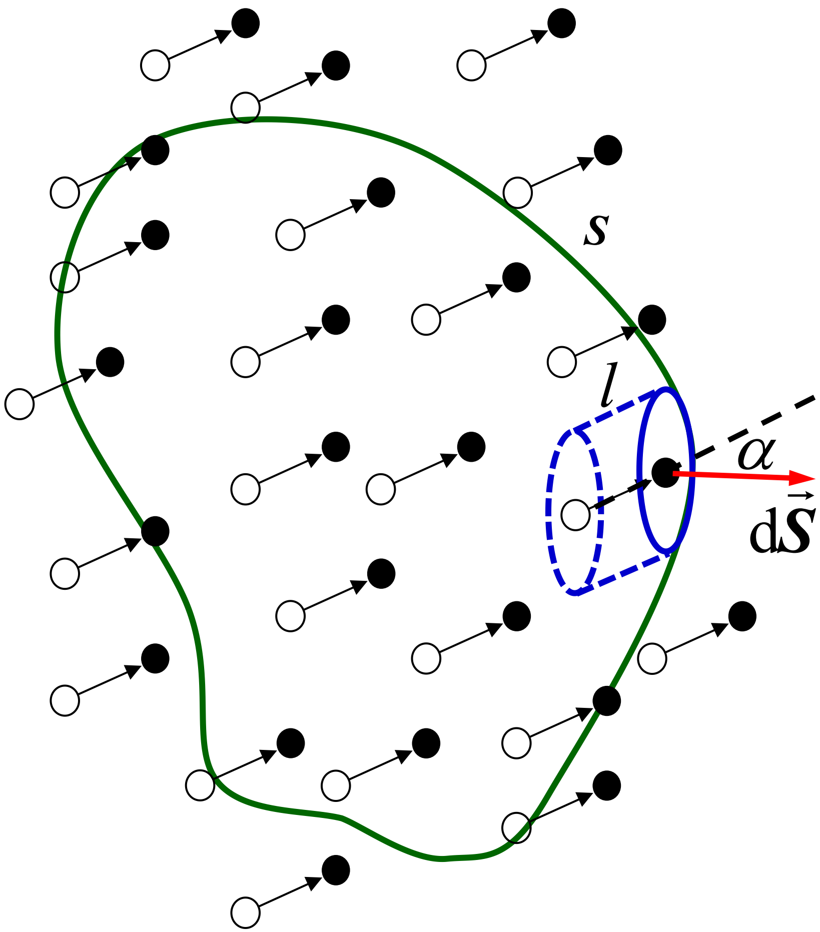
6.3.2 Polarization intensity and polarization charge

1. polarization

$$\vec{P} = \frac{\sum \vec{p}_i}{\Delta V}$$

For a uniformly polarized dielectric:

$$\vec{P} = n\vec{p} = nq\vec{l}$$



2. Polarization strength and the polarized charge

Volume of the dV:

$$\mathrm{d}V = l\,\mathrm{d}S\cos\alpha$$

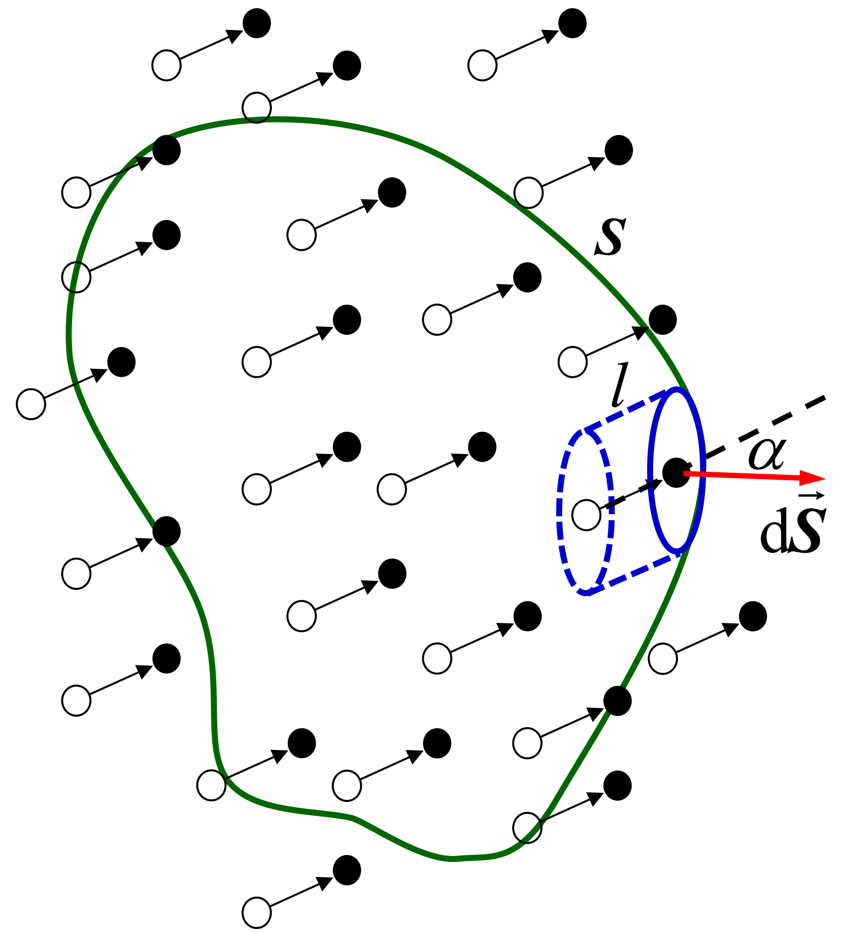
Number of molecules within the volume of the dV:

$$n\,\mathrm{d}V = nl\,\mathrm{d}S\cos\alpha$$

Value of the positive charge:

$$\mathrm{d}Q = nq\mathrm{d}V = nql\mathrm{d}S\cos\alpha = \vec{P}\cdot\mathrm{d}\vec{S}$$
$$= \vec{P}\cdot\vec{e}_{\mathrm{n}}\mathrm{d}S$$

A net equal ectopic charge in dV $-\mathrm{d}Q$



In the volume V surrounded by the entire closed surface S, the bulk bound charge q'is

$$q' = \oiiint_V -\mathrm{d}Q = -\oiint_S \vec{P}\cdot\mathrm{d}\vec{S}$$

Facial-bound charge density

$$\sigma' = \frac{\mathrm{d}Q}{\mathrm{d}S} = \vec{P}\cdot\vec{e}_{\mathrm{n}}$$

Suggesting that the higher the polarization, $\vec{P}\uparrow$ More dense is the bound charge surface on the dielectric surface $\sigma'\uparrow$

6.3.3 Polarization law of the dielectric

When the polarization reaches the stability, the experiment shows that: in the isotropic linear dielectric, the spot is not too strong, yes

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

χ_e **-Dielectric, and the electric polarizability rate**

$\vec{E}$ In the dielectric, the total electric field strength includes both the external $\vec{E}_0$ electric field and the additional electric field generated by the polarized charge. $\vec{E}'$

$\vec{P}$ ′, And these $\vec{E}'$ quantities $\vec{E}$ are mutually dependent and mutually restricted.

6.3.4 A Gaussian law in the presence of a dielectric

When there is a medium, the Gauss theorem in the original vacuum still holds

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0}(q_0 + q')$$

S internal charge (free, bound)

$$q' = \oiiint_V -\mathrm{d}Q = -\oiint_S \vec{P} \cdot \mathrm{d}\vec{S}$$

$$\oint_S \varepsilon_0 \vec{E} \cdot \mathrm{d}\vec{S} + \oint_S \vec{P} \cdot \mathrm{d}\vec{S} = q_0$$

$$\oint_S \left(\varepsilon_0 \vec{E} + \vec{P}\right) \cdot \mathrm{d}\vec{S} = q_0$$

A Gaussian theorem with a medium

$$\oint_S \vec{D} \cdot \mathrm{d}\vec{S} = \sum_i q_{0i}$$

Free charge inside the S

The potential shift vector $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

Uniform and homogeneous medium

$$\vec{D} = \varepsilon_0 \vec{E}(1 + \chi_e)$$

Uniform and homogeneous medium

$$\vec{D} = \varepsilon_0 \vec{E}(1+\chi_e)$$

a surname $\varepsilon_r = (1+\chi_e)$ Is the relative dielectric constant (relative capacitance)

$\varepsilon = \varepsilon_0 \varepsilon_r$ **It is a dielectric constant**

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

For vacuum $\chi_e = 0$ $\varepsilon_r = 1$ $\varepsilon = \varepsilon_0$ Is $\vec{D} = \varepsilon_0 \vec{E}$

Gaussian theorem with medium $\oint_S \vec{D} \cdot d\vec{S} = \sum_i q_{0i}$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

pay attention

Ask for media first $\boxed{\vec{D} \to \vec{E} \to \varphi, U}$

If a polarized charge is required $\boxed{\vec{E} \to P \to \sigma'}$

$$\left.\begin{aligned} P &= (\varepsilon_r - 1)\varepsilon_0 E \\ \sigma' &= P \end{aligned}\right\} \sigma' = (\varepsilon_r - 1)\varepsilon_0 E$$

Example 1 Place a relative capacitance dielectric $\varepsilon_r = 3$ between two plates of a parallel plate capacitor apart $d = 1\mathrm{mm}$ between the plates. Before putting in, the potential difference between the two plates is $1000\mathrm{V}$. Try to find the electric field strength E in the dielectric, the electric polarization strength P , the charge surface density of the plate and the dielectric, and the electric displacement D in the dielectric.

separate: $$E_0 = U/d = \frac{1000}{10^{-3}}\mathrm{V\cdot m^{-1}} = 10^6\mathrm{V\cdot m^{-1}} = 10^3\mathrm{kV\cdot m^{-1}}$$

$$E = E_0/\varepsilon_r = 3.33\times10^2\mathrm{kV\cdot m^{-1}}$$

$$P = (\varepsilon_r - 1)\varepsilon_0 E = 5.89\times10^{-6}\mathrm{C\cdot m^{-2}}$$

$$\sigma_0 = \varepsilon_0 E_0 = 8.85\times10^{-6}\mathrm{C\cdot m^{-2}}$$

$$\sigma' = P = 5.89\times10^{-6}\mathrm{C\cdot m^{-2}}$$

$$D = \varepsilon_0\varepsilon_r E = \varepsilon_0 E_0 = \sigma_0 = 8.85\times10^{-6}\mathrm{C\cdot m^{-2}}$$

§ 6.5

Electrostatic energy of the charged systems

6.5.1 Interaction energy of the point-charge system

Let the charge system composed of n stationary charges, which carry each charge infinitely away from each other to the existing position, and the external force overcomes the electrostatic interaction energy (mutual energy) of the electrostatic force between them

Potential energy zero

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i \varphi_i$$

Where: it is φ_i q_i The place is q_i Potential potential other than the charge

reduction

1. The simplest case: two point charges q and Q

The potential energy of the point charge q in the electric field of Q

$$W = q\varphi_Q = \frac{qQ}{4\pi\varepsilon_0 r} = Q\varphi_q$$

The potential energy of Q in the electric field of q is also expressed

Namely, the electrostatic energy of a charge system composed of Q and q

It can be written as:

$$W = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}\right) = \frac{1}{2}(q\varphi_q + Q\varphi_q)$$

2. There are n point charges in the space

induction

When only q_1And q_2For two charges, the electrostatic energy is

$$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

The introduction of a third electric charge

$$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \boxed{\left(\frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}\right)}$$

Change in the electrostatic energy induced by the introduction of a third point charge

$$W=\frac{1}{2}q_1\left(\frac{q_2}{4\pi\varepsilon_0 r_{12}}+\frac{q_3}{4\pi\varepsilon_0 r_{13}}\right)$$

$$+\frac{1}{2}q_2\left(\frac{q_1}{4\pi\varepsilon_0 r_{21}}+\frac{q_3}{4\pi\varepsilon_0 r_{23}}\right)$$

$$+\frac{1}{2}q_3\left(\frac{q_1}{4\pi\varepsilon_0 r_{31}}+\frac{q_2}{4\pi\varepsilon_0 r_{32}}\right)$$

$$W=\frac{1}{2}q_1(\varphi_{21}+\varphi_{31})+\frac{1}{2}q_2(\varphi_{12}+\varphi_{32})+\frac{1}{2}q_3(\varphi_{13}+\varphi_{23})$$
$$=\frac{1}{2}q_1\varphi_1+\frac{1}{2}q_2\varphi_2+\frac{1}{2}q_3\varphi_3$$

Introducing a fourth electric charge

$$W=\frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}+\left(\frac{q_1q_3}{4\pi\varepsilon_0 r_{13}}+\frac{q_2q_3}{4\pi\varepsilon_0 r_{23}}\right)$$

$$+\boxed{\left(\frac{q_1q_4}{4\pi\varepsilon_0 r_{14}}+\frac{q_2q_4}{4\pi\varepsilon_0 r_{24}}+\frac{q_3q_4}{4\pi\varepsilon_0 r_{34}}\right)}$$

Change in the electrostatic energy induced by the introduction of the fourth point charge

Repeat the above process to get the electrostatic energy of the system composed of n point charges

$$W=\sum_{i=1}^{n}\frac{1}{2}q_i\varphi_i$$

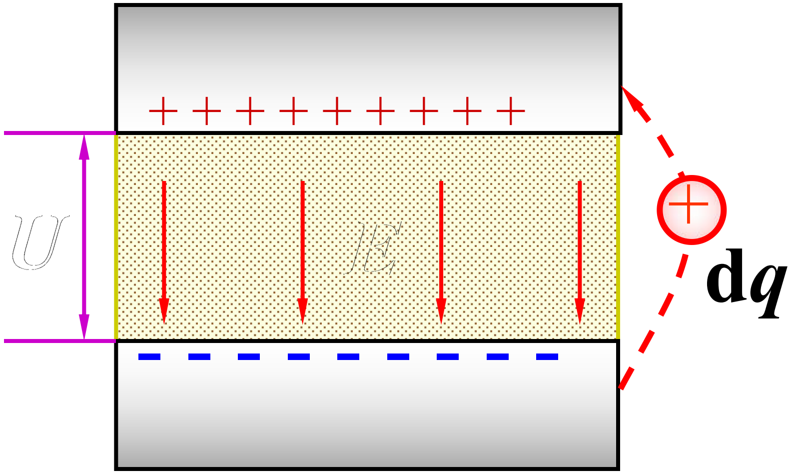
6.5.2 Electric energy of the capacitors

$$\mathrm{d}A = U\mathrm{d}q = \frac{q}{C}\mathrm{d}q$$

$$A = \frac{1}{C}\int_0^Q q\,\mathrm{d}q = \frac{Q^2}{2C}$$

$$C = \frac{Q}{U}$$

$$A = \frac{1}{2}QU = \frac{1}{2}CU^2$$



Electric energy stored by the capacitors

$$W_e = \frac{Q^2}{2C} = \frac{1}{2}QU = \frac{1}{2}CU^2$$

6.5.3 Electrostatic energy during the continuous charge distribution

Charge element system

The electrostatic energy of the charged body

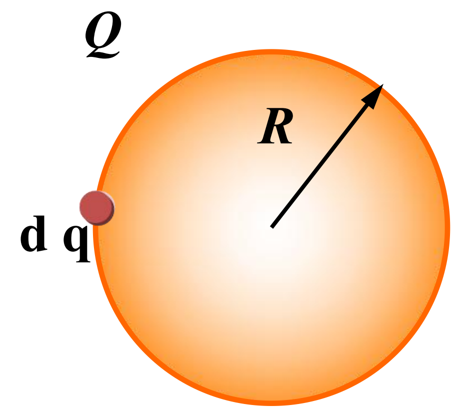
(self energy)

$$W = \frac{1}{2}\int_q \varphi \mathrm{d}q$$

Example 1 A uniform charged sphere (R, Q) with its electrostatic energy:

$$W = \frac{1}{2}\int_q \varphi \mathrm{d}q = \frac{1}{2}\int_q \frac{Q}{4\pi\varepsilon_0 R}\mathrm{d}q$$

$$= \frac{Q}{8\pi\varepsilon_0 R}\int_q \mathrm{d}q = \frac{Q^2}{8\pi\varepsilon_0 R}$$



6.5.4 Energy of the electrostatic field

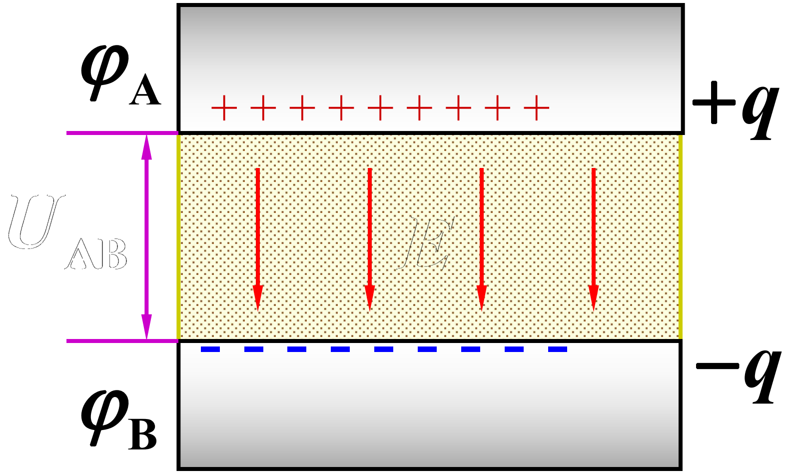
The static static energy of the system is:

$$W_e = \frac{1}{2}q\varphi_A - \frac{1}{2}q\varphi_B$$

$$= \frac{1}{2}qU_{AB} = \frac{1}{2}CU_{AB}^2$$

$$= \frac{q^2}{2C} = \frac{q^2 d}{2\varepsilon_0 S}$$

$$= \boxed{\frac{1}{2}\varepsilon_0 E^2} V = w_e V$$



$$E = \frac{q}{\varepsilon_0 S} \qquad V = Sd$$

Electric field energy density

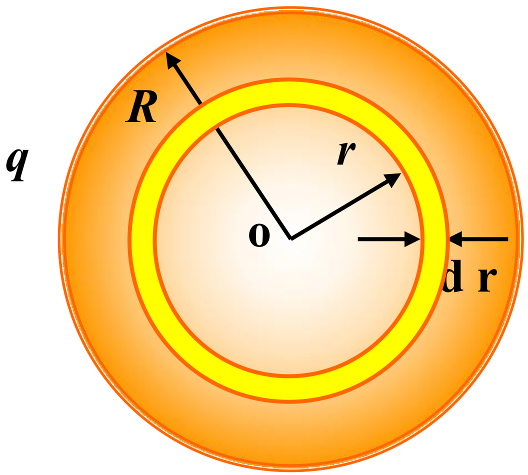
$$w_e = \frac{1}{2}\varepsilon_0 E^2$$

Total energy of the electric field of a charged system:

$$W = \int_V w_e \mathrm{d}V = \int_V \frac{\varepsilon_0 E^2}{2} \mathrm{d}V$$

The energy of a charged system represented by the concept of a field

Example 1 A uniform charged sphere (R, q) in vacuum can find the electrostatic energy of the charged system.



$$\mathrm{d}V = 4\pi r^2 \mathrm{d}r$$

$$W = \int_V w_e \mathrm{d}V = \int_0^\infty \frac{\varepsilon_0 E^2}{2} 4\pi r^2 \mathrm{d}r$$

$$= \int_0^R \frac{\varepsilon_0 E_1^2}{2} 4\pi r^2 \mathrm{d}r + \int_R^\infty \frac{\varepsilon_0 E_2^2}{2} 4\pi r^2 \mathrm{d}r$$

$$W=\int_0^R \frac{\varepsilon_0}{2}\left(\frac{qr}{4\pi\varepsilon_0 R^3}\right)^2 4\pi r^2 \mathrm{d}r+\int_R^{\infty} \frac{\varepsilon_0}{2}\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2 \mathrm{d}r$$

$$=\frac{3q^2}{20\pi\varepsilon_0 R}$$