HW 2

1. (15pt) Prove that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$

$$P(A\cup B\cup C)=P\big(A\cup(B\cup C)\big)=P(A)+P(B\cup C)-P\big(A\cap(B\cup C)\big)$$
$$=P(A)+P(B)+P(C)-P(BC)-P\big(A\cap(B\cap C)\big)$$
$$P\big(A\cap(B\cap C)\big)=P(AB\cup AC)=P(AB)+P(AC)-P(ABC)$$

Therefore:

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(AB)-P(AC)-P(BC)+P(ABC)$$

2. (15pt) Two cards are drawn one right after the other from a standard deck (52 cards, no jokers) without replacement. Find each of the following probabilities.

   (a) (5pt) The 2nd card is red, given that the 1st card is red.

$$P(2nd\ card\ is\ red|1st\ card\ is\ red)=\frac{P(2nd\ card\ is\ red\cap 1st\ card\ is\ red)}{P(1st\ card\ is\ red)}$$
$$=\frac{\frac{26}{52}\times\frac{25}{51}}{\frac{26}{52}}=\frac{25}{51}$$

   (b) (5pt) The 2nd card is a face card, given the 1st card is a jack.

$$P(2nd\ card\ is\ a\ face|1st\ card\ is\ a\ jack)$$
$$=\frac{P(2nd\ card\ is\ a\ face\cap 1st\ card\ is\ a\ jack)}{P(1st\ card\ is\ a\ jack)}=\frac{\frac{4}{52}\times\frac{11}{51}}{\frac{4}{52}}=\frac{11}{51}$$

   (c) (5pt) The 2nd card is an ace, given the 1st card is not an ace.

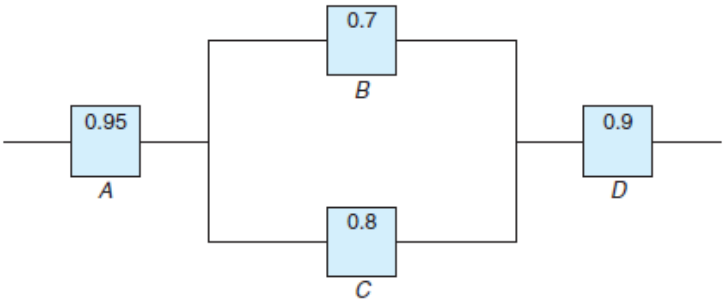
$$P(2nd\ card\ is\ an\ ace|1st\ card\ is\ not\ an\ ace)$$
$$=\frac{P(2nd\ card\ is\ an\ ace\cap 1st\ card\ is\ not\ an\ ace)}{P(1st\ card\ is\ not\ an\ ace)}=\frac{\frac{48}{52}\times\frac{4}{51}}{\frac{48}{52}}$$
$$=\frac{4}{51}$$

3. (15pt) Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 30% for method A and 10% for method B. Method B is more expensive, however, and hence is used only 20% of the time. Method A is used the other 80% of the time. A worker is taught the skill by one of the two methods, but he fails to learn it correctly. What is the probability that he was taught by using method A?

Let F be the failure of learning the skill. Using Bayes' rule,

$$P(A|F) = \frac{P(A)P(F|A)}{P(A)P(F|A) + P(B)P(F|B)} = \frac{80\% \times 30\%}{80\% \times 30\% + 20\% \times 10\%} = \frac{12}{13}$$

4. (15pt) Suppose the diagram of an electrical system is as given in the figure below. What is the probability that the system works? Assume the components fail independently.



$$P = (0.95)[1 - (1 - 0.7)(1 - 0.8)](0.9)$$

5. (20pt) A construction company employs two sales engineers. Engineer 1 does the work of estimating cost for 70% of jobs bid by the company. Engineer 2 does the work for 30% of jobs bid by the company. It is known that the error rate for engineer 1 is such that 0.02 is the probability of an error when he does the work, whereas the probability of an error in the work of engineer 2 is 0.04. Suppose a bid arrives and a serious error occurs in estimating cost. Which engineer would you guess did the work? Explain and show all work.

   Define events:
   1: engineer 1, P(1) = 0.7,
   2: engineer 2, P(2) = 0.3.
   E: an error has occurred in estimating cost, P(E|1) = 0.02 and P(E|2) = 0.04.
   Using Bayes' rule:

$$P(1|E) = \frac{P(1)P(E|1)}{P(1)P(E|1) + P(2)P(E|2)} = 0.5385, \qquad P(2|E) = 1 - 0.5385 = 0.4615$$

   So more likely engineer 1 did the job.

6. (20pt) During bad economic times, industrial workers are dismissed and are often replaced by machines. The history of 100 workers whose loss of employment is attributable to technological advances is reviewed. For each of these individuals, it is determined if he or she was given an alternative job within the same company, found a job with another company in the same field, found a job in a new field, or has been unemployed for 1 year. In addition, the union status of each worker is recorded. The following table summarizes the results.

	Union	Nonunion
Same Company	40	15
New Company (same field)	13	10
New Field	4	11
Unemployed	2	5

(a) (10pt) If the selected worker found a job with a new company in the same field, what is the probability that the worker is a union member?

$$P\left(Union\ member \middle| New\ company\ (same\ field)\right) = \frac{13}{13+10} = 0.5652$$

(b) (10pt) If the worker is a union member, what is the probability that the worker has been unemployed for a year?

$$P(Unemployed|Union\ member) = \frac{2}{40+13+4+2} = 0.034$$