§ 5.4

Potential

5.4.1 Electrostatic field force does work circuital theorem of electrostatic field

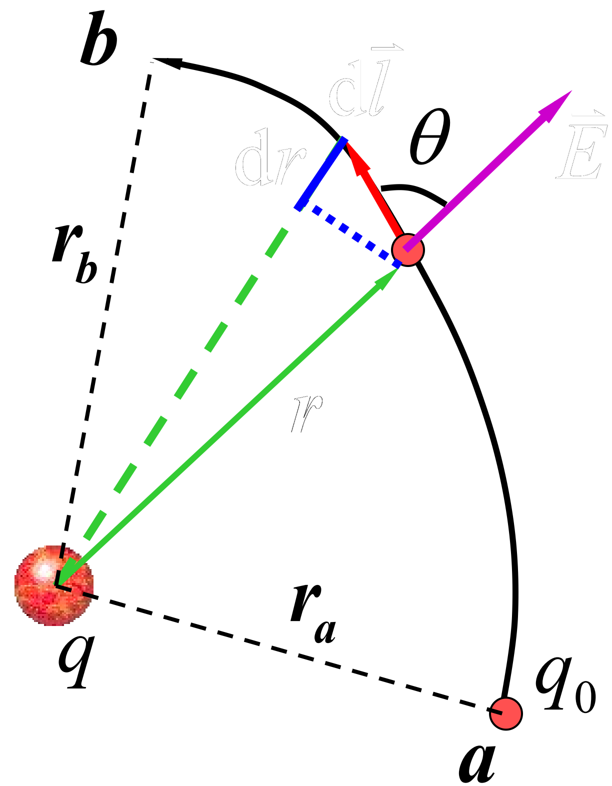
1. Work done by the electrostatic field force

1. The electric field of the point-electric charge

$$\mathrm{d}A = q_0\vec{E}\cdot\mathrm{d}\vec{l} = \frac{qq_0}{4\pi\varepsilon_0 r^3}\vec{r}\cdot\mathrm{d}\vec{l} = \frac{qq_0}{4\pi\varepsilon_0 r^2}\mathrm{d}r$$

$$A = q_0\int_L \vec{E}\cdot\mathrm{d}\vec{l} = \frac{qq_0}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{\mathrm{d}r}{r^2}$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$



$$\vec{r}\cdot\mathrm{d}\vec{l} = r\mathrm{d}l\cos\theta = r\mathrm{d}r$$

Results: W is only with q_0 The position is not related to the path.

2. Electric field of arbitrary charge (regarded as a point charge system)

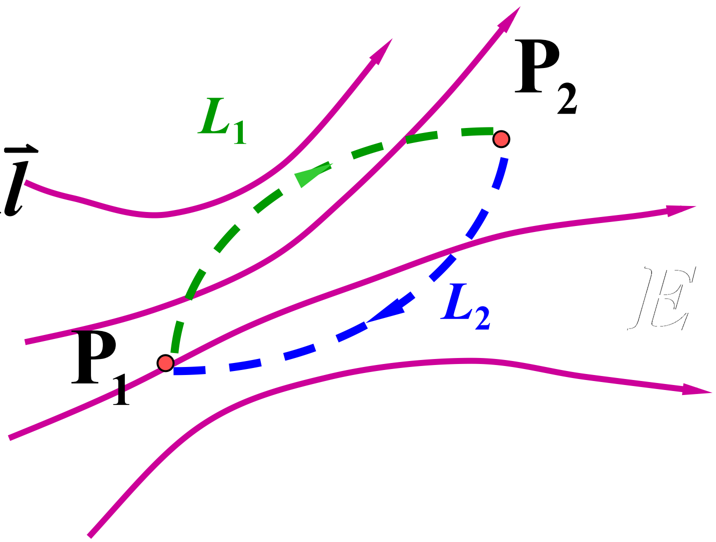
$$\vec{E} = \sum_i \vec{E}_i \Rightarrow A = q_0 \int_l \vec{E} \cdot \mathrm{d}\vec{l} = \sum_i q_0 \int_l \vec{E}_i \cdot \mathrm{d}\vec{l}$$

Conclusion: Electrostatic field force does work and path-independent —— conservative force

2. Loop theorem for the electrostatic field

$$\oint_L \vec{E} \cdot \mathrm{d}\vec{l} = \int_{P_1(L_1)}^{P_2} \vec{E} \cdot \mathrm{d}\vec{l} + \int_{P_2(L_2)}^{P_1} \vec{E} \cdot \mathrm{d}\vec{l}$$

$$= \int_{P_1(L_1)}^{P_2} \vec{E} \cdot \mathrm{d}\vec{l} - \int_{P_1(L_2)}^{P_2} \vec{E} \cdot \mathrm{d}\vec{l}$$



$$\boxed{\oint_L \vec{E} \cdot \mathrm{d}\vec{l} = 0}$$

$$\int_{P_1(L_1)}^{P_2} \vec{E} \cdot \mathrm{d}\vec{l} = \int_{P_1(L_2)}^{P_2} \vec{E} \cdot \mathrm{d}\vec{l}$$

5.4.2 Potential difference and electric potential

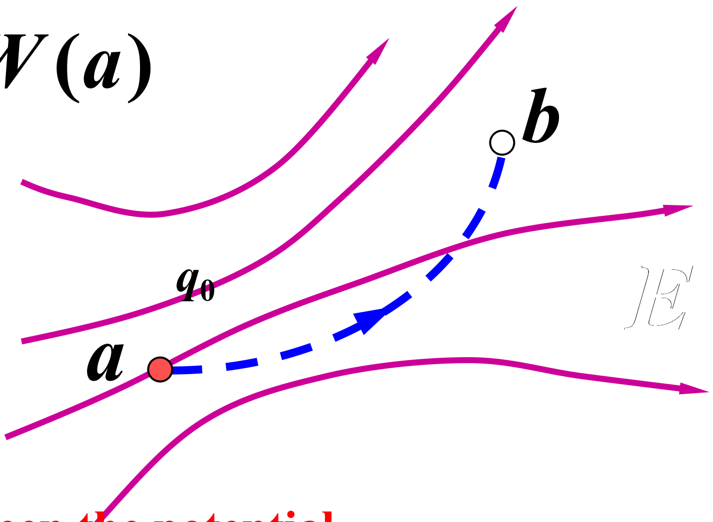
1. energy of position

The static field is the conservative field, and the electrostatic field force is the conservative force. The work done by the electrostatic field force is equal to the reduction of the charge potential energy (the negative value of the increment).

$$A_{ab} = \int_a^b q_0 \vec{E} \cdot \mathrm{d}\vec{l} = W(a) - W(b) = -\Delta W$$

$$A_{ab} \begin{cases} > 0, & W(b) < W(a) \\ < 0, & W(b) > W(a) \end{cases}$$

2. potential



$$\int_a^b \vec{E} \cdot \mathrm{d}\vec{l} = \frac{W(a) - W(b)}{q_0}$$

$$= \varphi_a - \varphi_b$$ The difference between the potential

$$\varphi_a = \int_a^b \vec{E}\cdot \mathrm{d}\vec{l} + \varphi_b$$

$$\varphi_b = 0$$

$$\varphi_a = \int_a^{\text{The potential zero}} \vec{E}\cdot \mathrm{d}\vec{l}$$

♦ Physical significance: the work of the electrostatic field force when the unit positive test charge is moved from point A to point zero of the potential.

Zero–point selection method of the electric potential

(1) The charge is distributed in a limited space. Order is then the $\varphi_\infty = 0$

$$\varphi_a = \int_a^\infty \vec{E}\cdot \mathrm{d}\vec{l}$$

(2), the charge is infinite and long-distributed Order is then the $\varphi_b = 0$

$$\varphi_a = \int_a^b \vec{E}\cdot \mathrm{d}\vec{l}$$

(3) In practical problems, the earth is often chosen to have zero electric potential.

pay attention to

The potential difference is absolute and independent of the choice of the zero point of the potential;

The potential size is relative and is related to the choice of the zero point of the potential.

♦ **Work of the electrostatic field force**

$$A_{ab} = q_0\left(\varphi_a - \varphi_b\right)$$

♦ **Unit: volt (V)**

Energy units in atomic physics:

$$1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J}$$

5.4.3, the calculation of the electric potential

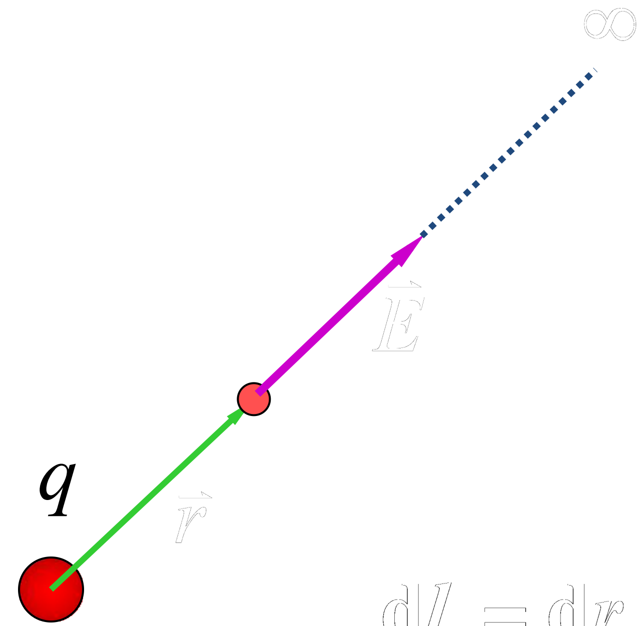
1. The electric potential of the point charge

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r$$

a surname $\varphi_\infty = 0$

$$\varphi = \int_r^\infty \vec{E}\cdot \mathrm{d}\vec{l} = \int_r^\infty E\,\mathrm{d}r$$

$$= \int_r^\infty \frac{q}{4\pi\varepsilon_0 r^2}\mathrm{d}r$$



$$\mathrm{d}l = \mathrm{d}r$$

$$\boxed{\varphi = \frac{q}{4\pi\varepsilon_0 r}}$$

$$\begin{cases} q > 0, & \varphi > 0 \\ q < 0, & \varphi < 0 \end{cases}$$

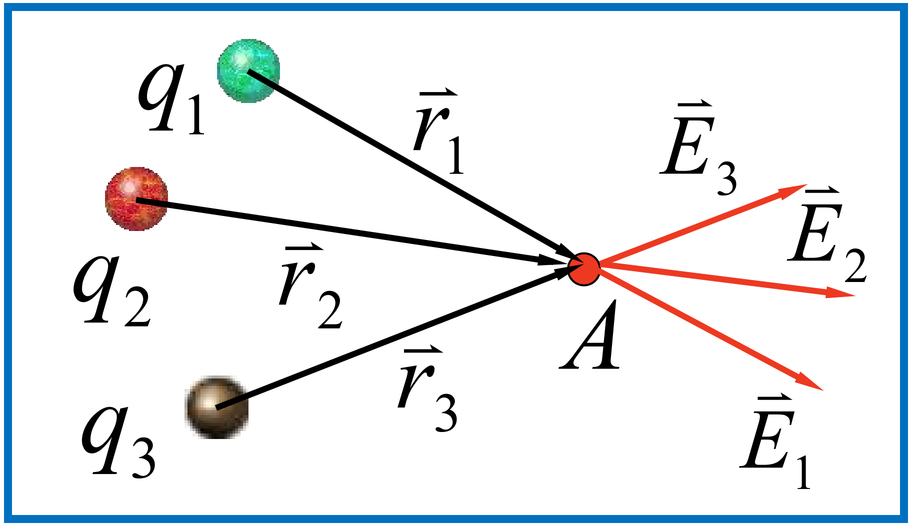
2. The electric potential of the point-charge system

$$\vec{E} = \sum_i \vec{E}_i$$

$$\varphi = \int_P^\infty \vec{E} \cdot \mathrm{d}\vec{l} = \sum_i \int_P^\infty \vec{E}_i \cdot \mathrm{d}\vec{l}$$

$$\boxed{\varphi = \sum_i \varphi_i = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}}$$

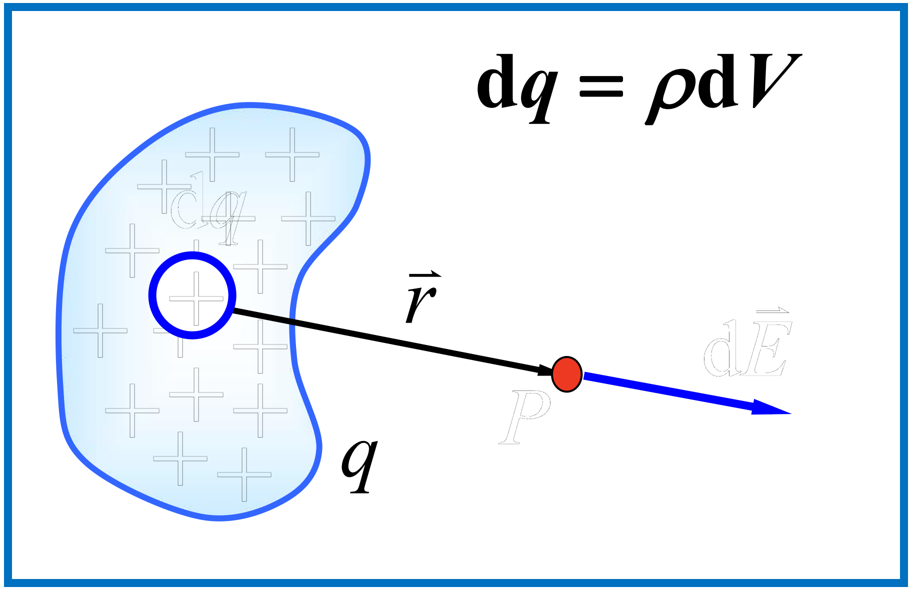
The superposition principle of the electric potential



3. The electric potential of a continuous charged body

$$\mathrm{d}\varphi = \frac{\mathrm{d}q}{4\pi\varepsilon_0 r}$$

$$\boxed{\varphi = \int \frac{\mathrm{d}q}{4\pi\varepsilon_0 r}}$$



4. The calculation of the electric potential

1) If the distribution function of the electric field E in the integral path is known,

By definition: $$\varphi = \int_P^{\infty} \vec{E} \cdot \mathrm{d}\vec{l}$$

Range: field strength with Gaussian theorem.

2) Use the point-charge potential

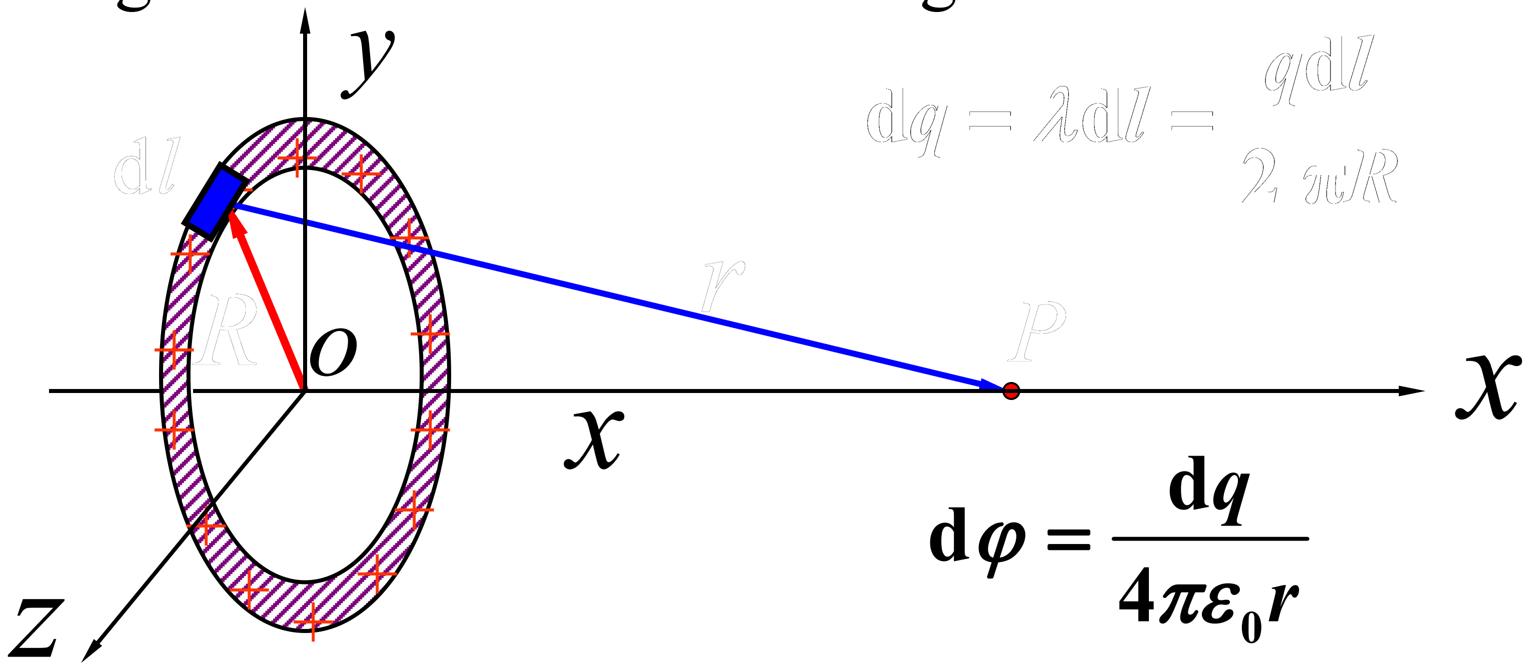
$$\varphi = \frac{q}{4\pi\varepsilon_0 r}$$

And the potential superposition principle

$$\varphi = \int \frac{\mathrm{d}q}{4\pi\varepsilon_0 r} \text{、} \quad \varphi = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}$$

Conditions: finite large charged body, choose infinite distance potential is zero.

Example 1 The positive charge q is uniformly distributed over a thin circle of radius R. Find the potential of the axis of the ring from the center of the ring at x.

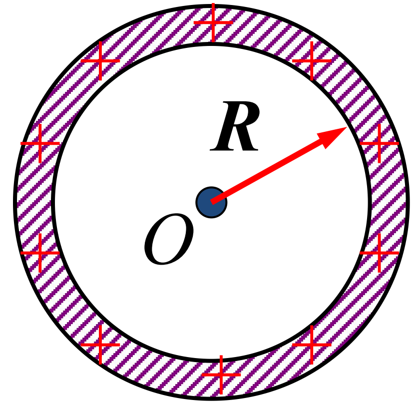


$$\mathrm{d}q = \lambda \mathrm{d}l = \frac{q\mathrm{d}l}{2\pi R}$$

$$\mathrm{d}\varphi = \frac{\mathrm{d}q}{4\pi\varepsilon_0 r}$$

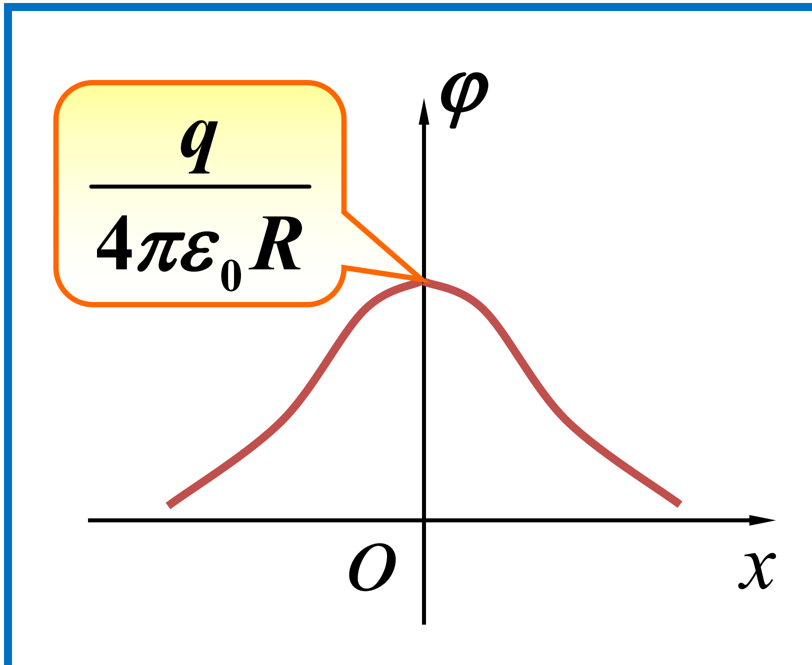
$$\varphi = \frac{1}{4\pi\varepsilon_0 r}\int \mathrm{d}q = \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

$$\boxed{\varphi = \frac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}}$$

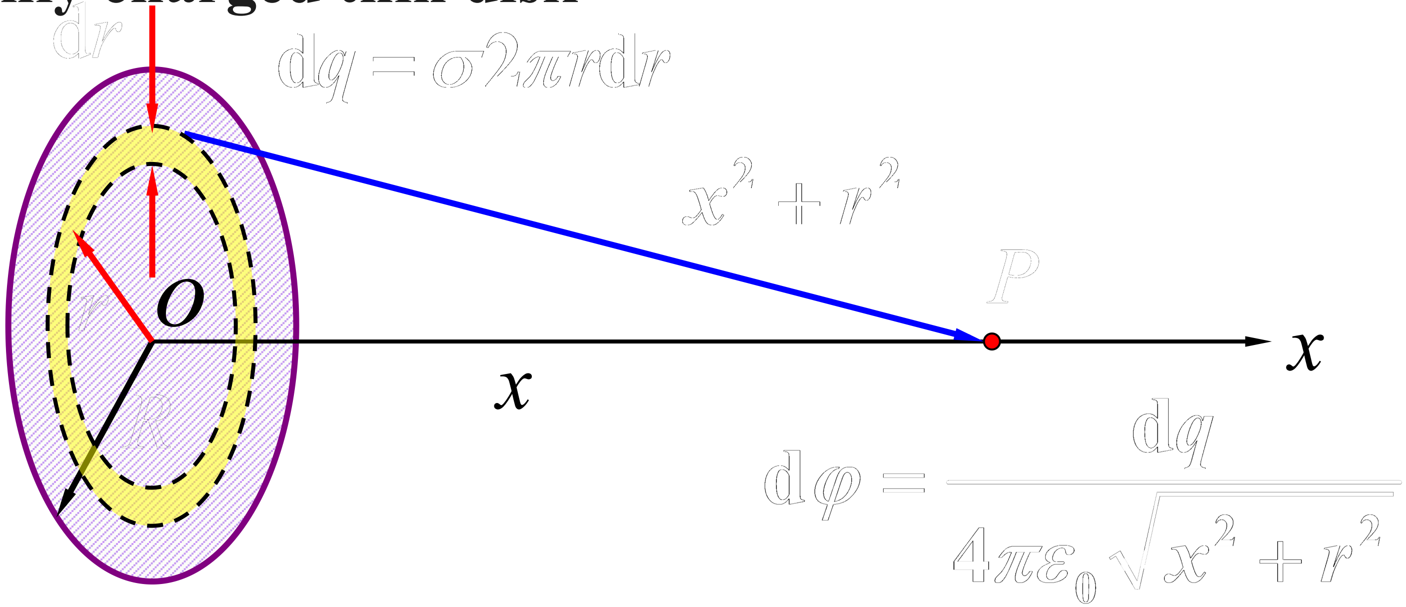


discuss

$$\begin{cases} x = 0, & \boxed{\varphi_0 = \dfrac{q}{4\pi\varepsilon_0 R}} \\ x >> R, & \varphi = \dfrac{q}{4\pi\varepsilon_0 x} \end{cases}$$



Example 2 The electric potential on the axis of a uniformly charged thin disk



$$\varphi = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma 2\pi r \mathrm{d}r}{\sqrt{x^2+r^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R^2} - x\right)$$

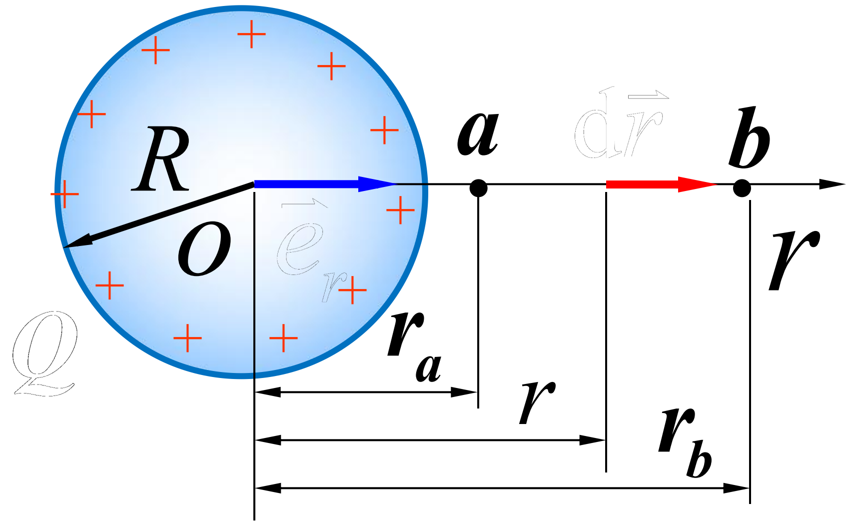
(Point-charge potential)

$$x \gg R \Rightarrow \sqrt{x^2+R^2} \approx x + \frac{R^2}{2x} \Rightarrow \varphi \approx Q/4\pi\varepsilon_0 x$$

Example 3 The electric potential of a uniformly charged spherical shell

In vacuum, there is a charged spherical shell with charged Q and radius R.

Try to find (1) the potential difference between two points outside the shell; (2) the potential difference between two points inside the shell; (3) the potential at any point outside the shell; (4) the potential at any point in the shell.



separate

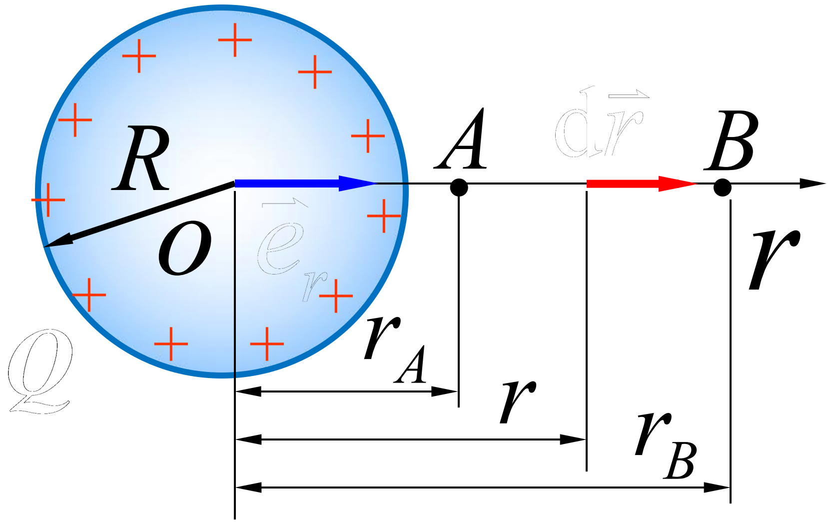
$$\begin{cases} r < R, & \vec{E}_1 = 0 \\ r > R, & \vec{E}_2 = \dfrac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r \end{cases}$$

(1) $$\varphi_a - \varphi_b = \int_{r_a}^{r_b} \vec{E}_2 \cdot \mathrm{d}\vec{r}$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{\mathrm{d}r}{r^2}\vec{e}_r \cdot \vec{e}_r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(2) $r < R$

$$\varphi_a - \varphi_b = \int_{r_a}^{r_b} \vec{E}_1 \cdot \mathrm{d}\vec{r} = 0$$



(3) $r > R$

a surname

$$r_b \to \infty, \quad \varphi_\infty = 0$$

cause

$$\varphi_a - \varphi_b = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

acquirability

$$\varphi_2(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

perhaps

$$\varphi_2(r) = \int_r^\infty \vec{E}_2 \cdot \mathrm{d}\vec{r} = \int_r^\infty \frac{Q}{4\pi\varepsilon_0 r^2}\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0 r}$$

(4) $r < R$

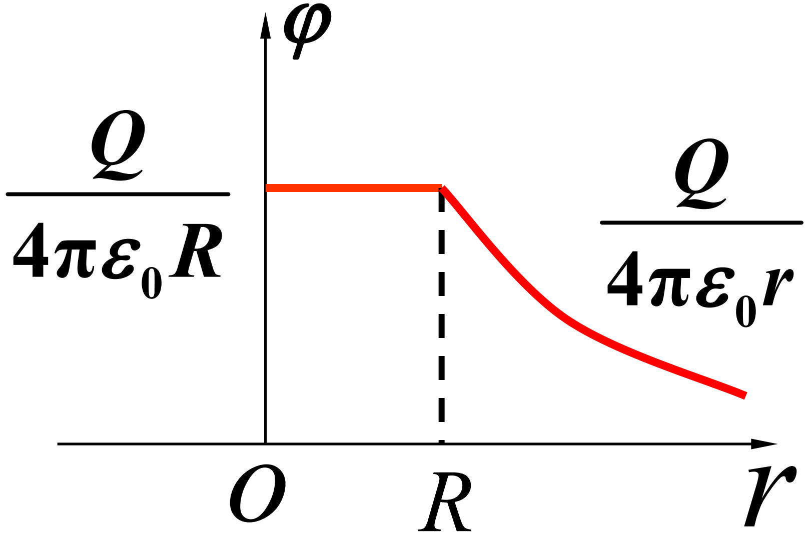
acquirability

cause $\varphi_2(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$ $\varphi(R) = \dfrac{Q}{4\pi\varepsilon_0 R} = \varphi_1$

perhaps

$$\varphi_1(r) = \int_r^R \vec{E}_1 \cdot \mathrm{d}\vec{r} + \int_R^\infty \vec{E}_2 \cdot \mathrm{d}\vec{r} = \frac{Q}{4\pi\varepsilon_0 R}$$

$$\begin{cases} \varphi_2(r) = \dfrac{Q}{4\pi\varepsilon_0 r} \\ \varphi_1(r) = \dfrac{Q}{4\pi\varepsilon_0 R} \end{cases}$$



Example 4 The potential of an "infinitely long" charged straight wire

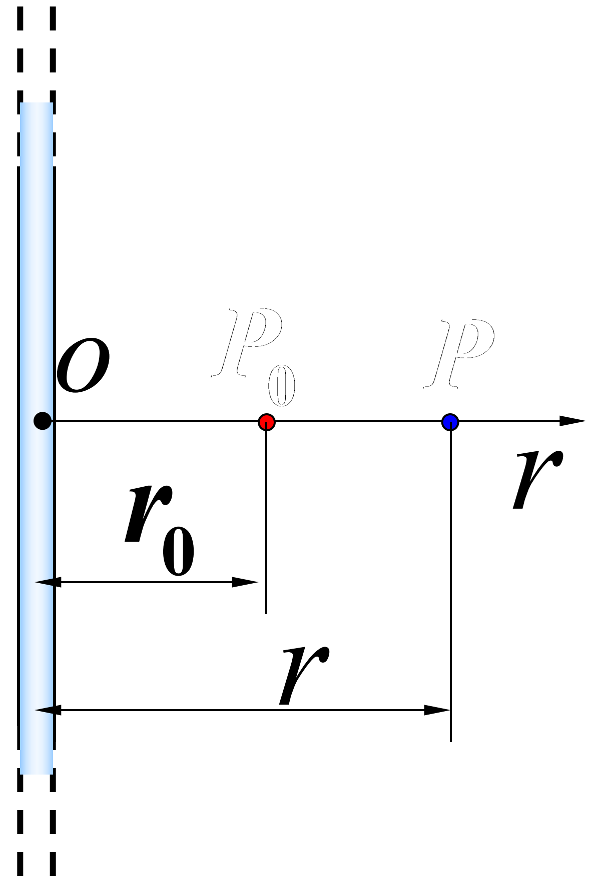
separate $\varphi_P = \int_P^{P_0} \vec{E} \cdot \mathrm{d}\vec{l} + \varphi_{P_0}$

a surname $\varphi_{P_0} = 0$

$$\varphi_P = \int_r^{r_0} \vec{E} \cdot \mathrm{d}\vec{r}$$

$$= \int_r^{r_0} \frac{\lambda}{2\pi\varepsilon_0 r} \vec{e}_r \cdot \mathrm{d}\vec{r}$$

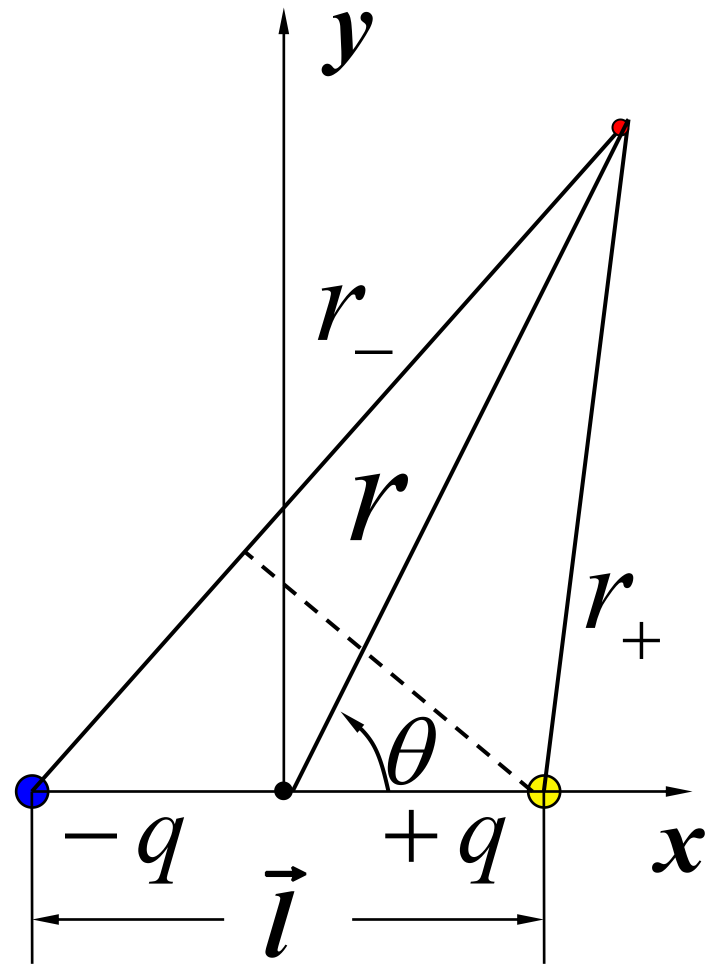
$$= \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_0}{r}$$



Can you choose $\varphi_\infty = 0$?

Example 5 Potential distribution of an electric dipole electric field.

$$\begin{cases} \varphi_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+} \\ \varphi_- = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_-} \end{cases} \qquad p = ql$$



$$\varphi = \varphi_+ + \varphi_- = \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_+ r_-}$$

$$\because l << r$$

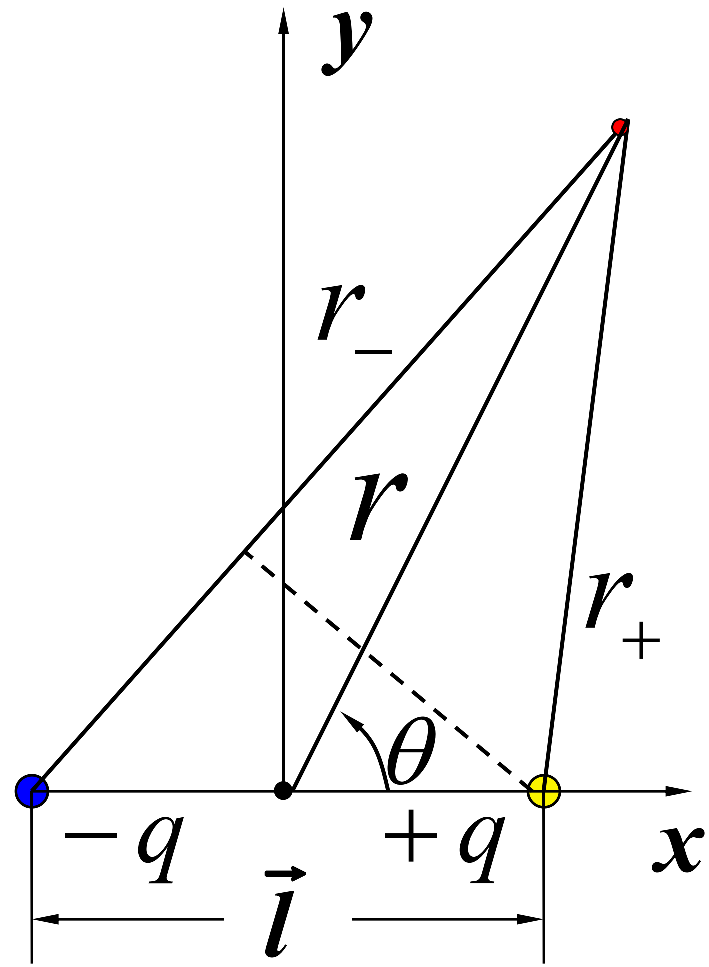
$$\therefore r_- - r_+ \approx l\cos\theta$$

$$r_- r_+ \approx r^2$$

$$\varphi=\varphi_{+}+\varphi_{-}=\frac{q}{4\pi\varepsilon_0}\frac{r_{-}-r_{+}}{r_{+}r_{-}}$$

$$\approx\frac{q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}=\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$



$$\begin{cases}\theta=0 & \varphi\approx\dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2}\\[2ex] \theta=\pi & \varphi\approx-\dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2}\\[2ex] \theta=\pi/2 & \varphi=0\end{cases}$$

5.4.4 Potential gradient

1. Equal potential surface (electric potential diagram method)

The surface formed by the joint connection is called the equal potential surface.

The potential difference is equal.

1. In the electrostatic field, when the charge moves along the equal potential surface, the electric field force does work

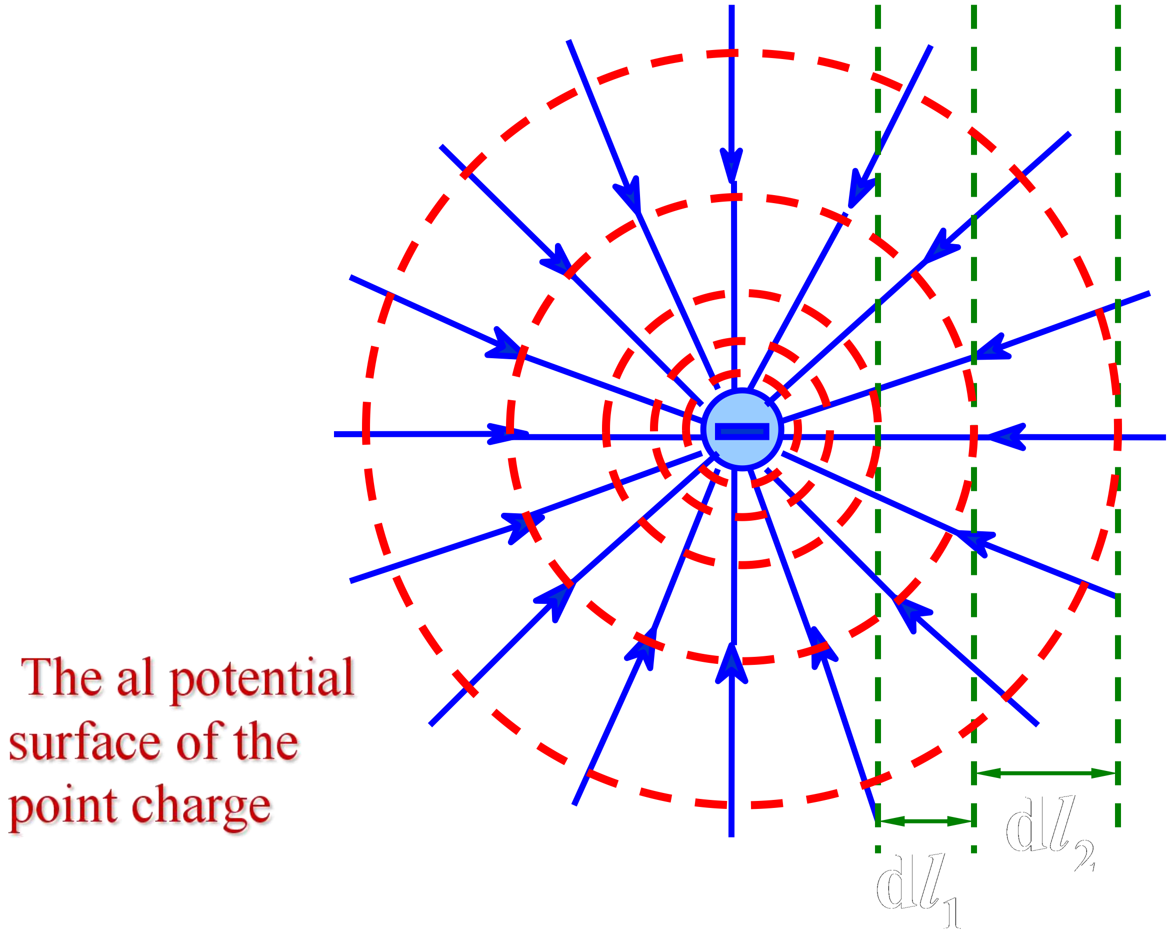
$$A_{ab} = q_0(\varphi_a - \phi_b) = \int_a^b q_0 \vec{E} \cdot \mathrm{d}\vec{l} = 0$$

2. In the electrostatic field, the electric field strength $\vec{E}$ is always perpendicular to the equipotential surface, that is, the electric field line is a curve cluster orthogonal to the equipotential surface.

$$A_{ab} = \int_a^b q_0 \vec{E} \cdot \mathrm{d}\vec{l} = 0$$

$$q_0 \neq 0 \quad \vec{E} \neq 0 \quad \mathrm{d}\vec{l} \neq 0 \qquad \boxed{\therefore \quad \vec{E} \perp \mathrm{d}\vec{l}}$$

♦ According to the regulation, the potential difference between any two adjacent isopotential surfaces in the electric field is equal, that is, the density of the isopotential surface can also represent the size of the field strength.

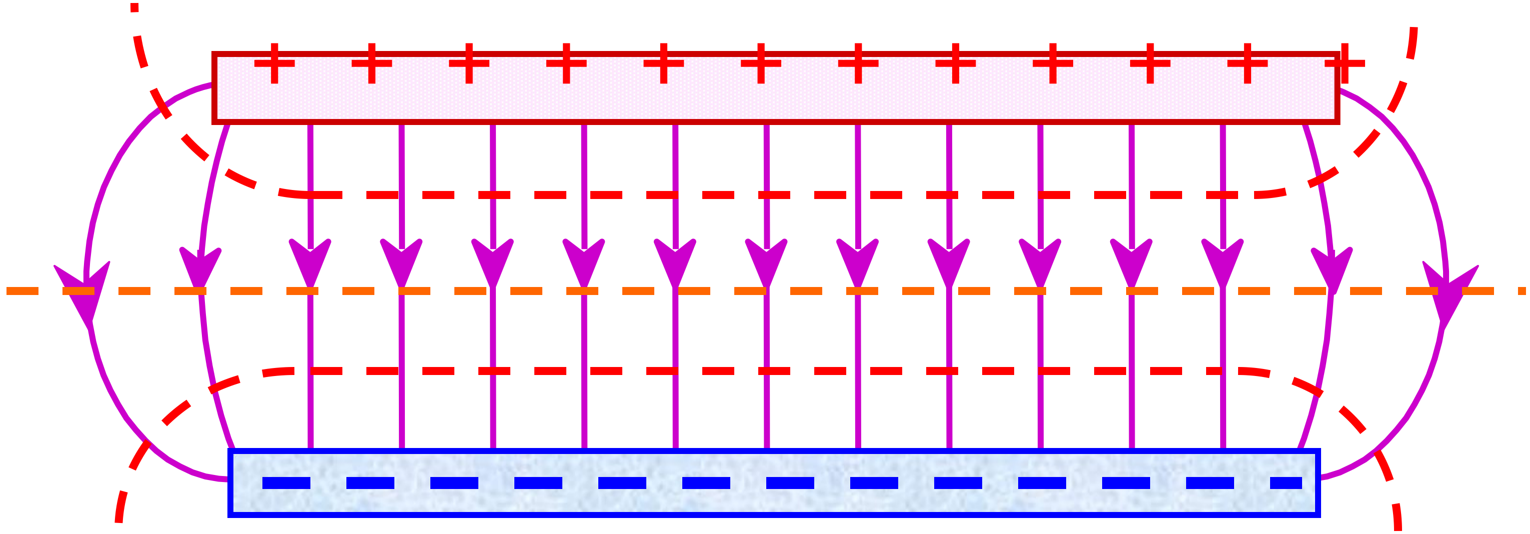


$$\varphi_a - \varphi_b = \int_a^b \vec{E} \cdot \mathrm{d}\vec{l}$$

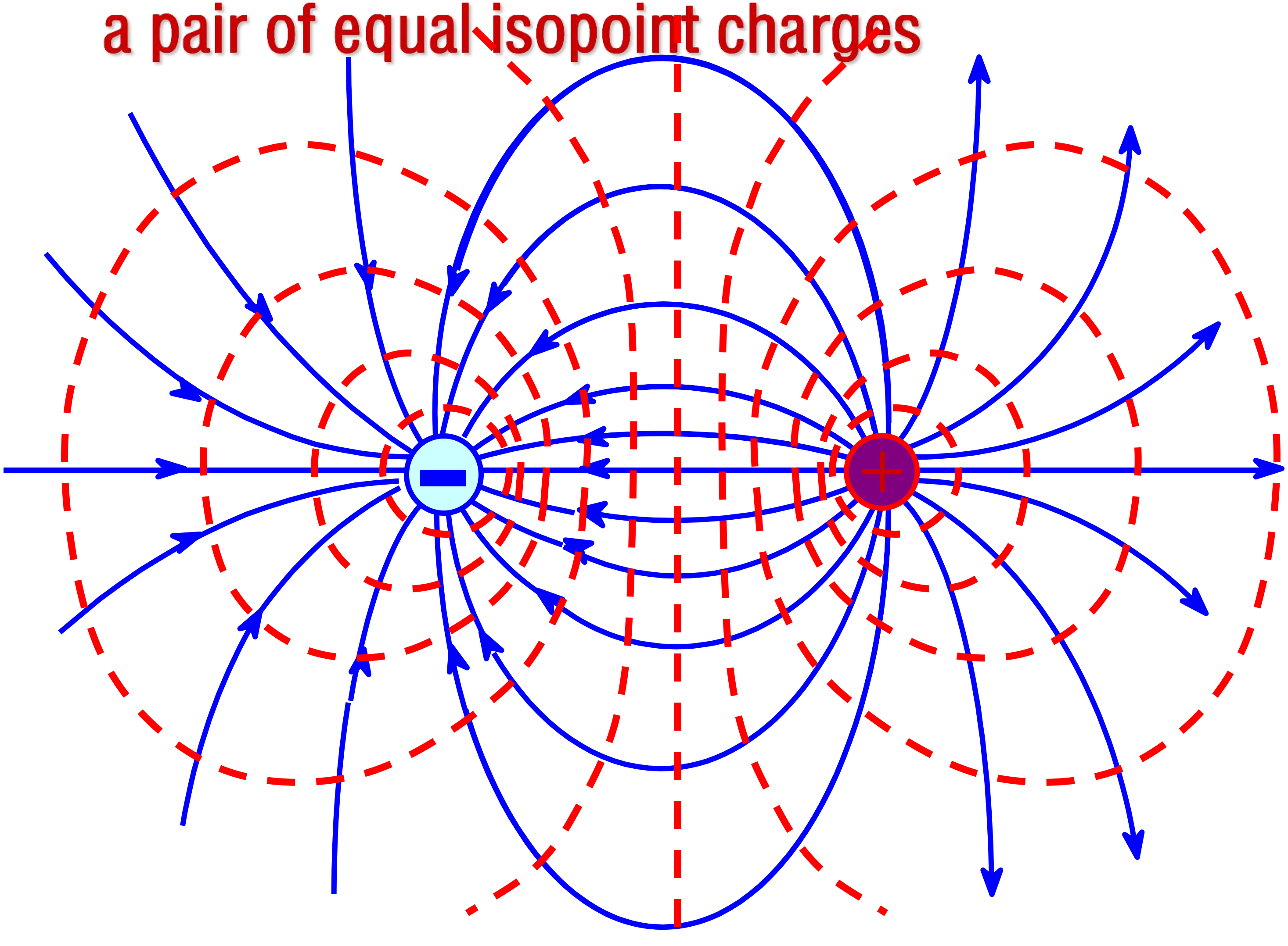
$$\mathrm{d}l_2 > \mathrm{d}l_1$$

$$\Rightarrow E_2 < E_1$$

The electric field line and the equipotential surface of two parallel charged plates



Electric field lines and equipotential surfaces of a pair of equal isopoint charges



2. Electric field strength and the electric potential gradient

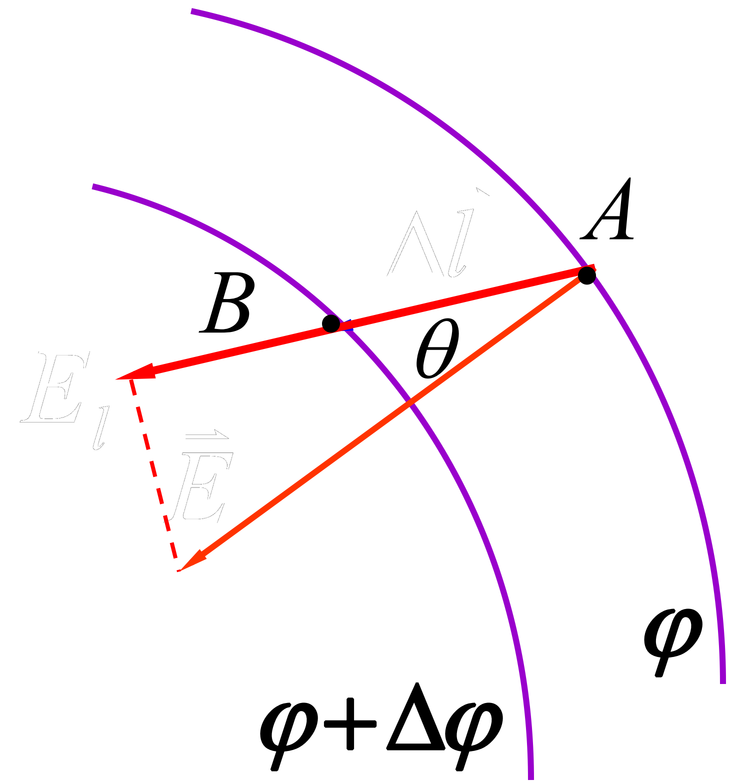
$$A = \vec{E} \cdot \Delta \vec{l} = E \cos\theta \Delta l$$

$$= \varphi - (\varphi + \Delta\varphi) = -\Delta\varphi$$

$$E \cos\theta = E_l$$

$$-\Delta\varphi = E_l \Delta l, \quad E_l = -\frac{\Delta\varphi}{\Delta l}$$

$$E_l = -\lim_{\Delta l \to 0} \frac{\Delta\varphi}{\Delta l} = -\frac{d\varphi}{dl}$$

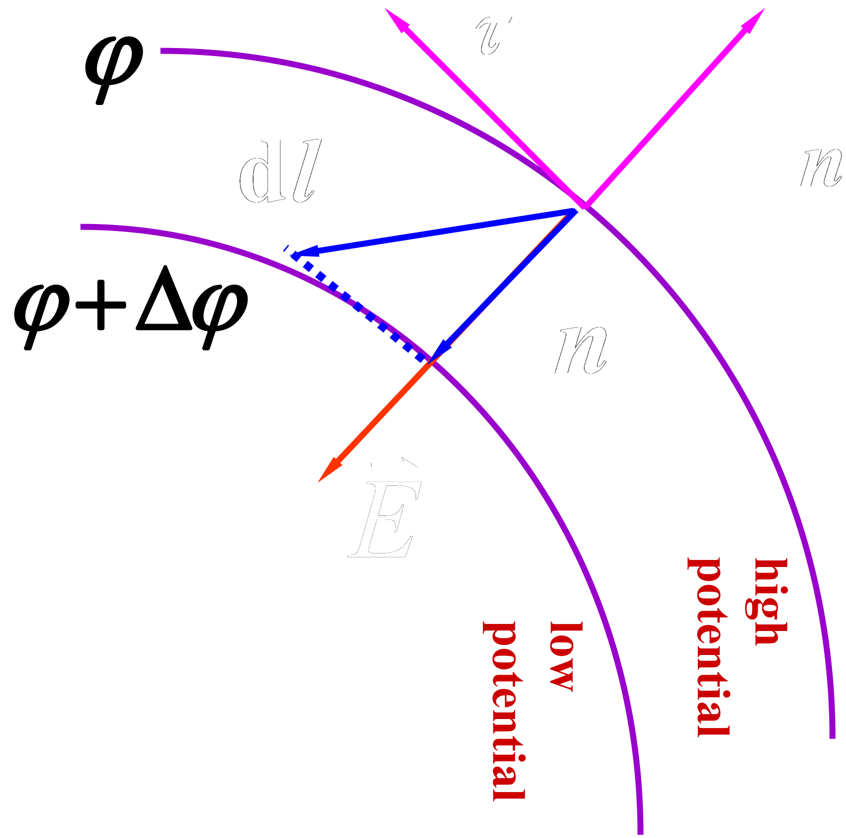


The strength of the electric field at a certain point along a certain direction is equal to the negative value of the rate of change of the electric potential along the unit length of that direction.

$$E_l = -\frac{\mathrm{d}\varphi}{\mathrm{d}l} \qquad E_\mathrm{n} = -\frac{\mathrm{d}\varphi}{\mathrm{d}n}$$

$$\because \mathrm{d}l > \mathrm{d}n \qquad \therefore E_\mathrm{n} > E_l$$

$$\vec{E} = -\frac{\mathrm{d}\varphi}{\mathrm{d}n}\vec{e}_\mathrm{n}$$



$$\left|\vec{E}\right| = \left|\frac{\mathrm{d}\varphi}{\mathrm{d}n}\right|$$

big or small

The direction $\vec{e}_\mathrm{n}$ is opposite, from the high potential to the low potential

Physical significance

(1) The intensity of the electric field at a point in space depends on the spatial variation rate of the potential in the field.

(2) The direction of the electric field strength, the constant index refers to the direction of the electric potential landing.

- In the rectangular coordinate system

$$\vec{E} = -\left(\frac{\partial \varphi}{\partial x}\vec{i} + \frac{\partial \varphi}{\partial y}\vec{j} + \frac{\partial \varphi}{\partial z}\vec{k}\right) = -\mathbf{grad}\,\varphi$$

nabla

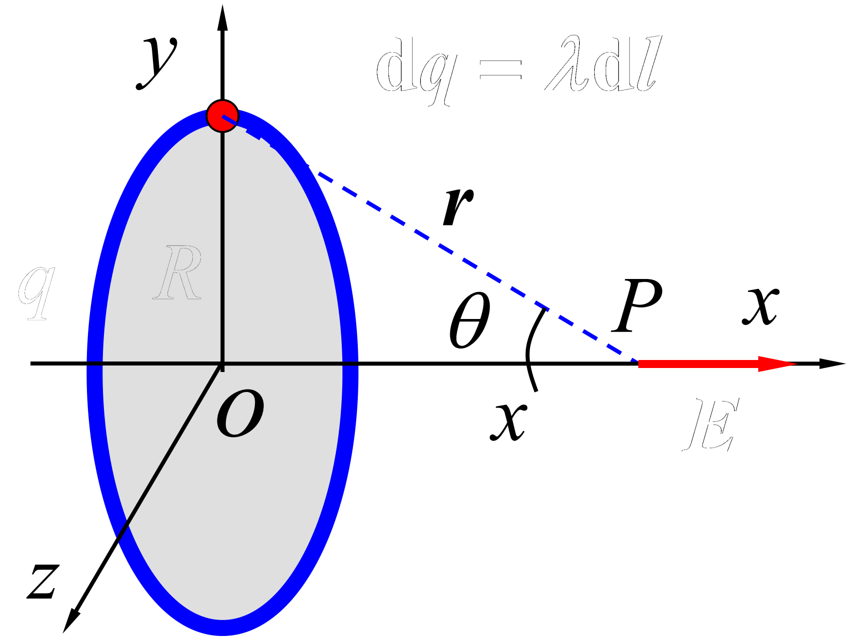
$$\vec{E} = -\nabla \varphi$$ **(potential gradient)**

- Provide a new way to find the electric field strength

The three methods sought:

- Using the superposition principle of the electric field strength
- **Using the Gaussian theorem**
- **Using the potential and electric field strength** *

Example 1*Find the electric field strength of the uniform charged fine circular ring axis.



$$\vec{E} = -\nabla \varphi$$

$$\varphi = \frac{q}{4\pi\varepsilon_0 (x^2 + R^2)^{1/2}}$$

$$E = E_x = -\frac{\partial \varphi}{\partial x}$$

$$= -\frac{\partial}{\partial x}\left[\frac{q}{4\pi\varepsilon_0 (x^2 + R^2)^{1/2}}\right]$$

$$= \frac{qx}{4\pi\varepsilon_0 (x^2 + R^2)^{3/2}}$$

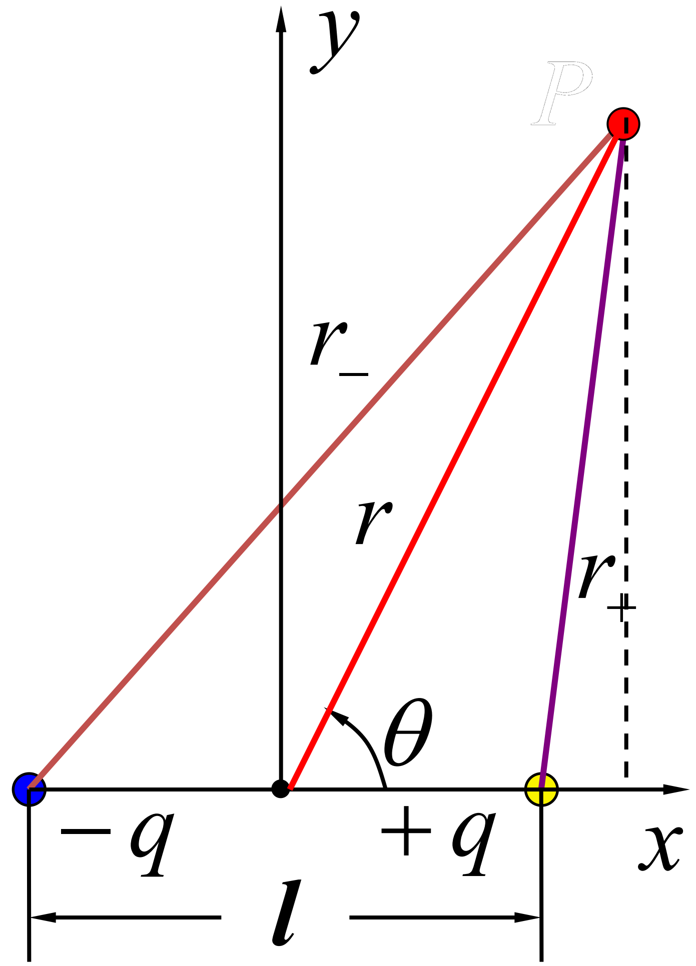
Example 2 * uses the field strength and the potential gradient to calculate the electric field strength distribution of the electric dipole.

$$\varphi = \frac{px}{4\pi\varepsilon_0 (x^2 + y^2)^{3/2}}$$

$$E_x = -\frac{\partial \varphi}{\partial x}$$

$$= -\frac{p}{4\pi\varepsilon_0} \frac{y^2 - 2x^2}{(x^2 + y^2)^{5/2}}$$

$$E_y = -\frac{\partial \varphi}{\partial y} = \frac{p}{4\pi\varepsilon_0} \frac{3xy}{(x^2 + y^2)^{5/2}}$$



$$\begin{cases} E_x = -\dfrac{p}{4\pi\varepsilon_0}\dfrac{y^2-2x^2}{(x^2+y^2)^{5/2}} \\ E_y = \dfrac{p}{4\pi\varepsilon_0}\dfrac{3xy}{(x^2+y^2)^{5/2}} \end{cases}$$

$$E = \sqrt{E_x^2 + E_y^2}$$

$$= \frac{p}{4\,\pi\varepsilon_0}\frac{(4x^2+y^2)^{1/2}}{(x^2+y^2)^2}$$

$$\begin{cases} y = 0 \qquad E = \dfrac{2p}{4\pi\varepsilon_0}\dfrac{1}{x^3} \\ x = 0 \qquad E = \dfrac{p}{4\pi\varepsilon_0}\dfrac{1}{y^3} \end{cases}$$