

Change in Voltage

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1 Introduction to Magnetism

- The Magnetic field is similar to a electric dipole field.

2 Notation

- Into the page : \times
- Out of the page : \bullet

3 Field Strength

$$B \propto \frac{1}{r^2} \quad (3.1)$$

4 Biot-Savart Law

$$B_{PC} = \frac{\mu_o}{4\pi} \frac{qv \sin \theta}{r^2} \quad (4.1)$$

Where

$$\mu_0 = 4\pi \times 10^{-7} Tm/A$$

q = Charge (C)

v = Velocity of Charge

θ = Angle between velocity vector and position vector

4.1 Magnetic Field For A Long Current Carrying wire

$$B_{PC} = \frac{\mu_o}{2\pi} \frac{I}{d} \quad (4.2)$$

4.2 Magnetic Field along a current loop axis

$$B_{PC} = \frac{\mu_o}{2} \frac{IR^2}{(z^2 + R^2)^{\frac{3}{2}}} \quad (4.3)$$

4.2.1 If $z=0$

$$B_{PC} = \frac{\mu_o}{2} \frac{I}{R^{\frac{3}{2}}} \quad (4.4)$$

If There is N Rings:

$$B_{PC} = \frac{\mu_o}{2} \frac{NIR^2}{(z^2 + R^2)^{\frac{3}{2}}} \quad (4.5)$$

5 Symmetry And Ampere's Law

They symmetry between Gauss's Law and Gauss's law:

$$\Phi_e = \oint E dA = \frac{Q_{in}}{\epsilon_0} \quad (5.1)$$

Line Integral

$$\oint T ds = n_1 ds_1 + n_2 ds_1 \quad (5.2)$$

Ampere's Law

When Magnetic Field is perpendicular to the Line

$$\vec{B} \cdot d\vec{s} = 0 \quad (5.3)$$

When the Magnetic Field is parallel to the Line

$$\oint \vec{B} \cdot d\vec{s} = Bl = B(2\pi d) \quad (5.4)$$

If we combine it with Biot-Savart Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \quad (5.5)$$

5.1 Ampere's Law Inside a Wire

In here we assume constant current density.

Ratio of Area:

$$\frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2} \quad (5.6)$$

Hence:

$$\oint \vec{B} \cdot d\vec{s} = \frac{\mu_0 r^2 I}{R^2} \quad (5.7)$$

Replace Line Integral

$$2\pi r B = \frac{\mu_0 r^2 I}{R^2} \quad (5.8)$$

Rearrangement results in:

$$B = \frac{\mu_0 I}{2\pi R^2} r \quad (5.9)$$

6 Solenoid

The Solenoid should have a diameter which is fair smaller than the length of the coil. They have such properties:

- The magnetic field inside a solenoid is uniform.
- Infinitely Long

6.1 Ampere's Law

From Biot-Savart Law:

$$\frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2} \quad (6.1)$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \quad (6.2)$$

6.2 Ampere's Law Inside a Wire

In here we assume constant current density.

Ratio of Area:

$$\frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2} \quad (6.3)$$

Hence:

$$\oint \vec{B} \cdot d\vec{s} = \frac{\mu_0 r^2 I}{R^2} \quad (6.4)$$

Replace Line Integral

$$2\pi r B = \frac{\mu_0 r^2 I}{R^2} \quad (6.5)$$

Rearrangement results in:

$$B = \frac{\mu_0 I}{2\pi R^2} r \quad (6.6)$$

7 Solenoid

The Solenoid should have a diameter which is fair smaller than the length of the coil. They have such properties:

- The magnetic field inside a solenoid is uniform.
- Infinitely Long

7.1 Ampere's Law

From Biot-Savart Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \quad (7.1)$$

Replace with line Integral:

$$Bl = \mu_0 NI \quad (7.2)$$

Hence:

$$B = \frac{\mu_0 NI}{l} = \mu_0 n I \quad (7.3)$$

n is the number of turns per unit length

7.2 Doughnut Shape

$$B = \frac{\mu_0 NI}{2\pi r} \quad (7.4)$$

8 Frequency of a Cyclotron

$$f = \frac{qB}{2\pi m} \quad (8.1)$$

9 Hall Effect

$$\Delta V_H = \frac{IB}{tne} \quad (9.1)$$

n = charge density. e = charge of an Electron.

10 Force on Wire Due To Magnetic Field

$$F = BIL \frac{\mu_0 I_1 I_2}{2\pi d}$$

11 Magnetic Moment

$$IA = Il2 = \mu \quad (11.1)$$

12 Torque due to Magnetic Field

$$\vec{\tau} \times \vec{B} = \mu B \sin \theta \quad (12.1)$$

13 Electromagnetic Induction

14 Eddy Currents

An current loop with a hole present acts like a capacitor with charges on either end of the hole. There IS Indeed an EMF even though no current will flow.

15 Faraday's Law

$$\varepsilon = \left| \frac{d\Phi_b}{dt} \right| = \left| \vec{B} \cdot \frac{d\vec{A}}{dt} + \vec{A} \cdot \frac{d\vec{B}}{dt} \right| \quad (15.1)$$

The direction of current is defined by Lenz's Law.

16 Induced Electric Field

If we divide the equation above into two parts. We can see that we have induced Electric-field which is non-Coulombic.

This is generated by a **changing** magnetic field. Such force will still exert the force $F = qe$ on an charge. (Same as Coulombic Electric Field)

We know that :

$$W = q \oint \vec{E} \cdot d\vec{s} \quad (16.1)$$

Hence we can write that:

$$\varepsilon = \oint \vec{E} \cdot d\vec{s} \quad (16.2)$$

We can also conclude that:

$$\oint \vec{E} \cdot d\vec{s} = A \left| \frac{d\vec{B}}{dt} \right| \quad (16.3)$$

$$(16.4)$$

If we have the pendulum we will have a Lorentz current which causes a Force.

17 Modified Faraday's Law

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad (17.1)$$

Where $ds = 2\pi r$ if the line is a circle.

18 Application of Faraday's Law

18.1 AC generator

18.1.1 EMF

$$\varepsilon = -N \frac{d\Phi_B}{dt} = -N \frac{dBA \cos(\omega t)}{dt} = NBA \sin(\omega t) \quad (18.1)$$

18.2 Transformer

It does not work with DC (DC current only causes a constant Magnetic Field.

Ratio:

$$\frac{V_1}{V_2} = N_1 N_2 \quad (18.2)$$

Current (Energy must be conserved):

$$V_1 I_1 = V_2 I_2 \quad (18.3)$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} \quad (18.4)$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1} \quad (18.5)$$

18.3 Metal Detector

The Transmitter coil causes a current in the receiver coil, but the metal can reduce the current in the receiver coil due to eddy currents.

18.4 Inductor

EMF:

$$V = -L \frac{dI}{dt} \quad (18.6)$$

Inductance

$$L = \frac{\mu_0 N^2 A}{l} \quad (18.7)$$

The negative signs indicates that it opposes the change in current.

18.5 Maxwell's Equations

Gauss's Law for Magnetism

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad (18.8)$$

Ampere-Maxwell's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} \quad (18.9)$$

Lorentz's Force Law

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \quad (18.10)$$

19 Electromagnetic Waves

An oscillating charge will create a changing magnetic and electric field which generates an EM Wave.

Poynting vector:

$$|\vec{S}| = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{EB}{\mu_0} \quad (19.1)$$

Hence

$$\frac{1}{2} \epsilon_0 E^2 = \frac{1}{2 \mu_0} B^2 \quad (19.2)$$