Exercise 3. MLS For Curves, Meshes and Images

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MATLAB R2016b version was used for coding and testing:

MathWorks, MATLAB R2016b (9.1.0.441655)
64-bit (maci64)

The ***code*** directory contains followings:

- ***main.m*** Script .m file for exercise.
- ***functions dir*** Function .m files
  - **SolveWithLP** Function .m file for solving a problem with the LP and simple testing.
  - **SplitProblem** Function .m file for split a parent problem into two children problems.
  - **FindBestCandidate** Function .m file for finding the best candidate between two children node.
  - **PushToStack** Function .m file for stack push operation.
  - **PopFromStack** Function .m file for stack pop operation.
  - **NewProblem** Function .m file for creating new problem (instance of struct).
  - **FindInliers** Function .m file for finding inliers with a input model.
  - and other sub-functions: **VisualizeMatch**, **SaveOptHistory**, **ComputeInlierLb**
- ***data dir*** data and image files provided.

For running exercise, adjust threshold parameter (default of 3 pixel) and run the ***main.m*** on the MATLAB environment. More details are stated in the *Running* section.

1 EXERCISE PART 1: CURVE AND SURFACE RECONSTRUCTION USING MLS

TODO In this exercise, branch and bound algorithm based on depth-first search(DFS) was implemented. The implementation steps are as follows:

- derivation of the equation of the MLS based surfaces
- evaluation and plotting of the derived $f(x)$
- logging and plotting the result of branch and bound algorithm

1.1 CURVE DERIVATION

1.2 CURVES PLOTTING

1.2.1 DESCRIPTION

1.2.2 RUNNING

1.2.3 RESULT

1.3 SMOOTHING MESHES

1.3.1 DESCRIPTION

1.3.2 RUNNING

1.3.3 RESULT

1.4 SPEEDING UP MESH SMOOTHING

1.4.1 DESCRIPTION

`https://github.com/jefferislab/MatlabSupport/tree/master/ann_wrapper`

1.4.2 RUNNING

1.4.3 RESULT

2 EXERCISE PART 2: IMAGE DEFORMATION USING MOVING LEAST SQUARES

2.0.1 BRANCH AND BOUND

Implemented branch and bound algorithm for finding optimal solution of the consensus set maximization problem. The depth-first search based on stack data structure was used. The stack has two operation **PopFromStack** and **PushToStack**.

The pseudo-code of the implementation is as follows:

Algorithm 1 Branch and bound

```
P0 ← whole problem space
init. opt ← variable for optimal solution (bound of inliers)
init. stack
push(stack, P0)

while stack is not empty do
    Parent = pop(stack)
    if Parent.ObjUpperBound < opt.LowerBound then
        continue; ← bad bound. does not contain optimum for sure
    end if

    if Parent.ObjLowerBound ≥ opt.LowerBound then
        opt = (Parent.ObjUpperBound, Parent.ObjLowerBound)
    end if

    if Parent.ObjUpperBound − Parent.ObjLowerBound < 1 then
        continue; ← Parent is a leaf node thus do not split
    end if

    (LeftChild, RightChild) = split(Parent)
    solveLP(LeftChild, RightChild) ← solve both problems by LP and test

    (BetterChild, WorseChild) = findbetter(LeftChild, RightChild)
    push(stack, WorseChild)
    push(stack, BetterChild) ← BetterChild should be on the top of stack
end while

opt ← now optimal solution
```

The several criteria/strategies of implementation were defined as follows

1. push the original problem $P0$ to the problem stack.
2. start iteration. The iteration of branch and bound terminates when the problem stack is empty
3. bad bound check criteria:
   - m^* is the highest lower bound of the number of inliers obtained so far.
   - if the upper cardinality bound of a problem $< m^*$ then there's no optimum for sure.
   - pop the problem from the stack without splitting.
4. optimal solution update criteria :
   - if the lower cardinality bound of a problem lb is $lb \geq$ the lower cardinality bound of the optimal solution found so far, then update the optimal solution as lb.
5. convergence criteria: if the lower and upper cardinality bound are nearer than 1, stop split.
   - iteration can be terminated because the cardinality bound converged to optimum.
   - but here, to check whether obtained optimum is indeed global optimum, do not terminate iteration but check remaining problems by continuing depth-first search.
6. split the problem space and branching: splitting in half along the longest dimension.
7. solve children problems and obtain the cardinality bound by the LP and simple test
   - compute the upper bound of the number of inliers: the LP with the formulation of (1.1.1) was exploited.
   - compute the lower bound of the number of inliers: simple test with the model Θ obtained by LP. Details are stated below.
8. after solving children problems, push *worse* child problem to the stack first, and then push *better* child:
   - *better* child is a problem with a larger lower cardinality bound. If two children have the same lower cardinality bound, then choose one with a larger upper cardinality bound.
   - as pushing two children, top of the stack is the *better* child now. Thus the *better* child will be popped in the next iteration step.
   - this strategy is for exploring *better* problem first for reducing running time.
9. iteration terminates with the global optimal cardinality bound.

SOLVING A PROBLEM The problem solving step is getting upper bound and lower bound of the number of inliers. Given a certain problem, solves the problem with the LP defined in (1.1.1). Then, the optimal solution of the relaxed problem (T_x^*, T_y^*) and the objective cost $c^T\mathbf{x}^*$ are obtained from:

$$\mathbf{x}^* = (T_x^*, T_y^*, z_1^*, \dots, z_n^*, w_{1x}^*, \dots, w_{nx}^*, w_{1y}^*, \dots, w_{ny}^*)^T \tag{2.1}$$

$$c^T\mathbf{x}^* = -z_1^* - z_2^* - \cdots - z_n^* \tag{2.2}$$

The (-)objective cost, $-c^T\mathbf{x}^*$ can be set as the upper bound of inliers since the the number of inliers of that problem space never exceeds $-c^T\mathbf{x}^*$.

Besides, the lower bound of inliers can be obtained by simple test with (T_x^*, T_y^*) and is set as the $card(S_I)$ as follows:

$$\begin{aligned} & card(S_I) \\ s.t. \quad & \left| x_i + T_x^* - x_i' \right| \le \delta, \forall i \in S_I \subseteq S \\ & \left| y_i + T_y^* - y_i' \right| \le \delta, \forall i \in S_I \subseteq S \end{aligned} \tag{2.3}$$

This was implemented as the function **SolveWithLP**.

2.1 RUNNING

Run the script ***main.m*** after setting the parameters.

- *threshold* : threshold for inliers. Default is 3 (px).
- *padding* : the width of black padding between left and right image for figure(1). Default is 10 (px).

2.2 RESULT

The optimal solution (T_x^*, T_y^*) found by branch and bound for the given problem is as follows:

Table 2.1: The global optimal solution obtained by branch and bound

T_x^*	T_y^*	cardinality bound
-232.00	-156.81	(15, 15.449)

Inlier indices are $(3, 8, 9, 15, 16, 20, 26, 31, 32, 34, 35, 40, 42, 45, 51)$. The identified inliers and plot of cardinality bounds are as follows. The red lines indicate outliers and the green lines indicate inliers:

Figure 2.1: The identified inlier and outlier correspondences

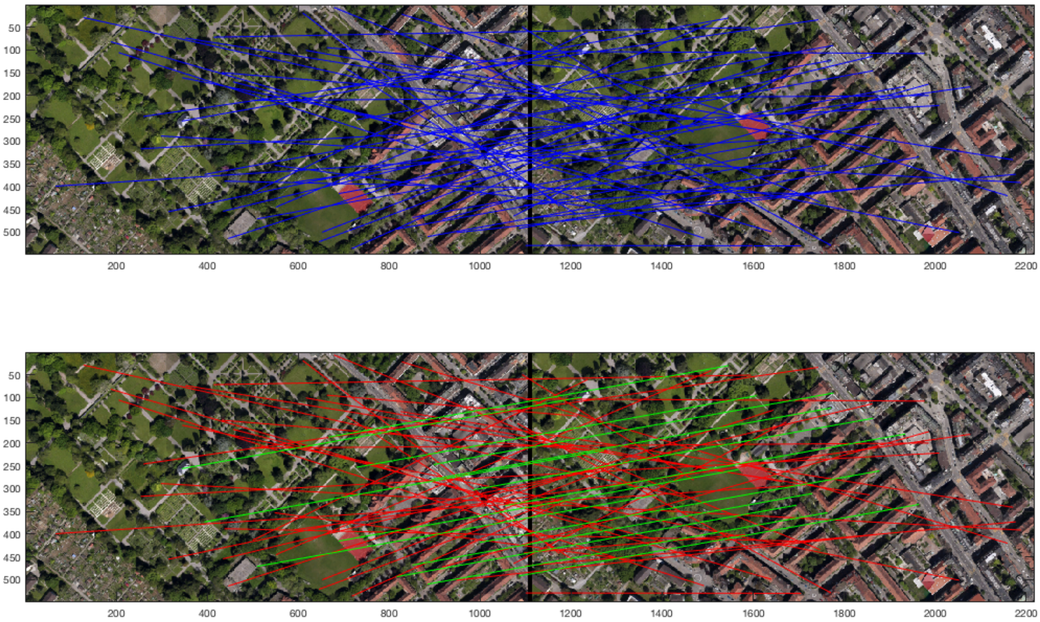
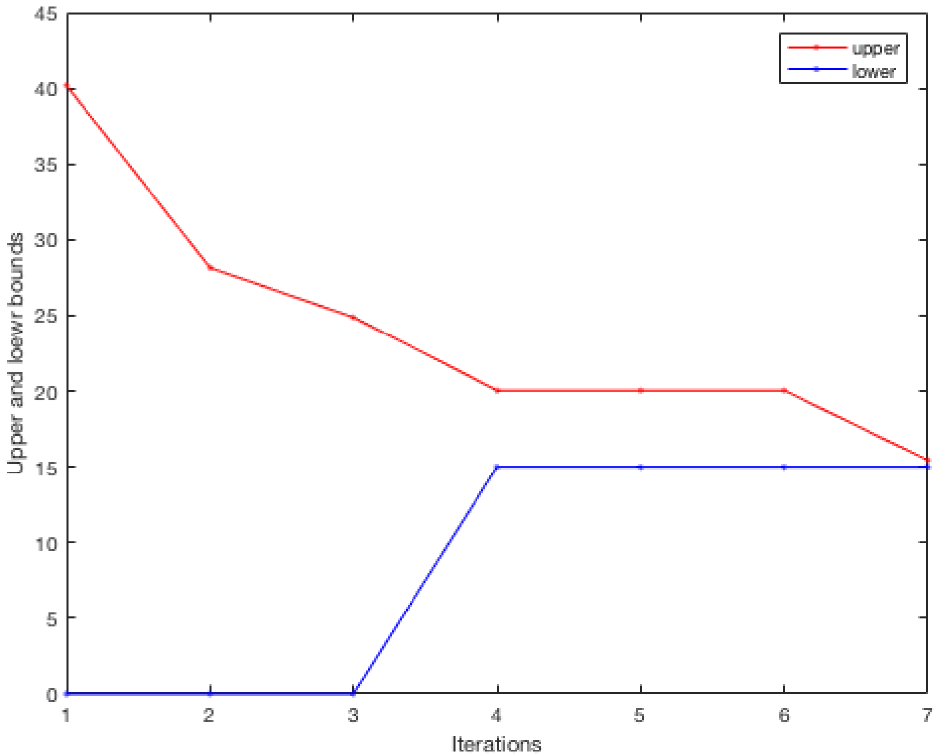


Figure 2.2: The convergence of the cardinality bounds



The branch and bound iteration can be described as the following binary tree illustration:

Figure 2.3: An illustration of DFS tree: one step before exploring $P11$

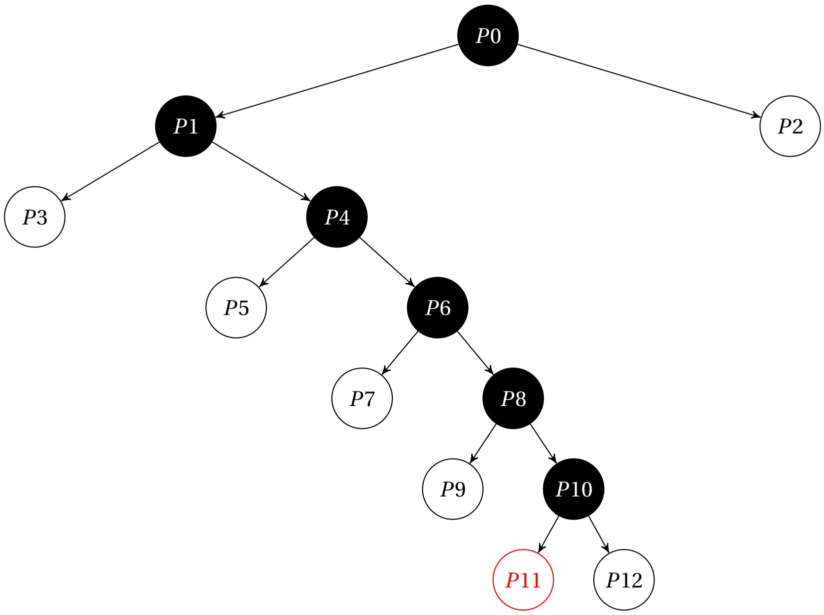


Figure 2.4: An illustration of DFS stack: one step before popping $P11$

P2	P3	P5	P7	P9	P12	P11

Figure 2.5: Problems

	Θ_{Lb}	Θ_{Ub}	Θ_{Opt}	ObjLb	ObjUb
P0	(-1104, -549)	(1104, 549)	(-140.94, -106.08)	0	40.17
P1	(-1104, -549)	(0, 549)	(-336.28, -74.21)	0	28.15
P2	(0, -549)	(1104, 549)	(432.47, 25.20)	0	11.52
P3	(-1104, -549)	(-552, 549)	(-665.11, -66.06)	0	5.05
P4	(-552, -549)	(0, 549)	(-238.00, -154.00)	0	24.88
P5	(-552, 0)	(0,549)	(-239.45, 198.5954)	0	5.56
P6	(-552, -549)	(-552, -549)	(0, 0)	15	20.04
P7	(-552, -549)	(-276, 0)	(-436.00, -259.00)	0	2.88
P8	(-276, -549)	(0, 0)	(-232.00, -158.18)	0	16.46
P9	(-276, -549)	(0, -275)	(-154.00, -401.97)	0	1.53
P10	(-276, -274)	(0, 0)	(-232.00, -156.94)	0	15.89
P11	(-276, -274)	(-138, 0)	(-232.00, -156.81)	15	15.44
P12	(-138, -274)	(0, 0)	(-38.00, -99.03)	0	1.37

As popping $P11$ from the stack, optimal solution is obtained but, since $P2, P3, P5, P7, P8, P12$ are still in the stack, algorithm does not terminate. However, These problems have *bad bound* i.e. have smaller ObjUb (upper bound of the card.) than $P11$'s ObjLb (lower bound of the cardinality), and cannot be a optimal for sure, thus are popped without splitting and the algorithm terminates.

2.3 Discussion

Some comments for the bound of cardinality:

- The lower bound of cardinality for optimal solution monotonically increases.
- The upper bound of cardinality for optimal solution not necessarily monotonically decreases.
  - if the first convergence (local optimal solution) is not the global optimal solution, then the upper bound of cardinality can be increased as continuing depth-first search.
- The global optimal solution is obtained by branch and bound algorithm.