

CS320 Programming Languages

Homework #8

Due : 13 May 2019 before 2:30PM

Student id: _____ Name: _____

Draw type derivations of the following expressions.

1. $\{\{\text{fun } \{x:\text{num } y:\text{num}\} \{+ x y\}\} 8\}$
2. $\{\{\text{if false } \{\text{fun } \{x:\text{num}\} x\} \{\text{fun } \{y:\text{num}\} 2\}\} 57\}$
3. $\{\{\text{fun } \{x:\text{num } y:(\text{num} \rightarrow \text{bool})\} \{y \{+ x 17\}\}\} 42 \{\text{fun } \{x:\text{num}\} \{> x 72\}\}\}$
where the type of $>$ is $(\text{num num} \rightarrow \text{bool})$

$$\begin{array}{c}
 1. \frac{\frac{[x:\text{num } y:\text{num}] \vdash x:\text{num} \quad [x:\text{num } y:\text{num}] \vdash y:\text{num}}{[x:\text{num } y:\text{num}] \vdash \{+ x y\}:\text{num}}}{\phi \vdash \{\text{fun } \{x:\text{num } y:\text{num}\} \{+ x y\}\}:(\text{num num} \rightarrow \text{num})} \quad \phi \vdash 8:\text{num} \\
 \hline
 \phi \vdash \{\{\text{fun } \{x:\text{num } y:\text{num}\} \{+ x y\}\} 8\}:\text{no type}
 \end{array}$$

$$\begin{array}{c}
 2. \phi \vdash \text{false}:\text{bool} \quad \frac{[x:\text{num}] \vdash x:\text{num} \quad [y:\text{num}] \vdash 2:\text{num}}{\phi \vdash \{\text{fun } \{x:\text{num}\} x\}:(\text{num} \rightarrow \text{num})} \quad \phi \vdash \{\text{fun } \{y:\text{num}\} 2\}:(\text{num} \rightarrow \text{num})} \\
 \hline
 \phi \vdash \{\text{if false } \{\text{fun } \{x:\text{num}\} x\} \{\text{fun } \{y:\text{num}\} 2\}\}:(\text{num} \rightarrow \text{num}) \\
 \hline
 \phi \vdash \{\{\text{if false } \{\text{fun } \{x:\text{num}\} x\} \{\text{fun } \{y:\text{num}\} 2\}\} 57\}:\text{num}
 \end{array}$$

$$\begin{array}{c}
 3. \frac{[x:\text{num } y:(\text{num} \rightarrow \text{bool})] \vdash x:\text{num} \quad [x:\text{num } y:(\text{num} \rightarrow \text{bool})] \vdash 17:\text{num}}{[x:\text{num } y:(\text{num} \rightarrow \text{bool})] \vdash \{+ x 17\}:\text{num}} \\
 \hline
 [x:\text{num } y:(\text{num} \rightarrow \text{bool})] \vdash y \{+ x 17\}:\text{bool} \\
 \hline
 \phi \vdash \{\text{fun } \{x:\text{num } y:(\text{num} \rightarrow \text{bool})\} \{y \{+ x 17\}\}\}:(\text{num } (\text{num} \rightarrow \text{bool}) \rightarrow \text{bool}) \\
 \hline
 \frac{[x:\text{num}] \vdash x:\text{num} \quad [x:\text{num}] \vdash 72:\text{num}}{[x:\text{num}] \vdash \{> x 72\}:\text{bool}} \\
 \hline
 \phi \vdash \{\{\text{fun } \{x:\text{num } y:(\text{num} \rightarrow \text{bool})\} \{y \{+ x 17\}\}\} \{> x 72\}\}:(\text{num } (\text{num} \rightarrow \text{bool}) \rightarrow \text{bool}) \\
 \hline
 \phi \vdash \{\{\text{fun } \{x:\text{num } y:(\text{num} \rightarrow \text{bool})\} \{y \{+ x 17\}\}\} \{> x 72\}\} 42:\text{bool}
 \end{array}$$