Quality Evaluation of Crowdsensed Fingerprints for Indoor Localization

***Abstract*—**

I. Introduction

A. Indoor localization

The past decade has witnessed a flourishing of indoor localization systems based on wireless techniques [1], where the fingerprinting based methodology has been widely adopted due to its convenient deployability [?], [?]. The fingerprinting based indoor localization system has two phases: In the offline phase, the site surveyor observes the received signal strength (RSS) of Wi-Fi access points (APs) termed as RSS fingerprints at each reference point, and submit the fingerprints and the location information of the reference point to the localization database; in the online phase, a user needs localization service could submit the observed fingerprints to the database, which then returns the location of the reference point that matches the fingerprints best as the estimated location of the user.

The fingerprinting based method utilizes Wi-Fi APs widely existing in buildings and has no need for other dedicated infrastructure; however, the site survey in the offline phase requires substantial efforts, which is hardly accomplished by any single entity. The recent advances of fingerprinting localization systems utilize mobile crowdsensing approach to collect fingerprints [?], [?], [?], [?], [?], [3]. Mobile crowdsensing is a cost-effective approach to collect large scale data for mobile applications, where individuals with hand-held mobile devices collectively contribute sensing data so that information of certain events could be retrieved [?], [?]. Although sensing participants could receive certain rewards for the efforts and resources spent on the sensing activity, the cost of mobile crowdsensing is still much lower than deploying the dedicated sensing networks [?].

As the crowdsensing data are collected by unprofessional participants with non-dedicated equipment, the sensing data obtained are usually with considerable noise. The quality of the sensing data is the crux for evaluating contribution of the participants, which is the vitally important for effective utilizing rewards to incentivize participants to accomplish sensing tasks satisfactorily. However, how to evaluate the quality of the crowdsensing data is a challenging issue, because there is no ground truth for the collected data to be compared with. Efforts have been made to evaluate the crowdsensing data quality [?], and the task allocation scheme [?], [?]. and incentive mechanisms considering the data quality are proposed.

While the efforts have been made for the evaluation of the quality of the data, the RSS data procurement still remains not fully studied yet. Some quality-driven incentive mechanism[?], [?], [?], [?], [?] were proposed, however, the state of the art method for crowdsensing data collection still focus on the incentive of workers and the utility of the platform. The economical problem is considered in [?], however, the budget constraint of the platform and is not included. Besides, all the work listed above do not consider the situation when data is coming in a sequential order and only available in each round. How to acquire the high-quality data that is in a sequential order given the limited budget still requires more thorough investigation, which is the focus of this work. Our motivation is two-fold. On one hand, the existing work for sequential data procurement in the literature [12]do not work well for the situation of indoor localization; on the other hand, we want to build a concrete measurement of RSS data specifically for the active learning mechanism. Our contributions are as following.

First, we design an effective way to measure the quality of RSS data through the analysis of the probability of localization error. With such a measurenent, we are able to know what kind of data we should procure. And we thus convert the problem into the basic framework of how to acquire the RSS data of best quality with the lowest costs.

Second, we generalize the mechanism to a more general case where the incoming cost of data is adversarially given. *Using the technique of varitional calculus, we give an upper regret bound.](do not know whether this should remain* The mechanism is robust in most indoor-localization situations, even the prior knowledge of the costs is not well understood and the noise in the crowdsensing data is rather arbitrary. And we further provide a most economical data procure mechanism when the accuracy of RSS data is required.

The remaining of the paper is organized as following. The system structure and settings are given in section III. The measurement RSS data quality is presented in Section IV. SectionV gives the abstract definition for the online data procurement mechanism. Section **??** describes the mechanism under the simple assumption that the costs data are drawn in a distribution. Section VI presents a generalized and more robust mechanism for the RSS data procurement. Section VIII gives our simulations and experiments for the mechanism we given before.

II. Related Work

During the inception of indoor localization research with WLAN as the background, NNSS outputs the location minimizing that distance as the ultimate estimation of the users location. However, NNSS still dangles the possibility of accuracy enhancement in that it fails to realize the joint location estimation from multiple APs [?].

In order to take advantage of these multiple APs, Chintalapudi *et al.* bring up an algorithm entitled as EZ localization, whose main contribution occurs in estimating mobile devices without any pre-deployment support of multiple APs [?]. EZ will learn from those acquired fingerprints, which reflect the value of mean and standard deviation of the RSSes corresponding to different APs, during the collection phase. The key point of EZ is that it is erected on the fact that the physics of wireless propagation constrain the fingerprints reported to the server, and it models these constraints and couples them with a genetic algorithm to get the final solution.

One of the common and convenient approaches to collect fingerprints is crowdsourcing, in which any user with a mobile device like smart phone can acquire data while walking freely, without the requirement for users to have any technical training. Wu *et al.* design a localizing system LiFS, combining indoor localization with crowdsourcing and bypassing the conventional site survey process [?], [?]. They initially place several landmarks in the physical space, and then harness information from user motions and pinned sensors in smart phone to set up a sample space with high dimension. The materialization of this sample space relies on Multidimensional Scaling(MDS) algorithm, visualizing the information of similarities and dissimilarities concealed in data [?]. Meanwhile, since the high-dimensional space generated by MDS can be applied to characterize the physical space as well, the estimation of a users location can be derived via comparing physical space and sample space with high dimension.

More utilization of crowdsourcing method has been revealed. Rai *et al.* develop a system called Zee to enable zero-effort crowdsourcing, which denotes that no explicit effort on the part of users is needed [?]. While a mobile device is traversing indoors scanning Wi-Fi signals, Zee leverages inner sensors of the device to track the device itself. Shen *et al.* present a crowdsourcing based system *Walkie-Markie* [?] to generate indoor pathway maps from the user contributed data. The central idea of the system is to exploit Wi-Fi-Marks defined by Wi-Fi RSS features in the indoor space, so that crowdsourced data can be fused. Luo *et al.* propose a self-calibrating participatory indoor localization system [?], which requires no prior knowledge about the building and user intervention including the floor planning. Additionally, Crowdsourcing approach has also been applied to various domains such as transportation [5], environment surveillance [6], [7] and location based service [?], [8].

Nevertheless, the above work has placed a premium on localizing accuracy while simplified the fingerprint procurement phase, just concluding this procedure as crowdsourcing. In fact, more complex situations should not be omitted in realistic fingerprint acquirement. For instance, we need to hire fingerprint samplers to collect fingerprints to construct our fingerprint database, so we have to use budget to purchase these samples from them. Moreover, our budget is limited practically, therefore probably we cannot buy all sampled fingerprints. How should we offer our price to sampled fingerprints from each sampler, reaching our goal for maximizing localizing accuracy with the limited budget? Perhaps samplers carry their sampled fingerprints in a batch or a queue, so what is the optimal purchasing strategy for these two conditions respectively? Zhang *et al.* cope with the situation where an auction is modeled to incentivize a batch of workers to label some binary tasks with a budget constraint [?], while in our work we study the scenario that the samplers come in a queue, formally an online way defined below, with fingerprints to be traded, and budget constraint is concerned as well.

A. Incentive design

In realistic situations, data providers are not always willing to sell their data to us for sundry reasons such as dissatisfying with the price we offer or worrying about data privacy. To cope with this problem, an incentive mechanism should be designed to stimulate data providers to supply us with their data, with means like offering compensation for the participation of data providers. Furthermore, given that the data we acquire are usually needed to be accurate enough to be used in specific industrial scenarios like indoor localization, incentive mechanism should meet the requirement of good quality of collected data.

With regard to the data quality of incentive design, Jin *et al.* introduce a key metric, quality of information(QoI), which generally evinces the quality of users sensory data but whose definition varies among different applications [?]. Taking QoI into consideration, the incentive mechanism can acquire data with higher quality making for further study like better identification for problems of medical devices [?]. Peng *et al.* bring up an incentive mechanism which gives impetus to participants to offer sensory data with high quality [?]. Their concrete recipe can be abstracted as follows. Their mechanism firstly measures each participant's effort in contributing data by an effort matrix. Then the mechanism calculates the quality of every user's data and figure out the user's efficient contribution on the foundation of the effort matrix. Finally the mechanism supplies participants with rewards in correspondence with their effective contributions.

In terms of new facets refreshing incentive mechanism research, Tham and Luo take timeliness of contributions into consideration [?]. Specifically, They assume that the usefulness of data contributed by a user will go downhill with time and may ultimately be of no worth. By incorporating the temporal factor into incentive mechanism, they render the mechanism more analogous to realistic scenes. For example, one practical situation where some employees only get salaries a month later is in need of time consideration. Kawajiri *et al.* also provide a novel framework, what is called Steered Crowdsensing, which aims to level up the quality of acquired data directly rather than the data size, pinpointing the problem that monetary pressure and time consumption perhaps ascend to an unbearable extent in reality when the quantity of data is up-scaled [?].

Nevertheless, the work mentioned above in this subsection except [?] does not fit the incentive mechanism into our concrete context of indoor localization, which is not necessarily suitable in our work. Concerning our particular research background, how can incentive mechanism be suitably utilized in the data procurement phase of localizing? Wen *et al.* tailors quality-based incentive mechanism into a Wi-Fi fingerprint-based indoor localization system [**?**]. Moreover they present a stochastic model to assess the reliability of sensed data, crystallizing the quality of data in indoor localization. In detail, They convert the unreliability of data into that of the user's sense of locality which can be profiled by experiments in advance. The profile of a user's sense of locality reflects the probability of a user's incorrectness of locality. The probabilistic information is then harnessed to discover the data with highest reliability. This reliability scheme is contained within their incentive mechanism. Kawajiri *et al.* also test their Steered Crowdsensing framework under the background of indoor localization [**?**]. However, their models function in offline conditions, differing from our online ones, whose problem setting is that the data providers come in a queue instead of a batch and data buyers need to decide whether to purchase the coming data at once.

B. Online learning

Online learning is one of the dominant mathematical models our work utilizes. It is carried out in the situation where there are continuous question-and-answer rounds and a question comes up in each round. The learner is required to predict an answer of the question in current round. After the answer is given, the correct answer will be presented to the learner. The learner will suffer a loss reflecting the discrepancy between this prediction and the true answer, then information about this round such as the loss will be noted down and the learner continues to the next round [11]. One of the most notable properties of the learners prediction in a round is that it can be depended on historical information learned from previous rounds so that a more reasonable answer will be given in this round. For instance, one can offer a prediction in t-th round which minimizes the sum of previous $t-1$ loss functions, rendering it as an optimization problem [12]. In fact, online learning problems are tied tightly with online optimization [11].

There are assorted theoretical studies pertinent to online learning and optimization. Shalev-Shwartz sorts out a refined survey about the theory [11]. He summarizes classical methods to solve online convex optimization such as Follow-the-Regularized-Leader(FTRL) and Online-to-Batch Conversions(OBC), with kernel theorems proved. Furthermore he presents some typical applications of online learning methods composed of online classification, multi-armed bandit problem and so on. Zinkevich introduces an effective algorithm: Generalized Infinitesimal Gradient Ascent (GIGA), which formulates a common form of online optimization algorithm [10]. However, these fundamental analyses do not touch the usage of online learning and optimization in practical models, which means that the classical methods need proper modifications before fitting realistically with specific objectives managed.

Execution of online learning theory in realistic models has been materialized. Abernethy *et al.* embed the theory into online data procurement [12]. The concrete problem setting is as follows. A data collector has to purchase data points with a limited budget, and the mechanism posts a price π of the coming data to an agent in every round. After that the agent acquires the true cost C from the data provider and compare it with π, if $\pi > C$ then the transaction is achieved with data, cost and loss learned by the mechanism, or else the deal fails with nothing learned. The terminate goal is to propose a pricing strategy reaching the lowest regret, which represents the difference between loss of our pricing hypothesis and that of the theoretically optimal one. They apply the FTRL and OBC to settle this problem with the final result a regret bound of $O(T/\sqrt{B})$, in which T denotes number of rounds and B is budget. However, this work makes some assumptions controversial in reality for simplification due to the inaccessibility to the final answer of original problem, such as if a deal is closed the mechanism will only pay the reporting cost of the data provider in lieu of practically the price reported itself. Whats more, they only endow abstract meanings of parameters in their work, thus for more specific work like indoor localization the concrete meaning of data, cost, loss and so on should be clarified.

In our work we carry out some renovations over the framework of [12] adjusting to our indoor localization background. We inject particular meanings to the parameters in our context. Data is the received signal strength(RSS) offered by signal samplers, and cost is the money we should pay for RSS values from samplers, while loss is the localizing error we suffer using data we have purchased. Moreover the prediction in each round is the estimated RSS value given by the mechanism in accordance with sampled RSS values, while regret is the distance between estimation and true value of RSS at this location. Deeper technical modifications of the model in [12] will be displayed in the next subsection.

C. Other mathematical tools

More mathematical tools involved in our analyses are to be discussed. Firstly, Abernethy *et al.* meet the problem that since budget is limited and probably the excessively expensive data come, it is improper to purchase all arriving data [12]. However, the situation is that no matter the mechanism achieves the trade in a round or not, there will be a loss. Therefore if the mechanism solely decides the prediction by losses of purchased data, there will be a bias of the overall loss since data not acquired generate losses as well. Hence they adopt a tactic called importance-weighting which gives an unbiased estimation of the whole loss by using losses of data procured. Theoretical analyses of importance-weighting can be discovered in [13]. Specifically in [12] they transform the primal loss function of acquired data in t-th round via dividing it by the probability q_t of making this deal, forming a new loss function as the input of FTRL, while inputing 0 if the deal fails. However when q_t is too small, the variance of estimation will mount largely and the regret bound in their model will increase dramatically as well. In our work we adapt the importance-weighting technique into a more reasonable one to inhibit the possibility of the surge of variance and regret bound.

Another research point lies in the budget constraint of the online optimization problem. In [12], Abernethy *et al.* simplify the optimization by loosing the budget constraint, clearing out the integral part of the expression of cost expectation by inequality zooming. However, it leaves the probability that the optimal result output by mechanism in [12] may not make the regret bound tight enough, or even the output is infeasible in the original problem setting for constraint relaxing. In order to reaching a more precise solution, we directly solve the initial optimization problem without erasing the integral part by means of calculus variation technique. Calculus variation views functions as the decision variables in optimization problems with functional theories combined, and the advantage of it is that it can derive the exact solution of the form of functions as decision variables corresponding to the optimal value of the objective function [14], [15]. In our work the decision variable is the cumulative distribution functions reflecting the probability of buying data in each round. With calculus variation applied, we derive the accurate pricing strategy for this problem.

III. System Model

In this section we describe our system model and give out the problem formulation.

As shown in Figure **??**, we present a mobile crowdsensing system consisted of *Data Purchaser*, *Trading Platform* and several *Workers*. For the purpose of performing accurate indoor localization in region $\mathcal{V}$, the data purchaser has to build the corresponding *Fingerprint Database* of *Received Signal Strength*(RSS). Therefore the data purchaser releases tasks of collecting data–RSS value on the platform. For a specific location $s \in \mathcal{V}$, we use $\mathbb{W}_s = \{w_1, w_2, ..., w_{N_s}\}$ to denote the corresponding applicants set. To simplify the notation, we omit the identification of s in almost all the rest of this paper. Without loss of generality, we mainly focus on workers with the same location's data. It's rational that data purchaser need to buy several data points at one location since the RSS value is not constant, in fact it obeys some probability distribution, we assume that its probability density function is $\mathcal{D}(\cdot)$. Consequently, we need several amounts of samples to learn the distribution, more specifically, to estimate the mean value of RSS.

At the very beginning, the data purchaser needs to submit his *Pricing Mechanism* $\mathbb{M}$ to the platform. Here we consider the most nature trading scenario: these N workers arrive in a sequential way with his data x_i. Once agents i arrives, he submit his bid c_i to the platform and the platform compute its price p_i using a mechanism $\mathbb{M}$. If $p_i \geq b_i$ then agents w_i accept this transaction: the platform pays b_i to worker i and receives data x_i, otherwise the worker reject the transaction and the platform receives null signal.

TABLE I
NOTATIONS

Notation	Remark
$\mathcal{V}$	Indoor location region
s	A specific location
$\mathbb{W}_s = \{w_1, \cdots, w_{N_s}\}$	Applications set for location s
$\mathcal{D}(\cdot)$	Probability density function
$\mathbb{M}$	Pricing mechanism
w_i	The i_{th} worker
x_i, b_i, p_i	w_i's data, bid, and corresponding price

IV. Analysis of two-dimensiion localization

A. One-Time Measurement for Single AP in Two-Dimension Space

Hereinafter we are devoted to the analysis of one-time measurement for a single AP in two-dimension physical space. Although our research object has been upgraded from a corridor to a room, a more complex one, the kernel thought of our analysis is analogical to that in one-dimension condition, with solely difference in addressing the problem of physical space in a two-dimension Cartesian Coordinate. Ultimately we derive the expression of P_{high} and P_{low} in the sample space of two-dimension physical space, which form $[P_{low}, P_{high}]$, as the feasible interval for judging a users' location, and furthermore study the possible localizing error incurred by imperfect data.

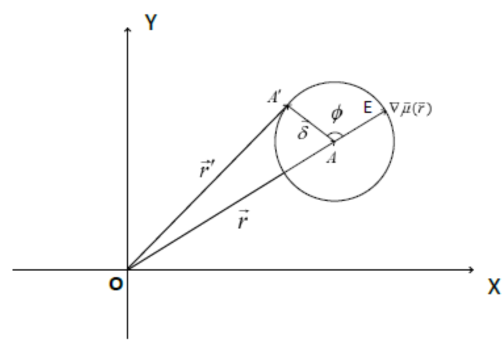

Fig. 1. Description.

1) Maximum Likelihood Estimation in two-dimension physical space: Figure 1 illustrates our model in two-dimension physical space. We designate the AP we sample as the original point O of the coordinate system, and denote A as as the users' actual location which we focus on estimating. Assume that the distribution of RSS around A are circular symmetric , therefore we can define region E as a circle with radius δ, and measurement in it means that the user is estimated at A while outside it indicates at other possible locations rather than A. Note that if we consider any one of the straight lines through A, then the distribution and localization are identical to those in one-dimension situation. Then we define the vector from O to A in physical space as $\vec{r}$ and to any location A' on the boundary of E as $\vec{r}'$. Therefore, $\vec{\delta} = \vec{r}' - \vec{r}$ in Figure 1 can be clearly deemed as the difference between user's real location and our estimation result, *i.e.*, the accuracy of our localization. In addition, since O is the AP we consider, the gradient of RSS at A (*i.e.*, $\nabla\vec{\mu}(\vec{r})$ in Figure 1) must be the same direction as $\overrightarrow{OA}$. We also define φ as the inserted angle between $\vec{\delta}$ and $\nabla\vec{\mu}(\vec{r})$, $\varphi \in (-\pi, \pi]$, where $\varphi < 0$ denotes the absolute value of φ is less than π clockwise (*i.e.*, starting from $\nabla\vec{\mu}(\vec{r})$ to $\vec{\delta}$ while $\varphi > 0$ indicates the opposite.

Similar as one-dimension condition, MLE can be the judgment of whether a user is located at A in two-dimension physical space based on our localization system. For single-measurement situation the MLE can be expressed as:

$$f_{\vec{r}}(P) \geq f_{\vec{r}+\vec{\delta}}(P), \tag{1}$$

where $f_{\vec{r}}(P)$ denotes the probability distribution function of RSS pertaining to the access point P at location $\vec{r}$. However, unlike in one-dimension condition $\vec{\delta}$ is reduced to a scalar δ without directions (φ) taken into consideration, two-dimension problem requires MLE satisfy every single direction mapped by every possible value of $\varphi \in (-\pi, \pi]$. Hence, φ should be embedded in our analysis.

In order to render our explanation simpler and clearer, we rotate the coordinate system to a point where the new x-axis covers $\overrightarrow{OA}$ (Figure 2). Meanwhile $\nabla\vec{\mu}(\vec{r})$ is also co-linear with the x-axis. Then aiming to derive the specific expression of MLE, we firstly make some assumptions:

(1) The radius value δ should be small enough since we intend to meet high localizing accuracy (distinguishing different locations with very short distances from each other). Therefore, E can be viewed as a very small region in physical space.
(2) $|\nabla\vec{\mu}(\vec{r})|$ equals to the difference between $\mu(\vec{r}+\vec{\delta})$ and $\mu(\vec{r})$ in which $\vec{\delta}$ is co-linear with the new x-axis. Hereinafter in this section we denote ∇ as $|\nabla\vec{\mu}(\vec{r})|$ to simplify the mathematical expression.
(3) Every line perpendicular to $\nabla\vec{\mu}(\vec{r})$ represents that all locations on this line inside E have uniform RSS value. Factually according to the direct correlation between distance and RSS, the equi-value delineation of RSS should be a circle rather than a straight line. Nevertheless, due to the fact that E is small enough, the equi-value curve inside E can be approximated as a line segment.
(4) The distribution of RSS strength in two-dimension physical space should be a two-dimension normal distribution. However given that every line vertical to the new x-axis represents an identical RSS value for all locations on it, the normal distribution is degraded to one dimension if we consider the distribution on the new x-axis.

It is clear that the RSS value of $\vec{r}$ (*i.e.*, $\mu(\vec{r})$) corresponds the largest measuring probability when $\vec{r}$ is which we aim to locate. **Original Graph in Section 4.1**. Hence we can derive

$$f_{\vec{r}}(P) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu(\vec{r}))^2}{2\sigma^2}}, \tag{2}$$

where σ represents the intrinsic noise. Combining with Assumption 3 and 4, we can further derive $f_{\vec{r}+\vec{\delta}}(P)$ predicated on the representative condition in Figure 2.

$$f_{\vec{r}+\vec{\delta}}(P) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-(\mu(\vec{r})+\nabla\cos\varphi))^2}{2\sigma^2}} \tag{3}$$

Then to meet the conditions of MLE, we have:

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu(\vec{r}))^2}{2\sigma^2}} \geq \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-(\mu(\vec{r})+\nabla\cos\varphi))^2}{2\sigma^2}} \tag{4}$$

for all $\varphi \in (-\pi, \pi]$. Simplify it we will have:

$$(x-\mu(\vec{r}))^2 \geq (x-(\mu(\vec{r})+\nabla\cos\varphi))^2 \tag{5}$$

In order to gain clearer insight into this inequality, Figure 3 is displayed as follows in which the sample space is concerned.

In Figure 3, the endpoints of this line segment represent the extreme conditions that φ equals to 0 and π. Analogical to one-dimension analysis, we can derive

$$P_{high} = \frac{\mu(\vec{r})+(\mu(\vec{r})+\nabla\cos\varphi)}{2} = \mu(\vec{r}) + \frac{\nabla\cos\varphi}{2} \tag{6}$$

$$P_{low} = \frac{\mu(\vec{r})+(\mu(\vec{r})-\nabla\cos\varphi)}{2} = \mu(\vec{r}) - \frac{\nabla\cos\varphi}{2} \tag{7}$$

which evinces that our localizing estimation will be judged to A if and only if P satisfies $P_{low} \leq P \leq P_{high}$. Here we limit $\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ to ensure $P_{high} \geq P_{low}$, if $\varphi \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$ then we can exchange the expression of P_{high} and P_{low} to maintain $P_{high} \geq P_{low}$. **—Rule 1)** Since the **equality(n)** holds for any $\varphi \in (-\pi, \pi]$, thus if fall into all possible intervals $[P_{low}, P_{high}]$ then it will be classified into location A. The extreme condition is that while $\varphi = \frac{\pi}{2}$, $P_{high} = P_{low}$, indicating that P will be judged to A if and only if $P = P_{high} = P_{low}$. That is to say our measurement must estimate the users location exactly at r, the real location. However, as error exists all the time in measurement and estimation, it is practically impossible to reach this extreme condition. Since that our previous analysis cannot be directly applied to realistic problems.

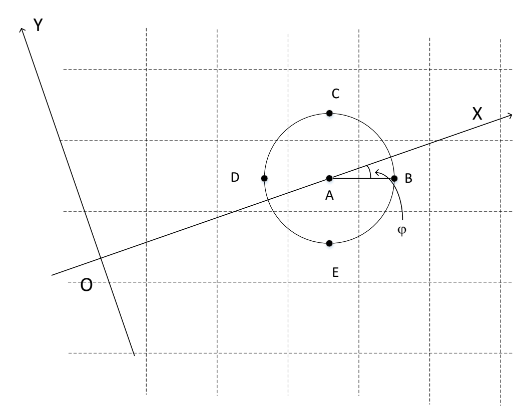

Fig. 2. Description.

However, noticing that practical indoor localization divides a room into many small square blocks and denotes the center of each block as a possible estimated location (*i.e.*, if an estimation falls into one of these blocks, then the location we determine this user is the center of this block.) Therefore, it is not necessarily for us to consider all locations on the boundary of E as analysis above. Instead we may solely concentrate on four blocks adjacent to our target block (A is the center), and denote each center of these four block as B, C, D, E, displayed in Figure 4. Under this circumstances, we will only have to meet four MLE constraints which relax our feasible solution from a single point P to an interval. We will discuss it in detail with Figure 5.

Figure 5 is similar to Figure 3 and we define parameters in accordance with Figure 4. Then based on previous analysis we can easily write the simplified MLE constraints of B, C, D, E:

$$\begin{cases} (x-\mu(\vec{r}))^2 \geq (x-(\mu(\vec{r})+\nabla\cos\phi))^2 \\ (x-\mu(\vec{r}))^2 \geq (x-(\mu(\vec{r})+\nabla\sin\phi))^2 \\ (x-\mu(\vec{r}))^2 \geq (x-(\mu(\vec{r})-\nabla\cos\phi))^2 \\ (x-\mu(\vec{r}))^2 \geq (x-(\mu(\vec{r})-\nabla\sin\phi))^2 \end{cases} \tag{8}$$

Therefore we can draw out Figure 6 in light of Figure 3. Then according to above analysis, it is clear that we can derive the feasible interval as

$$I = \left[\mu(\vec{r}) - \frac{\nabla\min\{\cos\phi,\sin\phi\}}{2}, \mu(\vec{r}) + \frac{\nabla\min\{\cos\phi,\sin\phi\}}{2}\right] \tag{9}$$

where $\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and follows Rule 1. It demonstrates that a user is measured in block A if and only if the estimation of him falls into I. Thus the reliability of our one-time measurement analysis of two-dimension physical space can be interpreted as:

$$\begin{aligned} R &= \int_{P_{low}}^{P_{high}} f_r(P)dP \\ &= \int_{\mu(\vec{r})-\frac{\nabla\min\{\cos\phi,\sin\phi\}}{2}}^{\mu(\vec{r})+\frac{\nabla\min\{\cos\phi,\sin\phi\}}{2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu(\vec{r}))^2}{2\sigma^2}} dx \end{aligned} \tag{10}$$

2) Imperfect Data: Then we extend our study to the influence of imperfect data in a two-dimension physical space. Recall in one-dimension analysis the imperfectness of received data is described as a deviation of line segment in the sample space. Therefore although our physical space has upgraded to two dimension, our sample space is still in one dimension as given above and what we receive are data of RSS, reflected directly in sample space. Thus we can imitate the method to derive error analysis similar as that of one-dimension situation.

We still set our target region as and consider A abut to B, C, D, E abut to A. Firstly it is reasonable to assume that the probability that a user is located at any location in A to be identical, so the distribution of user's true location in is a uniform one, whose probability distribution function is:

$$pf_A(Q) = \begin{cases} \frac{1}{S}, & Q \in A \\ 0, & Q \notin A \end{cases} \tag{11}$$

where A denotes the target region, S represents its area and Q means a user's true location. Note that every Q in two-dimension physical space can be mapped to the one-dimension sample space according to Assumption 3. Now we set the user's true location in physical space is Q_0 and its corresponding point in sample space is X_0. Figure 7 illustrates the influence of imperfectness of data procured, which results in the deviation of feasible interval I in sample space from its true range. As in Figure 7 we denote the right endpoint of true range as X_1 and deviated range as X_2, hence we can discover if a point X_0 falls into interval (X_1, X_2), then it will be falsely estimated: X_0 should have been classified into region B while because of the deviation it is determined to be in region A. The same goes for the left side of region A, the boundary point of A and C. Given our situation, then we can derive the probability of localizing error as:

$$P(err) = \iint_A f_A(Q)P(err|X=X_0)dxdy \tag{12}$$

$$= 2\iint_A f_A(Q)\int_{X_1}^{X_2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(X-X_0)^2}{2\sigma^2}} dXdxdy \tag{13}$$

V. Low-cost data purchasing problem

In this section, we will give some preliminary about the task of our mechanism and prevailing principle used in statistical machine learning. We abstractly define the problem of the designing of the effective mechanism to acquire the RSS information collected by the crowds.

A. Preliminaries

We first give the concept of loss and regret. The loss function that reflects the data quality is defined in the space $H \times Z \to R$, where H is the hypothesis class and Z is the space of the objects. We expect the loss function to get its minimum value when the data is exactly the ideal data. In our setting, the hypothesis h is the mean value and variance of RSS fingerprinting data, and the hypothesis class H is our expected internal of mean value and variance.

$$f_t(h_t) = \tag{14}$$

After we acquire the loss function, we give the concept of the regret function.

$$R(T) = \sum_{t=1}^{T} f_t(h_t) - \min_{h^* \in H} \sum_{t=1}^{T} f_t(h_t^*) \tag{15}$$

where h^* is the optimal choice, causing the least loss in our solution space H. The regret function reflexes how the data deviate from the desired value, the real mean and variance of RSS. We also make some assumptions for this problem

1) the agents have nothing to do with the costs
2) the
3) the

B. Online Gradient Descent

Online learning is a widely used learning paradign. The goal of online learning is to produce the best hypothesis when data is in sequential order We here use the classical Online Gradient Descent(OGD) algorithm to work as the Online Algorithms. It has been proved that the OGD has an upper bound of regret of $O(\sqrt{T})$, which ensures that the average regret tends to zero when T goes to infinite. There are also many kinds of other Online Algorithms which can be found in refOLServey, etc. The OGD is described as following. In each time t, we obtain a h_t according to

$$h_t = h_{t-1} - \eta \nabla f_t(h_{t-1}) \tag{16}$$

.

C. Importance Weighting technique

In tradational online learning problem, all the data will be used to produce the total regret. In our low-cost purchasing problem, the mechanism do not get access to data and obtain a loss in each time t. the estimation of loss is $E(\sum_{t=0}^{T} \delta_t f_t) = \sum_{t=0}^{T} q_t f_t$, where δ_t is the function showing whether the data is procured. However, the definition of regret in [15] still includes all the loss in each time t, whether it has been used or not. In order to get an unbiased estimator of the regeret, we define

$$\hat{f}_t(h) = \begin{cases} \frac{f_t(h_t)}{q_t} & data \quad access \quad to \quad RPM \\ 0 & else \end{cases} \tag{17}$$

.

D. Problem definition

We consider that the data collected through crowdsensing coming in a sequence of $d_1,,,d_T$, with each of them contains a cost $c_1,,,c_T$. We should design a pricing mechanism that can choose how much we should pay for the data. However, we have no means to know either the quality of data is good enough for localization or there will be a better one coming in the sequence. Under the framework of online machine learning, we formally define our RSS data Procure Mechanism (RPM) is defined as following.

Definition 1. *Given a sequence of data $d_1, ..., d_T$ coming in time $1,,,,,T$ with each data possessing a posted price c_t, $c_t \in [0, M]$.*

1) *The RPM post a hypthesis h_t from OGD*
2) *The RPM post a price p_t according to a distribution G_t over $[0, M]$.*
3) *If the $p_t > c_t$ agent accepted the price, the RPM send the loss function $f(h_t)/q_t$ back to the OGD and pay for the posted price p_t. If $p_t < c_t$the agent rejected the price, the mechanism send a null data to the OGD .*

The mechanism outputs a final hypothesis $\overline{h} \in H$

And clearly, our kernel problem is to find the best distribution G_t used for the mechanism to post its price

E. online batch to conversion

The meachanism and online learning algorithm produces a sequence of hypothesis $h_1,,,h_T$. The main goal of our algorithm is to get the best hypothesis $\overline{h}$, the mean value and variance of RSS, from the sequence. One simple aproach is to average every hypothesis h_t acquired in each time t.

$$\overline{h} = \sum_{t=1}^{n} h_t \tag{18}$$

It has been proved that ...,

VI. The main setting: regret minimization senario

In this senario, the mechanism has a fixed budget. The main purpose of the mechanism is to get a high accuracy of localization informaiton, which is consistent with our definition of loss function and regret.

A. Estimate the upper bound of regret

We will first find the upper bound of the regret produced by RPM defined in **??**. In normal case, the regret bound of OGD is

$$\frac{||h||^2}{2\eta} + \eta \sum_{t=1}^{T} \nabla f_t(h_t)^2 \tag{19}$$

, which is a well known result. Under the importance weighting framework, we give the regret bound in the following lemma

Lemma 1. *The regret bound produced by RPM in 1 is bounded by*

$$R(h) \leq \frac{||h||^2}{2\eta} + \eta E(\sum_{t=1}^{T} \frac{\nabla f_t(h_t)^2}{q_t}) \tag{20}$$

. The 1 is quite easy to be proved under our setting that the loss function f_t is of strong convexity.

B. Derivation of the Regeret Minimization Problem

In each time t, the RPM need to post a price p according to a distribution g in order to get a minimum regret, we thus reduce the problem of designing a mechanism into an optimization problem

$$\begin{aligned} \min \sum_{i=1}^{n} \frac{\nabla {f_i}^2}{1 - F_i(c_i)} \\ s.t. \quad \sum_{i=1}^{n} \int_{c_i}^{M} x dF_i(x) \leq B \end{aligned} \tag{21}$$

where $\forall c_i, 0 \leq c_i \leq M$,and$F(0) = 0, F(M) = 1$

Theorem 1. *The optimal solution of the optimization problem [**??**] is in the form*

$$F_t(c) = \begin{cases} 1 - \frac{\nabla f_t}{\sqrt{\lambda c - \beta}} & c \in (\frac{\nabla f_t^2 + \beta}{\lambda}, M] \\ 0 & else \end{cases} \tag{22}$$

Proof: We first give our function space $V = \{y | y(0) = 0, y(M) = 1\}$. And we denote our cost function as

$$M(F_1,,,F_n) = \sum_{i=1}^{n} \frac{\alpha_i}{1 - F_i(c_i)}.$$

Then the augmented Lagrange function is derived as

$$J(F_1,,,F_n,\lambda) = M(F_1,,,F_n) + \lambda(\sum_{i=1}^{n}\int_{c_i}^{M} x dF_i(x) - B)$$

According to the Gateaux Deravative, we obtain that for $\forall \hat{F} \in V$

$$\delta J|_{F_t}(\hat{F}_t - F_t) = \int_{c_t}^{M} (-\frac{\alpha_t}{(1-F_t(c_t))^2} + \lambda x)(\hat{f}(x) - f(x))dx$$

if $\overline{F}$ is the local minimum, then we have

$$\delta J(\hat{F}_t - \overline{F_t}) \geq 0$$

holds for every $\hat{F} \in V$.Noticing that

$$\int_0^M f_t(x) - f(x)dx = 0$$

We must have

$$-\frac{\alpha_t}{(1-F_t(c_t))^2} + \lambda x \geq 0$$

hold on every where on $[c_t, M]$ thus we obtain that

$$F_t(c) = \begin{cases} 1 - \frac{\nabla f_t}{\sqrt{\lambda c - \beta}} & c \in (\frac{\nabla f_t^2 + \beta}{\lambda}, M] \\ 0 & else \end{cases} \tag{23}$$

■

One major difficulty for the is the determination of the determination of β and λ Noticing that $F(x)$ is not continuous, according to Stieltjes Integral, we rewrite the constraint as following

$$\begin{aligned} &\sum_{t=1}^{T}(\int_{c_t}^{M} x dF_t(x)) \\ =&\sum_{t=1}^{T}(\int_{c_t}^{M} x f_i(x)dx + (1 - F_i(M)M) \\ \leq&\sum_{t=1}^{T} \nabla f_t(\frac{2}{\lambda}\sqrt{\lambda M - \beta} + \frac{c_t}{\sqrt{\lambda c_t - \beta}} - \frac{2}{\lambda}\sqrt{\lambda c_t - \beta}) \\ \leq& B \end{aligned}$$

The Stieltjes Integral here has its practical significance. Because we assume that the cost lies between $[0, M]$, in other word, the mechanism do not accept any price higher than M, thus for all posted price c that are higher than M, the mechanism will only pay M instead of c.

Now since we get the solution of the F_t, the remaining work is to determine the parameters λ and β, we go back to our initial optimizationproblem that minimize the regret bound. The Lagrangian is thus given as follows

$$\begin{aligned} L(\mu,\beta,\lambda) = \sum_t \Bigg(&\nabla f_t\Big(\sqrt{\lambda c_t - \beta} + \mu(\frac{2}{\lambda}\sqrt{\lambda M - \beta} \\ &+ \frac{c_t}{\sqrt{\lambda c_t - \beta}} - \frac{2}{\lambda}\sqrt{\lambda c_t - \beta})\Big)\Bigg) - \mu B \end{aligned} \tag{24}$$

According to the complementary relaxation condition, $\mu \neq 0$, which means that the constraint condition in **??** for the optimal solution is strict. To get the analytic solution of the optimal value of β and λ is infeasible , thus we use the numeric solution for the equation **??**.

C. Analysis of the result

We first simply set the $\beta = 0$ for a special case. Through simple calculation, we can have an estimation of λ_0 as following

$$\lambda_0 = \frac{T}{B}(2\theta_0 - \theta) \tag{25}$$

where $\theta_0 = \frac{1}{T}\sum_t \nabla f_t \sqrt{M}$, $\theta = \frac{1}{T}\sum_t \nabla f_t \sqrt{c_t}$ Since that $\partial L/\partial\beta > 0$, $\partial L/\partial\lambda < 0$, we obtain that the optimal solution (β^*, λ^*) statisfy that $\beta^* > \beta^0$, $\lambda^* < \lambda_0$ Thus we have the estimate of the upper bound of the regret of RPM in theorem 2.

Theorem 2. *The regret of RPM in produced by the algorithm in* **??** *is bounded by*

$$Regret < O(\frac{T}{\sqrt{B}}(2\theta_0 - \theta)\alpha) \tag{26}$$

where $\alpha = O(\sqrt{1 - \frac{\beta B^2}{T\theta^2}})$

Proof: To be determined ■

One problem in this situation is that we may not get enough prior knowledge to both c_t and ∇f_t. One way to solve the problem is that we initially set β to a fixed value and λ to a very small value, e.g. 0.0001. Then in each time t, we update the value of λ with

$$\theta_0^{(t)} = \sum_{i=1}^{t-1} \frac{\nabla f_t(h_t)}{t-1}\sqrt{M} \tag{27}$$

$$\lambda^{(t)} = \frac{T^2}{B^2 M}\theta_0^2 + \frac{\beta}{M} \tag{28}$$

VII. The budget minimization senario

In this senario, we consider the problem to find the most money-saving way to acquire the data inorder to achieve a satisfactory regret bound R. The objective function is in the form of a integral, which is not an easy problem of the classical optimization problem. Besides, to solve the very exact form of the budget . Thus we use the of the budget B as following

$$\sum_t c_t q_t \leq B \leq \sum_t M q_t \tag{29}$$

Thus what we have to do is to solve the optimization problem of the form

$$\begin{aligned} &\min_{q_t} c_t \\ &s.t. \sum_t \frac{\nabla f_t^2}{q_t} \leq R \\ &0 \leq q_t \leq 1 \end{aligned} \tag{30}$$

A. The optimal mechanism

Consider the convexity of the objective function, we give the Lagrangian

$$L=\sum_t c_t q_t-\lambda(-\sum_t \frac{\nabla f_t^2}{q_t}+R-\sum_t \mu_t(1-q_t)) \quad (31)$$

The optimal K-T condition of the problem **??** is

$$\frac{\partial L}{\partial q_t}=c_t-\lambda[\frac{\nabla f_t^2}{q_t^2}]-\mu_i=0 \quad (32)$$

when $q_t=1$, we get $u_i\neq 0$, when $q_t\neq 1$, $\mu_i=0$, thus we have

$$q_t=\min\{1,\sqrt{\frac{\lambda}{c_t}}\nabla f_t\} \quad (33)$$

According to our constraint condition

$$\sum_t \sqrt{\frac{c_t}{\lambda}}\nabla f_t\leq R \quad (34)$$

we can get an approximation of the $\sqrt{\lambda}$ through simple calculation

$$\sqrt{\lambda}=\frac{T}{R}\theta \quad (35)$$

where we use θ to denote the term $\frac{1}{T}\sum_t\sqrt{c_t}\nabla f_t$. Since 33 holds for $\forall c_t$, and c_t is arbitrarily given.We may assume that the convoluted distribution function of the price mechanism is of the form

$$F_t(c)=1-\sqrt{\frac{\lambda}{c}}\nabla f_t \quad (36)$$

And the PDF is

$$f(c)=\frac{1}{2}\sqrt{\frac{\lambda}{c^3}}\nabla f_t \quad (37)$$

B. result analysis

Now we can make a relatively more precise estimate the budget B

$$E(B)=\sum_t\int_{c_t}^{M} cf(c)dc \quad (38)$$

$$=\frac{T^2}{R}\theta\varphi \quad (39)$$

where $\varphi=\sum_t\frac{1}{T}\nabla f_t(\sqrt{M}-\sqrt{c_t})$

VIII. Experiments and Simulations

In this section, we conduct the experiments and simulations to validate the ,,,of our system model and data procurement mechanism. We use the data collected in FoxCom Shanghai,where we tested the RSS value of 10 AP in 13 different locations, the distribution of the location and AP points are shown in Figure**??**. Since that the workers collected our data did not ased for reward to us, we simply simulate the costs of the data through a normal distribution with mean value of 0.5 and variance of 1.

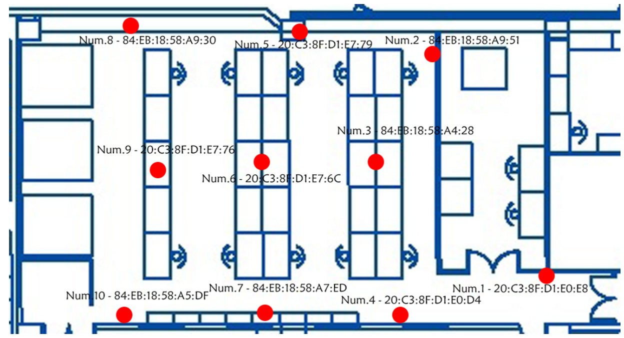

Fig. 3. Location Description.

IX. Conclusion and Future Work

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