Solution2

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1 Problem

In this senario, we consider the problem to find the most money-saving way to acquire the data inorder to achieve a satisfactory regret bound R. The objective function is in the form of a integral, which is not an easy problem of the classical optimization problem. Besides, to solve the very exact form of the budget . Thus we use the of the budget B as following

$$\sum_t c_t q_t \leq B \leq \sum_t M q_t \tag{1}$$

Thus what we have to do is to solve the optimization problem of the form

$$\min_{q_t} c_t \tag{2}$$

$$s.t. \sum_t \frac{\nabla f_t^2}{q_t} \leq R \tag{3}$$

$$0 \leq q_t \leq 1 \tag{4}$$

2 Solution

Consider the convexity of the objective function, we give the Lagrangian

$$L = \sum_t c_t q_t - \lambda(-\sum_t \frac{\nabla f_t^2}{q_t} + R - \sum_t \mu_t(1 - q_t)) \tag{5}$$

The optimal K-T condition of the problem **??** is

$$\frac{\partial L}{\partial q_t} = c_t - \lambda[\frac{\nabla f_t^2}{q_t^2}] - \mu_i = 0 \tag{6}$$

when $q_t = 1$, we get $u_i \neq 0$, when $q_t \neq 1$, $\mu_i = 0$, thus we have

$$q_t = \min\{1, \sqrt{\frac{\lambda}{c_t}} \nabla f_t\} \tag{7}$$

According to our constraint condition

$$\sum_t \sqrt{\frac{c_t}{\lambda}} \nabla f_t \leq R \tag{8}$$

$$\begin{aligned}
\sqrt{\lambda} &\geq \frac{\sum_t \sqrt{c_t} \nabla f_t}{R} && (9)\\
&= \frac{T}{R}(\frac{1}{T}\sum_t \sqrt{c_t}\nabla f_t) && (10)\\
&= \frac{T}{R}\theta && (11)
\end{aligned}$$

where we use θ to denote the term $\frac{1}{T}\sum_t \sqrt{c_t}\nabla f_t$. Since **??** holds for $\forall c_t$, and c_t is arbitrarily given.We may assume that the convoluted distribution function of the price mechanism is of the form

$$F_t(c) = 1 - \sqrt{\frac{\lambda}{c}} \nabla f_t \tag{12}$$

And the PDF is

$$f(c) = \frac{1}{2}\sqrt{\lambda} c^3 \nabla f_t \tag{13}$$

3 Analysis

Now we can make a relatively more precise estimate the budget B

$$\begin{aligned}
B &= \sum_t \int_{c_t}^{M} cf(c)dc && (14)\\
&= \sum_t \int_{c_t}^{M} \frac{1}{2}\sqrt{\frac{\lambda}{c}} \nabla f_t dc && (15)\\
&= \sum_t \sqrt{\lambda} \nabla f_t (\sqrt{})M - \sqrt{c_t}) && (16)\\
&= \frac{T^2}{R}(\frac{1}{T}\sum_t \sqrt{c_t}\nabla f_t)(\sum_t \frac{1}{T}\nabla f_t(\sqrt{M} - \sqrt{c-t})) && (17)\\
&= \frac{T^2}{R}\theta\varphi && (18)
\end{aligned}$$

where $\varphi = \sum_t \frac{1}{T}\nabla f_t(\sqrt{M} - \sqrt{c_t})$