Geometric Continuous-Curvature Path Planning for Automatic Parallel Parking

Hélène Vorobieva*, Nicoleta Minoiu-Enache*, Sébastien Glaser†, and Saïd Mammar‡
*Renault, 1 avenue du Golf, 78288 Guyancourt, France
Email: helene.vorobieva@renault.com and nicoleta.minoiu-enache@renault.com
†IFSTTAR, LIVIC, 14 route de la Minière, Bâtiment 824, Satory, 78000 Versailles, France
Email: sebastien.glaser@ifsttar.fr
‡IBISC, Université d'Evry Val d'Essonne, 91025 Evry, France
Email: said.mammar@ifsttar.fr

***Abstract*—This paper deals with path planning for car-like vehicle in parallel parking problems. Our path planning method uses simple geometry of the vehicle kinematic model. The presented strategy consists in two parts: create a simple geometric path for the parallel parking in one or more maneuvers, formed by circle arcs and then transform it to a continuous-curvature path with the use of clothoids. Accordingly, a car-like vehicle can follow such path without reorienting its front wheels at stop. The proposed planning method is independent of the initial position and of the orientation of the vehicle. Control inputs for steering angle and longitudinal velocity are generated so that the vehicle can park by following them.**

I. INTRODUCTION

A. Motivation and state of the art solutions for parallel parking

Since parking spots have become very narrow in big cities, even experimented drivers need to be very cautious when maneuvering the vehicle. This often leads to minor scratches on the car and increases traffic jam by multiple repositioning. Therefore, automatic parking is a solution to reduce stress and increase driver's comfort and safety. Drivers also expect from an automatic parking to be fast, predictable and as less demanding to the tires as possible. In this paper, we consider parallel parking problem, which is particularly demanding for the driver.

There are many methods to tackle the trajectory generation for the parking problem:
- Some methods use reference functions. For example, [1] presents a two step method using Lyapunov function to stabilize the vehicle in the parking spot. In [2] the authors propose the optimization of two parameters to find steering angles and durations of commands to execute the parking maneuvers. These methods strongly depend on gains and parameters chosen for the functions, which can be difficult to adjust and not certainly lead to correct parking maneuvers.
- Other methods are based on fuzzy logic [3] or neural network [4] to learn human technique. They are however limited to human experts knowledge and are difficult to generalize.
- Several methods are based on two phases path planning, for example [5] and [6]: creation of collision-free path by a lower-level geometric planner that ignores the motion constraints and subdivisions of this path to create an admissible path. An optimization routine can reduce the length of the path. Very often these methods present continuous-curvature paths, but the methods are more adapted for general path planning and become very complicated when applied to the parking problem.
- Recent geometric methods are based on admissible collision-free circular arcs, which lead the vehicle in the parking spot in one trial [7], [8], [9] or in several trials [10] if the parking spot is too narrow. The trajectories created with these methods are constituted of circle arcs and involve easy geometrical equations. Whereas in the method proposed in [8] the minimum parking spot depends on the initial position of the vehicle, the authors of [7] and [9] propose a minimal parking spot, which only depends on the characteristics of the vehicle. In [10], a generalization of the geometric method to parking in several trials, is provided if the parking in one trial is not possible.

B. Continuous-curvature paths

The geometric approach based on retrieving a vehicle from parking spot and reversing this procedure to solve the parking problem ([7], [9], [10]) is particularly instinctive and adapted to the parking problem. However, in [7], [9] and [10] the curvature of this type of path is discontinuous. When the vehicle tracks such a path it has to stop at each circle arc to reorient its front wheels. This can be undesirable for two reasons. First, in these methods the first maneuver is constituted of two circle arcs with the vehicle moving backward. In [11] a time-comparison is performed between paths composed of circle arcs and paths using quintic polynomials. This study shows, that if the vehicle has to stop at the change of circle arcs, it involves and undesirable time delay. Second, steering at stop is particularly demanding to the steering column and induces a faster wear of the tires. Curvature continuity of the generated path is, therefore, a desirable property.

Continuous-curvature paths can be created from smooth curves, which can be divided into two categories. There are curves with coordinates that can be expressed in a closed-form, for example B-splines [12], polar splines [13] or quintic polynomials [14]. However it appears that the most popular are parametric curves for which curvature is a function of their arc length, for example clothoids [15], cubic spirals [16] or intrinsic splines [17].

An admissible path for a real vehicle has to include a constraint on the curvature (maximal steering angle) and on

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its derivative: the later is upper bounded to reflect the fact that the vehicle can only reorient its front wheels with a finite velocity. It appears also that paths with continuous curvature, upper-bounded curvature value and rate can be tracked with a high accuracy by real vehicles [18]. Clothoid paths can answer to those three constraints [19], [20]. Moreover, [21] proved that the shortest path between two vehicles configurations with a constraint on the curvature rate is made up of line segments and clothoid arcs of maximum curvature derivative. Another argument on favor of the clothoids is that these curves describe very well the behavior of car-like vehicles. Indeed, if a vehicle has a constant velocity and steers its wheels with a constant angular velocity, the middle of the rear track follows a clothoidal path. In consequence, in order to create continuous-curvature paths for car-like vehicles, the clothoids seems to be a relevant solution.

C. Content of the article

In this work, we combine simplicity of geometric path planning with continuous-curvature path using clothoids. The methodology of simple geometric path planning as in [10] is used to generate a path composed of circle arcs. Then each circle arc is transformed into a sequence of clothoid, circle arc, clothoid in order to create a collision free continuous-curvature path. In this work no obstacle is taken into consideration except for the limits of the parking spot. A collision free path means hence a path for which the borders of the parking spot are not reached. The initial position and orientation of the vehicle are not imposed but known and the parking is possible as long as the length and width of the parking spot are bigger than those of the vehicle.

In the next section, the vehicle properties are depicted and a preliminary study on geometric path planning and clothoid turns is presented. Section 3 presents the continuous-curvature geometric path planning. In section 4, the generation of reference trajectories is outlined. Finally, the section 5 is devoted the conclusion.

II. Vehicle properties and preliminary study

A. Vehicle properties

1) Vehicle's model: In this paper front wheels steering vehicles are studied. The model chosen for simulations is the Renault Fluence ZE. Table I, Table II, and Fig. 1 show the notations, the parameters and the values used here. The vehicle is represented by its bounding rectangle, including the outside rear-view mirrors. The front track is approximated to have the same length as the rear one.

TABLE I. Vehicle's model

Parameters	*Notation*	*Value*
Wheelbase	a	2701 mm
Track	2b	1537 mm
Front overhang	d_{front}	908 mm
Rear overhang	d_{rear}	1114 mm
Distance from the left, right wheel to the left, right side of the vehicle (exterior mirrors folded)	d_l,d_r	136 mm
Maximum left, right steering angle	δ_{lmax},δ_{rmax}	38 degree

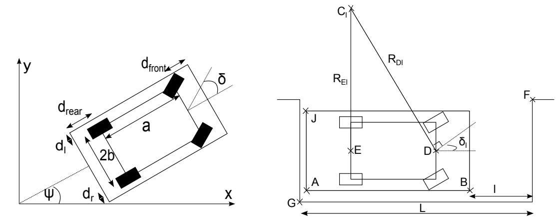

Fig. 1. Vehicle in global (x, y, ψ)-coordinates (left) and in a parking spot (right)

2) Geometric properties: The parking maneuver is a low-speed movement. Consequently the Ackerman steering is considered with the four wheels rolling without slipping, around the instantaneous center of rotation. Different turning radius are calculated. With E and D being respectively the center of the rear and the front track, it yields:

$$R_E = a/\tan\delta \quad , \quad R_D = a/\sin\delta \tag{1}$$

For example, to have R_{El}, we take δ_l. Minimum radius is obtained with the maximum steering angle: for example, to have R_{Elmin}, we take δ_{lmax}. With B and A being respectively the right front and the right rear extremities of the vehicle, applying the Pythagorean Theorem it results:

$$\begin{aligned} R_{Bl} &= \sqrt{(R_{El}+b+d_r)^2+(a+d_{front})^2} \\ R_{Ar} &= \sqrt{(R_{Er}+b+d_l)^2+{d_{rear}}^2} \end{aligned} \tag{2}$$

B. Geometric path planning with circle arcs

In [22] the authors showed that optimal paths for a vehicle going forwards and backwards can be obtained with circles of minimal radius. In consequence, we study here geometric paths constituted when possible by circle arcs of minimal admissible radius.

A particularly interesting approach is presented by [7] and [9], based on retrieving a vehicle from the parking spot and reversing the created path. The path is constituted by two circle arcs with the minimal admissible radius. The circles are created according to the Ackerman model. In [10], the method is generalized to the parking in several maneuvers if the parking spot is too narrow to park in one maneuver. To create this path, the authors consider the vehicle in the parking spot and define a retrieving path like a human driver would do it and reverse this path. This retrieving is composed by forward and backward moves until the vehicle can retrieve. When a forward move allows the vehicle to retrieve without collision, the concerned circle arc is considered and a feasible

TABLE II. Notations

Meaning	*Notation*
Distance between 2 points (ex: A, B)	d_{AB}
Circle of center C and radius R	$C(C, R)$
Left, right instantaneous center of rotation	C_l, C_r
Distance between a point (ex: A) and C_l, C_r	R_{Al}, R_{Ar}
Length of the parking spot	L
Absolute value of the steering angle (right, left)	δ (δ_r, δ_l)
Orientation of the vehicle	ψ

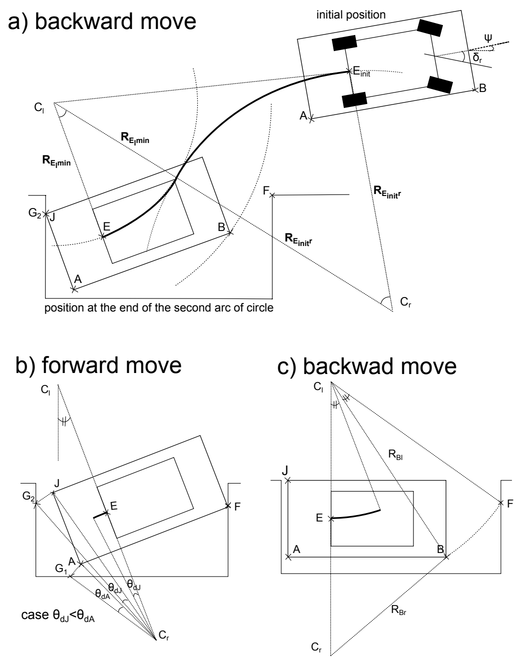

Fig. 2. Parking in three maneuvers with the "reversed" geometric method

circle arc connecting this arc by a tangential point to the real initial position is calculated. In Fig. 2, an example of parking in three maneuvers is presented. The vehicle follows the path in the order a), b), c) but the path was created in the inverse order. This method allows parking in any parking spot as long as its length and width is bigger than those of the vehicle.

Due to its good results, in this paper the "reversed" geometric method will be compared to a new method, which provides continuous curvature paths.

C. Clothoid turns

In this subsection, we briefly describe general properties of clothoids applied to path planning for vehicles [20].

A clothoid is a curve whose curvature $\kappa = 1/R$ varies linearly with its arc length L: $\kappa(s) = \sigma L + \kappa(0)$, σ is the sharpness of the clothoid. It is also commonly used to define a clothoid by its parameter A with $A^2 = RL$, where $A = 1/\sigma$.

Without loss of generality, consider the start configuration of the vehicle $q_i = (x_i = 0, y_i = 0, \psi_i = 0, \delta_i = 0)$. The vehicle is moving with constant positive longitudinal velocity and with constant steering velocity to the left. The vehicle is then describing a clothoid with the following properties.

The configuration for any position q_e of the vehicle at a distance L_e from the initial configuration is (see Fig. 3):

$$q_e = \begin{cases} x_e = A\sqrt{\pi}C_f(\frac{L_e}{\pi A}) \\ y_e = A\sqrt{\pi}S_f(\frac{L_e}{\pi A}) \\ \psi_e = \frac{A^2}{2R^2} \\ \delta_e = 1/R_e \end{cases} \tag{3}$$

where A is the parameter of the clothoid depending on the longitudinal and steering velocities, $R_e = A^2/L_e$ and C_f and S_f are the Fresnel integrals: $C_f(x) = \int_0^x \cos\frac{\pi}{2}u^2\,du$ and $S_f(x) = \int_0^x \sin\frac{\pi}{2}u^2\,du$.

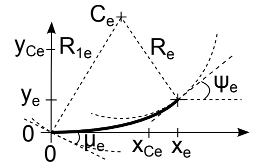

Fig. 3. Clothoid turns

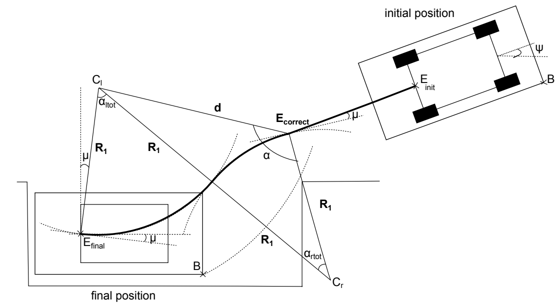

Fig. 4. Path for a parking in one maneuver

The center of the circular arc C_e, located at a distance R_e from a configuration q_e, in the direction normal to ψ_e is:

$$\begin{aligned} x_{C_e} &= x_{R_e} - R_e \sin\psi_e \\ y_{C_e} &= y_{R_e} + R_e \cos\psi_e \end{aligned} \tag{4}$$

In addition, we define the radius R_{1e} and the angle μ_e between the orientation of q_i and the tangent to the circle of center C_e and radius R_{1e}:

$$\begin{aligned} R_{1e} &= \sqrt{{x_{Ce}}^2 + {y_{Ce}}^2} \\ \mu_e &= \arctan x_{C_e}/y_{C_e} \end{aligned} \tag{5}$$

III. Path planning

A. Strategy

The first step of the parking path generation is to create a geometric path composed of circle arcs, with a parking in one or more maneuvers. Methods presented in [10] could be used. In these methods, we need to define the radius of the circle arcs. To satisfy the criteria of μ-tangency, which will be explained later, the radius of circles for left and right steering should be the same. Lets call R_1 this radius, associated to the middle of the rear track E. It will be calculated in the next section (see Fig. 5).

In these geometrical methods, according to the Ackerman model with front wheels steering, the instantaneous center of rotation is located on the same line than the rear track. This means, that the orientation of the vehicle is tangent in E to the circle arc of the geometric path.

In our case, to use clothoids, the orientation of the vehicle situated on the circles of the geometric path presents always an angle μ with the tangent to the circle in E. In consequence, in our method this constraint is taken in account during the geometric path generation: the instantaneous centers of rotation are transformed by the rotation of center E and angle μ. The figure 4 shows the path generated by a geometric method taking into account the μ-tangency criteria, for the parking in one maneuver.

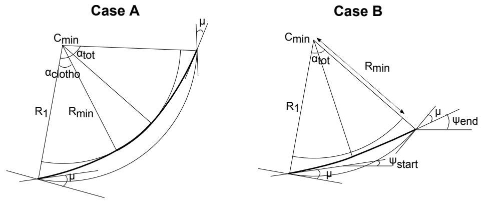

Fig. 5. Replacing of a circle arc by an A or B sequence

Let call α_{clotho} the angle formed by a clothoid started with a null curvature and finished with a curvature $1/R_{min}$ where $R_{min} = max(R_{Elmin}, R_{Ermin})$. An associated clothoid of parameter A_{min} and length L_{min} will be calculated in the next section.

In the geometric methods, the vehicle has to travel along a circle arc during an angle α_{tot}. In our method, two cases are considered (see Fig. 5).

1) Case A $\alpha_{tot} \geq 2\alpha_{clotho}$: The circle arc of the geometric method is replaced by the following sequence:

- Clothoid of parameter A_{min}, of length L_{min}, of initial null curvature and of end curvature equal to $1/R_{min}$;
- Circle arc of radius R_{min} and of angle $\alpha_{tot} - 2\alpha_{clotho}$;
- Clothoid of parameter A_{min}, of length L_{min}, of initial curvature equal to $1/R_{min}$ and of end null curvature.

2) Case B $2\alpha_{clotho} > \alpha_{tot} \geq 2\mu$: The circle arc of the geometric method is replaced by the following sequence:

- Clothoid of parameter A_{new}, of length L_{new}, of initial null curvature and of end curvature equal to $L_{new}/{A_{new}}^2$;
- Clothoid of parameter A_{new}, of length L_{new}, of initial curvature equal to $L_{new}/{A_{new}}^2$ and of end null curvature.

A_{new} and L_{new} have to respect the μ-tangency criteria and are classically calculated as follows [19]:

$$A_{new} = \sqrt{\frac{{R_1}^2 \sin\frac{\beta}{2} + \mu^2}{\pi(\cos\frac{\beta}{2} C_f(\sqrt{\frac{\beta}{\pi}}) + \sin\frac{\beta}{2} S_f(\sqrt{\frac{\beta}{\pi}}))^2}} \quad (6)$$

$$L_{new} = A_{new}\sqrt{\beta} \quad (7)$$

where $\beta = (\psi_{end} - \psi_{start}) mod 2\pi$ is the deflection of the turn.

3) Case C $\alpha_{tot} < 2\mu$: In this case it is no more possible to use the method of the case B but a left steering is still possible. A clothoid of parameter A_{min} and of length L_{caseC} is calculated to satisfy the following criteria: $\alpha_{tot} = 2\alpha_{clothocaseC}$ and in the end position it is no more possible to go forward (or backward, depending on the case) without a collision. As this calculus includes parametric curves with Fresnel integrals, an approached solution is considered. Finally, the following sequence is used:

- Clothoid of parameter A_{min}, of length L_{caseC}, of initial null curvature and of end curvature equal to $L_{caseC}/{A_{min}}^2$;
- Clothoid of parameter A_{min}, of length L_{caseC}, of initial curvature equal to $L_{caseC}/{A_{min}}^2$ and of end null curvature.

B. Calculus of useful parameters

R_1 represents the distance from the point E at the beginning of a circle arc of the geometric path to the instantaneous center of rotation C. To make the most of the maximal steering angles of the vehicle, we want this radius R_1 to be as small as possible.

Lets define t_{min} the necessary time to steer from null steering to maximal steering: $t_{min} = \frac{\delta_{max}}{v_\delta}$, where v_δ is the maximal desired steering velocity. The minimal associated length of the path is then defined as $L_{min} = v_{longi} t_{min}$, where v_{longi} is the maximal desired longitudinal velocity.

The parameter A_{min} of the used clothoid is then:

$${A_{min}}^2 = R_{min} L_{min} \quad (8)$$

R_1 is then calculated using Eq. 3 - 5.

C. Starting point of the first circle arc

Hypothesis: the vehicle begins the parking maneuver with straight wheels.

In [10], the first circle arc could be of non-minimal radius. Here, with respect of the criteria of μ-tangency, this radius has to be R_1. In consequence, the vehicle has to go backward or forward without steering from its initial position E_{init} until it arrives at a point $E_{correct}$ on a circle of radius R_1 connected with a tangential point to the second circle (see Fig. 4). The coordinates of $E_{correct}$ can be calculated applying the Al-Kashi theorem to the triangle $C_l C_r E_{correct}$:

$$3{R_1}^2 - d^2 + 2dR_1 \cos\alpha = 0 \quad (9)$$

with

$$\begin{cases} d = \sqrt{(x_{Cl} - x_{E_{correct}})^2 + (y_{Cl} - y_{E_{correct}})^2} \\ \alpha = \arccos\frac{|y_{E_{correct}} - y_{Cl}|}{d} - \mu + \psi_{E_{init}} \\ y_{E_{correct}} = y_{E_{init}} - (x_{E_{init}} - x_{E_{correct}})\tan\psi_{E_{init}} \end{cases} \quad (10)$$

D. Conditions of feasibility

By construction, the created path takes into account the geometry, the steering and velocity limitations of the vehicle. In consequence, the created path is admissible for the vehicle.

Geometric path planning presents conditions of non collision as conditions of non collision of extremities of the vehicle with the spot borders, when the point E follows circle arcs of radius R_1. In the solution presented here, those conditions have to be modified. Without loss of generality, we consider the feasibility of the parking in one trial. During the parking maneuver, the vehicle travels between two extremal circles of center C_l and of radius R_{min} or R_1. This creates a lower and upper bounds of the minimal length L_{min} of the parking spot, with which the parking in one trial is possible (Eq. 11). L_{limmin} and L_{limmax} decrease when R_1 decreases. By construction, R_1 decreases when the maximal desired

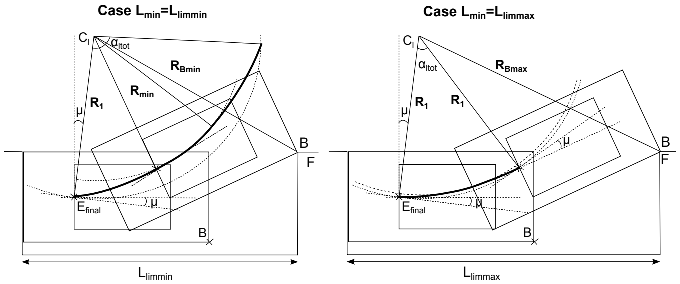

Fig. 6. Cases with lower (left) and upper (right) bounds for the minimal length of the parking spot for the parking in one maneuver

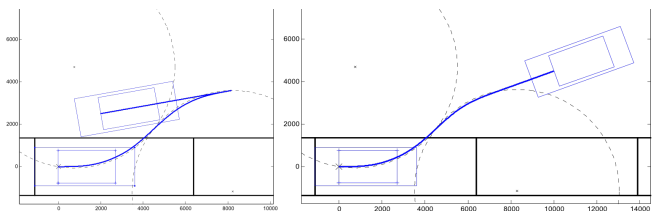

Fig. 7. Simulation of the parking for two different initial configurations

longitudinal velocity v_{longi} decreases and the maximal desired steering velocity v_δ increases.

$$L_{limmin} \leq L_{min} \leq L_{limmax} \tag{11}$$

where

$$\begin{aligned} L_{limmin} = \quad & d_{rear} + R_1 \sin\mu \\ & + \sqrt{{R_{Bmin}}^2 - (R_1 \cos\mu - b - d_l)^2} \\ L_{limmax} = \quad & d_{rear} + R_1 \sin\mu \\ & + \sqrt{{R_{Bmax}}^2 - (R_1 \cos\mu - b - d_l)^2} \end{aligned} \tag{12}$$

with

$$\begin{aligned} R_{Bmin} &= \sqrt{{R_{min}}^2 + d_{EB} - 2R_{min}d_{EB}\cos(\pi/2 + \alpha)} \\ R_{Bmax} &= \sqrt{{R_1}^2 + d_{EB} - 2R_1 d_{EB}\cos(\pi/2 + \alpha + \mu)} \\ \alpha &= \arccos\frac{a + d_{front}}{d_{EB}} \end{aligned} \tag{13}$$

Cases with $L_{min} = L_{limmin}$ and with $L_{min} = L_{limmax}$ are presented in Fig. 6.

E. Simulations and discussion

Simulations on Matlab were performed with the model indicated in the section 2. The Fig. 7 shows two simulations of the present method for different initial orientations for a parking in one trial, with a first forward (left) or backward (right) motion.

It is interesting to compare the minimal length of the parking spot in one trial with a geometric path formed by circle arcs and with the present method. The minimal length for parking in n precise maneuvers can be calculated by generalization. Like in [9], the minimal length L_{arc} for a parking in one maneuver for a geometric method with circle arcs is:

$$L_{arc} = d_{rear} + \sqrt{R_B^2 - (R_{min} - b - d_l)^2} \tag{14}$$

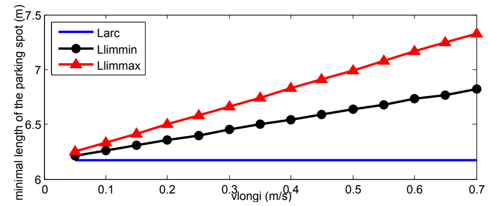

Fig. 8. Variation of minimal length of the parking spot for $v_\delta = 20^o/s$

The Fig. 8 shows the variation of L_{limmin} and L_{limmax} in function of the longitudinal velocity for $v_\delta = 20^o/s$. These lengths are bigger than L_{arc} but considerably decrease when v_{longi} decreases. Nevertheless, these minimal length are acceptable considering that the parking in several maneuvers is possible and that continuous-curvature path is preferable.

IV. Generation of Reference Trajectories

To make the vehicle follow the generated path, control signals of the steering angle δ and longitudinal velocity v need to be built. They are generated for the middle of rear track, in consequence each radius concerned in the circle arcs can be calculated by $R = a/\tan\delta$.

These commands are open-loop in the (x, y, ψ)-coordinates. The steering column and the engine are controlled to execute these commands to provide the desired path and orientation of the vehicle.

The lengths of each sequence L_{seq} is calculated from the lengths of the clothoids and of the circle arcs L_{arc}. The length L_{line} for the first movement in a straight line is easily deduced from the calculated positions of the point E.

A. Control signal of the longitudinal velocity

To generate longitudinal velocity, the same method as in [9] is used. The needed time to reach a desired maximal velocity or to reach null velocity from the maximal velocity v_{max} is $t_1 = v_{max}/\gamma_{des}$, where γ_{des} is the desired acceleration for the parking maneuver. During t_1, the vehicle goes over the distance $d_1 = \gamma_{des}{t_1}^2/2$. Then, the vehicle stays at constant speed v_{max} during the time $t_2 = (l_{arc} - 2d_1)/v_{max}$.

The time control signal for each forward or backward motion is:
$\forall t \in (0, 2t_1 + t_2)$

$$v(t) = \begin{cases} k_v \gamma_{des} t & t \in [0, t_1] \\ k_v v_{max} & t \in [t_1, t_1 + t_2] \\ -k_v \gamma_{des} t & t \in [t_1 + t_2, 2t_1 + t_2] \end{cases} \tag{15}$$

where $k_v = \pm 1$ corresponds to forward (+1) or backward (-1) motion.

B. Control signal of the steering angle

The velocity control allows to calculate the covered distance for each time step. Therefore, a distance control of the steering angle equivalent to a time control is established. This method is comparable to the time-scaling method presented in [23].

- For the initial movement in a straight line:

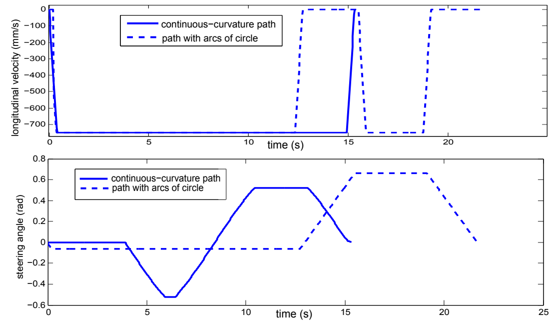

Fig. 9. Simulation of the control for the parking in one maneuver

$$\forall d \in (0, L_{line}) \qquad \delta(d) = 0 \tag{16}$$

- For a sequence case A (Eq. 17), case B (Eq. 18) and case C (Eq. 19) as defined in section IIIA:

$\forall d \in (0, L_{seq})$

$$\delta(d) = \begin{cases} k_\delta * |\arctan \frac{a}{R}| & d \in [0, L_{min}] \\ k_\delta * |\arctan \frac{a}{R_{min}}| & d \in [L_{min}, L_{min} + L_{arc}] \\ k_\delta * |\arctan \frac{a}{R_i}| & d \in [L_{min} + L_{arc}, L_{seq}] \end{cases} \tag{17}$$

$$\delta(d) = \begin{cases} k_\delta * |\arctan \frac{a}{R_{new}}| & d \in [0, L_{new}] \\ k_\delta * |\arctan \frac{a}{R_{newi}}| & d \in [L_{new}, L_{seq}] \end{cases} \tag{18}$$

$$\delta(d) = \begin{cases} k_\delta * |\arctan \frac{a}{R}| & d \in [0, L_{caseC}] \\ k_\delta * |\arctan \frac{a}{R_i}| & d \in [L_{caseC}, L_{seq}] \end{cases} \tag{19}$$

where $k_\delta = \pm 1$ corresponds to a left (+1) or right steering (-1), $R = \frac{A^2}{d}$, $R_i = \frac{A^2}{L_{seq}-d}$, $R_{new} = \frac{{A_{new}}^2}{d}$ and $R_{newi} = \frac{{A_{new}}^2}{L_{seq}-d}$

C. Simulations and discussion

The Fig. 9 compares simulated examples of parking in one trial for the present method and for a geometric method with circle arcs with same maximal longitudinal and steering velocities, with an initial position similar to the right part of Fig. 7. With the present method, in this example, the parking is 30% faster.

V. Conclusion

A new parallel parking path planning with continuous-curvature paths was proposed outgoing from the geometric path planning. This method is valid independently of the initial position and the orientation of the vehicle and offers the possibility to park the vehicle as long as its length and width are smaller than those of the parking spot. This method allows a faster parking than the geometric method with circle arcs. Moreover, the steering being done during movement, there are less solicitation on the steering column and less wear of the tires. Simulations were performed and in the future, the method will be tested on a real vehicle. This method can also be generalized for other parking configurations.

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