Estimation of Deterministic and Stochastic IMU Error Parameters

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***Abstract*— Inertial Measurement Units, the main component of a navigation system, are used in several systems today. IMU's main components, gyroscopes and accelerometers, can be produced at a lower cost and higher quantity. Together with the decrease in the production cost of sensors it is observed that the performances of these sensors are getting worse. In order to improve the performance of an IMU, the error compensation algorithms came into question and several algorithms have been designed. Inertial sensors contain two main types of errors which are deterministic errors like scale factor, bias, misalignment and stochastic errors such as bias instability and scale factor instability. Deterministic errors are the main part of error compensation algorithms. This study explains the methodology of how the deterministic errors are defined by 27 state static and 60 state dynamic rate table calibration test data and how those errors are used in the error compensation model. In addition, the stochastic error parameters, gyroscope and bias instability, are also modeled with Gauss Markov Model and instant sensor bias instability values are estimated by Kalman Filter algorithm. Therefore, accelerometer and gyroscope bias instability can be compensated in real time. In conclusion, this article explores how the IMU performance is improved by compensating the deterministic end stochastic errors. The simulation results are supported by real IMU test data.**

Keywords: Inertial Measurement Unit, gyroscope, accelerometer, Kalman filter.

I. INTRODUCTION

Navigation is the art of getting from one place to another, safely and efficiently [1]. From past to present, several tools and systems such as compasses, maps, sun, stars were used for navigation. In today's world these tools have been replaced by electronic equipments such as sensors, antennas, etc. These electronic equipments form the basis of the modern navigation systems. Inertial Navigation Systems and Global Positioning Systems can be shown as an example for the modern navigation systems. Nowadays several types of INS, GPS and integrated INS/GPS are used in different platforms such as aircrafts, ships, guided missiles and UAVs.

GPS acquires and processes satellite signals to calculate navigation parameters such as position, velocity and attitude, according to the received signals. GPS always need satellite signals and this is the major drawback of GPS. However, INS uses IMU outputs to construct position velocity and attitude by processing the navigation equations. Therefore IMUs are the major part of inertial navigation systems.

An IMU is a device, which is used to measure linear acceleration and angular rate. Inertial measurement units contain two types of sensor, accelerometer and gyroscope. An accelerometer measures linear acceleration about its sensitivity axis and integrated acceleration measurements are used to calculate velocity and position. Besides a gyroscope measures angular rate about its sensitivity axis and gyroscope outputs are used to maintain orientation in space.

The cost of an IMU increases when the sensor performance requirements increase. The major reasons for the cost increase can be explained in two ways. The first reason is the highly skilled production line requirement and the second reason is the decrease in the percentage of utilizable sensor in the batch. Therefore, in order to improve the performance of inertial sensors, the calibration algorithms and the error compensation models were researched and developed. Thereby both low-cost and high-performance IMUs could be produced.

The main objective of this article is to develop methods in order to estimate deterministic and stochastic error parameters of MEMS based inertial measurement units. Additionally, improving the performance of IMUs is aimed by using these estimated parameters. Therefore an error calibration algorithm is implemented and estimated parameters are used in this algorithm.

II. IMU ERROR MODEL

Inertial navigation systems need acceleration and angular rate measurements in the x, y and z- directions to calculate attitude, position and velocity. Therefore, inertial measurement units contain three accelerometer and three gyroscope. For this reason, IMU error model is determined with equation (1) and (2) [3,4].

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$$\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} = \begin{bmatrix} 1+S_x+\delta S_x & M_{xy} & M_{xz} \\ M_{yx} & 1+S_y+\delta S_y & M_{yz} \\ M_{zx} & M_{zy} & 1+S_y+\delta S_y \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} B_x+\delta B_x \\ B_y+\delta B_y \\ B_z+\delta B_z \end{bmatrix} + \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \quad (1)$$

$\tilde{w}_x$: gyroscope output $\quad$ $\tilde{a}_x$: accelerometer output

w_x: actual angular rate $\quad$ a_x: actual acceleration

S_x: scale factor error $\quad$ δS_x: scale factor instability

B_x: bias $\quad$ δB_x: bias instability

B_{Gx}: g-dep bias coeff. $\quad$ n_x: sensor noise

$M_{xy}, M_{xz}, M_{yx}, M_{yz}, M_{zx}, M_{zy}$: misalignment errors

$$\begin{bmatrix} \tilde{w}_x \\ \tilde{w}_y \\ \tilde{w}_z \end{bmatrix} = \begin{bmatrix} 1+S_x+\delta S_x & M_{xy} & M_{xz} \\ M_{yx} & 1+S_y+\delta S_y & M_{yz} \\ M_{zx} & M_{zy} & 1+S_y+\delta S_y \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} + \begin{bmatrix} B_x+\delta B_x \\ B_y+\delta B_y \\ B_z+\delta B_z \end{bmatrix} +$$

$$\begin{bmatrix} B_{gx} & 0 & 0 \\ 0 & B_{gy} & 0 \\ 0 & 0 & B_{gz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \quad (2)$$

A. Deterministic Error Parameters

Accelerometer bias can be defined as the accelerometer output at zero g [14]. It means that when no input acceleration is applied to the sensor, measured acceleration presents bias. Unit of accelerometer bias is mili-g. Gyroscope bias is the gyroscope output in the absence of an applied angular rate. It means that when no input angular rate is applied to the sensor, measured angular rate presents bias. Unit of gyroscope bias is deg/h. The bias includes fixed terms, temperature induced variations, turn-on to turn-on variations and in-run variations [15]. Fixed terms of bias and temperature induced variations in bias can be estimated by laboratory calibration tests and estimated fixed bias and temperature induced variations are used as an input of error compensation algorithms.

Scale factor error is errors in the ratio of a change in the output signal to a change in the input acceleration/angular rate which is to be measured [3]. The magnitude of the scale factor is expressed in parts per million (ppm) or percent. The scale factor error includes fixed terms, temperature induced variations, asymmetry and nonlinearity error parts. The major parts of scale factor error are fixed terms and temperature induced variations. Similar to the bias, scale factor errors can be estimated by laboratory calibration tests and estimated error parameters constitute input of error compensation or calibration algorithms.

Accelerometers and gyroscopes should be mounted orthogonal to observe true measurements about its sensitivity axis. But mechanical components cannot be produced perfectly and these components cannot be mounted perfectly. This will cause a nonorthogonality between the IMU axes and this nonorthogonality creates a scale factor effect on measurements. Any movements in any axis causes a change in the other axes depending on the magnitude of misalignment. The unit of misalignment is mili-radian and the misalignment error can be determined by laboratory calibration tests and used in error compensation algorithms.

Structure of the MEMS gyroscope is affected by acceleration. Therefore this effect causes an offset in the output signal. This bias component is proportional to the acceleration magnitude which is applied about the measurement axis [3]. Therefore there is a relationship between acceleration and gyroscope measurement and this magnitude of this relationship is determined with g-dependent bias coefficient. Like other deterministic errors, g-dependent bias coefficients can be determined by laboratory tests and these coefficients are used in calibration algorithms. Unit of the g-dependent bias is deg/h/g.

B. Stochastic Error Parameters

Stochastic errors are the random errors that occur due to random variations of bias or scale factor drift over time and random sensor noise [6]. Random variations in bias and scale factor are the low frequency components of the stochastic errors. The sensor noise is also high frequency components of the stochastic errors. The most important feature of stochastic errors is there may not be any direct relationship between input and output [4].The source of the stochastic errors are flicker noise in the electronics and interference effects on signals. Allan variance tests and autocorrelation analysis are performed to determine the stochastic characterization of inertial sensors. In addition to that several random processes exist for modeling stochastic errors.

Bias instability occurs due to change in bias during a run [3]. In other words, bias instability represents the variations in bias which change with time. Bias instability can be characterized by Allan variance and autocorrelation analysis and modeled by using results of these tests and analysis. Various methods (random processes) such as Random Walk Model, Gauss-Markov Model, Random Constant Model and Autoregressive Model are used to model stochastic errors. First order Gauss-Markov Model is the most selected and the most appropriate random process for modeling bias instability [4,5]. Therefore the random process Gauss- Markov Model was used to model gyro and accelerometer bias instability in this study. Stochastic model of the bias instability is presented in equation (3), (4) and (5). The bias instability of sensors is denoted by 1σ value. It means that this error type has Gaussian distribution.

$$x_k = e^{-dt/T_c} x_{k-1} + w_k \quad (3)$$

$$\sigma_{x_k}^2 = \frac{\sigma_{w_k}^2}{1-e^{-2dt/T_c}} \quad , \textit{bias instability variance} \quad (4)$$

$$\sigma_{w_k}^2 = \sigma_{x_k}^2 \left(1-e^{-2dt/T_c}\right) , \textit{driven noise variance} \quad (5)$$

Scale factor instability represents the variations in scale factor which change with time. Scale factor instability characterization test is different from the bias instability characterization test method. Scale factor instability characterization needs long term dynamical rate test and the effect of the scale factor instability is not very observable and quite negligible. Therefore, this error component excluded from the scope of this study.

Effect of sensor noise can be reduced by filtering. Low pass filters are designed according to sensor noise bandwidth and noise power. Random sensor noise is modeled as a zero mean white noise in the IMU error models. And these system specific filters are used in error compensation algorithms. Unit of the random sensor noise density is deg/h/ $\sqrt{Hz}$ or deg/s/ $\sqrt{Hz}$ and the random noise density is represented by 1σ value.

III. ERROR COMPENSATION MODEL

The aim of deterministic error compensation algorithm is compensating the effect of deterministic error parameters of an IMU. A deterministic error compensation algorithm was developed and implemented within the scope of this article. Output of the compensation model was used as input of the stochastic error estimation algorithm.

The error model of the three axis accelerometer and gyroscope were given in equation (1) and (2). IMU deterministic error compensation algorithms are based on inverse of these models. Deterministic error compensation model depends on equation (6) and (7).

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} 1+S_x & M_{xy} & M_{xz} \\ M_{yx} & 1+S_y & M_{yz} \\ M_{zx} & M_{zy} & 1+S_z \end{bmatrix}^{-1} \begin{bmatrix} \tilde{a}_x - B_x \\ \tilde{a}_y - B_y \\ \tilde{a}_z - B_z \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \hat{w}_x \\ \hat{w}_y \\ \hat{w}_z \end{bmatrix} = \begin{bmatrix} 1+S_x & M_{xy} & M_{xz} \\ M_{yx} & 1+S_y & M_{yz} \\ M_{zx} & M_{zy} & 1+S_z \end{bmatrix}^{-1} \left[\begin{bmatrix} \tilde{w}_x - B_x \\ \tilde{w}_y - B_y \\ \tilde{w}_z - B_z \end{bmatrix} - \begin{bmatrix} B_{gx} & 0 & 0 \\ 0 & B_{gy} & 0 \\ 0 & 0 & B_{gz} \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} \right] \quad (7)$$

IV. DETERMINISTIC ERROR ESTIMATION ALGORITHM

Multi-position static tests and multi-rate dynamic tests are performed to estimate deterministic errors of accelerometers and gyroscopes. Contents of these tests must be designed according to the specifications of the sensors.

A. *Multi-Position Static Tests*

Multi-position test is performed to extract accelerometer scale factor error, misalignment and bias. In addition to that gyroscope bias and g-dependent bias coefficients can be extracted by using multi-position static tests.

IMU is placed on the rate table or flight motion simulator with a fixture. The accelerometer and gyroscope data is collected about 3 axis at 27 different positions. Local gravity is the reference input in this tests and the magnitude of acceleration which is sensed by the accelerometer is changed in each position by changing the axis position relative to local gravity. The angle between the measurement axis and the gravity vector is changed with 22.5 degree. Therefore the components of gravity vector can be observed on the other axes. This test method supplies to scan 1g to -1g acceleration range with more input. In this way estimated error parameters which are close to actual error parameters can be obtained. Furthermore bias and g-dependent bias coefficients of gyroscopes can be determined more accurately.

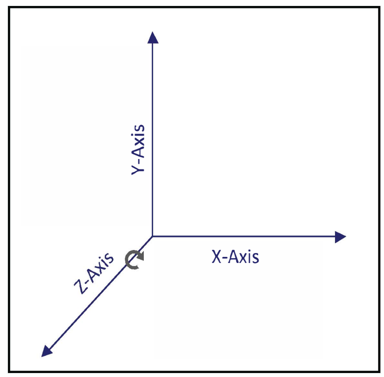

Figure 1. IMU measurement axis configuration

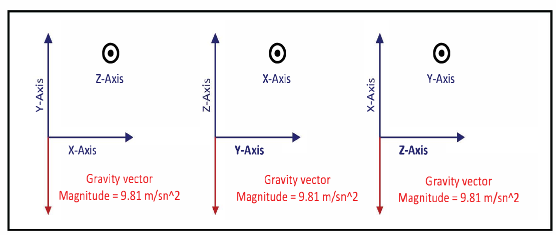

Figure 2. Multi-Position Test Configurations

IMU is turned about z axis when x axis multi position static test is performed. Similarly, IMU is turned around x axis when y axis multi position static test is performed and, IMU is turned about/around y axis when z axis multi position static test is performed.

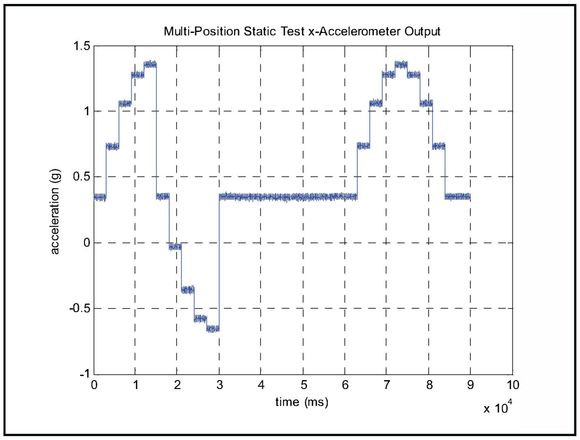

Figure 3. Multi-position static test x acc output

Fig.3 shows the accelerometer measurements which are observed during the test period.

B. *Multi-Rate Dynamic Tests*

This test method is performed to extract gyroscope scale factor error and misalignment. Like multi-position static test, special test equipments like rate table, FMS are required to perform multi-rate dynamic test. IMU is placed on the rate table or flight motion simulator with a fixture. 3 axis gyroscope data is collected during the test period. Each axis test procedure has 20 different states. All states represent different angular rate. These states include 10 positive and 10 negative angular rate and vary between 0 and 200 deg/s. Fig. 4 presents multi-rate dynamic test gyroscope measurements.

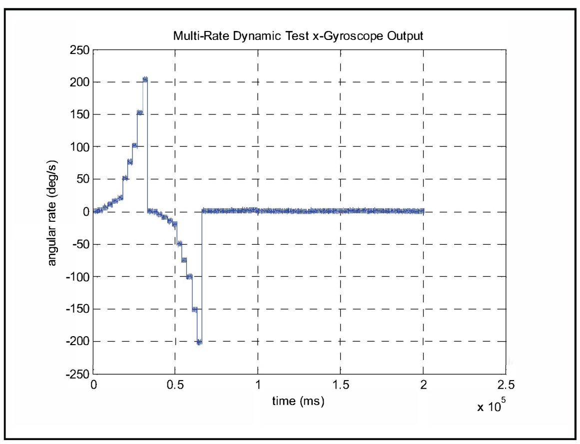

Figure 4. Multi-rate dynamic test x gyro output

C. *Data Processing Algorithm for Multi-Position Static and Multi-Rate Dynamic Tests*

Processing steps are explained below:

1. Decomposing position states and angular rates from test data

2. Taking the average value of each position and rate.

3. Creating the true and measurement matrices for least square fitting. (Each measurement has equal weight to calculate calibration parameters)

4. Implementing least squares fitting and calculating error (calibration) parameters.

Deterministic error estimation depends on least squares fitting method implementation. Multi position test data is used to determine accelerometer bias, scale factor error and misalignment parameters. Additionally, gyroscope bias and g-dependent bias parameters are also extracted from multi-position static test data.

Least squares fitting formulation for accelerometer and gyroscope are given in equations (8) and (9).

$$P_{acc} = M_{acc} T_{acc}^{\ T} (T_{acc} T_{acc}^{\ T})^{-1} \tag{8}$$

$$P_{acc} = \begin{bmatrix} 1+S_x & M_{xy} & M_{xz} & B_x \\ M_{yx} & 1+S_y & M_{yz} & B_y \\ M_{zx} & M_{zy} & 1+S_z & B_z \end{bmatrix}$$, matrix of estimated accelerometer error parameters

M_{acc} = accelerometer measurement matrix

T_{acc} = reference input matrix

$$P_{gyr} = M_{gyr} T_{gyr}^{\ T} (T_{gyr} T_{gyr}^{\ T})^{-1} \tag{9}$$

$$P_{gyr} = \begin{bmatrix} B_{gx} & 0 & 0 & B_x \\ 0 & B_{gy} & 0 & B_y \\ 0 & 0 & B_{gz} & B_z \end{bmatrix}$$, matrix of estimated gyroscope error parameters for static tests

$$P_{gyr} = \begin{bmatrix} 1+S_x & M_{xy} & M_{xz} & B_x \\ M_{yx} & 1+S_y & M_{yz} & B_y \\ M_{zx} & M_{zy} & 1+S_z & B_z \end{bmatrix}$$, matrix of estimated gyroscope error parameters for dynamic tests

M_{gyr} = gyroscope measurement matrix

T_{gyr} = reference input matrix

In order to observe the effects of IMU parameters, error model was simulated. Real MEMS gyroscope and accelerometer error parameters were fed into the simulation. Multi-position static test and multi-rate dynamic tests were simulated. Additionally, output of the simulation was processed and deterministic errors were estimated by using least squares fitting method. Comparative table of real and estimated error parameters are presented in Table 1 and Table 2.

TABLE I. ESTIMATED GYROSCOPE ERROR PARAMETERS

Gyro	Bias	Scale Factor Error	Misalignment	g-dependent bias
x real	1.1464 deg/s	10817 ppm	xy 7.5879 xz 3.3769 mrad	0.05 deg/s/g
x estimated	1.1457 deg/s	10802 ppm	xy 7.5812 xz 3.3749 mrad	0.0522 deg/s/g
y real	1.1241 deg/s	9124 ppm	yx 5.6617 yz 6.3705 mrad	0.05 deg/s/g
y estimated	1.140 deg/s	9079 ppm	yx 5.6634 yz 6.3827 mrad	0.0488 deg/s/g
z real	1.6820 deg/s	15529 ppm	zx 7.5009 zy 3.8084 mrad	0.05 deg/s/g
z estimated	1.6756 deg/s	15584 ppm	zx 7.5156 zy 3.7935 mrad	0.0526 deg/s/g

TABLE II. ESTIMATED ACCELEROMETER ERROR PARAMETERS

Acc	Bias	Scale Factor Error	Misalignment
x real	349.66 mg	3916 ppm	xy 3.9824 xz 0.6601 mrad
x estimated	349.6557 mg	4003 ppm	xy 3.8995 xz 0.5115 mrad
y real	346.36 mg	3284 ppm	yx 1.6875 yz 0.8487 mrad
y estimated	346.2070 mg	3106 ppm	yx 1.4454 yz 0.9496 mrad
z real	250.18 mg	4733 ppm	zx 2.5776 zy 2.7814 mrad
z estimated	249.9346 mg	4964 ppm	zx 2.5641 zy 2.7139 mrad

The raw data (output of the IMU error model) is calibrated with IMU error compensation model by using the estimated deterministic error parameters. After the calibration process, the difference between the calibrated data and the reference data (IMU error model input data) was calculated to evaluate the performance of the error compensation algorithm. This difference represents the IMU's total residual error. The magnitude of the residual errors vary depending on the difference between the actual and the estimated error parameters. Furthermore, the magnitude of bias instability and random sensor noise also affect the magnitude of residual errors.

In addition to simulation studies a tactical grade commercial IMU was used to perform real experiments. The commercial IMU which was used in the experiments contains three identical MEMS accelerometer and three identical MEMS gyroscope.

Multi-position static test and multi-rate dynamic test were performed respectively. The IMU data which was collected during the multi-position static test and multi-rate dynamic test were used to estimate deterministic errors of the IMU.

Acutronic 3-axis rate table was used to simulate reference position and angular rate. In addition, test computer and power supply were used in the experimental tests.

Deterministic error parameters of the IMU were estimated by processing the collected test data accordingly deterministic error estimation algorithm. Error compensation (calibration) simulation was run by using estimated deterministic error parameters and calibrated IMU outputs were acquired. And it was observed that deterministic error estimation and compensation provided 99% improvement in accelerometer performance. Similarly, 95% improvement was ensured in gyroscope performance. Cfollected raw IMU data and calibrated IMU data shows that the error compensation (calibration) model outputs oscillate around the reference test states.

V. STOCHASTIC ERROR COMPENSATION ALGORITHM

As mentioned before gyroscope and accelerometer bias instabilities are modeled with first order Gauss-Markov process. An IMU has three accelerometers and three gyroscopes for measuring linear acceleration and angular rate about x, y and z axis. Therefore six sensors' bias instabilities were estimated by using Kalman filter and the state vector contains six states and these states represent the bias instability of x gyroscope, y gyroscope, z gyroscope, x accelerometer, y accelerometer and z accelerometer respectively. The bias instability model of an inertial sensor was given with equation (3). That equation was developed for six sensor (3 gyroscope and 3 accelerometer) implementation and developed system model is presented in equation (8).

$$\underbrace{\begin{bmatrix} \delta B_{gx_k} \\ \delta B_{gy_k} \\ \delta B_{gz_k} \\ \delta B_{ax_k} \\ \delta B_{ay_k} \\ \delta B_{az_k} \end{bmatrix}}_{x_k} = \underbrace{\begin{bmatrix} e^{-dt/T_{gxc}} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-dt/T_{gyc}} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-dt/T_{gzc}} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-dt/T_{axc}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-dt/T_{ayc}} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-dt/T_{azc}} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \delta B_{gx_{k-1}} \\ \delta B_{gy_{k-1}} \\ \delta B_{gz_{k-1}} \\ \delta B_{ax_{k-1}} \\ \delta B_{ay_{k-1}} \\ \delta B_{az_{k-1}} \end{bmatrix}}_{x_{k-1}} + \underbrace{\begin{bmatrix} gyrx\_bias\_w_k \\ gyry\_bias\_w_k \\ gyrz\_bias\_w_k \\ accx\_bias\_w_k \\ accy\_bias\_w_k \\ accz\_bias\_w_k \end{bmatrix}}_{w_{k-1}} \quad (8)$$

δB_{gx_k} : x gyro instability at time k , T_{gxc} : x gyro time constant

δB_{gy_k} : y gyro instability at time k , T_{gyc} : y gyro time constant

δB_{gz_k} : z gyro instability at time k , T_{gzc} : z gyro time constant

δB_{ax_k} : x acc instability at time k , T_{axc} : x acc time constant

δB_{ay_k} : x acc instability at time k , T_{ayc} : y acc time constant

δB_{az_k} : x acc instability at time k , T_{azc} : z acc time constant

$\delta B_{gx_{k-1}}$: x gyro instability at time $k-1$, $gyrx\_bias\_w_k$: x gyro driven noise

$\delta B_{gy_{k-1}}$: y gyro instability at time $k-1$, $gyry\_bias\_w_k$: y gyro driven noise

$\delta B_{gz_{k-1}}$: z gyro instability at time $k-1$, $gyrz\_bias\_w_k$: z gyro driven noise

$\delta B_{ax_{k-1}}$: x acc instability at time $k-1$, $accx\_bias\_w_k$: x acc driven noise

$\delta B_{ay_{k-1}}$: x acc instability at time $k-1$, $accy\_bias\_w_k$: y acc driven noise

$\delta B_{az_{k-1}}$: x acc instability at time $k-1$, $accz\_bias\_w_k$: z acc driven noise

Sensor measurements represent measurement vector and random sensor noises represent the measurement noise vector. After the compensation of deterministic errors, gyroscope and accelerometer bias instabilities can be observed from sensor measurements directly. Therefore, measurement conversion matrix C tuned as an identity matrix.

The error compensation model's outputs were used as measurements in Kalman filter algorithm. Therefore measurement matrix was selected as an identity matrix.

$$\underbrace{\begin{bmatrix} gyr\_x\_output_k \\ gyr\_y\_output_k \\ gyr\_z\_output_k \\ acc\_x\_output_k \\ acc\_y\_output_k \\ acc\_z\_output_k \end{bmatrix}}_{z_k} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \delta B_{gx_k} \\ \delta B_{gy_k} \\ \delta B_{gz_k} \\ \delta B_{ax_k} \\ \delta B_{ay_k} \\ \delta B_{az_k} \end{bmatrix}}_{x_k} + \underbrace{\begin{bmatrix} gyr\_x\_noise_k \\ gyr\_y\_noise_k \\ gyr\_z\_noise_k \\ acc\_x\_noise_k \\ acc\_y\_noise_k \\ acc\_z\_noise_k \end{bmatrix}}_{v_k} \tag{9}$$

All inertial sensors have some characteristic parameters such as time constant, random noise density and bias instability. In this study these parameters are very useful inputs for Kalman filter parameter tuning and algorithm initialization. For example, gyroscope and accelerometer random noise densities were used to determine measurement noise covariance, R . Similarly, 3σ value of gyroscope and accelerometer bias instabilities were used to constitute initial value of error covariance matrix, P . Finally, It was assumed that $\delta B_{gx_k} = \delta B_{gy_k} = \delta B_{gz_k} = \delta B_{ax_k} = \delta B_{ay_k} = \delta B_{az_k} = 0$ for k=0.

TABLE III. STOCHASTIC PROPERTIES OF THE IMU SENSORS

	Correlation Time (Time Constant)	**Bias Instability (1σ)**	**Sensor Noise Density (1σ)**
Gyroscopes	15 seconds	0.005 deg/s	0.0015 deg/s/ $\sqrt{Hz}$
Accelerometers	1 second	0.5 mg	0.00027 g/ $\sqrt{Hz}$

Stochastic properties of the simulated IMU are given in Table 3. Allan variance tests and autocorrelation analysis are performed to determine the stochastic characterization of inertial sensors.

State transition matrix, A, was calculated according to the stochastic properties, correlation time and bias instability, of the gyroscopes and accelerometers.

First of all, effect of bias instability was simulated in IMU error model and simulation outputs were used in Kalman filter algorithm. Real instability values were observed and recorded during the simulation period. In this way, estimated instability values and real instability values were compared. The following figures present the performance of the Kalman filter algorithm which was used to estimate sensor bias instability.

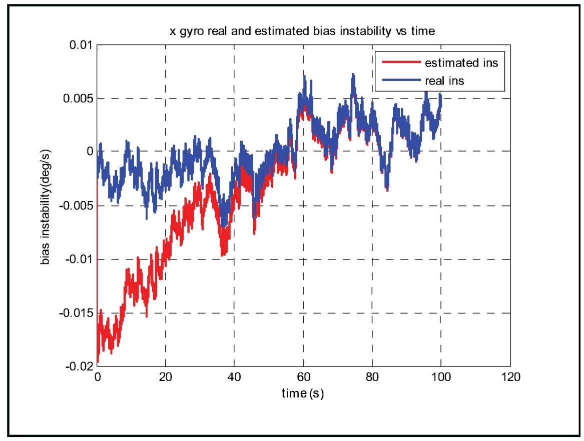

Figure 5. x gyroscope estimated bias instability

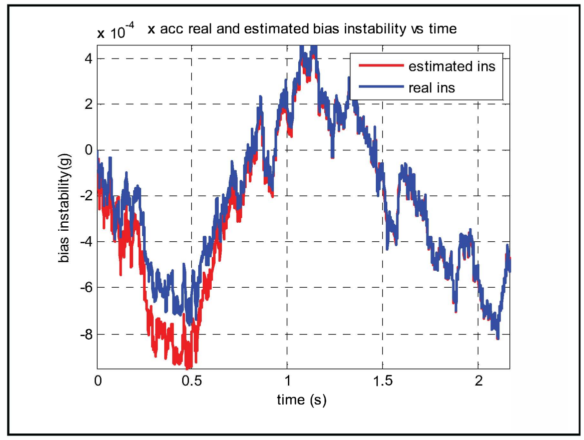

Figure 6. x accelerometer estimated bias instability

At the end of 1 second, accelerometer bias instability estimations converged to real accelerometer bias instability values. On the other hand, convergence of gyroscope instability took 60 seconds. Gyroscope's sensor noise density and correlation time are greater than the accelerometer's noise level and correlation time. It can be observed that, the performance of estimation is directly affected by noise density and correlation time.

As mentioned earlier, random sensor noise represents the uncertainty in sensor outputs. Therefore, an increase in sensor noise level causes an increase in convergence duration. Additionally, correlation time gives information about the relationship between successive values of sensor bias instability and the relationship between consecutive values of bias instability reduces when the sensor correlation time increases. This means that, an increase in correlation time leads an increase in convergence duration.

After the simulation studies real experiment was performed. Stochastic properties of the real simulated IMU are equal to simulated IMU's stochastic properties which are given in Table 3. IMU data was collected in a fixed position for 120 seconds to perform stochastic error estimation test.

It was observed that experimental test results are similar to simulated test results. Instantaneous values of the bias instabilities oscillates between -3σ and $+3\sigma$. (1σ value of the gyroscope or accelerometer bias instability is the specification of the sensor.) This results shows that performance of the estimation algorithm is consistent. Behavior of the simulation results and real experiment results are the same.

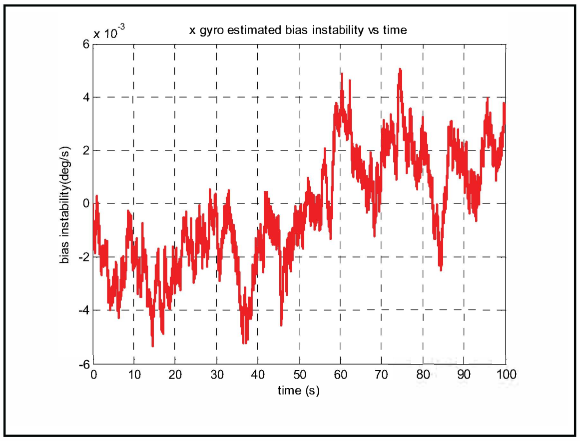

Figure 7. x gyro estimated bias instability (real data)

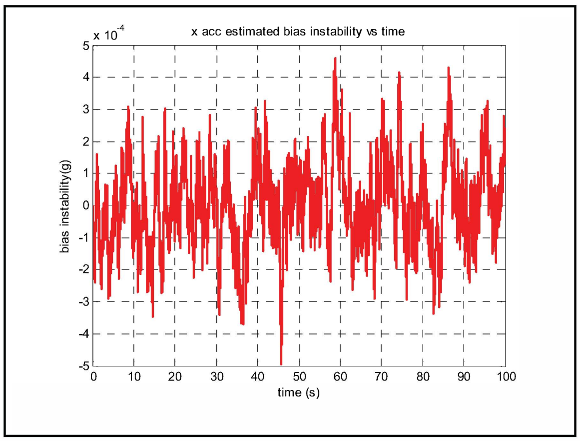

Figure 8. x acc estimated bias instability (real data)

VI. Conclusion

This study suggests some methods for estimation of deterministic and stochastic IMU error parameters. Estimation of deterministic error parameters and stochastic error parameters were investigated separately. Firstly deterministic error estimation methods were researched. In order to determine the optimum calibration method multi rate & position method were implemented and test results were discussed. According to the results, multi rate & position method determined as the optimum method for estimation of deterministic IMU errors. After estimation of deterministic error parameters, error compensation model was designed to fix deterministic error effects. After the correction of deterministic errors, the remaining errors, difference between reference inputs and error compensation model outputs, were defined as stochastic errors and these errors were divided into two groups. First group was defined as random noise and the second group was determined as gyroscope/accelerometer bias instability. The second group errors, gyroscope and bias instabilities, were estimated by using Kalman filter algorithm. Thereby effect of bias sensor instabilities of could be corrected. The purpose of all these studies is improving the performance of MEMS IMU.

Deterministic errors are the major part of IMU errors. Therefore, deterministic error estimation and compensation algorithm design is more critical than stochastic error estimation and compensation algorithm. However, bias instability error compensation becomes very crucial in long-term navigation. Because of this, deterministic and stochastic error compensation should be considered together.

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