- Let `S` be the set of all vertices on the shortest path found by Dijkstra's algorithm.
- Prove that the `distances[u]` is the minimum length between the vertex `u` and the target vertex `t`.

Proof: Base case

  - When `|S| = 1`, it is true.
- Induction case
  - Inductive hypothesis: Assume that it is true for `|S| = k > 1`.
  - For each target vertex t in S, distances [u] is the shortest path.
  - Given k>1 vertices. Currently distances[u] is the minimum length between u and t.
  - Let v be the next node added to the S. And let t-v be the chosen edge. Assume there is a shorter path than P (u-v). Let P' denote such a path.
  - However P' is already longer (or at least equal if b does not exist because edges have positive weights) than P as soon as it leaves S. Therefore it contradicts the assumption that P' is shorter than P. P and therefore dist[u] is the shortest.