MATH40082 Example Mini Task Solution

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1 Solution

This coursework calculates the value of the financial contract $\Pi(S,t)$ at time $t=0$ which has the formula

$$d_1(S,t) = \frac{\exp\left(\frac{S}{X}-1\right) - 1 + r(T-t)\sqrt{1+\left(\frac{\sigma^2}{r}\right)}}{\exp\left(\sigma\sqrt{T-t}\right)-1} \tag{1}$$

$$d_2(S,t) = \frac{\exp\left(\frac{S}{X}-1\right) - 1 - q(T-t)\exp\left(1-\frac{\sigma^2}{q}\right)}{\exp\left(\sigma\sqrt{T-t}\right)-1} \tag{2}$$

$$\Pi(S,t) = S\left(1+\frac{S}{X}\right)^{\frac{1}{2}} e^{-r(T-t)}N(d_1) - X\ln\left(1+\frac{X}{S}\right)e^{-q(T-t)}N(d_2) \tag{3}$$

where $T=2.5$, $X=3000$, $r=0.0356$, $q=0.0371$, $\sigma=0.1625$, and $N(x)$ is the standard normal cumulative distribution function. For the below values of asset S in column 1 of Table 1, the results of the above equations are found and given in columns 2, 3, and 4, of Table 1.

S	d_1	d_2	$\Pi(S,t=0)$
2250.0000	-0.3541	-1.1774	707.8276
2400.0000	-0.2178	-1.0411	888.7286
2550.0000	-0.0745	-0.8978	1099.6322
2700.0000	0.0761	-0.7472	1340.9128
2850.0000	0.2345	-0.5888	1611.3212
3000.0000	0.4009	-0.4224	1907.7393
3150.0000	0.5759	-0.2474	2225.1785
3300.0000	0.7599	-0.0634	2557.1256
3450.0000	0.9533	0.1300	2896.2789
3600.0000	1.1567	0.3334	3235.6085
3750.0000	1.3704	0.5471	3569.5328

Table 1: The calculated value of the contract Π at time $t=0$ for different values of the underlying asset S.

2 Code

My code for the pricing method used to make Table 1 is in Listing 1. Note: the imports and output commands have been excluded as requested.

Listing 1: Code for contract pricing, mini task-1 coursework.

```
# Importing libraries
import math
from scipy.stats import norm

# Function to calculate the price of a financial contract given relevant parameters
def financial_contract_price(S, t, T, X, r, q, sigma):

  d1 = (math.exp((S/X) - 1) - 1 + r*(T - t)*math.sqrt(1 + ((sigma**2) / r))) /(math.exp(
      sigma*math.sqrt(T - t)) - 1)
  d2 = (math.exp((S/X) - 1) - 1 - q*(T - t)*math.exp(1 - ((sigma**2) / q))) / (math.exp(
      sigma * math.sqrt(T - t)) - 1)
  Pi = S*((1 + (S/X))**0.5)*math.exp(-r*(T - t))*norm.cdf(d1) - X*math.log(1 + (X/S))*math.
      exp(-q*(T - t))*norm.cdf(d2)

  # Return values to be printed
  return [S, d1, d2, Pi]

# Main to run the program
def main():
  # Given list of S values
  S_list = [2250, 2400, 2550, 2700, 2850, 3000, 3150, 3300, 3450, 3600, 3750]

  print('S | d1 | d2 | Pi')
  print('-----------------')

  # Loop through the list of S values and calculate the contract price for each
  for S in S_list:
    t = 0
    T = 2.5
    X = 3000
    r = 0.0356
    q = 0.0371
    sigma = 0.1625

    contract_price_i = financial_contract_price(S, t, T, X, r, q, sigma)

    # Print the values of S, d1, d2 and Pi
    print('%.4f | %.4f | %.4f | %.4f' % (S, contract_price_i[1], contract_price_i[2],
        contract_price_i[3]))

# Run main function
if __name__ == '__main__':
  main()
```