Linear Algebra II - Homework 9

1. Let $X = \{(2,2), (-1,2), (1,1), (-2,3)\}$ and A be the matrix which rows are elements of X.

(1) Find the SVD and the compact SVD of A.
(2) Draw the image of the projection of X onto the best-fit 1-subspace on $\mathbb{R}^2$.
(3) Find the first principal component of X.
(4) Draw the image of the projection of X onto the best-fit affine 1-subspace on $\mathbb{R}^2$.

2. Let $X = \{(0,-1), (4,-2), (3,-2), (1,0), (2,0)\}$ and A be the matrix which rows are elements of X.

(1) Find the compact SVD of A.
(2) Draw the image of the projection of X onto the best-fit 1-subspace on $\mathbb{R}^2$.
(3) Find the first principal component of X.
(4) Draw the image of the projection of X onto the best-fit affine 1-subspace on $\mathbb{R}^2$.

3. Given a real quadratic form $Q(\vec{x}^t) = \vec{x}^t A \vec{x}$ on $\mathbb{R}^n$ where A is a real symmetric matrix. For a two dimensional subspace W of $\mathbb{R}^n$, define

$$Q(W) := Q(\vec{w}_1) + Q(\vec{w}_2).$$

where $\{\vec{w}_1, \vec{w}_2\}$ is an orthonormal basis of W.

(1) Show that $Q(W)$ is well-defined. In other words, if $\{\vec{v}_1, \vec{v}_2\}$ is another orthonormal basis of W, then $Q(\vec{w}_1) + Q(\vec{w}_2) = Q(\vec{v}_1) + Q(\vec{v}_2)$.
(2) Let λ_1 and λ_2 be the largest two eigenvalues of A and $\vec{v}_1$ and $\vec{v}_2$ be corresponding unit eigenvectors. Let V be the subspace spanned by $\vec{v}_1$ and $\vec{v}_2$. Show that $Q(V)$ is maximal among all two dimensional subspaces.

4. Apply SVD to the image of Lenna. Print your code and the resulted image using the first 50 singular values.
Remark: Click the red words to link to the image or you can also find the image on Google by the key words ”lenna wiki”.