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# Linear Algebra II - Homework 5

**1.** In $\mathbb{C}^2$ apply Gram-Schmidt method to the set $\{\left(\begin{smallmatrix} 2 \\ 2-i \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 \\ 1-i \end{smallmatrix}\right)\}$ to obtain an unitary basis.

**2.** Find an orthonormal/unitary basis to diagonalize the following matrices.

$$(1)\ \begin{pmatrix} 3 & 2 \\ 2 & 0 \end{pmatrix} \qquad (2)\ \begin{pmatrix} 10 & -2-2i \\ -2+2i & 8 \end{pmatrix} \qquad (3)\ \begin{pmatrix} 5 & 5 & 2 \\ 5 & 5 & 2 \\ 2 & 2 & 8 \end{pmatrix} \qquad (4)\ \begin{pmatrix} 4 & -1 & -i \\ -1 & 4 & i \\ i & -i & 4 \end{pmatrix}$$

**3.** Let $T_1$ and $T_2$ be two linear transformation on the complex inner product space $V$. Suppose $\langle \vec{v}, T_1\vec{w}\rangle = \langle T_2\vec{v}, \vec{w}\rangle$ for all $\vec{v}, \vec{w} \in V$. Show that we always have $\langle T_1\vec{v}, \vec{w}\rangle = \langle \vec{v}, T_2\vec{w}\rangle$ for all $\vec{v}, \vec{w} \in V$.

**4.** Let $V$ be the set of real continuous function on $[0, 1]$ with the inner product defined by

$$\langle f(x), g(x)\rangle = \int_0^1 f(x)g(x)dx.$$

Show that $\left\{\sqrt{2}\cos(2\pi x)\middle| n \in \mathbb{N}\right\} \cup \left\{\sqrt{2}\sin(2\pi x)\middle| n \in \mathbb{N}\right\}$ is an orthonormal set.

**5.** In the same setting as (4) apply Gram-Schmidt method to the set of functions $\{1, x, x^2\}$ to obtain an orthogonal set.

**6.** Find a $2 \times 2$ complex symmetric matrix which is not diagonalizable.

**7.** Find all bases $\alpha$ of $\mathbb{R}^2$ such that the inner product induced by $\alpha$ is equal to the standard inner product.

**8.** Give an example that $A$ is a real symmetric matrix, but $PAP^{-1}$ is not symmetric for some invertiable matrix $P$.