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Linear Algebra II - Homework 4

In this homework, T is a linear transformation from V to V.

1. Let $A = \begin{pmatrix} 0 & 0 & 2 & -1 \\ -2 & 3 & 2 & -1 \\ -3 & 0 & 5 & -1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$, $f_A(x) = (x-3)^3(x-2)$, $\ker(A-2I) = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

$\ker(A-3I) = \text{span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$, and $\ker(A-3I)^2 = \text{span}\left\{ \begin{pmatrix} -1 \\ 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$.
Find a form and a Jordan basis of A.

2. Let $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & -2 \end{pmatrix}$ with $f_A(x) = (x+1)^3(x-1)$ Find a Jordan form and a Jordan basis of A.

3. Let $A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 2 \\ 3 & -2 & -1 & 3 \end{pmatrix}$ with $f_A(x) = (x-1)^2x^2$. Find a Jordan form and a Jordan basis of A.

4. Let $A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$. Find the formula of A^m for all positive integer m.

5. Let $A = \begin{pmatrix} -3 & 2 & -1 \\ 0 & -1 & 1 \\ 4 & -4 & 3 \end{pmatrix}$. Find the formula of A^m for all positive integer m.

6. Suppose $f_T(x) = (x-3)^4(x-2)^3$ and $m_T(x) = (x-3)^2(x-2)^2$. Find all possible Jordan forms of T.

7. Suppose $\dim V = 4$ and $T^2(T+1) \equiv 0$. Find all possible Jordan forms of T.

8. Find three square matrices A, B, C satisfying tr$(ABC) \neq$tr(BAC).