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Linear Algebra II - Homework 2

In this homework, T is a linear transformation from V to V.

1. For (1) $A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & -2 & 2 & -3 \\ 0 & -1 & 0 & -1 \end{pmatrix}$ (2) $A = \begin{pmatrix} 4 & 1 & 2 & 5 \\ -1 & -2 & -1 & -3 \\ 1 & 3 & 3 & 4 \\ -1 & 0 & -1 & -1 \end{pmatrix}$ (3) $A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & -2 & -1 & -2 \\ 0 & -1 & -1 & -1 \\ -1 & 2 & 1 & 1 \end{pmatrix}$,

   - Find the generalized image and the generalized kernel of A.
   - Find the generalized eigenspace of A.
   - Find the minimal polynomial of A.

2. For any $f(x) \in F[x]$, show that the image and the kernel of $f(T)$ are both T-invariant subspace.

3. Let λ be an eigenvalue of T. Show that the characteristic polynomial of T restricted on $E_\infty(\lambda)$ is of the form $(x-\lambda)^d$.

4. Suppose we have $f_T(x) = \prod_{i=1}^k (x-\lambda_i)^{m_i}$ so that V admits a decomposition $\oplus_{i=1}^k E_\infty(\lambda_i)$. Let P_i be the projection from V onto $E_\infty(\lambda_i)$ with respect to this decomposition. Show that P_i and T commute for all i. (Hint: Express P_i as a polynomial of T.)

5. Suppose $T^4 - T^2 \equiv 0$.
   (1) What are the possible eigenvalues of T?
   (2) Find a T with the smallest possible size such that T satisfying the assumption and T is not diagonalizable.