Deep learning practice homework

October 17, 2018

Polynomial regression

Find the coefficient of the polynomial function

$$y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + ... + w_M x^M = \sum_{j=0}^{M} w_j x^j \tag{1}$$

using the error function

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2 \tag{2}$$

We can rewrite (1) as

$$y(x, \mathbf{w}) = \mathbf{w}^{\mathbf{T}} \phi(x) \tag{3}$$

or

$$\mathbf{y} = \Phi(x)\mathbf{w} \tag{4}$$

Polynomial regression

Rewrite the error function

$$E(\mathbf{w}) = \frac{1}{2}(\mathbf{t} - \Phi(x)\mathbf{w})^T(\mathbf{t} - \Phi(x)\mathbf{w}) \tag{5}$$

Obtain the optimal values of the coefficient

$$\mathbf{w}^* = (\Phi(x)^T\Phi(x))^{-1}(\Phi(x)^T\mathbf{t}) \tag{6}$$

Then evaluate the root-mean-square error

$$E_{RMS} = \sqrt{\frac{2E(\mathbf{w}^*)}{N}} \tag{7}$$

D-dimensional polynomial regression

Find the coefficient of the polynomial function

$$y(x, \mathbf{w}) = w_0 + \sum_{i=1}^{D} w_i x_i + \sum_{i=1}^{D} \sum_{j=1}^{D} w_{ij} x_i x_j + \sum_{i=1}^{D} \sum_{j=1}^{D} \sum_{k=1}^{D} w_{ijk} x_i x_j x_k \tag{8}$$

We can still use the previous method to find the optimal coefficients

$$y(x, \mathbf{w}) = \mathbf{w}^{\mathbf{T}} \phi(x) \tag{9}$$

Bayesian linear regression

In this problem, we consider different basis function for $y(x, \mathbf{w}) = \mathbf{w}^\mathbf{T}\phi(x)$.

1. $$\phi_j(x) = \exp -\frac{(x-\mu_j)^2}{2s^2} \tag{10}$$

   with $\mu_j = \frac{j - \frac{(M-1)}{2}}{\frac{(M-1)}{2}}$, $M = 9$, and $s = 1$.

2. $$\phi_j(x) = \sigma(\frac{x-\mu_j}{s}) \tag{11}$$

   with $\mu_j = \frac{2j}{M}$ and $\sigma(a) = (1 + \exp(-a))^{-1}$

Bayesian linear regression

Evaluate the mean vector and covariance matrix.

$$\mathbf{m_N} = \beta \mathbf{S_N} \Phi^T \mathbf{t} \tag{12}$$

$$\mathbf{S_N^{-1}} = \alpha^{-1} \mathbf{I} + \beta \Phi^T \mathbf{t} \tag{13}$$

Then we can sample the coefficients from the distribution

$$p(\mathbf{w}|\mathbf{t}) = \mathcal{N}(\mathbf{w}|\mathbf{m_N}, \mathbf{S_N}) \tag{14}$$