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## Epilogue

Bornons ici cette carrière.  
Les longs ouvrages me font peur.  
Loin d'épuiser une matière,  
On n'en doit prendre que la fleur.

La Fontaine, *Fables*, VI

Et pour ceux qui joignent le bon sens avec l'étude, lesquels seuls je souhaite  
pour mes juges [...]

Descartes, *Discours de la méthode*

Quant au mouvement en lui-même, je vous le déclare avec humilité, nous  
sommes impuissants à le définir.

Balzac, *La Peau de chagrin*

– This excursion, Reader, ends here. I hope the journey has been pleasant and instructive.

We attempt to build market models for hedging instruments – the underlying along with vanilla options, or a subset of them, or variance swaps, or yet other convex payoffs – such that (a) the P&L of a hedged option position is of the typical gamma/theta form, with payoff-independent break-even levels, (b) these break-even levels are – at least partially – in our control.

We could consider higher-order contributions but, in practice, managing the risk of all second-order greeks at a book level is a formidable task already.

Vanilla options are not quite independent instruments, however; they are related to one another and to the underlying itself through their terminal condition. At order one in volatility of volatility, for example, the SSR of homogeneous stochastic volatility models is related to the decay of the AMTF skew.

Characterizing the restrictions that the initial configuration of vanilla implied volatilities places on their future evolution is an unsettled issue, that may be more easily addressed by modeling other convex payoffs, for example the power payoffs of Chapter 4.

Even achieving objective (a) is not as straightforward, as highlighted by our analysis of mixed models in Chapter 12. Expressing the instantaneous volatility as the product of local and stochastic volatility components, a seemingly reasonable and innocuous ansatz, generally leads to non-functional models, for lack of a proper breakdown of the carry P&L.

This serves as a reminder that derivatives modeling does not start with the assumption of a process for  $S_t$ , but with clearly articulated modeling objectives

relating to observable quantities, regardless of how these objectives are achieved mathematically.

A commonplace statement one has been hearing at recent quant conferences is that the age of modeling is now over, that quantitative finance has become but a tedious form of accounting, the preserve of a new order of adjusters – xxA-quants: CVA-quants, FVA-quants, KVA-quants, etc.

Focusing on derivatives' risks, on their proper modeling and the meaningfulness of model-generated prices – the things being adjusted – is the symptom of an old-fashioned, outdated, mindset.

I respectfully dissent. Product risks and modeling choices are still begging for a proper understanding; forty years after Black-Scholes, work on the next generation of models has just started.

What about calibration, a deceptive notion we should strive to abolish? No one says that they “calibrate” the spot value in the Black-Scholes model: a model should naturally take as inputs the market values of instruments used as hedges. Then, *parameters* should be chosen so as to generate the desired break-even levels for the carry P&L.

Should they be calibrated to market prices? This is meaningful only if the difference between their calibrated and realized values can be materialized as the P&L of an actual trading strategy, a rare occurrence.

Therefore, Reader, you will do well to resist the compulsion of calibration and the addictive psychological reward that comes with it.

With these last words of encouragement, Reader, I bid you farewell.

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# Stochastic Volatility Modeling

Packed with insights, Lorenzo Bergomi's **Stochastic Volatility Modeling** explains how stochastic volatility is used to address issues arising in the modeling of derivatives, including:

- *Which trading issues do we tackle with stochastic volatility?*
- *How do we design models and assess their relevance?*
- *How do we tell which models are usable and when does calibration make sense?*

This manual covers the practicalities of modeling local volatility, stochastic volatility, local-stochastic volatility, and multi-asset stochastic volatility. In the course of this exploration, the author, *Risk*'s 2009 Quant of the Year and a leading contributor to volatility modeling, draws on his experience as head quant in Société Générale's equity derivatives division. Clear and straightforward, the book takes readers through various modeling challenges, all originating in actual trading/hedging issues, with a focus on the practical consequences of modeling choices.

## Features

- Covers forward-start options, variance swaps, options on realized variance, timer options, VIX futures and options, and daily cliquets
- Includes an in-depth study of the dynamics of the local volatility model, its carry P&L, and its delta
- Surveys the uncertain volatility model and its usage
- Discusses the parametrization of local-stochastic volatility and multi-asset stochastic volatility models
- Characterizes the links between static and dynamics features of stochastic volatility models
- Contains a wealth of unpublished results and insights



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