PAPA*: Path-Aware Parallel A*

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Abstract

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Fancy Stuff

Hello world.

Algorithm 1 $bound(s)$

$g_{front} := \infty$
$s' :=$ first node in $OPEN \cup BE$
$g_{back} := g(s) + f(s') - f(s) + (2\epsilon - w - 1)c_l$
while $g_{back} < g(s) \leq g_{front}$ **do**
  $g_{front} := \min(g_{front},\ g_p(s') + \epsilon h(s', s))$
  $s' :=$ node following s' in $OPEN \cup BE$
  $g_{back} := g(s) + f(s') - f(s) + (2\epsilon - w - 1)c_l$
end while
return $\min(g_{front},\ g_{back})$

Throughout this paper, we assume all edge costs are bounded below by c_l, that $\epsilon \geq 1$ and $0 \leq w \leq \epsilon$.

Definition 1 (TODO: don't need these definitions)**.** *We say a state s is independent of s' if $g(s) \leq g_p(s') + \epsilon h(s', s)$. We say $s \in OPEN$ is safe to expand if s is independent of all $s' \in OPEN \cup BE$.*

Lemma 1. *At all times, for all states s, s':*

$$g(s) + f(s') - f(s) + (2\epsilon - w - 1)c_l \leq g_p(s') + \epsilon h(s', s).$$

Proof.

$$\begin{aligned} & g(s) + f(s') - f(s) + (2\epsilon - w - 1)c_l \\ = \;& g(s') + w(h(s') - h(s)) + (2\epsilon - w - 1)c_l \\ \leq \;& g(s') + wh(s', s) + (2\epsilon - w - 1)c_l \\ \leq \;& g(s') + \epsilon h(s', s) + (w - \epsilon)c_l + (2\epsilon - w - 1)c_l \\ = \;& g(s') + \epsilon h(s', s) + (\epsilon - 1)c_l \\ \leq \;& g_p(s') + \epsilon h(s', s) \end{aligned}$$

□

Algorithm 2 PAPA*

LOCK
while $g(s_{goal}) > bound(s_{goal})$ **do**
  remove an s from $OPEN$ that has the smallest $f(s)$ among all states in $OPEN$ with $g(s) \leq bound(s)$ and let $g_{bound} := bound(s)$
  if such an s does not exist **then**
    UNLOCK
    wait until $OPEN$ or BE change
    LOCK
    continue
  end if
  insert s into BE
  insert s into $CLOSED$
  UNLOCK
  $S := getSucessors(s)$
  LOCK
  for all $s' \in S$ **do**
    if s' has not been generated yet **then**
      $f(s') := g(s') := g_p(s') := \infty$
    end if
    if $s' \notin CLOSED$ **then**
      $g_p(s') = \min(g_p(s),\ g_{bound} + \epsilon c(s, s'))$
      if $g(s') > g(s) + c(s, s')$ **then**
        $g(s') = g(s) + c(s, s')$
        $f(s') = g(s') + wh(s')$
        $bp(s') = s$
        insert/update s' in(to) $OPEN$ with key $f(s)$
      end if
    end if
  end for
  remove s from BE
end while

Lemma 2. *For all states s, $bound(s) \leq \epsilon g^*(s)$. Hence, upon expanding s, $g(s) \leq \epsilon g^*(s)$.*

Proof. We proceed inductively by the order in which states are expanded. By construction, $bound(s)$ is bounded above by $g_p(s') + \epsilon h(s', s)$ for states s' which are checked in the loop. As for the remaining states $s' \in OPEN \cup BE$, the algorithm ensures that $bound(s) \leq g(s) + f(s') - f(s) + (2\epsilon - w - 1)c_l$ for these by using a minimum representative. By Lemma 1, it follows that

$$bound(s) \leq \min_{s' \in OPEN \cup BE} g_p(s') + \epsilon h(s', s).$$

We complete the proof by showing that the latter expression is bounded above by $\epsilon g^*(s)$. Fix any optimal path to s, and let s' be the first node on it which is in $OPEN \cup BE$. Let s_p be the predecessor of s' on this path. By the induction hypothesis, $g(s_p) \leq \epsilon g^*(s_p)$. Therefore,

$$\begin{aligned} \epsilon g^*(s) &= \epsilon \left(g^*(s_p) + c(s_p, s') + c^*(s', s)\right) \\ &\geq g(s_p) + \epsilon c(s_p, s') + \epsilon h(s', s) \\ &\geq g_p(s') + \epsilon h(s', s) \end{aligned}$$

□

TODO: clear up initial case where $s = s' = s_{start}$.

Algorithm 3 $meanbound(s)$ with $w \leq 1$

s' := first node in $OPEN \cup BE$
return $g(s) + f(s') - f(s) + (1 - w)c_l + 2(\epsilon - 1)c_m$

When c_l is very small but we have a good estimate for the mean edge cost, we can maximize parallelism by using $meanbound(s)$ in place of $bound(s)$. This variant of PAPA* is inspired by (Klein and Subramanian 1997).

Lemma 3. *For all states s, $bound(s) \leq g^*(s) + (\epsilon - 1)c_m k(s)$ where $k(s)$ is the least number of edges used in a minimum-cost path to s. In particular, if c_m is not more than the mean cost of the edges along the minimum-cost path, then upon expansion, $g(s) \leq \epsilon g^*(s)$.*

Proof. We proceed inductively by the order in which states are expanded. Fix any optimal path to s, and let s' be the first node on it which is in $OPEN \cup BE$. Let s_p be the predecessor of s' on this path. By the induction hypothesis,

$$\begin{aligned} & g^*(s) + (\epsilon - 1)c_m k(s) \\ = \ & g^*(s_p) + c(s_p, s') + c^*(s', s) + (\epsilon - 1)c_m k(s) \\ \geq \ & g(s_p) + c(s_p, s') + h(s', s) + (\epsilon - 1)c_m(k(s) - k(s_p)) \\ \geq \ & g(s') + h(s', s) + 2(\epsilon - 1)c_m \\ \geq \ & g(s') + wh(s', s) + (1 - w)c_l + 2(\epsilon - 1)c_m \\ \geq \ & g(s') + w(h(s') - h(s)) + (1 - w)c_l + 2(\epsilon - 1)c_m \\ = \ & g(s) + f(s') - f(s) + (1 - w)c_l + 2(\epsilon - 1)c_m \\ = \ & bound(s) \end{aligned}$$

□

References

Klein, P. N., and Subramanian, S. 1997. A randomized parallel algorithm for single-source shortest paths. *Journal of Algorithms* 25(2):205–220.