



Minimum Path Cover - Using DAG

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Definition Of Minimum Path Cover DAG

The minimum path cover of a directed graph is a path cover containing the fewest possible paths joining the vertices of the graph. These paths can link any two vertices with any length size (≥ 0).

Let, DAG $G = (V, A)$ whose 'n' vertices are named $V = \{1, \dots, n\}$. Construct a bipartite graph $G = (V \cup V, E)$ with $V = \{1, \dots, n\}$ - where $\{v, w\}$ belongs to E iff (v, w) belongs to A . With G so constructed, call the maximal-matching procedure on G to find a maximal matching $M^* \leq E$ for G . Suppose this matching has $|M^*| = m^*$ edges. Now grow a set of paths P in G , as mentioned in algorithm.

Algorithm Of Minimum Path Cover in DAG

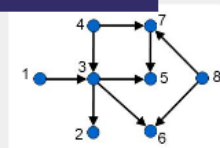
- 1) $P \leftarrow \text{NULL}$
- 2) Repeat until P covers every vertex
- 3) Choose any "v" that belongs to V (set of Vertices) such that v belongs to P
- 4) GrowAPath (v)
- 5) Return P

Procedure GrowAPath (v)

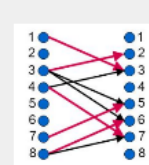
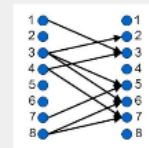
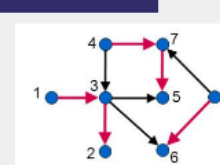
- 6) $p \leftarrow v$
- 7) while v is matched to some vertex w , set $p \leftarrow w$ and $v \leftarrow w$
- 8) $P \leftarrow P \cup \{p\}$

Visual Representation

Problem



Solution



Steps of verification

1. Every vertex output is connected at least once with the input of other vertex
2. There are no unnecessary path joints in the graph.
3. The initial integrity of the directed graph is not compromised.

Minimum path cover can only be used in dag, otherwise it would be considered as shortest path cover problem

References: <http://mradowan.github.io/algorithms/2014/05/02/flows-cuts-and-matchings/>
<https://www.coursehero.com/file/p1i9k5b/Problem-7-Finding-a-minimum-path-cover-in-a-DAG-A-path-cover-of-a-directed/>

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Definition Of Minimum Path Cover DAG

A path cover of a directed graph is a path cover containing the fewest possible paths joining the vertices of the graph. These paths can link any two vertices (if the distance between them is 0).

Suppose 'n' vertices are named $V=\{1, \dots, n\}$. Construct a bipartite graph $G=(V \cup V, E)$ with $V=\{1, \dots, n\}$ - where $\{v, w\}$ belongs to E iff (v, w) belongs to A .
Call the maximal-matching procedure on G to find a maximal matching $M^* \subseteq E$ for G . Suppose this matching has $|M^*|=m^*$ edges. Now grow a set of paths as mentioned in algorithm.

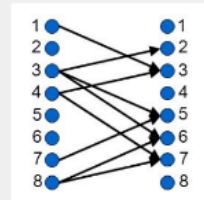
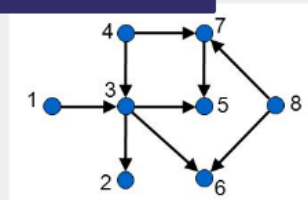
Minimum Path Cover in DAG

Every vertex v belongs to V (set of Vertices) such

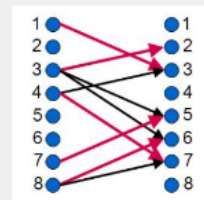
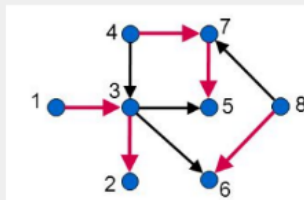
For every vertex v , if there is some vertex w , set $p \rightarrow w$ and

Visual Representation

Problem



Solution



Steps of verification

1. Every vertex output is connected at least once with the input of other vertex
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