Student Information

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Answer 1

a)

$E(X)$ **for Blue die:**

$E(X) = (2 \cdot \frac{4}{6}) + (3 \cdot \frac{1}{6}) + (4 \cdot \frac{1}{6})$
$E(X) = 2.5$

$E(X)$ **for Yellow die:**

$E(X) = (1 \cdot \frac{2}{6}) + (2 \cdot \frac{2}{6}) + (3 \cdot \frac{2}{6})$
$E(X) = 2$

$E(X)$ **for Red die:**

$E(X) = (1 \cdot \frac{2}{8}) + (2 \cdot \frac{2}{8}) + (3 \cdot \frac{3}{8}) + (5 \cdot \frac{1}{8})$
$E(X) = 2.5$

b)

$E(X)$ **for "2 red and 1 yellow" rolls:**

We can take the expected values of a single roll of Red and Yellow die from part (a):
$E(X) = (2 \cdot 2.5) + (1 \cdot 2)$
$E(X) = 7$

$E(X)$ **for "2 yellow and 1 blue" rolls:**

We can take the expected values of a single roll of Yellow and Blue die from part (a):
$E(X) = (2 \cdot 2) + (1 \cdot 2.5)$
$E(X) = 6.5$

Therefore, to maximize the total value we will take the option of "2 red and 1 yellow" rolls as it's expected value is higher.

c)

We can note now that the E(X) for Blue die is 4. So now:

$E(X)$ **for "2 yellow and 1 blue" rolls:**

$E(X) = (2 \cdot 2) + (1 \cdot 4)$
$E(X) = 8$

Therefore, now we will take the option of "2 yellow and 1 blue" rolls as it's expected value is higher.

d)

It is a conditional probability where,

P(B) is the probability that value of the die rolled is 3.

P(A) is the probability that the die rolled is Red.

And we need to find $P(A|B)$. Therefore, we can use Bayes Rule, $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

From Law of Total Probability, $P(B) = P(B|A) \cdot P(A) + P(B|\overline{A}) \cdot P(A)$

The probability to choose Red out of the three colors is: $P(A) = \frac{1}{3}$

The probability that we get a 3, Given the die color is Red is: $P(B|A) = \frac{3}{8}$

The probability that we get a 3, Given the die color is not Red is: $P(B|\overline{A}) = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$

Combining all these, we get :

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\overline{A}) \cdot P(A)}$$

Now we plug in values into this equation.

$$P(A|B) = \frac{\frac{3}{8} \cdot \frac{1}{3}}{\frac{3}{8} \cdot \frac{1}{3} + (\frac{2}{6} + \frac{1}{6}) \cdot \frac{1}{3}} = \frac{3}{7}$$

Therefore, If it is known that the value of the die is 3, the probability that the rolled die is red is $\mathbf{\frac{3}{7}}$.

e)

when a single red die and a single yellow die are rolled together the combinations that will get us 6 are:

Red	5	3
Yellow	1	3
Probability	$\frac{1}{8} \cdot \frac{2}{6} = \frac{1}{24}$	$\frac{3}{8} \cdot \frac{2}{6} = \frac{1}{8}$

Hence, When we add the probabilities of these 2 possible options, we get:
P(X) = $\frac{1}{24} + \frac{1}{8} = \frac{1}{6}$
Where X is the probability that the total value will be 6 when a single red die and a single yellow die are rolled together.

Answer 2

a)

We can see this probability directly from the table, no electric outages in Ankara and two electric outages in Istanbul is **0.17**.

b)

We can see from the Table 1 that there can't be two electric outages in Ankara therefore, the probability is **0**.

c)

There are two possible ways that the total outages are equal to 2 in both Ankara and Istanbul. Therefore:
$P(X) = P(0,2) + P(1,1) = 0.17 + 0.11 = \mathbf{0.28}$
Where X is two electric outages in total.

d)

To get the probability of a single electric outage in Ankara. We add up all the probabilities where there is a single electric outage in Ankara and i electric outages in Istanbul. Therefore:
$P(X) = P(1,0) + P(1,1) + P(1,2) + P(1,3) = 0.12 + 0.11 + 0.22 + 0.15 = \mathbf{0.6}$
Where X is a single electric outage in Ankara.

e)

Let Z = A + I be the total number of electric outages. To find the distribution of Z, we first identify its possible values, then find the probability of each value. We see that Z can be as small as 0 and as large as 4. Then,
$P_Z(0) = P(0,0) = 0.08$
$P_Z(1) = P(0,1) + P(1,0) = 0.13 + 0.12 = 0.25$
$P_Z(2) = P(0,2) + P(1,1) = 0.17 + 0.11 = 0.28$
$P_Z(3) = P(0,3) + P(1,2) = 0.02 + 0.22 = 0.24$
$P_Z(4) = P(1,3) = 0.15$
We can also verify that $\sum_z P_Z(z) = 1$

f)

We do the check of whether their joint pmf factors into a product of marginal pmfs. We see that $P_{(A,I)}(0,0) = 0.08$ indeed equals $P_A(0) \cdot P_I(0) = (0.4)(0.2) = 0.08$. Next, $P_{(A,I)}(0,1) = 0.13$ whereas, whereas $P_A(0) \cdot P_I(10) = (0.4)(0.24) = 0.096$. We do not need to check any further as we have found a pair of A and I that does not obey the formula for independent random variables. Hence, We can conclude that the electric outages in Ankara and Istanbul are **dependent**.