Student Information

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Answer 1

a)

We test the Null hypothesis $(H_0) : \mu = 7$ against an Alternative hypothesis $(H_A) : \mu > 7$ which is a one-sided right-tail alternative as we are interested in the fact that ratings are significantly higher than 7 or not.
Using Test statistic and $\mu_0 = 7$, $\sigma = 1.4$, n = 17 and $\hat{\mathrm{X}} = 7.8$ we get the equation:
$Z = \frac{\hat{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{7.8-7}{\frac{1.4}{\sqrt{17}}} = 2.356$
Now to get the acceptance region,
$1 - \alpha = 0.95$
we need $\alpha = 0.05$ (We don't divide by two because this is only a single sided test)
Hence, $z_{0.05} = 1.645$,
Now, as we can see from the right-tail test that we only accept the Null hypothesis if $Z < 1.645$, and as we can see that $Z = 2.356 \geq 1.645$ this means that we can reject the Null Hypothesis. There is enough evidence in favour of the Alternate hypothesis. Thus we can safely conclude that the average grade is significantly greater than 7. Therefore, with 95% confidence the customer service will be regarded as successful.

b)

As now the mean should change, we first calculate that:
Sum of grades(with 10) $= 7.8 \cdot 17 = 132.6$
Sum of grades(with 1 instead of 10) $= 132.6 - 10 + 1 = 132.6 - 9 = 123.6$
Mean of grades(with 1 instead of 10) $= \frac{123.6}{17} = 7.271$
Hence, We recalculate the test statistic:
$Z = \frac{\hat{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{7.271-7}{\frac{1.4}{\sqrt{17}}} = 0.795$
Now, we see that $Z = 0.795 < 1.645$ this means that we can accept the Null Hypothesis. There is insufficient evidence in favour of the Alternate hypothesis. Thus we can safely conclude that the average grade is NOT significantly greater than 7. Therefore, with 95% confidence the customer service will NOT be regarded as successful.

c)

As now n = 45 and mean should change, we first calculate that:
Sum of grades(with 10) $= 7.8 \cdot 45 = 351$
Sum of grades(with 1 instead of 10) $= 351 - 10 + 1 = 351 - 9 = 342$
Mean of grades(with 1 instead of 10) $= \frac{342}{45} = 7.60$

Hence, We recalculate the test statistic:
$Z = \frac{\hat{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{7.60-7}{\frac{1.4}{\sqrt{45}}} = 2.875$
Now, we see that $Z = 2.875 \geq 1.645$ this means that we can reject the Null Hypothesis. Therefore, with 95% confidence the customer service will be regarded as successful.

So no, the mistake doesn't affect the customer success anymore as we can safely conclude that the average grade is still significantly greater than 7 even after the mistake. This tells us that having a relatively large sample size helps us make sure that mistakes/errors do not affect the end result. Therefore, larger sample size means more reliable results.

d)

When we set the threshold of success at 8, Null hypothesis $(H_0) : \mu = 8$ against an Alternative hypothesis $(H_A) : \mu > 8$. As we can observe, all our mean values in previous parts are in fact less than 8 i.e $(7.271 < 7.6 < 7.8 < 8)$. This means, every time, we will accept the Null hypothesis and this means that the customer service will not be regarded as successful. We can conclude this due to the fact that the value of our test statistic falls drastically(It is negative for the above parts) and is always less than 1.645. This means we have insufficient evidence for the alternate hypothesis because our test statistic values are not greater than 1.645 and instead lie in the acceptance region of Null hypothesis.

Answer 2

To see if the new vaccine protects for a longer duration we have:
Let New vaccine use the subscript 1, therefore, $\hat{x}_1 = 6.2$, $s_1 = 1.5$ and Old vaccine use the subscript 2, therefore, $\hat{x}_2 = 5.8$, $s_1 = 1.1$ and $n = n_1 = n_2 = 55$.

Null hypothesis $(H_0) : \mu_{new} = \mu_{old}$ against an Alternative hypothesis $(H_A) : \mu_{new} > \mu_{old}$ that is one-sided right-tail test.
Now, we can use the Z-test:
$Z = \frac{\hat{x}_1-\hat{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$
So plugging in values, we get:
$Z = \frac{6.2-5.8}{\sqrt{\frac{1.5^2}{55}+\frac{1.1^2}{55}}} = 1.595$
Now, Using standard normal distribution, $P(Z \geq 1.595) = 0.055$. Since, this P-value is greater than $\alpha = 0.05$, We can't reject our null hypothesis. The evidence against H_0 is not sufficient and the evidence to conclude Alternative hypothesis $(H_A) : \mu_{new} > \mu_{old}$ is insufficient. To conclude, With 5% level of significance, we can't state that the new vaccine protects for a longer duration.

Answer 3

a)

We can note that margin of error can be computed by:
$margin = z_{\frac{\alpha}{2}}\sqrt{\frac{(p)(1-p)}{n}}$
Red party's support probability = 0.48.
Blue party's support probability = 0.37.
Using 95% confidence we know that $z_{\frac{\alpha}{2}} = 1.960$
Hence, Red party's margin of error = $1.960\sqrt{\frac{(0.48)(0.52)}{400}} = 0.0490$
And, Blue party's margin of error = $1.960\sqrt{\frac{(0.37)(0.63)}{400}} = 0.0473$

b)

As we saw above, $margin = z_{\frac{\alpha}{2}}\sqrt{\frac{(p)(1-p)}{n}}$
estimated lead probability = 0.11
Using 95% confidence we know that $z_{\frac{\alpha}{2}} = 1.960$
Hence, estimated lead's margin of error = $1.960\sqrt{\frac{(0.11)(0.89)}{400}} = 0.0307$

c)

We can say that margin of error of Red party is greater than margin of error of Blue party. To prove this, we can estimate the unknown standard error by:
$\mathrm{s}(\hat{\mathrm{p}}) = \sqrt{\frac{(\hat{p})(1-\hat{p})}{n}}$
For Red party: $\mathrm{s}(\hat{\mathrm{p}}) = \sqrt{\frac{(0.48)(0.52)}{400}} = 0.0250$
For Blue party: $\mathrm{s}(\hat{\mathrm{p}}) = \sqrt{\frac{(0.37)(0.63)}{400}} = 0.0241$
We can clearly see that the standard error (standard deviation) of the Red party is greater, Therefore, Red party's margin of error is greater.

d)

As we do an increase in the sample size, the margin of error seems to decrease for both the parties. We can note that the margin of errors will become:
Red party's margin of error = $1.960\sqrt{\frac{(0.48)(0.52)}{1800}} = 0.0231$
And, Blue party's margin of error = $1.960\sqrt{\frac{(0.37)(0.63)}{1800}} = 0.0223$
We can see that the margin of error's square is inversely proportional to the sample size that we take. But because both margin of errors of Red and Blue party will be scaled down with the same proportion, the Red party's margin of error will still be greater than Blue party's.