Middle East Technical University Department of Computer Engineering

CENG 223

Discrete Computational Structures

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Homework 3

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Question 1

By Fermat's Little Theorem,
$2^{10} \equiv 4^{10} \equiv 6^{10} \equiv 8^{10} \equiv 10^{10} \equiv 1 \mod 11$.
Hence, by using the above stated results:
$2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110} \equiv 2^2 + 4^4 + 6^6 + 8^0 + 10^0 \equiv 4 + 2^8 + 6^6 + 1 + 1 \equiv 4 + 3 + 5 + 1 + 1 \equiv 14 \equiv 3 \mod 11$
Therefore, we have proved that $(2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110}) \mod 11 \equiv 3$

Question 2

We know that $gcd(a, b)$ is equal to $gcd(a, c)$ when $a > b$ and c is the remainder when a is divided by b.
Therefore we can use Euclid's algorithm to find $gcd(5n + 3, 7n + 4)$:
$(7n + 4) = 1 \times (5n + 3) + (2n + 1)$ (because $(7n + 4) > (5n + 3)$)
$(5n + 3) = 2 \times (2n + 1) + (n + 1)$
$(2n + 1) = 1 \times (n + 1) + n$
$(n + 1) = 1 \times n + 1$
$n = n \times 1 + 0$
Therefore, $gcd(5n + 3, 7n + 4) = n$

Question 3

If there is an integer k such that $m^2 = n^2 + kx$, then $kx = m^2 - n^2$. Hence, $x|m^2 - n^2$ and now we can expand $m^2 - n^2$ to $(m + n)(m - n)$. Now it follows that $x|(m + n)(m - n)$.
Now we can use Euclid's Division Lemma which states that if x is a prime number, then $x|ab$ implies $x|a$ and $x|b$.
Therefore, from this lemma it is then seen that if $x|(m + n)(m - n)$ then it implies that $x|(m + n)$ and $x|(m - n)$.

Question 4

We are going to use Mathematical induction to prove that the above statement holds for all $n \geq 1$.
Basic Step: P(1) is TRUE since $1 = \frac{1(3-1)}{2} = 1$.
Inductive step: Assume true for P(k). The inductive hypothesis is then $1+4+7+\ldots+(3k-2) = \frac{k(3k-1)}{2}$ and prove that P(k+1) holds true meaning it is $1+4+7+\ldots+(3(k+1)-2) = \frac{(k+1)(3(k+1)-1)}{2}$.
We see that $1+4+7+\ldots+(3(k+1)-2) = (1+4+7+...+(3k-2))+(3k+1)$
Therefore, P(k+1):
$1+4+7+\ldots+(3(k+1)-2) = \frac{k(3k-1)}{2} + (3k+1)$
$= \frac{k(3k-1)+2(3k+1)}{2}$
$= \frac{3k^2-k+6k+2}{2}$
$= \frac{3k^2+3k+2k+2}{2}$
$= \frac{(k+1)(3k+2)}{2}$
$= \frac{(k+1)(3(k+1)-1)}{2}$
Hence, we have shown that P(k + 1) follows from P(k), so by using Mathematical induction on P(k) we prove it is true for all $n \geq 1$.