Student Information

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Answer 1

a)

This is a uniform distribution, with the parameters a = 60 and b = 180.
Therefore, the probability density function $f(x) = \frac{1}{(b-a)} for(a \leq x \leq b) = \frac{1}{(180-60)} = \frac{1}{(120)}$ for $60 \leq x \leq 180$

b)

Mean $= \frac{b+a}{2} = \frac{180+60}{2} = 120$.
Variance $= \frac{(b-a)^2}{12} = \frac{(180-60)^2}{12} = 1200$.
Standard deviation $= \sqrt{Variance} = \sqrt{1200} = 34.641$

c)

We need to find the probability of $P(90 < x < 120)$.
We get the probability of $P(x < 120) = \frac{120-60}{180-60} = 0.5$
We get the probability of $P(x < 90) = \frac{90-60}{180-60} = 0.25$
Therefore, $P(90 < x < 120) = 0.5 - 0.25 = 0.25$

d)

Here, We find the probability of $P(150 < x < 180)$ when it is given that $P(120 < x < 180)$.
This can noted as a conditional probability so we use the formula, $P(A|B) = \frac{P(A \cap B)}{P(B)}$
$P(150 < x < 180|120 < x < 180) = \frac{P(150<x<180 \cap 120<x<180)}{P(120<x<180)} = \frac{P(150<x<180)}{P(120<x<180)}$.
$P(150 < x < 180) = \frac{180-150}{180-60} = 0.25$
$P(120 < x < 180) = \frac{180-120}{180-60} = 0.5$
$P(150 < x < 180|120 < x < 180) = \frac{0.25}{0.5} = 0.5$

Answer 2

a)

Mean $= n \times p = 500 \times 0.02 = 10$
Standard deviation $= \sqrt{n \times p \times (1-p)} = \sqrt{500 \times 0.02 \times (1 - 0.02)} = \sqrt{9.8} = 3.1305$

b)

Now that we have $\mu = 10$ and $\sigma = 3.1305$. We can apply the Central Limit Theorem along with continuity correction because of using Normal approximation to Binomial distribution.
$P(Y < 8) = P(Y < 7.5) = P(\frac{X-10}{3.1305} < \frac{7.5-10}{3.1305}) = \Phi(-0.79859) = 0.212$

c)

Apply the Central Limit Theorem along with continuity correction because of using Normal approximation to Binomial distribution.
$P(Y > 15) = 1 - P(Y \leq 15.5) = 1 - P(\frac{X-10}{3.1305} < \frac{15.5-10}{3.1305}) = 1 - \Phi(1.7569) = 1 - 0.9605 = 0.0395$

d)

$P(7 < Y) = P(\frac{X-10}{3.1305} < \frac{6.5-10}{3.1305}) = \Phi(-1.1180) = 0.1317$
$P(14 \leq Y) = P(\frac{X-10}{3.1305} < \frac{14.5-10}{3.1305}) = \Phi(1.4375) = 0.9247$
$P(7 \leq Y \leq 14) = 0.9247 - 0.1317 = 0.793$

Answer 3

a)

If we know that the skyscraper was hit today, then we want to find the the probability that the skyscraper will not be hit within the next year. This is an exponential probability with t=1 and with $\lambda = 1$(lightning strikes a skyscraper about once a year).
$f_T(t = 1) = \lambda e^{-\lambda t} = 1 \times e^{-1\times 1} = e^{-1} = 0.3679$

b)

If we know that the skyscraper was not hit in a year, then we want to find the probability that the skyscraper will not be hit within the next year as well. We use the memoryless property of the exponential distribution which states that having waited for time 't' gets forgotten, and it does not affect the future waiting time.Therefore, t = 1 and $\lambda = 1$.
$f_T(t = 1) = \lambda e^{-\lambda t} = 1 \times e^{-1\times 1} = e^{-1} = 0.3679$