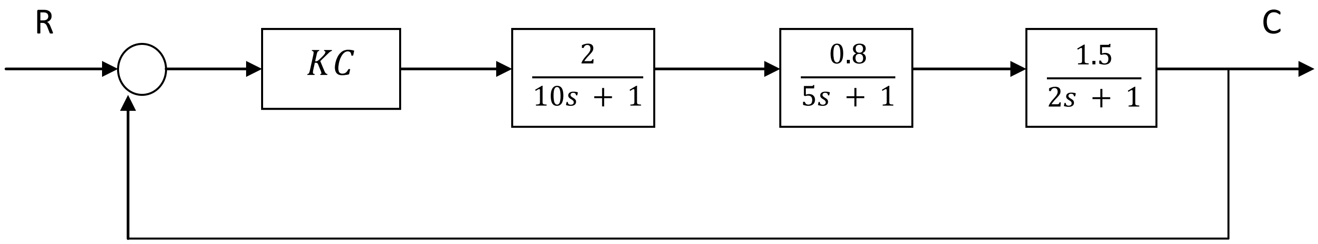
QUESTION 8.5

Consider the process shown below. Calculate the maximum controller gain ($K_{c,max}$) using Bode analysis. What is the phase margin at $K_{c,max}$



SOLUTION

a) Calculating for the maximum controller gain.

The characteristic equation is; 1 + GH =0

Where H = 1

$G=G_1G_2G_3G_4$

$G1=K_C$

$G2=\frac{2}{10S+1}$ Therefore $G = \frac{2.4kc}{(10s+1)(5s+1)(2s+1)}$

$G3=\frac{0.8}{5S+1}$

$G4=\frac{1.5}{2S+1}$

$1 + GH = 1 + \frac{2.4KC}{(10S+1)(5S+1)(2S+1)} = 0$ .... (i)

Simplifying .... (i) above, gives

$\Longrightarrow (10s+1)(5s+1)(2s+1) + 2.4K_C = 0$

$100s^3 + 80s^2 + 17s + 1 + 2.4K_C = 0$

let $s = i\omega$

$-100\, i\omega^3 - 30\omega^2 + 17i\omega + 1 + 2.4K_C = 0 + 0i$

Equating real and imaginary part on both sides of the equation above gives

For the imaginary part;

$-100\omega^3 + 17\omega = 0$

$\therefore 100\omega^2 = 17$

$\omega = \sqrt{\frac{17}{100}} = 0.4123$

For the real part;

$-35\, \omega^2 + 1 + 2.4\, K_C = 0$

$K_C = \frac{80\omega^2 - 1}{2.4} = \frac{80(0.4123)^2 - 1}{2.4} = 5.2497$ $\therefore K_{C,max} = 5.2497$

b) Calculating for phase margin at K_{Cmax}

Phase margin is 180 - the phase angle in degrees for which A.R = 1

The forward transfer function is given as,

$G_{(S)} = \frac{2.4Kc}{(10s+1)(5s+1)(2s+1)}$ but Kc = 5.2497 $\therefore$ $G_{(S)} = \frac{12.6}{(10S+1)(5S+1)(2S+1)}$

$G_1 = 12.6$ A.R = 12.6 $\phi = 0$

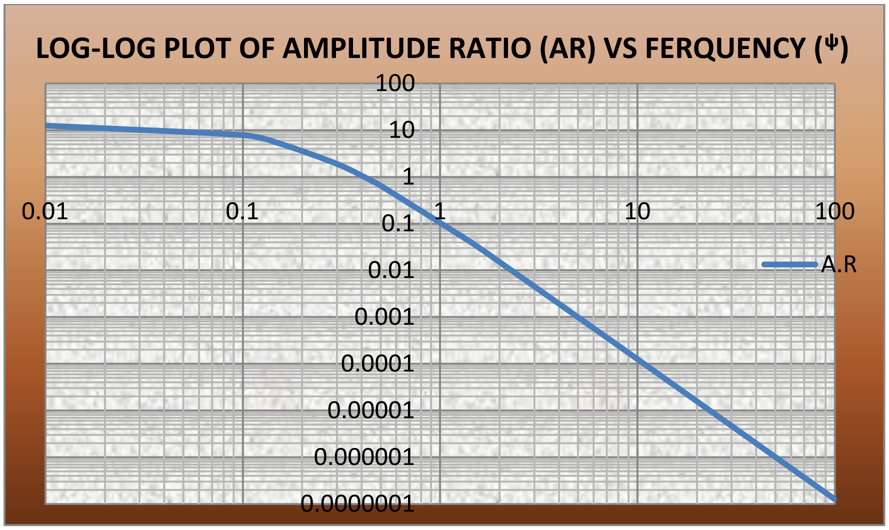
$G_2 = \frac{1}{10S + 1}$ A.R = $\frac{1}{\sqrt{100\omega^2+1}}$ $\phi = -\tan^{-1} 10\,\omega$

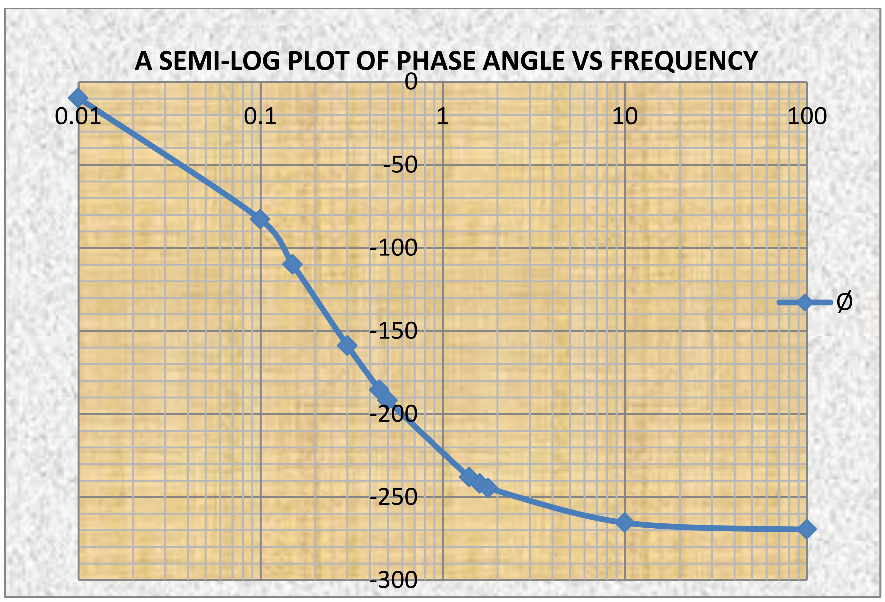
$G_3 = \frac{1}{5S + 1}$ A.R = $\frac{1}{\sqrt{25\omega^2+1}}$ $\phi = -\tan^{-1} 5\,\omega$

$G_4 = \frac{1}{2S + 1}$ A.R = $\frac{1}{\sqrt{4\omega^2+1}}$ $\phi = -\tan^{-1} 2\,\omega$

/A.R/ = $\frac{12.6}{\sqrt{(100\omega^2+1)(25\omega^2+1)(4\omega^2+1)}}$ /ϕ/ = $-\tan^{-1} 10\,\omega - \tan^{-1} 5\,\omega - \tan^{-1} 2\,\omega$

The Bode plot was done on excel spread sheet as shown below;





From the plot the phase lag is -106.57^0

Phase margin = 180 - phase lag = 180 - |-106.57|

Phase margin = 73.43^0

QUESTION 8.10

The block diagram of a conventional feedback control system contains the following transfer functions;

$G_C = K_C\left(1 + \frac{1}{5S}\right)$; $G_V = 1$; $G_M = \frac{1}{S+1}$; $G_P = G_L = \frac{5e^{-2S}}{10S=1}$;

Where G_c= controller transfer function; G_v = valve transfer function; G_p = Process transfer function; G_m = feedback transfer function and K_p = proportional gain

a) Plot the Bode diagram of the open loop system

b) For what value of Kc is the system stable?

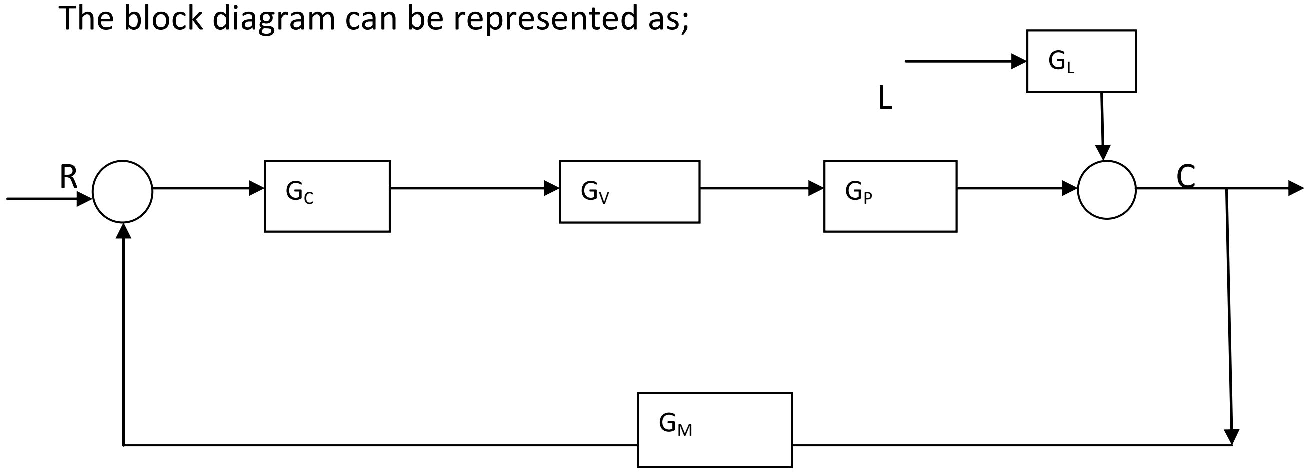
c) If Kc = 0.2, What is the phase margin?

d) What value of Kc will result in a gain margin of 1.7?

e) repeat (b) using Routh stability criterion.

SOLUTION

The block diagram can be represented as;



To obtain the closed loop relationship between R and C, we use the principle of superposition and set L = 0

From our conical form, $\frac{C}{R} = \frac{G}{1+GH}$; Where $G = G_C G_V G_P$ $H = G_M$

$G = K_C\left(1+\frac{1}{5S}\right)\left(\frac{5e^{-2S}}{10S+1}\right) = K_C\left(\frac{5S+1}{5S}\right)\left(\frac{5e^{-2s}}{10s+1}\right)$

8.10 a) The open loop transfer function is 'GH'

$GH = G(s) = \frac{Kc(5s+1)e^{-2s}}{s(s+1)(10s+1)}$

$G_1 = Kc$ A.R = Kc $\phi = 0$

$G_2 = 5s + 1$ A.R = $\sqrt{25\omega^2+1}$ $\phi = \tan^{-1} 5\,\omega$

$G_3 = e^{-2s}$ A.R = 1 $\phi = \frac{-360\omega}{\pi}$

$G_4 = \frac{1}{s}$ A.R = $\frac{1}{\omega}$ $\emptyset = -90$

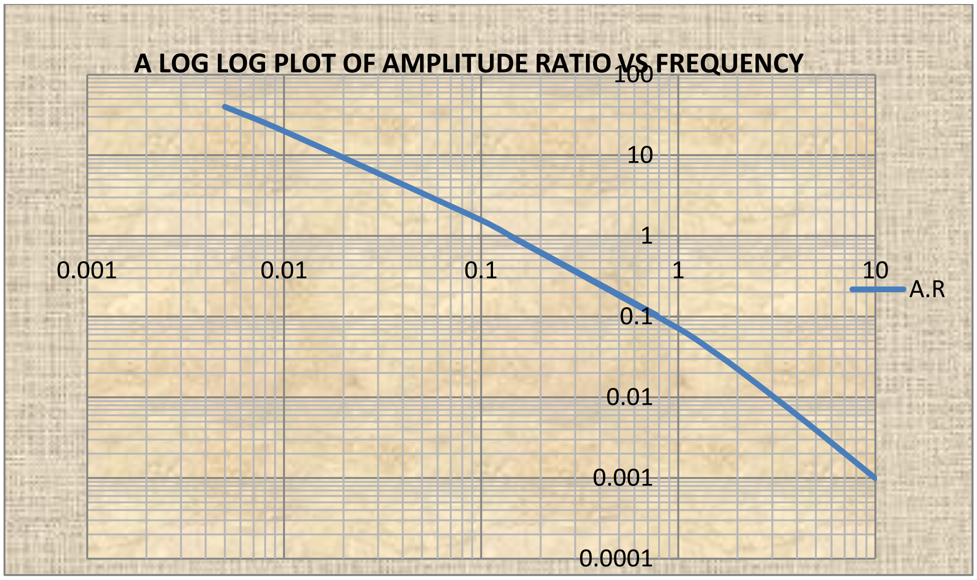
$G_5 = \frac{1}{s+1}$ A.R = $\frac{1}{\sqrt{1+\omega^2}}$ $\emptyset = -\tan^{-1}\omega$

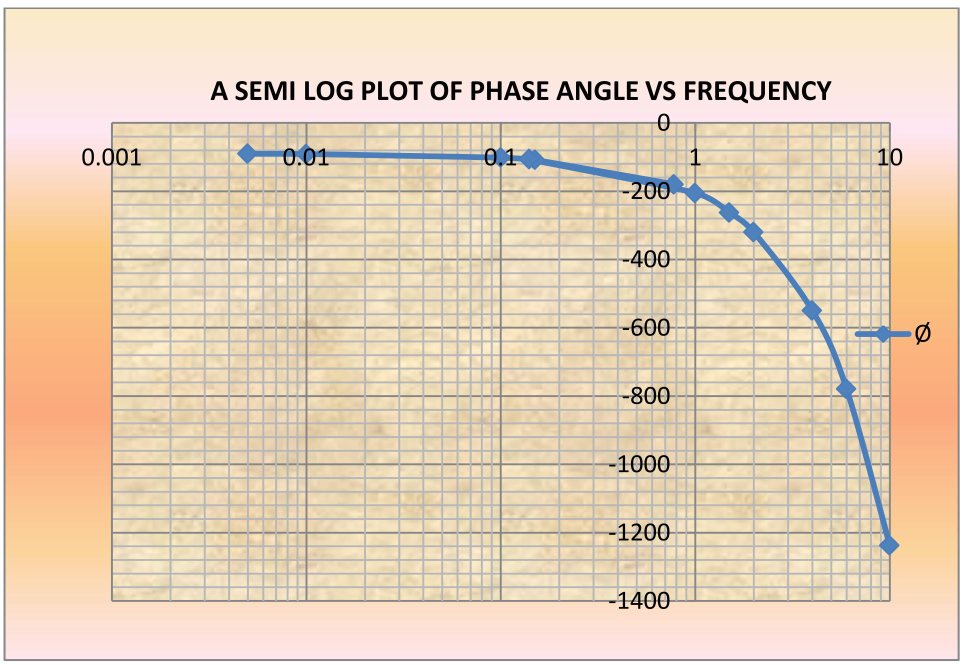
$G_6 = \frac{1}{10s+1}$ A.R = $\frac{1}{\sqrt{100\omega^2+1}}$ $\emptyset =$ -$\tan^{-1} 10\omega$

The normalized amplitude ratio is $\frac{A.R}{Kc} = \frac{\sqrt{25\omega^2+1}}{\omega\sqrt{(\omega^2+1)(100\omega^2+1)}}$ ....*

$\emptyset = \tan^{-1} 5\omega - \left(90 + \tan^{-1}\omega + \tan^{-1} 10\omega + \frac{360\omega}{\pi}\right)$ ....* *

The Bode plot was done using Excel spread sheet as shown below





8.10 b) $G = K_C\left(1 + \frac{1}{5S}\right)\left(\frac{5e^{-2S}}{10S+1}\right) = K_C\left(\frac{5S+1}{5S}\right)\left(\frac{5e^{-2s}}{10s+1}\right)$

But $e^{-2s} = \frac{1-s}{1+s}$ $\quad \therefore G = \frac{Kc(5s+1)(1-s)}{s(1+s)(10s+1)}$

recall; 1 + GH = 0 (characteristic equation) $\rightarrow 1 + \frac{Kc(5s+1)(1-s)}{s(1+s)^2(10s+1)} = 0$

$s(1+s)^2(10s+1) + K_c(5s+1)(1-s) = 0$ .... (ii)

Expanding equation (ii) above;

$10s^4 + 21s^3 + 12s^2 + s + 4K_cs^2 + K_c = 0$

Let s = iω

$10\omega^4 - 21i\omega^3 - 12\omega^2 + i\omega + K_C(4i\omega + 5\omega^2 + 1) = 0 + 0i$

Equating the real and complex part on both sides of the equation above gives;

for the real part; $10\omega^4 - 12\omega^2 + 5K_c\omega^2 + K_c = 0$ .... (iii)

for complex part; $-21\omega^2 + \omega + 4K_C = 0$ .... (iv)

From equation (iv), $\omega = \sqrt{\frac{1+4Kc}{21}}$ .... (v)

Substituting the value of ω in equation (v) into equation (iii) gives,

$10\left(\frac{1+4Kc}{21}\right)^2 - 12\frac{(1+4Kc)}{21} + 5K_c(1+4K_c) + 21K_c = 0$

$= \frac{580Kc^2}{21} - \frac{382Kc}{21} - \frac{242}{21} = 0$

$580K_C^2 - 382K_C - 242 = 0$ .... (vi)

from .... (vi) above; KC = 1.0544, ω = 0.4985

8.10 c) If K_c= 0.2, what is the phase margin?

Phase margin = 180 - phase lag

Phase lag is the angle in degree at the frequency for which AR = 1

∴ From ....*

$A.R = \frac{0.2\sqrt{25\omega^2+1}}{\omega\sqrt{(\omega^2+1)}(100\omega^2+1)} = 1$

$\therefore \omega\sqrt{(\omega^2+1)(100\omega^2+1)} - 0.2\sqrt{(25\omega^2+1)} = 0$ ....(vii)

Using What-If analysis (Goal seek) on excel, ω in .... (vii) was gotten as Phase lag $= \emptyset|\omega$

substituting ω = 0.1404 into .... $**$ gives phase lag = 106.573°

∴ Phase margin = 180 - 106.573 = 73.427°

8.10 d) What value of K_c will result is a gain margin of 1.7?

Gain margin is the reciprocal of the amplitude ratio at the cross over frequency

The cross over frequency is the frequency for which the phase angle is -180

=∴ from .... $**$ -180 = $\tan^{-1} 5\omega - (90 + \tan^{-1} \omega + \tan^{-1} 10\omega + \frac{360\omega}{\pi})$

Using What-If analysis (Goal seek) in excel the cross over frequency was gotten from the equation above as ω_{co} = 0.7785 rad/min

A.R = 1/1.7 = 0.58823

substituting the values of A.R = 0.58823 and ω_{co} = 0.7785 rad/min into .... $*$

K_c = 0.6896

8.10 e) Using Roulth's stability criterion, the characteristic equation from ....(ii)

is $s(1+s)^2(10s+1) + K_c(5s+1)(1-s) = 0$ .... (ii)

Expanding .... (ii) above;

$10s^4 + 21s^3 + 12s^2 + s + 4K_cs^2 + K_c = 0$

s^4	10	$12 - 5K_c$	K_c
s^3	21	$1 + 4K_C$	0
s^2	a1	a2	0
s^1	b1	b2	

s^0 c1 c2

a1 = $\frac{21(12-5Kc)-10(1+4Kc)}{21} = \frac{242-145Kc}{21}$; a2= (21Kc)/21 -10(0) = Kc

similarly, b1 = $\frac{5082-16842Kc-12100Kc^2}{242-145Kc}$; b2 =

c1= a2 = Kc

s^0 Kc $\geq$ 0

$\therefore$ 0.2548 $\leq$ Kc $\geq$ 1.6689

QUESTION 8.12

Consider a unity feedback system with the closed loop characteristic equation;

$$1 + K_c\frac{0.8e^{-0.25s}}{(5.1s+1)(1.2s+1)} = 0$$

a) Calculate analytically the exact values of the magnitude and phase lag on the Bode plot for the specific case in which Kc = 7.5 and $\omega = 0.8\ rad/min$

SOLUTION

$$G(s) = K_c\frac{0.8e^{-0.25s}}{(5.1s+1)(1.2s+1)}$$

G_1 = 0.8Kc A.R = 0.8Kc = 6 $\emptyset = 0$

$G_2 = e^{-0.25s}$ A.R = 1 $\emptyset = \frac{-45\omega}{\pi}$

$G_3 = \frac{1}{\sqrt{(5.1s+1)}}$ A.R = $\frac{1}{\sqrt{26.01\omega^2+1}}$ $\emptyset = \tan^{-1} 5.1\,\omega$

$G_4 = \frac{1}{\sqrt{(1.2s+1)}}$ A.R = $\frac{1}{\sqrt{1.44\omega^2+1}}$ $\emptyset = \tan^{-1} 1.2\ \omega$

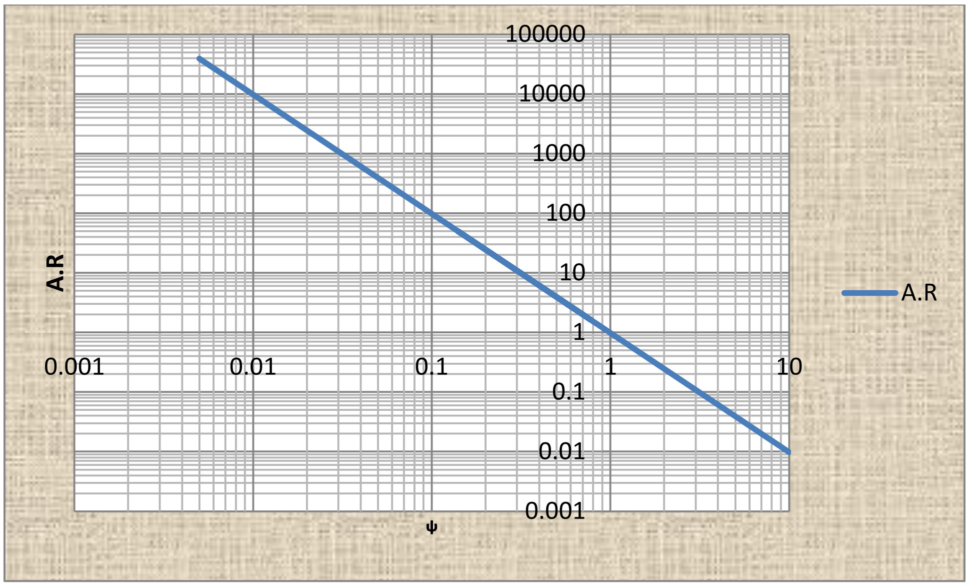
A.R = $\frac{6}{\sqrt{(26.01\omega^2+1)(1.44\omega^2+1)}}$ 1

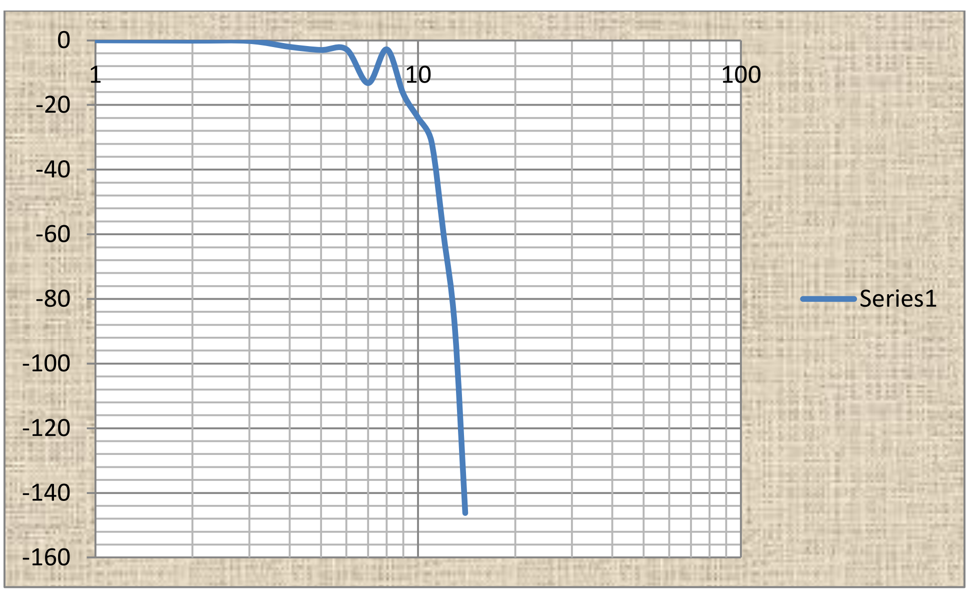
$\emptyset = \frac{-45\omega}{\pi}$ - $\tan^{-1} 5.1\,\omega$ - $\tan^{-1} 1.2\ \omega$ 2

inputting the value of $\omega = 0.8\ rad/min$ into ...... 1 and ....2 yields

A.R = 1.0304 and $\emptyset = -131.5184°$

The Bode plot was done on Excel spread sheet and its shown below





8.12 b) i) What is the critical gain of the system?

The characteristic equation is

$1 + K_c\frac{0.8e^{-0.25s}}{(5.1s+1)(1.2s+1)} = 0$

$\therefore$ (5.1s+1)(1.2s+1)(1+0.125s) + 0.8Kc(1-0.125s) = 0

$-0.765s^3 + 6.9075s^2 + 6.425s + 1 + 0.8Kc = 0$

$s = i\omega$

$-0.765i\omega^3 - 6.9075\omega^2 + 6.425i\omega + 1 + 0.8K_c - 0.1i\omega K_c = 0 + 0i$

equating coefficients

$-0.765\omega^3 + 6.425\omega - 0.1\omega K_c = 0$

$- 6.9075\omega^2 + 1 + 0.8K_c = 0$

solving the above two equation simultaneously; K_{cmax}=33.48

8.12b) ii What is the critical gain if there is no dead time?

The characteristic equation would be;

$1 + K_c\frac{0.8}{(5.1s+1)(1.2s+1)} = 0$

(5.1s+1)(1.2s+1)+0.8Kc = 0

$6.12s^2 + 6.3s + 1 + 0.8K_c = 0$

let $s = i\omega$

$6.12\omega^2 + 6.3i\omega + 1 + 0.8Kc = 0 + 0i$

equating coefficients

$6.3\omega = 0$; $\omega = 0$ and $K_c = -0.25$

8.12 c) What is the proportional gain when the desired gain margin is 1.7?

Gain margin is the reciprocal of the amplitude ratio at the cross over frequency which is the frequency at which the phase lag is 180

$\emptyset = \frac{-45\omega}{\pi}$ - $\tan^{-1} 5.1\,\omega$ - $\tan^{-1} 1.2\ \omega$ 2

$\therefore\ -180 = \frac{-45\omega co}{\pi}$ - $\tan^{-1} 5.1\,\omega co$ - $\tan^{-1} 1.2\ \omega co$

using What-If analysis on excel, the value of ω_{co} was gotten as 1.9843 rad/min

A.R| ω_{co}= $\frac{1}{1.7} = 0.5882$

A.R =$\frac{0.8Kc}{\sqrt{(26.01\omega^2+1)(26.01\omega^2+1)}}$ 1

substituting the value of A.R| ω_{co}= $\frac{1}{1.7} = 0.5882$ and ω_{co} =1.9843 rad/min into the equation above, gives Kc = 19.31

8.12 d) What is the proportional gain when the desired phase margin is 30?

Phase margin = 180 - phase lag at A.R = 1

phase lag =180 - phase margin = 180 - 30 = 150

phase lag = -150

substituting phase lag = -150 into

$\emptyset = \frac{-45\omega}{\pi}$ - $\tan^{-1} 5.1\,\omega$ - $\tan^{-1} 1.2\ \omega$

$$\omega = 1.1314$$

At the phase margin A.R = 1, substituting this and $\omega = 1.1314$ into

A.R =$\frac{0.8Kc}{\sqrt{(26.01\omega^2+1)(26.01\omega^2+1)}}$ 1

gives Kc = 12.3434