

MATH 036

Insights into Mathematics

Sample Syllabus

Description

This course will provide students the mathematical background and quantitative skills in various mathematical applications in such areas which are related to voting, fair divisions which includes apportionment methods, and the understanding and application of basic graph theory such as Euler and Hamilton circuits. This course may be used by students from non-technical majors to satisfy 3 credits of their General Education Quantification (GQ) requirement. This course does not serve as a prerequisite for any mathematics courses and should be treated as a terminal course.

Objectives

Upon successful completion of MATH 036, the student should be able to:

The Mathematics of Voting

- Preference Ballots and Schedules
 - Construct a preference ballot
 - Apply the Multiplication Rule to count total possible preference ballot outcomes
 - Construct a preference schedule
 - Apply the Transitive Property to determine voting preferences
- The Plurality Method
 - Given a preference schedule, identify a majority candidate and a plurality candidate
 - Given a preference schedule, apply the plurality method to determine a winner
 - Determine the number of points a candidate needs to when when partial results are given
 - Describe when the plurality method is appropriate to use
 - Determine if a voting method violates the majority criterion.

- Plurality with Elimination Method
 - Use the Plurality with Elimination Method to determine a winner
 - Describe when the Plurality with Elimination Method is appropriate to use
 - Understand the Monotonicity Criterion and determine if a voting method supports or violates the Monotonicity Criterion
- Borda Count Method
 - Use the Borda Count Method to determine a winner
 - Determine the number of points given out in one ballot
 - Determine the maximum number of points that a candidate can receive
 - Determine the minimum number of points a candidate can receive
 - Determine the total number of points awarded when there are n candidates
 - Determine the winner of an election when partial results are given
 - Describe when the Borda Count Method is appropriate to use
 - Understand the Monotonicity Criterion and determine if a voting method supports or violates the Monotonicity Criterion
- Pairwise Comparisons Method
 - Determine the total number of Pairwise Comparisons when there are n candidates
 - Use the Pairwise Comparison Method to determine a winner
 - Determine appropriate methods to break ties
 - Determine a winner when partial results are given
 - Describe when the Pairwise Comparison Method is appropriate to use
 - Understand the Condorcet Criterion and determine if a voting method supports or violates the Condorcet Criterion
- Applications
 - Construct a round-robin tournament (or pool play) to determine a winner
 - Construct a table to identify when the 4 different methods can violate the 4 different criteria
 - Calculate the sum of all integers from 1 to L
 - Read the constitution of the United States and learn the method for determining the President of the United States

- Explain Arrow's Impossibility Theorem

Weighted Voting

- Weighted Voting Systems
 - Learn and correctly use the terminology for weighted voting systems: players, weights, quotas
 - Identify dictators, dummies, veto power or some power players in a weighted voting system
 - Identify trends in players powers as the quota ranges from its lowest value to its highest value
- Banzhaf Power Index
 - Learn and correctly use the terminology for the Banzhaf Power Index: coalition, winning coalition, critical player
 - Calculate the Banzhaf Power Index for a weighted voting system
 - Calculate the total number of coalitions for a weighted voting system
- Shapely-Shubik Power Index
 - Learn and correctly use the terminology for the Shapely-Shubik Power Index: sequential coalition, winning coalition, pivotal player
 - Calculate the Shapely-Shubik Power Index for a weighted voting system
 - Calculate the total number of sequential coalitions for a weighted voting system
- Applications
 - Use and calculate factorial notation
 - Determine the number of coalitions and sequential coalitions for n players in a weighted voting system and determine which power index method would be most efficient
 - Calculate the Banzhaf and Shapely Shubik Power Index for a voting system with 4 players and one player determines the tie
 - Learn how the Vice-president's power in the Senate relates to an individual senator's power.

Fair Division

- Fair Shares
 - Learn and use the terminology of fair division game: goods, players, value system
 - Identify whether a fair division game is continuous, discrete or mixed
 - Identify when a player has received a fair share
- Divider-Chooser Method
 - Divide an item into fair shares based on a player's value system
 - Explain the Divider Chooser Method
 - Use the Divider-Chooser Method to divide an item
 - Identify when this is an appropriate method to use
- Lone-Divider Method
 - Explain the Lone-Divider Method
 - Use the Lone-Divider Method to divide goods
 - Determine a fair division using the Lone-Divider Method, with missing information
 - Identify when this is an appropriate method to use
- Lone-Chooser Method
 - Explain the Lone-Chooser Method
 - Use the Lone-Chooser Method to divide goods
 - Identify when this is an appropriate method to use
- Method of Sealed Bids
 - Explain the Method of Sealed Bids
 - Use the Method of Sealed Bids to divide goods
 - Identify when this is an appropriate method to use
- Method of Markers
- Explain the Method of Markers
 - Use the Method of Markers to divide goods
 - Determine a fair division when the value of the items are related to each other
 - Identify when this is an appropriate method to use
- Applications
 - Divider-Chooser Method for a donut
 - Method of Markers for candy

The Mathematics of Apportionment

- Apportionment Problems
 - Learn and use the terminology of apportionment: states, seats, populations, standard divisor, standard quota
 - Calculate the standard divisor and standard quotas
 - Calculate the number of seats and standard divisor given the standard quotas
- Hamilton's Method and the Quota Rule
 - Explain Hamilton's Method of Apportionment
 - Calculate an apportionment using Hamilton's Method
 - Explain the Alabama Paradox
 - Explain the New States Paradox
 - Explain the Population Paradox
- Jefferson's Method
 - Explain Jefferson's Method of Apportionment
 - Calculate an apportionment using Jefferson's Method
 - Explain how Jefferson's Method can violate the quota rule
- Adam's Method
 - Explain Adam's Method of Apportionment
 - Calculate an apportionment using Adam's Method
 - Explain how Adam's Method can violate the quota rule
- Webster's Method
 - Explain Webster's Method of Apportionment
 - Calculate an apportionment using Webster's Method
 - Explain how Webster's Method can violate the quota rule
- Applications
 - Construct a chart to show the different apportionment methods indicating violations that can occur
 - Read the US Constitution and learn the apportionment that is required
 - Learn what is the current method used for apportioning the representatives for the House of Representatives

Euler Circuits

- Graph Concepts and Terminology
 - Learn and use the terminology for graphs: vertex, edge degree of vertex, same graph representations, loops, multiple edges, path, circuit
 - Recognize an Euler Path
 - Recognize an Euler Circuit
- Graph Models
 - Construct a graph to represent relationships within a set of objects
 - Construct a graph to represent a city or neighborhood
- Euler's Theorems
 - Use Euler's Theorem to determine if a graph has an Euler Path
 - Use Euler's Theorem to determine if a graph has an Euler Circuit
 - Use Euler's Theorem to determine the relationship between the sum of the degrees of the vertices and the number of edges
- Fleury's Theorem
 - Use Fleury's Algorithm to determine an Euler circuit.
 - Determine the edges that have to be traveled to complete an Euler circuit
- Eulerizing Graphs
 - Determine an optimal eulerization for a graph
 - Determine an optimal semi-eulerization for a graph
- Applications
 - Determine if a graph is an open or closed unicursal tracing
 - Determine an Euler circuit for an applied graph

The Traveling Salesman Problem

- Hamilton Circuits and Hamilton Paths
 - Learn and use the terminology for Hamilton circuits and paths: weight of edges, paths and circuits
 - Calculate the weight for a Hamilton circuit and a Hamilton Path

- Complete Graphs
 - Identify a complete graph
 - Given a complete graph, determine the degree of each vertex and the total number of edges
 - Identify a mirror image circuit
 - Calculate the number of Hamilton circuits for a complete graph
- Traveling Salesman Problems
 - Identify when a problem is an example of a Traveling Salesman
 - Differentiate between a Hamilton Circuit(Path) and an Euler Circuit(Path)
- Brute-Force Method
 - Calculate the number of circuits required to determine the optimal solution
 - Use the Brute-Force Method to determine the optimal solution given a graph
 - Use the Brute-Force Method to determine the optimal solution given a table
- Nearest Neighbor and Repetitive Nearest Neighbor Method
 - Calculate the number of circuits required to determine the optimal solution
 - Use the Nearest Neighbor Method to determine the optimal solution given a graph
 - Use the Nearest Neighbor Method to determine the optimal solution given a table
 - Use the Repetitive Nearest Neighbor to determine the optimal solution given a graph
 - Use the Repetitive Nearest Neighbor to determine the optimal solution given a table
- The Cheapest Link Method
 - Calculate the number of circuits required to determine the optimal solution
 - Use the Cheapest Link to determine the optimal solution given a table
 - Use the Cheapest Link to determine the optimal solution given a graph

- Applications
 - Determine which methods are optimal or approximate
 - Determine which methods are efficient or inefficient
 - Perform computations with factorial notation

Textbook

Excursions in Modern Mathematics, 7th ed., Prentice Hall 2010. Peter Tannenbaum.

Course Schedule

Unit	Topic(s)
1	The mathematics of voting and counting votes
2	Fair games and congressional apportionment
3	Introduction to graph theory and routing applications

Weekly Work

Watch assigned mini lecture videos and attend a weekly lecture (or review the recorded session). Suggested problems will be assigned, but will not be submitted. Most weeks will also have a specific assignment which is due. These "specific assignments" will vary depending upon the curriculum material of the week.

Exams

A midterm exam follows each of the three units. The midterms associated with Unit 1 and Unit 3 are NOT proctored, however they are timed exams and will be done via Canvas. The second midterm and final exam will be more comprehensive and are proctored exams. These will also be done via Canvas assessment, but your secured proctor will need to provide the password to unlock the exam.

Grading

Assignments	Points Each	Total Points
12 Weekly Assignments (Top 10 Counted)	10 pts / each	100 pts
2 Midterm Exams (Unproctored)	50 pts / each	100 pts
1 Midterm Exam (Proctored)	100 pts	100 pts
1 Final Exam (Proctored / Comprehensive)	200 pts	200 pts
Total		500 pts

Grading Scale

Letter Grade	% Score	Total Points
A	90-100%	448-500
B	80-89%	398-447
C	70-79%	348-397
D	60-69%	298-347
F	0-59%	0-298

Arranging a Proctor

You will need to secure a proctor in order to take exams in this course. A proctor will not automatically be assigned to you; rather, you must make the necessary contacts to secure a professional who will serve in this capacity.

- Contact a person who meets the qualifications and ask him or her to proctor your exam.
- Student Services must approve your proctor before any exams can be taken. Please see instructions for securing a [suitable proctor](#) . While many proctors will serve on a voluntary basis, you are responsible for paying any expenses incurred in retaining a proctor.

- You must complete a Proctor Information Form and submit the completed form with the proctor verification documentation. Note: If your proctor has been previously approved by the World Campus during a prior course within two years, you do not need to obtain verification. World Campus retains proctor information on file for two years.
- If your proctor does not meet the required specifications, Student Services will notify you within 5 to 7 business days.
- You will need to complete an Exam Request Form for each exam. Contact your proctor to confirm the date, time, and location of your exam(s). Complete the form 3 weeks prior to your scheduled exam to allow for processing the request and mailing exams to proctors. If you are located outside the United States, in order to allow adequate time for mailing, please plan to request your exam earlier than 3 weeks prior to the exam week.
- Contact Student Services if you cannot take a scheduled exam.

Academic Integrity

Academic integrity is the pursuit of scholarly activity in an open, honest and responsible manner. Academic integrity is a basic guiding principle for all academic activity at The Pennsylvania State University, and all members of the University community are expected to act in accordance with this principle. Consistent with this expectation, the University's Code of Conduct states that all students should act with personal integrity, respect other students' dignity, rights and property, and help create and maintain an environment in which all can succeed through the fruits of their efforts.

Academic integrity includes a commitment by all members of the University community not to engage in or tolerate acts of falsification, misrepresentation or deception. Such acts of dishonesty violate the fundamental ethical principles of the University community and compromise the worth of work completed by others.

Accommodating Disabilities

Penn State welcomes students with disabilities into the University's educational programs. Every Penn State campus has an office for students with disabilities. The [Student Disability Resources \(SDR\) website](#) provides contact information for every Penn State campus . For

further information, please visit [Student Disability Resources website](#).

In order to receive consideration for reasonable accommodations, you must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: [See documentation guidelines](#) . If the documentation supports your request for reasonable accommodations, your campus disability services office will provide you with an accommodation letter. Please share this letter with your instructors and discuss the accommodations with them as early as possible. You must follow this process for every semester that you request accommodations.

Counseling and Psychological Services

Many students at Penn State face personal challenges or have psychological needs that may interfere with their academic progress, social development, or emotional wellbeing. The university offers a variety of confidential services to help you through difficult times, including individual and group counseling, crisis intervention, consultations, online chats, and mental health screenings. These services are provided by staff who welcome all students and embrace a philosophy respectful of clients' cultural and religious backgrounds, and sensitive to differences in race, ability, gender identity and sexual orientation.

- [Counseling and Psychological Services at University Park \(CAPS\)](#): 814-863-0395
- [Counseling and Psychological Services at Commonwealth Campuses](#)
- Penn State Crisis Line (Available 24 hrs, 7 days a week): 877-229-6400
- Crisis Text Line (Available 24 hrs, 7 days a week): Text LIONS to 741741

Educational Equity / Report Bias

Penn State takes great pride to foster a diverse and inclusive environment for students, faculty, and staff. Acts of intolerance, discrimination, or harassment due to age, ancestry, color, disability, gender, gender identity, national origin, race, religious belief, sexual

orientation, or veteran status are not tolerated and can be reported through Educational Equity via the [Report Bias website](#).