Axiom Quick Reference (January 2008)

Command Line

(pathname)

)clear all – clear workspace

)display op \(\forall \text{function} \rangle - \text{function arguments}\)

)set message autoload off – quietly load algebra

)set message bottom on – show selection process

)set stream calculate 20 – number of terms to calculate

)show (domain) – list all functions

)spool (filename) – start save session

)spool – close spool file

)trace (domain))math – trace execution

)quit – exit Axiom

)read $\langle \text{filename} \rangle [\text{.input}] - \text{evaluate a file}$

)sys (command line) – execute command

_ continues input lines or escapes chars a_ b = "a b"

% is last value

%%(n) is *n*th value

- and ++ start comment lines

Programming

assignment: var := value

x := 3

conditional: if \(\rangle \text{pred} \rangle \text{then \(\text{truecase} \rangle \text{else \(\text{falsecase} \) \)

if (2 > 4) then 4 else 5

loop: for \(\rangle \text{pred} \rangle \text{repeat \(\rangle \text{block} \rangle \)

for i in 1..5 repeat print i

while i < 3 repeat (print i ; i:=i+1)

function: $f(x) = x^2$

 $f(x)==x^2$

anon. function: g:=x +-> x+1 $g(3) \rightarrow 4$

Indentation is significant:

f(x) == (x > 3 => x ; 0)

f(x) ==

 $x > 3 \Rightarrow x$

Basic constants and functions

 $\pi = \%$ pi e = %e i = %i $\infty = \%$ infinity

 $+\infty=\%$ plusInfinity $-\infty=\%$ minusInfinity numeric(%pi) = 3.1415926535897932385

Functions: sin cos tan sec csc cot sinh cosh tanh

sech csch coth log ln exp

ab = a*b $\frac{a}{b} = a/b$ $a^b = a^b$ $\sqrt{x} = sqrt(x)$

 $\sqrt[n]{x} = x^{(1/n)} |x| = abs(x) \log_b(x) = \log(x)/\log(b)$

Operations on expressions

factor(...) expand(...)simplify(...)

Symbolic equations: f(x)=g(x)

Solve f(x) = q(x): solve(f(x)=g(x),x)

 $solve([x^2*y-1,x*y^2-2],.01)$

 $\rightarrow [[y = 1.5859375, x = 0.79296875]]$

complexSolve($[x^2*y-1,x*y^2-2],1/1000$)

radicalSolve($[x^2/a+a+y^3-1,a*y+a+1],[x,y]$)

 $\sum_{i=1}^{n} f(i) = \texttt{reduce(+,[f(i) for i in k..n])}$

 $\prod f(i) = \text{reduce}(*, [f(i) \text{ for } i \text{ in } k..n])$

Pattern Matching

 $logrule:=rule log(x)+log(y) == log(x*y) \rightarrow$ $\log(y) + \log(x) + \%B = \log(x y) + \%B$

 $f:=\log \sin x + \log x \rightarrow \log(\sin(x)) + \log(x)$

 $logrule f \rightarrow log(x sin(x))$

Calculus

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} f(x), x=a$$

 $\lim f(x) = \lim (f(x), x=a, "left")$

 $\lim f(x) = \lim f(x), x=a, "right"$

 $\lim f(x) = \lim f(x), x=\text{plusInfinity}$

 $limit(sin(x)/x,x=\%plusInfinity) \rightarrow 0$

complexLimit($\sin(x)/x$, x=%infinity) \rightarrow "failed"

 $\frac{d}{dx}(f(x)) = D(f(x), x)$

 $\frac{\partial}{\partial x}(f(x,y)) = D(f(x,y),x)$

 $\int f(x)dx = integrate(f(x),x)$

 $\int_a^b f(x)dx = integrate(f(x), x=a..b)$

Series

x:=series 'x

y:= $\sin(x) \to x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + O(x^9)$ coefficient(y,3) $\rightarrow -\frac{1}{6}$

taylor(f(x), x=a)

laurent(x/log(x),x=1)puiseux(sqrt(sec(x)),x=3*%pi/2)

2D graphics

draw(cos(5*t/8),t=0..16*%pi,coordinates==polar)

f(t:SF):SF == sin(3*t/5)

g(t:SF):SF == sin(t)

draw(curve(f,g),0..%pi)

 $draw(x^2+y^3-1=0,x,y,range==[-1..1,-1..1])$

v1:=draw(Gamma(i),i=-4.2..4,adaptive==true)

v2:=draw(1/Gamma(i),i=-4.2..4,adaptive==true)

putGraph(v2,getGraph(v1,1),2)

makeViewport2D(v2)

options: adaptive clip toScale curveColor pointColor unit range coordinates

3D graphics

m(u:SF,v:SF):SF == 1

draw(m,0..2*%pi,0..%pi,coordinates==spherical)

options: title style colorFunction coordinates tubeRadius tubePoints var1Steps var2Steps space

Discrete math

|x| = floor(x) [x] = ceiling(x)

Remainder of n divided by k = rem(n,k), k|n iff n%k==0

 $\binom{x}{m} = \text{binomial}(x,m)$ n! = factorial(n)

 $\phi(n) = \text{eulerPhi}(n)$ Tuples: (1,'Hello,x)

Type Conversions

 $r:=(2/3)*x^2-y+4/5 \rightarrow -y+\frac{2}{3}x^2+\frac{4}{5}$

Type: Polynomial Fraction Integer

r::FRAC POLY INT $\rightarrow \frac{-15y+10x^2+12}{15}$

Type: Fraction Polynomial Integer

 $s := (3+4*\%i)/(7+3*\%i) \rightarrow \frac{33}{58} + \frac{19}{58}\%i$

s::FRAC COMPLEX INT $\rightarrow \frac{3+4\%i}{7+3\%i}$

Equation

eq1:= $3*x+4*v=5 \rightarrow 4u + 3x = 5$

eq2:= $2*x+2*v=3 \rightarrow 2y+2x=3$

lhs eq $1 \to 4y + 3x$

rhs eq $1 \rightarrow 5$

 $eq1+eq2 \to 6y + 5x = 8$

Factored

 $g := factor(4312) \rightarrow 2^37^211$ unit $g \to 1$ numberOfFactors $g \rightarrow 3$ $nthFactor(g,2) \rightarrow 7$ $nthExponent(g,2) \rightarrow 2$ $nthFlag(g,2) \rightarrow "prime"$ $map(factor, 55739/2520) \rightarrow \frac{139401}{2^33^257}$

List

 $a := [1,2,3,4] \rightarrow [1,2,3,4]$ $b := [3,4,5,6] \rightarrow [3,4,5,6]$ append(a,b) \rightarrow [1, 2, 3, 4, 3, 4, 5, 6] $cons(10,a) \rightarrow [10,1,2,3,4]$ empty? $a \rightarrow false$ $a.2 \rightarrow 2$ $a.2 := 99 \rightarrow [1, 99, 3, 4]$ reverse $b \rightarrow [6, 5, 4, 3]$

MakeFunction

expr:=(x+a)^3 $\rightarrow x^3 + 3ax^2 + 3a^2x + a^3$ function(expr,f,x) \rightarrow f $f(2) \rightarrow a^3 + 6a^2 + 12a + 8$ function(expr,g,a) \rightarrow g $g(2) \rightarrow x^3 + 6x^2 + 12x + 8$

Matrix

A:=matrix([[1,2],[3,4]]) $\rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ determinant $A \rightarrow -2$ $v:=vector([1,2]) \rightarrow [1,2]$ $A*v \to [5, 11]$ $A^{-1} \rightarrow \left[\begin{array}{cc} 2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{array}\right]$ $transpose(A) \rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

nrows $A \rightarrow 2$ ncols $A \rightarrow 2$

nullity $A \to 0$

rank $A \rightarrow 2$ trace $A \rightarrow 5$

Polynomial

x+1 yields Type Polynomial Integer z-2.3 yields Type Polynomial Float y^2-z+3/4 yields Type Polynomial Fraction Integer $p:=(y-1)^2*x*z \to (xy^2 - 2xy + x)z$ $q:=(y-1)*x*(z+5) \rightarrow (xy-x)z + 5xy - 5x$ $\gcd(p,q) \to xy - x$ mainVariable p $\rightarrow z$ variables $p \to [z, y, x]$ $degree(p,y) \rightarrow 2$ totaldegree p $\rightarrow 4$ $eval(p,x,w) \rightarrow (wy^2 - 2wy + w)z$ $D(p,x) \to (y^2 - 2y + 1)z$ integrate(p,x) $\rightarrow (\frac{1}{2}x^2y^2 - x^2y + \frac{1}{2}x^2)z$

PrimeField

x:PrimeField(7):= $5 \rightarrow 5$ $x^3 \rightarrow 6$ $1/x \rightarrow 3$

Set

 $s:=brace([1,2,3,4,5]) \rightarrow \{1,2,3,4,5\}$ $t:=brace([2,3,5,7]) \rightarrow \{2,3,5,7\}$ $intersect(s,t) \rightarrow \{2,3,5\}$ union(s,t) $\rightarrow \{1, 2, 3, 4, 5, 7\}$ $difference(s,t) \rightarrow \{1,4\}$ insert! $(7,s) \rightarrow \{1,2,3,4,5,7\}$ remove! $(7,s) \rightarrow \{1, 2, 3, 4, 5\}$ $\{1,2,1,a\} = brace([1,2,1,'a]) (= \{1,2,a\})$ $\{f(x): x \in X, x > 0\} \approx \operatorname{brace}([f(x) \text{ for x in X | x>0}])_{\operatorname{resultant}(p,q) \to 0}$

Special Functions

[fibonacci(k) for k in 0..] \rightarrow [0,1,1,2,3,5,...] [legendre(i,11) for i in 0..5] \rightarrow [0,1,-1,1,1,1] $[jacobi(i,15) \text{ for i in } 0..5] \rightarrow [0,1,1,0,1,0]$ [eulerPhi i for i in 1..] \rightarrow [1,1,2,2,4,2,...] [moebius Mu i for i in 1..] \rightarrow [1,- 1,- 1,0,- 1,1,...] $E1(0.01) \rightarrow 4.0379295765381134$ $Gamma(0.01) \rightarrow 99.432585119150588$

Stream

)set streams calculate 6

ints := [i for i in 1..] \rightarrow [1,2,3,4,5,6,...] ints.20 \rightarrow 20 [i for i in ints | odd? i] \rightarrow [1,3,5,7,9,11,...] String

creation: s:= "Hello" concatenate "He" "llo" \rightarrow "Hello" s(1)='H' s.1='H' s(2..3)='e1's(4..)='lo'split("hi there",char " ") → ["hi", "there"] prefix?("He","Hello") \rightarrow true substring?("ll","Hello",3) \rightarrow true

TwoDimensionalArray

creation: arr:ARRAY2 INT:=new(2,3,0) $\rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ nrows arr $\rightarrow 2$ $ncols arr \rightarrow 3$ $setelt(arr,1,1,17) \rightarrow \begin{bmatrix} 17 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $arr(1,1) \rightarrow 17$

Univariate Polynomial

 $q:UP(x,INT):=(1-6*x+9*x^2)^2$ leadingCoefficient p \rightarrow 18 degree $p \rightarrow 3$ reductum p $\rightarrow 60x^2 - 46x + 8$ $gcd(p,q) \to 9x^2 - 6x + 1$ $lcm(p.q) \rightarrow 162x^5 + 432x^4 - 756x^3 + 408x^2 - 94x + 8$ $p(2) \rightarrow 300$ (used as function)

creation: $p:UP(x,INT):=(3*x-1)^2*(2*x+8)$

 $D(p) \rightarrow 54x^2 + 120x - 46$ (derivative)

Vector

creation: $v := vector([1,2,3,4,5]) \rightarrow [1,2,3,4,5]$ length: $\#v \rightarrow 5$ access: $v.2 \rightarrow 2$ add: $v+v \rightarrow [2, 4, 6, 8, 10]$ multiply: $5*v \rightarrow [5, 10, 15, 20, 25]$ assign: $v.2 := 7 \rightarrow [1, 7, 3, 4, 5]$