$T = 0.75$. Using the alternative tree-building procedure, we set $\Delta t = 0.25$ (3 steps) and the probabilities on each branch to 0.5, so that

$$u = e^{(0.06-0.10-0.0016/2)0.25+0.04\sqrt{0.25}} = 1.0098$$

$$d = e^{(0.06-0.10-0.0016/2)0.25-0.04\sqrt{0.25}} = 0.9703$$

The tree for the exchange rate is shown in Figure 21.11. The tree gives the value of the option as \$0.0026.

Trinomial Trees

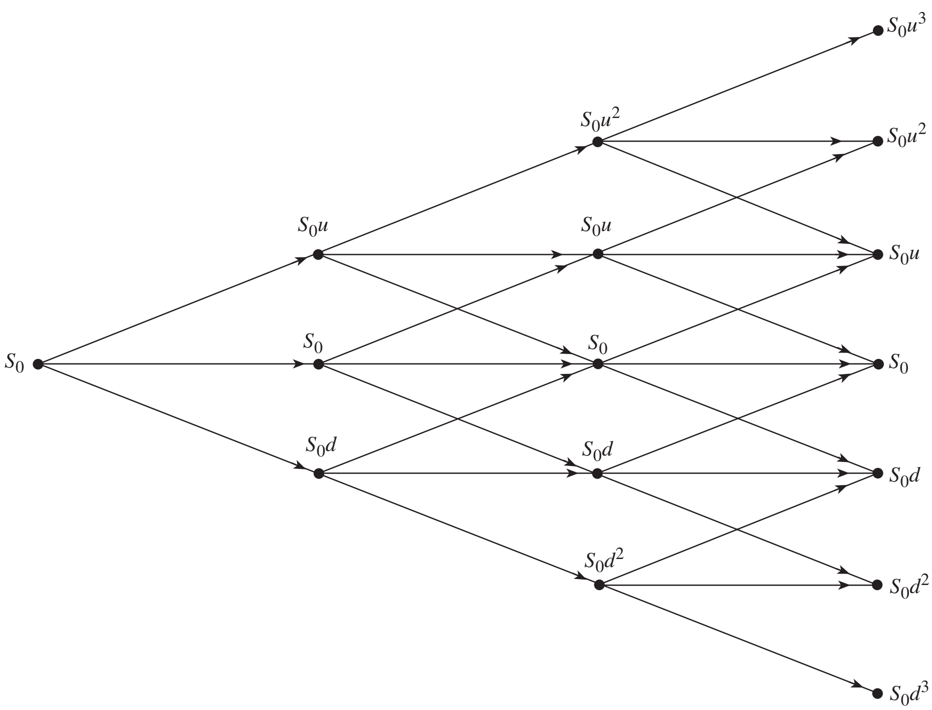
Trinomial trees can be used as an alternative to binomial trees. The general form of the tree is as shown in Figure 21.12. Suppose that p_u, p_m, and p_d are the probabilities of up, middle, and down movements at each node and Δt is the length of the time step. For an asset paying dividends at a rate q, parameter values that match the mean and standard deviation of changes in ln S are

$$u = e^{\sigma\sqrt{3\Delta t}}, \quad d = 1/u$$

$$p_d = -\sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - q - \frac{\sigma^2}{2}\right) + \frac{1}{6}, \quad p_m = \frac{2}{3}, \quad p_u = \sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - q - \frac{\sigma^2}{2}\right) + \frac{1}{6}$$

Calculations for a trinomial tree are analogous to those for a binomial tree. We work from the end of the tree to the beginning. At each node we calculate the value of

Figure 21.12 Trinomial stock price tree.



exercising and the value of continuing. The value of continuing is

$$e^{-r\Delta t}(p_u f_u + p_m f_m + p_d f_d)$$

where f_u, f_m, and f_d are the values of the option at the subsequent up, middle, and down nodes, respectively. The trinomial tree approach proves to be equivalent to the explicit finite difference method, which will be described in Section 21.8.

Figlewski and Gao have proposed an enhancement of the trinomial tree method, which they call the *adaptive mesh model*. In this, a high-resolution (small-Δt) tree is grafted onto a low-resolution (large-Δt) tree.[9] When valuing a regular American option, high resolution is most useful for the parts of the tree close to the strike price at the end of the life of the option.

21.5 TIME-DEPENDENT PARAMETERS

Up to now we have assumed that r, q, r_f, and σ are constants. In practice, they are usually assumed to be time dependent. The values of these variables between times t and $t + \Delta t$ are assumed to be equal to their forward values.[10]

To make r and q (or r_f) a function of time in a Cox–Ross–Rubinstein binomial tree, we set

$$a = e^{[f(t)-g(t)]\Delta t} \tag{21.11}$$

for nodes at time t, where $f(t)$ is the forward interest rate between times t and $t + \Delta t$ and $g(t)$ is the forward value of q (or r_f) between these times. This does not change the geometry of the tree because u and d do not depend on a. The probabilities on the branches emanating from nodes at time t are:[11]

$$p = \frac{e^{[f(t)-g(t)]\Delta t} - d}{u - d} \tag{21.12}$$

$$1 - p = \frac{u - e^{[f(t)-g(t)]\Delta t}}{u - d}$$

The rest of the way that we use the tree is the same as before, except that when discounting between times t and $t + \Delta t$ we use $f(t)$.

Making the volatility, σ, a function of time in a binomial tree is more difficult. Suppose $\sigma(t)$ is the volatility used to price an option with maturity t. One approach is to make the length of each time step inversely proportional to the average variance rate during the time step. The values of u and d are then the same everywhere and the tree recombines. Define the $V = \sigma(T)^2 T$, where T is the life of the tree, and define t_i as the end of the ith time step. For N time steps, we choose t_i to satisfy $\sigma(t_i)^2 t_i = iV/N$ and set $u = e^{\sqrt{V/N}}$ with $d = 1/u$. The parameter p is defined in terms of u, d, r, and q as for a constant volatility. This procedure can be combined with the procedure just mentioned for dealing

[9] See S. Figlewski and B. Gao, "The Adaptive Mesh Model: A New Approach to Efficient Option Pricing," *Journal of Financial Economics*, 53 (1999): 313–51.

[10] The forward dividend yield and forward variance rate are calculated in the same way as the forward interest rate. (The variance rate is the square of the volatility.)

[11] For a sufficiently large number of time steps, these probabilities are always positive.