

## ÁLGEBRA

### Operaciones aritméticas

$$a(b + c) = ab + ac$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

### Exponentes y radicales

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

### Factorización de polinomios notables

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

### Teorema del binomio

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2$$

$$+ \dots + \binom{n}{k}x^{n-k}y^k + \dots + nxy^{n-1} + y^n$$

$$\text{donde } \binom{n}{k} = \frac{n(n-1) \cdots (n-k+1)}{1 \cdot 2 \cdot 3 \cdots k}$$

### Fórmula cuadrática

$$\text{Si } ax^2 + bx + c = 0, \text{ entonces } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

### Desigualdades y valor absoluto

Si  $a < b$  y  $b < c$ , entonces  $a < c$ .

Si  $a < b$ , entonces  $a + c < b + c$ .

Si  $a < b$  y  $c > 0$ , entonces  $ca < cb$ .

Si  $a < b$  y  $c < 0$ , entonces  $ca > cb$ .

Si  $a > 0$ , entonces

$$|x| = a \text{ significa } x = a \text{ o } x = -a$$

$$|x| < a \text{ significa } -a < x < a$$

$$|x| > a \text{ significa } x > a \text{ o } x < -a$$

## GEOMETRÍA

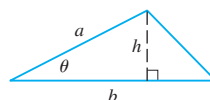
### Fórmulas geométricas

Fórmulas para área  $A$ , circunferencia  $C$  y volumen  $V$ :

Triángulo

$$A = \frac{1}{2}bh$$

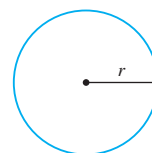
$$= \frac{1}{2}ab \sin \theta$$



Círculo

$$A = \pi r^2$$

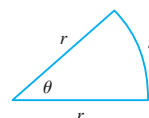
$$C = 2\pi r$$



Sector de círculo

$$A = \frac{1}{2}r^2\theta$$

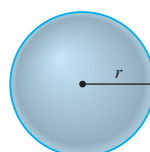
$$s = r\theta \text{ (}\theta \text{ en radianes)}$$



Esfera

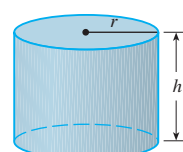
$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$



Cilindro

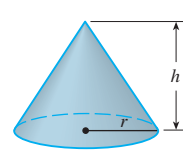
$$V = \pi r^2 h$$



Cono

$$V = \frac{1}{3}\pi r^2 h$$

$$A = \pi r \sqrt{r^2 + h^2}$$



### Fórmulas de distancia y de punto medio

Distancia entre  $P_1(x_1, y_1)$  y  $P_2(x_2, y_2)$ :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Punto medio de } \overline{P_1P_2}: \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

### Rectas

Pendiente de la recta que pasa por  $P_1(x_1, y_1)$  y  $P_2(x_2, y_2)$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Ecuación de punto-pendiente de la recta que pasa por  $P_1(x_1, y_1)$  con pendiente  $m$ :

$$y - y_1 = m(x - x_1)$$

Ecuación de intersección-pendiente de la recta con pendiente  $m$  e intersección  $b$  con el eje  $y$ :

$$y = mx + b$$

### Círculos

Ecuación del círculo con centro  $(h, k)$  y radio  $r$ :

$$(x - h)^2 + (y - k)^2 = r^2$$

## TRIGONOMETRÍA

### Medida de un ángulo

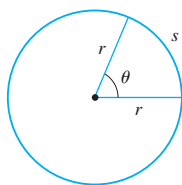
$$\pi \text{ radianes} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$s = r\theta$$

( $\theta$  en radianes)



### Trigonometría de ángulo recto

$$\sin \theta = \frac{\text{op}}{\text{hip}}$$

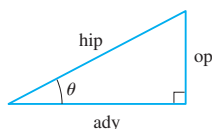
$$\csc \theta = \frac{\text{hip}}{\text{op}}$$

$$\cos \theta = \frac{\text{ady}}{\text{hip}}$$

$$\sec \theta = \frac{\text{hip}}{\text{ady}}$$

$$\tan \theta = \frac{\text{op}}{\text{ady}}$$

$$\cot \theta = \frac{\text{ady}}{\text{op}}$$



### Funciones trigonométricas

$$\sin \theta = \frac{y}{r}$$

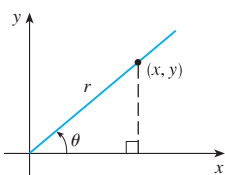
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

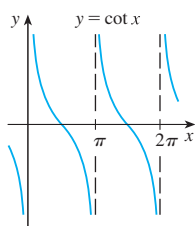
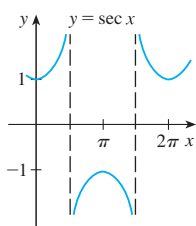
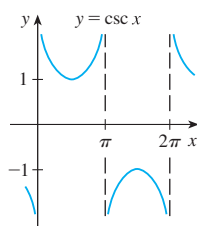
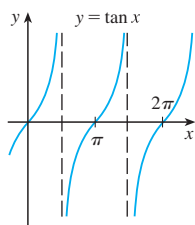
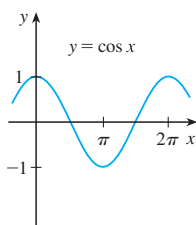
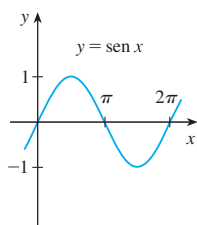
$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



### Gráficas de funciones trigonométricas



### Funciones trigonométricas de ángulos importantes

$\theta$	radianes	$\sin \theta$	$\cos \theta$	$\tan \theta$
$0^\circ$	0	0	1	0
$30^\circ$	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$45^\circ$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$60^\circ$	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$90^\circ$	$\pi/2$	1	0	—

### Identidades fundamentales

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

### La ley de senos

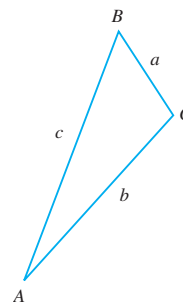
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### La ley de cosenos

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



### Fórmulas de adición y sustracción

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

### Fórmulas de ángulo doble

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

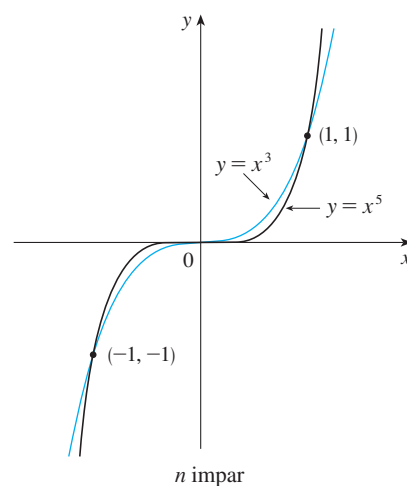
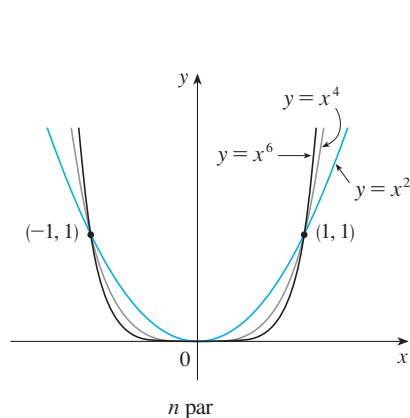
### Fórmulas de semiángulo

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

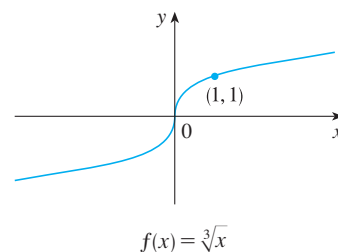
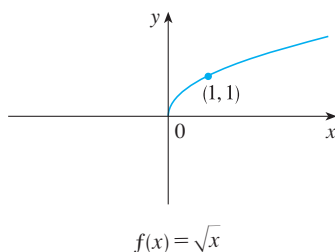
## FUNCIONES ESPECIALES

### Funciones de potencias $f(x) = x^a$

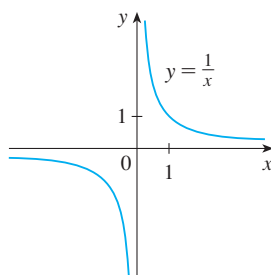
i)  $f(x) = x^n$ ,  $n$  es entero positivo



ii)  $f(x) = x^{1/n} = \sqrt[n]{x}$ ,  $n$  es entero positivo



iii)  $f(x) = x^{-1} = \frac{1}{x}$

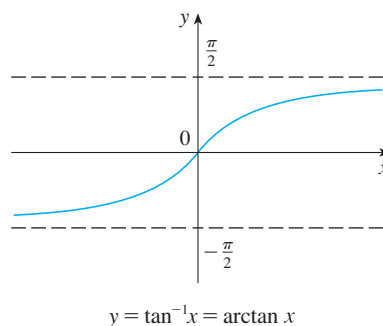


### Funciones trigonométricas inversas

$$\arcsen x = \sin^{-1}x = y \iff \sin y = x \quad y \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$\arccos x = \cos^{-1}x = y \iff \cos y = x \quad y \quad 0 \leq y \leq \pi$$

$$\arctan x = \tan^{-1}x = y \iff \tan y = x \quad y \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$



$$\lim_{x \rightarrow -\infty} \tan^{-1}x = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow \infty} \tan^{-1}x = \frac{\pi}{2}$$

## FUNCIONES ESPECIALES

### Funciones exponenciales y logarítmicas

$$\log_a x = y \iff a^y = x$$

$$\ln x = \log_e x, \text{ donde } \ln e = 1$$

$$\ln x = y \iff e^y = x$$

#### Ecuaciones de cancelación

$$\log_a(a^x) = x \quad a^{\log_a x} = x$$

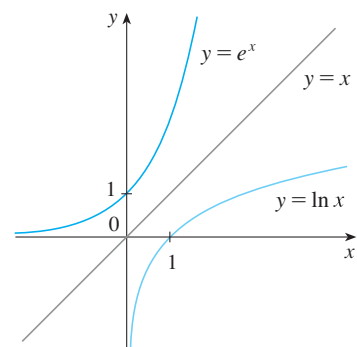
$$\ln(e^x) = x \quad e^{\ln x} = x$$

#### Leyes de los logaritmos

$$1. \log_a(xy) = \log_a x + \log_a y$$

$$2. \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$3. \log_a(x^r) = r \log_a x$$

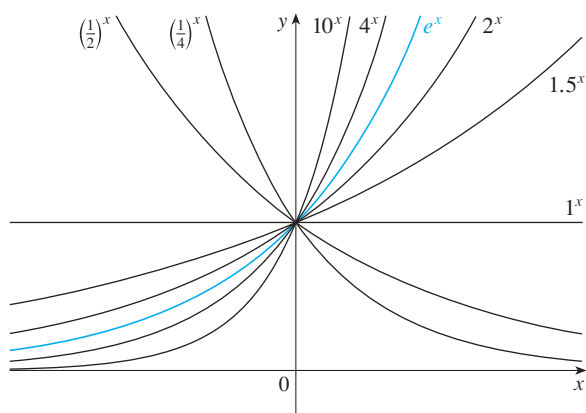


$$\lim_{x \rightarrow -\infty} e^x = 0$$

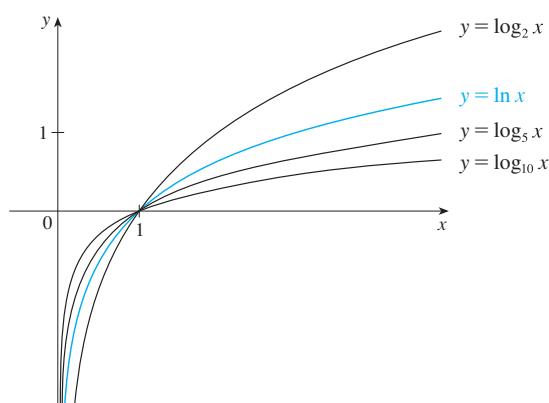
$$\lim_{x \rightarrow \infty} e^x = \infty$$

$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

$$\lim_{x \rightarrow \infty} \ln x = \infty$$



Funciones exponenciales



Funciones logarítmicas

### Funciones hiperbólicas

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

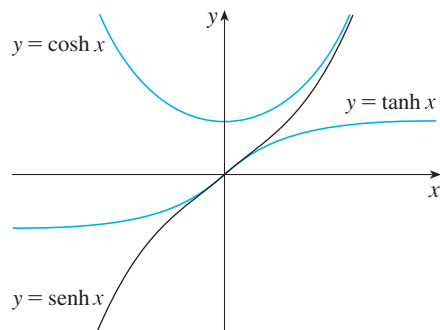
$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{coth} x = \frac{\cosh x}{\sinh x}$$



### Funciones hiperbólicas inversas

$$y = \sinh^{-1} x \iff \sinh y = x$$

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$y = \cosh^{-1} x \iff \cosh y = x \quad y \geq 0$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$$

$$y = \tanh^{-1} x \iff \tanh y = x$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$



## REGLAS DE DIFERENCIACIÓN

### Fórmulas generales

1.  $\frac{d}{dx}(c) = 0$
2.  $\frac{d}{dx}[cf(x)] = cf'(x)$
3.  $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
4.  $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
5.  $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$  (regla del producto)
6.  $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$  (regla del cociente)
7.  $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$  (regla de la cadena)
8.  $\frac{d}{dx}(x^n) = nx^{n-1}$  (regla de potencias)

### Funciones exponenciales y logarítmicas

9.  $\frac{d}{dx}(e^x) = e^x$
10.  $\frac{d}{dx}(a^x) = a^x \ln a$
11.  $\frac{d}{dx} \ln |x| = \frac{1}{x}$
12.  $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

### Funciones trigonométricas

13.  $\frac{d}{dx}(\sin x) = \cos x$
14.  $\frac{d}{dx}(\cos x) = -\sin x$
15.  $\frac{d}{dx}(\tan x) = \sec^2 x$
16.  $\frac{d}{dx}(\csc x) = -\csc x \cot x$
17.  $\frac{d}{dx}(\sec x) = \sec x \tan x$
18.  $\frac{d}{dx}(\cot x) = -\csc^2 x$

### Funciones trigonométricas inversas

19.  $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
20.  $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$
21.  $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
22.  $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$
23.  $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$
24.  $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

### Funciones hiperbólicas

25.  $\frac{d}{dx}(\sinh x) = \cosh x$
26.  $\frac{d}{dx}(\cosh x) = \sinh x$
27.  $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$
28.  $\frac{d}{dx}(\text{csch } x) = -\text{csch } x \coth x$
29.  $\frac{d}{dx}(\text{sech } x) = -\text{sech } x \tanh x$
30.  $\frac{d}{dx}(\coth x) = -\text{csch}^2 x$

### Funciones hiperbólicas inversas

31.  $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}$
32.  $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$
33.  $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$
34.  $\frac{d}{dx}(\text{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2+1}}$
35.  $\frac{d}{dx}(\text{sech}^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$
36.  $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2}$

## TABLA DE INTEGRALES

## Formas básicas

1.  $\int u \, dv = uv - \int v \, du$
2.  $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3.  $\int \frac{du}{u} = \ln |u| + C$
4.  $\int e^u \, du = e^u + C$
5.  $\int a^u \, du = \frac{a^u}{\ln a} + C$
6.  $\int \sin u \, du = -\cos u + C$
7.  $\int \cos u \, du = \sin u + C$
8.  $\int \sec^2 u \, du = \tan u + C$
9.  $\int \csc^2 u \, du = -\cot u + C$
10.  $\int \sec u \tan u \, du = \sec u + C$
11.  $\int \csc u \cot u \, du = -\csc u + C$
12.  $\int \tan u \, du = \ln |\sec u| + C$
13.  $\int \cot u \, du = \ln |\sin u| + C$
14.  $\int \sec u \, du = \ln |\sec u + \tan u| + C$
15.  $\int \csc u \, du = \ln |\csc u - \cot u| + C$
16.  $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C, \quad a > 0$
17.  $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
18.  $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$
19.  $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$
20.  $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

Formas que involucran  $\sqrt{a^2 + u^2}$ ,  $a > 0$ 

21.  $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
22.  $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
23.  $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$
24.  $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$
25.  $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$
26.  $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
27.  $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
28.  $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29.  $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$



## TABLA DE INTEGRALES

### Formas que involucran $\sqrt{a^2 - u^2}$ , $a > 0$

30.  $\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$
31.  $\int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$
32.  $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$
33.  $\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$
34.  $\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$
35.  $\int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$
36.  $\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$
37.  $\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$
38.  $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$

### Formas que involucran $\sqrt{u^2 - a^2}$ , $a > 0$

39.  $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$
40.  $\int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$
41.  $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$
42.  $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$
43.  $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$
44.  $\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$
45.  $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$
46.  $\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$

## TABLA DE INTEGRALES

Formas que involucran  $a + bu$ 

$$47. \int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$$

$$48. \int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$$

$$49. \int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$50. \int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$51. \int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$$

$$52. \int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$53. \int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left( a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$$

$$54. \int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$$

$$55. \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$$

$$56. \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$$

$$57. \int \frac{du}{u \sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \quad \text{si } a > 0$$

$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \quad \text{si } a < 0$$

$$58. \int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u \sqrt{a + bu}}$$

$$59. \int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u \sqrt{a + bu}}$$

$$60. \int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[ u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$$

$$61. \int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$$

$$62. \int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$$





## TABLA DE INTEGRALES

### Formas trigonométricas

$$63. \int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

$$64. \int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$65. \int \tan^2 u \, du = \tan u - u + C$$

$$66. \int \cot^2 u \, du = -\cot u - u + C$$

$$67. \int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$$

$$68. \int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$$

$$69. \int \tan^3 u \, du = \frac{1}{2}\tan^2 u + \ln |\cos u| + C$$

$$70. \int \cot^3 u \, du = -\frac{1}{2}\cot^2 u - \ln |\sin u| + C$$

$$71. \int \sec^3 u \, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln |\sec u + \tan u| + C$$

$$72. \int \csc^3 u \, du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln |\csc u - \cot u| + C$$

$$73. \int \sin^n u \, du = -\frac{1}{n}\sin^{n-1}u \cos u + \frac{n-1}{n} \int \sin^{n-2}u \, du$$

$$74. \int \cos^n u \, du = \frac{1}{n}\cos^{n-1}u \sin u + \frac{n-1}{n} \int \cos^{n-2}u \, du$$

$$75. \int \tan^n u \, du = \frac{1}{n-1}\tan^{n-1}u - \int \tan^{n-2}u \, du$$

$$76. \int \cot^n u \, du = \frac{-1}{n-1}\cot^{n-1}u - \int \cot^{n-2}u \, du$$

$$77. \int \sec^n u \, du = \frac{1}{n-1}\tan u \sec^{n-2}u + \frac{n-2}{n-1} \int \sec^{n-2}u \, du$$

$$78. \int \csc^n u \, du = \frac{-1}{n-1}\cot u \csc^{n-2}u + \frac{n-2}{n-1} \int \csc^{n-2}u \, du$$

$$79. \int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

$$80. \int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

$$81. \int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$82. \int u \sin u \, du = \sin u - u \cos u + C$$

$$83. \int u \cos u \, du = \cos u + u \sin u + C$$

$$84. \int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

$$85. \int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

$$86. \int \sin^n u \cos^m u \, du = -\frac{\sin^{n-1}u \cos^{m+1}u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2}u \cos^m u \, du$$

$$= \frac{\sin^{n+1}u \cos^{m-1}u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2}u \, du$$

### Formas trigonométricas inversas

$$87. \int \sin^{-1}u \, du = u \sin^{-1}u + \sqrt{1-u^2} + C$$

$$88. \int \cos^{-1}u \, du = u \cos^{-1}u - \sqrt{1-u^2} + C$$

$$89. \int \tan^{-1}u \, du = u \tan^{-1}u - \frac{1}{2}\ln(1+u^2) + C$$

$$90. \int u \sin^{-1}u \, du = \frac{2u^2-1}{4}\sin^{-1}u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$91. \int u \cos^{-1}u \, du = \frac{2u^2-1}{4}\cos^{-1}u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$92. \int u \tan^{-1}u \, du = \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C$$

$$93. \int u^n \sin^{-1}u \, du = \frac{1}{n+1} \left[ u^{n+1} \sin^{-1}u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

$$94. \int u^n \cos^{-1}u \, du = \frac{1}{n+1} \left[ u^{n+1} \cos^{-1}u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

$$95. \int u^n \tan^{-1}u \, du = \frac{1}{n+1} \left[ u^{n+1} \tan^{-1}u - \int \frac{u^{n+1} du}{1+u^2} \right], \quad n \neq -1$$

## TABLA DE INTEGRALES

## Formas exponenciales y logarítmicas

$$96. \int u e^{au} du = \frac{1}{a^2} (au - 1) e^{au} + C$$

$$97. \int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$98. \int e^{au} \sen bu du = \frac{e^{au}}{a^2 + b^2} (a \sen bu - b \cos bu) + C$$

$$99. \int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sen bu) + C$$

$$100. \int \ln u du = u \ln u - u + C$$

$$101. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$102. \int \frac{1}{u \ln u} du = \ln |\ln u| + C$$

## Formas hiperbólicas

$$103. \int \sinh u du = \cosh u + C$$

$$104. \int \cosh u du = \sinh u + C$$

$$105. \int \tanh u du = \ln \cosh u + C$$

$$106. \int \coth u du = \ln |\sinh u| + C$$

$$107. \int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$$

$$108. \int \operatorname{csch} u du = \ln \left| \tanh \frac{1}{2} u \right| + C$$

$$109. \int \operatorname{sech}^2 u du = \tanh u + C$$

$$110. \int \operatorname{csch}^2 u du = -\coth u + C$$

$$111. \int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$112. \int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$$

Formas que involucran  $\sqrt{2au - u^2}$ ,  $a > 0$ 

$$113. \int \sqrt{2au - u^2} du = \frac{u - a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left( \frac{a - u}{a} \right) + C$$

$$114. \int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1} \left( \frac{a - u}{a} \right) + C$$

$$115. \int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1} \left( \frac{a - u}{a} \right) + C$$

$$116. \int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left( \frac{a - u}{a} \right) + C$$

$$117. \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1} \left( \frac{a - u}{a} \right) + C$$

$$118. \int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1} \left( \frac{a - u}{a} \right) + C$$

$$119. \int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1} \left( \frac{a - u}{a} \right) + C$$

$$120. \int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$