PRACTICO 1

1)

a)

$$f(x) = \ln(x+1)$$
$$f'(x) = \frac{1}{x+1} = (x+1)^{-1}$$
$$f''(x) = (-1)(x+1)^{-2}$$
$$f'''(x) = 2(x+1)^{-3}$$
$$f''''(x) = (-1) . 6 (x+1)^{4}$$

$$f^{n}(x) = \frac{(-1)^{n+1}(n-1)!}{(x+1)^{n}}$$

Por lo tanto:

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n-1)!}{n!} \cdot x^{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot x^{n}}{n}$$

Además:

$$f^{(n+1)}(t) = \frac{(-1)^{n+2}\, n!}{(t+1)^{n+1}} = \frac{(-1)^{n} \cdot n!}{(t+1)^{n+1}}$$

Entonces:

$$R_{n,0}(x) = \frac{(-1)^{n} \cdot n!}{(T+1)^{n+1}} \cdot \frac{x^{n+1}}{(n+1)!}$$

b) De $\ln(1{,}5)$ podemos usar que $a=0$, $x=0{,}5$. Por lo tanto:

$$R_{n,0}\left(\frac{1}{2}\right)=\frac{(-1)^n\, n!}{(t+1)^{n+1}}\cdot\frac{\left(\frac{1}{2}\right)^{n+1}}{(n+1)!}\ , \quad \left|R_{n,0}\left(\frac{1}{2}\right)\right|=\frac{\left(\frac{1}{2}\right)^{n+1}}{(t+1)^{n+1}(n+1)}$$

Por la serie de taylor de a), $n\geq 1$. Entonces:

$$n\geq 1$$
$$n+1\geq 2\geq 1$$
$$\frac{1}{n+1}\leq 1$$

Además:

$$\frac{1}{n+1}>0$$

Por otra parte, $t\in\left(0,\frac{1}{2}\right)$:

$$0<t<\frac{1}{2}$$
$$1<t+1<\frac{3}{2}$$
$$1^{n+1}<(t+1)^{n+1}$$
$$1>\frac{1}{(t+1)^{n+1}}$$

Y además

$$\frac{1}{(t+1)^{n+1}}>0$$

Por ende:

$$\left|R_{n,0}\left(\tfrac{1}{2}\right)\right| = \frac{1}{n+1}\cdot\frac{1}{(\xi+1)^{n+1}}\cdot\left(\frac{1}{2}\right)^{n+1} < \left(\frac{1}{2}\right)^{n+1} < 10^{-10}$$

$$\left(\frac{1}{2}\right)^{n+1} < 10^{-10}$$

$$(n+1)\cdot \ln\left(\tfrac{1}{2}\right) < \ln\left(10^{-10}\right)$$

$$n+1 > \frac{\ln(10^{-10})}{\ln(\frac{1}{2})}$$

$$n > \frac{\ln(10^{-10})}{\ln(\frac{1}{2})} - 1 = 32{,}22$$

$$\boxed{n = 33}$$

2

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = -\frac{1}{x^2}$$

$$f'''(x) = \frac{2}{x^3}$$

$$f''''(x) = -\frac{6}{x^4}$$

$$f^{n}(x) = \frac{(-1)^{n+1}(n-1)!}{x^n}$$

$$f^{n}(a) = \frac{(-1)^{n+1}(n-1)!}{1^n} = (-1)^{n+1}(n-1)!$$

La serie de Taylor de $\ln(x)$ centrada en $a=1$:

$$f(x)=\sum_{n=1}^{\infty}\frac{f^{n}(a)}{n!}(x-a)^n=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n-1)!}{n!}\cdot(x-1)^n=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-1)^n}{n}$$

Por definición de resto de Taylor y por Teorema de Lagrange para el resto:

$n=1000$

$$\cdot f(x)=\sum_{i=1}^{1000}\frac{(-1)^{i+1}(x-1)^i}{i!}+R_{1000,1}(x)$$

$$\cdot R_{1000,1}(x)=\frac{f^{100}(t)}{100!}\cdot(x-1)^{1001} \quad , \quad t\in(x,1) \text{ ó } t\in(1,x)$$

Entonces:

$$f^{1001}(t)=\frac{(1001-1)!\,(-1)^{1001+1}}{t^{1001}}=\frac{1000!}{t^{1001}}$$

$$R_{1000,1}(x)=\frac{1000!}{t^{1001}(1001)!}(x-1)^{1001}=\frac{1000!\,(x-1)^{1001}}{t^{1001}\cdot 1000!\,1001}$$

$$=\frac{(x-1)^{1001}}{t^{1001}\cdot 1001}$$

Como $f(x)=\ln(x)$ y queremos aproximar $\ln(2)$:

$$R_{1000,1}(2)=\frac{1^{1001}}{t^{1001}\cdot 1001}=\frac{1}{t^{1001}\cdot 1001} \quad , \quad t\in(1,2)$$

Por otro lado:

$$1 < T < 2$$
$$1^{1001} < T^{1001}$$
$$1 = \frac{1}{1^{1001}} > \frac{1}{T^{1001}}$$

Finalmente

$$R_{1000,1}(2) = \frac{1}{T^{1001} \cdot 1001} < \frac{1}{1001}$$

3 $$\ln(e+x) - 1 \stackrel{?}{=} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \cdot \left(\frac{x}{e}\right)^n$$

$$\left.\begin{aligned} f(x) &= \ln(e+x) - 1 \\ f'(x) &= \frac{1}{e+x} \\ f''(x) &= -\frac{1}{(e+x)^2} \\ f'''(x) &= \frac{2}{(e+x)^3} \\ f''''(x) &= -\frac{6}{(e+x)^4} \end{aligned}\right\} \begin{aligned} f^{n}(x) &= \frac{(-1)^{n-1} \cdot (n-1)!}{(x+e)^n} \\ f^{n}(a) &= \frac{(-1)^{n-1}(n-1)!}{(a+e)^n} \end{aligned}$$

Su serie de Taylor centrada en $a=0$:

$$f^{n}(0) = \frac{(-1)^{n-1}(n-1)!}{e^n}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)!}{e^n \cdot n!} \cdot x^n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \cdot \left(\frac{x}{e}\right)^n$$

Es decir que:

$$\ln(e+x)-1=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\cdot\left(\frac{x}{e}\right)^n \qquad \forall x\in I \text{, } I \text{ es el intervalo de convergencia}$$

Veamos dónde converge:

$$\lim_{n\to\infty}\left|\frac{(-1)^n}{(n+1).e^{n+1}}:\frac{(-1)^{n-1}}{n.e^n}\right|=\lim_{n\to\infty}\frac{n.e^n}{(n+1)e^{n+1}}$$

$$=\lim_{n\to\infty}\frac{n}{(n+1).e}=e.\lim_{n\to\infty}\frac{n}{n+1}=e$$

$$\therefore I=(-e,e)$$

Veamos los extremos:

$\underline{x=-e}$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}.(-1)^n=\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}=-\sum_{n=1}^{\infty}(-1)^{2n}\cdot\frac{1}{n}$$

$$=-\sum_{n=1}^{\infty}\frac{1}{n}$$ $\therefore$ diverge por serie armónica.

$\underline{x=e}$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}1^n}{n}=-\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$

Crit. series alternante:

$$\frac{1}{n} \overset{?}{\geq} \frac{1}{n+1}$$

$$n+1 \geq n$$

$1 \geq 0$ no se cumple $\forall n$

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

$\therefore$ Converge

Finalmente:

$$\boxed{I = (-e, e]}$$

4

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f''(x) = -\frac{1}{4\sqrt{x^3}}$$

$$f'''(x) = \frac{3}{8\sqrt{x^5}}$$

$$f''''(x) = -\frac{15}{16\sqrt{x^7}}$$

$$f^{n}(x) = \frac{(-1)^{n+1} \cdot \prod_{k=0}^{n-2} (2k+1)}{2^n \, x^{n-\frac{1}{2}}}$$

$$f^{n}(1) = \frac{(-1)^{n+1} \cdot \prod_{k=0}^{n-2} (2k+1)}{2^n}$$

~~[illegible]~~ Entonces:

$$f(x) = \sum_{j=0}^{n} \frac{(-1)^{j+1} \cdot \left(\prod_{k=0}^{j-2} (2k+1)\right)(x-1)^n}{2^j \; j!} + R_{n,1}(x)$$

Si encontramos n Talque $|R_{n,1}(x)| \leq 10^{-10}$ queda demostrado.

$$|R_{n,1}(x)| = \frac{\left(\prod_{k=0}^{n-1}(2k+1)\right) \cdot (x-1)^{n+1}}{(n+1)!\, 2^{n+1} . T^{n+\frac{1}{2}}} \quad , \quad T \in (x,1) \text{ ó } T \in (1,x)$$

En este caso $x = 0{,}9999999995$

$$|R_{n,1}(0{,}9999999995)| = \frac{\left(\prod_{k=0}^{n-1}(2k+1)\right)(-0{,}0000000005)^{n+1}}{(n+1)!\, 2^{n+1} . T^{n+\frac{1}{2}}}$$

con $T \in (0{,}9999999995 \, , \, 1)$

Probemos con $n=1$:

$$|R_{1,1}(0{,}9999999995)| = \frac{(-0{,}0000000005)^2}{2 . 2^2 . T} = \frac{2{,}5 . 10^{-19}}{8 . T^{\frac{3}{2}}}$$

Notemos que:

$$0{,}9999999995 < T$$
$$(\text{''})^{\frac{3}{2}} < T^{\frac{3}{2}}$$
$$(\text{''})^{-\frac{3}{2}} > T^{-\frac{3}{2}}$$

Entonces:

$$|R_{1,1}(\text{''})| < \frac{2{,}5 . 10^{-19} (0{,}9999999995)^{-\frac{3}{2}}}{8} \approx 3{,}125 . 10^{-33} < 10^{-10}$$

5

Para encontrar la velocidad de convergencia

- $\lim_{n\to\infty} \dfrac{|X_{n+1}-X_*|}{|X_n-X_*|} = C$ $\begin{cases} 0<C<1 \rightsquigarrow \text{lineal} \\ C=0 \rightsquigarrow \text{Superlineal} \end{cases}$

- $\lim_{n\to\infty} \dfrac{|X_{n+1}-X_*|}{|X_n-X_*|^2} = C>0 \rightsquigarrow$ cuadrática
  $\hookrightarrow$ finito

a)

$$\lim_{n\to\infty} 1+\frac{1}{2^n} = 1+\frac{1}{\infty} = 1$$ ✓ converge a 1

Veamos si es cuadrática:

$$\lim_{n\to\infty} \frac{\left|1+\frac{1}{2^n 2}-1\right|}{\left|1+\frac{1}{2^n}-1\right|^2} = \lim_{n\to\infty} \frac{1}{2^n 2} \div \frac{1}{2^n 2^n} = \lim_{n\to\infty} \frac{2^n}{2} = \infty$$

∴ No es cuadrática

Por otro lado:

$$\lim_{n\to\infty} \frac{\left|1+\frac{1}{2^n 2}-1\right|}{\left|1+\frac{1}{2^n}-1\right|} = \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2}$$

Dado que $0<\frac{1}{2}<1$ ∴ Es lineal.

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b)

$$\lim_{n\to\infty} 1+\frac{1}{2^{2^n}} = 1+\frac{1}{2^{2^\infty}} = 1+\frac{1}{2^\infty} = 1+0 = 1$$

Veamos si es cuadrática

$$|X_{n+1}-X_*| = \frac{1}{2^{2^{n+1}}} = \frac{1}{2^{2^n \cdot 2}} = \frac{1}{(2^2)^{2n}} = \frac{1}{4^{2^n}}$$

$$|X_n - X_*|^2 = \frac{1}{(2^{2^n})^2} = \frac{1}{2^{2^n} \cdot 2^{2^n}} = \frac{1}{2^{2^n+2^n}} = \frac{1}{2^{2\cdot 2^n}} = \frac{1}{4^{2^n}}$$

$$\lim_{n\to\infty} \frac{|X_{n+1}-X_*|}{|X_n-X_*|^2} = \lim_{n\to\infty} \frac{1}{4^{2^n}} : \frac{1}{4^{2^n}} = 1$$

$\therefore$ Es cuadrática

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$$\sum_{K=0}^{n} X^K = \frac{1}{1-X} + o(X^n)$$

$$\underbrace{\sum_{K=0}^{n} X^K - \frac{1}{1-X}}_{\alpha} = o(\underbrace{X^n}_{\beta})$$

Si $\lim_{x\to 0} \frac{\alpha}{\beta} = 0$ se cumple.

Notemos que: $\sum_{K=0}^{n} X^K = 1+X+X^2+\cdots+X^n$

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Entonces:

$$\lim_{x \to 0} \frac{x^0 + x^1 + \cdots + x^n - \frac{1}{1-x}}{x^n} = \lim_{x \to 0} \frac{\frac{(1-x)(1+x+\cdots+x^n) - 1}{1-x}}{x^n}$$

$$= \lim_{x \to 0} \frac{1 - x + x - x^2 + x^2 - \cdots - x^n + x^n - x^{n+1} - 1}{(1-x)\, x^n} = \lim_{x \to 0} -\frac{x^{n+1}}{(1-x) \cdot x^n}$$

$$= \lim_{x \to 0} -\frac{x}{1-x} = 0 \quad \therefore \text{ Si se cumple}$$

7

Por hipótesis: $|E| \leq C \cdot |x^n|$ cuando $x \to 0$

$|E| \leq C \cdot \frac{|x^n|}{|x^q|}$ con $n \geq q \geq 1$ cuando $x \to 0$

Siendo $m = n - q$:

$|E| \leq C \cdot |x^m|$ cuando $x \to 0$

Además:

$$\left.\begin{array}{l} q \geq 1 \\ q \geq 0 \\ 0 \geq -q \\ n \geq n-q \\ \boxed{n \geq m} \end{array}\right\} \begin{array}{l} n \geq q \\ n - q \geq 0 \\ m \geq 0 \rightsquigarrow \text{no negativo} \end{array}$$

8

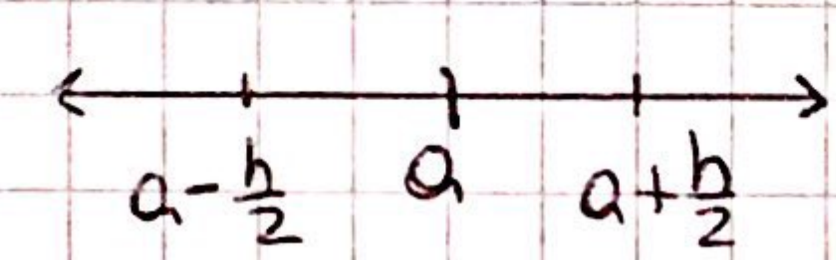
Tenemos que demostrar que:

$$|R_{n,a}(x)| \le C h^{n+1} \qquad \text{cuando } h \to 0$$

función "suave": Tiene todas las derivadas n-ésimas que necesites

$$|R_{n,a}(x)| = \left|\frac{f^{n+1}(t)\cdot(x-a)^{n+1}}{(n+1)!}\right|, \qquad t\in(x,a) \text{ ó } t\in(a,x)$$

que se pueda aproximar en un intervalo de longitud h significa que:



Continuando:

$$|R_{n,a}(x)| = \frac{|f^{n+1}(t)|\cdot|(x-a)^{n+1}|}{|(n+1)!|}$$

Notemos que:

- $(n+1)! = C_1$
- $|f^{n+1}(t)| \le C_2$, pues es una función continua, por lo tanto está acotada.

Por último:

$$a-\frac{h}{2}\leq x\leq a+\frac{h}{2}$$

$$-\frac{h}{2}\leq x-a\leq \frac{h}{2}$$

$$|x-a|\leq \frac{h}{2}$$

$$|x-a|^{n+1}\leq \left(\frac{h}{2}\right)^{n+1}$$

$$|(x-a)^{n+1}|\leq \underbrace{\frac{1}{2^{n+1}}}_{\text{es una constante}}\cdot h^{n+1}$$

$$|(x-a)^{n+1}|\leq C_3\cdot h^{n+1}$$

Por lo tanto:

$$|R_{n,a}(x)|=\frac{|f^{n+1}(t)|\,|(x-a)^{n+1}|}{|(n+1)!|}\leq \frac{C_1.C_3}{C_2}\cdot h^{n+1}=C.h^{n+1}$$

9

a) Si existe C, r tal que $\frac{|f(x)|}{|g(x)|}\leq c$ cuando $x\to 0$ se cumple:

$$\frac{1}{x^2}\leq c.\frac{1}{x}\quad \text{cuando } x\to 0$$

$$\frac{x}{x^2}\leq c\quad x\to 0$$

$$\frac{1}{x}\leq c\quad x\to 0$$

Notemos que $\frac{1}{x}$ es decreciente, si tomamos $r=1$, $c=1$:

$\frac{1}{1}=1\leq 1$ ✓ $\therefore$ Verdadero

b)

$$\frac{n+1}{n^2} \leq C \cdot \frac{1}{n} \qquad n \to \infty$$

$$\frac{n(n+1)}{n^2} \leq C$$

$$\frac{n+1}{n} \leq C$$

$$1 + \frac{1}{n} \leq C$$

↳ es decreciete

Se cumple con $r=1$, $C=2$ pues:

$$1+1=2 \leq 2 \ ✓ \qquad \therefore \text{Verdadero}$$

c)

$$\lim_{x \to 0} \frac{\frac{1}{x}}{\frac{1}{x^2}} = \lim_{x \to 0} x = 0 \qquad \therefore \text{Verdadero}$$

d) $\cos(x) - 1 + \frac{x^2}{2} = O(x^4)$ cuando $x \to 0$

Notemos que $\cos(x) = 1 + 0 - \frac{x^2}{2!} + 0 + \frac{x^4}{4!} \cdots = 1 - \frac{x^2}{2!} + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$

Por lo tanto:

$$\cos(x) = 1 - \frac{x^2}{2} + R_3(x)$$

Si encontramos c tal que $c \geq \frac{|f(x)|}{|g(x)|}$ queda demostrado.

Entonces:

$$\frac{|\cos(x)-1+\frac{x^2}{2}|}{|x^4|} = \frac{|1-\frac{x^2}{2}+R_{3,0}(x)-1+\frac{x^2}{2}|}{|x^4|} = \left|\frac{R_{3,0}(x)}{x^4}\right|$$

$$= \left|\frac{\frac{\cos(t).x^4}{4!}}{x^4}\right| \qquad \text{Con } t\in(0,x) \ \underline{v}\ t\in(x,0)$$

$$= \left|\frac{\cos(t)}{4!}\right| = \frac{|\cos(t)|}{4!} \leq \frac{1}{4!} = C$$

$\hookrightarrow$ pues $-1 \leq \cos(t) \leq 1 \quad \forall t$

10

a)

$$\sin(x) = T_{1,0}(x) + R_{1,0}(x)$$

$f(x) = \sin(x) \qquad = 0$

$f'(x) = \cos(x) \qquad = 1$

$f''(x) = -\sin(x) \qquad = 0$

$f'''(x) = -\cos(x) \qquad = -1$

$f''''(x) = \sin(x) \qquad = 0$

$$R_{1,0}(x) = \frac{\cos(t).\ x^2}{2!}$$

$$|R_{1,0}(x)| \leq \frac{x^2}{2!} \leq \frac{1}{2}10^{-14} \implies x^2 \leq 10^{-14}$$

$$|x| \leq 10^{-7}$$

$$\implies x \in [-10^{-7}, 10^{-7}]$$

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b)

$$\left|\frac{\sin(x)-x}{x^3}\right| \overset{?}{\leq} C$$

$$\sin(x) = T_{2,0}(x) + R_{2,0}(x) = x + 0 + R_{2,0}(x) = x + \frac{f^{3}(t) \cdot x^3}{3!} = x + \frac{-\cos(t) \cdot x^3}{3!}$$

Para $t \in \begin{matrix}[0, x] \\ \circ \\ [x, 0]\end{matrix}$

Luego:

$$\left|\frac{\sin(x)-x}{x^3}\right| \leq \frac{\left|x + \frac{-\cos(t) \cdot x^3}{3!} - x\right|}{|x^3|} \leq \frac{\left|\frac{x^3}{3!}\right|}{|x^3|} \leq \frac{1}{6}$$

$\therefore$ QED

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$$fl(u) = 0,37215 \cdot 10^1$$

$$fl(v) = 0,37202 \cdot 10^1$$

$$fl(fl(u) - fl(v)) = fl(0,00013 \cdot 10^1) = fl(0,13 \cdot 10^{-2}) = 0,00130 \cdot 10^0$$

$$e_r = \frac{|0,001248121 - 0,00130|}{|0,001248121|} \simeq 0,04156\ldots$$

$$e_{r\%} = (100 e_r)\% \approx 4,15\%$$

$$fl(-b+\sqrt{b^2-4ac}) = fl(40+39{,}99) = fl(79{,}99) = 0{,}7999 \cdot 10^2$$

$$(\alpha)\ fl(-b-\sqrt{b^2-4ac}) = fl(40-39{,}99) = fl(0{,}01) = 0{,}1 \cdot 10^{-1}$$

$$fl(2a) = 0{,}2 \cdot 10^{-1}$$

$$fl\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) = fl\left(\frac{0{,}7999 \cdot 10^2}{0{,}2 \cdot 10^1}\right) = fl\left(\frac{7{,}999}{0{,}2}\right)$$

$$= fl(39{,}995) = 0{,}4000 \cdot 10^2 = 40$$

$$fl\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = fl\left(\frac{0{,}1 \cdot 10^{-1}}{0{,}2 \cdot 10^{-1}}\right) = fl(0{,}5) = 0{,}5000 \cdot 10^0$$

$$e_{r_{x_1}} = \frac{|39{,}9937490231 - 40|}{|39{,}9937490231|} \simeq 0{,}00015629834$$

$$e_{r_{x_2}} = \frac{|0{,}0062509768 6 - 0{,}5|}{|0{,}00625097686|} \simeq 78{,}98\ldots$$

Notemos que en (α) se produce una resta de números próximos.

Un método más optimo:

Si $b>0 \leadsto x_1 = (-b-\sqrt{b^2-4ac}) : 2a$

Si $b<0 \leadsto x_1 = (-b+\sqrt{b^2-4ac}) : 2a$

Luego $x_2 = \dfrac{c}{a \cdot x_1}$

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- Sumarle 0,000009, es como sumarle 0,00001 por redondeo
- Sumarle 0,0000049 es como sumarle 0
- ~~Sumarle~~

$\therefore fl(1.0 + x) = 1.0 \quad \forall |x| \leq 0,0000049 < 5 \cdot 10^{-6}$

13

a) Para evitar la resta:

$$(\alpha+x)^n - \alpha^n = \left(\sum_{i=0}^{n} \binom{n}{i} \alpha^{n-i} \cdot x^i\right) - \alpha^n = \alpha^n + \left(\sum_{i=1}^{n} \binom{n}{i} \cdot \alpha^{n-i} x^i\right) - \alpha^n$$

$$= \sum_{i=1}^{n} \binom{n}{i} \alpha^{n-i} \cdot x^i$$

b) Hay que evitar resta de números próximos:

$$a - \sqrt{a^2 - x} = a - \sqrt{a^2 - x} \cdot \frac{a + \sqrt{a^2 - x}}{a + \sqrt{a^2 - x}}$$

$$= \frac{a^2 - (a^2 - x)}{a + \sqrt{a^2 - x}} = \frac{x}{a + \sqrt{a^2 - x}}$$

$$= \frac{x}{a + \sqrt{x} \cdot \sqrt{\frac{a^2}{x} - 1}}$$

c)

$$\ln(a+x) - \ln(a) = \ln\left(\frac{a+x}{a}\right)$$

d) $\sin(a+x) - \sin(a-x)$

$= [\sin(a)\cos(x) + \sin(x)\cos(a)] - [\sin(a)\cos(x) - \sin(x)\cos(a)]$

$= 2\sin(x)\cos(a)$

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$$\frac{40 \pm \sqrt{40^2 - 1}}{2} \begin{cases} 39{,}9937490231 \\ 0{,}00625097686 \end{cases}$$

$fl(a) = 0{,}1 \cdot 10^1$

$fl(b) = -0{,}4 \cdot 10^1$

$fl(c) = 0{,}25 \cdot 10^0$

$fl(a \cdot c) = fl(0{,}25) = 0{,}25 \cdot 10^0$

$fl(4 \cdot a \cdot c) = fl(4 \cdot 0{,}25) = 0{,}1 \cdot 10^1$

$fl(b \cdot b) = fl(40 \cdot 40) = fl(1600) = 0{,}16 \cdot 10^4$

$fl(b^2 - 4ac) = fl(0{,}16 \cdot 10^4 - 0{,}1 \cdot 10^0) = fl(1599) = 0{,}1599 \cdot 10^4$

$fl(\sqrt{b^2 - 4ac}) = fl(\sqrt{0{,}1599} \cdot 10^2) = fl(0{,}399874998046 \cdot 10^2)$

$= 0{,}3999 \cdot 10^2$

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Noter que $b<0$, entonces $x_1 = 40$

$$fl(a \cdot x_1) = fl(1 \cdot 40) = 40$$

$$fl\left(\frac{c}{a \cdot x_1}\right) = fl\left(\frac{0{,}25}{40}\right) = fl(0{,}00625) = 0{,}625 \cdot 10^{-2}$$

$$e_{r_{x_2}} = \left| \frac{0{,}0062509 7686 - 0{,}00625}{0{,}00625097686} \right| \approx 0{,}000156273 17$$

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