Definiciones

martes, 16 de marzo de 2021 10:42

Definición de límite:

$$\lim_{n\to\infty} x_n = L \Leftrightarrow \left\langle \forall \varepsilon > 0 : \langle \exists \mathrm{N} : \langle \forall n > N : |x_n - L| < \varepsilon \rangle \rangle \right\rangle$$

$$\lim_{n\to\infty} x_n = L \Leftrightarrow \left\langle \forall \varepsilon > 0 : \langle \exists \mathrm{N} : \langle \forall n > N : x_n \in (L + \varepsilon, L - \varepsilon) \rangle \rangle \right\rangle$$

Polinomio de Taylor de orden n centrado en a de f:
Sea:
f con n derivadas

$$\mathrm{T}f_{n,a}(x) = \sum_{k=0}^{n} \frac{f^k(a)}{k!}(x-a)^k$$

Resto de Taylor de orden n centrado en a de f:
Sea:
f con n derivadas

$$\mathrm{R}f_{n,a}(x) = f(x) - \sum_{k=0}^{n} \frac{f^k(a)}{k!}(x-a)^k$$

Orden de convergencia:
Sea:
x_n una suseción que converge a x_*
$q \neq 1$

x_n converge linalmente $\Leftrightarrow \langle \exists c \in (0,1), N \in \mathbb{N} : \langle \forall n \in \mathbb{N}_{\geq N} : |x_{n+1} - x_*| \leq c|x_n - x_*| \rangle \rangle$

x_n converge superlinalmente $\Leftrightarrow \left\langle \exists \{\varepsilon_n\}, N \in \mathbb{N}: \lim_{n\to\infty} \varepsilon_n = 0 : \langle \forall n \in \mathbb{N}_{\geq N} : |x_{n+1} - x_*| \leq \varepsilon_n |x_n - x_*| \rangle \right\rangle$

x_n converge con orden $q \Leftrightarrow \langle \exists c \in \mathbb{R}_{>0}, N \in \mathbb{N} : \langle \forall n \in \mathbb{N}_{\geq N} : |x_{n+1} - x_*| \leq c|x_n - x_*|^q \rangle \rangle$

O grande y o chica:

Sean:
$\{x_n\}, \{\alpha_n\}$ sucesiones

$$x_n = O(\alpha_n) \Leftrightarrow \langle \exists C > 0, r : \langle \forall n > r : |x_n| \leq C|\alpha_n| \rangle \rangle$$
$$x_n = o(\alpha_n) \Leftrightarrow \left\langle \exists \{\varepsilon_n\}, N : \lim_{n\to\infty} \varepsilon_n = 0 \land \varepsilon_n \geq 0 : |x_n| \leq \varepsilon_n |\alpha_n| \right\rangle$$

Sean:
f, g funciones

$$f(x) = O\big(g(x)\big) \text{ cuando } x \to \infty \Leftrightarrow \langle \exists C > 0, r : \langle \forall x \geq r : |f(x)| \leq C|g(x)| \rangle \rangle$$
$$f(x) = o\big(g(x)\big) \text{ cuando } x \to \infty \Leftrightarrow \lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$$
$$f(x) = O\big(g(x)\big) \text{ cuando } x \to x_0 \Leftrightarrow \langle \exists C > 0, \varepsilon : \langle \forall x \in (x_0 - \varepsilon, x_0 + \varepsilon) : |f(x)| \leq C|g(x)| \rangle \rangle$$
$$f(x) = o\big(g(x)\big) \text{ cuando } x \to x_0 \Leftrightarrow \lim_{x\to x_0} \frac{f(x)}{g(x)} = 0$$

Error:
Sea r un número aproximado por $\tilde{r}$

Error absoluto: $\Delta r = |r - \tilde{r}|$
Error relativo: $\delta r = \frac{\Delta r}{|r|} = \frac{|r-\tilde{r}|}{|r|}$

Teoremas

martes, 16 de marzo de 2021 10:15

Serie de Taylor centrada en a:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}(x-a)^{n}$$

Teorema de Lagrange:
Sea:
$\mathbb{I}$ un intervalo de $\mathbb{R}$
$f : \mathbb{I} \to \mathbb{R}$ tal que existe f^{n+1} en $\mathbb{I}$
$a, x \in \mathbb{I}$

$$\exists t \text{ entre } a \text{ y } x : \mathrm{R}f_{n,a}(x) = \frac{f^{n+1}(t)}{(n+1)!}(x-a)^{n+1}$$

Orden de convergencia:
Sean:
x_n una susceción que converge a x_*

$$x_n \text{ converge linealmente} \Leftrightarrow \left\langle \exists c \in (0,1) : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} = c \right\rangle$$

$$x_n \text{ converge superlinealmente } \Leftrightarrow \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} = 0$$

$$x_n \text{ converge cuadraticamente} \Leftrightarrow \left\langle \exists c \in (0,\infty) : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|^2} = c \right\rangle$$

O grande y o chica:

Sean:

$x_*, x_0 \in \mathbb{R}$

x_n una susesión que converge a x_*

$f, g : \mathbb{R} \to \mathbb{R}$

$$x_n = O(\alpha_n) \Leftrightarrow \left\langle \exists c \in (0,\infty) : \lim_{n\to\infty} \frac{|x_n|}{|\alpha_n|} = c \right\rangle$$

$$x_n = o(\alpha_n) \Leftrightarrow \lim_{n\to\infty} \frac{|x_n|}{|\alpha_n|} = 0$$

$$f(x) = O\big(g(x)\big) \text{ cuando } x \to \infty \Leftrightarrow \left\langle \exists c \in (0,\infty) : \lim_{x\to\infty} \frac{|f(x)|}{|g(x)|} = c \right\rangle$$

$$f(x) = O\big(g(x)\big) \text{ cuando } x \to x_0 \Leftrightarrow \left\langle \exists c \in (0,\infty) : \lim_{x\to x_0} \frac{|f(x)|}{|g(x)|} = c \right\rangle$$

Orden de convergencia:

Según wikipedia:
q ≠ 1

x_n converge con orden $q \Leftrightarrow \left\langle \exists c \in (0,\infty) : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|^q} = c \right\rangle$

x_n converge linealmente $\Leftrightarrow \left\langle \exists c \in (0,1) : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} = c \right\rangle$

$\langle \exists c \in (0,1), N : \langle \forall n \geq N : |x_{n+1} - x_*| \leq c|x_n - x_*| \rangle \rangle \Leftrightarrow \left\langle \exists c \in (0,1) : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} = c \right\rangle$

Demostración:

Demostración:
Ida (⇒):

$$\langle \exists c \in (0,1), N : \langle \forall n \geq N : |x_{n+1} - x_*| \leq c|x_n - x_*| \rangle \rangle$$
$$\Leftrightarrow \left\langle \exists c \in (0,1), N : \left\langle \forall n \geq N : \frac{|x_{n+1} - x_*|}{|x_n - x_*|} \leq c \right\rangle \right\rangle$$
$$\Rightarrow \left\langle \exists c \in (0,1), N : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} \leq c \right\rangle$$

Vuelta (⇐):

$$\left\langle \exists c \in (0,1) : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} = c \right\rangle$$
⇔ ⦃Definición de límite⦄
$$\left\langle \exists c \in (0,1) : \left\langle \forall \varepsilon > 0 : \left\langle \exists \mathrm{N} : \left\langle \forall n > N : \frac{|x_{n+1} - x_*|}{|x_n - x_*|} \in (c - \varepsilon, c + \varepsilon) \right\rangle \right\rangle \right\rangle \right\rangle$$
⇒ ⦃En particular para un ε tal que c + ε < 1⦄
$$\left\langle \exists c \in (0,1) : \left\langle \exists \mathrm{N} : \left\langle \forall n > N : \frac{|x_{n+1} - x_*|}{|x_n - x_*|} < c + \varepsilon \right\rangle \right\rangle \right\rangle$$
⇒ ⦃$c + \varepsilon \in (0,1)$⦄

$$\left\langle \exists L \in (0,1) : \left\langle \exists \mathrm{N} : \left\langle \forall n > N : \frac{|x_{n+1} - x_*|}{|x_n - x_*|} < L \right\rangle \right\rangle \right\rangle$$

$$\Leftrightarrow \left\langle \exists c \in (0,1), N : \langle \forall n > N : |x_{n+1} - x_*| < c|x_n - x_*| \rangle \right\rangle$$

$$\Rightarrow \left\langle \exists c \in (0,1), N : \langle \forall n > N : |x_{n+1} - x_*| \leq c|x_n - x_*| \rangle \right\rangle$$

x_n converge superlinalmente $\Leftrightarrow \left\langle \exists \{\varepsilon_n\}, N \in \mathbb{N}: \lim\limits_{n\to\infty} \varepsilon_n = 0 : \langle \forall n \in \mathbb{N}_{\geq N} : |x_{n+1} - x_*| \leq \varepsilon_n |x_n - x_*| \rangle \right\rangle$

$$\left\langle \exists \{\varepsilon_n\}, N \in \mathbb{N}: \lim_{n\to\infty} \varepsilon_n = 0 : \langle \forall n \in \mathbb{N}_{\geq N} : |x_{n+1} - x_*| \leq \varepsilon_n |x_n - x_*| \rangle \right\rangle$$

$$\Leftrightarrow \left\langle \exists \{\varepsilon_n\}, N \in \mathbb{N}: \lim_{n\to\infty} \varepsilon_n = 0 : \left\langle \forall n \in \mathbb{N}_{\geq N} : \frac{|x_{n+1} - x_*|}{|x_n - x_*|} \leq \varepsilon_n \right\rangle \right\rangle$$

$$\Leftrightarrow \left\langle \exists \{\varepsilon_n\} : \lim_{n\to\infty} \varepsilon_n = 0 : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} \leq \lim_{n\to\infty} \varepsilon_n \right\rangle$$

$$\Leftrightarrow \left\langle \exists \{\varepsilon_n\} : \lim_{n\to\infty} \varepsilon_n = 0 : \lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} \leq 0 \right\rangle$$

$$\lim_{n\to\infty} \frac{|x_{n+1} - x_*|}{|x_n - x_*|} = 0$$

Resumen

viernes, 25 de junio de 2021 15:24

1)

martes, 16 de marzo de 2021 10:05

a) $f(x) = \ln(x+1)$

$$f'(x) = (x+1)^{-1}$$

$$f''(x) = (-1)(x+1)^{-2}$$

$$f'''(x) = (-1)^2(2!)(x+1)^{-3}$$

$$f^{(n)}(x) = (n-1)!\,(-1)^{n-1}(x+1)^{-n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= \ln(0+1) + \sum_{n=1}^{\infty} \frac{(n-1)!\,(-1)^{n-1}(0+1)^{-n}}{n!} x^n$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \cdot x^n$$

$\exists\, t$ entre 0 y x : $Rf_{n,0}(x) = \dfrac{f^{n+1}(t)}{(n+1)!} x^{n+1}$

$$Rf_{n,0}(x) = \frac{n!\,(-1)^n (t+1)^{-n-1}}{(n+1)!} x^{n+1}$$

$$Rf_{n,0}(x) = \frac{(-1)^n}{(n+1)(t+1)^{n+1}} x^{n+1}$$

b)

$$\exists t \in \left(0, \tfrac{1}{2}\right) : Rf_{n,0}\left(\tfrac{1}{2}\right) = \frac{(-1)^n}{(n+1)(t+1)^{n+1}} \left(\frac{1}{2}\right)^{n+1}$$

$$\Rightarrow \exists t \in \left(0, \tfrac{1}{2}\right) : \left|Rf_{n,0}\left(\tfrac{1}{2}\right)\right| = \left|\frac{(-1)^n}{(n+1)(t+1)^{n+1}} \left(\frac{1}{2}\right)^{n+1}\right|$$

$$\Rightarrow \exists t \in \left(0, \tfrac{1}{2}\right) : \left|Rf_{n,0}\left(\tfrac{1}{2}\right)\right| = \frac{1}{(n+1)(t+1)^{n+1} 2^{n+1}}$$

$$\Downarrow \quad \left|\begin{array}{l} 0 < t \\ \Rightarrow 1 < t + 1 \\ \Rightarrow 1 < (t+1)^{n+1} \\ \Rightarrow \dfrac{1}{(t+1)^{n+1}} < 1 \\ \Rightarrow \dfrac{1}{(n+1)(t+1)^{n+1} 2^{n+1}} < \dfrac{1}{(n+1) 2^{n+1}} \end{array}\right.$$

$$\left|Rf_{n,0}\left(\tfrac{1}{2}\right)\right| < \frac{1}{(n+1) 2^{n+1}}$$

Nesesito el menor n tal que $\left| R f_{n,0}\left(\frac{1}{2}\right) \right| < 10^{-10}$

Asi que busco el menor n tal que

$$\frac{1}{(n+1)\,2^{n+1}} < 10^{-10}$$

$$10^{10} < (n+1)\,2^{n+1}$$

2)

martes, 16 de marzo de 2021 12:20

$$\ln'(x) = x^{-1}$$

$$\ln''(x) = (-1)\, x^{-2}$$

$$\ln'''(x) = 2!\,(-1)^{2}\, x^{-3}$$

$$\ln^{n}(x) = (n-1)!\,(-1)^{n-1}\, x^{-n}$$

$$\exists t \in (1,2) : \quad R\ln_{1000,1}(2) = \frac{f^{1001}(t)}{1001!}\,(2-1)^{1001}$$

$$\Rightarrow \exists t \in (1,2) : \quad \left|R\ln_{1000,1}(2)\right| = \left|\frac{1000!\,(-1)^{1000}\,t^{-1001}}{1001!}\,1^{1001}\right|$$

$$\Rightarrow \exists t \in (1,2) : \quad \left|R\ln_{1000,1}(2)\right| = \frac{1}{1001\cdot t^{1001}}$$

$$\Downarrow \quad \left|\begin{array}{l} 1 < t \\ \Rightarrow 1 < t^{1001} \\ \Rightarrow \dfrac{1}{1001\cdot t^{1001}} < \dfrac{1}{1001} \end{array}\right.$$

$$\left|R\ln_{1000,1}(2)\right| < \frac{1}{1001}$$

3)

martes, 16 de marzo de 2021 12:34

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

$$f''(x) = \frac{1}{2} \cdot \left(-\frac{1}{2}\right) x^{-\frac{3}{2}}$$

$$f'''(x) = \frac{1}{2} (-1)^{3-1} \frac{1}{2} \cdot \frac{3}{2} x^{-\frac{5}{2}}$$

$$f''''(x) = \frac{(-1)^{4-1}}{2^4} \cdot 5 \cdot 3 \; x^{-\frac{7}{2}}$$

$$f^{(5)}(x) = \frac{(-1)^{5-1}}{2^5} \cdot 7 \cdot 5 \cdot 3 \cdot x^{-\frac{9}{2}}$$

$$f(x) = \sqrt{x}$$

$$f^{(n)}(x) = \frac{(-1)^{n-1}}{2^n} \cdot \left(\prod_{k=0}^{n-2} (2k+1)\right) x^{-\frac{2n-1}{2}}$$

$$T f_{n,1}(x) = \sum_{k=0}^{n} \frac{f^{(k)}(1)}{k!} (x-1)^k$$

$$\exists t \in (0{,}9999999995, 1) \; : \; R f_{1,1}(0{,}9999999995) = \frac{f^{(1+1)}(t)}{(1+1)!} (0{,}9999999995)^{1+1}$$

$$\Rightarrow \exists t \in (0{,}9999999995, 1) \; : \; \left|R f_{1,1}(0{,}9999999995)\right| = \left| \frac{\frac{1}{2} \cdot \left(-\frac{1}{2}\right) t^{-\frac{3}{2}}}{2} (0{,}9999999995)^2 \right|$$

$$\Rightarrow \exists t \in (0{,}9999999995, 1) \; : \; \left|R f_{1,1}(0{,}9999999995)\right| = \frac{(0{,}9999999995)^2}{8 \, t^{\frac{3}{2}}}$$

$$\Downarrow \quad \left| \begin{array}{l} t < 1 \\ \Rightarrow t^{\frac{1}{2}} < 1 \\ \Rightarrow \dfrac{(0{,}9999999995)^2}{8} < \dfrac{(0{,}9999999995)^2}{8\, t^{\frac{1}{2}}} \end{array} \right.$$

$$\left| \mathbb{R}f_{1,1}(0{,}9999999995) \right| > \frac{(0{,}9999999995)^2}{8}$$

$$\frac{(0{,}9999999995)^2}{8} > 10^{-10}$$

$$\Rightarrow \left| \mathbb{R}f_{1,1}(0{,}9999999995) \right| > 10^{-10}$$

4a)

$$\lim_{n\to\infty} \left(1 + \left(\tfrac{1}{2}\right)^n\right)$$
$$= \lim_{n\to\infty} \left(1 + \tfrac{1}{2^n}\right)$$
$$= 1 + 0$$
$$= 1$$

Velocidad de convergencia:

$1 + \left(\frac{1}{2}\right)^n$ converge linealmente

$$\iff \exists c \in (0,1), N : \left\langle \forall n \geq N : \underbrace{\left|1 + \tfrac{1}{2^{n+1}} - 1\right| \leq c\left|1 + \tfrac{1}{2^n} - 1\right|} \right\rangle$$

$$\Updownarrow \qquad \frac{1}{2^{n+1}} \leq \frac{c}{2^n} \iff \frac{1}{2} \leq c$$

$$\exists c \in (0,1) : \tfrac{1}{2} \leq c$$
$\iff$ True

$$\left|1 + \tfrac{1}{2^{n+1}} - 1\right| \leq c\left|1 + \tfrac{1}{2^n} - 1\right|$$
$$\iff \frac{1}{2^{n+1}} \leq \frac{c}{2^n}$$
$$\iff \frac{1}{2} \leq c$$
$$\Rightarrow 1 + \frac{1}{2^n} \text{ converge linealmente } \Big\} \exists c \in ($$

$1 + \left(\frac{1}{2}\right)^n$ no converge super linealmente

$$\iff \nexists \{\varepsilon_n\}, N : \lim_{n\to\infty} \varepsilon_n = 0 : \left\langle \forall n \geq N : \underbrace{\left|1 + \left(\tfrac{1}{2}\right)^{n+1} - 1\right| \leq \varepsilon_n \left|1 + \left(\tfrac{1}{2}\right)^n - 1\right|} \right\rangle$$

$$\Updownarrow \qquad \frac{1}{2} \leq \varepsilon_n$$

$$\nexists \{\varepsilon_n\}, N : \lim_{n\to\infty} \varepsilon_n = 0 : \left\langle \forall n \geq N : \tfrac{1}{2} \leq \varepsilon_n \right\rangle \quad \tfrac{1}{2} \leq \varepsilon_n \Rightarrow \lim_{n\to\infty} \varepsilon_n \geq \tfrac{1}{2}$$
$\iff$ True

$$\left|1 + \tfrac{1}{2^{n+1}} - 1\right| \leq \varepsilon_n\left|1 + \tfrac{1}{2^n} - 1\right|$$
$$\iff \frac{1}{2} \leq \varepsilon_n$$
$$\Rightarrow \lim_{n\to\infty} \varepsilon_n \geq \frac{1}{2}$$
$$\Rightarrow \lim_{n\to\infty} \varepsilon_n \neq 0$$
$$\Rightarrow 1 + \frac{1}{2^n} \text{ no converge super lina}$$

4b) $x_n = 1 + \frac{1}{2^{2^n}}$

$$\lim_{n\to\infty} = 1 + \frac{1}{2^{2^n}}$$
$$= 1 + 0 = 1$$

$$\left|1 + \tfrac{1}{2^{2^{n+1}}} - 1\right| \leq \varepsilon_n \left|1 + \tfrac{1}{2^n} - 1\right|$$
$$\iff \frac{1}{2^{2^n \cdot 2}} \leq \frac{\varepsilon_n}{2^n}$$
$$\iff \frac{1}{2^{2^n} \cdot 2^{2^n}} \leq \frac{\varepsilon_n}{2^n}$$
$$\iff \frac{1}{2^{2^n}} \leq \varepsilon_n$$
$$\Rightarrow \exists \{\varepsilon_n\} : \lim_{n\to\infty} \varepsilon_n = 0 : |x_{n+1} - 1| \leq \varepsilon_n |x_n - 1| \quad \Big\} \varepsilon_n \text{ particular } \varepsilon_n = \frac{1}{2^{2^n}}$$
$\Rightarrow x_n$ Converge superlinealmente

$$\left|1 + \tfrac{1}{2^{2^{n+1}}} - 1\right| \leq c \left|1 + \tfrac{1}{2^n} - 1\right|^2$$
$$\iff \frac{1}{2^{2^n} \cdot 2^{2^n}} \leq c \frac{1}{(2^n)^2}$$

$0,1)$: $\frac{1}{2} \leq ($

lmente

$\Leftrightarrow 1 \leqslant C$

$\Rightarrow x_n$ converge cuadráticamente (al menos)

4c) $x_n = 1 + \frac{1}{n^n}$

$\lim_{n\to\infty}\left(1+\frac{1}{n^n}\right)$

$= 1 + 0$

$= 1$

$$\left|1+\frac{1}{(n+1)^{n+1}}-1\right| \leqslant \varepsilon_n \left|1+\frac{1}{n^n}-1\right|$$

$$\Leftrightarrow \frac{1}{(n+1)^n(n+1)} \leqslant \varepsilon_n \frac{1}{n^n}$$

$$\Leftrightarrow \frac{n^n}{(n+1)^n(n+1)} \leqslant \varepsilon_n$$

Tomo $\varepsilon_n = \frac{n^n}{(n+1)^n(n+1)}$

$\lim_{n\to\infty}\frac{n^n}{(n+1)^n(n+1)} = 0$

$\Downarrow$ Comparación $\Uparrow$

$n > 0$

$\Rightarrow \frac{n}{n+1} < 1$

$\Rightarrow \left(\frac{n}{n+1}\right)^n < 1$

$\Rightarrow \frac{n^n}{(n+1)^n}\cdot\frac{1}{n+1} < \frac{1}{n+1}$) $\frac{1}{n+1} > 0$

$\lim_{n\to\infty}\frac{1}{n+1} = 0$

$\Leftrightarrow 0 = 0$

$\exists\{\varepsilon_n\} : \lim_{n\to\infty}\varepsilon_n = 0 : |x_{n+1}-1| \leqslant \varepsilon_n |x_n - 1|$

$\Rightarrow x_n$ converge super linealmente

$$\left|1+\frac{1}{(n+1)^{n+1}}-1\right| \leqslant C\left|1+\frac{1}{n^n}-1\right|^2$$

$$\Leftrightarrow \frac{1}{(n+1)^{n+1}} \leqslant C\,\frac{1}{n^n\cdot n^n}$$

5)

jueves, 18 de marzo de 2021 12:06

$$\sum_{k=0}^{n} x^k = \frac{1}{1-x} + o(x^n) \quad \text{cuando } x \to 0$$

$$\Leftrightarrow \sum_{k=0}^{n} x^k - \frac{1}{1-x} = o(x^n) \quad \text{cuando } x \to 0$$

$$\Leftrightarrow \sum_{k=0}^{n} x^k - \sum_{k=0}^{\infty} x^k = o(x^n) \quad \text{cuando } x \to 0$$

$$\Leftrightarrow \sum_{k=0}^{n} x^k - \left(\sum_{k=0}^{n} x^k + \sum_{k=n+1}^{\infty} x^k \right) = o(x^n) \quad \text{cuando } x \to 0$$

$$\Leftrightarrow \sum_{k=n+1}^{\infty} x^k = o(x^n) \quad \text{cuando } x \to 0$$

$$\Leftrightarrow \{\!\{\text{Definición } o\}\!\}$$

$$\lim_{x\to 0} \frac{\sum_{k=n+1}^{\infty} x^k}{x^n} = 0$$

$$\Leftrightarrow \lim_{x\to 0} \left(\sum_{k=n+1}^{\infty} \frac{x^k}{x^n} \right) = 0$$

$$\Leftrightarrow \lim_{x\to 0} \left(\sum_{k=n+1}^{\infty} x^{k-n} \right) = 0$$

$$\Leftrightarrow \sum_{k=n+1}^{\infty} 0^{k-n} = 0$$

$$\Leftrightarrow 0 = 0$$

6)

jueves, 18 de marzo de 2021 13:39

Sea:

f una función suave

$h > 0$

$x \in \left(a - \frac{h}{2}, a + \frac{h}{2}\right)$

$\mathrm{R}f_{n,a}(x) = O(h^{n+1})$ cuando $h \to 0$

$$\left\langle \exists t \text{ entre } a \text{ y } x : \mathrm{R}f_{n,a}(x) = \frac{f^{(n+1)}(t)}{(n+1)!}(x-a)^{n+1} \right\rangle$$

Trabajo con el termino usando ese t:

$$\mathrm{R}f_{n,a}(x) = \frac{f^{(n+1)}(t)}{(n+1)!}(x-a)^{n+1}$$

$$\Rightarrow \left|\mathrm{R}f_{n,a}(x)\right| = \left|\frac{f^{(n+1)}(t)}{(n+1)!}(x-a)^{n+1}\right|$$

$$\Rightarrow \lim_{h\to 0} \frac{\left|\mathrm{R}f_{n,a}(x)\right|}{h^{n+1}} = \lim_{h\to 0} \frac{\left|f^{(n+1)}(t)\,(x-a)^{n+1}\right|}{(n+1)!\ h^{n+1}}$$

$$\Rightarrow \{\!\!\{\text{Acoto: } x - a < h\}\!\!\}$$

$$\lim_{h\to 0} \frac{\left|\mathrm{R}f_{n,a}(x)\right|}{h^{n+1}} < \lim_{h\to 0} \frac{\left|f^{(n+1)}(t)\,h^{n+1}\right|}{(n+1)!\ h^{n+1}}$$

$$\Leftrightarrow \lim_{h\to 0}\frac{\left|\mathrm{R}f_{n,a}(x)\right|}{h^{n+1}} < \frac{\left|f^{(n+1)}(t)\right|}{(n+1)!}$$

$$\Rightarrow \lim_{h\to 0}\frac{\left|\mathrm{R}f_{n,a}(x)\right|}{h^{n+1}} \in (0,\infty)$$

$$\Rightarrow \mathrm{R}f_{n,a}(x) = O(h^{n+1}) \quad \text{cuando } h \to 0$$

7)

jueves, 18 de marzo de 2021 12:07

a) $\frac{1}{x^2} = O\left(\frac{1}{x}\right)$ cuando $x \to 0$

Definición O

$$\Longleftrightarrow \left\langle \exists c > 0, \varepsilon : \left\langle \forall x \in (-\varepsilon, \varepsilon) : \left|\frac{1}{x^2}\right| \leqslant c \left|\frac{1}{x}\right| \right\rangle\right\rangle$$

$$\Longleftrightarrow \left\langle \exists c > 0, \varepsilon : \left\langle \forall x \in (-\varepsilon, \varepsilon) : \left|\frac{1}{x}\right| \leqslant c \right\rangle\right\rangle$$

$$\Longleftrightarrow \neg \left\langle \forall c > 0, \varepsilon : \left\langle \exists x \in (-\varepsilon, \varepsilon) : \left|\frac{1}{x}\right| > c \right\rangle\right\rangle$$

$$\Longleftarrow \neg \left\langle \forall c > 0, \varepsilon : \left|\frac{1}{|c+1| \min |\varepsilon/2|}\right| > c \right\rangle$$

$$\Longleftrightarrow \neg \left\langle \forall c > 0, \varepsilon : \frac{1}{c} > (c+1) \min |\varepsilon/2| \right\rangle$$

$$\Longleftarrow \neg \left\langle \forall c > 0 : \frac{1}{c} > c+1 \right\rangle$$

$$\Longleftrightarrow \neg \left\langle \forall c > 0 : 1 > c^2 + c \right\rangle$$

$$\Longleftrightarrow \neg \text{True}$$

$$\Longleftrightarrow \text{False}$$

$$\lim_{x \to 0} \frac{\left|\frac{1}{x}\right|}{\left|\frac{1}{x^2}\right|}$$

$$= \lim_{x \to 0} \left|\frac{1}{x}\right|$$

$$= \infty$$

$$\Rightarrow \boxed{\frac{1}{x^2} \neq O\left(\frac{1}{x}\right) \quad \text{cuando} \quad x \to 0}$$

b) $\frac{n+1}{n^2} = O\left(\frac{1}{n}\right)$ cuando $n \to \infty$

$$\lim_{n \to \infty} \frac{\left|\frac{n+1}{n^2}\right|}{\left|\frac{1}{n}\right|}$$

$$= \lim_{n \to \infty} \frac{n+1}{n}$$

$$= \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)$$

$$= 1 + 0 = 1$$

$$\Rightarrow \boxed{\frac{n+1}{n^2} = O\left(\frac{1}{n}\right) \quad \text{cuando} \quad n \to \infty}$$

c)

$$\lim_{x \to 0} \frac{\left|\frac{1}{x}\right|}{\left|\frac{1}{x^2}\right|}$$

$$= \lim_{x \to 0} |x|$$

$$= 0$$

$$\Rightarrow \frac{1}{x} = o\left(\frac{1}{x^2}\right) \quad \text{cuando} \quad x \to 0$$

d)

$$\lim_{x \to 0} \frac{\left|\cos(x) - 1 + \frac{x^2}{2}\right|}{|x^4|}$$

$$= \left| \lim_{x \to 0} \frac{\cos(x) - 1 + x^2}{x^4} \right|$$

$$\overset{L'H \frac{0}{0}}{=} \left| \lim_{x \to 0} \frac{-\sin(x) + 2x}{4x^3} \right|$$

$$\overset{L'H \frac{0}{0}}{=} \left| \lim_{x \to 0} \frac{-\cos(x) + 2}{12x^2} \right|$$

$x \to 0 \Rightarrow -\cos(x) + 2 \to 1$
$\Rightarrow 12x^2 \to 0$

$$= |-\infty|$$

$$= \infty$$

$$\Rightarrow \boxed{\cos(x) - 1 + \frac{x^2}{2} \neq o(x^4) \quad \text{cuando } x \to 0}$$

8)

viernes, 19 de marzo de 2021 16:22

$$\operatorname{sen}(x) = x + o(x^3) \quad \text{cuando} \quad x \to 0$$

$$\Longleftrightarrow \operatorname{sen}(x) - x = o(x^3) \quad \text{cuando} \quad x \to 0$$

$$\lim_{x \to 0} \frac{|\operatorname{sen}(x) - x|}{|x^3|}$$

$$= \left| \lim_{x \to 0} \frac{\operatorname{sen}(x) - x}{x^3} \right|$$

$$\overset{L'H \frac{0}{0}}{=} \left| \lim_{x \to 0} \frac{\cos(x) - 1}{3x^2} \right|$$

$$\overset{L'H \frac{0}{0}}{=} \left| \lim_{x \to 0} \frac{-\operatorname{sen}(x)}{6x} \right|$$

$$\overset{L'H \frac{0}{0}}{=} \left| \lim_{x \to 0} \frac{-\cos(x)}{6} \right|$$

$$= \frac{1}{6}$$

$$\Rightarrow \sin(x) - x = O(x^3) \quad \text{cuando} \quad x \to 0$$

9)

viernes, 19 de marzo de 2021 20:22

$$u = 3.721478693 \simeq 0.37214 \cdot 10^{1}$$

$$v = 3.720230572 \simeq 0.37202 \cdot 10^{1}$$

$$u - v \simeq (0.37214 - 0.37202) \cdot 10^{1}$$

$$= (0.00012) \cdot 10^{1}$$

$$= 0.0012 = 0.12 \cdot 10^{-2}$$

$$\delta(u-v) = \frac{|0.001248121 - 0.0012|}{0.001248121} \simeq 0.0385547555$$

10)

martes, 23 de marzo de 2021 09:24

a) $(\alpha + x)^n - \alpha^n$

$$= -\alpha^n + \sum_{k=0}^{n} \binom{n}{k} \alpha^k x^{n-k}$$

$$= -\alpha^n + \alpha^n + \sum_{k=0}^{n-1} \binom{n}{k} \alpha^k x^{n-k}$$

$$= \sum_{k=0}^{n-1} \binom{n}{k} \alpha^k x^{n-k}$$

b) $\alpha - \sqrt{\alpha^2 - x}$

$$= \alpha - \sqrt{\alpha^2 - x} \; \frac{\alpha + \sqrt{\alpha^2 - x}}{\alpha + \sqrt{\alpha^2 - x}}$$

$$= \frac{\alpha^2 - (\alpha^2 - x)}{\alpha + \sqrt{\alpha^2 - x}}$$

$$= \frac{x}{\alpha + \sqrt{\alpha^2 - x}}$$

c) $\ln(\alpha + x) - \ln(\alpha)$

$$= \ln\left(\frac{\alpha + x}{\alpha}\right)$$

$$= \ln\left(1 + \frac{x}{\alpha}\right)$$

d) $\sin(\alpha + x) - \sin(\alpha - x)$

$$= \sin(\alpha)\cos(x) + \cos(\alpha)\sin(x) - \sin(\alpha)\cos(\alpha) + \cos(\alpha)\sin(x)$$

$$= 2 \cdot \cos(\alpha) \cdot \sin(x)$$

11)

martes, 23 de marzo de 2021 11:01

Orden creciente

$$x_5 + x_4 = 0.9876 \cdot 10^{-3} + 0.4667 \cdot 10^{-2}$$

$$= 0.9876 \cdot 10^{-3} + 4.667 \cdot 10^{-3}$$

$$= 5.6546 \cdot 10^{-3}$$

$$\simeq 0.5655 \cdot 10^{-2}$$

$$\overline{(x_5 + x_4)} + x_3 = 0.5655 \cdot 10^{-2} + 0.3441 \cdot 10^{-2}$$

$$= 0.9096 \cdot 10^{-2}$$

$$+ x_2 = 0.9096 \cdot 10^{-2} + 0.3543 \cdot 10^{1}$$

$$= 0.9096 \cdot 10^{-2} + 354.3 \cdot 10^{-2}$$

$$= 355.2096 \cdot 10^{-2}$$

$$\simeq 0.3552 \cdot 10^{1}$$

$$+ x_1 = 0.3552 \cdot 10^{1} + 0.1234 \cdot 10^{2}$$

$$= 0.3552 \cdot 10^{1} + 1.234 \cdot 10^{1}$$

$= 0.3552 \cdot 10^1 + 1.234 \cdot 10^1$

$= 1.5892 \cdot 10^1$

$\simeq 0.1589 \cdot 10^2$

$\bar{x} = 15.89$

$$\delta x = \frac{|15.8330646 - 15.89|}{15.8330646}$$
$$= \frac{284677}{79165323}$$
$$\simeq 0.00359598103$$

Orden decreciente:

$x_1 + x_2 = 0.1234 \cdot 10^2 + 0.3453 \cdot 10^1$

$= 1.5793 \cdot 10^1$

$\simeq 0.1579 \cdot 10^2$

$+ x_3 = 0.1579 \cdot 10^2 + 0.3441 \cdot 10^{-1}$

$0.0003441 \cdot 10^2$

$= 0.1582441 \cdot 10^2$

$$\simeq 0.1582 \cdot 10^2$$

$$+ x_4 = 0.1582 \cdot 10^2 + 0.4667 \cdot 10^{-2}$$

$$= 0.15824667 \cdot 10^2$$

$$\simeq 0.1582 \cdot 10^2$$

$$+ x_5 = 0.1582 \cdot 10^2 + 0.9876 \cdot 10^{-3}$$

$$0.000009876 \cdot 10^2$$

$$= 0.158209876 \cdot 10^2$$

$$\simeq 0.1582 \cdot 10^2$$

$$\delta x = \frac{|15.8330646 - 15.82|}{15.8330646}$$
$$= \frac{65323}{79165323}$$
$$\simeq 0.00082514663$$

Es mejor el orden decreciente

12)

martes, 23 de marzo de 2021 11:51

$$x^2 - 40x + \frac{1}{4}$$

$$\frac{40 \pm \sqrt{40^2 - 4 \cdot 1 \cdot \frac{1}{4}}}{2 \cdot 1} = \frac{40 \pm \sqrt{1599}}{2} \approx \begin{cases} 39.9937490231 \\ 0.00625097686 \end{cases}$$

$$fl\left(\frac{40 - \sqrt{40^2 - 4 \cdot 0.25}}{2}\right)$$

$$= fl\left(\frac{0.4 \cdot 10^2 - \sqrt{(0.4 \cdot 10^2)^2 - (0.4 \cdot 10^1) \cdot (0.25 \cdot 10^0)}}{0.2 \cdot 10^1}\right)$$

$$= fl\left(\frac{0.4 \cdot 10^2 - \sqrt{0.16 \cdot 10^4 - 0.1 \cdot 10^1}}{0.2 \cdot 10^1}\right)$$

$0.1 \cdot 10^1 = 0.0001 \cdot 10^4$

$$= fl\left(\frac{0.4 \cdot 10^2 - \sqrt{0.1599 \cdot 10^4}}{0.2 \cdot 10^1}\right)$$

$$= fl\left(\frac{0.4 \cdot 10^2 - 0.39987\ldots \cdot 10^2}{0.2 \cdot 10^1}\right)$$

$$= fl\left(\frac{0.4 \cdot 10^2 - 0.3999 \cdot 10^2}{0.2 \cdot 10^1}\right)$$

$$= fl\left(\frac{0.0001 \cdot 10^2}{0.2 \cdot 10^1}\right)$$

$$= fl\left(\frac{0.1 \cdot 10^{-1}}{0.2 \cdot 10^1}\right)$$

$$= fl\left(\frac{0.001 \cdot 10^1}{0.2 \cdot 10^1}\right)$$

$$= fl(0.0005 \cdot 10^1)$$

$$= 0.5000 \cdot 10^{-2}$$

$$\delta x_1 = \frac{|0.00625097686 - 0.005|}{0.00625097686}$$

$\simeq 0.20013$

Método alternativo:

$$\frac{-b-\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}$$

$$=\frac{\left(-b-\sqrt{b^2-4\cdot a\cdot c}\right)\left(-b+\sqrt{b^2-4\cdot a\cdot c}\right)}{2\cdot a\left(-b+\sqrt{b^2-4\cdot a\cdot c}\right)}$$

$$=\frac{b^2-b^2+4\cdot a\cdot c}{2\cdot a\left(-b+\sqrt{b^2-4\cdot a\cdot c}\right)}$$

$$=\frac{2c}{-b+\sqrt{b^2-4\cdot a\cdot c}}$$

$fl\left(-\sqrt{40^2-4\cdot 1\cdot 0.25}\right)=0.3999\cdot 10^2$ (calculado antes

$fl(-b+0.3999\cdot 10^2)=fl(0.4000\cdot 10^2+0.3999\cdot 10^2)$

$=fl(0.7999\cdot 10^2)=0.7999\cdot 10^2)$

$fl(2c)=fl(0.2000\cdot 10^1\cdot 0.25\cdot 10^0)$

$=fl(0.5\cdot 10^0)=0.5000\cdot 10^0$

$fl\left(\frac{0.5000\cdot 10^0}{0.7999\cdot 10^2}\right)=fl\left(\frac{0.5}{79.99}\right)$

$=fl(0.0060205 00\ldots)$

$=0.6021\cdot 10^{-2}$

.500. | 7.999
−479.94 | 0.0060 20500X (x>0)
020.0.6
−15 9.98
04 0.6.2
−399.95
006.25

$$\delta x_1=\frac{|0.00625097686-0.006021|}{0.00625097686}$$
$$=\frac{11498843}{312548843}\simeq 0.03679$$

13)

miércoles, 24 de marzo de 2021 16:24

13a) $f_i(2.25 \cdot 9.30) = f_i(2092.5)$

$= 2092.50$

$$\begin{array}{r} 2.25 \\ 9.3 \\ \hline 675 \\ 20.25 \\ \hline 20.925 \end{array}$$

$f_i\left(\frac{20.92\,50}{100}\right) = f_i(0.20925)$

$= 0.21$

13b) $F_i(0.21 \cdot 14\,000\,000) = F_i(2\,940\,000)$

13c) Podría hacer:

$$\frac{2.25 \cdot 14\,000\,000 \cdot 9.3}{1000}$$

Le daria:

$$fi(2.25 \cdot 14\,000\,000) = fi(31\;500\;000)$$

$$= 31\,500\,000.00$$

$$fi(31\,500\,000 \cdot 9.3) = 292\;950\,000$$

$$fi\left(\frac{292\;950\,000}{100}\right) = 2\;929\;500$$

14)

miércoles, 24 de marzo de 2021 16:52

a)

$$fl(0.98765) = 0.9877 \cdot 10^{0}$$

$$fl(0.012424) = 0.1242 \cdot 10^{-1}$$

$$fl(0.0065432) = 0.6543 \cdot 10^{-2}$$

$$fl(0.9877 \cdot 10^{0} + 0.1242 \cdot 10^{-1}) = fl(0.99012 \cdot 10^{0})$$
$$= 0.9901 \cdot 10^{0}$$

$$fl(0.9901 \cdot 10^{0} - 0.6543 \cdot 10^{-2}) = fl(0.983557 \cdot 10^{0})$$
$$= 0.9836 \cdot 10^{0}$$

$\neq$

$$fl(0.1242 \cdot 10^{-1} - 0.6543 \cdot 10^{-2}) = fl(0.05877 \cdot 10^{-1})$$
$$= 0.5877 \cdot 10^{-2}$$

$$fl(0.9877 \cdot 10^{0} + 0.5877 \cdot 10^{-2}) = fl(0.993577 \cdot 10^{0})$$
$$= 0.9936 \cdot 10^{0}$$

b)

$$fl(4.2832) = 0.4283 \cdot 10^{1}$$

$$fl(4.2821) = 0.4282 \cdot 10^{1}$$

$$fl(5.7632) = 0.5763 \cdot 10^{1}$$

$$fl(0.4283 \cdot 10^{1} - 0.4282 \cdot 10^{1}) = fl(0.0001 \cdot 10^{1})$$
$$= 0.1000 \cdot 10^{-3}$$

$$fl(0.1000 \cdot 10^{-3} \cdot 0.5763 \cdot 10^{1}) = fl(0.5763 \cdot 10^{-3})$$
$$= 0.5763 \cdot 10^{-3}$$

$$fl(0.4283 \cdot 10^{1} \cdot 0.5763 \cdot 10^{1}) = fl(0.23682\ldots \cdot 10^{2})$$

$$\begin{array}{r} 4283 \\ * \; 5763 \\ \hline 12849 \end{array} \qquad \begin{array}{r} 4282 \\ * \; 5763 \\ \hline \end{array}$$

$$fl(0.4283\cdot 10^1 \cdot 0.5763 \cdot 10^1) = fl(0.23682\ldots \cdot 10^2)$$

$$= 0.2368 \cdot 10^2$$

$$fl(0.4282\cdot 10^1 \cdot 0.5763 \cdot 10^1) = fl(0.24677\ldots \cdot 10^2)$$

$$= 0.2468 \cdot 10^2$$

$$fl(0.2368 \cdot 10^2 - 0.2468 \cdot 10^2) = fl(-0.01 \cdot 10^2$$

$$= -0.1 \cdot 10^1$$

$\neq$

4283
*5763
12849
25698
29981
21415
24682929

4282
*5763
12846
25692
29974
21410
24677166

15) ❌

miércoles, 24 de marzo de 2021 17:51

$$P(4.71) = (4.71)^3 - 6 * (4.71)^2 + 3 * 4.71 - 0.149$$
$$= -14.636489$$

$$Fl(x) = 0.471 \cdot 10^1$$

$$x^2 \simeq Fl((0.471 \cdot 10^1 \cdot 0.471 \cdot 10^1) = Fl(0.221841 \cdot 10^2)$$
$$= 0.222 \cdot 10^2$$

$$\begin{array}{r} 471 \\ * \; 471 \\ \hline 0471 \\ 3297 \\ 1884 \\ \hline 221841 \end{array} \qquad \begin{array}{r} 222 \\ * \; 471 \\ \hline 0222 \\ 1554 \\ 0888 \\ \hline 104562 \end{array}$$

$$x^3 \simeq Fl(0.222 \cdot 10^2 \cdot 0.471 \cdot 10^1) = Fl(0.104562 \cdot 10^3)$$
$$= 0.105 \cdot 10^3$$

$$6x^2 \simeq Fl(0.600 \cdot 10^1 \cdot 0.222 \cdot 10^2) = Fl(0.1332 \cdot 10^3)$$
$$= 0.133 \cdot 10^3$$

$$3x \simeq Fl(0.300 \cdot 10^1 \cdot 0.471 \cdot 10^1) = Fl(0.1413 \cdot 10^2)$$
$$= 0.141 \cdot 10^2$$

$$x^3 - 6x^2 \simeq Fl(0.105 \cdot 10^3 - 0.133 \cdot 10^3) = Fl(-0.028 \cdot 10^3)$$
$$= -0.280 \cdot 10^2$$

$$x^3 - 6x^2 + 3x \simeq Fl(-0.280 \cdot 10^2 + 0.141 \cdot 10^2) = Fl(-0.139 \cdot 10^2)$$
$$= -0.139 \cdot 10^2$$

$$x^3 - 6x^2 + 3x - 0.149 \simeq Fl(-0.139 \cdot 10^2 - 0.149 \cdot 10^1) = Fl(-13.9 - 0.149)$$
$$= Fl(-14.049)$$
$$= -0.140 \cdot 10^2$$

$$\delta P(4.71) = \frac{|-14.636489 + 14|}{|-14.636489|}$$
$$= \frac{90927}{2090927}$$
$$\simeq 0.04349$$

$$x-6 \simeq fl(0.477 \cdot 10^1 - 0.600 \cdot 10^1) = fl(-0.129 \cdot 10^1)$$
$$= -0.129 \cdot 10^1$$

$$(x-6) \cdot x \simeq fl(-0.129 \cdot 10^1 \cdot 0.477 \cdot 10^1) = fl(-0.060759 \cdot 10^2)$$
$$= -0.608 \cdot 10^1$$

$$(x-6) \cdot x + 3 \simeq fl(-0.608 \cdot 10^1 + 0.300 \cdot 10^1) = fl(-0.308 \cdot 10^1)$$
$$= -0.308 \cdot 10^1$$

$$((x-6)x+3)x \simeq fl(-0.308 \cdot 10^1 \cdot 0.477 \cdot 10^1) = fl(-0.145068 \cdot 10^2)$$
$$= -0.146 \cdot 10^2$$

$$((x-6)x+3)x - 0.149 \simeq fl(-0.146 \cdot 10^2 - 0.149 \cdot 10^0) = fl(-0.14749 \cdot 10^2)$$
$$= -0.148 \cdot 10^2$$

$$\begin{array}{r} 477 \\ * \; 129 \\ \hline 4239 \\ 0942 \\ 0477 \\ \hline 060759 \end{array}$$

$$\begin{array}{r} 477 \\ 308 \\ \hline 3768 \\ 0000 \\ 1413 \\ \hline 145068 \end{array}$$

$$\delta P(x) = \frac{|-14.636489 + 14.8|}{|-14.636489|}$$
$$= \frac{163511}{14636489}$$
$$\simeq 0.01117$$

Con el esquema de Horner da mejor resultado,
y además es mas rápido