Definiciones

jueves, 8 de julio de 2021 15:04

Aproximación discreta de funciones

Sea:

$f : \mathbb{R} \rightarrow \mathbb{R}$

$x_1, x_2, \dots, x_m \in \mathbb{R}$

$p \in \mathbb{R}_n[x]$

p es la mejor aproximación de grado n por cuadrados mínimos de f en $x_1, x_2, \dots, x_m$

$$\Leftrightarrow \left\langle \forall q \in \mathbb{R}_n[x] : \sum_{i=1}^{m} \left(f(x_i) - p(x_i)\right)^2 \leq \sum_{i=1}^{m} \left(f(x_i) - q(x_i)\right)^2 \right\rangle$$

Funciones de peso y funciones ortogonales:

Sea:

$\omega : \mathbb{R} \rightarrow \mathbb{R}$

I un intervalo de $\mathbb{R}$

ω es una función de peso en $I \Leftrightarrow \langle \forall x \in I : \omega(x) \geq 0 \rangle \wedge \langle \forall J :$ J es un subintervalo de $I : \langle \exists x \in J : \omega(x) \neq 0 \rangle \rangle$

Aclaraciones:

La parte de después del $\wedge$ está diciendo que en todo subintervalo de I, la función no es constantemente 0

Teoremas y demostraciones

lunes, 3 de mayo de 2021 12:47

Cuadrados mínimos:

Sea:

$f : \mathbb{R} \to \mathbb{R}$

$x_1, x_2, \ldots, x_m \in \mathbb{R}$

$p_n(x) = \sum_{j=0}^{n} a_j x^j$

$$a_0, a_1, \ldots, a_n \text{ minmizan} \left(f(x_i) - p_n(x_i)\right)^2 \Leftrightarrow \begin{bmatrix} \sum_{i=0}^{m} x_i^{0+0} & \sum_{i=0}^{m} x_i^{0+1} & \cdots & \sum_{i=0}^{m} x_i^{0+n} \\ \sum_{i=0}^{m} x_i^{1+0} & \sum_{i=0}^{m} x_i^{1+1} & \cdots & \sum_{i=0}^{m} x_i^{1+n} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{m} x_i^{n+0} & \sum_{i=0}^{m} x_i^{n+1} & \cdots & \sum_{i=0}^{m} x_i^{n+n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{m} x_i^0 f(x_i) \\ \sum_{i=0}^{m} x_i^1 f(x_i) \\ \vdots \\ \sum_{i=0}^{m} x_i^n f(x_i) \end{bmatrix}$$

Demostración:

$$\min_{a_0, a_1, \ldots, a_n} \sum_{i=0}^{m} \left(f(x_i) - p_n(x_i)\right)^2$$

$$\min_{a_0, a_1, \ldots, a_n} \sum_{i=0}^{m} \left(f(x_i) - \sum_{j=0}^{n} a_j x_i^j \right)^2$$

$$\forall k \leq n : \frac{\partial}{\partial a_k} \sum_{i=0}^{m} \left(f(x_i) - \sum_{j=0}^{n} a_j x_i^j \right)^2 = 0$$

$$\forall k \leq n : \sum_{i=0}^{m} \frac{\partial}{\partial a_k} \left(f(x_i) - \sum_{j=0}^{n} a_j x_i^j \right)^2 = 0$$

$$\forall k \leq n : \sum_{i=0}^{m} \left(2 \left(f(x_i) - \sum_{j=0}^{n} a_j x_i^j \right) \frac{\partial}{\partial a_k} \sum_{j=0}^{n} a_j x_i^j \right) = 0$$

$$\forall k \leq n : \sum_{i=0}^{m} \left(2 \left(f(x_i) - \sum_{j=0}^{n} a_j x_i^j \right) x_i^k \right) = 0$$

$$\forall k \leq n : \sum_{i=0}^{m} \left(2 x_i^k f(x_i) - 2 x_i^k \sum_{j=0}^{n} a_j x_i^j \right) = 0$$

$$\forall k \leq n : 2\sum_{i=0}^{m} 2x_i^k f(x_i) - 2\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^k a_j x_i^j = 0$$

$$\forall k \leq n : \sum_{j=0}^{n} a_j \sum_{i=0}^{m} x_i^k x_i^j = \sum_{i=0}^{m} 2x_i^k f(x_i)$$

$$\forall k \leq n : \sum_{j=0}^{n} a_j \sum_{i=0}^{m} x_i^{k+j} = \sum_{i=0}^{m} x_i^k f(x_i)$$

$$\begin{bmatrix} \sum_{i=0}^{m} x_i^{0+0} & \sum_{i=0}^{m} x_i^{0+1} & \cdots & \sum_{i=0}^{m} x_i^{0+n} \\ \sum_{i=0}^{m} x_i^{1+0} & \sum_{i=0}^{m} x_i^{1+1} & \cdots & \sum_{i=0}^{m} x_i^{1+n} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{m} x_i^{n+0} & \sum_{i=0}^{m} x_i^{n+1} & \cdots & \sum_{i=0}^{m} x_i^{n+n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{m} x_i^0 f(x_i) \\ \sum_{i=0}^{m} x_i^1 f(x_i) \\ \vdots \\ \sum_{i=0}^{m} x_i^n f(x_i) \end{bmatrix}$$

Con integrales:

$$p_n(x) = \sum_{j=0}^{n} a_j x^j$$

Entre b y c

$$\min_{a_0, a_1, \ldots, a_n} \int_b^c \left(p_n(x) - f(x)\right)^2 \mathrm{d}x$$

$$\min_{a_0, a_1, \ldots, a_n} \int_b^c \left(\left(p_n(x)\right)^2 - 2p_n(x)f(x) + \left(f(x)\right)^2\right) \mathrm{d}x$$

$$\min_{a_0, a_1, \ldots, a_n} \int_b^c \left(p_n(x)\right)^2 \mathrm{d}x - \int_b^c 2p_n(x)f(x)\,\mathrm{d}x + \int_b^c \left(f(x)\right)^2 \mathrm{d}x$$

$$\frac{\partial}{\partial a_k}\left(\int_b^c \left(p_n(x)\right)^2 \mathrm{d}x - \int_b^c 2p_n(x)f(x)\,\mathrm{d}x + \int_b^c \left(f(x)\right)^2 \mathrm{d}x\right) = 0$$

$$\frac{\partial}{\partial a_k}\int_b^c \left(p_n(x)\right)^2 \mathrm{d}x - \frac{\partial}{\partial a_k}\int_b^c 2p_n(x)f(x)\,\mathrm{d}x = 0$$

$$\frac{\partial}{\partial a_k}\int_b^c \left(\sum_{j=0}^{n} a_j x^j\right)^2 \mathrm{d}x - \frac{\partial}{\partial a_k}\int_b^c 2\left(\sum_{j=0}^{n} a_j x^j\right) f(x)\,\mathrm{d}x = 0$$

$$\int_b^c \frac{\partial}{\partial a_k}\left(\sum_{j=0}^{n} a_j x^j\right)^2 \mathrm{d}x - 2\sum_{j=0}^{n}\frac{\partial}{\partial a_k} a_j \int_b^c x^j f(x)\,\mathrm{d}x = 0$$

$$\int_b^c \left(2\sum_{j=0}^{n} a_j x^j\, x^k\right) \mathrm{d}x - 2\int_b^c x^k f(x)\,\mathrm{d}x = 0$$

$$\sum_{j=0}^{n} a_j \int_b^c x^{j+k}\,\mathrm{d}x - \int_b^c x^k f(x)\,\mathrm{d}x = 0$$

$$\sum_{j=0}^{n} a_j \left[\frac{x^{j+k+1}}{j+k+1}\right]_{x=b}^{x=c} - \int_b^c x^k f(x)\,\mathrm{d}x = 0$$

$$\sum_{j=0}^{n} a_j \left(\frac{c^{j+k+1}-b^{j+k+1}}{j+k+1}\right) = \int_b^c x^k f(x)\,\mathrm{d}x$$

$$\begin{bmatrix} \int_b^c x^{0+0}\,\mathrm{d}x & \int_b^c x^{1+0}\,\mathrm{d}x & \cdots & \int_b^c x^{n+0}\,\mathrm{d}x \\ \int_b^c x^{0+1}\,\mathrm{d}x & \int_b^c x^{1+1}\,\mathrm{d}x & \cdots & \int_b^c x^{n+1}\,\mathrm{d}x \\ \vdots & \vdots & \ddots & \vdots \\ \int_b^c x^{0+n}\,\mathrm{d}x & \int_b^c x^{1+n}\,\mathrm{d}x & \cdots & \int_b^c x^{n+n}\,\mathrm{d}x \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \int_b^c x^0 f(x)\,\mathrm{d}x \\ \int_b^c x^1 f(x)\,\mathrm{d}x \\ \vdots \\ \int_b^c x^n f(x)\,\mathrm{d}x \end{bmatrix}$$

Generalización con integrales
Sean:

$B_n = \{b_0, b_1, \dots, b_n\}$ una base ordenada de $\mathbb{R}_n[x]$ (polinomios de grado $\leq n$)
$a_0, a_1, \dots, a_n \in \mathbb{R}$

$$p_n(x) = \sum_{j=0}^{n} a_j b_j$$

Busco minimizar $\int_b^c \left(p_n(x) - f(x)\right)^2 \mathrm{d}x$:

$$\min_{a_0,a_1,\dots,a_n} \int_b^c \left(p_n(x) - f(x)\right)^2 \mathrm{d}x$$

$$\min_{a_0,a_1,\dots,a_n} \int_b^c \left(p_n(x)^2 - 2p_n(x)f(x) + f(x)^2\right) \mathrm{d}x$$

$$\min_{a_0,a_1,\dots,a_n} \int_b^c p_n(x)^2 \,\mathrm{d}x - \int_b^c 2p_n(x)f(x)\,\mathrm{d}x + \int_b^c f(x)^2 \,\mathrm{d}x$$

Estos $a_0, a_1, \dots, a_n$ tienen que cumplir $\forall k \in \mathbb{N}_{\leq n}$:

$$\frac{\partial}{\partial a_k}\left(\int_b^c p_n(x)^2 \,\mathrm{d}x - \int_b^c 2p_n(x)f(x)\,\mathrm{d}x + \int_b^c f(x)^2 \,\mathrm{d}x\right) = 0$$

Desarrollo esto:

$$\frac{\partial}{\partial a_k}\left(\int_b^c p_n(x)^2 \,\mathrm{d}x - \int_b^c 2p_n(x)f(x)\,\mathrm{d}x + \int_b^c f(x)^2 \,\mathrm{d}x\right) = 0$$

$$\frac{\partial}{\partial a_k}\int_b^c p_n(x)^2 \,\mathrm{d}x - \frac{\partial}{\partial a_k}\int_b^c 2p_n(x)f(x)\,\mathrm{d}x = 0$$

$$\frac{\partial}{\partial a_k}\int_b^c \left(\sum_{j=0}^n a_j b_j(x)\right)^2 \mathrm{d}x - \frac{\partial}{\partial a_k}\int_b^c 2\left(\sum_{j=0}^n a_j b_j(x)\right) f(x)\,\mathrm{d}x = 0$$

$$\int_b^c \frac{\partial}{\partial a_k}\left(\sum_{j=0}^n a_j b_j(x)\right)^2 \mathrm{d}x - 2\sum_{j=0}^n \frac{\partial}{\partial a_k} a_j \int_b^c b_j(x) f(x)\,\mathrm{d}x = 0$$

$$\int_b^c \left(2\sum_{j=0}^n a_j b_j(x)\, b_k(x)\right) \mathrm{d}x - 2\int_b^c b_k(x) f(x)\,\mathrm{d}x = 0$$

$$\sum_{j=0}^n a_j \int_b^c b_j(x) b_k(x)\,\mathrm{d}x = \int_b^c b_k(x) f(x)\,\mathrm{d}x$$

Esto forma el sistema de ecuaciones:

$$
\begin{bmatrix}
\int_b^c b_0(x)b_0(x)\,\mathrm{d}x & \int_b^c b_1(x)b_0(x)\,\mathrm{d}x & \cdots & \int_b^c b_n(x)b_0(x)\,\mathrm{d}x \\
\int_b^c b_0(x)b_1(x)\,\mathrm{d}x & \int_b^c b_1(x)b_1(x)\,\mathrm{d}x & \cdots & \int_b^c b_n(x)b_1(x)\,\mathrm{d}x \\
\vdots & \vdots & \ddots & \vdots \\
\int_b^c b_0(x)b_n(x)\,\mathrm{d}x & \int_b^c b_1(x)b_n(x)\,\mathrm{d}x & \cdots & \int_b^c b_n(x)b_n(x)\,\mathrm{d}x
\end{bmatrix}
\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}
=
\begin{bmatrix}
\int_b^c b_0(x)f(x)\,\mathrm{d}x \\
\int_b^c b_1(x)f(x)\,\mathrm{d}x \\
\vdots \\
\int_b^c b_n(x)f(x)\,\mathrm{d}x
\end{bmatrix}
$$

Unisidad de la mejor aproximación

Sea:

$f : \mathbb{R} \to \mathbb{R}$

$x_1, x_2, \dots, x_m \in \mathbb{R}$ distintos entre si

$n < m$

$$\langle \exists!\, p \in \mathbb{R}_n[x] \ : \ p \text{ es la mejor aprixmación de grado } n \text{ por cuadrados mínimos de } f \text{ en } x_1, x_2, \dots, x_m \rangle$$

$$
\det \begin{pmatrix}
\sum_{i=1}^{m} x_i^{0+0} & \sum_{i=1}^{m} x_i^{0+1} & \cdots & \sum_{i=1}^{m} x_i^{0+n} \\
\sum_{i=1}^{m} x_i^{1+0} & \sum_{i=1}^{m} x_i^{1+1} & \cdots & \sum_{i=1}^{m} x_i^{1+n} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^{m} x_i^{n+0} & \sum_{i=1}^{m} x_i^{n+1} & \cdots & \sum_{i=1}^{m} x_i^{n+n}
\end{pmatrix} \neq 0
$$

$\Leftrightarrow$

$$\operatorname{rango}\begin{pmatrix} \sum_{i=1}^{m} x_i^0 x_i^0 & \sum_{i=1}^{m} x_i^0 x_i^1 & \cdots & \sum_{i=1}^{m} x_i^0 x_i^n \\ \sum_{i=1}^{m} x_i^1 x_i^0 & \sum_{i=1}^{m} x_i^1 x_i^1 & \cdots & \sum_{i=1}^{m} x_i^1 x_i^n \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{m} x_i^n x_i^0 & \sum_{i=1}^{m} x_i^n x_i^1 & \cdots & \sum_{i=1}^{m} x_i^n x_i^n \end{pmatrix} = n+1$$

$\Leftrightarrow$

$$\operatorname{rango}\left(\begin{bmatrix} x_1^0 & x_2^0 & \cdots & x_m^0 \\ x_1^1 & x_2^1 & \cdots & x_m^1 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^n & x_2^n & \cdots & x_m^n \end{bmatrix}\begin{bmatrix} x_1^0 & x_1^1 & \cdots & x_1^n \\ x_2^0 & x_2^1 & \cdots & x_2^n \\ \vdots & \vdots & \ddots & \vdots \\ x_m^0 & x_m^1 & \cdots & x_m^n \end{bmatrix}\right) = n+1$$

$\Leftrightarrow$

$$\operatorname{rango}\left(\begin{bmatrix} x_1^0 & x_1^1 & \cdots & x_1^n \\ x_2^0 & x_2^1 & \cdots & x_2^n \\ \vdots & \vdots & \ddots & \vdots \\ x_m^0 & x_m^1 & \cdots & x_m^n \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} x_1^0 & x_1^1 & \cdots & x_1^n \\ x_2^0 & x_2^1 & \cdots & x_2^n \\ \vdots & \vdots & \ddots & \vdots \\ x_m^0 & x_m^1 & \cdots & x_m^n \end{bmatrix}\right) = n+1$$

$\Leftrightarrow \{\operatorname{rango}(A^{\mathrm{T}}) = \operatorname{rango}(A) \;\Rightarrow\; \operatorname{rango}(A^{\mathrm{T}}A) = \operatorname{rango}(A)\}$

$$\operatorname{rango}\begin{pmatrix} x_1^0 & x_1^1 & \cdots & x_1^n \\ x_2^0 & x_2^1 & \cdots & x_2^n \\ \vdots & \vdots & \ddots & \vdots \\ x_m^0 & x_m^1 & \cdots & x_m^n \end{pmatrix} = n+1$$

$\Leftrightarrow \{n < m\}$

$$\left\{\begin{bmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_m^0 \end{bmatrix}, \begin{bmatrix} x_1^1 \\ x_2^1 \\ \vdots \\ x_m^1 \end{bmatrix}, \ldots, \begin{bmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_m^n \end{bmatrix}\right\} \text{ es LI}$$

$\Leftarrow$

$$\left\{\begin{bmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_m^0 \end{bmatrix}, \begin{bmatrix} x_1^1 \\ x_2^1 \\ \vdots \\ x_m^1 \end{bmatrix}, \ldots, \begin{bmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_m^n \end{bmatrix}, \ldots, \begin{bmatrix} x_1^{m-1} \\ x_2^{m-1} \\ \vdots \\ x_m^{m-1} \end{bmatrix}\right\} \text{ es LI}$$

$\Leftrightarrow$

$$\det\begin{pmatrix} x_1^0 & x_1^1 & \cdots & x_1^{m-1} \\ x_2^0 & x_2^1 & \cdots & x_2^{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ x_m^0 & x_m^1 & \cdots & x_m^{m-1} \end{pmatrix} \neq 0$$

$\Leftrightarrow$ {Matriz de Vandermonde}

True

Independencia lineal entre polinomios de distinto grado:
Sea:
$\Omega \subset \mathbb{R}[\mathrm{x}]$ de tamaño finito

$$\langle \forall \varphi, \eta \in \Omega : \varphi \neq \eta : \ \text{grado}(\varphi) \neq \text{grado}(\eta) \rangle \ \Rightarrow \ \Omega \text{ es LI}$$

Demostración por indución en el tamaño de Ω:

Caso base ($\Omega = \{\}$):

Es cierto porque todo conjunto vacío es LI.

Caso inductivo para $|\Omega| = n + 1$ suponiendo que vale para conjuntos de tamaño n:
Demostración suponiendo el antecedente y la hipótesis inductiva:

Ω es LI
$\Leftrightarrow$ ⦃Sea: $p = \text{maxarg}_{q \in \Omega} \ \text{grado}(q)$ (p es único por antecedente)⦄
$\Omega - \{p\}$ es LI $\wedge$ $p \notin \text{gen}(\Omega - \{p\})$
$\Leftrightarrow$ ⦃$\Omega - \{p\}$ es LI por HI, sea: $\Omega - \{p\} = \{\varphi_1, \varphi_2, \dots, \varphi_n\}$⦄

$$\text{True} \wedge \left\langle \nexists c_1, c_2, \dots, c_n \in \mathbb{R} : p = \sum_{j=1}^{n} c_j \varphi_j \right\rangle$$

$\Leftarrow$ ⦃Sea $k = \text{grado}(p)$, $\text{coef}_k(p) \neq 0$⦄

$$\left\langle \nexists c_1, c_2, \dots, c_n \in \mathbb{R} : \text{coef}_k\left(\text{grado}\left(\sum_{j=1}^{n} c_j \varphi_j \right) \right) \neq 0 \right\rangle$$

$\Leftarrow$

$$\langle \forall j \in \mathbb{N}_{\leq n} : \ \text{coef}_k(\varphi_j) = 0 \rangle$$

$\Leftrightarrow$

$$\langle \forall \varphi \in \Omega - \{p\} : \ \text{coef}_k(\varphi) = 0 \rangle$$

$\Leftarrow$

$$\langle \forall q \in \Omega - \{\mathrm{p}\} : \ \text{grado}(q) < k \rangle$$

$\Leftarrow$ ⦃$p = \text{maxarg}_{q \in \Omega} \ \text{grado}(q)$, p es único⦄
True

$a, b \in \mathbb{R}$
ω una función de peso en $[a, b]$
Producto interno entre p y q: $\Phi(p, q) = \int_a^b \omega(x) p(x) q(x)\, \mathrm{d}x$

$$B_k = \frac{\int_a^b x\omega(x)\left(\varphi_{k-1}(x)\right)^2 \mathrm{d}x}{\int_a^b \omega(x)\left(\varphi_{k-1}(x)\right)^2 \mathrm{d}x}$$

$$C_k = \frac{\int_a^b x\omega(x)\varphi_{k-1}(x)\varphi_{k-2}(x)\mathrm{d}x}{\int_a^b \omega(x)\left(\varphi_{k-2}(x)\right)^2 \mathrm{d}x}$$

$\varphi_0(x) = 1$
$\varphi_1(x) = x - B_1$
$\varphi_k(x) = (x - B_k)\varphi_{k-1}(x) - C_k\varphi_{k-2}(x)$

$\{\varphi_0, \varphi_1, \dots, \varphi_n\}$ es ortoganal respecto a Φ

Demostración por indución:

Caso recursivo para $n \geq 2$, suponiendo que vale para $n - 1$:

$j \in \mathbb{N}_{<n}$

$\Phi(\varphi_n, \varphi_j) = 0$:

$$\int_a^b \omega(x)\varphi_n(x)\varphi_j(x)\,\mathrm{d}x$$

$\Phi(\varphi_n, \varphi_{n-1}) = 0$:

$$\begin{aligned}
&\int_a^b \omega(x)\varphi_n(x)\varphi_{n-1}(x)\,\mathrm{d}x \\
&= \int_a^b \omega(x)\big((x - B_n)\varphi_{n-1}(x) - C_n\varphi_{n-2}(x)\big)\varphi_{n-1}(x)\,\mathrm{d}x \\
&= \int_a^b \omega(x)x\varphi_{n-1}(x)\varphi_{n-1}(x)\,\mathrm{d}x - B_n\int_a^b \omega(x)\varphi_{n-1}(x)\varphi_{n-1}(x)\mathrm{d}x - C_n\int_a^b \omega(x)\varphi_{n-2}(x)\varphi_{n-1}(x)\,\mathrm{d}x \\
&= \int_a^b \omega(x)x\big(\varphi_{n-1}(x)\big)^2\mathrm{d}x - \frac{\int_a^b x\omega(x)\big(\varphi_{n-1}(\mathrm{x})\big)^2\mathrm{d}x}{\int_a^b \omega(x)\big(\varphi_{n-1}(\mathrm{x})\big)^2\mathrm{d}x}\int_a^b \omega(x)\big(\varphi_{n-1}(x)\big)^2 dx - C_n\int_a^b \omega(x)\varphi_{n-2}(x)\varphi_{n-1}(x)\,dx \\
&= -C_n\int_a^b \omega(x)\varphi_{n-2}(x)\varphi_{n-1}(x)\,dx
\end{aligned}$$

$= \{$Hipótesis inductiva$\}$

0

1)

lunes, 3 de mayo de 2021 12:42

1a)

$$P(x) = a + bx$$

-0.1+1.1+
1.9+3.2+
3.8+5+6+
7.3+8.1+
8.9

Por teorema:

$$\begin{bmatrix} \sum_{j=0}^{9} x_j^0 & \sum_{j=0}^{9} x_j^1 \\ \sum_{j=0}^{9} x_j^1 & \sum_{j=0}^{9} x_j^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{9} x_j^0 y_j \\ \sum_{j=1}^{9} x_j^1 y_j \end{bmatrix}$$

$$\begin{bmatrix} \sum_{j=0}^{9} 1 & \sum_{j=0}^{9} j \\ \sum_{j=0}^{9} j & \sum_{j=0}^{9} j^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{9} y_j \\ \sum_{j=1}^{9} j y_j \end{bmatrix}$$

$$\begin{bmatrix} 10 & 45 \\ 45 & 285 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 45.2 \\ 286.8 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 45 \\ 45 & 285 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 45.2 \\ 286.7 \end{bmatrix}$$

$$P(x) = -\frac{13}{550} + \frac{833}{825} x$$

$$p(x) = -\frac{13}{550} + \frac{833}{825}x$$

1b)

$$P(x) = a + bx + cx^2$$

Por teorema:

$$\begin{bmatrix} \sum_{j=0}^{4} x_j^0 & \sum_{j=0}^{4} x_j^1 & \sum_{j=0}^{4} x_j^2 \\ \sum_{j=0}^{4} x_j^1 & \sum_{j=0}^{4} x_j^2 & \sum_{j=0}^{4} x_j^3 \\ \sum_{j=0}^{4} x_j^2 & \sum_{j=0}^{4} x_j^3 & \sum_{j=0}^{4} x_j^4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{m} x_j^0 y_j \\ \sum_{j=0}^{m} x_j^1 y_j \\ \sum_{j=0}^{m} x_j^2 y_j \end{bmatrix}$$

$$\begin{bmatrix} 5 & 9 & 47 \\ 9 & 47 & 243 \\ 47 & 243 & 1379 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 44 \\ 175.5 \\ 1030.7 \end{bmatrix}$$

p(x) = 22333/5020 - 2507/502*x + 7441/5020*x²

2)

martes, 4 de mayo de 2021 09:10

Sea:

$x_0, x_1, \dots, x_n \in \mathbb{R}$

$f : \mathbb{R} \to \mathbb{R}$

$p, q \in \mathbb{R}_n[x]$

p interpola a f en $x_0, x_1, \dots, x_n$

q es la mejor aproximación de grado n por cuadrados minimos de f en $x_1, x_2, \dots, x_m$

$$p = q$$

Demostración:

p interpola a f en $x_0, x_1, \dots, x_n$

$\Rightarrow$

$\langle \forall i \in \{0,1, \dots, n\} : p(x_i) = f(x_i) \rangle$

$\Rightarrow$

$$\sum_{i=0}^{n} \left(p(x_i) - f(x_i)\right)^2 = 0$$

$\Rightarrow$ {0 es menor igual que cualquier cosa}

$$\left\langle \forall r \in \mathbb{R}_n[x] : \sum_{i=0}^{n} \left(p(x_i) - f(x_i)\right)^2 \leq \sum_{i=0}^{n} \left(r(x_i) - f(x_i)\right)^2 \right\rangle$$

$\Rightarrow$ {Definición de mejor aproximación por cuadados mínimos}

p es la mejor aproximación de grado n por cuadrados mínimos de f en $x_1, x_2, \dots, x_m$

$\Rightarrow$ {Unisidad de la mejor aproximación}

$p = q$

3)

martes, 4 de mayo de 2021 09:26

$$\min_{a_0} \sum_{i=1}^{n} f\big((x_i) - p_0(x_i)\big)^2$$

$$\min_{a_0} \sum_{i=1}^{n} (f(x_i) - a_0)^2$$

$$\frac{\partial}{\partial a_0} \sum_{i=1}^{n} (f(x_i) - a_0)^2 = 0$$

$$\sum_{i=1}^{n} 2(f(x_i) - a_0)(-1) = 0$$

$$\sum_{i=1}^{n} f(x_i) = \sum_{i=1}^{n} a_0$$

$$\sum_{i=1}^{n} f(x_i) = n a_0$$

$$\frac{\sum_{i=1}^{n} f(x_i)}{n} = a_0$$

4)

martes, 4 de mayo de 2021 09:39

$$\ln(f(x)) = \ln(a) + bx$$

$8.1^{-1} * 1.1 * (0.5)^2$

8.1*3*1.1*0.5

Aproximo linealmente ln(a) y b

Por teorema:

$$\begin{bmatrix} \sum_{j=0}^{3} x_j^0 & \sum_{j=0}^{3} x_j^1 \\ \sum_{j=0}^{3} x_j^1 & \sum_{j=0}^{3} x_j^2 \end{bmatrix} \begin{bmatrix} \ln(a) \\ b \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{3} x_j^0 \ln(y_j) \\ \sum_{j=0}^{3} x_j^1 \ln(y_j) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} \ln(a) \\ b \end{bmatrix} = \begin{bmatrix} \ln(13.365) \\ \ln\left(\frac{11}{324}\right) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} \ln(a) \\ b \end{bmatrix} = \begin{bmatrix} \ln(13.365) \\ \ln\left(\frac{11}{324}\right) \end{bmatrix}$$

$$\ln(a) = \frac{\ln\left(\frac{140633637507}{2000000}\right)}{10}$$

$$\ln(a) = \frac{\ln\left(\frac{140633637507}{2000000}\right)}{10}$$

$$a = e^{\frac{\ln\left(\frac{140633637507}{2000000}\right)}{10}}$$

$$a = \sqrt[10]{\frac{140633637507}{2000000}} \simeq 3.05285$$

$$b = \frac{\ln\left(\frac{275}{3188646}\right)}{10} \simeq -0.935839$$

$$f(x) = \sqrt[10]{\frac{140633637507}{2000000}}\, e^{\left(\frac{\ln\left(\frac{275}{3188646}\right)}{10}x\right)}$$

5)

martes, 4 de mayo de 2021 10:30

$$\ln(-f(x)) = a x^2 + b x + c$$

Busco a, b y c

$1.1^1 * 0.9 * 0.5^4$

Por teorema:

$$\begin{bmatrix} \sum_{j=0}^{3} x_j^0 & \sum_{j=0}^{3} x_j^1 & \sum_{j=0}^{3} x_j^2 \\ \sum_{j=0}^{3} x_j^1 & \sum_{j=0}^{3} x_j^2 & \sum_{j=0}^{3} x_j^3 \\ \sum_{j=0}^{3} x_j^2 & \sum_{j=0}^{3} x_j^3 & \sum_{j=0}^{3} x_j^4 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{3} x_j^0 \ln(-y_j) \\ \sum_{j=0}^{3} x_j^1 \ln(-y_j) \\ \sum_{j=0}^{3} x_j^2 \ln(-y_j) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} \ln(0.198) \\ \ln(9/44) \\ \ln(0.061875) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} \ln(0.198) \\ \ln\left(\frac{9}{44}\right) \\ \ln(0.061875) \end{bmatrix}$$

$$c = \frac{\ln\left(\frac{2062626683436}{11920928955078125}\right)}{20} \approx -0.4331$$

$$b = \frac{\ln\left(\frac{1530550080}{285311670611}\right)}{20} \approx -0.2614$$

$$a = \frac{\ln\left(\frac{55}{36}\right)}{4} \approx 0.10595$$

6)

martes, 4 de mayo de 2021 11:11

1523	880	898	960	618	709
1500	900	800	1050	650	690

p(x) = a + bx

Por teorema:

$$\begin{bmatrix} 6 & 1523 + 880 + 898 + 960 + 618 + 709 \\ 1523 + 880 + 898 + 960 + 618 + 709 & 1523^2 + 880^2 + 898^2 + 960^2 + 618^2 + 709^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1500 + 900 + 800 + 1050 + 650 + 690 \\ 1523 * 1500 + 880 * 900 + 898 * 800 + 960 * 1050 + 618 * 650 + 709 * 690 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5588 \\ 5588 & 5706538 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5590 \\ 5693810 \end{bmatrix}$$

$$a = \frac{20634285}{753371} \approx 27.38927$$

$$b = \frac{731485}{753371} \approx 0.97095$$

$$p(x) = \frac{20634285}{753371} + \frac{731485}{753371}x$$

$$p(1150) = \frac{861842035}{753371} \approx 1143.9809$$

7)

martes, 4 de mayo de 2021 11:33

7a)

$$f(x) = x^2 + 3x + 2$$
$$p(x) = a + bx$$

$$\min_{a,b} \int_0^1 \left(p(x) - f(x)\right)^2 \mathrm{d}x$$

Por teorema:

$$\begin{bmatrix} \int_0^1 x^0 \,\mathrm{d}x & \int_0^1 x^1 \,\mathrm{d}x \\ \int_0^1 x^1 \,\mathrm{d}x & \int_0^1 x^2 \,\mathrm{d}x \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \int_0^1 x^0 f(x)\mathrm{d}x \\ \int_0^1 x^1 f(x)\mathrm{d}x \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{23}{6} \\ \frac{9}{4} \end{bmatrix}$$

$$b = 4$$
$$a = \frac{11}{6}$$

$$p(x) = \frac{11}{6} + 4x$$

7b)

$f(x) = x^2 + 3x + 2$

$p(x) = a + bx$

Por teorema:

$$\begin{bmatrix} \int_{-1}^{1} x^0 \, dx & \int_{-1}^{1} x^1 \, dx \\ \int_{-1}^{1} x^1 \, dx & \int_{-1}^{1} x^2 \, dx \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \int_{-1}^{1} x^0 f(x) dx \\ \int_{-1}^{1} x^1 f(x) dx \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{14}{3} \\ 2 \end{bmatrix}$$

$a = \frac{7}{3}$

$b = 3$

$$p(x) = \frac{7}{3} + 3x$$

7c)

$f(x) = e^x$

$p(x) = a + bx$

Por teorema:

$$\begin{bmatrix} \int_0^2 x^0 \, dx & \int_0^2 x^1 \, dx \\ \int_0^2 x^1 \, dx & \int_0^2 x^2 \, dx \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \int_0^2 x^0 f(x) dx \\ \int_0^2 x^1 f(x) dx \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 2 & \frac{8}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} e^2 - 1 \\ e^2 + 1 \end{bmatrix}$$

$$b = 3$$

$$a = \frac{e^2 - 7}{2} \approx 0.19453$$

$$p(x) = \frac{e^2 - 7}{2} + 3x$$

8)

martes, 4 de mayo de 2021 19:53

$$f(x) = a\cos(x) + b\sin(x)$$

$$\min_{a,b} \sum_{i=0}^{10} \left(y_i - f(x_i)\right)^2$$

$$\min_{a,b} \sum_{i=0}^{10} \left(y_i - \left(a\cos(x_i) + b\sin(x_i)\right)\right)^2$$

$$\min_{a,b} \sum_{i=0}^{10} \left(y_i^2 - 2y_i\left(a\cos(x_i) + b\sin(x_i)\right) + \left(a\cos(x_i) + b\sin(x_i)\right)^2\right)$$

$$\min_{a,b} \sum_{i=0}^{10} \left(y_i^2 - 2y_i a\cos(x_i) + 2y_i b\sin(x_i) + a^2\cos^2(x_i) + 2a\cos(x_i)\, b\sin(x_i) + b^2\sin^2(x_i)\right)$$

$$\frac{\partial}{\partial a} \sum_{i=0}^{10} \left(y_i^2 - 2y_i a\cos(x_i) + 2y_i b\sin(x_i) + a^2\cos^2(x_i) + 2a\cos(x_i)\, b\sin(x_i) + b^2\sin^2(x_i)\right) = 0$$

$$\sum_{i=0}^{10} \left(-2y_i\cos(x_i) + 2a\cos^2(x_i) + 2\cos(x_i)\, b\sin(x_i)\right) = 0$$

$$a\sum_{i=0}^{10} \cos^2(x_i) + b\sum_{i=0}^{10} \cos(x_i)\sin(x_i) = \sum_{i=0}^{10} y_i\cos(x_i)$$

$$\frac{\partial}{\partial b} \sum_{i=0}^{10} \left(y_i^2 - 2y_i a\cos(x_i) + 2y_i b\sin(x_i) + a^2\cos^2(x_i) + 2a\cos(x_i)\, b\sin(x_i) + b^2\sin^2(x_i)\right) = 0$$

$$\sum_{i=0}^{10} \left(2y_i\sin(x_i) + 2a\cos(x_i)\sin(x_i) + 2b\sin^2(x_i)\right) = 0$$

$$a\sum_{i=0}^{10} \cos(x_i)\sin(x_i) + b\sum_{i=0}^{10} \sin^2(x_i) = -\sum_{i=0}^{10} y_i\sin(x_i)$$

$$\begin{bmatrix} \sum_{i=0}^{10} \cos^2(x_i) & \sum_{i=0}^{10} \cos(x_i)\sin(x_i) \\ \sum_{i=0}^{10} \cos(x_i)\sin(x_i) & \sum_{i=0}^{10} \sin^2(x_i) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{10} y_i\cos(x_i) \\ -\sum_{i=0}^{10} y_i\sin(x_i) \end{bmatrix}$$

$$\begin{bmatrix} \sum_{i=0}^{10} \cos^2(i) & \sum_{i=0}^{10} \cos(i)\sin(i) \\ \sum_{i=0}^{10} \cos(i)\sin(i) & \sum_{i=0}^{10} \sin^2(i) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{10} y_i\cos(i) \\ -\sum_{i=0}^{10} y_i\sin(i) \end{bmatrix}$$

$$
\begin{bmatrix}
\sum_{j=0}^{10} \cos^2(j) & \sum_{j=0}^{10} \cos(j)\sin(j) \\
\sum_{j=0}^{10} \cos(j)\sin(i) & \sum_{j=0}^{10} \sin^2(j)
\end{bmatrix}
\begin{bmatrix} a \\ b \end{bmatrix}
$$

$$
= \begin{bmatrix}
\cos(0)\,1.8 + \cos(1)\,3.2 + \cos(2)\,2.1 + \cos(3)\,(-1) + \cos(4)\,(-3.3) + \cos(5)\,(-2.7) + \cos(7)\,3.3 + \cos(8)\,2.8 + \cos(9)\,(-0.1) + \cos(10)\,(-3) \\
-(\sin(0)\,1.8 + \sin(1)\,3.2 + \sin(2)\,2.1 + \sin(3)\,(-1) + \sin(4)\,(-3.3) + \sin(5)\,(-2.7) + \sin(7)\,3.3 + \sin(8)\,2.8 + \sin(9)\,(-0.1) + \sin(10)\,(-3))
\end{bmatrix}
$$

Aproximadamente:

$$
\begin{bmatrix} 5.99557 & 0.32325 \\ 0.32325 & 5.00143 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \simeq \begin{bmatrix} 9.72499 \\ -16.07677 \end{bmatrix}
$$

$a \simeq 1.80161$

$b \simeq -3.33088$

9) 🛠️

jueves, 6 de mayo de 2021 09:17

10)

jueves, 6 de mayo de 2021 09:17

Linal:

$$p(x) = a + bx$$

Por teorema:

$$\begin{bmatrix} \int_{-1}^{1} x^0 \mathrm{d}x & \int_{-1}^{1} x^1 \mathrm{d}x \\ \int_{-1}^{1} x^1 \mathrm{d}x & \int_{-1}^{1} x^2 \mathrm{d}x \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \int_{-1}^{1} x^0 e^x \mathrm{d}x \\ \int_{-1}^{1} x^1 e^x \mathrm{d}x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} e - \frac{1}{e} \\ \frac{2}{e} \end{bmatrix}$$

$$a = \frac{e}{2} - \frac{1}{2e} \approx 1.1752$$

$$b = \frac{3}{e} \approx 1.10364$$

$$p(x) = \frac{e}{2} - \frac{1}{2e} + \frac{3}{e} x$$

Cuadrática:

$$p(x) = a + bx + cx^2$$

Por teorema:

$$\begin{bmatrix} \int_{-1}^{1} x^0 \mathrm{d}x & \int_{-1}^{1} x^1 \mathrm{d}x & \int_{-1}^{1} x^2 \mathrm{d}x \\ \int_{-1}^{1} x^1 \mathrm{d}x & \int_{-1}^{1} x^2 \mathrm{d}x & \int_{-1}^{1} x^3 \mathrm{d}x \\ \int_{-1}^{1} x^2 \mathrm{d}x & \int_{-1}^{1} x^3 \mathrm{d}x & \int_{-1}^{1} x^4 \mathrm{d}x \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \int_{-1}^{1} x^0 e^x \mathrm{d}x \\ \int_{-1}^{1} x^1 e^x \mathrm{d}x \\ \int_{-1}^{1} x^2 e^x \mathrm{d}x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & \frac{2}{5} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} e - \frac{1}{e} \\ \frac{2}{e} \\ e - \frac{5}{e} \end{bmatrix}$$

$$a = \frac{33}{4e} - \frac{3e}{4} \approx 0.99629$$

$$c = \frac{15e}{4} - \frac{105}{4e} \approx 0.53672$$

$$b = \frac{3}{e} \approx 1.10364$$

$$p(x) = \frac{33}{4e} - \frac{3e}{4} + \frac{3}{e}x + \left(\frac{15e}{4} - \frac{105}{4e}\right)x^2$$

Cuadrática con Legendre:

Base ordenada: $\mathrm{B} = \{b_0, b_1, b_2\} = \left\{1, x, x^2 - \frac{1}{3}\right\}$

$$p(x) = a_0 + a_1 x + a_2 \left(x^2 - \frac{1}{3}\right)$$

Por teorema:

$$\begin{bmatrix} \int_{-1}^{1} b_0(x)b_0(x)\mathrm{d}x & \int_{-1}^{1} b_1(x)b_0(x)\mathrm{d}x & \int_{-1}^{1} b_2(x)b_0(x)\mathrm{d}x \\ \int_{-1}^{1} b_0(x)b_1(x)\mathrm{d}x & \int_{-1}^{1} b_1(x)b_1(x)\mathrm{d}x & \int_{-1}^{1} b_2(x)b_1(x)\mathrm{d}x \\ \int_{-1}^{1} b_0(x)b_2(x)\mathrm{d}x & \int_{-1}^{1} b_1(x)b_2(x)\mathrm{d}x & \int_{-1}^{1} b_2(x)b_2(x)\mathrm{d}x \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \int_{-1}^{1} b_0(x)f(x)\mathrm{d}x \\ \int_{-1}^{1} b_1(x)f(x)\mathrm{d}x \\ \int_{-1}^{1} b_2(x)f(x)\mathrm{d}x \end{bmatrix}$$

$$\begin{bmatrix} \int_{-1}^{1} 1\,\mathrm{d}x & \int_{-1}^{1} x\,\mathrm{d}x & \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)\mathrm{d}x \\ \int_{-1}^{1} x\,\mathrm{d}x & \int_{-1}^{1} x^2\,\mathrm{d}x & \int_{-1}^{1} x\left(x^2 - \frac{1}{3}\right)\mathrm{d}x \\ \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)\mathrm{d}x & \int_{-1}^{1} x\left(x^2 - \frac{1}{3}\right)\mathrm{d}x & \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)^2\mathrm{d}x \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \int_{-1}^{1} e^x\mathrm{d}x \\ \int_{-1}^{1} e^x\mathrm{d}x \\ \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)e^x\mathrm{d}x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{8}{45} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} e - \frac{1}{e} \\ \frac{2}{e} \\ \frac{2e}{3} - \frac{14}{3e} \end{bmatrix}$$

$$a_0 = \frac{e}{2} - \frac{1}{2e}$$

$$a_1 = \frac{3}{e}$$

$$a_2 = \frac{15e}{4} - \frac{105}{4e}$$

$$\begin{aligned} p(x) &= \frac{e}{2} - \frac{1}{2e} + \frac{3}{e}x + \left(\frac{15e}{4} - \frac{105}{4e}\right)\left(x^2 - \frac{1}{3}\right) \\ &= \frac{33}{4e} - \frac{3e}{4} + \frac{3}{e}x + \left(\frac{15e}{4} - \frac{105}{4e}\right)x^2 \end{aligned}$$

11)

jueves, 6 de mayo de 2021 09:42

$$\int_{-1}^{1} x^2 p(x) q(x) \mathrm{d}x$$

Aplico el teorema:

Sea:

$$B_k = \frac{\int_{-1}^{1} x x^2 \left(\Phi_{\mathrm{k}-1}(x)\right)^2 \mathrm{d}x}{\int_{-1}^{1} x^2 \left(\Phi_{\mathrm{k}-1}(x)\right)^2 \mathrm{d}x} = \frac{\int_{-1}^{1} x^3 \left(\Phi_{\mathrm{k}-1}(x)\right)^2 \mathrm{d}x}{\int_{-1}^{1} x^2 \left(\Phi_{\mathrm{k}-1}(x)\right)^2 \mathrm{d}x}$$

$$C_k = \frac{\int_{-1}^{1} x x^2 \Phi_{k-1}(x) \Phi_{k-2}(x) \mathrm{d}x}{\int_{-1}^{1} x^2 \left(\Phi_{k-2}(x)\right)^2 \mathrm{d}x} = \frac{\int_{-1}^{1} x^3 \Phi_{k-1}(x) \Phi_{k-2}(x) \mathrm{d}x}{\int_{-1}^{1} x^2 \left(\Phi_{k-2}(x)\right)^2 \mathrm{d}x}$$

$$\Phi_0(x) = 1$$
$$\Phi_1(x) = x - B_1$$
$$\Phi_2(x) = (x - B_2)\Phi_1(x) - C_2 \Phi_0(x)$$

$$\begin{aligned} B_1 &= \frac{\int_{-1}^{1} x^3 \left(\Phi_0(x)\right)^2 \mathrm{d}x}{\int_{-1}^{1} x^2 \left(\Phi_0(x)\right)^2 \mathrm{d}x} \\ &= \frac{\int_{-1}^{1} x^3 \, \mathrm{d}x}{\int_{-1}^{1} x^2 \, \mathrm{d}x} \\ &= \frac{\frac{1^4}{4} - \frac{(-1)^4}{4}}{\int_{-1}^{1} x^2 \, \mathrm{d}x} \\ &= 0 \end{aligned}$$

$$\Phi_1(x) = x - 0$$
$$\Phi_1(x) = x$$

$$B_2 = \frac{\int_{-1}^{1} x^3 \left(\Phi_1(x)\right)^2 \mathrm{d}x}{\int_{-1}^{1} x^2 \left(\Phi_1(x)\right)^2 \mathrm{d}x}$$
$$= \frac{\int_{-1}^{1} x^3 x^2 \mathrm{d}x}{\int_{-1}^{1} x^2 x^2 \mathrm{d}x}$$
$$= \frac{\int_{-1}^{1} x^5 \,\mathrm{d}x}{\int_{-1}^{1} x^4 \,\mathrm{d}x}$$
$$= 0$$

$$C_2 = \frac{\int_{-1}^{1} x^3 \Phi_1(x)\Phi_0(x) \mathrm{d}x}{\int_{-1}^{1} x^2 \left(\Phi_0(x)\right)^2 \mathrm{d}x}$$
$$= \frac{\int_{-1}^{1} x^4 \,\mathrm{d}x}{\int_{-1}^{1} x^2 \mathrm{d}x}$$
$$= \frac{\frac{1^5}{5} - \frac{(-1)^5}{5}}{\frac{1^3}{3} - \frac{(-1)^3}{3}}$$
$$= \frac{3}{5}$$

$$\Phi_2(x) = (x - B_2)\Phi_1(x) - C_2\Phi_0(x)$$
$$\Phi_2(x) = (x - B_2)x - C_2$$
$$\Phi_2(x) = x^2 - xB_2 - C_2$$
$$\Phi_2(x) = x^2 - x0 - \frac{3}{5}$$
$$\Phi_2(x) = x^2 - \frac{3}{5}$$

Queda:

$$\Phi_0(x) = 1$$
$$\Phi_1(x) = x$$
$$\Phi_2(x) = x^2 - \frac{3}{5}$$