Guía 10

1. De la semántica denotacional eager y normal de las expresiones **True** $\vee$ 0 y **True** $\vee$ $\Delta\Delta$.

Normal $\iota_{\underline{\ell}} = \iota_{norm}(\iota_e)$

$$\llbracket True \vee 0 \rrbracket\eta = \llbracket if\ True\ then\ True\ else\ 0 \rrbracket\eta$$
$$= (\lambda b \in V_{bool}.\ if\ b\ then\ \iota_{\underline{bool}} True\ else\ \llbracket 0 \rrbracket\eta)_{bool*}\ \llbracket True \rrbracket\eta$$
$$= (\lambda b \in V_{bool}.\ if\ b\ then\ \iota_{\underline{bool}} True\ else\ \llbracket 0 \rrbracket\eta)_{bool*}\ \iota_{\underline{bool}} True$$
$$= (\lambda b \in V_{bool}.\ if\ b\ then\ \iota_{\underline{bool}} True\ else\ \llbracket 0 \rrbracket\eta)\ True$$
$$= (if\ True\ then\ \iota_{\underline{bool}} True\ else\ \llbracket 0 \rrbracket\eta)$$
$$= \iota_{\underline{bool}} True$$

$$\llbracket True \vee \Delta\Delta \rrbracket\eta = \llbracket if\ True\ then\ True\ else\ \Delta\Delta \rrbracket\eta$$
$$= (\lambda b \in V_{bool}.\ if\ b\ then\ \iota_{\underline{bool}} True\ else\ \llbracket \Delta\Delta \rrbracket\eta)_{bool*}\ \llbracket True \rrbracket\eta$$
$$= (\lambda b \in V_{bool}.\ if\ b\ then\ \iota_{\underline{bool}} True\ else\ \llbracket \Delta\Delta \rrbracket\eta)_{bool*}\ \iota_{\underline{bool}} True$$
$$= (\lambda b \in V_{bool}.\ if\ b\ then\ \iota_{\underline{bool}} True\ else\ \llbracket \Delta\Delta \rrbracket\eta)\ True$$
$$= (if\ True\ then\ \iota_{\underline{bool}} True\ else\ \llbracket \Delta\Delta \rrbracket\eta)$$
$$= \iota_{\underline{bool}} True$$

Eager

$$\llbracket True \vee 0 \rrbracket\eta = (\lambda b \in V_{bool}.\ (\lambda b' \in V_{bool}.\ \iota_{\underline{bool}}(b \vee b'))_{bool*}\ \llbracket 0 \rrbracket\eta)_{bool*}\ \llbracket True \rrbracket\eta$$
$$= (\lambda b \in V_{bool}.\ (\lambda b' \in V_{bool}.\ \iota_{\underline{bool}}(b \vee b'))_{bool*}(\llbracket 0 \rrbracket\eta))_{bool*}\ \iota_{\underline{bool}} True$$
$$= (\lambda b \in V_{bool}.\ (\lambda b' \in V_{bool}.\ \iota_{\underline{bool}}(b \vee b'))_{bool*}(\llbracket 0 \rrbracket\eta))\ True$$
$$= (\lambda b' \in V_{bool}\ \iota_{\underline{bool}}(True \vee b'))_{bool*}(\llbracket 0 \rrbracket\eta)$$
$$= (\lambda b' \in V_{bool}\ \iota_{\underline{bool}}(True \vee b'))_{bool*}\ \iota_{\underline{int}}\ 0$$
$$= tyerr$$

$$\llbracket True \vee \Delta\Delta \rrbracket\eta = (\lambda b \in V_{bool}.\ (\lambda b' \in V_{bool}.\ \iota_{\underline{bool}}(b \vee b'))_{bool*}\ \llbracket \Delta\Delta \rrbracket\eta)_{bool*}\ \llbracket True \rrbracket\eta$$
$$= (\lambda b \in V_{bool}.\ (\lambda b' \in V_{bool}.\ \iota_{\underline{bool}}(b \vee b'))_{bool*}(\llbracket \Delta\Delta \rrbracket\eta))_{bool*}\ \iota_{\underline{bool}} True$$

$= (\lambda b \in V_{bool}. (\lambda b' \in V_{bool}\ \iota_{bool}\ (b \vee b'))_{\perp\!\!\perp bool*} (\llbracket \Delta\Delta \rrbracket \eta))\ true$

$= (\lambda b' \in V_{bool}\ \iota_{bool}\ (True \vee b'))_{\perp\!\!\perp bool*} (\llbracket \Delta\Delta \rrbracket \eta)$

$= (\lambda b' \in V_{bool}\ \iota_{bool}\ (True \vee b'))_{\perp\!\!\perp bool*}\ \perp$

$= \perp$

2. Encuentre ecuaciones semánticas sencillas para las siguientes expresiones, considerando los casos eager y normal.
   a) $\llbracket (\lambda x.e)e' \rrbracket \eta$
   b) $\llbracket we \rrbracket [\eta | w : \iota_{\text{fun}} f]$

a) Eager

$\llbracket (\lambda x.e)e' \rrbracket \eta = (\lambda f \in V_{fun}\ f_*(\llbracket e' \rrbracket \eta))_{fun*} (\llbracket (\lambda x.e) \rrbracket \eta)$

$= (\lambda f \in V_{fun}\ f_*(\llbracket e' \rrbracket \eta))_{fun*}\ \iota_{fun} (\lambda z \in V.\ \llbracket e \rrbracket [\eta | x : z])$

$= (\lambda f \in V_{fun}\ f_*(\llbracket e' \rrbracket \eta)) (\lambda z \in V.\ \llbracket e \rrbracket [\eta | x : z])$

$= (\lambda z \in V\ \llbracket e \rrbracket [\eta | x : z])_* (\llbracket e' \rrbracket \eta)$

Normal

$\llbracket (\lambda x.e)\ e' \rrbracket \eta = (\lambda f \in V_{fun}.\ f(\llbracket e' \rrbracket \eta))_{fun*} (\llbracket \lambda x.e \rrbracket \eta)$

$= (\lambda f \in V_{fun}.\ f(\llbracket e' \rrbracket \eta))_{fun*}\ \iota_{fun} (\lambda d \in D.\ \llbracket e \rrbracket [\eta | x : d])$

$= (\lambda f \in V_{fun}.\ f(\llbracket e' \rrbracket \eta)) (\lambda d \in D\ \llbracket e \rrbracket [\eta | x : d])$

$= (\lambda d \in D.\ \llbracket e \rrbracket [\eta | x : d]) (\llbracket e' \rrbracket \eta)$

$= \llbracket e \rrbracket [\eta | x : \llbracket e' \rrbracket \eta]$

b) Eager

$\llbracket we \rrbracket [\eta | w\ \iota_{fun} f] = (\lambda f' \in V_{fun}.\ f'_*(\llbracket e \rrbracket [\eta | w : \iota_{fun} f]))_{fun*} (\llbracket w \rrbracket [\eta | w : \iota_{fun} f])$

$= (\lambda f' \in V_{fun}.\ f'_*(\llbracket e \rrbracket [\eta | w : \iota_{fun} f]))_{fun*}\ \iota_{norm} ([\eta | w : \iota_{fun} f]\ w)$

$= (\lambda f' \in V_{fun}.\ f'_* \llbracket e \rrbracket [\eta | w : \iota_{fun} f])_{fun*}\ \iota_{norm} (\iota_{fun} f)$

$= (\lambda f' \in V_{fun}.\ f'_* \llbracket e \rrbracket [\eta | w : \iota_{fun} f])\ f$

$= f_* \llbracket e \rrbracket [\eta | w : \iota_{fun} f]$

Normal (en este caso $w : \iota_{fun} f$ para que tipe la expresión)

$$[\![we]\!][\eta | w : \iota_{fun} f] = (\lambda f' \in V_{fun}.\ f'([\![e]\!][\eta | w : \iota_{fun} f]))_{fun^*} ([\![w]\!][\eta | w : \iota_{fun} f])$$
$$= (\lambda f' \in V_{fun}.\ f'([\![e]\!][\eta | w : \iota_{fun} f]))_{fun^*}\ \iota_{fun} f$$
$$= (\lambda f' \in V_{fun}.\ f'([\![e]\!][\eta | w : \iota_{fun} f]))\ f$$
$$= f([\![e]\!][\eta | w : \iota_{fun} f])$$

3. Calcular la semántica denotacional eager y normal de las expresiones $\langle \mathbf{True} + 0, \Delta\Delta \rangle$ y $\langle \Delta\Delta, \mathbf{True} + 0 \rangle$.

Eager

$$[\![\langle True + 0, \Delta\Delta \rangle]\!]\eta = (\lambda z_1 \in V\ (\lambda z_2 \in V\ \iota_{norm}(\iota_{tuple}\langle z_1, z_2 \rangle))_* ([\![\Delta\Delta]\!]\eta))_* ([\![True + 0]\!]\eta)$$
$$= (\lambda z_1 \in V\ (\lambda z_2 \in V\ \underline{\iota_{tuple}}\langle z_1, z_2 \rangle)_* ([\![\Delta\Delta]\!]\eta))_*$$
$$(\lambda i \in V_{int}.\ (\lambda i' \in V_{int}.\ \underline{\iota_{int}}(i + i'))_{int^*} ([\![0]\!]\eta))_{int^*} ([\![True]\!]\eta)$$
$$= (\lambda z_1 \in V\ (\lambda z_2 \in V\ \underline{\iota_{tuple}}\langle z_1, z_2 \rangle)_* ([\![\Delta\Delta]\!]\eta))_*$$
$$(\lambda i \in V_{int}.\ (\lambda i' \in V_{int}.\ \underline{\iota_{int}}(i + i'))_{int^*} ([\![0]\!]\eta))_{int^*}\ \underline{\iota_{bool}}\ True$$
$$= (\lambda z_1 \in V\ (\lambda z_2 \in V\ \underline{\iota_{tuple}}\langle z_1, z_2 \rangle)_* ([\![\Delta\Delta]\!]\eta))_*$$
$$tyerr$$
$$= tyerr$$

$$[\![\langle \Delta\Delta, True + 0 \rangle]\!]\eta = (\lambda z_1 \in V\ (\lambda z_2 \in V\ \iota_{norm}(\iota_{tuple}\langle z_1, z_2 \rangle))_* ([\![True + 0]\!]\eta))_* ([\![\Delta\Delta]\!]\eta)$$
$$= (\lambda z_1 \in V\ (\lambda z_2 \in V\ \underline{\iota_{tuple}}\langle z_1, z_2 \rangle)_* ([\![True + 0]\!]\eta))_*\ \bot$$
$$= \bot$$

Normal

$$[\![\langle True + 0, \Delta\Delta \rangle]\!]\eta = \underline{\iota_{tuple}}\langle [\![True + 0]\!]\eta, [\![\Delta\Delta]\!]\eta \rangle$$
$$= \underline{\iota_{tuple}}\langle (\lambda i \in V_{int}.\ (\lambda i' \in V_{int}.\ \underline{\iota_{int}}(i + i'))_{int^*} ([\![0]\!]\eta))_{int^*} ([\![True]\!]\eta), \bot \rangle$$
$$= \underline{\iota_{tuple}}\langle (\lambda i \in V_{int}.\ (\lambda i' \in V_{int}.\ \underline{\iota_{int}}(i + i'))_{int^*} ([\![0]\!]\eta))_{int^*}\ \underline{\iota_{bool}}\ True, \bot \rangle$$
$$= \underline{\iota_{tuple}}\langle tyerr, \bot \rangle$$

$$[\![\langle \Delta\Delta, True + 0 \rangle]\!]\eta = \underline{\iota_{tuple}}\langle [\![\Delta\Delta]\!]\eta, [\![True + 0]\!]\eta \rangle$$

$$= \iota_{tuple} \ \langle \bot, (\lambda i \in V_{int}. (\lambda i' \in V_{int}. \ \iota_{int}(i+i'))_{int*}(⟦0⟧\eta))_{int*}(⟦True⟧\eta) \rangle$$

$$= \iota_{tuple} \langle \bot, (\lambda i \in V_{int}. (\lambda i' \in V_{int}. \ \iota_{int}(i+i'))_{int*}(⟦0⟧\eta))_{int*} \iota_{bool} True \rangle$$

$$= \iota_{tuple} \ \langle \bot, ty_{err} \rangle$$

4. De la semántica denotacional normal de las expresiones $e.1$ y $(e.2).1$, donde e es la expresión que dio en el ejercicio 7 del práctico 9.

$e = rec(\lambda w \ \langle 0, w \rangle)$ Suponiendo que los índices de tupla empiezan en 1.

$$⟦e⟧\eta = ⟦rec \ (\lambda w \ \langle 0, w \rangle)⟧\eta = (\lambda f \in V_{fun} \ Y f)_{fun*}(⟦\lambda w \ \ \langle 0, w \rangle⟧\eta)$$

$$= (\lambda f \in V_{fun} \ Y f)_{fun*} \ \iota_{fun}(\lambda d \in D. \ ⟦\langle 0, w \rangle⟧[\eta | w : d])$$

$$= (\lambda f \in V_{fun} \ Y f) \ (\lambda d \in D. \ \iota_{tuple} \langle ⟦0⟧[\eta|w:d], ⟦w⟧[\eta|w:d] \rangle)$$

$$= (\lambda f \in V_{fun} \ Y f) \ (\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, [\eta|w:d] \, w \rangle)$$

$$= Y \ (\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, d \rangle)$$

$$F = (\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, d \rangle)$$

$$Y F = F(Y F)$$

$$Y F = (\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, d \rangle) \ Y(\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, d \rangle)$$

$$= \iota_{tuple} \langle \iota_{int} 0, Y(\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, d \rangle) \rangle$$

$$⟦e.1⟧\eta = (\lambda t \in V_{tuple} \begin{cases} t.1 & \text{si } 1 \leq \# t \\ ty_{err} & \text{c.c} \end{cases})_{tuple*} ⟦e⟧\eta$$

$$= (\lambda t \in V_{tuple} \begin{cases} t.1 & \text{si } 1 \leq \# t \\ ty_{err} & \text{c.c} \end{cases}) \langle \iota_{int} 0, Y(\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, d \rangle) \rangle$$

$\# t = 2$ caso 1.

$$= \langle \iota_{int} 0, Y(\lambda d \in D. \ \iota_{tuple} \langle \iota_{int} 0, d \rangle) \rangle. 1$$

$$= \iota_{int} 0$$

$$⟦(e.2).1⟧ = (\lambda t \in V_{tuple} \begin{cases} t.1 & \text{si } 1 \leq \# t \\ ty_{err} & \text{c.c} \end{cases})_{tuple*} ⟦e.2⟧\eta$$

$$= \left(\lambda t\in V_{tuple} \begin{cases} t.1 & \text{si } 1\le \#t \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right)_{tuple*}$$

$$\left(\lambda t'\in V_{tuple} \begin{cases} t'.2 & \text{si } 2\le \#t' \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right)_{tuple*} [\![e]\!]\eta$$

$$= \left(\lambda t\in V_{tuple} \begin{cases} t.1 & \text{si } 1\le \#t \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right)_{tuple*}$$

$$\left(\lambda t'\in V_{tuple} \begin{cases} t'.2 & \text{si } 2\le \#t' \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right)_{tuple*}$$

$$\iota_{tuple}\langle \iota_{int}\, 0,\ Y(\lambda d\in D.\ \iota_{tuple}\langle \iota_{int}\, 0, d\rangle)\rangle$$

$$= \left(\lambda t\in V_{tuple} \begin{cases} t.1 & \text{si } 1\le \#t \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right)_{tuple*}$$

$$\left(\lambda t'\in V_{tuple} \begin{cases} t'.2 & \text{si } 2\le \#t' \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right) \langle \iota_{int}\, 0,\ Y(\lambda d\in D.\ \iota_{tuple}\langle \iota_{int}\, 0, d\rangle)\rangle$$

$$= \left(\lambda t\in V_{tuple} \begin{cases} t.1 & \text{si } 1\le \#t \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right)_{tuple*}$$

$$Y(\lambda d\in D.\ \iota_{tuple}\langle \iota_{int}\, 0, d\rangle)$$

$$= \left(\lambda t\in V_{tuple} \begin{cases} t.1 & \text{si } 1\le \#t \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right)_{tuple*} \qquad YF = F(YF)$$

$$\iota_{tuple}\langle \iota_{int}\, 0,\ Y(\lambda d\in D.\ \iota_{tuple}\langle \iota_{int}\, 0, d\rangle)\rangle$$

$$= \left(\lambda t\in V_{tuple} \begin{cases} t.1 & \text{si } 1\le \#t \\ \mathrm{tyerr} & \text{c.c} \end{cases}\right) \langle \iota_{int}\, 0\ \ Y(\lambda d\in D.\ \iota_{tuple}\langle \iota_{int}\, 0, d\rangle)\rangle$$

$$= \iota_{int}\, 0$$

5. Suponga que e es una expresión cerrada. Considere las siguientes expresiones:

$$\begin{array}{ll} \textbf{letrec}\ \mathsf{f} \equiv \lambda x.\ \textbf{if}\ e\ \textbf{then}\ 1\ \textbf{else}\ \mathsf{f}\ x & \textbf{in}\ \mathsf{f}\ 0 \\ \textbf{letrec}\ \mathsf{f} \equiv \lambda x.\ \textbf{if}\ e\ \textbf{then}\ \textbf{True}\ \textbf{else}\ \mathsf{f}\ x & \textbf{in}\ \mathsf{f}\ 0 + 1 \end{array}$$

calcular la semántica denotacional eager y normal directa considerando por separado los casos $[\![e]\!]\eta = \iota_{\mathsf{norm}}\,(\iota_{\mathsf{bool}}\,\mathsf{V})$ y $[\![e]\!]\eta = \iota_{\mathsf{norm}}\,(\iota_{\mathsf{bool}}\,\mathsf{F})$.