# Definiciones

domingo, 2 de junio de 2024 19:13

Semántica dotacional de lo de la guía 9

**Versión eager:**

$V_{int} = \mathbb{Z}$
$V_{bool} = \{T, F\}$
$V_{fun} = [V \to D]$
$V_{tuple} = V^*$
$V = V_{int} + V_{bool} + V_{fun} + V_{tuple}$
$D = (V + \{\text{error}\} + \{typeerror\})_\perp$

$err = \iota_\perp \iota_{\text{error}}$
$tyerr = \iota_\perp \iota_{\text{typeerror}}$
$\iota_{norm} : V \to D$

Para $\theta \in \{inf, bool, fun, tuple\}$:

$\iota_\theta : V_\theta \to V$
$\iota_{\underline{\theta}} = \iota_{norm} \circ \iota_\theta$

Si $f : V \to D$:

$f_*(\iota_{norm} v) = f\ v$
$f_*\ x = x$

$f_\theta(\iota_\theta\ x) = f\ x$
$f_\theta(\iota_{\theta'}\ x) = tyerr$
$f_\theta\ x = x$

$f_{\theta*} = (f_\theta)_*$

$Env = \langle var \rangle \to V$

$$\llbracket\cdot\rrbracket \in \langle exp\rangle \to Env \to D$$

$$\llbracket v\rrbracket\eta = \iota_{norm}(\eta\ v)$$

$$\llbracket e_0\, e_1\rrbracket\eta = \left(\lambda f\,.\, f_*(\llbracket e_1\rrbracket\eta)\right)_{fun*}(\llbracket e_0\rrbracket\eta)$$

$$\llbracket\lambda v.\, e\rrbracket\eta = \iota_{\underline{fun}}(\lambda x\, \llbracket e\rrbracket[\eta \mid v : x])$$

$$\llbracket\lfloor i\rfloor\rrbracket\eta = \iota_{int}\ i$$

$$\llbracket\lfloor b\rfloor\rrbracket\eta = \iota_{bool}\ b$$

$$\llbracket -e\rrbracket\eta = \left(\lambda i\,.\,\iota_{\underline{int}}(-i)\right)_{int*}(\llbracket e\rrbracket\eta)$$

Para $\oplus \in \{+,-,*\}$:

$$\llbracket e_0 \oplus e_1\rrbracket\eta = \left(\lambda i_0\,.\left(\lambda i_1\,.\,\iota_{\underline{int}}(i_0 \oplus i_1)\right)_{int*}(\llbracket e_1\rrbracket\eta)\right)_{int*}(\llbracket e_0\rrbracket\eta)$$

Para $\oplus \in \{/,\%\}$:

$$\llbracket e_0 \oplus e_1\rrbracket\eta = \left(\lambda i_0\,.\left(\lambda i_1\,.\begin{cases} i_1 \neq 0 & \to \quad \iota_{\underline{int}}(i_0 \oplus i_1) \\ \text{si no} & \to \quad err \end{cases}\right)_{int*}(\llbracket e_1\rrbracket\eta)\right)_{int*}(\llbracket e_0\rrbracket\eta)$$

$$\llbracket\neg e\rrbracket\eta = \left(\lambda b\,.\,\iota_{\underline{bool}}(\neg b)\right)_{bool*}(\llbracket e\rrbracket\eta)$$

Para $\oplus \in \{\wedge,\vee,\to,\leftrightarrow\}$:

$$\llbracket e_0 \oplus e_1\rrbracket\eta = \left(\lambda b_0\,.\left(\lambda b_1\,.\,\iota_{\underline{bool}}(b_0 \oplus b_1)\right)_{bool*}(\llbracket e_1\rrbracket\eta)\right)_{bool*}(\llbracket e_0\rrbracket\eta)$$

$$\llbracket\text{if } e \text{ then } e_0 \text{ else } e_1\rrbracket\eta = \left(\lambda b\,.\begin{cases} b & \to \quad \llbracket e_0\rrbracket\eta \\ \text{si no} & \to \quad \llbracket e_1\rrbracket\eta \end{cases}\right)_{bool*}(\llbracket e\rrbracket\eta)$$

$$\llbracket\langle e_0, \dots, e_{n-1}\rangle\rrbracket\eta = \begin{cases} \exists i < n\ \llbracket e_i\rrbracket\eta \neq \iota_{norm} d & \to \quad \left\llbracket e_{\min_i \llbracket e_i\rrbracket\eta \neq \iota_{norm} d}\right\rrbracket\eta \\ \text{si no} & \to \quad \iota_{\underline{tuple}}\langle\llbracket e_0\rrbracket\eta, \dots, \llbracket e_{n-1}\rrbracket\eta\rangle \end{cases}$$

$$\llbracket e.\lfloor k\rfloor\rrbracket\eta = \left(\lambda t\,.\begin{cases} k < |t| & \to \quad \iota_{norm} t_k \\ \text{si no} & \to \quad tyerr \end{cases}\right)_{tuple*}(\llbracket e\rrbracket\eta)$$

$$\llbracket\text{letrec } f \equiv \lambda x.\, e \text{ in } e'\rrbracket\eta = \llbracket e'\rrbracket\left[\eta \mid f : \iota_{fun}\ g\right]$$

Donde:

$$F\ h\ z = \llbracket e\rrbracket\left[\eta \mid f : \iota_{fun} h, x : z\right]$$

$$g = \mathbf{Y}_{V_{fun}}\ F$$

## Versión normal:

$V_{int} = \mathbb{Z}$
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$V = V_{int} + V_{bool} + V_{fun} + V_{tuple}$
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$f_\theta(\iota_\theta\ x) = f\ x$
$f_\theta(\iota_{\theta'}\ x) = tyerr$
$f_\theta\ x = x$

$f_{\theta *} = (f_\theta)_*$

$Env = \langle var \rangle \to D$

$[\![\cdot]\!] \in \langle exp \rangle \to Env \to D$
$[\![v]\!]\eta = \iota_{norm}(\eta\ v)$
$[\![e_0\ e_1]\!]\eta = (\lambda f\ .\ f([\![e_1]\!]\eta))_{fun*}([\![e_0]\!]\eta)$

$$[\![\lambda v\,.\,e]\!]\eta = \underline{\iota_{fun}}(\lambda x\,.\,[\![e]\!][\eta \mid v : x])$$

$$[\![\lfloor i \rfloor]\!]\eta = \iota_{int}\ i$$

$$[\![\lfloor b \rfloor]\!]\eta = \iota_{bool}\ b$$

$$[\![-e]\!]\eta = \left(\lambda i\,.\,\underline{\iota_{int}}(-i)\right)_{int*}\ ([\![e]\!]\eta)$$

Para $\oplus \in \{+,-,*\}$:

$$[\![e_0 \oplus e_1]\!]\eta = \left(\lambda i_0\,.\left(\lambda i_1\,.\,\underline{\iota_{int}}(i_0 \oplus i_1)\right)_{int*}\ ([\![e_1]\!]\eta)\right)_{int*}\ ([\![e_0]\!]\eta)$$

Para $\oplus \in \{/,\%\}$:

$$[\![e_0 \oplus e_1]\!]\eta = \left(\lambda i_0\,.\left(\lambda i_1\,.\begin{cases} i_1 \neq 0 & \rightarrow \quad \underline{\iota_{int}}(i_0 \oplus i_1) \\ \text{si no} & \rightarrow \quad err \end{cases}\right)_{int*}\ ([\![e_1]\!]\eta)\right)_{int*}\ ([\![e_0]\!]\eta)$$

$$[\![\neg e]\!]\eta = \left(\lambda b\,.\,\underline{\iota_{bool}}(\neg b)\right)_{bool*}\ ([\![e]\!]\eta)$$

$$[\![e_0 \wedge e_1]\!]\eta = \left(\lambda b_0\,.\begin{cases} b_0 & \rightarrow \quad [\![e_1]\!]\eta \\ \text{si no} & \rightarrow \quad \iota_{bool} F \end{cases}\right)_{bool*}\ ([\![e_0]\!]\eta)$$

$$[\![e_0 \vee e_1]\!]\eta = \left(\lambda b_0\,.\begin{cases} \neg b_0 & \rightarrow \quad [\![e_1]\!]\eta \\ \text{si no} & \rightarrow \quad \iota_{bool} T \end{cases}\right)_{bool*}\ ([\![e_0]\!]\eta)$$

$$[\![e_0 \rightarrow e_1]\!]\eta = \left(\lambda b_0\,.\begin{cases} b_0 & \rightarrow \quad [\![e_1]\!]\eta \\ \text{si no} & \rightarrow \quad \iota_{bool} T \end{cases}\right)_{bool*}\ ([\![e_0]\!]\eta)$$

$$[\![e_0 \leftrightarrow e_1]\!]\eta = \left(\lambda b_0\,.\begin{cases} b_0 & \rightarrow \quad [\![e_1]\!]\eta \\ \text{si no} & \rightarrow \quad [\![\neg e_1]\!]\eta \end{cases}\right)_{bool*}\ ([\![e_0]\!]\eta)$$

$$[\![\text{if } e \text{ then } e_0 \text{ else } e_1]\!]\eta = \left(\lambda b\,.\begin{cases} b & \rightarrow \quad [\![e_0]\!]\eta \\ \text{si no} & \rightarrow \quad [\![e_1]\!]\eta \end{cases}\right)_{bool*}\ ([\![e]\!]\eta)$$

$$[\![\langle e_0, \ldots, e_{n-1} \rangle]\!]\eta = \underline{\iota_{tuple}}\langle [\![e_0]\!]\eta, \ldots, [\![e_{n-1}]\!]\eta \rangle$$

$$[\![e.\lfloor k \rfloor]\!]\eta = \left(\lambda t\,.\begin{cases} k < |t| & \rightarrow \quad t_k \\ \text{si no} & \rightarrow \quad tyerr \end{cases}\right)_{tuple*}\ ([\![e]\!]\eta)$$

$$[\![\text{rec } e]\!]\eta = (\lambda f\,.\,\mathbf{Y}_D f)_{fun*}\ ([\![e]\!]\eta)$$

# 1)

domingo, 2 de junio de 2024 20:43

1. De la semántica denotacional eager y normal de las expresiones **True** $\vee$ 0 y **True** $\vee$ $\Delta\Delta$.

## Eager:

$T[\![\text{True} \vee 0]\!]\eta$

$$= \left(\lambda b_0 . \left(\lambda b_1 . \underline{\iota_{bool}}(b_0 \oplus b_1)\right)_{bool*} ([\![0]\!]\eta)\right)_{bool*} (T[\![\text{True}]\!]\eta)$$

$$= \left(\lambda b_0 . \left(\lambda b_1 . \underline{\iota_{bool}}(b_0 \oplus b_1)\right)_{bool*} \iota_{int} 0\right)_{bool*} \iota_{bool} \mathrm{T}$$

$$= (\lambda b_0 . tyerr)_{bool*} \iota_{bool} \mathrm{T}$$

$$= tyerr$$

$T[\![\text{True} \vee \Delta\Delta]\!]\eta$

$$= \left(\lambda b_0 . \left(\lambda b_1 . \underline{\iota_{bool}}(b_0 \oplus b_1)\right)_{bool*} ([\![\Delta\Delta]\!]\eta)\right)_{bool*} (T[\![\text{True}]\!]\eta)$$

$$= \left(\lambda b_0 . \left(\lambda b_1 . \underline{\iota_{bool}}(b_0 \oplus b_1)\right)_{bool*} \perp\right)_{bool*} \iota_{bool} \mathrm{T}$$

$$= (\lambda b_0 . \perp)_{bool*} \iota_{bool} \mathrm{T}$$

$$= \perp$$

## Normal:

$T[\![\text{True} \vee 0]\!]\eta$

$$= \left(\lambda b_0 . \begin{cases} \neg b_0 & \rightarrow & [\![0]\!]\eta \\ \text{si no} & \rightarrow & \iota_{bool} T \end{cases}\right)_{bool*} (T[\![\text{True}]\!]\eta)$$

$$= \left(\lambda b_0 . \begin{cases} \neg b_0 & \rightarrow & [\![0]\!]\eta \\ \text{si no} & \rightarrow & \iota_{bool} T \end{cases}\right) T$$

$$= \iota_{bool} T$$

$T[\![\text{True} \vee \Delta\Delta]\!]\eta$

$$= \left(\lambda b_0 . \begin{cases} \neg b_0 & \rightarrow & [\![\Delta\Delta]\!]\eta \\ \text{si no} & \rightarrow & \iota_{bool} T \end{cases}\right)_{bool*} (T[\![\text{True}]\!]\eta)$$

$$= \left(\lambda b_0 . \begin{cases} \neg b_0 & \rightarrow & [\![\Delta\Delta]\!]\eta \\ \text{si no} & \rightarrow & \iota_{bool} T \end{cases}\right) T$$

$$= \iota_{bool} T$$

# 2)

lunes, 3 de junio de 2024 19:27

2. Encuentre ecuaciones semánticas sencillas para las siguientes expresiones, considerando los casos eager y normal.
   a) $[\![(\lambda x.e)e']\!]\eta$
   b) $[\![we]\!][\eta|w : \iota_{\text{fun}} f]$

## Eager:

a)

$$[\![(\lambda x.e)e']\!]\eta$$
$$= (\lambda f\,.\,f_*([\![e']\!]\eta))_{fun*}([\![\lambda x.e]\!]\eta)$$
$$= (\lambda f\,.\,f_*([\![e']\!]\eta))_{fun*}\left(\underline{\iota_{fun}}(\lambda y\ [\![e]\!][\eta \mid x : y])\right)$$
$$= (\lambda y\ [\![e]\!][\eta \mid x : y])_*([\![e']\!]\eta)$$

b)

$$[\![we]\!][\eta \mid w : \iota_{fun} f]$$
$$= \left(\lambda g\,.\,g_*([\![e]\!][\eta \mid e : \iota_{fun} f])\right)_{fun*}([\![w]\!][\eta \mid w : \iota_{fun} f])$$
$$= \left(\lambda g\,.\,g_*([\![e]\!][\eta \mid e : \iota_{fun} f])\right)_{fun*}\iota_{norm}(\iota_{fun} f)$$
$$= f_*([\![e]\!][\eta \mid e : \iota_{fun} f])$$

## Normal:

a)

$$[\![(\lambda x.e)e']\!]\eta$$
$$= (\lambda f\,.\,f([\![e']\!]\eta))_{fun*}([\![\lambda x.e]\!]\eta)$$
$$= (\lambda f\,.\,f([\![e']\!]\eta))_{fun*}\left(\underline{\iota_{fun}}(\lambda y\ [\![e]\!][\eta \mid x : y])\right)$$
$$= (\lambda y\ [\![e]\!][\eta \mid x : y])([\![e']\!]\eta)$$
$$= [\![e]\!][\eta \mid x : [\![e']\!]\eta]$$

b)

$$[\![we]\!][\eta \mid w : \underline{\iota_{fun}} f]$$
$$= \left(\lambda g\,.\,g\left([\![e]\!][\eta \mid w : \underline{\iota_{fun}} f]\right)\right)_{fun*}\left([\![w]\!][\eta \mid w : \underline{\iota_{fun}} f]\right)$$
$$= \left(\lambda g\,.\,g\left([\![e]\!][\eta \mid w : \underline{\iota_{fun}} f]\right)\right)_{fun*}\left(\iota_{norm}\left(\underline{\iota_{fun}} f\right)\right)$$

$$= f\left(e[\![\eta \mid w : \iota_{\underline{fun}} f]\!]\right)$$

# 3)

lunes, 3 de junio de 2024 19:38

3. Calcular la semántica denotacional eager y normal de las expresiones $\langle \textbf{True} + 0, \Delta\Delta \rangle$ y $\langle \Delta\Delta, \textbf{True} + 0 \rangle$.

Eager:

$$[\![\langle \text{True} + 0, \Delta\Delta \rangle]\!]\eta$$
$$= ([\![\text{True} + 0]\!]\eta = tyerr)$$
$$tyerr$$

$$[\![\langle \Delta\Delta, \text{True} + 0 \rangle]\!]\eta$$
$$= ([\![\Delta\Delta]\!]\eta = \perp)$$
$$\perp$$

Normal:

$$[\![\langle \text{True} + 0, \Delta\Delta \rangle]\!]\eta$$
$$= \underline{\iota_{tuple}} \langle [\![\text{True} + 0]\!]\eta, [\![\Delta\Delta]\!]\eta \rangle$$
$$= \underline{\iota_{tuple}} \langle tyerr, \perp \rangle$$

$$[\![\langle \Delta\Delta, \text{True} + 0 \rangle]\!]\eta$$
$$= \underline{\iota_{tuple}} \langle [\![\Delta\Delta]\!]\eta, [\![\text{True} + 0]\!]\eta \rangle$$
$$= \underline{\iota_{tuple}} \langle \perp, tyerr \rangle$$

# 4) 🛠️

lunes, 3 de junio de 2024 20:09

4. De la semántica denotacional normal de las expresiones $e.1$ y $(e.2).1$, donde $e$ es la expresión que dio en el ejercicio 7 del práctico 9.

$e = \text{rec}(\lambda f\,.\langle 0, f\rangle)$

$$[\![e.0]\!]\eta$$
$$= (\lambda\langle z_0, z_1\rangle\,.\,\iota_{norm} z_0)_{tuple*}([\![e]\!]\eta)$$

= (

$$[\![e]\!]\eta$$
$$= (\lambda g\,.\,\mathbf{Y}_D\, g)_{fun*}([\![\lambda f\,.\langle 0, f\rangle]\!]\eta)$$
$$= (\lambda g\,.\,\mathbf{Y}_D\, g)_{fun*}\left(\iota_{\underline{fun}}(\lambda x\,.[\![\langle 0, f\rangle]\!][\eta \mid f : x])\right)$$
$$= \mathbf{Y}_D(\lambda x\,.[\![\langle 0, f\rangle]\!][\eta \mid f : x])$$
$$= \mathbf{Y}_D\left(\lambda x\,.\,\iota_{\underline{tuple}}\langle [\![0]\!][\eta \mid f : x], [\![f]\!][\eta \mid f : x]\rangle\right)$$
$$= \mathbf{Y}_D\left(\lambda x\,.\,\iota_{\underline{tuple}}\langle 0, x\rangle\right)$$
$$= \iota_{\underline{tuple}}\left\langle 0, \mathbf{Y}_D\left(\lambda x\,.\,\iota_{\underline{tuple}}\langle 0, x\rangle\right)\right\rangle$$

)

$$(\lambda\langle z_0, z_1\rangle\,.\,\iota_{norm} z_0)\left(\iota_{\underline{tuple}}\left\langle 0, \mathbf{Y}_D\left(\lambda x\,.\,\iota_{\underline{tuple}}\langle 0, x\rangle\right)\right\rangle\right)$$
$$= \iota_{norm} 0$$

# 5) 🛠️

lunes, 3 de junio de 2024 20:09

5. Suponga que $e$ es una expresión cerrada. Considere las siguientes expresiones:

$$\textbf{letrec } \mathsf{f} \equiv \lambda x.\ \textbf{if } e \textbf{ then } 1 \textbf{ else } \mathsf{f}\ x \quad \textbf{in } \mathsf{f}\ 0$$
$$\textbf{letrec } \mathsf{f} \equiv \lambda x.\ \textbf{if } e \textbf{ then True else } \mathsf{f}\ x \quad \textbf{in } \mathsf{f}\ 0+1$$

calcular la semántica denotacional eager y normal directa considerando por separado los casos $[\![e]\!]\eta = \iota_{\mathsf{norm}}(\iota_{\mathsf{bool}}\,\mathsf{V})$ y $[\![e]\!]\eta = \iota_{\mathsf{norm}}(\iota_{\mathsf{bool}}\,\mathsf{F})$.

Asumo $f, x \notin FV(e)$

Eager, $[\![e]\!] = \iota_{\underline{bool}}(T)$:

$[\![\text{letrec } f \equiv (\lambda x.\, if\ e \text{ then } 1 \text{ else } f\ x) \text{ in } f\ 0]\!]\eta$

= (Sea $F\ h\ z = [\![if\ e \text{ then } 1 \text{ else } f\ x]\!][\eta \mid f : \iota_{fun} h, x : z]$)

$[\![f\ 0]\!][\eta \mid f : \iota_{fun}(\mathbf{Y}_{V_{fun}}\ F)]$

$= \left(\lambda g.\, g_* ([\![0]\!][\eta \mid f : \iota_{fun}(\mathbf{Y}_{V_{fun}}\ F)])\right)_{fun*} ([\![f]\!][\eta \mid f : \iota_{fun}(\mathbf{Y}_{V_{fun}}\ F)])$

$= \left(\lambda g.\, g_*(\iota_{\underline{int}}0)\right)_{fun*} \left(\iota_{fun}(\mathbf{Y}_{V_{fun}}\ F)\right)$

$= \mathbf{Y}_{V_{fun}}\ F(\iota_{\underline{int}}0)$

$= F(\mathbf{Y}_{V_{fun}}\ F)(\iota_{\underline{int}}0)$

$= [\![if\ e \text{ then } 1 \text{ else } f\ x]\!][\eta \mid f : \iota_{fun}(\mathbf{Y}_{V_{fun}}\ F), x : \iota_{\underline{int}}0]$

$= \iota_{\underline{int}}1$

# 6) 🛠️

martes, 4 de junio de 2024 20:13

6. Enuncie un teorema de corrección de la evaluación respecto a la semántica denotacional; explique cómo lo probaría y (*) pruebe el caso para la regla del operador de recursión normal.

# 7)

martes, 4 de junio de 2024 20:13

7. Utilizando la semántica denotacional normal ¿Qué opinas de las siguientes afirmaciones? La siguiente meta-expresión define un elemento del dominio $D$ y además existe una expresión $e$ tal que su semántica denotacional está definida por esta.

$$\iota_{\mathsf{norm}}(\iota_{\mathsf{fun}}(\lambda b_0 \in D.(\iota_{\mathsf{norm}}(\iota_{\mathsf{fun}}\left(\lambda b_1 \in D. \begin{cases} \iota_{\mathsf{norm}}(\iota_{\mathsf{bool}}\mathsf{V}) & b_0 = \iota_{\mathsf{norm}}(\iota_{\mathsf{bool}}\mathsf{V}) \\ \iota_{\mathsf{norm}}(\iota_{\mathsf{bool}}\mathsf{V}) & b_1 = \iota_{\mathsf{norm}}(\iota_{\mathsf{bool}}\mathsf{V}) \\ b_0 \vee_{\mathsf{bool}*} b_1 & c.c. \end{cases}\right)))))$$

donde

$$b_0 \vee_{\mathsf{bool}*} b_1 = (\lambda b_0' \in V_{\mathsf{bool}}.(\lambda b_1' \in V_{\mathsf{bool}}.\iota_{\mathsf{norm}}(\iota_{\mathsf{bool}}(b_0' \vee b_1')))_{\mathsf{bool}\,*} b_1)_{\mathsf{bool}\,*} b_0$$

¿Valen estas afirmaciones para la semántica denotacional eager?

Si define un elemento de $D$

No hay ninguna expreción que lo tenga como semántica por este caso:

$[\![b_0]\!] = tyerr$

$[\![b_1]\!] = \perp$

Si $tyerr$ y $err$ fueran lo mismo que $\perp$ posiblemente se podría pero no se como

Para lograrlo la idea sería hacer un interprete del propio cálculo lambda que tome dos expresiones y valla ejecutando un paso de cada una