Parcial 1 - 2016-05-06

Lenguajes y Compiladores

1. Determinar si es verdadero o falso. Justificar la respuesta.
   a) Sean p, q predicados. Si $[\![p]\!] = [\![q]\!]$, entonces para toda sustitución δ se tiene $[\![p/\delta]\!] = [\![q/\delta]\!]$.
   b) Sea Ω el dominio del lenguaje imperativo con fallas y output. Si σ es un estado, entonces existe una cadena interesante que tiene como supremo a $\iota_{out}(1, \iota_{term}\sigma)$.
   c) Sea $f, g \in \mathbf{Z} \to \mathbf{Z}_\perp$. Entonces existe $h \in \mathbf{Z} \to \mathbf{Z}_\perp$ tal que $f \leq h$ y $g \leq h$.
   d) En el lenguaje imperativo simple, si $[\![c]\!]\sigma = \langle \mathbf{abort}, \sigma' \rangle$, entonces

$$[\![\mathbf{catchin}\ c\ \mathbf{with}\ c']\!]\sigma = [\![c; c']\!]\sigma.$$

(a) Falso, sea σ un estado tal que $\sigma y = 5$, $\sigma z = 2$

$p = x := y$, $q = x := 5$

$[\![p]\!]\sigma = [\![x := y]\!]\sigma = [\sigma | x : [\![y]\!]\sigma] = [\sigma | x : 5]$

$[\![q]\!]\sigma = [\![x := 5]\!]\sigma = [\sigma | x : 5]$

Pero dado δ con $\delta y = z$

$[\![p/\delta]\!]\sigma = [\![x/\delta := y/\delta]\!]\sigma = [\![x := z]\!]\sigma = [\sigma | x : [\![z]\!]\sigma] = [\sigma | x : 2]$

$[\![q/\delta]\!]\sigma = [\![x/\delta := 5]\!]\sigma = [\![x := 5]\!]\sigma = [\sigma | x : 5]$

(b) Falso, para que $\iota_{out}(1, \iota_{term}\sigma)$ sea supremo de una cadena los elementos de la cadena deben ser de la forma $\iota_{out}(1, \iota_{term}\sigma')$ con $\sigma' \in \Sigma_\perp$ pero como $\Sigma_\perp$ es el orden llano $\iota_{out}(1, \iota_{term}\sigma)$ solo es mayor que $\iota_{out}(1, \perp)$.

(c) Falso, si $\mathbb{Z}$ es el orden discreto y f, g son ambas funciones totales cualquier función h va a ser incomparable a f y g.

(d) Falso, si $c' = \mathbf{skip}$

$[\![\mathbf{catchin}\ c\ \mathbf{with}\ c']\!]\sigma =$

$[\![c']\!]_+ ([\![c]\!]\sigma) =$

$[\![c']\!]_+ \langle \mathbf{abort}, \sigma' \rangle =$

$[\![c']\!]\sigma' = [\![\mathbf{skip}]\!]\sigma' = \sigma'$

$[\![c; c']\!]\sigma =$

$[\![c']\!]_* ([\![c]\!]\sigma) =$

$[\![c']\!]_* \langle \mathbf{abort}, \sigma' \rangle =$

$\langle \mathbf{abort}, \sigma' \rangle$

2. Considere el lenguaje aplicativo con fallas, output e input. Analice utilizando la semántica denotacional la equivalencia entre los siguientes comandos:
   a) **newvar** $v := e$ **in** $?v; !v \;\equiv\; ?v; !v$
   b) Si $FA\ c \cap FA\ c' = \emptyset$ entonces $c; c' \equiv c'; c$

a) $[\![\text{newvar } v := e \text{ in } ?v; !v]\!]\sigma =$

$(\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger ([\![?v; !v]\!][\sigma | v := e]) =$

$(\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger ([\![!v]\!]_* [\![?v]\!][\sigma | v := e]) =$

$(\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger ([\![!v]\!]_* \iota_{in}(\lambda n. \iota_{term}[\sigma | v : e | v : n])) =$

$(\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger ([\![!v]\!]_* \iota_{in}(\lambda n. \iota_{term}[\sigma | v : n])) =$

$(\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger \iota_{in}(\lambda n\ [\![!v]\!]_* \iota_{term}[\sigma | v : n]) =$

$(\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger \iota_{in}(\lambda n\ \iota_{out}(\sigma v, \iota_{term}[\sigma | v : n])) =$

$\iota_{in}(\lambda n\ (\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger \iota_{out}(n, \iota_{term}[\sigma | v : n])) =$

$\iota_{in}(\lambda n\ \iota_{out}(n, (\lambda\sigma'. [\sigma' | v : \sigma v])_\dagger \iota_{term}[\sigma | v : n])) =$

$\iota_{in}(\lambda n\ \iota_{out}(n, \iota_{term}[[\sigma | v : n] : v : \sigma v])) =$

$\iota_{in}(\lambda n\ \iota_{out}(n, \iota_{term}\sigma)).$

$[\![?v; !v]\!]\sigma =$

$[\![!v]\!]_* ([\![?v]\!]\sigma) =$

$[\![!v]\!]_* \iota_{in}(\lambda n\ \iota_{term}[\sigma | v : n]) =$

$\iota_{in}(\lambda n\ [\![!v]\!]_* \iota_{term}[\sigma | v : n]) =$

$\iota_{in}(\lambda n\ \iota_{out}(\sigma v, \iota_{term}[\sigma | v : n]) =$

$\iota_{in}(\lambda n\ \iota_{out}(n, \iota_{term}[\sigma | v : n])$

No son equivalentes.

b) Falso sea $c = x := 1$ $c' = y := x$ con $\sigma x = 5$

$[\![c; c']\!]\sigma =$

$[\![c']\!]_* ([\![c]\!]\sigma) =$

$[\![y := x]\!]_* ([\![x := 1]\!]\sigma) =$

$[\![y := x]\!]_* [\sigma | x : 1] =$

$[[\sigma | x : 1] | y : \sigma x] =$

$[\sigma | x:1 | y:1]$.

$[\![c';c]\!]\sigma =$

$[\![c]\!]_* ([\![c']\!]\sigma) =$

$[\![x:=1]\!]_* ([\![y:=x+1]\!]\sigma) =$

$[\![x:=1]\!]_* [\sigma | y:\sigma x+1] =$

$[\![x:=1]\!]_\sigma [\sigma | y:6] =$

$[\sigma | y:6 | x:1]$

3. Considere el lenguaje imperativo simple.
   a) Dé la semántica denotacional de **while** b **do** c.
   b) Pruebe que la función F que define la semántica de **while** b **do** c es continua.
   c) De ejemplo de un comando c de la forma **while** b **do** c tal que $[\![c]\!] = F^3 \perp_{\Sigma\to\Sigma_\perp}$ pero $[\![c]\!] \neq F^2 \perp_{\Sigma\to\Sigma_\perp}$.

a) $[\![\text{while } b \text{ do } c]\!]_* = \bigsqcup_{i=0}^{\infty} F^i \perp \sigma$

$$F\,w\,\sigma = \begin{cases} w_* [\![c]\!]\sigma & \text{si } [\![b]\!]\sigma \\ \sigma & \text{si } \neg[\![b]\!]\sigma \end{cases}$$

b) Sean g, h funciones tales que $g \sqsubseteq h$. Primero veamos que F es monótona.

si $\neg[\![b]\!]\sigma$ $\quad F g \sigma = \sigma = F h \sigma$

si $[\![b]\!]\sigma$ $\quad F g \sigma = g_* [\![c]\!]\sigma \qquad F h \sigma = h_* [\![c]\!]\sigma$

- $[\![c]\!]\sigma = \perp$

$g_* [\![c]\!]\sigma = g\perp \leq h\perp = h_* [\![c]\!]\sigma$

- caso $[\![c]\!]\sigma = \sigma'$

$g_* [\![c]\!]\sigma = g\sigma' \leq h\sigma' = h_* [\![c]\!]\sigma$

Ahora veamos que es continua

$$F(\sup\{f_i\}_{i\in\mathbb{N}})\sigma = \begin{cases} (\sup\{f_i\}_{i\in\mathbb{N}})_* [\![c]\!]\sigma & \text{si } [\![b]\!]\sigma \\ \sigma & \text{si } \neg[\![b]\!]\sigma \end{cases}$$

si $\neg [\![b]\!]\sigma$

$$F(\sup\{f_i\}_{i\in N})\sigma = \sigma = \sup\{F f_i \sigma\}_{i\in N}.$$

si $[\![b]\!]\sigma$

$$F(\sup\{f_i\}_{i\in N})\sigma = (\sup\{f_i\}_{i\in N})_\perp [\![c]\!]\sigma$$
$$= \sup\{f_{i\perp} [\![c]\!]\sigma\}_{i\in N}$$
$$= \sup\{F f_i \sigma\}_{i\in N}$$
$$= \sup\{F f_i\}_{i\in N}\sigma$$

(c) $c = \text{while } x\geq 0 \wedge x<3 \text{ do } x:=x+1$

$$[\![c]\!] = \bigsqcup_{i=0}^{\infty} F^i \perp_{\Sigma\to\Sigma_\perp}$$

$$F_w\sigma = \begin{cases} w_\perp [\![x:=x+1]\!]\sigma & \sigma x\geq 0 \wedge \sigma x<3 \\ \sigma & c.c \end{cases}$$

$$F^0 \perp_{\Sigma\to\Sigma_\perp}\sigma = \perp_{\Sigma\to\Sigma_\perp}$$

$$F\perp_{\Sigma\to\Sigma_\perp}\sigma = \begin{cases} \perp_{\Sigma\to\Sigma_\perp\perp}[\![x:=x+1]\!]\sigma & \sigma x\geq 0 \wedge \sigma x<3 \\ \sigma & c.c \end{cases}$$

$$F\perp_{\Sigma\to\Sigma_\perp}\sigma = \begin{cases} \perp & \sigma x\geq 0 \wedge \sigma x<3 \\ \sigma & c.c \end{cases}$$

$$F\perp_{\Sigma\to\Sigma_\perp}\sigma = \begin{cases} (F\perp_{\Sigma\to\Sigma_\perp})[\sigma|x:\sigma x+1] & \sigma x\geq 0 \wedge \sigma x<3 \\ \sigma & c.c \end{cases}$$

$$F\perp_{\Sigma\to\Sigma_\perp}\sigma = \begin{cases} \perp & \sigma x\geq 0 \wedge \sigma x<3 \wedge (\sigma x+1\geq 0 \wedge \sigma x+1<3) \\ [\sigma|x:\sigma x+1] & \sigma x\geq 0 \wedge \sigma x<3 \wedge (\sigma x+1<0 \vee \sigma x+1\geq 3) \\ \sigma & c.c \end{cases}$$

$$F\perp_{\Sigma\to\Sigma_\perp}\sigma = \begin{cases} \perp & \sigma x\geq 0 \wedge \sigma x<2 \\ [\sigma|x:\sigma x+1] & \sigma x\geq 2 \wedge \sigma<3 \Rightarrow \sigma x=2 \\ \sigma & c.c \end{cases}$$

$$F\perp_{\Sigma\to\Sigma_\perp}\sigma = \begin{cases} \perp & \sigma x\geq 0 \wedge \sigma x<2 \\ [\sigma|x:3] & \sigma x=2 \\ \sigma & c.c \end{cases}$$

$$F^2 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} (F \perp_{\Sigma \to \Sigma_\perp})[\sigma | x : \sigma x + 1] & \sigma x \geq 0 \wedge \sigma x < 3 \\ \sigma & c.c \end{cases}$$

$$F^2 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} \perp & \sigma x \geq 0 \wedge \sigma x < 3 \wedge \sigma x + 1 \geq 0 \wedge \sigma x + 1 < 2 \\ [\sigma | x : 3] & \sigma x \geq 0 \wedge \sigma x < 3 \wedge (\sigma x + 1 = 2) \\ [\sigma | x : \sigma x + 1] & \sigma x \geq 0 \wedge \sigma x < 3 \wedge (\sigma x + 1 < 0 \vee \sigma x + 1 > 2) \\ \sigma & c.c \end{cases}$$

$$F^2 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} \perp & \sigma x \geq 0 \wedge \sigma x < 1 \Rightarrow \sigma x = 0 \\ [\sigma | x : 3] & \sigma x = 1 \\ [\sigma | x : \sigma x + 1] & \sigma x > 1 \wedge \sigma x < 3 \Rightarrow \sigma x = 2 \\ \sigma & c.c \end{cases}$$

$$F^2 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} \perp & \sigma x = 0 \\ [\sigma | x : 3] & \sigma x = 1, 2 \\ \sigma & \sigma x < 0 \vee \sigma x \geq 3 \end{cases}$$

$$F^3 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} (F^2 \perp_{\Sigma \to \Sigma_\perp})[\sigma | x : \sigma x + 1] & \sigma x \geq 0 \wedge \sigma x < 3 \\ \sigma & c.c \end{cases}$$

$$F^3 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} \perp & \sigma x \geq 0 \wedge \sigma x < 3 \wedge \sigma x + 1 = 0 \quad \times \\ [\sigma | x : 3] & \sigma x \geq 0 \wedge \sigma x < 3 \wedge \sigma x + 1 = 1, 2 \Rightarrow \sigma x = 0, 1 \\ [\sigma | x : \sigma x + 1] & \sigma x \geq 0 \wedge \sigma x < 3 \wedge (\sigma x + 1 < 0 \vee \sigma x + 1 \geq 3) \Rightarrow \sigma x = 2 \\ \sigma & c.c \end{cases}$$

$$F^3 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} [\sigma | x : 3] & \sigma x = 0, 1 \\ [\sigma | x : \sigma x + 1] & \sigma x = 2 \\ \sigma & c.c \end{cases}$$

$$F^3 \perp_{\Sigma \to \Sigma_\perp} \sigma = \begin{cases} [\sigma | x : 3] & \sigma x \in \{0, 1, 2\} \\ \sigma & \sigma x \notin \{0, 1, 2\} \end{cases}$$

Claramente $\bigsqcup_{i=0}^{\infty} F^i \perp_{\Sigma \to \Sigma_\perp} \sigma = F^3 \perp_{\Sigma \to \Sigma_\perp} \sigma = [\![c]\!]\sigma$