Parcial 1 - 06/05/2015

Lenguajes y compiladores

1. Complete las siguientes igualdades, expresando de la forma más sencilla posible el resultado, sin efectuar ningún cálculo. Considere el lenguaje que corresponde en cada caso.

   a) $[\![\forall x. x/0 = 0]\!]\sigma =$
   b) $[\![x := 1; \textbf{while true do skip}]\!]\sigma =$
   c) $[\![x := 1; \textbf{newvar } x := 0 \textbf{ in } (\textbf{fail};\ y := x)]\!]\sigma =$
   d) $[\![x := 1; \textbf{newvar } x := 0 \textbf{ in } (!x;\ \textbf{fail};\ y := x)]\!]\sigma =$
   e) $[\![x := 1; \textbf{newvar } x := 0 \textbf{ in } (?x;\ !x\,; \textbf{fail};\ y := x)]\!]\sigma =$

a) $[\![\forall x. x/0 = 0]\!]\sigma = \langle abort, \sigma\rangle$

b) $[\![x := 1; \text{while true do skip}]\!]\sigma = \perp$

c) $[\![x := 1; \text{newvar } x := 0 \text{ in } (\text{fail}; y := x)]\!]\sigma = \langle abort, [\sigma | x : 1]\rangle$

d) $[\![x := 1; \text{newvar } x := 0 \text{ in } (!x; \text{fail}; y := x)]\!]\sigma = \iota_{out}(0, \iota_{abort}[\sigma | x : 1])$

e) $[\![x := 1; \text{newvar } x := 0 \text{ in } (?x; !x; \text{fail}; y := x)]\!]\sigma = \iota_{in}(\lambda n.\ \iota_{out}(n, \iota_{abort}[\sigma | x : 1]))$

2. Calcule la semántica denotacional del programa dado en el item *c)* del ejercicio 1.

c) $[\![x := 1; \text{newvar } x := 0 \text{ in } (\text{fail}; y := x)]\!]\sigma =$

$[\![\text{newvar } x := 0 \text{ in } (\text{fail}; y := x)]\!]_* ([\![x := 1]\!]\sigma) =$

$[\![\text{newvar } x := 0 \text{ in } (\text{fail}; y := x)]\!] [\sigma | x : 1] =$

$(\lambda \sigma'. [\sigma' | x : 1])_\dagger ([\![\text{fail}; y := x]\!] [\sigma | x : 1 | x : 0]) =$

$(\lambda \sigma'. [\sigma' | x : 1])_\dagger ([\![y := x]\!]_* [\![\text{fail}]\!] [\sigma | x : 0]) =$

$(\lambda \sigma'. [\sigma' | x : 1])_\dagger ([\![y := x]\!]_* \langle abort, [\sigma | x : 0]\rangle) =$

$(\lambda \sigma'. [\sigma' | x : 1])_\dagger \langle abort, [\sigma | x : 0]\rangle =$

$\langle abort, (\lambda \sigma'. [\sigma' | x : 1]) [\sigma | x : 0]\rangle =$

$\langle abort, [\sigma | x : 0 | x : 1]\rangle =$

$\langle abort, [\sigma | x : 1]\rangle$.

3. *a*) Defina de la manera más clara posible el supremo de una cadena de funciones en el dominio $D \to D'$, donde D y D' son dos dominios.

b) Pruebe que la función $F : (\mathbf{Z} \to \mathbf{Z}_\perp) \to (\mathbf{Z} \to \mathbf{Z}_\perp)$ preserva el orden.

$$F f n = \begin{cases} n & n = 0, 1, 2 \\ 1 + f(n-3) & n > 1 \\ f n & n < 0 \end{cases}$$

c) Dé un ejemplo de una función $F : (\mathbf{Z} \to \mathbf{Z}_\perp) \to (\mathbf{Z} \to \mathbf{Z}_\perp)$ qué satsfaga que su menor punto fijo es $F^3 \perp_{\mathbf{Z}\to\mathbf{Z}_\perp}$

a) Sea $f_1 \le f_2 \le \dots$ una cadena donde $f_i \in D \to D'$ el supremo de la cadena es una función $g \in D \to D'$ tal que

para cada f_i $i \in \mathbb{N}$, $x \in D$: $f_i x = g x$ o $f_i x = \perp$ y $g x \neq \perp$

por ende $f_i x \le g x$

b) Veamos que la función es monótona. Sean $g, h \in \mathbb{Z} \to \mathbb{Z}_\perp$ tales que $g \sqsubseteq h$

Si $n = 0, 1, 2$

$$F g\, n = n = F h\, n$$

Si $n > 2$

$$F g\, n = 1 + g(n-3) \le 1 + h(n-3) = F h\, n$$

Si $n < 0$

$$F g\, n = g(n) \le h(n) = F h\, n$$

c)

$$F f n = \begin{cases} n & n = 0 \\ f(n-1) & n > 0 \wedge n \le 2 \\ f(n) & n \ge 3 \vee n < 0 \end{cases}$$

$$F^0 \perp n = \perp$$

$$F \perp n = \begin{cases} n & n = 0 \\ \perp(n-1) & n > 0 \wedge n \le 2 \\ \perp(n) & n \ge 3 \end{cases}$$

$$F \perp n = \begin{cases} n & n = 0 \\ \perp & n > 0 \wedge n \le 2 \\ \perp & n \ge 3 \end{cases}$$

$$F^2 \bot n = \begin{cases} n & n=0 \\ (F\bot)(n-1) & n>0 \wedge n\le 2 \\ (F\bot)(n) & n\ge 3 \end{cases}$$

$$F^2 \bot n = \begin{cases} n & n=0 \\ n-1 & n>0 \wedge n\le 2 \wedge n-1=0 \Rightarrow n=1 \\ \bot & n>0 \wedge n\le 2 \wedge n-1>0 \wedge n-1\le 2 \Rightarrow n>1 \wedge n\le 2 \\ \bot & n>0 \wedge n\le 2 \wedge n-1\ge 3 \Rightarrow n=2 \\ n & n\ge 3 \wedge n=0 \quad \times \\ \bot & n\ge 3 \wedge n>0 \wedge n\le 2 \quad \times \\ \bot & n\ge 3 \wedge n\ge 3 \quad \checkmark \end{cases}$$

$$F^2 \bot n = \begin{cases} n & n=0 \\ n-1 & n=1 \\ \bot & n>1 \wedge n\le 2 \\ \bot & n\ge 3 \end{cases}$$

$$F^2 \bot n = \begin{cases} 0 & n=0,1 \\ \bot & (n>1 \wedge n\le 2) \vee n\ge 3 \end{cases}$$

$$F^3 \bot n = \begin{cases} n & n=0 \\ (F^2\bot)(n-1) & n>0 \wedge n\le 2 \\ (F^2\bot)(n) & n\ge 3 \end{cases}$$

$$F^3 \bot n = \begin{cases} n & n=0 \quad \checkmark \\ 0 & n>0 \wedge n\le 2 \wedge n-1=0,1 \Rightarrow n=1,2 \\ \bot & n>0 \wedge n\le 2 \wedge ((n-1>1 \wedge n-1\le 2) \vee n-1\ge 3) \quad \times \\ 0 & n=0,1 \wedge n\ge 3 \quad \times \\ \bot & n\ge 3 \wedge ((n>1 \wedge n\le 2) \vee n\ge 3) \Rightarrow n\ge 3 \end{cases}$$

$$F^3 \bot n = \begin{cases} 0 & n\in\{0,1,2\} \\ \bot & n\ge 3 \end{cases}$$

$$F^4 \bot n = \begin{cases} n & n=0 \\ F^3\bot(n-1) & n>0 \wedge n\le 2 \\ F^3\bot(n) & n\ge 3 \end{cases}$$

$$F^4 \perp n = \begin{cases} n & n=0 \\ 0 & n>0 \wedge n\leq 2 \wedge n-1=0,1,2 \Rightarrow n=1,2 \\ \perp & n>0 \wedge n\leq 2 \wedge n-1\geq 3 \quad \times \\ 0 & n\geq 3 \wedge n=0,1,2 \quad \times \\ \perp & n\geq 3 \wedge n\geq 3 \Rightarrow n\geq 3 \end{cases}$$

$$F^4 \perp n = \begin{cases} n & n=0 \\ 0 & n=1,2 \\ \perp & n\geq 3 \end{cases}$$

$$F^4 \perp n = \begin{cases} 0 & n=0,1,2 \\ \perp & n\geq 3 \end{cases}$$

Claramente como $F^3 \perp n = F^4 \perp n$ $\quad \bigsqcup_{i=0}^{\infty} F^i \perp n = F^3 \perp n$

Falto agregar el condicional $x<0$ pero claramente

$$\bigsqcup_{i=0}^{\infty} F^i \perp n = \begin{cases} 0 & n=0,1,2 \\ \perp & n\geq 3 \vee n<0 \end{cases}$$