TSN1101
Computer Architecture and Organization

Section A (Digital Logic Design)

Lecture A-03

Introduction to Digital Logic and Boolean Algebra

TOPIC COVERAGE IN THE LECTURE (1)…

- Introductory Digital Concepts
  - Analog Vs Digital Quantities
  - Analog Vs Digital Electronic Systems – Examples
  - Advantages and Limitations of Digital Systems
  - Hybrid Systems
  - Logic levels – Positive and Negative Logic
  - Digital Waveforms – Period, Frequency, Duty cycle
  - Timing Diagram
  - Types of Data Transfer – Serial Vs Parallel

TOPIC COVERAGE IN THE LECTURE (2)...

- Logic Gates - NOT, AND, OR, NAND, NOR, XOR, XNOR
  - Standard Logic Symbols
  - Truth Tables
  - Logic expression
  - Logical operation
  - Timing Diagram
  - Application examples
- IC Gates
  - DIP, Pin configurations

TOPIC COVERAGE IN THE LECTURE (3)

- Boolean Algebra
  - Laws and Rules of Boolean Algebra
  - Demorgan’s theorems
- Boolean Analysis of Logic circuits
  - Evaluation of logic circuit output
  - Constructing a truth table for a logic circuit
- Simplification using Boolean Algebra

Introduction to Digital Logic and Boolean Algebra – Part 1 of 3

Introductory Digital Concepts

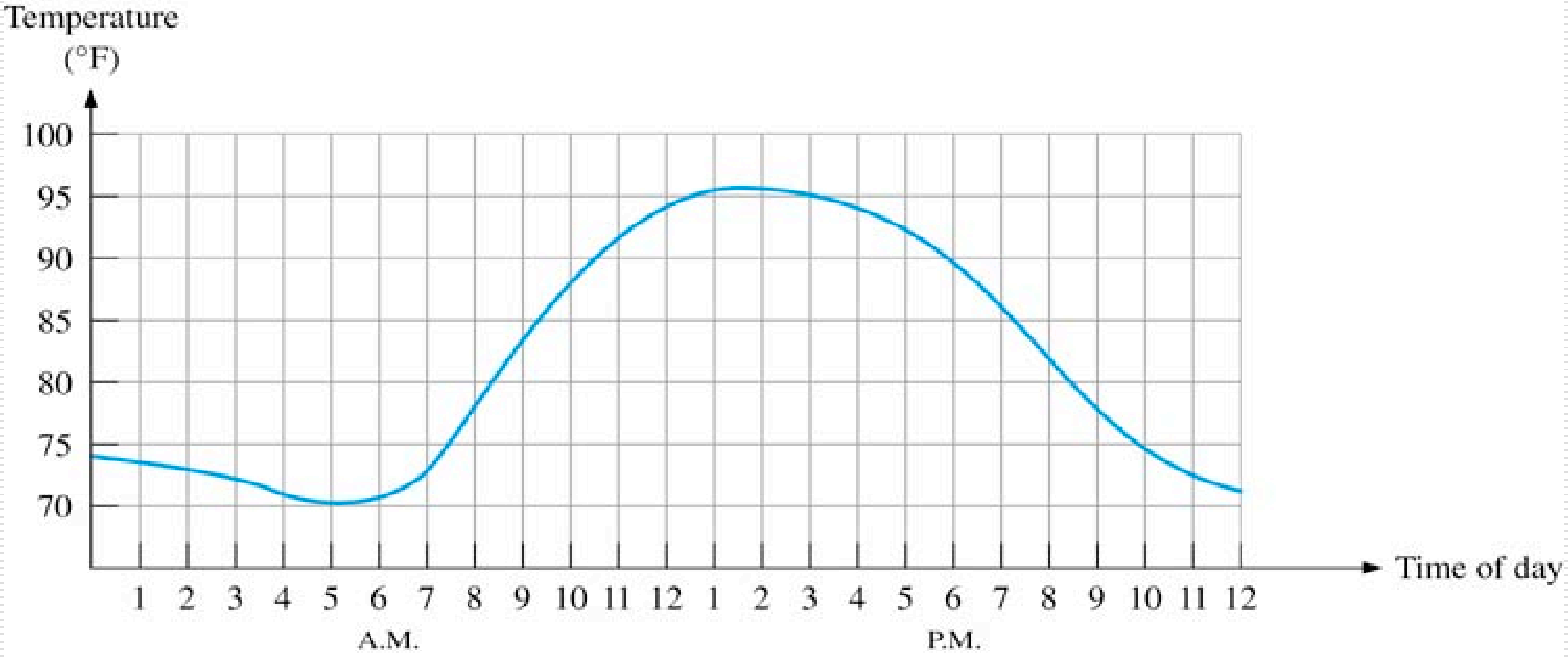
- Digital Vs Analog Systems

TOPIC COVERAGE
- PART 1 of 3

- Analog Vs Digital Quantities
- Analog Vs Digital Electronic Systems – Examples
- Advantages and Limitations of Digital Systems
- Hybrid Systems
- Logic levels – Positive and Negative Logic
- Digital Waveforms – Period, Frequency, Duty cycle
- Timing Diagram
- Types of Data Transfer – Serial Vs Parallel

Analog Quantities

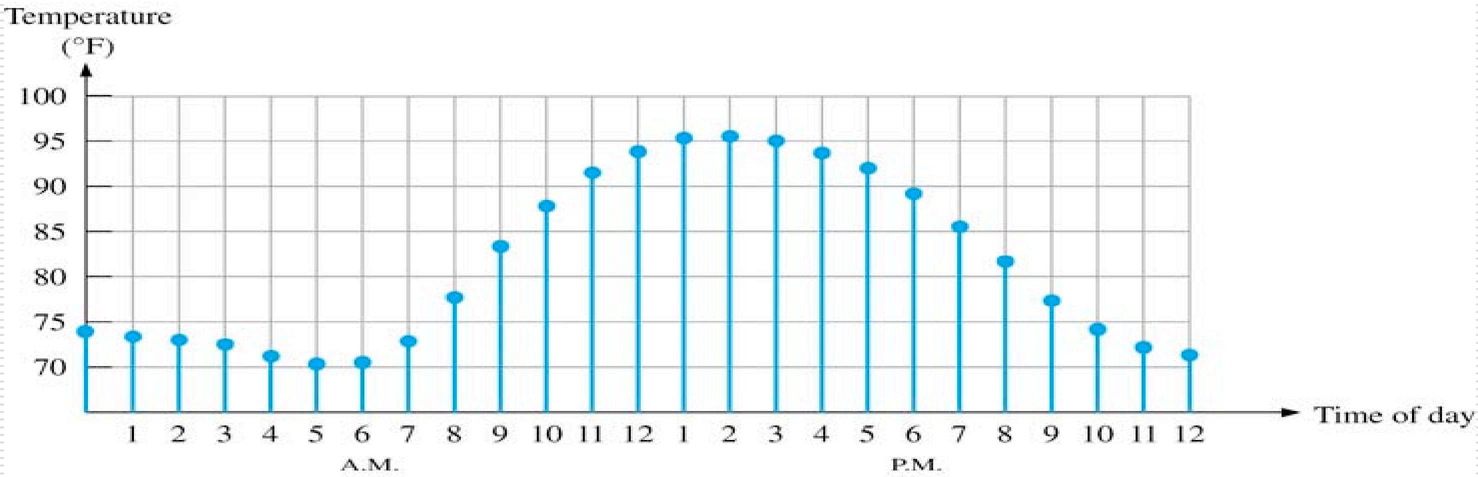
An **analog quantity** is one having continuous values.
Examples: Temperature, Pressure, Level, Position, Volume, Voltage, Current



Graph of an Analog Quantity – Temperature Vs Time

Digital Quantities

A **digital quantity** is one having a discrete set of values.
Examples: Digital Watch reading – (*Time of the day in minutes/seconds)*
Number of coins
Human population of a city (it changes with the time)
People travel from/to the city

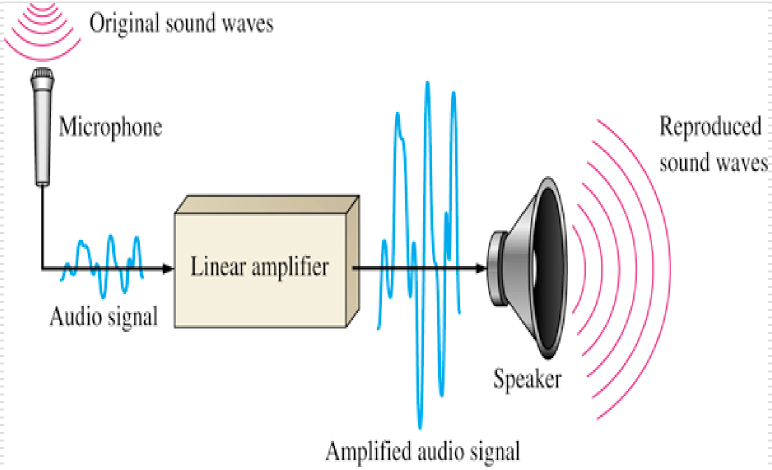


Sampled-value representation of Temperature Vs Time

Analog Electronic System – An Example

- Analog system
  - *A combination of devices that manipulate values represented in an analog form*
    - Here the variable is allowed to take any value in a specified range.
  - An example: A basic audio public address system.

    Sound through a microphone causes voltage changes in proportion to the amplitude of the sound waves.



Analog Electronic System
– More Examples

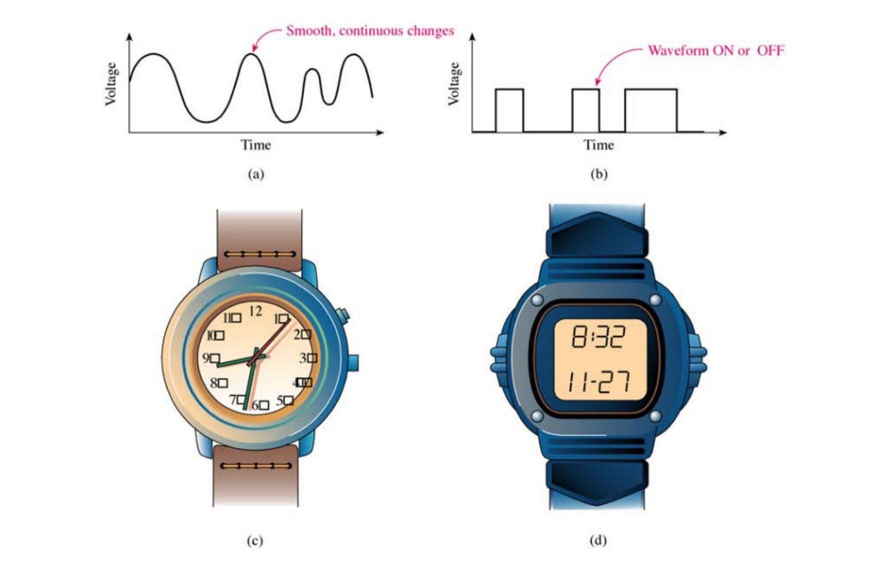
- Automobile speedometer changes with speed
  - It can have any value between zero and say, 100mph
- Mercury thermometer varies over a range of values with temperature.
  - Height of the column of mercury is proportionalto the room temperature.
  - Level of the mercury represents the value of the temperature
- Magnetic Tape recording and playback equipment

Digital System
- Examples

- Digital system
  - *A combination of devices that manipulate values represented in digital form.*

Examples:

- Digital Computer
- Handheld Calculator
- Digital Watch
- Telephone system
- Digital audio and video equipment



Analog Watch and Digital Watch

Advantages of Digital over Analog

- Data Processing and Transmission – more efficient and reliable
- Data Storage – more compact storage and greater accuracy and clarity in reproduction
- Ease of design – In switching circuits, only the range in which the voltage or current fall is important not the exact values
- Accuracy and precision are easier to maintain – In analog systems, voltage and current signals are affected by temperature, humidity but in digital systems, info. does not degrade
- Easy Programmable operation
- Less affected by noise - since exact value is not important in digital systems
- Ease of fabrication on IC chips – analog devices cannot be economically integrated.

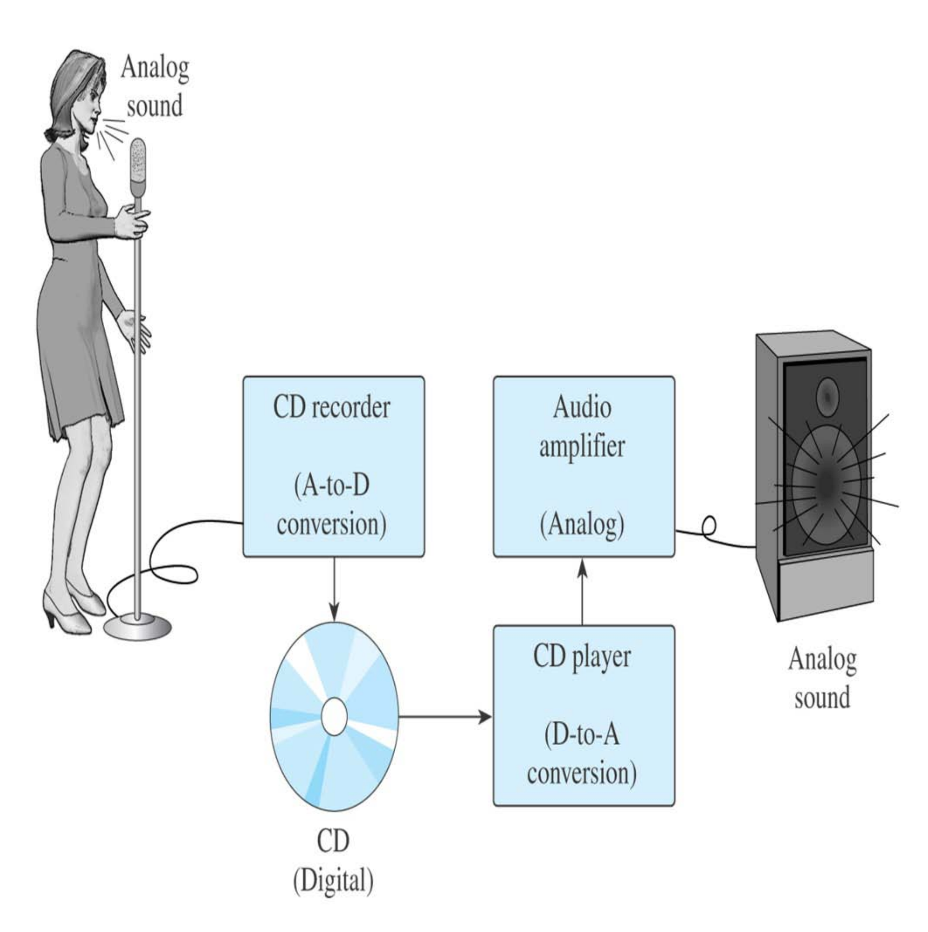
Limitations of Digital Techniques

- The real world is analog
- The analog nature of the world requires a time consuming conversion process:
  - Convert analog inputs to digital
  - Process (operate on) the digital information
  - Convert the digital output back to analog

Digital and Analog electronics together - Examples

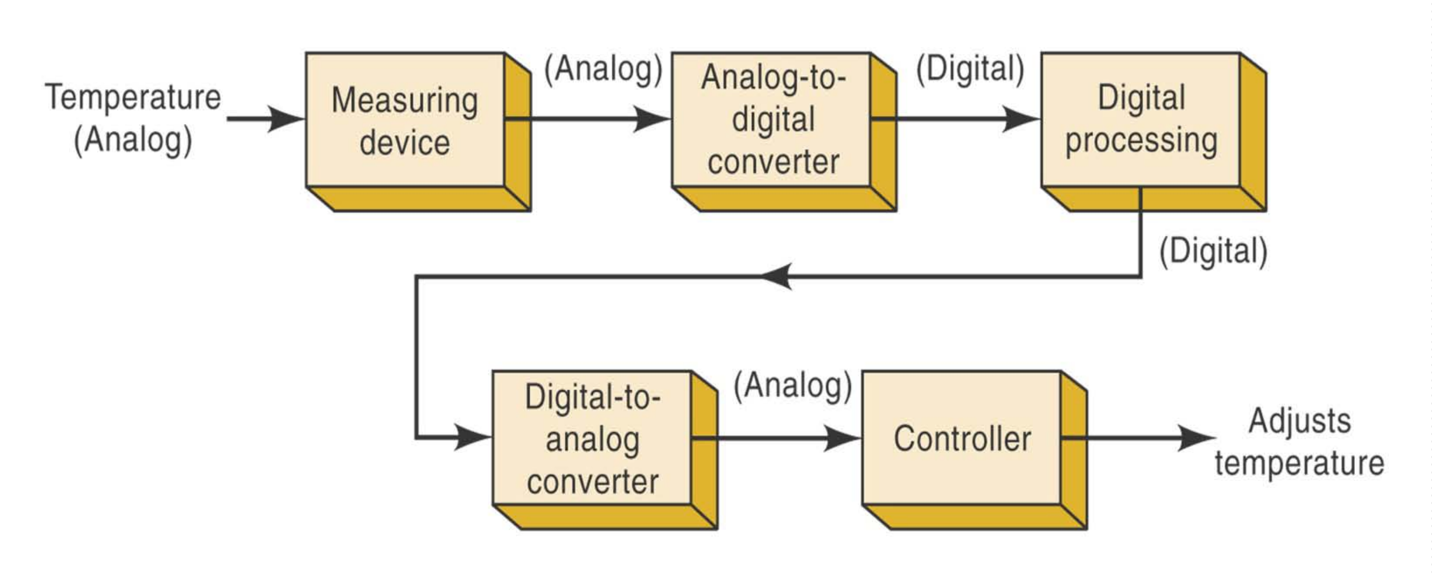
Example 1: The audio CD is a typical hybrid (combination) system.

- Analog sound is converted into analog voltage.
- Analog voltage is changed into digital through an ADC in the recorder.
- Digital information is stored on the CD .
- At playback the digital information is changed into analog by a DAC in the CD player.
- The analog voltage is amplified and used to drive a speaker that produces the original analog sound.



Digital and Analog electronics together -Examples

Example 2: Temperature Control System



Binary Digits and Logic Levels

- Positive Logic

HIGH = 1 Low = 0

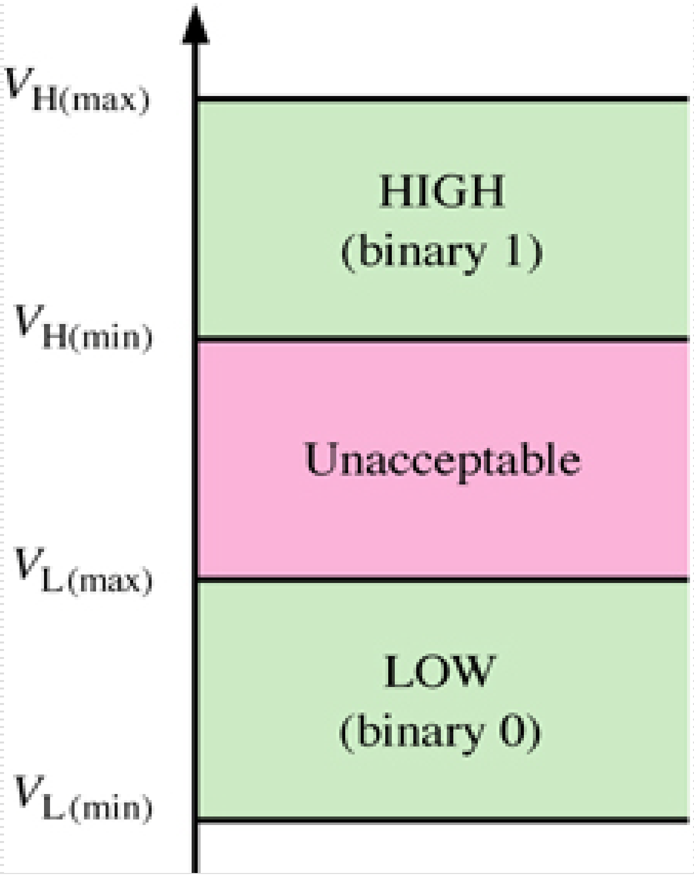
Logic Levels - The voltages used to represent a 1 and a 0

Ex: For TTL , HIGH=2V to 5 V

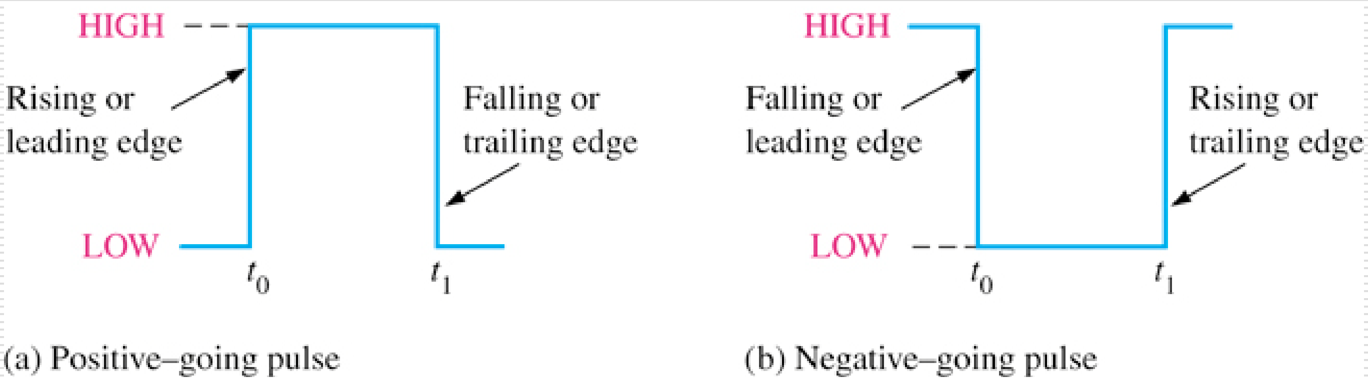
LOW=0 V to 0.8 V

- Negative logic

High =0 Low =1



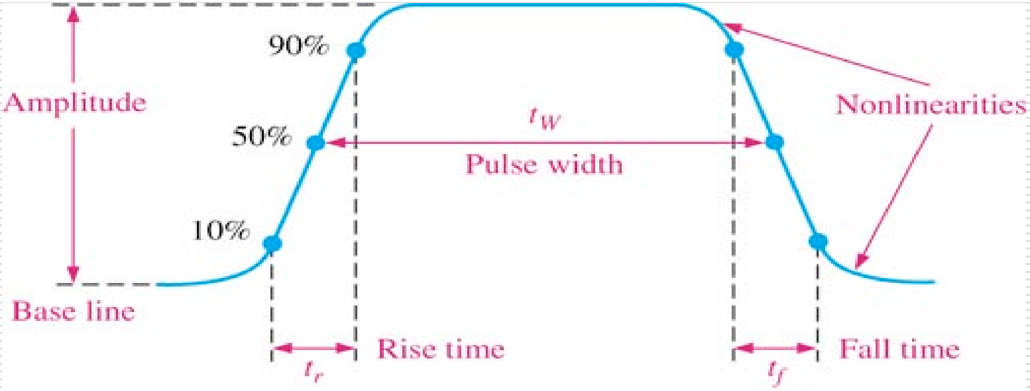
Digital Waveforms – Ideal Pulse



IDEAL PULSES

- Digital Waveforms is made up of a series of pulses
- Digital Waveforms consist of voltage levels that are changing back and forth between HIGH and LOW states.

Digital Waveforms
– Non-ideal pulse



NONIDEAL PULSE CHARACTERISTICS

Rise Time – measured from 10% of the pulse amplitude to 90% of the pulse amplitude
Fall Time - measured from 90% of the pulse amplitude to 10% of the pulse amplitude
Pulse Width – Time interval between the 50% points on the rising and falling edges

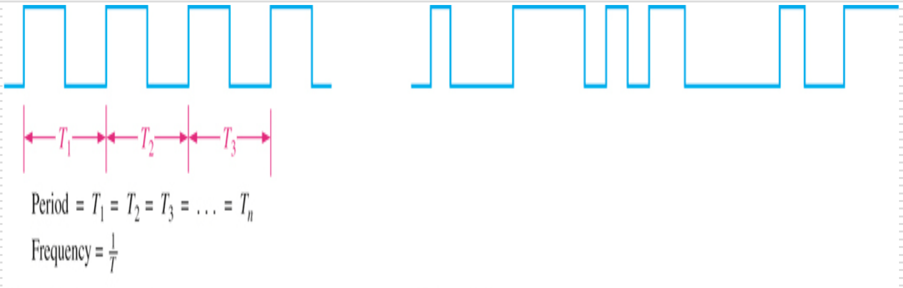
Digital Waveforms-Characteristics - Periodic Vs Non-periodic

•Periodic pulse waveform

One that repeats itself at a fixed interval, called a period

• Non-periodic pulse waveform

Composed of pulses of randomly differing time interval between pulses (pulse width)



(a) Periodic (square wave)
(b) Nonperiodic

Periodic Digital Waveforms-Characteristics - Period Vs Frequency

- Frequency (f) is the rate at which it repeats itself - measured in cycles per second or Hertz (Hz)
- Period (T) is the time required for a periodic waveform to repeat itself
  - measured in seconds
- Relationship between frequency and period

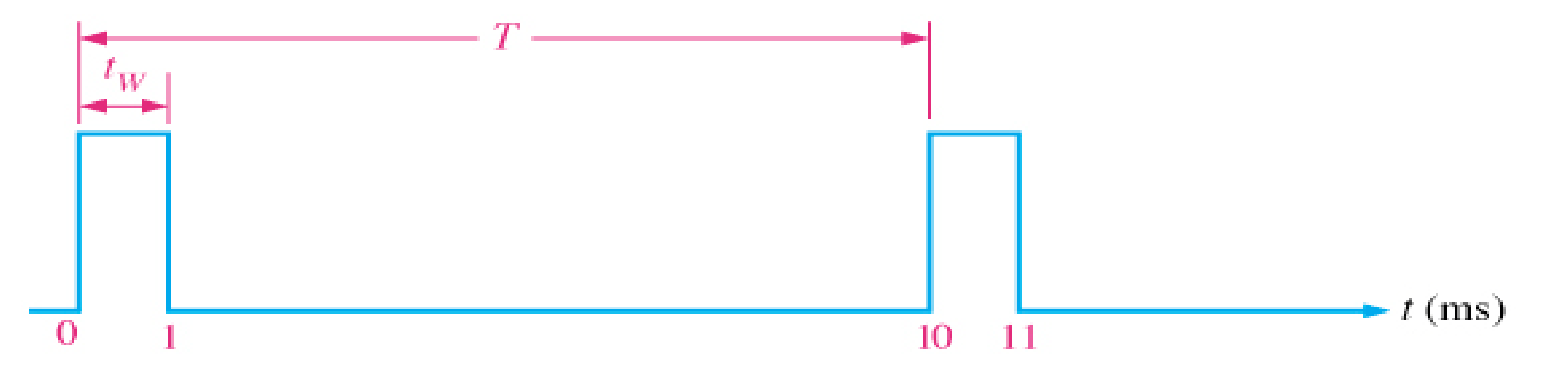
$$f = 1/T$$

$$T = 1/f$$

Periodic Digital Waveforms-Characteristics - Duty Cycle

Duty cycle is the ratio of the pulse width to the period and expressed as a percentage

$$\text{Duty cycle} = (t_w / T) \times 100\%$$

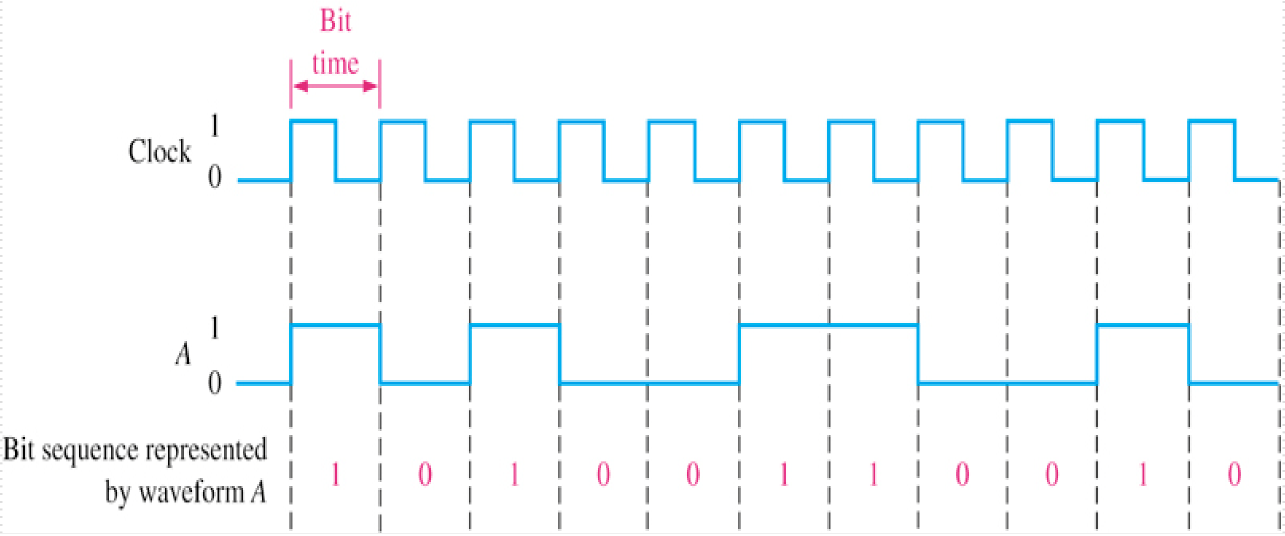


Example: For the above periodic waveform, determine the following

a. Period b. Frequency c. duty cycle

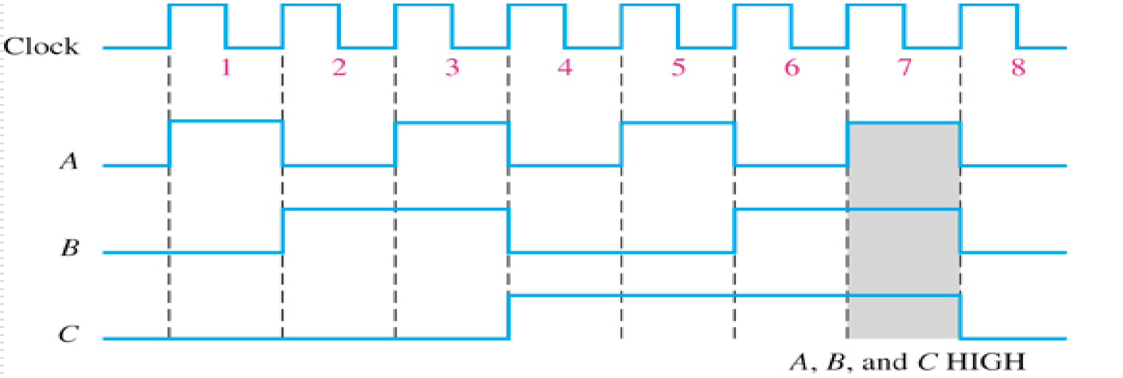
Solution: Period = 10 ms, Frequency = 100 Hz , Duty cycle = 10%

Representation of Bit Sequence



- The clock is a periodic waveform in which each interval between pulses (period) equals the time for one bit (bit time)
- Here waveform A level change occurs at the leading edge of the clock waveform

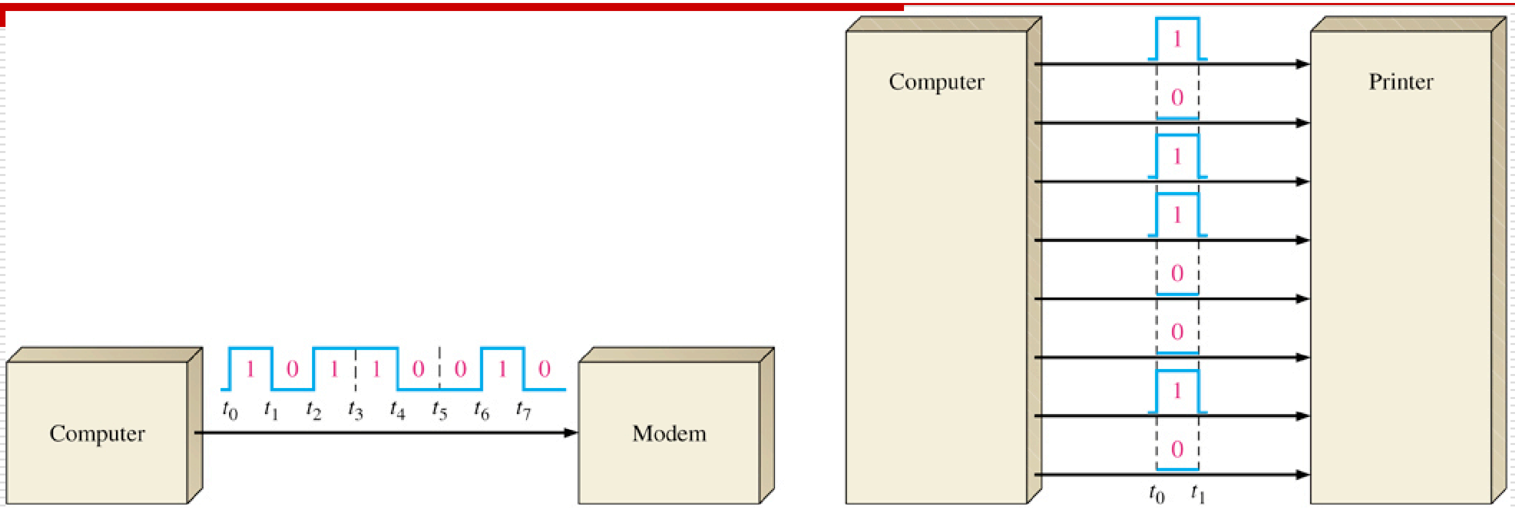
Timing Diagrams



- a graph of digital waveforms showing the actual relationship of two or more waveforms and how each waveform changes in relation to the others
- show voltage versus time.
- Horizontal scale represents regular intervals of time beginning at time zero.
- used to show how digital signals change with time.
- The oscilloscope and logic analyzer are used to produce timing diagrams.

Here waveforms A, B, and C are HIGH only during bit time 7

Binary Data Transfer
- Two types

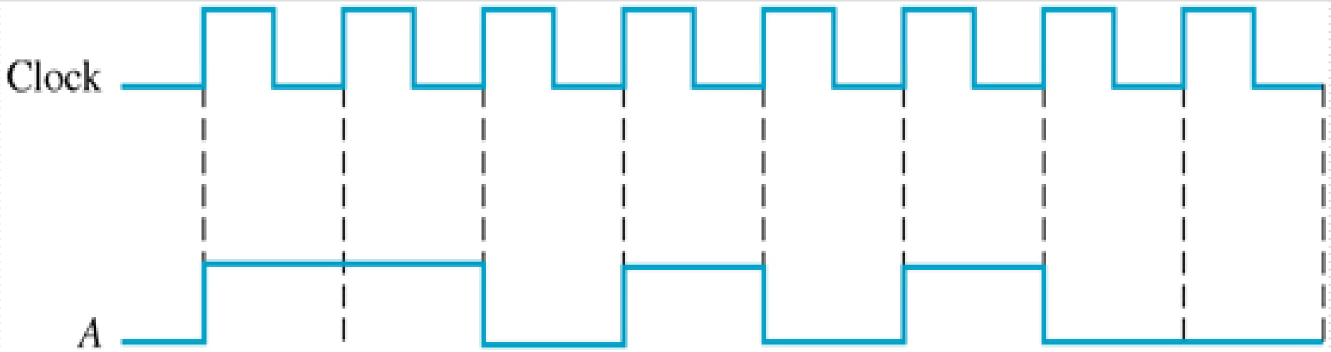


(a) Serial transfer of 8 bits of binary data from computer to modem. Interval t_0 to t_1 is first.

(b) Parallel transfer of 8 bits of binary data from computer to printer. The beginning time is t_0.

- Serial Transfer - Sent one bit at a time along a single line
  Advantage: only one line is required
  Disadvantage: It takes longer to transfer a given number of bits
- Parallel Transfer - all the bits in a group are sent out on separate lines at the same time
  Advantage: Speed of transfer – more
  Disadvantage: More lines are required

Binary Data Transfer
- Example



Problem:

Determine the total time required to serially transfer the eight bits contained in waveform A and indicate the sequence of bits. The 100kHz is used as reference. What is the total time to transfer the same eight bits in parallel

Solution:

Period T=1/f = 10 microseconds

Total time required for serial transfer = 8 T = 80 microseconds

Bit Sequence = 11010100

Total time required for parallel transfer = 1T =10 microseconds

Problems

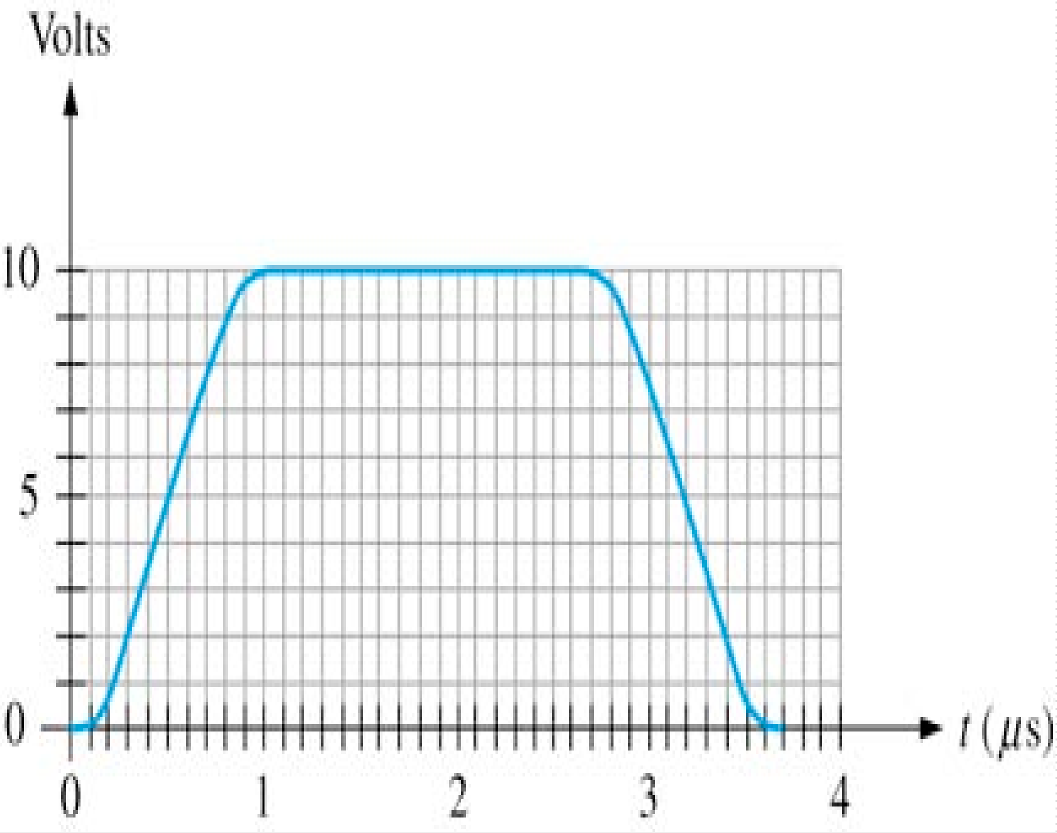
Problem:

Determine the following:

a. Rise time b. fall time

c. Pulse width

d. amplitude



Solution:

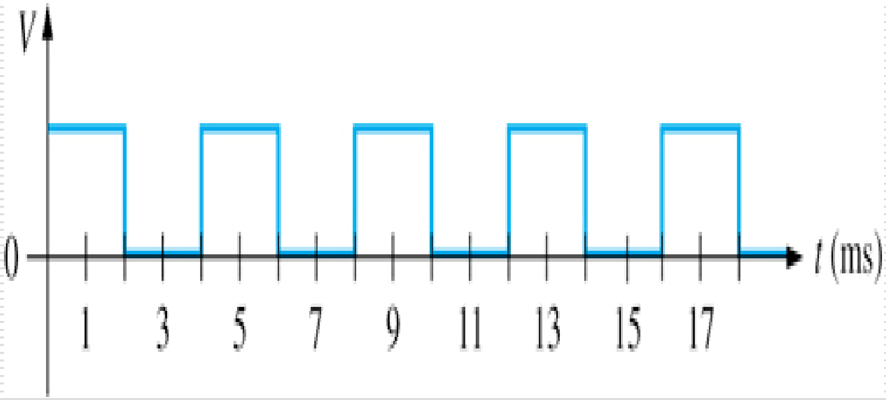
a. Rise Time = 550 ns
b. Fall time = 600 ns
c. Pulse Width = 2.7 microseconds
d. Amplitude = 10 V

Problems

Problem:

Determine the

a. Period
b. Frequency
c. Duty cycle
d. Determine the waveform is periodic or non-periodic



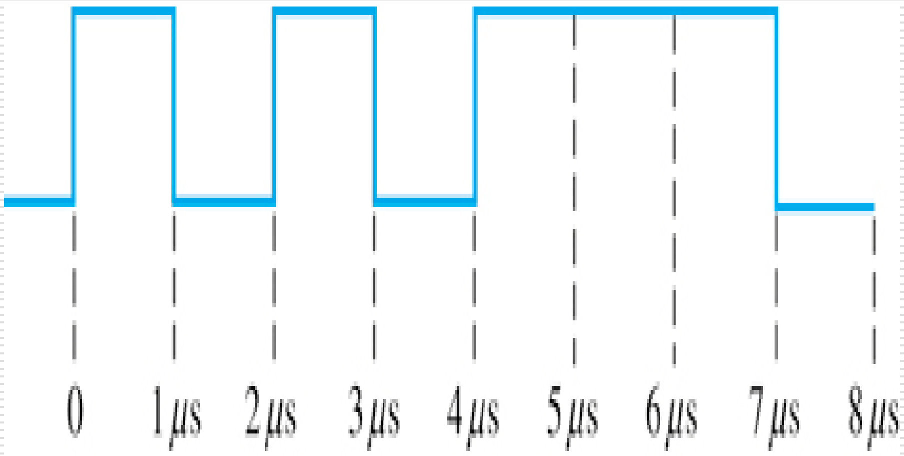
Solution:

a. Period = 4 ms
b. Frequency = 250 Hz
c. Duty cycle=(2ms/4ms)×100 %
   = 50 %
d. The waveform is periodic since it repeats at a fixed interval

Problems

Problem:

a. Determine the bit sequence
b. Determine the total serial transfer time for the eight bits
c. Determine the total parallel transfer time



Solution:

a. 10101110
b. Each bit time = 1 microsecond
   Serial transfer time = (8 bits) × 1 microsecond/bit = 8 microseconds
   Parallel transfer time = 1 bit time =1 microsecond

Introduction to Digital Logic and Boolean Algebra – Part 2 of 3

Logic Gates

- NOT, AND, OR, NAND, NOR, XOR, XNOR

TOPIC COVERAGE
- PART 2 of 3

- Logic Gates - NOT, AND, OR, NAND, NOR, XOR, XNOR
  - Standard Logic Symbols
  - Truth Tables
  - Logic expression
  - Logical operation
  - Timing Diagram
  - Application examples
- IC Gates
  - DIP, Pin configurations

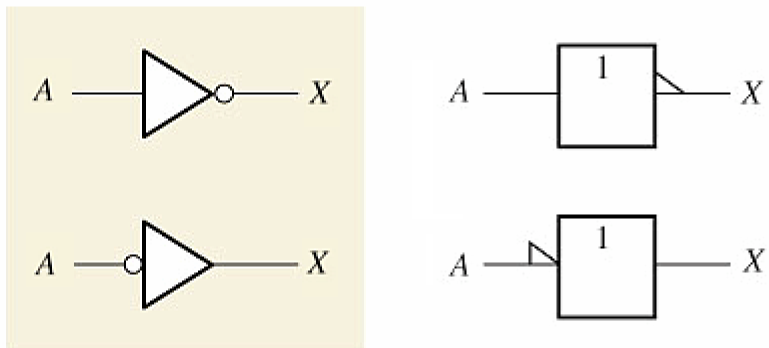
Logic Gates - Introduction

- Logic gates are the building blocks of computers.
- Most of the functions in a computer are implemented with logic gates used on a very large scale.
- For example, a Microprocessor, which is the main part of the computer, is made of up of hundreds of thousands of logic gates

The Inverter (NOT gate)

The Inverter (NOT gate)

- Symbol, Truth Table, Boolean Expression, Logical Operation, Timing Diagram



Distinctive shape symbols　　　　Rectangular outline symbols

Basic Logical Function:

NOT gate can have only one input and performs **logical inversion or complementation**.

Logical operation:

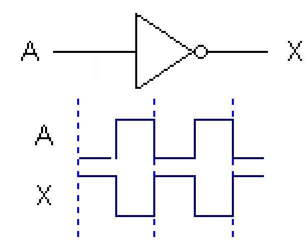
The output of an inverter is always the complement (opposite) of the input.

A	X
0	1
1	0

$X = \overline{A}$

Truth table　　**Boolean expression**

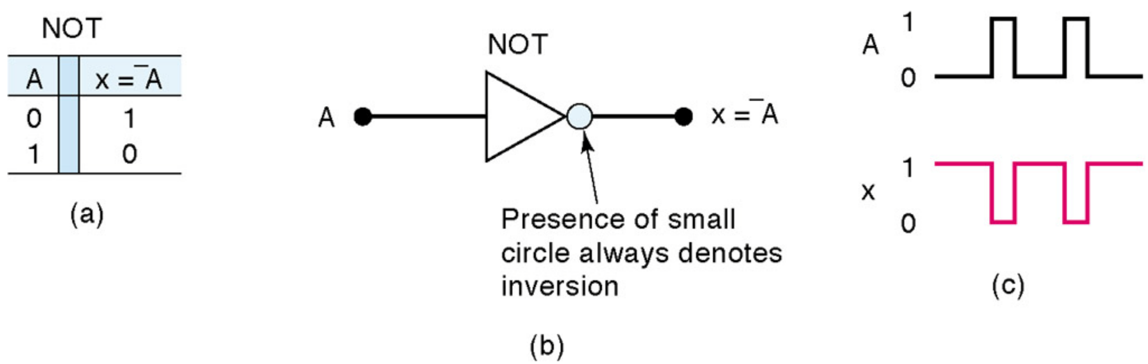
0 = LOW
1 = HIGH



Pulsed waveforms / Timing Diagram

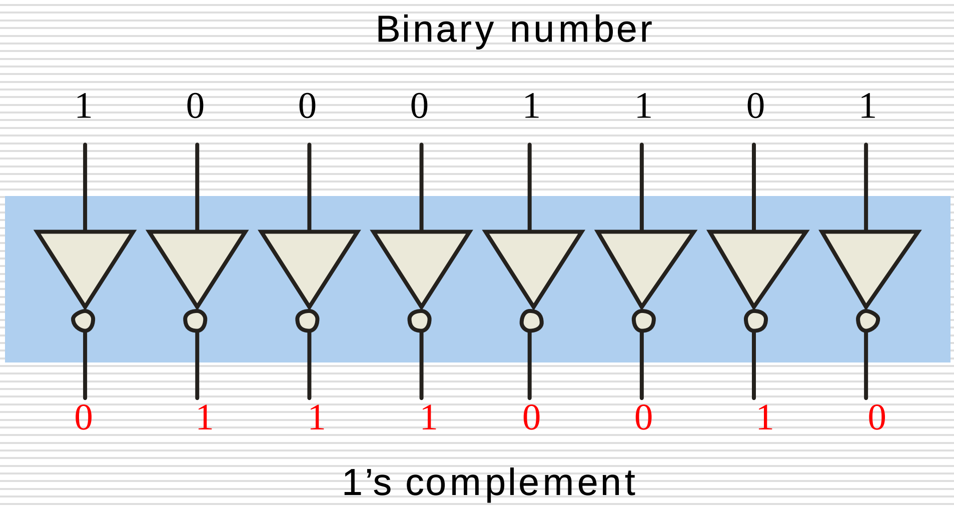
The Inverter (NOT gate)
- Problem

The output of the INVERTER is connected to the input of a second INVERTER. Determine the output level of the second INVERTER for each level of input A



The Inverter
- An example application

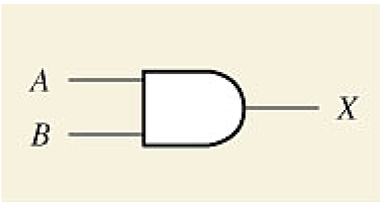
A group of inverters can be used to form the 1’s complement of a binary number



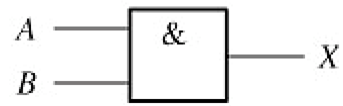
The AND Gate

The AND Gate

- Symbol, Truth Table, Boolean Expression, Logical operation, Timing Diagram



Distinctive shape symbol



Rectangular outline symbol

Basic Logical Function:

An AND gate can have two or more inputs and performs **logical multiplication**.

Logical Operation:

The output of an AND gate is HIGH only when all inputs are HIGH.

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

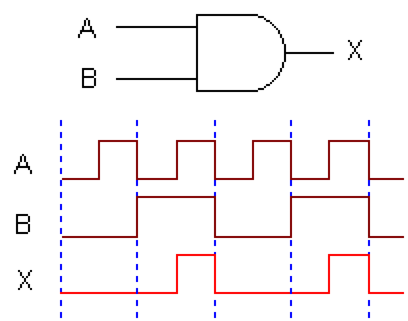
Truth table

(2-input AND gate)

0 = LOW
1 = HIGH

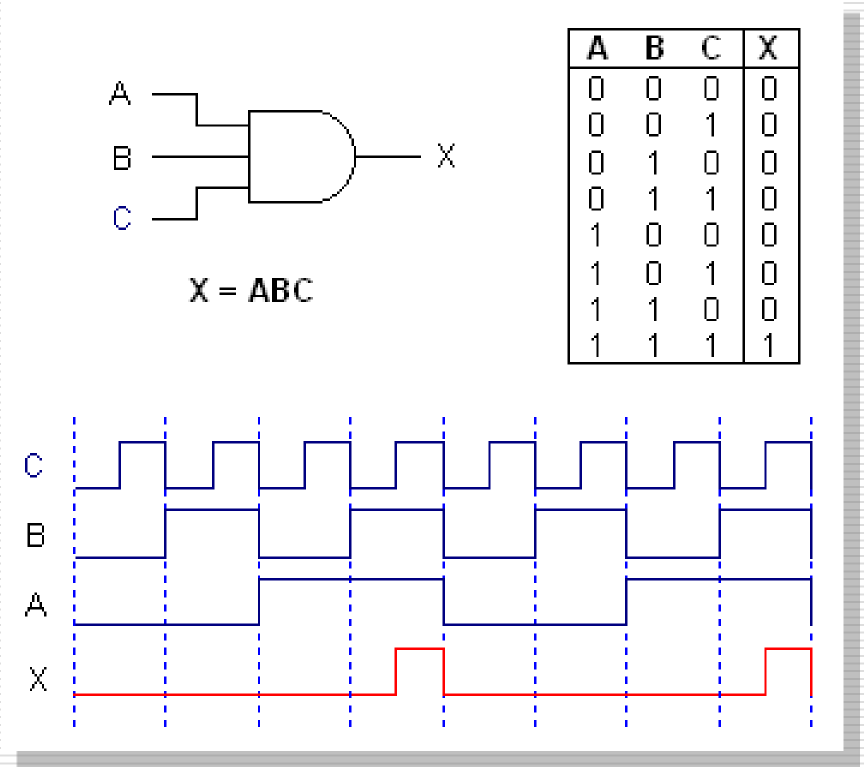
$X = AB$

Boolean expression



Timing Diagram

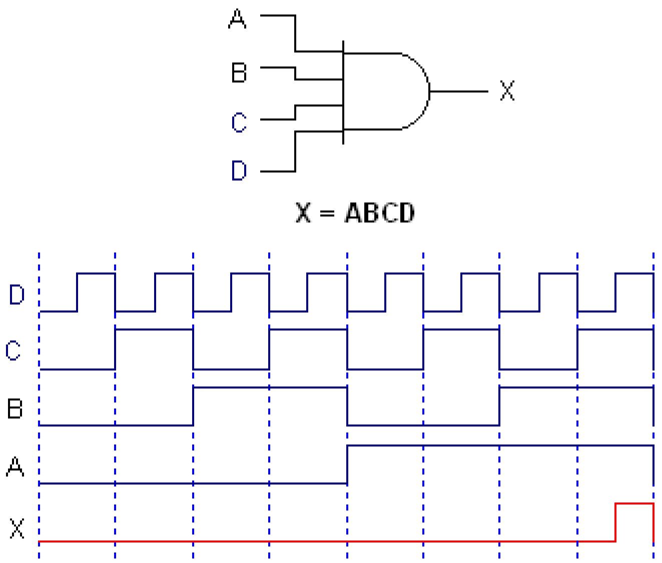
The AND gate (3-inputs)



A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

3-Input AND Gate

The AND gate (4-inputs)



A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

4-Input AND Gate

- The total number of possible combinations of binary inputs to a gate is determined by the following formula:

$$N = 2^n$$

where N is the number of possible input combinations and n is the number of input variables.

For two input variables:

$N = 2^2 = 4$ combinations

For three input variables:

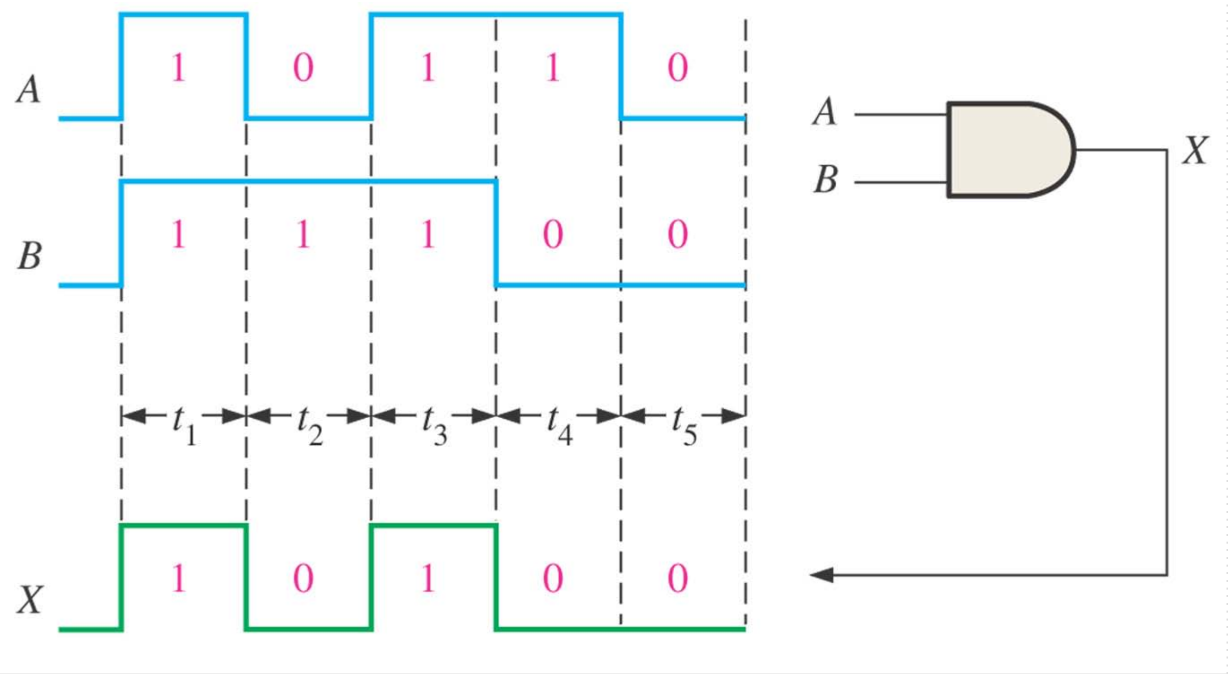
$N = 2^3 = 8$ combinations

For four input variables:

$N = 2^4 = 16$ combinations

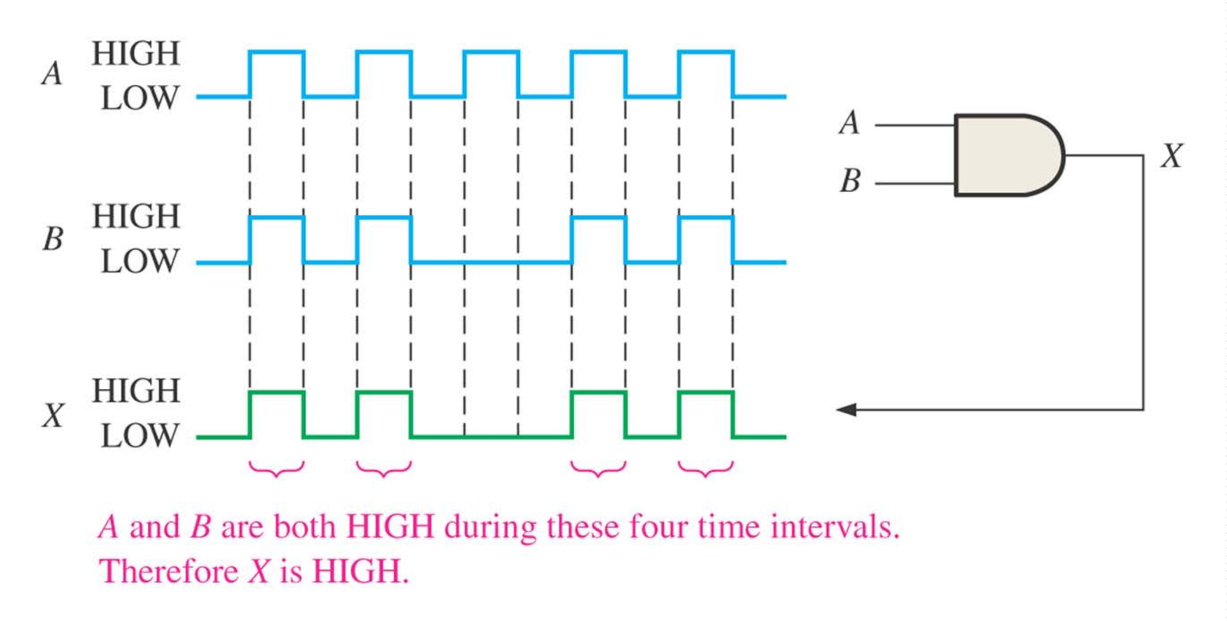
The AND gate
- Problem 1

If two waveforms, A and B are applied to AND gate inputs as shown in figure below, what is the resulting output waveform?



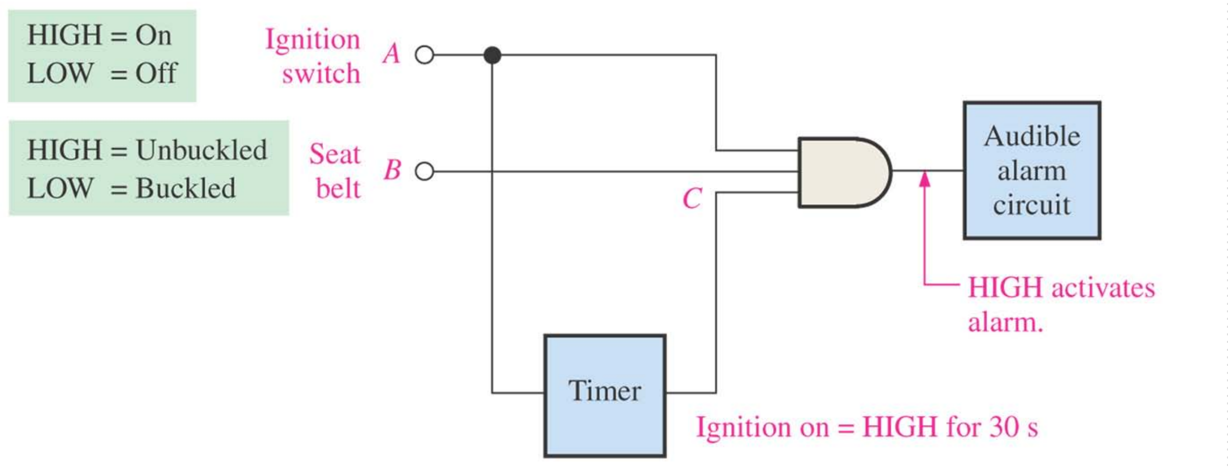
The AND gate
- Problem 2

If two waveforms, A and B are applied to AND gate inputs as shown in figure below, what is the resulting output waveform?



The AND gate
- An example application

- A common application of the AND gate is to enable (to allow) the passage of a signal (pulse waveform) from one point to another at certain times and to inhibit (prevent) the passage at other times.
- Example : A seat belt Alarm system



An AND gate is used to detect when the ignition switch is ON and the seat belt is unbuckled.
If the ignition switch is on, a HIGH is produced on input A of the AND gate.
If the seat belt is not properly buckled, a HIGH is produced on input B.
Also, when the ignition switch is turned on, a timer is started that produces a HIGH on input C for 30s.
If all three conditions exist- that is, if the ignition is on and the seat belt is unbuckled and timer is running- the output of AND gate is HIGH and audible alarm is energized to remind the driver.

The AND gate
- An example application

- The AND operation is used in computer programming as a selective mask.
- If you want to retain certain bits of a binary number but reset the other bits to 0, you could set a mask with 1’s in the position of the retained bits.

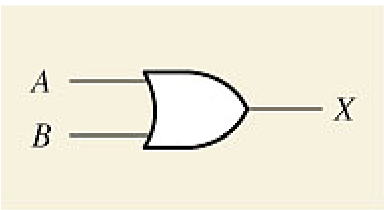
Example: If the binary number 10100011 is ANDed with the mask 00001111, what is the result?

Ans: 00000011

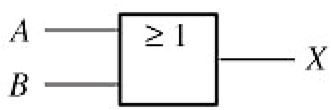
The OR Gate

The OR Gate

- Symbol, Truth Table, Boolean Expression, Logical operation, Timing Diagram



Distinctive shape symbol



Rectangular outline symbol

Basic Logical Function:
The OR gate can have two or more inputs and performs **logical addition**.

Logical Operation:

The output of an OR gate is LOW only when all inputs are LOW.

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

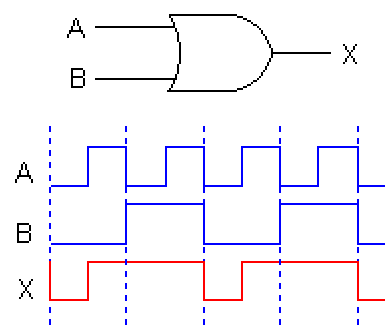
Truth table

(2-input OR gate)

0 = LOW
1 = HIGH

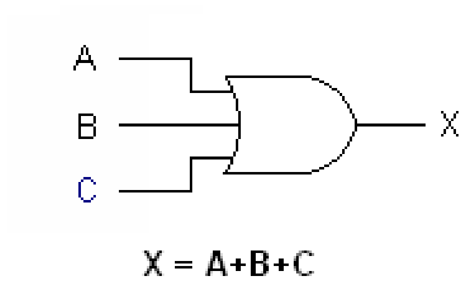
$X = A + B$

Boolean expression

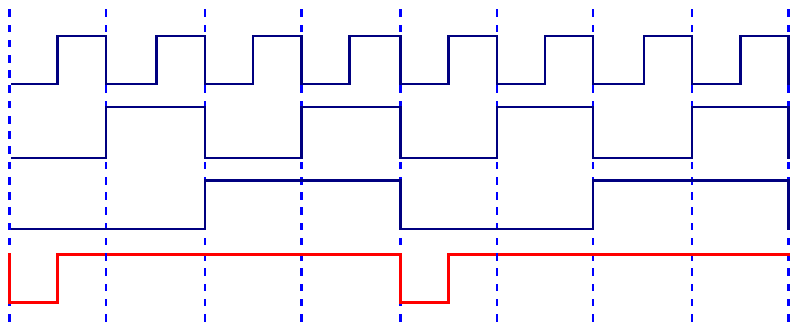


Timing Diagram

The OR gate (3-inputs)



A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



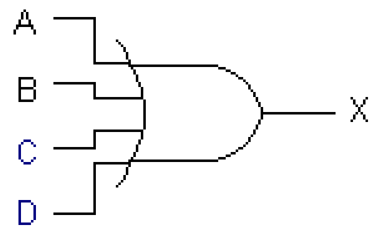
3-Input OR Gate

Note:

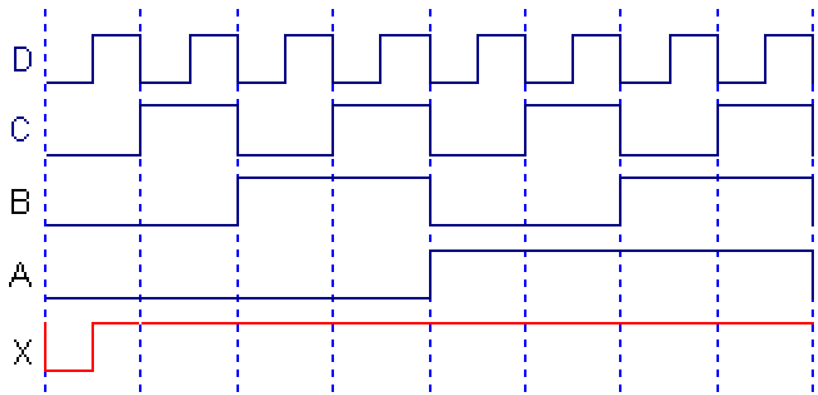
Boolean addition differs from binary addition in the cases where two or more 1s are added.

There is no carry in Boolean addition.

The OR gate (4-inputs)



$$X = A+B+C+D$$

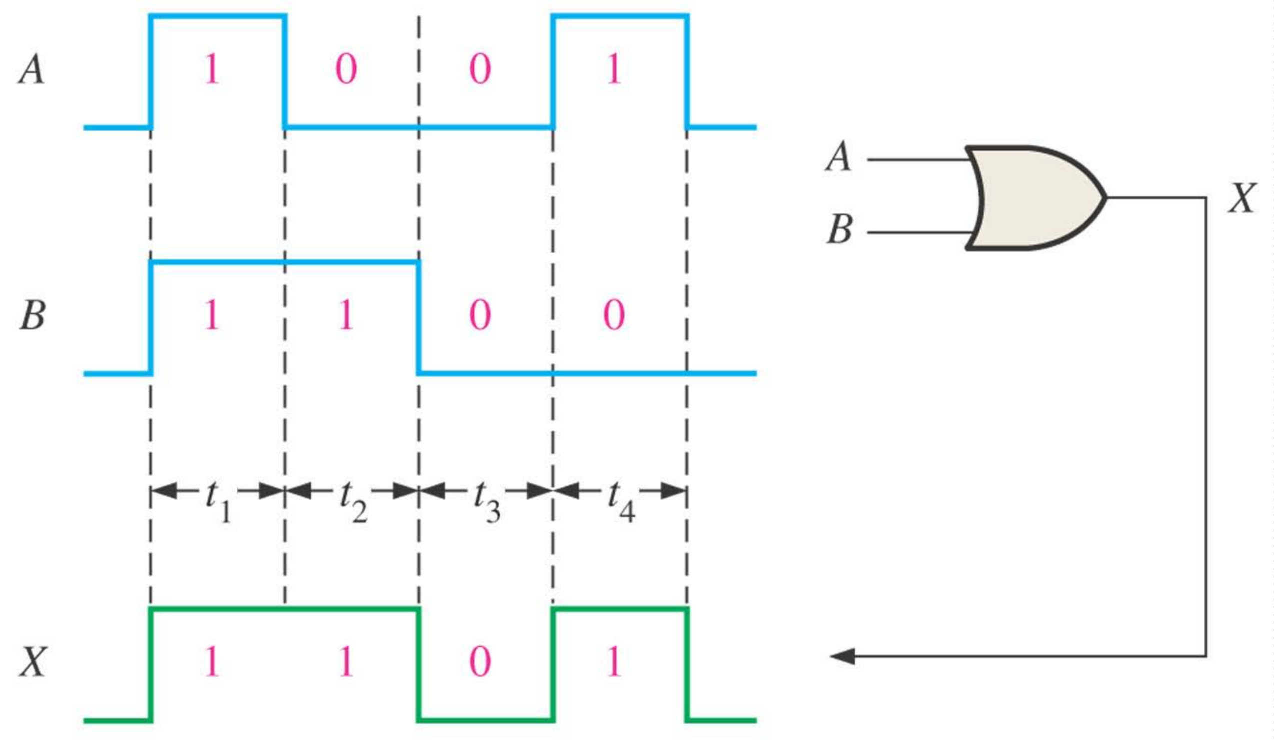


A	B	C	D	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

4-Input OR Gate

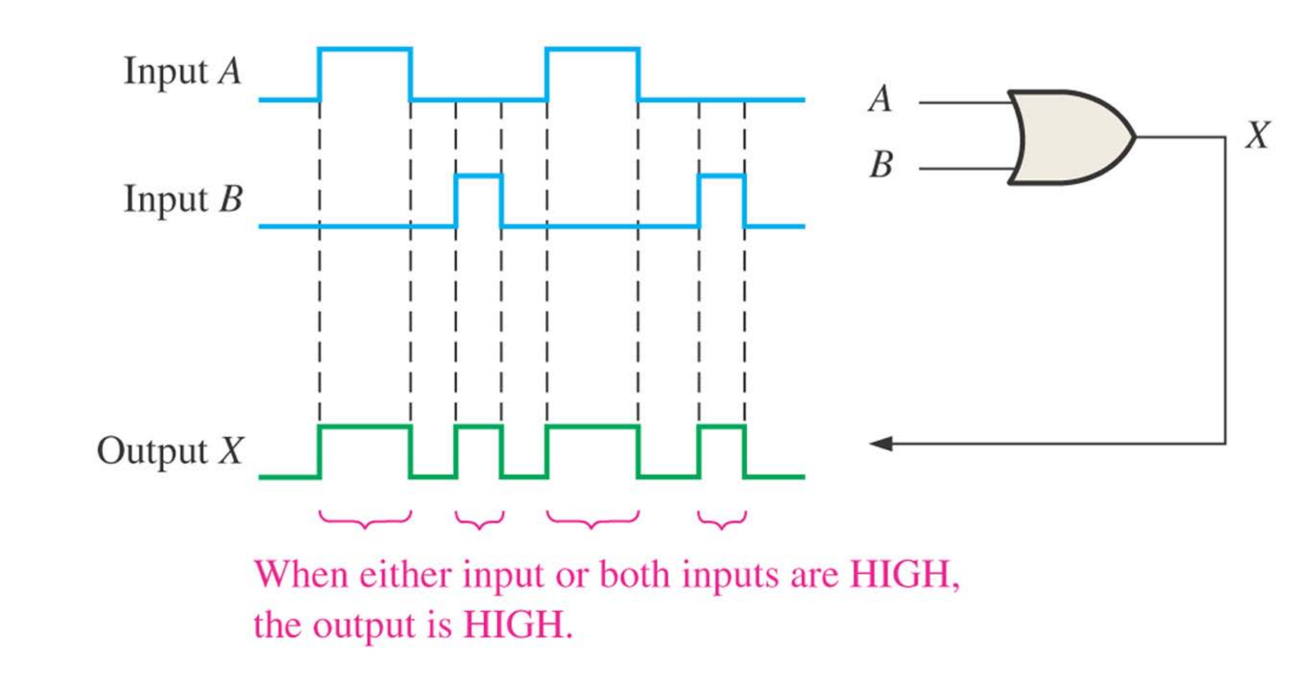
The OR gate
- Problem 1

If two waveforms, A and B are applied to OR gate inputs as shown in figure below, what is the resulting output waveform?



The OR gate
- Problem 2

If two waveforms, A and B are applied to OR gate inputs as shown in figure below, what is the resulting output waveform?



The OR gate
- An example application

- The OR operation can be used in computer programming to set certain bits of a binary number to 1.

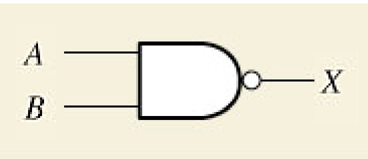
Example: What will be the result if you OR an ASCII upper case letter with the binary number 0100000?

Ans: The resulting letter will be ASCII lower case.

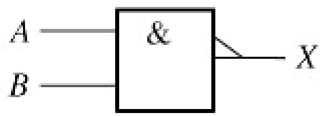
The NAND Gate

The NAND Gate

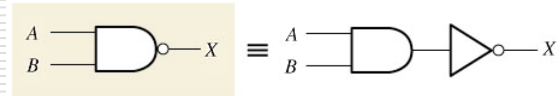
- Symbol, Truth Table, Boolean Expression, Logical operation, Timing Diagram



Distinctive shape symbol



Rectangular outline symbol



Basic Logical Function:

The NAND gate can have two or more inputs and performs **inverse** of **logical multiplication**.

Logical Operation:

The output of a NAND gate is LOW only when all inputs are HIGH

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

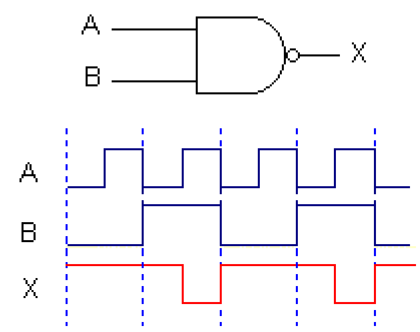
Truth table

(2-input NAND Gate)

0 = LOW
1 = HIGH

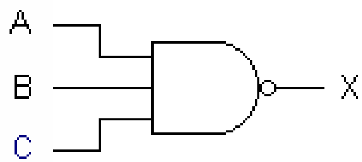
$$X = \overline{AB}$$

Boolean expression



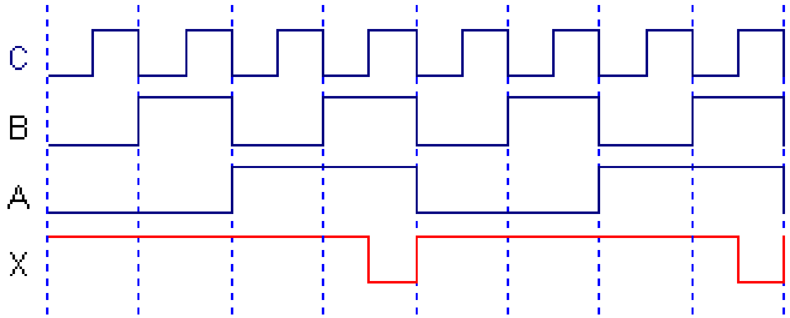
Timing Diagram

The NAND gate (3-inputs)



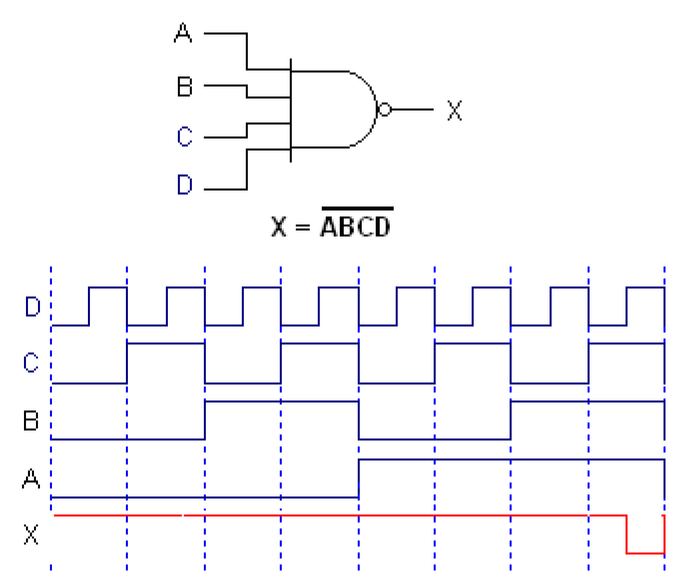
$X = \overline{ABC}$

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



3-Input NAND Gate

The NAND gate (4-inputs)

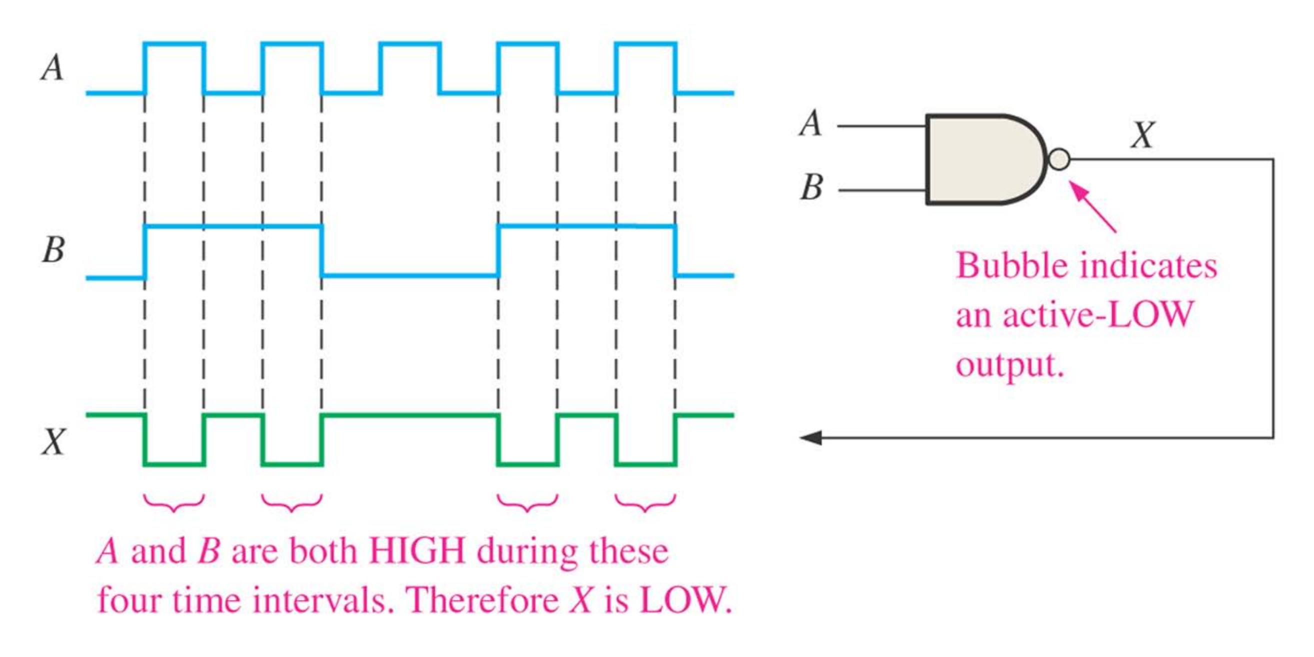


A	B	C	D	X
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

4-Input NAND Gate

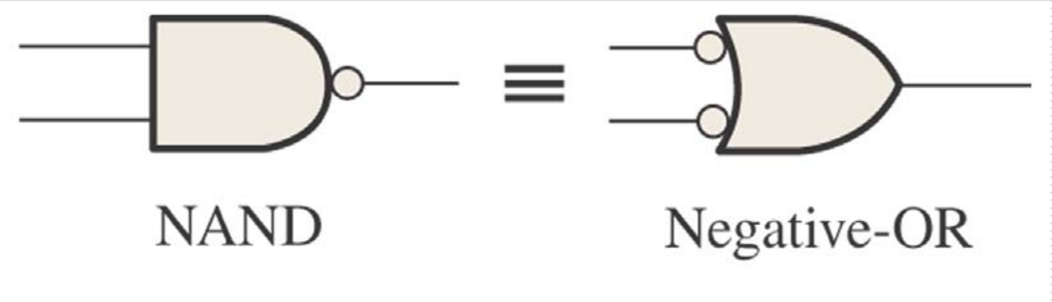
The NAND gate
- Problem

If two waveforms, A and B are applied to NAND gate inputs as shown in figure below, what is the resulting output waveform?



The NAND Gate
- Negative-OR Equivalent operation

- A NAND gate produces HIGH output, when one or more inputs are LOW.
- From this view point, a NAND gate can be used for an OR operation that requires one or more LOW inputs to produce a HIGH output. This aspect of NAND operation is referred to as negative-OR

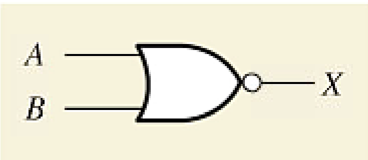


Standard symbols representing the two equivalent operations of a NAND gate

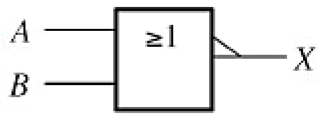
The NOR Gate

The NOR Gate

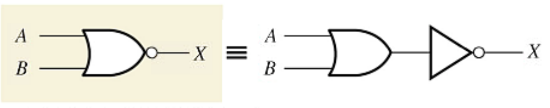
- Symbol, Truth Table, Boolean Expression, Logical operation, Timing Diagram



Distinctive shape symbol



Rectangular outline symbol



Basic Logical Function:

The NOR gate can have two or more inputs and performs **inverse** of **logical addition**.

Logical Operation:

The output of a NOR gate is HIGH only when all inputs are LOW

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

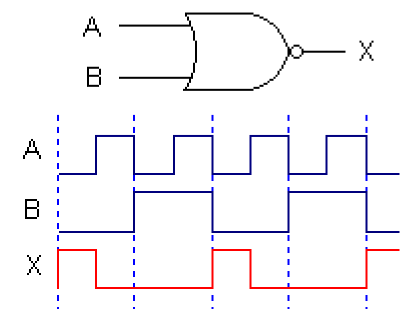
$$X = \overline{A + B}$$

Boolean expression

Truth table

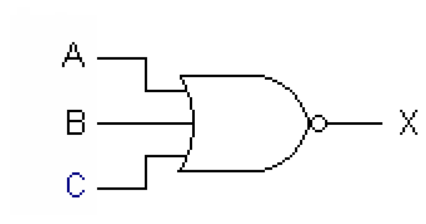
(2-input NOR Gate)

0 = LOW
1 = HIGH



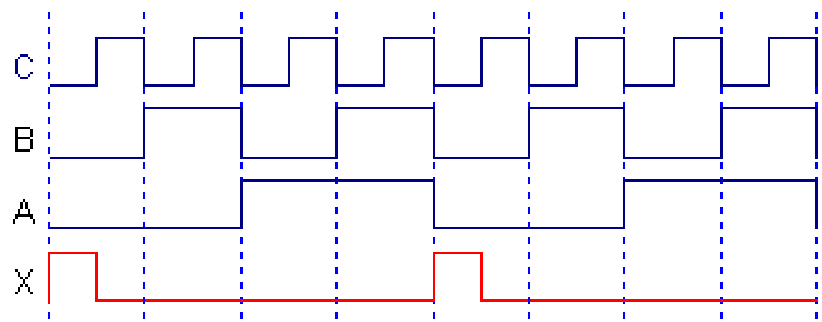
Timing Diagram

The NOR gate (3-inputs)



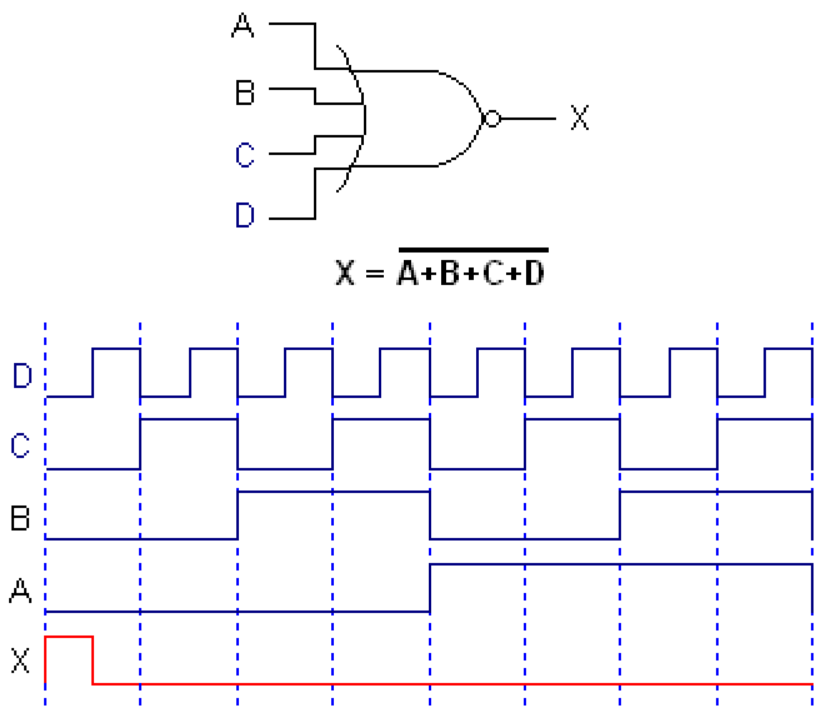
$$X = \overline{A+B+C}$$

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0



3-Input NOR Gate

The NOR gate (4-inputs)



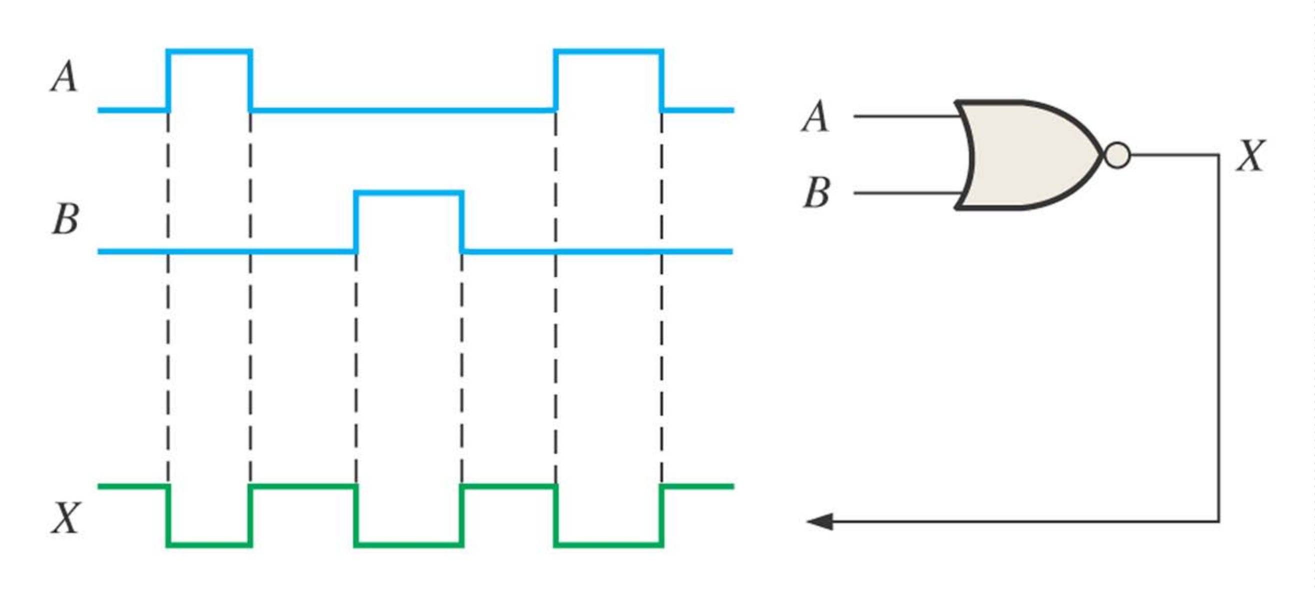
$$X = \overline{A+B+C+D}$$

A	B	C	D	X
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

4-Input NOR Gate

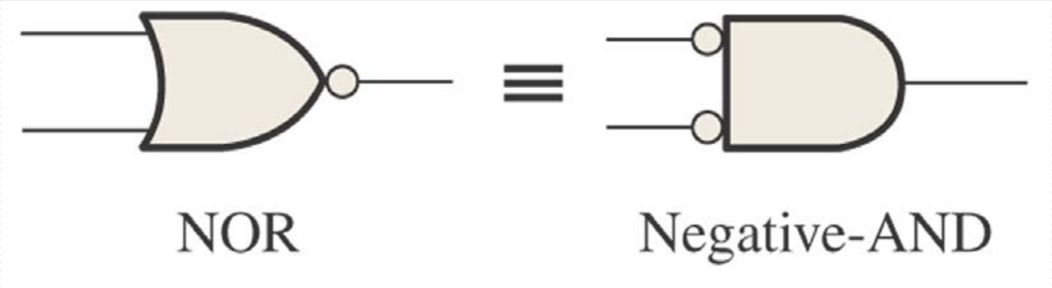
The NOR gate
- Problem

If two waveforms, A and B are applied to NOR gate inputs as shown in figure below, what is the resulting output waveform?



The NOR Gate
- Negative-AND Equivalent operation

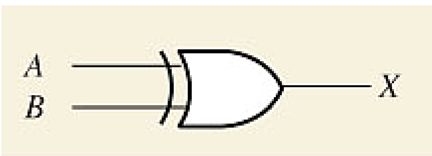
- A NOR gate produces HIGH output, when all inputs are LOW.
- From this view point, a NOR gate can be used for an AND operation that requires all LOW inputs to produce a HIGH output. This aspect of NOR operation is referred to as negative-AND



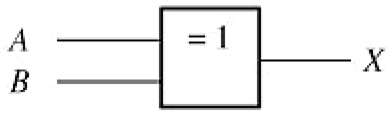
Standard symbols representing the two equivalent operations of a NOR gate

The XOR Gate

The XOR (Exclusive-OR) Gate
- Symbol, Truth Table, Boolean Expression, Logical operation, Timing Diagram



Distinctive shape symbol



Rectangular outline symbol

Basic Logical Function:

The XOR gate can have two or more inputs and performs **ODD function** (output is equal to 1 if the input variables have an odd number of 1's).

Logical Operation:

The output of XOR gate is HIGH whenever the two inputs are different (2-input XOR gate)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

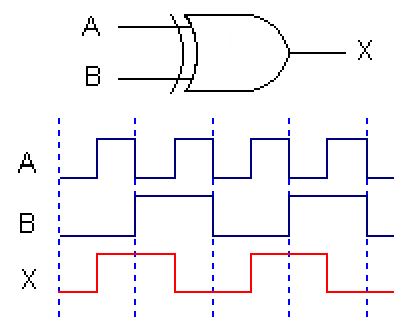
Truth table

(2-input XOR gate)

0 = LOW
1 = HIGH

$$X = A \oplus B$$

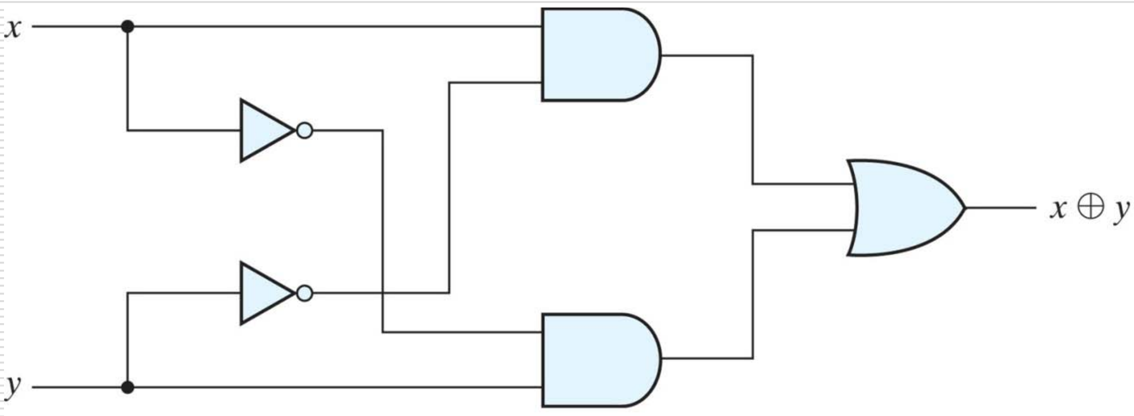
Boolean expression



Timing Diagram

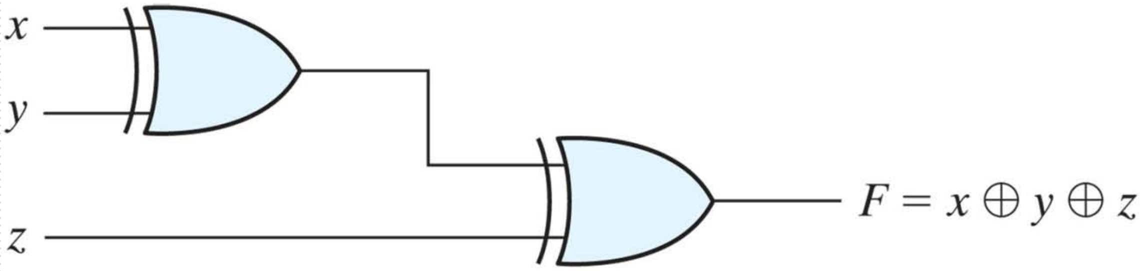
The XOR gate (2-inputs)
- using AND-OR-NOT Gates

$$x \oplus y = \bar{x}\, y + x\, \bar{y}$$

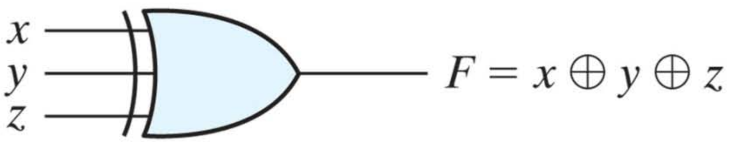


(a) Exclusive-OR with AND–OR–NOT gates

The XOR gate (3-inputs)



(a) Using 2-input gates



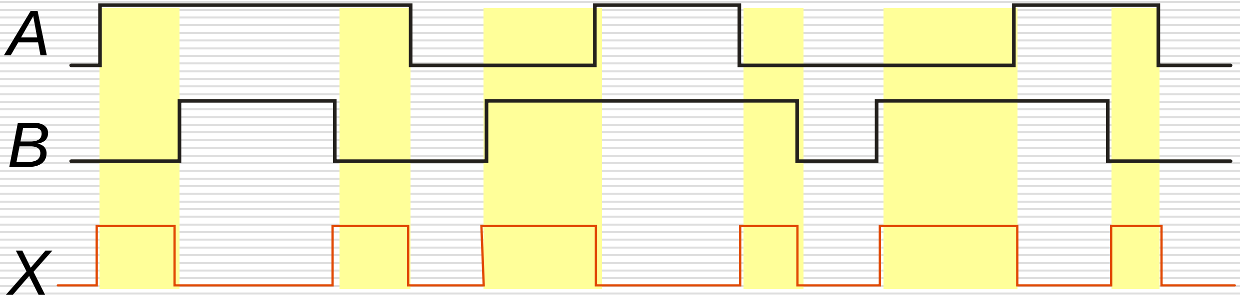
(b) 3-input gate

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(c) Truth table

The XOR gate
- Problem

If two waveforms, A and B are applied to XOR gate inputs as shown in figure below, what is the resulting output waveform?

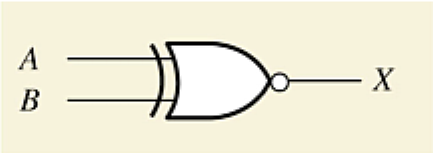


If the *A* and *B* waveforms are both inverted for the above waveforms, how is the output affected?

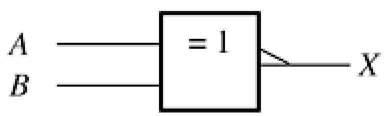
Ans: There is no change in the output.

The XNOR Gate

The XNOR (Exclusive-NOR) Gate
- Symbol, Truth Table, Boolean Expression, Logical operation, Timing Diagram



Distinctive shape symbol



Rectangular outline symbol

Basic Logical Function:
The XNOR gate can have two or more inputs and performs **EVEN function** (output is equal to 1 if the input variables have an even number of 1's).

Logical Operation:

The output of XNOR gate is HIGH whenever the two inputs are identical
(2-input XNOR gate - can be called as equivalence gate)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

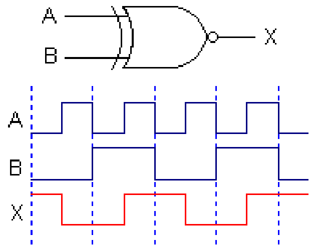
Truth table

(2-input XNOR Gate)

0 = LOW
1 = HIGH

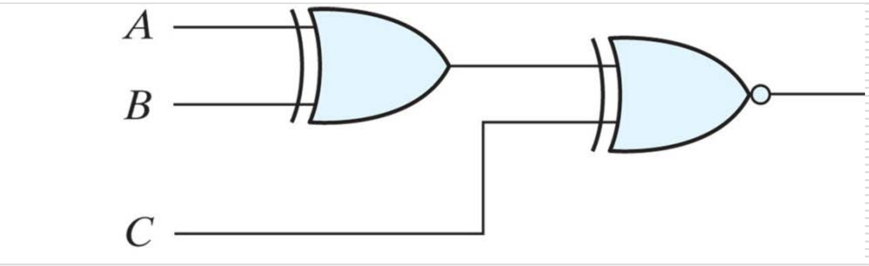
$$X = \overline{A \oplus B}$$

Boolean expression



Timing Diagram

The XNOR gate (3-inputs)



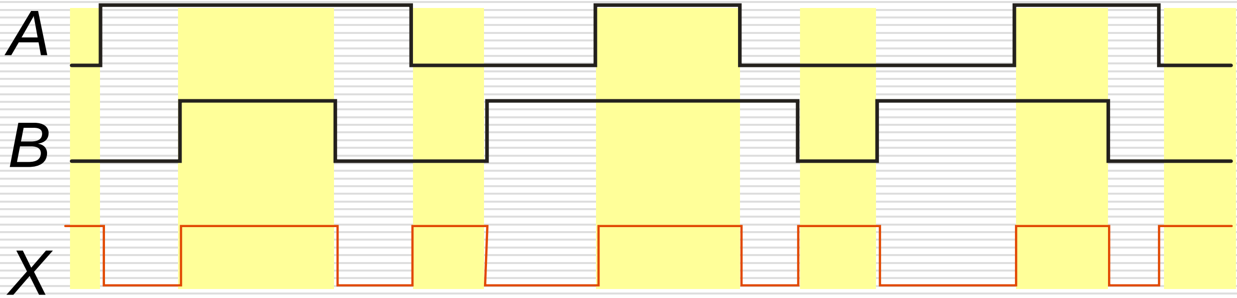
3-input even function

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Truth Table

The XNOR gate
- Problem

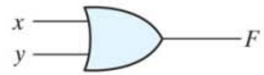
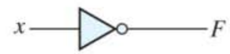
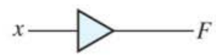
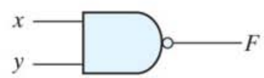
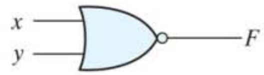
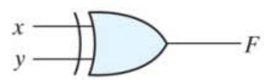
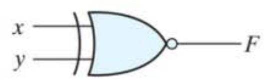
If two waveforms, A and B are applied to XNOR gate inputs as shown in figure below, what is the resulting output waveform?



If the *A* waveform is inverted but *B* remains the same, how is the output affected?

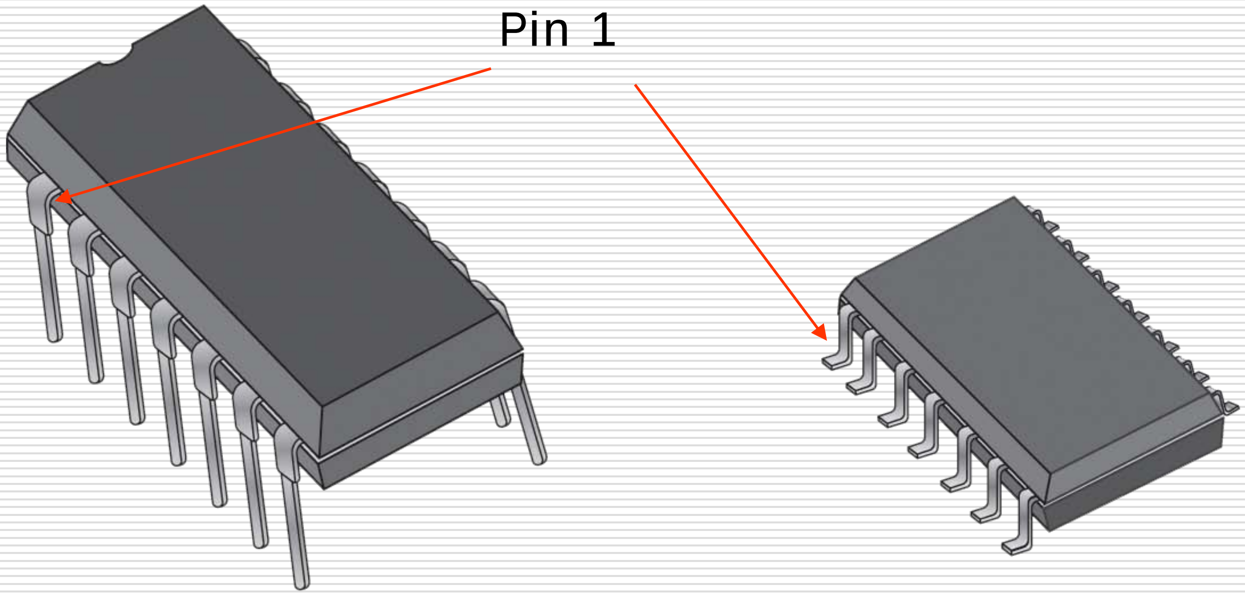
Ans: The output will be inverted

Basic Logic Gates (Summary)

Name	Graphic symbol	Algebraic function	Truth table
AND		$F = x \cdot y$	x y | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = x + y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
Inverter		$F = x'$	x | F 0 | 1 1 | 0
Buffer		$F = x$	x | F 0 | 0 1 | 1
NAND		$F = (xy)'$	x y | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = (x + y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1

Integrated Circuit (IC) Packages

DIP and surface mount chips

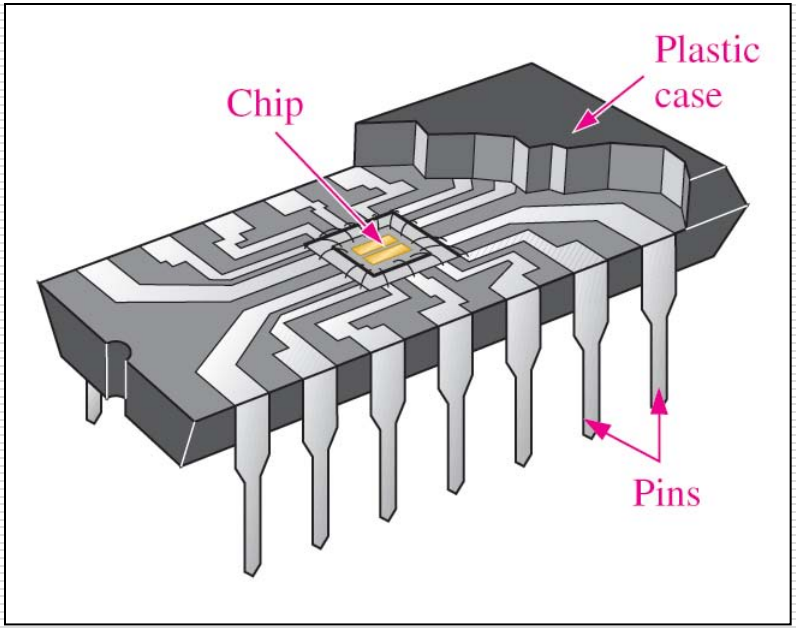


Dual in-line package

Small outline IC (SOIC)

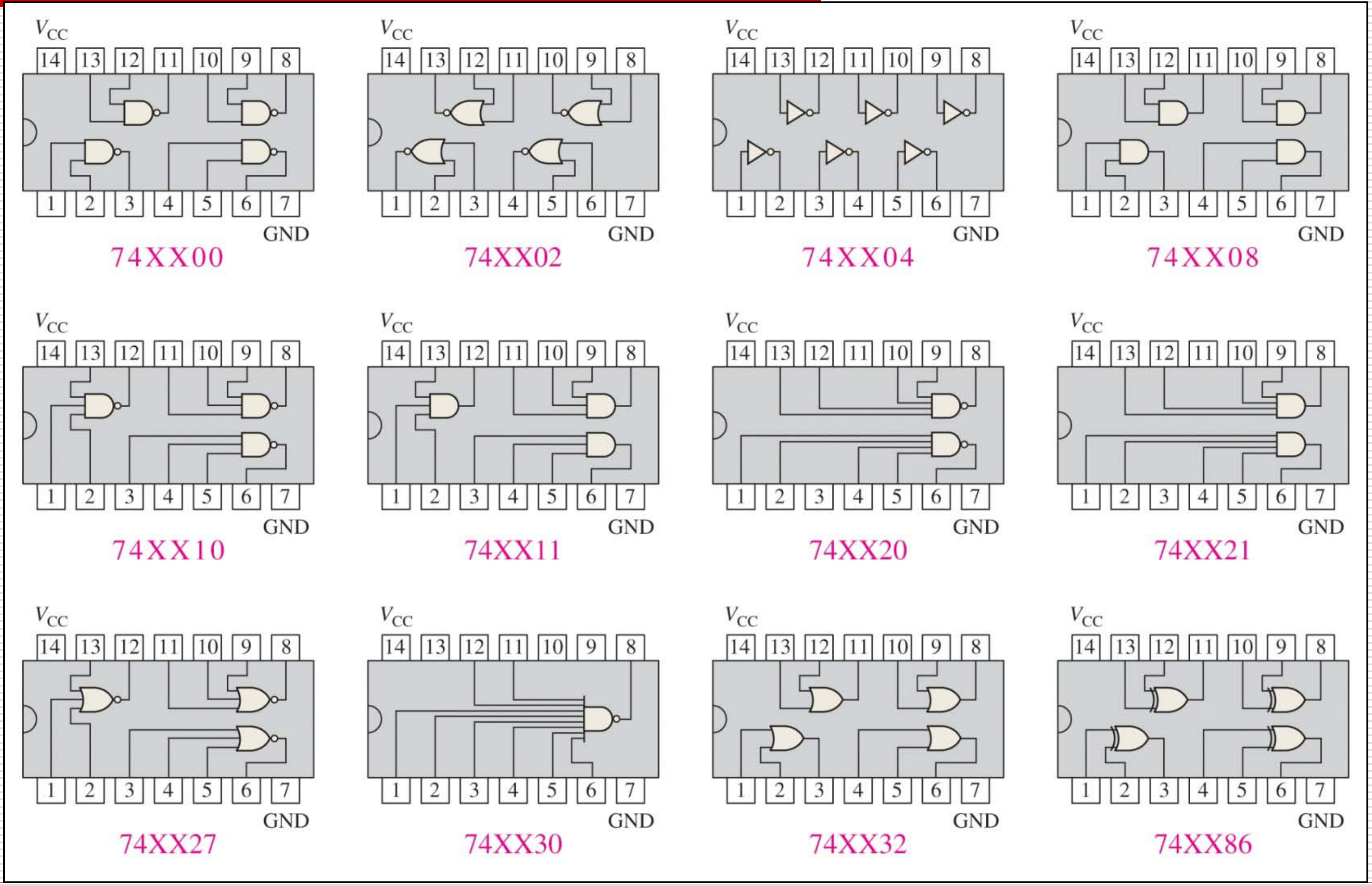
Dual-In-line Package Chip

Cutaway view of a DIP (**D**ual-**I**n-line **P**ackage) chip:



The TTL series, available as DIPs, are popular for laboratory experiments with logic.

Pin configuration diagrams
- Common fixed-function IC gates



Introduction to Digital Logic and Boolean Algebra – Part 3 of 3

Boolean Algebra

- **Laws, Rules and Theorems**
- **Analysis of Logic Circuits**
- **Simplification**

TOPIC COVERAGE
- PART 3 of 3

- Boolean Algebra
  - Laws and Rules of Boolean Algebra
  - Demorgan's theorems
- Boolean Analysis of Logic circuits
  - Evaluation of logic circuit output
  - Constructing a truth table for a logic circuit
- Simplification using Boolean Algebra

BOOLEAN ALGEBRA
– Rules, Laws, and Theorems

Boolean Algebra

- Boolean algebra is the mathematics of digital systems developed by George Boole in 1854.

- A basic knowledge of Boolean algebra is necessary to the study and analysis of logic circuits.

- Variable, complement and literal are terms used in Boolean algebra.

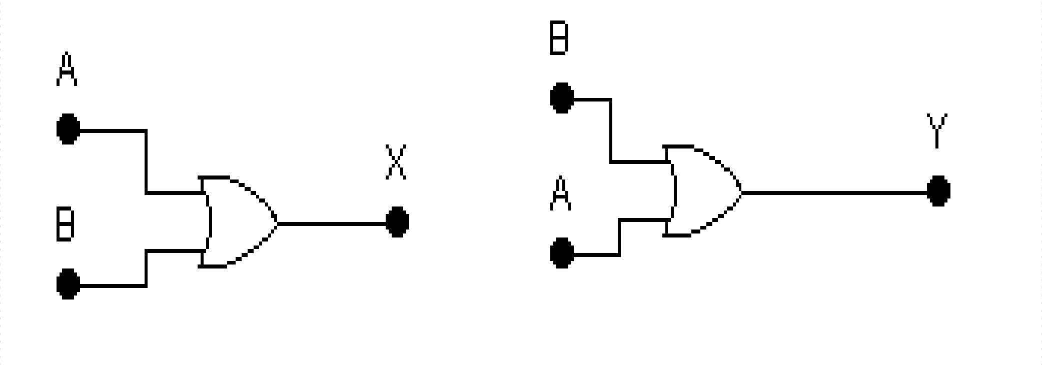
  - A **variable** is a symbol used to represent logical quantity. Any single variable can have a 1 or a 0 value.
  - The **complement or inverse of a variable** is indicated by bar or prime symbol
  - A **literal** is a variable or its complement.

Laws of Boolean Algebra

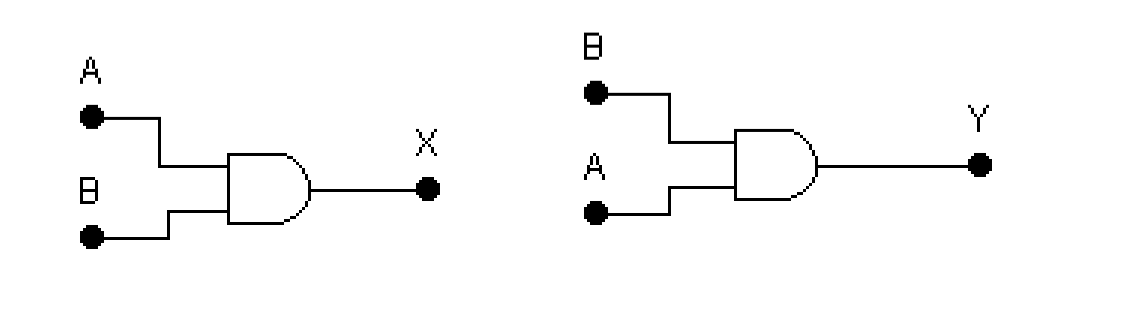
- Commutative Law
- Associative Law
- Distributive Law

Laws of Boolean Algebra

- **Commutative Law** of Addition: **A + B = B + A**

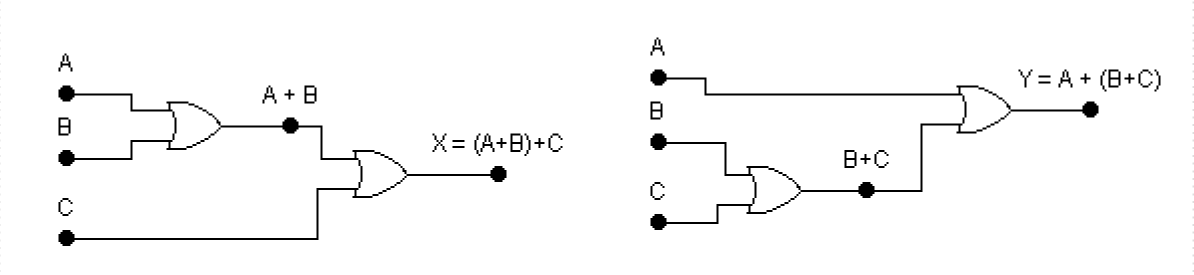


- **Commutative Law** of Multiplication: **A . B = B .A**

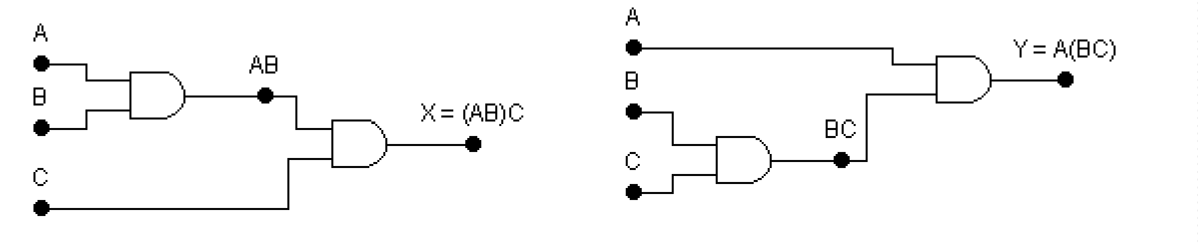


Laws of Boolean Algebra

- **Associative Law** of Addition: **A + (B + C) = (A + B) + C**



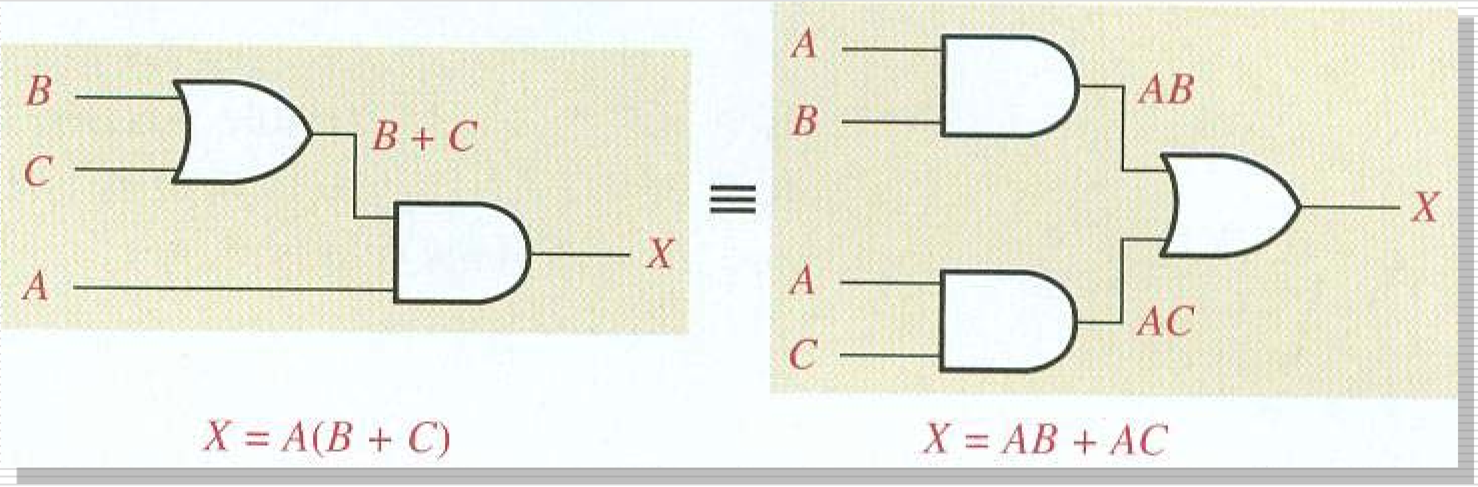
- **Associative Law** of Multiplication: **A . (B . C) = (A . B) . C**



Laws of Boolean Algebra

- **Distributive Law:**

$$A(B + C) = AB + AC$$

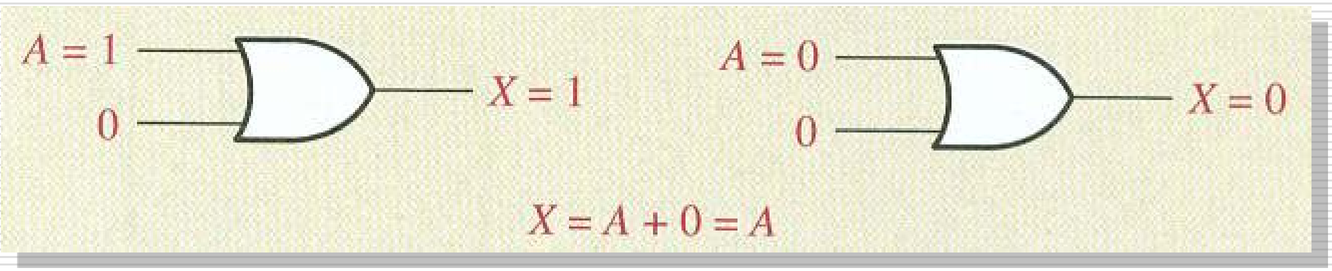


Rules of Boolean Algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Rules of Boolean Algebra

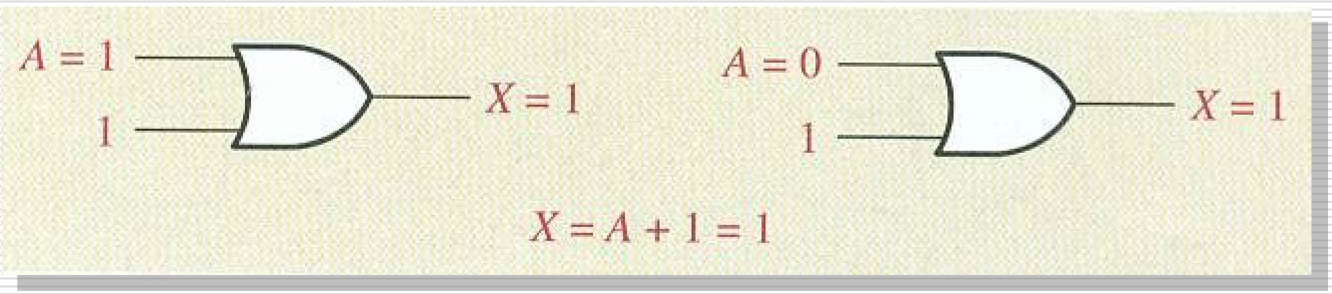
- Rule 1



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

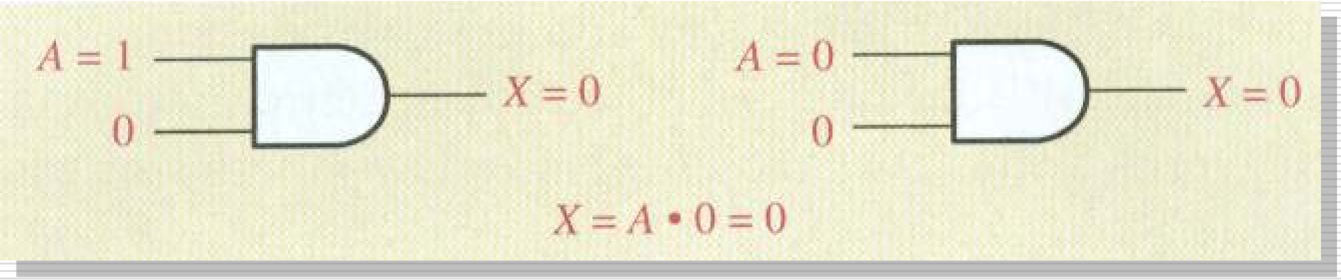
OR Truth Table

- Rule 2

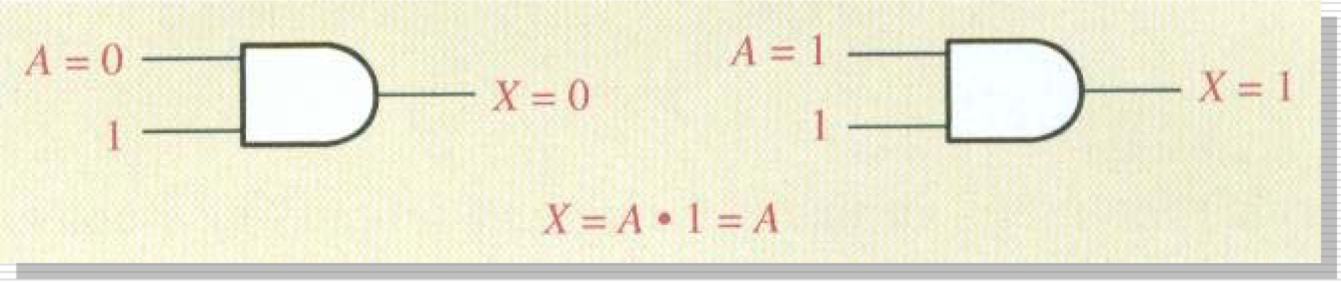


Rules of Boolean Algebra

- Rule 3



- Rule 4

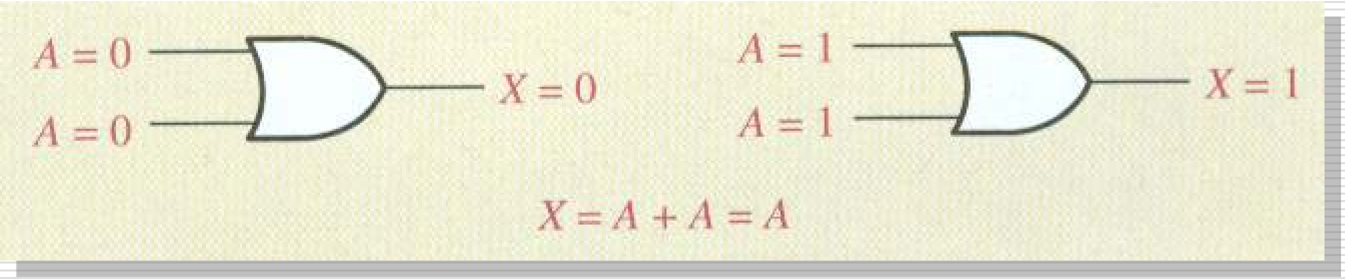


A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

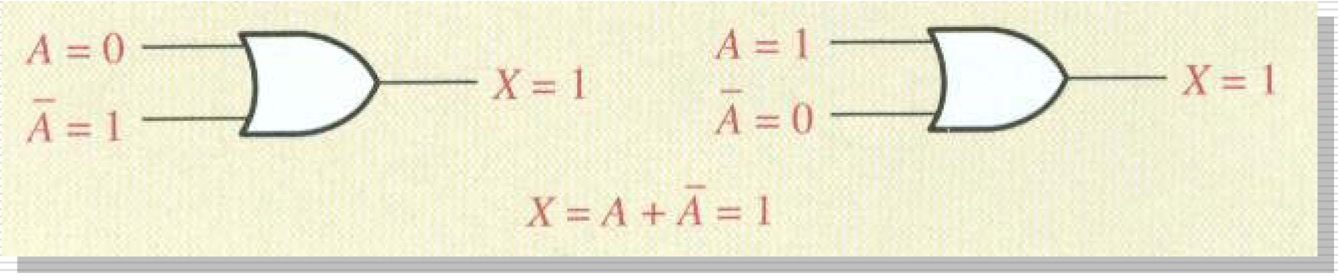
AND Truth Table

Rules of Boolean Algebra

- Rule 5



- Rule 6

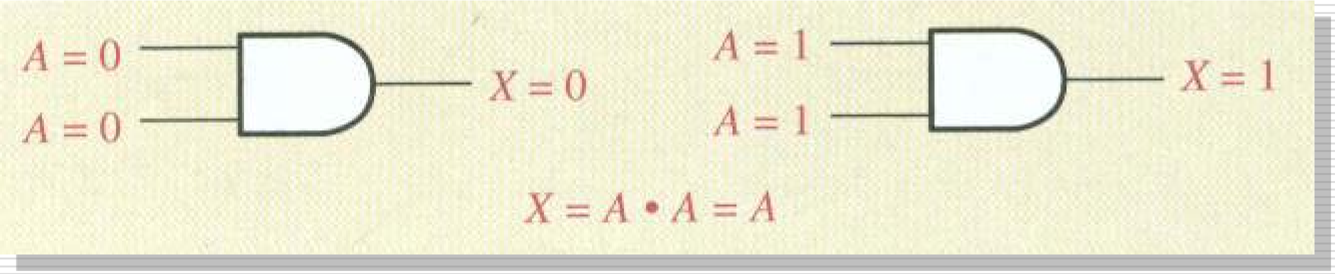


A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Rules of Boolean Algebra

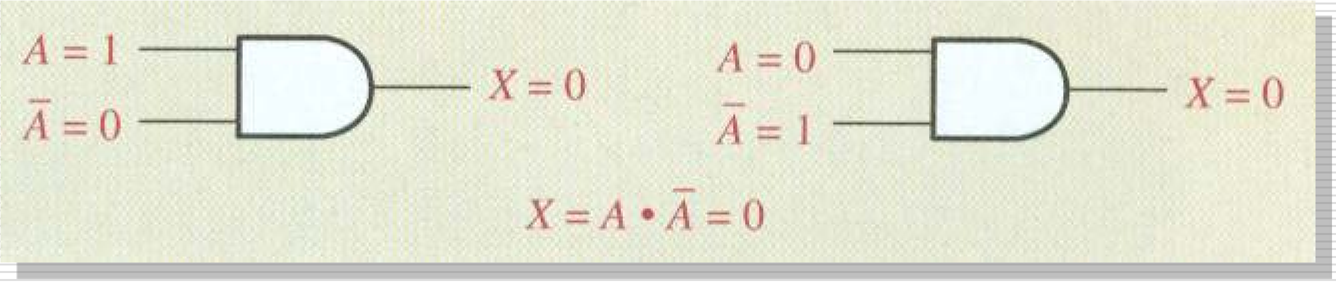
- Rule 7



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

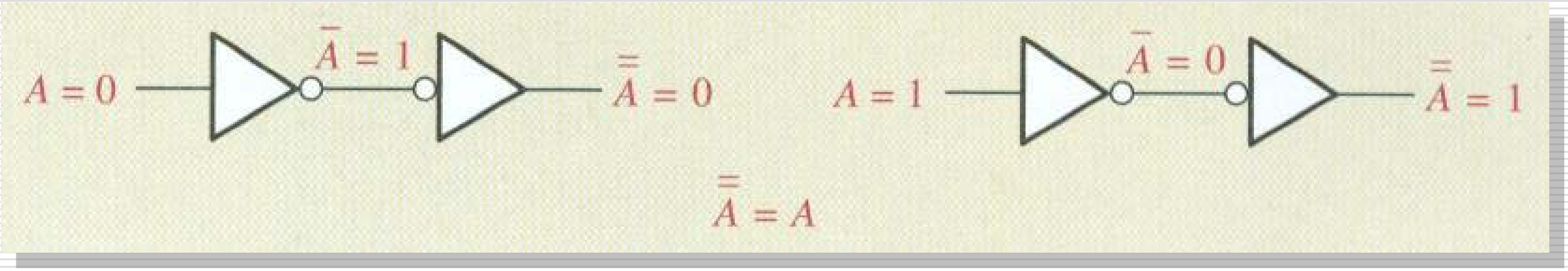
AND Truth Table

- Rule 8



Rules of Boolean Algebra

- Rule 9

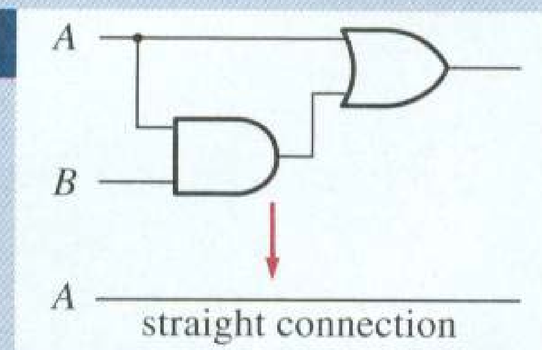


Rules of Boolean Algebra

- Rule 10: $A + AB = A$

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

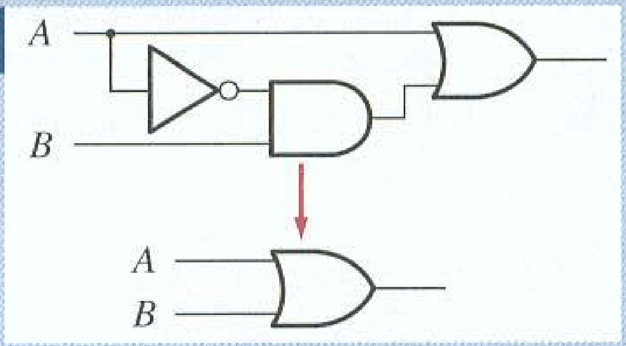
OR Truth Table

Rules of Boolean Algebra

- Rule 11: $A + \overline{A}B = A + B$

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

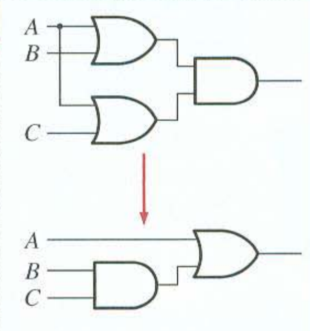
OR Truth Table

Rules of Boolean Algebra

- Rule 12: $(A + B)(A + C) = A + BC$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

De Morgan's Theorems

De Morgan's first theorem:

- **The complement of a product of variables is equal to the sum of the complements of the variables.**

Stated in another way,

- The complement of two or more variables **AND**ed is equivalent to the **OR** of the complements of the individual variables.

- The formula for expressing this theorem for two variables is:

$$\overline{X \,.\, Y} = \overline{X} + \overline{Y}$$

De Morgan's Theorems

De Morgan's second theorem:

- **The complement of a sum of variables is equal to the product of the complements of the variables.**

Stated in another way,

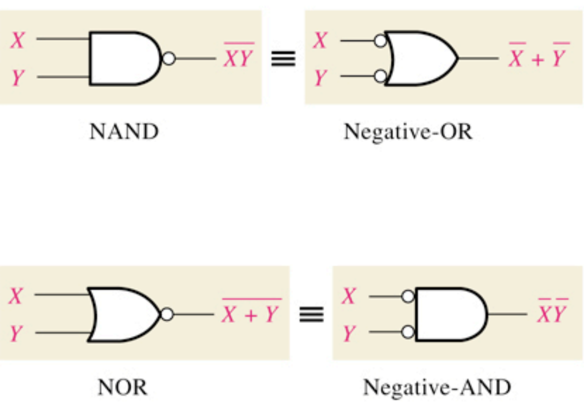
- The complement of two or more variables **Or**ed is equivalent to the **AND** of the complements of the individual variables

- The formula for expressing this theorem for two variables is:

$$\overline{X + Y} = \overline{X} \cdot \overline{Y}$$

De Morgan’s Theorem

- De Morgan’s theorems provide mathematical verification of the equivalency of the NAND and negative-OR gates and equivalency of the NOR and negative- AND gates.
- These theorems are extremely useful in simplifying expressions in which a product or sum of variables is inverted



Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X}+\overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Inputs		Output	
X	Y	$\overline{X+Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Gate equivalencies and corresponding truth tables

BOOLEAN ANALYSIS OF LOGIC CIRCUITS

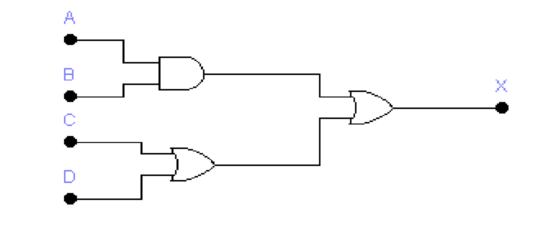
Boolean Analysis of Logic Circuits

The purpose of this section is to

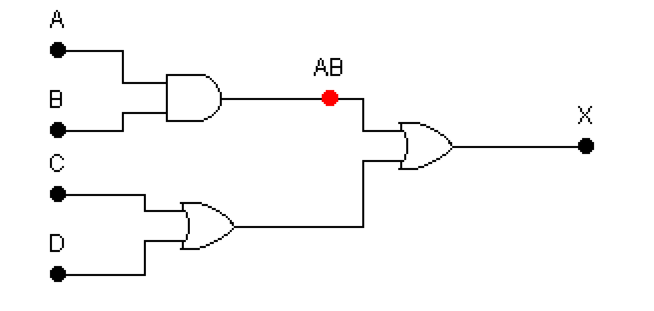
- Determine the boolean expression for a combination of gates.
- Evaluate the logic operation of a circuit from the boolean expression.
- Arrive at the simplified boolean algebra expression with the given logic.

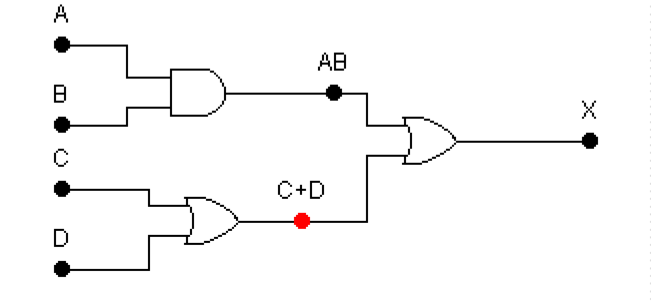
Determination of the boolean expression is done one gate at a time starting at the inputs and simplification is done using Boolean laws and rules

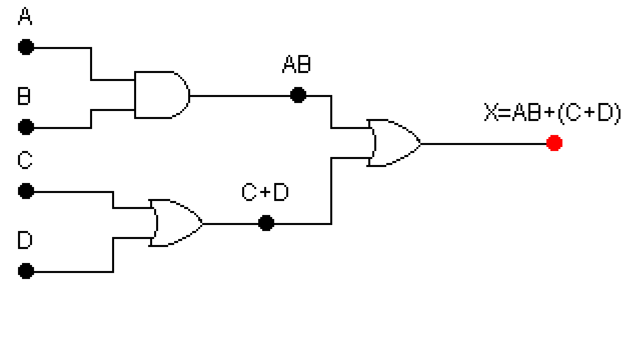
Example 1 : Problem



Solution: One gate at a time starting with the inputs



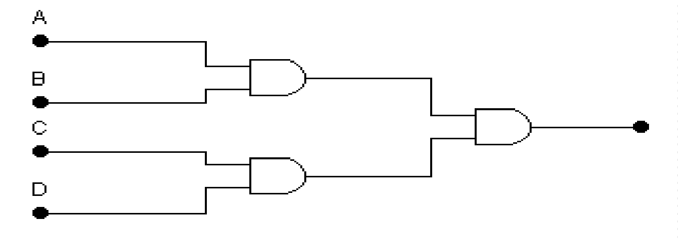




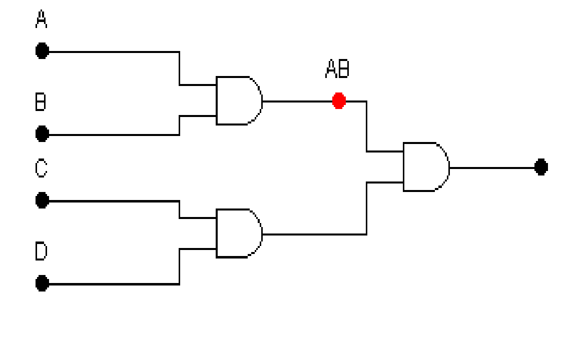
X= AB+(C+D)

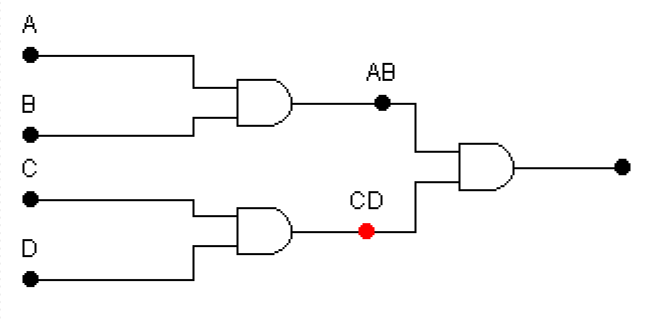
X= AB + C+ D

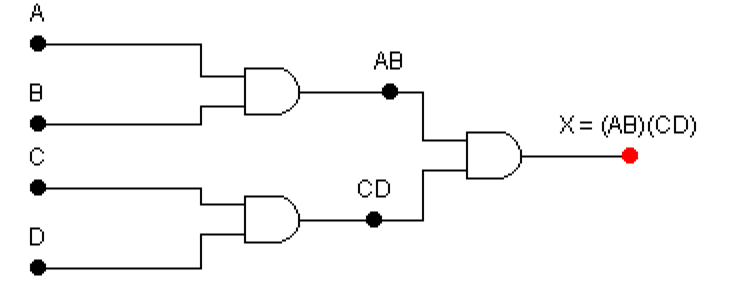
Example 2 : Problem



Solution:



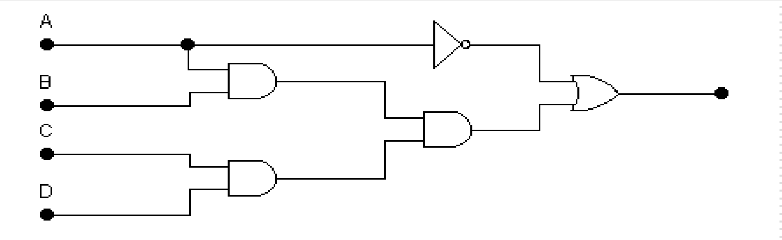




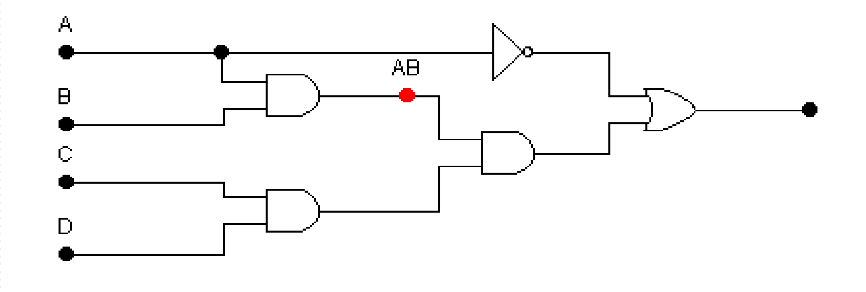
X = (AB)(CD)

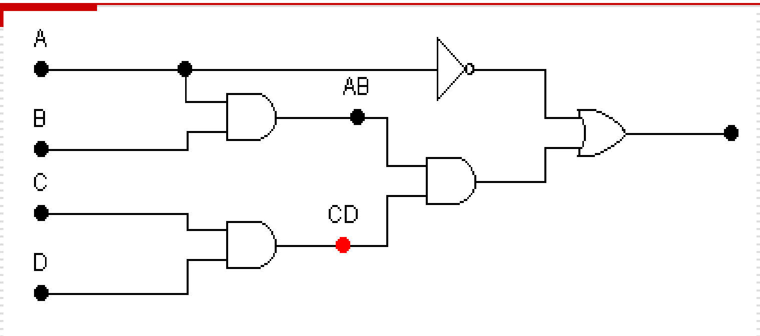
X = ABCD

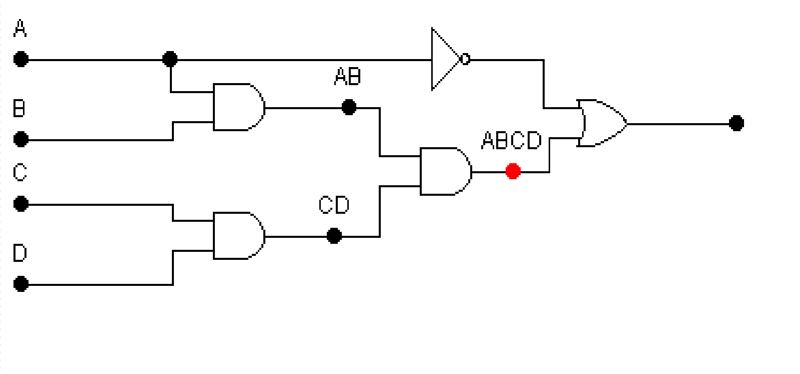
Example 3 : Problem

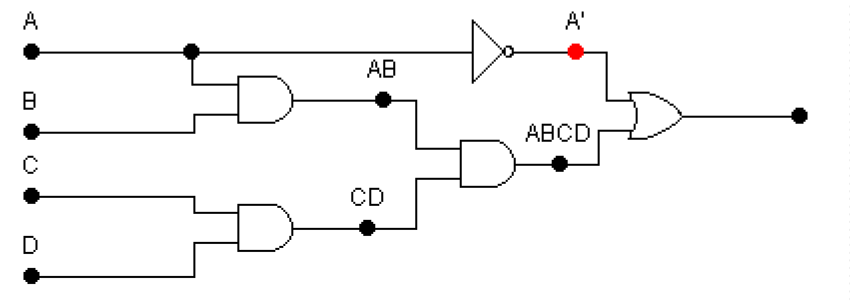


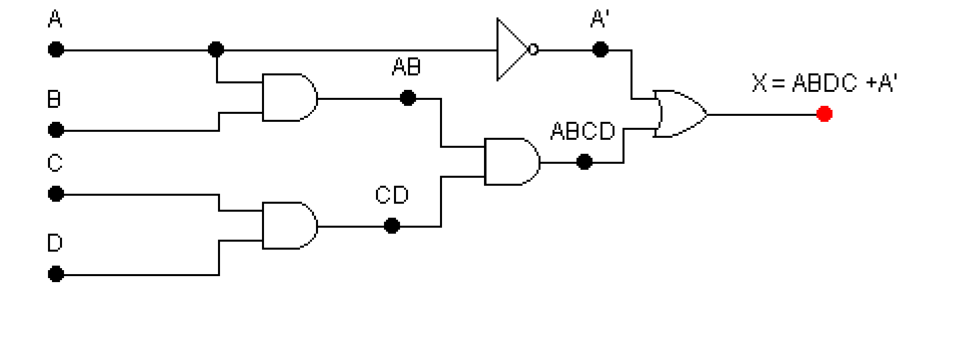
Solution:







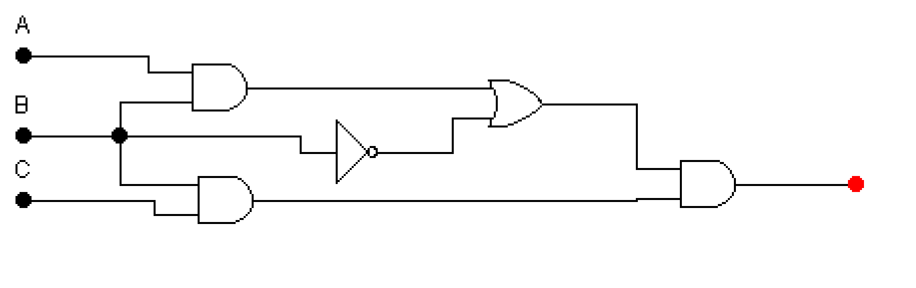




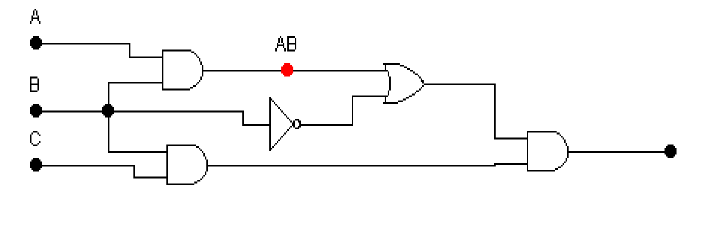
$$X = ABCD + \overline{A}$$

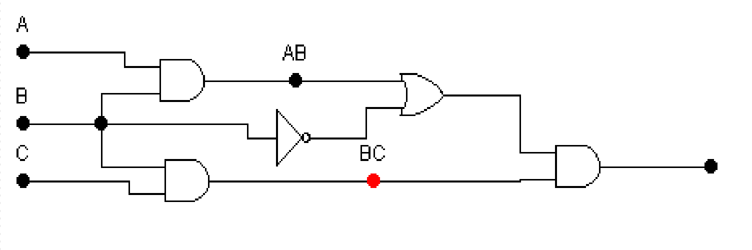
$$X = \overline{A} + BCD$$

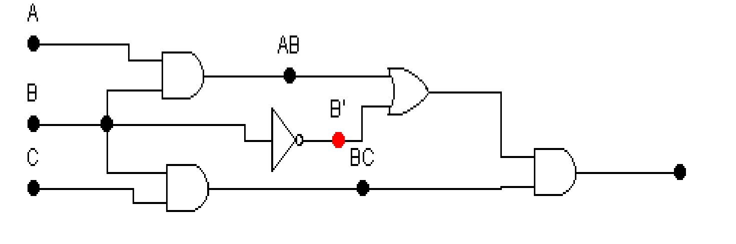
Example 4 : Problem

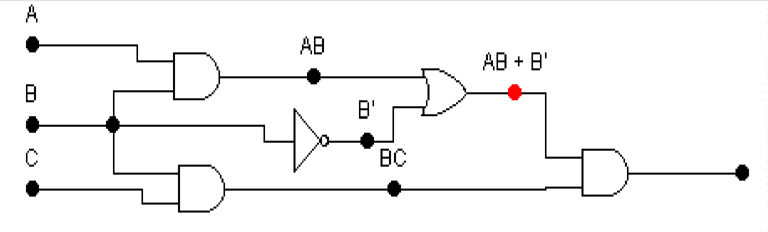


Solution:



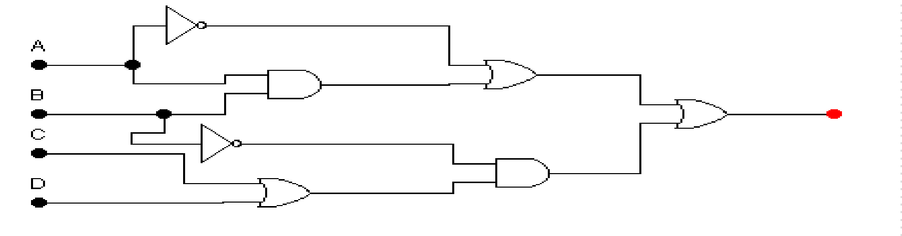




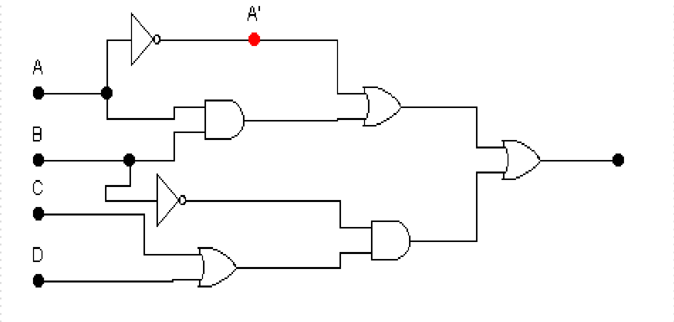


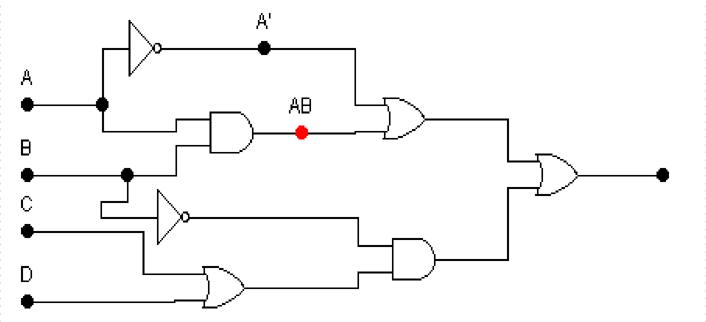
X = ABC

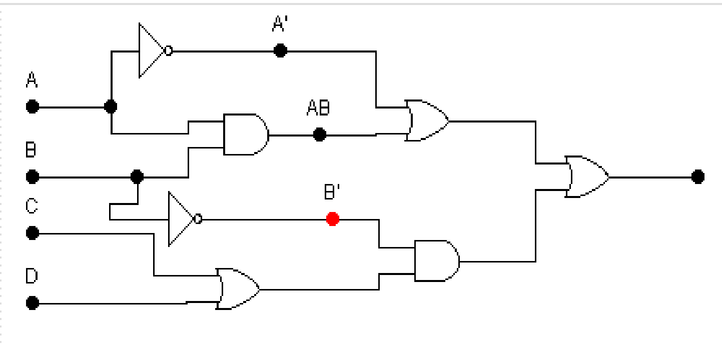
Example 5 : Problem

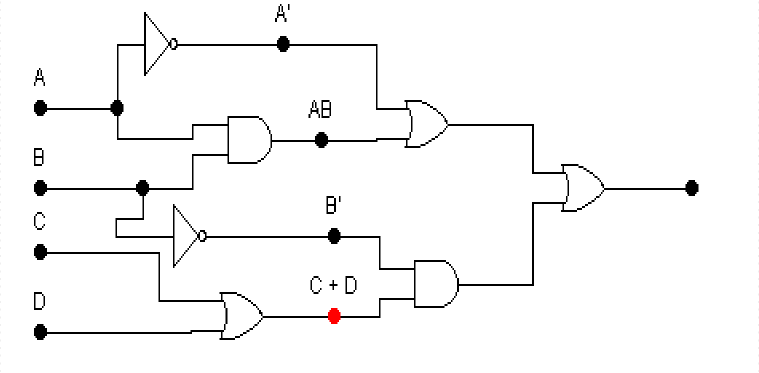


Solution:



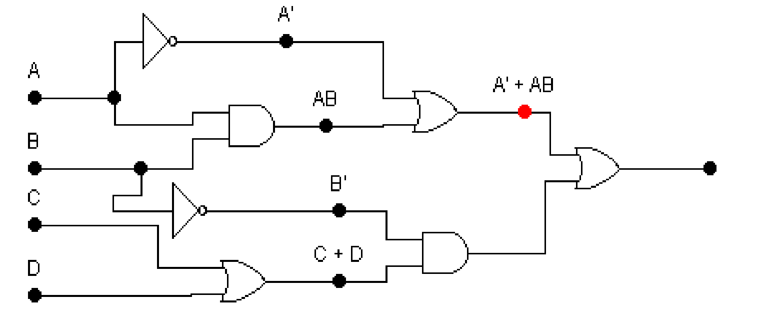


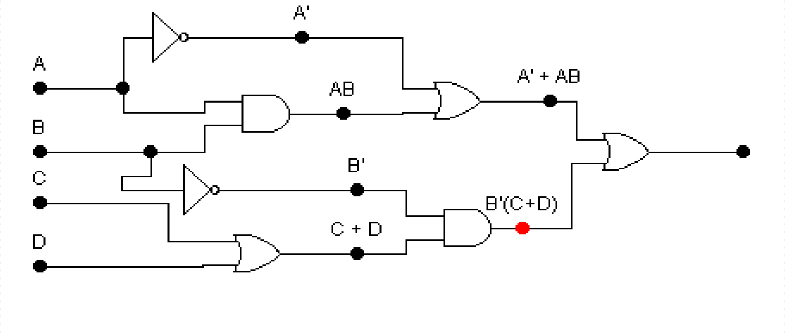


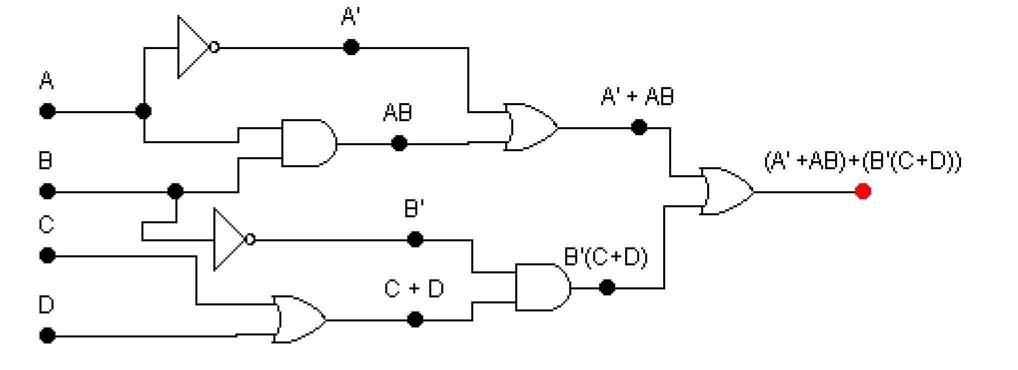


Example 5 : Problem

Solution (continued):







Example 5 : Problem

Solution (continued):

Simplification:

$$X = (\overline{A} + AB) + (\overline{B}(C+D))$$

$$X = (\overline{A} + B) + (\overline{B}(C + D))$$

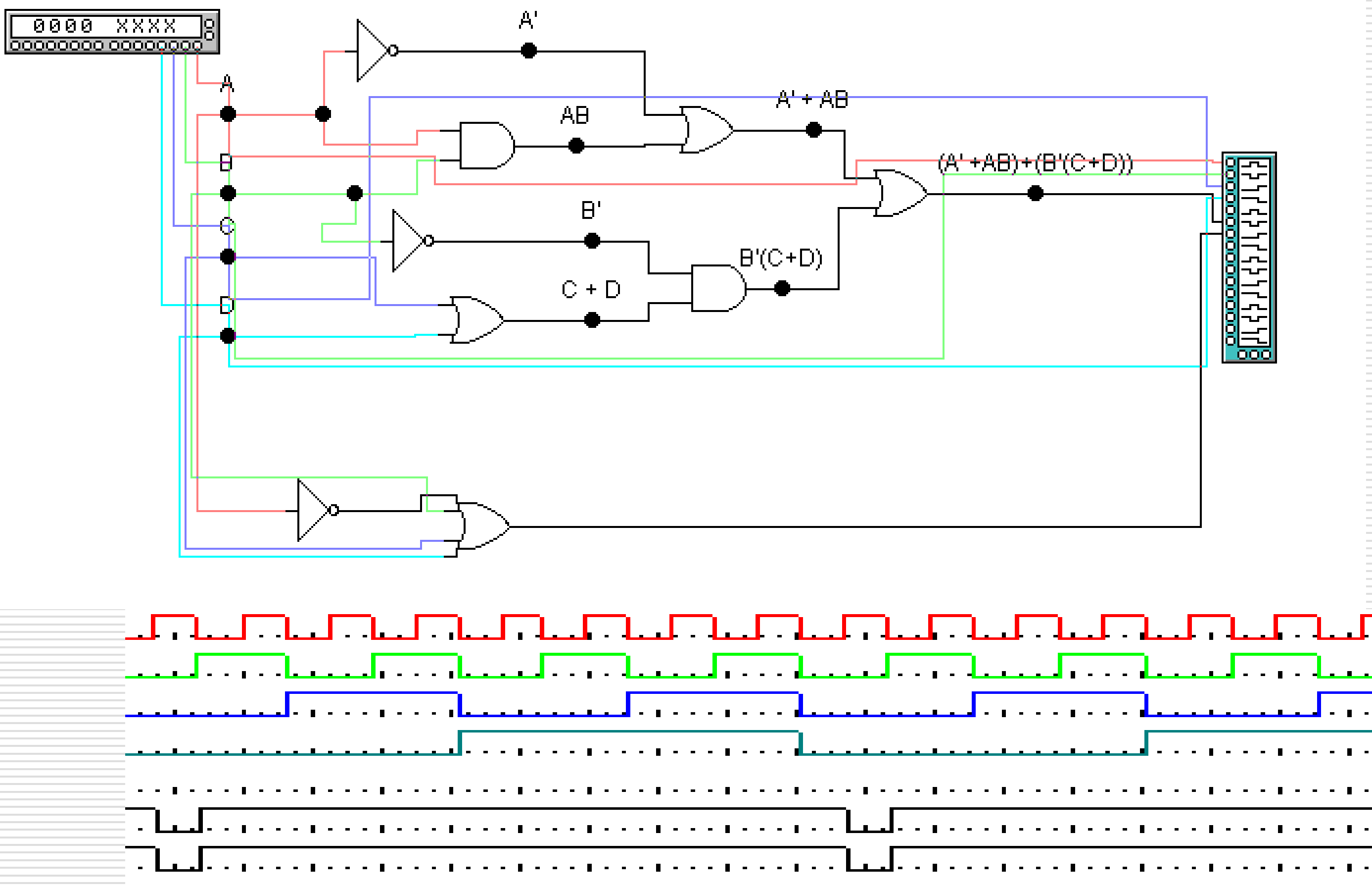
$$X = (\overline{A} + B) + (\overline{B}\,C + \overline{B}\,D)$$

$$X = \overline{A} + B + \overline{B}C + \overline{B}D$$

$$X = \overline{A} + B + C + \overline{B}\ D$$

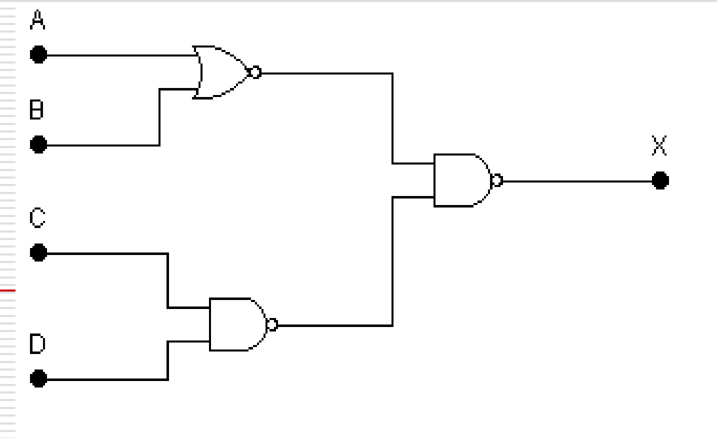
$$X = \overline{A} + B + C + D$$

Example 5 : Problem

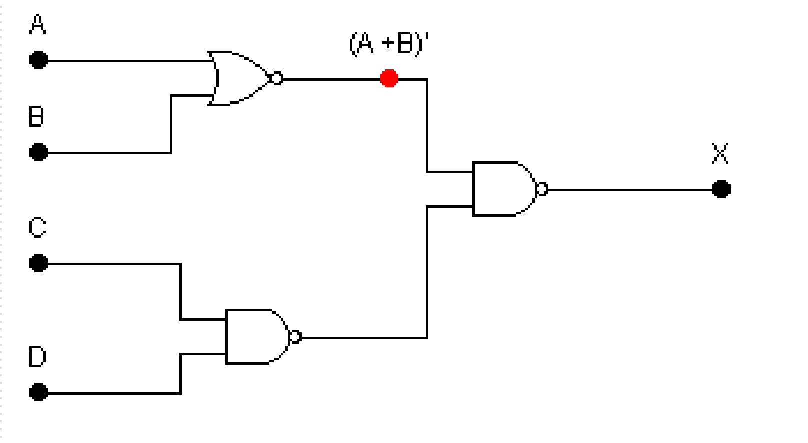


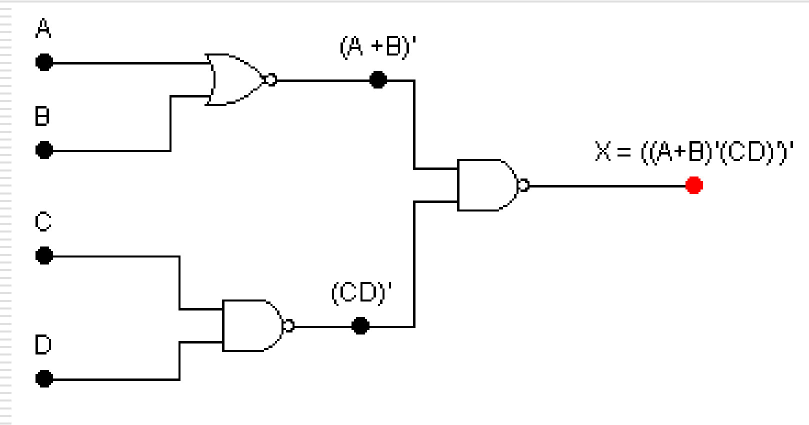
The circuits are different but the outputs are the same

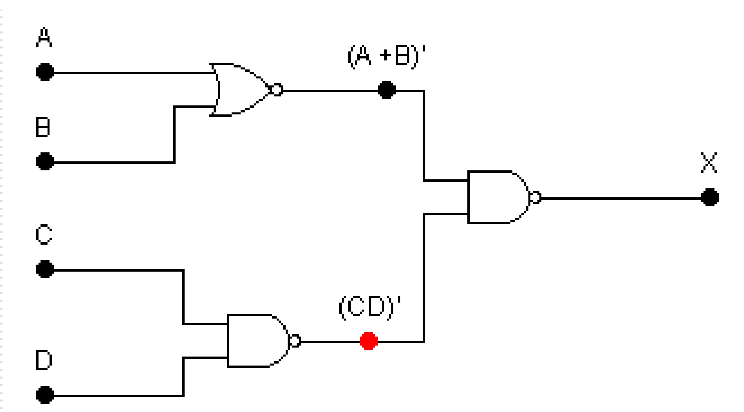
Example 6 : Problem



Solution:



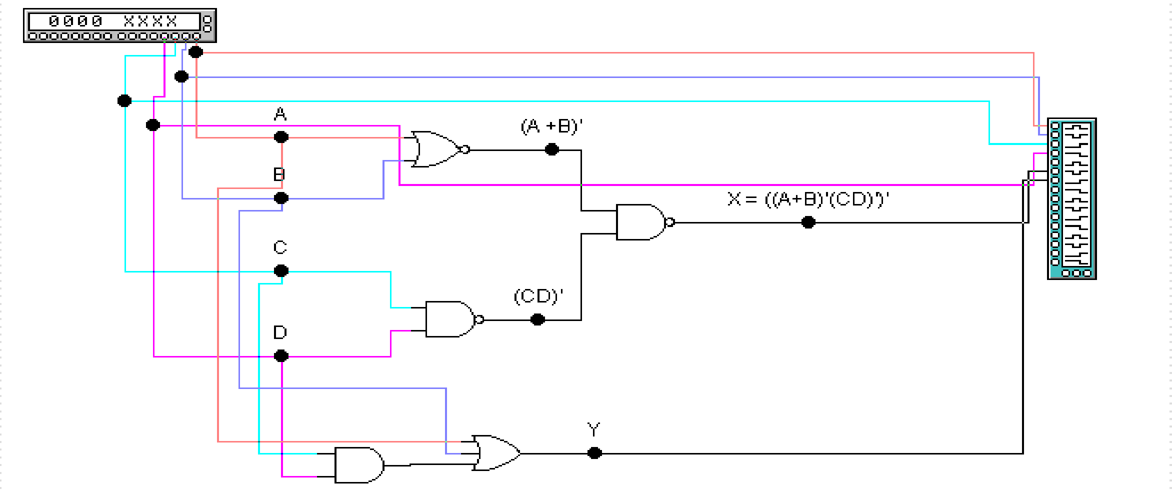


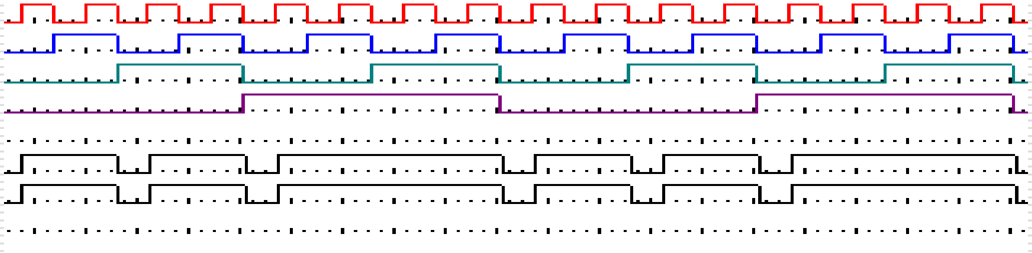


$$\overline{\overline{(A + B)}\,\overline{(CD)}} = \overline{\overline{A + B}} + \overline{\overline{CD}}$$

$$= A + B + CD$$

Example 6 : Problem





X and Y are the same

SIMPLIFICATION USING BOOLEAN ALGEBRA TECHNIQUES

Problem 1

Simplify: *A'B + A' B' C' D' + A B C D'*

$$
\begin{aligned}
A'B + A'B'C'D' + ABCD' &= A'(B + B'C'D') + ABCD' \\
&= A'(B + C'D') + ABCD' \\
&= B(A' + ACD') + A'C'D' \\
&= B(A' + CD') + A'C'D' \\
&= A'B + BCD' + A'C'D'
\end{aligned}
$$

Problem 2

Simplify: F = A’ B C + A B’ C + A B C’ + A B C

$$F = \bar{A}BC + A\bar{B}C + AB(\bar{C} + C)$$

$$F = \bar{A}BC + A\bar{B}C + AB(1)$$

$$F = \bar{A}BC + A\bar{B}C + AB$$

$$F = \bar{A}BC + A\bar{B}C + AB + ABC$$

$$F = \bar{A}BC + AC(\bar{B} + B) + AB$$

$$F = \bar{A}BC + AC + AB$$

$$F = \bar{A}BC + AC + AB + ABC$$

$$F = BC(\bar{A} + A) + AC + AB$$

F = B C + A C + A B

Problem 3

Simplify:

$W = [M + N'P + (R + ST)'][M + N'P + R + ST]$

Assume $X = M + N'P$ $Y = R + ST$

$W = (X + Y')(X + Y)$

$W = XX + XY + Y'X + Y'Y$

$W = X\cdot 1 + XY + XY' + 0$

$W = X + X(Y + Y') = X + X\cdot 1 = X$

$\mathbf{W = M + N'P}$

Problem 4

Express the complement $f'(w,x,y,z)$ of the following expression in a simplified form.

$$f(w,x,y,z) = wx(y'z + yz')$$

$$\begin{aligned}
f'(w,x,y,z) &= w' + x' + (y'z + yz')' \\
&= w' + x' + (y'z)'(yz')' \\
&= w' + x' + (y + z')(y' + z) \\
&= w' + x' + yy' + yz + z'y' + z'z \\
&= w' + x' + 0 + yz + z'y' + 0 \\
&= \mathbf{w' + x' + yz + y'z'}
\end{aligned}$$

Simplification using Boolean Algebra – More Problems

- Using Boolean Algebra Techniques, simplify the following expressions :

1. AB+A(B+C)+B(B+C)
2. [AB’(C+BD)+A’B’]C
3. A’BC+AB’C’+A’B’C’+AB’C+ABC
4. (AB+AC)’+A’B’C

References

- Slides adopted from the books
  Thomas L.Floyd, “Digital Fundamentals,” 10th Edition, Pearson Education International, 2009
  (ISBN10:0138146462/ISBN13:9780138146467)

- M.Morris Mano and Michael D. Ciletti, " Digital Design," 4th Edition, Pearson Education International, 2007
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- Ronald J.Tocci, Neal S.Widmer, and Gregory L.Moss, "Digital Systems- Principles and Application"- 10th Edition, Pearson Education International, 2007 (ISBN: 9780131725799)