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《流体力学》

习题解答

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目录

第一章 空气动力学基础知识

1.1 **解**：首先计算力距有

$$
\begin{aligned}
M_{z\infty} &= \int_{\mathrm{LE}}^{\mathrm{TE}} [(p_u\cos\theta + \tau_u\sin\theta)x - (p_u\sin\theta - \tau_u\cos\theta)y]\mathrm{d}s_u \\
&\quad + \int_{\mathrm{LE}}^{\mathrm{TE}} [(-p_l\cos\beta + \tau_l\sin\beta)x + (p_l\sin\beta + \tau_l\cos\beta)y]\mathrm{d}s_l \\
&= \int_0^c C_1 x\,\mathrm{d}x - \int_0^c C_2 x\,\mathrm{d}x = \frac{1}{2}(C_1 - C_2)c^2
\end{aligned}
$$

其次计算法向力有

$$
\begin{aligned}
N_\infty &= -\int_{\mathrm{LE}}^{\mathrm{TE}} (p_u\cos\theta + \tau_u\sin\theta)\mathrm{d}s_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l\cos\beta - \tau_l\sin\beta)\mathrm{d}s_l \\
&= -\int_0^c C_1\mathrm{d}x + \int_0^c C_2\mathrm{d}x \\
&= (C_2 - C_1)c
\end{aligned}
$$

最后利用压力中心坐标公式可得

$$
x_{cp} = -\frac{M_{z\infty}}{N_\infty} = -\frac{\frac{1}{2}(C_1 - C_2)c^2}{(C_2 - C_1)c} = \frac{c}{2}.\square
$$

1.2 **解**：(1)直接利用法向力和轴向力公式可得

$$
\begin{aligned}
N_\infty &= -\int_{\mathrm{LE}}^{\mathrm{TE}} (p_u\cos\theta + \tau_u\sin\theta)\mathrm{d}s_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l\cos\beta - \tau_l\sin\beta)\mathrm{d}s_l \\
&= -\int_0^1 [4\times 10^4(x-1)^2 + 5.4\times 10^4]\mathrm{d}x + \int_0^1 [2\times 10^4(x-1)^2 + 1.73\times 10^5]\mathrm{d}x \\
&= (-67333.3 + 179666.7)\mathrm{N} = 112333\mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
A_\infty &= \int_{\mathrm{LE}}^{\mathrm{TE}} (-p_u\sin\theta + \tau_u\cos\theta)\mathrm{d}s_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l\sin\beta + \tau_l\cos\beta)\mathrm{d}s_l \\
&= \int_0^1 288x^{-0.2}\mathrm{d}x + \int_0^1 731x^{-0.2}\mathrm{d}x \\
&= 1273.8\mathrm{N}.
\end{aligned}
$$

(2)直接利用升力、阻力与法向力、轴向力之间的转换公式可得

$$
\begin{bmatrix} L_\infty \\ D_\infty \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} N_\infty \\ A_\infty \end{bmatrix} = \begin{bmatrix} \cos 10^\circ & -\sin 10^\circ \\ \sin 10^\circ & \cos 10^\circ \end{bmatrix} \begin{bmatrix} 112333\mathrm{N} \\ 1273.8\mathrm{N} \end{bmatrix} = \begin{bmatrix} 110405.2\mathrm{N} \\ 20760.9\mathrm{N} \end{bmatrix}
$$

因此有 $L_\infty = 110405.2\mathrm{N}$，$D_\infty = 20760.9\mathrm{N}$.

(3)直接利用对前缘点的力距公式可得

$$
\begin{aligned}
M_{z\infty} &= \int_{\mathrm{LE}}^{\mathrm{TE}} [(p_u\cos\theta + \tau_u\sin\theta)x - (p_u\sin\theta - \tau_u\cos\theta)y]\mathrm{d}s_u \\
&\quad + \int_{\mathrm{LE}}^{\mathrm{TE}} [(-p_l\cos\beta + \tau_l\sin\beta)x + (p_l\sin\beta + \tau_l\cos\beta)y]\mathrm{d}s_l \\
&= \int_0^1 [4\times10^4(x-1)^2 + 5.4\times10^4]x\,\mathrm{d}x - \int_0^1 [2\times10^4(x-1)^2 + 1.73\times10^5]x\,\mathrm{d}x \\
&= (30333.3 - 88166.7)\mathrm{N\cdot m} = -57833.4\mathrm{N\cdot m}
\end{aligned}
$$

直接利用对前缘$\frac{1}{4}$弦长点的力距公式可得

$$
\begin{aligned}
M_{z\infty} &= \int_{\mathrm{LE}}^{\mathrm{TE}} \left[(p_u\cos\theta + \tau_u\sin\theta)\left(x - \frac{1}{4}\right) - (p_u\sin\theta - \tau_u\cos\theta)y\right]\mathrm{d}s_u \\
&\quad + \int_{\mathrm{LE}}^{\mathrm{TE}} \left[(-p_l\cos\beta + \tau_l\sin\beta)\left(x - \frac{1}{4}\right) + (p_l\sin\beta + \tau_l\cos\beta)y\right]\mathrm{d}s_l \\
&= \int_0^1 [4\times10^4(x-1)^2 + 5.4\times10^4]\left(x - \frac{1}{4}\right)\mathrm{d}x \\
&\quad - \int_0^1 [2\times10^4(x-1)^2 + 1.73\times10^5]\left(x - \frac{1}{4}\right)\mathrm{d}x \\
&= (13500 - 43250)\mathrm{N\cdot m} = -29750\mathrm{N\cdot m}.
\end{aligned}
$$

(4)直接利用压力中心坐标公式可得

$$
x_{cp} = -\frac{M_{z\infty}}{N_\infty} = -\frac{-57833.4\mathrm{N\cdot m}}{112333\mathrm{N}} = -0.514837\mathrm{m}.\square
$$

1.6 **解：**首先计算当地压强为

$$
C_p = \frac{p - p_\infty}{q_\infty} \Longrightarrow p = q_\infty C_p + p_\infty
$$

所以有

$$
\begin{aligned}
N_\infty &= -\int_{\mathrm{LE}}^{\mathrm{TE}} p_u\sin\varphi\,\mathrm{d}s_u + \int_{\mathrm{LE}}^{\mathrm{TE}} p_l\sin\varphi\,\mathrm{d}s_l = 0 \\
A_\infty &= \int_{\mathrm{LE}}^{\mathrm{TE}} p_u\cos\varphi\,\mathrm{d}s_u + \int_{\mathrm{LE}}^{\mathrm{TE}} p_l\cos\varphi\,\mathrm{d}s_l \\
&= 2r\left[\int_0^{\frac{\pi}{2}} (2q_\infty\cos^2\varphi + p_\infty)\cos\varphi\,\mathrm{d}\varphi + \int_{\frac{\pi}{2}}^{\pi} p_\infty\cos\varphi\,\mathrm{d}\varphi\right] \\
&= 2r\left[2q_\infty\int_0^{\frac{\pi}{2}} \cos^3\varphi\,\mathrm{d}\varphi + p_\infty\int_0^{\frac{\pi}{2}} \cos\varphi\,\mathrm{d}\varphi + p_\infty\int_{\frac{\pi}{2}}^{\pi} \cos\varphi\,\mathrm{d}\varphi\right] \\
&= 4rq_\infty\int_0^{\frac{\pi}{2}} \cos^3\varphi\,\mathrm{d}\varphi = 4rq_\infty\cdot\frac{2}{3} = \frac{8rq_\infty}{3}
\end{aligned}
$$

因此有

$$
D_\infty = A_\infty = \frac{8rq_\infty}{3}
$$

于是我们可以得到阻力系数为

$$c_d = \frac{D_\infty}{q_\infty c} = \frac{\frac{8rq_\infty}{3}}{q_\infty \cdot 2r} = \frac{4}{3}. \square$$

第二章 流体运动基本方程和基本规律

2.5 **解**：在笛卡尔坐标系下有流线方程

$$\frac{\mathrm{d}x}{u}=\frac{\mathrm{d}y}{v}$$

又因为有

$$\begin{bmatrix}u\\v\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}V_r\\V_\theta\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\3r\end{bmatrix}=\begin{bmatrix}-3r\sin\theta\\3r\cos\theta\end{bmatrix}=\begin{bmatrix}-3y\\3x\end{bmatrix}$$

带入可得

$$\frac{\mathrm{d}x}{-3y}=\frac{\mathrm{d}y}{3x}\Longrightarrow x\,\mathrm{d}x+y\,\mathrm{d}y=0\Longrightarrow \mathrm{d}(x^2+y^2)=0\Longrightarrow x^2+y^2=C.\square$$

2.6 **解**：在笛卡尔坐标系下有流线方程

$$\frac{\mathrm{d}x}{u}=\frac{\mathrm{d}y}{v}$$

又因为有

$$u=3x,\quad v=-3y$$

因此可得

$$\frac{\mathrm{d}x}{3x}=-\frac{\mathrm{d}y}{3y}\Longrightarrow x\,\mathrm{d}y+y\,\mathrm{d}x=\mathrm{d}(xy)=0\Longrightarrow xy=C.\square$$

2.13 **解**：积分形式的动量方程为

$$\frac{\partial}{\partial t}\oiiint_V(\rho\,\mathrm{d}V)\vec{V}+\oiint_S(\rho\vec{V}\cdot\mathrm{d}\vec{S})\vec{V}=\oiiint_V\rho\vec{f}\,\mathrm{d}\vec{V}-\oiint_S p\,\mathrm{d}\vec{S}+\vec{F}_{\text{visc}}$$

对于定常无黏流动问题(不计彻体力)，我们有

$$\frac{\partial}{\partial t}\oiiint_V(\rho\,\mathrm{d}V)\vec{V}=0,\quad \oiiint_V\rho\vec{f}\,\mathrm{d}\vec{V}=0,\quad \vec{F}_{\text{visc}}=0$$

因此方程可化简为

$$\oiint_S(\rho\vec{V}\cdot\mathrm{d}\vec{S})\vec{V}=-\oiint_S p\,\mathrm{d}\vec{S}$$

对于一维问题可以简化为

$$\oiint_S(\rho u\vec{u})\mathrm{d}\vec{S}=\oiiint_V\nabla\cdot(\rho u\vec{u})\mathrm{d}V=-\oiint_S p\,\mathrm{d}\vec{S}=-\oiiint_V\nabla\cdot p\,\mathrm{d}V$$

于是我们可以得到

$$\oiiint_V[\nabla\cdot(\rho u\vec{u})+\nabla\cdot p]\mathrm{d}V=0\Longrightarrow\nabla\cdot(\rho u\vec{u})=-\nabla\cdot p$$

由连续方程可得

$$\mathrm{d}(\rho u)=0$$

因此有

$$\rho u \, \mathrm{d}u = -\,\mathrm{d}p. \square$$

第三章 不可压无黏流

3.1 **解**：在喷嘴处有

$$Q = A_2V_2 = \frac{1}{4}\pi D_2^2 V_2$$

于是可得喷嘴处的流速为

$$V_2 = \frac{4Q}{\pi D_2^2}$$

假设流体为不可压理想定常流体，应用不可压连续流体方程有

$$A_1V_1 = A_2V_2 \Longleftrightarrow D_1^2V_1 = D_2^2V_2 \Longrightarrow \frac{V_1}{V_2} = \frac{D_2^2}{D_1^2}$$

取控制体，忽略管子中流体所受的重力和粘性力的影响，其中 p_1 和 p_1 为相对压强，那么有

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

其中 $p_2 = p_a = 0$，于是可得

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2}\rho V_2^2\left[1-\left(\frac{V_1}{V_2}\right)^2\right] = \frac{1}{2}\rho\left(\frac{4Q}{\pi D_2^2}\right)^2\left(1-\frac{D_2^4}{D_1^4}\right) = \frac{8\rho Q^2}{\pi^2 D_2^4}\left(1-\frac{D_2^4}{D_1^4}\right)$$

不计彻体力，那么有

$$\oiint_S \vec{V}\left(\rho\vec{V}\mathrm{d}\vec{S}\right) = -\oiint_S p\,\mathrm{d}\vec{S} - \boldsymbol{F}_B$$

所以有

$$\begin{aligned}
\boldsymbol{F}_B &= -\oiint_S p\,\mathrm{d}\vec{S} - \oiint_S \vec{V}\left(\rho\vec{V}\mathrm{d}\vec{S}\right) = p_1A_1 - p_2A_2 + \rho(V_1^2A_1 - V_2^2A_2) \\
&= \frac{8\rho Q^2}{\pi^2 D_2^4}\left(1-\frac{D_2^4}{D_1^4}\right)\cdot\frac{1}{4}\pi D_1^2 + \frac{1}{4}\pi\rho(V_1^2D_1^2 - V_2^2D_2^2) \\
&= \frac{2\rho Q^2 D_1^2}{\pi D_2^4}\left(1-\frac{D_2^4}{D_1^4}\right) + \frac{1}{4}\pi\rho\left(\frac{16Q^2}{\pi^2 D_2^4}\cdot\frac{D_2^4}{D_1^4}\cdot D_1^2 - \frac{16Q^2}{\pi^2 D_2^4}\cdot D_2^2\right) \\
&= \frac{2\rho Q^2 D_1^2}{\pi D_2^4}\left(1-\frac{D_2^4}{D_1^4}\right) + \frac{4\rho Q^2}{\pi}\left(\frac{D_1^2}{D_1^4} - \frac{D_2^2}{D_2^4}\right) \\
&= \frac{2\cdot 1000\cdot\left(\frac{1.5}{60}\right)^2\cdot 0.1^2}{\pi\cdot 0.03^4}\left(1-\frac{0.03^4}{0.1^4}\right) + \frac{4\cdot 1000\cdot\left(\frac{1.5}{60}\right)^2}{\pi}\left(\frac{0.1^2}{0.1^4} - \frac{0.03^2}{0.03^4}\right)\mathrm{N} \\
&= 4067.78\mathrm{N}
\end{aligned}$$

其方向为水平向左.□

3.2 **解**：由叠加原理可得

$$\begin{cases}
\phi = \phi_1 + \phi_2 = \dfrac{Q}{2\pi}\ln r + V_\infty r\cos\theta \\
\psi = \psi_1 + \psi_2 = \dfrac{Q}{2\pi}\theta + V_\infty r\sin\theta
\end{cases}$$

求导可得

$$V_r=\frac{\partial\phi}{\partial r}=\frac{Q}{2\pi r}+V_\infty\cos\theta,\ V_\theta=\frac{\partial\phi}{r\partial\theta}=-V_\infty\sin\theta$$

解得驻点

$$(r_0,\theta)=\left(\frac{Q}{2\pi V_\infty},\pi\right)$$

极限流线为

$$\psi|_{(r_0,\theta)}=\left(\frac{Q}{2\pi}\theta+V_\infty r\sin\theta\right)\bigg|_{(r_0,\theta)}=\frac{Q}{2}$$

过驻点的流线，即为半无限体的表面，其方程为

$$y=r\sin\theta=\frac{Q}{2\pi V_\infty}(\pi-\theta)$$

垂直分速度

$$v=\frac{\partial\phi}{\partial y}=\frac{Q}{2\pi}\frac{y}{x^2+y^2}=\frac{Q}{2\pi}\frac{\sin\theta}{r}=\frac{V_\infty\sin^2\theta}{\pi-\theta}$$

求导可得

$$\frac{\mathrm{d}v}{\mathrm{d}\theta}=\frac{2V_\infty\sin\theta\cos\theta(\pi-\theta)+V_\infty\sin^2\theta}{(\pi-\theta)^2}=\frac{V_\infty\sin^2\theta\left[\frac{2(\pi-\theta)}{\tan\theta}+1\right]}{(\pi-\theta)^2}=0$$

解得 $\frac{\tan\theta}{\theta-\pi}=2$ 或者 $\sin\theta=0$，于是有

(1)当 $\sin\theta=0$ 时，即 $\theta=0$ 的时候有 $v=0$(舍)；

(2)当 $\frac{\tan\theta}{\theta-\pi}=2$ 时，即 $\theta=-1.4626=-83.8°$ 时有

$$v=\frac{V_\infty\sin^2\theta}{\pi-\theta}=\frac{V_\infty\sin^2\theta}{-\frac{1}{2}\tan\theta}=-V_\infty\sin2\theta=-0.2147V_\infty$$

水平分速度

$$u=\frac{\partial\phi}{\partial x}=\frac{Q}{2\pi}\frac{x}{x^2+y^2}+V_\infty=\frac{Q}{2\pi}\frac{\cos\theta}{r}+V_\infty=\frac{V_\infty\sin\theta\cos\theta}{\pi-\theta}+V_\infty$$

该点处的合速度为

$$V=\sqrt{u^2+v^2}=\sqrt{\left(\frac{V_\infty\sin\theta\cos\theta}{\pi-\theta}+V_\infty\right)^2+\left(\frac{V_\infty\sin^2\theta}{\pi-\theta}\right)^2}$$

$$=V_\infty\sqrt{\frac{\sin^2\theta\cos^2\theta+2\sin\theta\cos\theta(\pi-\theta)+(\pi-\theta)^2+\sin^4\theta}{(\pi-\theta)^2}}$$

$$=V_\infty\sqrt{\frac{\sin^2\theta-2\sin\theta\cos\theta\cdot\frac{1}{2}\tan\theta}{(\pi-\theta)^2}+1}=V_\infty.\square$$

3.12 **解**：因为 $\psi=100y\left(1-\frac{25}{r^2}\right)+\frac{628}{2\pi}\ln\frac{r}{5}=100y\left(1-\frac{25}{x^2+y^2}\right)+\frac{628}{2\pi}\ln\frac{\sqrt{x^2+y^2}}{5}$，因此

求导可得

$$\begin{cases} u=\dfrac{\partial\psi}{\partial y}=100-\dfrac{2500\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}+\dfrac{628y}{2\pi\left(x^2+y^2\right)}=\dfrac{y}{625}\left(5000y+\dfrac{15700}{2\pi}\right) \\ v=-\dfrac{\partial\psi}{\partial x}=\dfrac{5000xy}{\left(x^2+y^2\right)^2}-\dfrac{628x}{2\pi\left(x^2+y^2\right)}=\dfrac{x}{625}\left(5000y+\dfrac{15700}{2\pi}\right) \end{cases}$$

解得驻点

$$(x,y)=(4.9750,\ -0.4997),\ (x,y)=(-4.9750,\ -0.4997)$$

即

$$\theta=-5.74°,\ \theta=174.26°$$

因为

$$\psi=100y\left(1-\frac{25}{r^2}\right)+\frac{628}{2\pi}\ln\frac{r}{5}=100y-\frac{2500y}{r^2}+\frac{628}{2\pi}\ln r+C$$

$$=V_\infty y-\frac{my}{r^2}+\frac{\Gamma_0}{2\pi}\ln r+C$$

比较系数可得

$$V_\infty=100\,\mathrm{m/s},\ \Gamma_0=628\,\mathrm{m^2/s}$$

由库塔-茹科夫斯基升力定理可得

$$L_\infty=\rho V_\infty\Gamma_0,\ R=0.\square$$

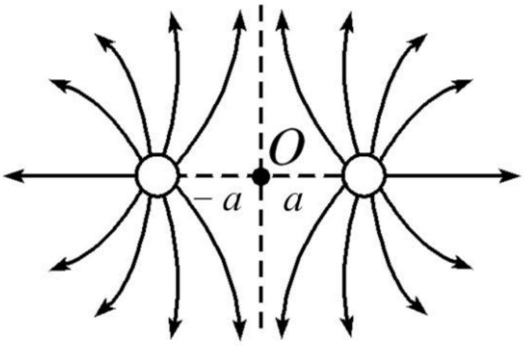
PPT.1 **解**：(1)点源的流函数为

$$\begin{cases} \phi_1=\dfrac{Q}{2\pi}\ln\sqrt{(x-a)^2+y^2} \\ \psi_1=\dfrac{Q}{2\pi}\arctan\dfrac{y}{x-a} \end{cases},\quad \begin{cases} \phi_2=\dfrac{Q}{2\pi}\ln\sqrt{(x+a)^2+y^2} \\ \psi_2=\dfrac{Q}{2\pi}\arctan\dfrac{y}{x+a} \end{cases}$$

由叠加原理可得

$$\begin{cases} \phi=\phi_1+\phi_2=\dfrac{Q}{2\pi}\ln\sqrt{\left[(x-a)^2+y^2\right]\left[(x+a)^2+y^2\right]} \\ \psi=\psi_1+\psi_2=\dfrac{Q}{2\pi}\left(\arctan\dfrac{y}{x-a}+\arctan\dfrac{y}{x+a}\right) \end{cases}$$

(2)图像如下



(3)一个平静的水面有两个相距为 $2a$ 的出水口向四周均匀排水的水的实际流动.□

第五章 高速可压流动

5.3 **解**：取高度为海平面有$\gamma=1.4$，$\rho=1.225\,\mathrm{kg/m^3}$，$p=101325\,\mathrm{Pa}$，$a=340.3\,\mathrm{m/s}$. 由等熵关系式可得

$$p_0=p\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

利用不可压流伯努利方程可得

$$V_1=\sqrt{\frac{2(p_0-p)}{\rho}}=\sqrt{\frac{2p\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}-1\right]}{\rho}}$$

$$=\sqrt{\frac{2\cdot 101325\cdot\left[\left(1+\frac{1.4-1}{2}\cdot 0.6^2\right)^{\frac{1.4}{1.4-1}}-1\right]}{1.225}}\,\mathrm{m/s}=213.486\,\mathrm{m/s}$$

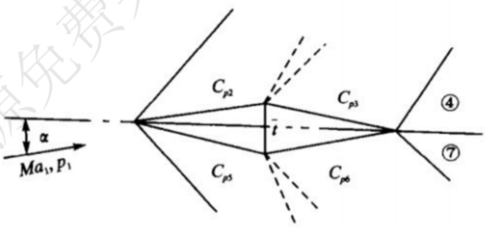
其真实速度V_2可由马赫数可得，即

$$V_2=aM=340.3\cdot 0.6\,\mathrm{m/s}=204.18\,\mathrm{m/s}$$

于是有

$$\Delta V=V_1-V_2=(213.486-204.18)\,\mathrm{m/s}=9.30582\,\mathrm{m/s}.\square$$

5.9 **解**：因为$\bar{t}=\frac{t}{c}=0.1$，所以菱形翼型夹角为



$$\beta=\arctan\frac{t}{c}=\arctan(0.1)=5.71°$$

波面 2：

因为超声速气流迎角为$\alpha=2°$，所以偏折角为

$$\varphi=\beta-\alpha=3.71°$$

已知$M_1=2$，利用偏折角和波角公式可得

$$\tan\varphi=\frac{M_1^2\sin^2\beta-1}{\left[1+M_1^2\left(\frac{\gamma+1}{2}-\sin^2\beta\right)\right]\tan\beta}\Longleftrightarrow\tan 3.71°=\frac{4\sin^2\beta-1}{(5.8-4\sin^2\beta)\tan\beta}$$

解得

$$\beta = 33.13°$$

则波后马赫数为

$$M_{2-3} = \frac{1}{\sin(\beta-\varphi)}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2\sin^2\beta}{\gamma M_1^2\sin^2\beta-\frac{\gamma-1}{2}}}$$

$$= \frac{1}{\sin(33.13°-3.71°)}\sqrt{\frac{1+0.8\sin^2(33.13°)}{5.6\sin^2(33.13°)-0.2}} = 1.87$$

所以可得

$$C_{p_2} = \frac{2}{\sqrt{M_1^2-1}}\varphi + \frac{(\gamma+1)M_1^4-4(M_1^2-1)}{2(M_1^2-1)^2}\varphi^2$$

$$= \frac{2}{\sqrt{2^2-1}}\cdot\frac{\pi\cdot 3.71}{180} + \frac{(1.4+1)\cdot 2^4-4\cdot(2^2-1)}{2\cdot(2^2-1)^2}\cdot\left(\frac{\pi\cdot 3.71}{180}\right)^2$$

$$= 0.081$$

波面 5：

因为超声速气流迎角为$\alpha = 2°$，所以偏折角为

$$\varphi = \beta + \alpha = 7.71°$$

已知$M_1 = 2$，利用偏折角和波角公式可得

$$\tan\varphi = \frac{M_1^2\sin^2\beta-1}{\left[1+M_1^2\left(\frac{\gamma+1}{2}-\sin^2\beta\right)\right]\tan\beta} \Longleftrightarrow \tan 7.71° = \frac{4\sin^2\beta-1}{(5.8-4\sin^2\beta)\tan\beta}$$

解得

$$\beta = 36.92°$$

则波后马赫数为

$$M_{2-6} = \frac{1}{\sin(\beta-\varphi)}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2\sin^2\beta}{\gamma M_1^2\sin^2\beta-\frac{\gamma-1}{2}}}$$

$$= \frac{1}{\sin(36.92°-7.71°)}\sqrt{\frac{1+0.8\sin^2(36.92°)}{5.6\sin^2(36.92°)-0.2}} = 1.72$$

所以可得

$$C_{p_5} = \frac{2}{\sqrt{M_1^2-1}}\varphi + \frac{(\gamma+1)M_1^4-4(M_1^2-1)}{2(M_1^2-1)^2}\varphi^2$$

$$= \frac{2}{\sqrt{2^2-1}}\cdot\frac{\pi\cdot 7.71}{180} + \frac{(1.4+1)\cdot 2^4-4\cdot(2^2-1)}{2\cdot(2^2-1)^2}\cdot\left(\frac{\pi\cdot 7.71}{180}\right)^2$$

$$= 0.182$$

波面 3：

偏转角$\theta=2\beta=11.42°$，利用马赫波的压强系数公式可得

$$C_{p_3}=-\frac{2}{\sqrt{M_{2-3}^2-1}}\theta+\frac{(\gamma+1)M_{2-3}^4-4(M_{2-3}^2-1)}{2(M_{2-3}^2-1)^2}\theta^2$$
$$=-\frac{2}{\sqrt{1.87^2-1}}\cdot\frac{\pi\cdot 11.42}{180}+\frac{(1.4+1)\cdot 1.87^4-4\cdot(1.87^2-1)}{2\cdot(1.87^2-1)^2}\cdot\left(\frac{\pi\cdot 11.42}{180}\right)^2$$
$$=-0.191$$

波面 6：

偏转角$\theta=2\beta=11.42°$，利用马赫波的压强系数公式可得

$$C_{p_6}=-\frac{2}{\sqrt{M_{2-6}^2-1}}\theta+\frac{(\gamma+1)M_{2-6}^4-4(M_{2-6}^2-1)}{2(M_{2-6}^2-1)^2}\theta^2$$
$$=-\frac{2}{\sqrt{1.72^2-1}}\cdot\frac{\pi\cdot 11.42}{180}+\frac{(1.4+1)\cdot 1.72^4-4\cdot(1.72^2-1)}{2\cdot(1.72^2-1)^2}\cdot\left(\frac{\pi\cdot 11.42}{180}\right)^2$$
$$=-0.217$$

5.11 **解：**取一矩形微元扰动面一维分析，以扰动面为参考系，设气流以ρ_1，V_1穿过波面后变为ρ_2，V_2，流体为不可压理想定常流体，应用不可压连续流体方程有

$$\rho_1 V_1 A=\rho_2 V_2 A\Longleftrightarrow V_2=\frac{\rho_1}{\rho_2}V_1$$

由定常无黏流动问题(不计彻体力)积分形式的动量方程可得

$$\oiint_S\left(\rho\vec{V}\cdot \mathrm{d}\vec{S}\right)\vec{V}=-\oiint_S p\,\mathrm{d}\vec{S}$$
$$\rho_2 V_2^2-\rho_1 V_1^2=-(p_2-p_1)$$

联立可得

$$V_1=\sqrt{\frac{\rho_2}{\rho_1}\cdot\frac{p_1-p_2}{\rho_1-\rho_2}}=\sqrt{\frac{\rho_1+\Delta\rho}{\rho_1}\cdot\frac{\Delta p}{\Delta\rho}}=\sqrt{\frac{\Delta p}{\Delta\rho}\left(1+\frac{\Delta\rho}{\rho_1}\right)}$$

于是可得

$$U_a=\sqrt{\frac{\Delta p}{\Delta\rho}\left(1+\frac{\Delta\rho}{\rho}\right)}$$

如果扰动很弱，有$\Delta\rho$，$\Delta p\to 0$，于是可导出声速公式

$$a=\sqrt{\frac{\Delta p}{\Delta\rho}}=\sqrt{\left(\frac{\partial p}{\partial\rho}\right)_s}=\sqrt{\left(\frac{\mathrm{d}p}{\mathrm{d}\rho}\right)_s}.\square$$

PPT.2 **解：**直接利用公式可得

$$M=\sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}}-1\right]}=\sqrt{5\left[\left(\frac{1.186}{1}\right)^{\frac{2}{7}}-1\right]}=0.5$$

由 Bernoulli 方程可得

$$V=\sqrt{\frac{2(p_0-p)}{\rho}}=\sqrt{\frac{2\cdot(1.186-1)\cdot 101325}{1.225}}\,\text{m/s}=175.413\,\text{m/s}$$

又因为声速公式有

$$a=\sqrt{\gamma\frac{p}{\rho}}=\sqrt{1.4\cdot\frac{1\cdot 101325}{1.225}}\,\text{m/s}=340.294\,\text{m/s}$$

所以马赫数为

$$M=\frac{V}{a}=\frac{175.413}{340.294}=0.52.\square$$