

# STA305 Assignment 4

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## 1 Description of the design

I decided to conduct an experiment to investigate the effect of three independent factors on the flight distance of a paper airplane. The three factors are Paper Type (Printed vs. Newspaper), Fold Type (Dart vs. Standard), and Launch Height (Standing vs. Sitting). My experiment follows a  $2^3$  factorial design, and I chose to conduct two replications, resulting in a total of 8 runs with 16 observations. The dependent variable of interest is the distance flown by the plane, measured in centimeters.

A factorial design will allow me to study the effect of the three factors with two levels each on the flight distance. Testing all the main and interaction effects will provide me with a more comprehensive understanding of the influence on the flight distance. However, given that a single replication is not reliable as it fails to provide estimates of error variance of the effects from replicated runs, I chose to include two replications to account for this.

I chose to investigate paper airplanes because, ever since I was a kid, I've been fascinated by how to make the best paper airplane that could fly farther than those of my friends. This assignment allows me to revisit my interest with a more analytical lens to determine which factors actually produce the greatest flight distance. I specifically consider the combinations of material, fold, and launch technique in my study. In particular, I am interested in determining whether there is a notable interaction effect. Specifically, I want to test whether paper type matters more when folded a certain way, as this is what I believed as a kid and my hypothesis is that it does.

## 2 Analysis of the data

We begin our analysis by summarizing the data in an appropriate table. I call the factor Paper Type "A" and let Newspaper be represented by a 1 when it's chosen and -1 when Printed paper is chosen. Similarly, I call the factor Fold Type "B" and let Standard fold be represented by a 1 when chosen and -1 when the Dart is chosen. Lastly, I call the factor Launch Height "C" and represent Sitting with a 1 and Standing with a -1. Table 1. summarizes the first 3 runs as a table while also including the estimated variances.

Table 1: Factorial Combinations

| Run | A  | B  | C  | AB | AC | BC | ABC | Rep1   | Rep2   | Var  |
|-----|----|----|----|----|----|----|-----|--------|--------|------|
| 1   | -1 | -1 | -1 | 1  | 1  | 1  | -1  | 507.48 | 504.31 | 5.02 |
| 2   | -1 | -1 | 1  | 1  | -1 | -1 | 1   | 498.24 | 502.62 | 9.59 |
| 3   | -1 | 1  | -1 | -1 | 1  | -1 | 1   | 423.83 | 424.98 | 0.66 |

To determine the main effect (the effect of a single factor on the flight distance independent of all other factors), I calculate the statistics given by  $ME(H) = \bar{y}(H+) - \bar{y}(H-)$  where H is the symbol for a given factor: A, B, and C in this case. Doing this for all factors yields the result of Both replication 1 and replication 2

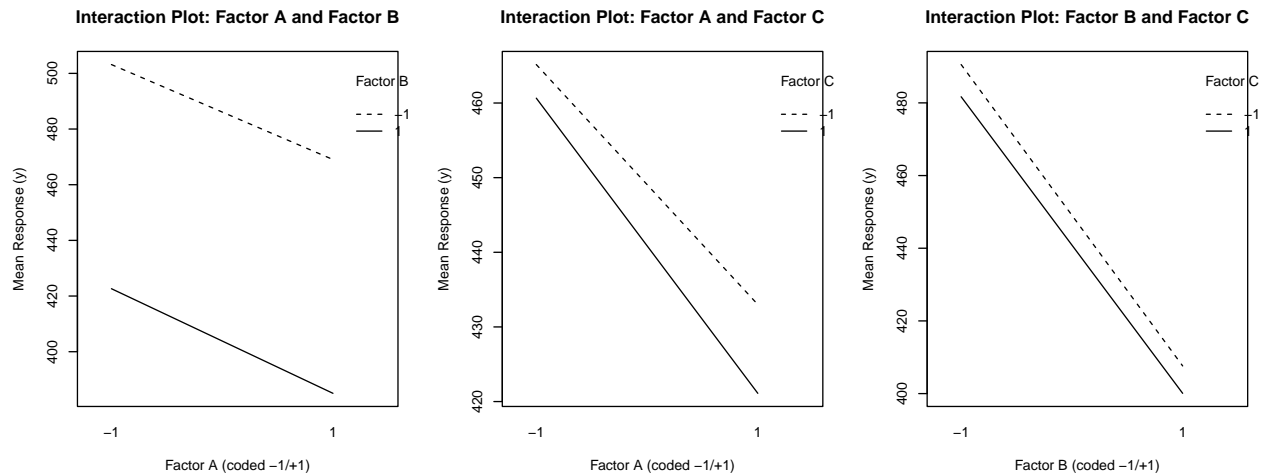
Table 2: Replication 1

| Factor | Mean_Positive | Mean_Negative | Main_Effect |
|--------|---------------|---------------|-------------|
| A      | 426.98        | 463.1125      | -36.1325    |
| B      | 404.83        | 485.2625      | -80.4325    |
| C      | 440.05        | 450.0425      | -9.9925     |

Table 3: Replication 2

| Factor | Mean_Positive | Mean_Negative | Main_Effect |
|--------|---------------|---------------|-------------|
| A      | 427.125       | 462.6875      | -35.5625    |
| B      | 402.860       | 486.9525      | -84.0925    |
| C      | 441.705       | 448.1075      | -6.4025     |

Given that all the main effects are negative for both of the two replications, the results are interpreted to mean that for factor A (Paper Type), choosing Newspaper leads to a shorter average flight distance compared to Printed paper. Similarly for factor B (Fold Type), choosing a Standard fold leads to a shorter average flight distance compared to a dart fold. Lastly, for factor C (Launch Height), choosing to launch the airplane while sitting leads to shorter flight distances compared to standing. Therefore, Printed paper, Dart folds, and Standing launches contributes more positively to the flight distance than their alternatives. The image below shows the interaction effects for the 3 factors.



The interaction plot for Factors A and B reveals evidence of a slight interaction effect. This means that going from printed to newspaper reduces the flight distance of the paper plane; however, the degree to which it decreases depends on whether one stands or sits. Standing reduces the decline in flight distance more than sitting. However, the interactions between A and C, as well as B and C, do not show any evidence of an interaction effect, given that the lines are parallel in the interaction plot. Additionally the pooled estimated variance for the effect which is calculated as the pooled estimated variance divided by 4 is equal to 27.83.

### 3 Conclusions

In this experiment I wanted to investigate whether the three factors (paper type, fold type, and launch height) affect the flight distance of a paper airplane. I used a  $2^3$  factorial design with two replication to test this effect. My analysis resulted in the conclusion that Printed paper, Dart folds, and Standing launches each contribute to greater flight distances compared to their alternatives. This is supported by the fact that all main effects were negative, indicating performance differences across levels. Additionally, observing the interactions revealed that only the interaction between paper type and fold type showed signs of a possible interaction effect. The other interactions did not suggest significant combined influence. Finally, the estimated pooled error variance was calculated to be 27.83, allowing for future confidence interval construction and significance testing.