BIOM-5405-Assig2

October 17, 2024

$\mathbf{Q}\mathbf{1}$

a

Plot histogram of T (temperature) for both SICK and HEALTHY patients on the same axis. Does temperature appear to be strongly correlated with PatientStatus (SICK vs.HEALTHY)?

Answer

So since the histogram of sick and healthy have a high overlapping area, temperature doesn't **strongly** correlate with Patient_Status but a **moderate** one. Though the SICK patients tend to have slightly higher temperatures overall, there is some variation in temperature between the two groups. It does not appear to be strongly correlated with patient status. Both SICK and HEALTHY patients exhibit a wide range of temperatures, which suggests that temperature alone may not be a strong predictor of whether a patient is sick or healthy.

I have also performed an additional quantitative test to check if my assumptions are correct using Pearson correlation.

Pearson Correlation: 0.442267537907381, p-value: 5.5025170014598154e-11

- A value of 0.44 suggests a moderate positive correlation between temperature and status of being sick or healthy. This means that, as temperature increases, the likelihood of a patient being classified as "SICK" also increases. However, the correlation is not very strong—there is still a significant amount of variability in patient status that is not explained by temperature alone.
- The p-value is extremely small. There is a very low probability that this correlation is due to random chance. Therefore, we can be confident that temperature is related to patient status, even though the **strength** of the relationship is **moderate**.

```
data = patient_data.copy()

data['Status_Binary'] = data['Patient_Status'].apply(lambda x: 1 if x == 'SICK' else 0)

# Calculate Pearson correlation
correlation, p_value = pearsonr(data['Status_Binary'], data['T'])

print(f"Pearson Correlation: {correlation}, p-value: {p_value}")
```

```
# Separate the data based on Patient_Status
sick_patients = data[data['Patient_Status'] == 'SICK']
healthy_patients = data[data['Patient_Status'] == 'HEALTHY']

# Plot the histograms
plt.figure(figsize=(10, 6))
plt.hist(sick_patients['T'], bins=20, alpha=0.7, label='SICK', color='red')
plt.hist(healthy_patients['T'], bins=20, alpha=0.7, label='HEALTHY', color='blue')
plt.xlabel('Temperature (T)')
plt.ylabel('Frequency')
plt.title('Histogram of Temperature for SICK vs HEALTHY Patients')
plt.legend()
plt.show()
```

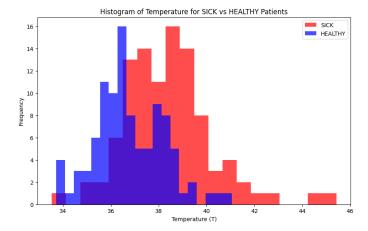


Figure 1: Q1a

b)

Now create an ordinal variable called T_ORD that maps T data as follows:

$$T \le 37 \implies T_{ord} = Normal$$

 $37 < T \le 38.5 \implies T_{ord} = Fever$

$$T > 38.5 \implies T_{ord} = Danger$$

Plot a histogram of T_ORD showing SICK and HEALTHY patients separately on the same axis. The x-axis categories should be ordered as NORMAL, FEVER, DANGER.

Code and Output

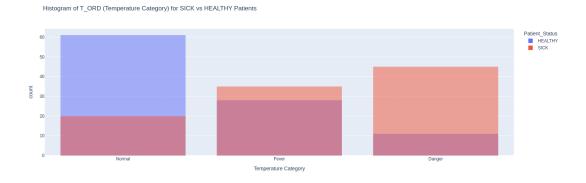


Figure 2: Q1b

c)

Create (and report) a contingency table for T_ORD vs. PatientStatus and use a X^2 test to check if T_ORD is significantly associated with PatientStatus. Report your null hypothesis H0 (15 words), your alternate hypothesis H1, your X^2 value, your degrees of freedom, your p-value, and your conclusion (75 words).

Answer

- Null Hypothesis (H0):
 - T_ORD and Patient_Status are independent.
- Alternate Hypothesis (H1):

T_ORD and Patient_Status are not independent, meaning there is a significant association between them.

Conclusion: Since the p-value is much smaller than the common threshold of 0.05, we can reject the null hypothesis.

There is a significant association between T_ORD and Patient_Status. (T_ORD) is strongly associated with whether they are healthy or sick.

Code and Output

```
# Create a contingency table for T_ORD vs Patient_Status
contingency_table = pd.crosstab(patient_data ['T_ORD'], patient_data['Patient_Status'])

# Perform the Chi-square test
chi2_stat, p_value, dof, expected = chi2_contingency(contingency_table)

print("Contingency_Table:")
print(contingency_table)
print("nChi-square Statistic:", chi2_stat)
print("Degrees of Freedom:", dof)
print("P-value:", p_value)
```

```
Patient_Status HEALTHY SICK
T_ORD
Normal 61 20
Fever 28 35
Danger 11 45

Chi-square Statistic: 42.173721340388006
Degrees of Freedom: 2
P-value: 6.951727985959681e-10
```

d)

Provide box plots for RR for each of the three T_ORD values pooling SICK and HEALTHY patients together. Based on the box plots, what can you infer about about the relationship between T_ORD and RR?

Answer

It can be observed that the median respiratory rate increases with higher temperature categories. This means that there is a relationship between increased temperature (fever or danger levels) and respiratory rate.

More analysis on the plots:

First, the overlap can be observed in all three categories.

- Normal: It has a narrow interquartile range (IQR), meaning most of the normal people (50% of the data) have the respiratory rate (RR) between [19.5 21]. From the upper whisker (75th percentile), we can see that there are some cases where normal people have a higher RR than 50% of the norm. We can also see some **outliers** in this category.
- Fever: The IQR is wider here, meaning 50% of the people labeled with fever have an RR between [20 22]. The upper and lower whiskers almost cover the entire RR range.
- Danger: 50% of the people have an RR between [20.7 22.5].

Although the RR increases as the temperature increases, these three categories have significant overlapping RR.

```
# Create a box plot using Plotly
fig = px.box(patient_data, x='T_ORD', y='RR', title='Box Plot of RR for Each T_ORD Category',
labels={'T_ORD': 'T_ORD (Temperature Categories)', 'RR': 'Respiratory Rate (RR)'}, points="all")
# Show the plot
fig.show()
```



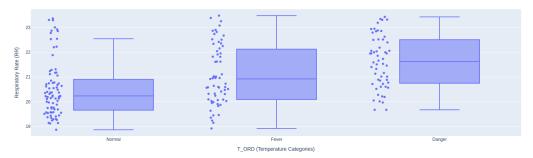


Figure 3: Q1d

e)

Compute the inter-quartile range and the "20% trimmed mean" of RR for both HEALTHY and SICK patients. (20% means dropping the top and bottom 10% of samples)

Answer

I have also plotted the probability distribution of Sick-labeled Respiratory Rate and healthy-labeled, Fig 4. The high variaty between Sick RR is highly noticable and can show that it is drawn from multinomial distribution.

```
def calculate_trimmed_mean_20(data): #(trimming 10% from both ends)
    return trim_mean(data, proportiontocut=0.1)

RR_sick = patient_data[patient_data['Patient_Status'] == 'SICK']['RR']
RR_healthy = patient_data[patient_data['Patient_Status'] == 'HEALTHY']['RR']

iqr_healthy = RR_healthy.quantile(0.75) - RR_healthy.quantile(0.25)
iqr_sick = RR_sick.quantile(0.75) - RR_sick.quantile(0.25)

print("IQR Healthy:", iqr_healthy)
print("IQR Sick:", iqr_sick)

trimmed_mean_healthy = calculate_trimmed_mean_20(RR_healthy)
trimmed_mean_sick = calculate_trimmed_mean_20(RR_sick)

print("20% Trimmed Mean Healthy:", trimmed_mean_healthy)
print("20% Trimmed Mean Sick:", trimmed_mean_sick)
```

```
IQR Healthy: 0.7953133422502212
IQR Sick: 1.6551857952876503
20% Trimmed Mean Healthy: 20.030955486181114
20% Trimmed Mean Sick: 21.975704253840554
```

```
df1 = RR_sick.copy()
df2 = RR_healthy.copy()
mean1 = df1.mean()
variance1 = df1.var()
mean2 = df2.mean()
variance2 = df2.var()
fig, axes = plt.subplots(1, 2, figsize=(14, 6))
sns.histplot(df1, bins=30, kde=True, stat='density', color='red', alpha=0.6, ax=axes[0])
axes[0].axvline(mean1, color='red', linestyle='--', label=f'Mean: {mean1:.2f}')
axes[0].axvline(mean1 + np.sqrt(variance1), color='green',
               linestyle='--', label=f'Mean + 1 Std Dev: {mean1 + np.sqrt(variance1):.2f}')
axes[0].axvline(mean1 - np.sqrt(variance1), color='green',
                linestyle='--', label=f'Mean - 1 Std Dev: {mean1 - np.sqrt(variance1):.2f}')
axes[0].set_title('Distribution of Sick RR')
axes[0].set_xlabel('RR')
axes[0].set_ylabel('Density')
axes[0].legend()
axes[0].grid()
sns.histplot(df2, bins=30, kde=True, stat='density', color='green', alpha=0.6, ax=axes[1])
axes[1].axvline(mean2, color='red', linestyle='--', label=f'Mean: {mean2:.2f}')
axes[1].axvline(mean2 + np.sqrt(variance2), color='green',
                linestyle='--', label=f'Mean + 1 Std Dev: {mean2 + np.sqrt(variance2):.2f}')
axes[1].axvline(mean2 - np.sqrt(variance2), color='green',
               linestyle='--', label=f'Mean - 1 Std Dev: {mean2 - np.sqrt(variance2):.2f}')
axes[1].set_title('Distribution of Health RR')
axes[1].set_xlabel('RR')
axes[1].set_ylabel('Density')
axes[1].legend()
axes[1].grid()
plt.tight_layout()
plt.show()
```

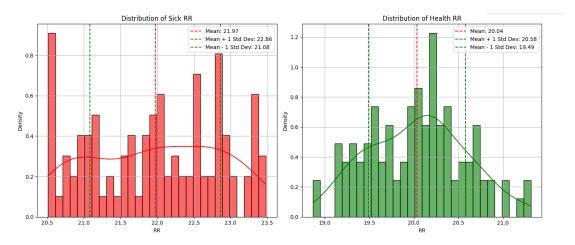


Figure 4: Q1e

f)

Using bootstrapping, compute the 90% confidence interval of the "20% trimmed mean" of RR for HEALTHY patients.

Follow Procedure 5.6 from Cohen's text:

- 1. Construct a distribution from K bootstrap samples for a statistic u;
- 2. Sort the values in the distribution;
- 3. The lower bound of the 90% confidence interval is the $(K \times 0.05)$ th value, the upper bound is the $(K \times 0.95)$ th value in the sorted distribution.

(Here, u is the observed trimmed mean and a bootstrap sample will consist of 200 samples drawn with replacement from RR.)

- i) Report your value of K, and your 90% confidence interval.
- ii) Does your estimate from part e) fall within the interval computed in part f)?
- iii) If you wanted this to fail (i.e., you want to come up with an interval that only occasionally contains the true value), what would you change?

Answer

The 90% confidence interval for the 20% trimmed mean of RR for HEALTHY patients, based on **1000** bootstrap samples, is:

For 200 samples

Lower bound: 19.96Upper bound: 20.10

For 100 samples

Lower bound: 19.93Upper bound: 20.12

Now, addressing the remaining parts:

- ii) The trimmed mean calculated in part (e) was 20.03, and it falls within this confidence interval. See the plot.
- iii) If you wanted to create a confidence interval that only occasionally contains the true value (making it less reliable), you could:
 - Reduce the confidence level, which would result in a narrower interval, increasing the chance that it excludes the true value.
 - Use fewer bootstrap samples (reduce K), which would increase variability in the results and make the confidence interval less precise.
 - Reduce the resampling size and force the samples to be less representative of the population.
 - Increasing the Trimming Proportion

```
def bootstrap_conf_interval(data, num_boots_samples, resampling_size,
                            actual_trimmed_mean ,
                            conf_l_l, conf_l_u):
 # Set the number of bootstrap samples (K)
 K = num_boots_samples
  # Generate K bootstrap samples and calculate the trimmed mean for each sample
 bootstrap_means = []
 for _ in range(K):
     bootstrap_sample = np.random.choice(data, size=resampling_size, replace=True)
      trimmed_mean = calculate_trimmed_mean_20(bootstrap_sample)
     bootstrap_means.append(trimmed_mean)
 # Sort the bootstrap means
 bootstrap_means_sorted = sorted(bootstrap_means)
 # Calculate the 90% confidence interval
 lower_bound = np.percentile(bootstrap_means_sorted, conf_l_l)
 upper_bound = np.percentile(bootstrap_means_sorted, conf_l_u)
 # Display the confidence interval
 print(lower_bound, upper_bound)
 # Plot the bootstrap distribution
 plt.figure(figsize=(10, 6))
 plt.hist(bootstrap_means, bins=30, color='skyblue', alpha=0.7, edgecolor='black')
 # Add vertical lines for the trimmed mean from part (e) and the confidence interval
 plt.axvline(x=actual_trimmed_mean, color='red', linestyle='--',
                label=f'Trimmed Mean (part e): {trimmed_mean_healthy:.2f}')
 plt.axvline(x=lower_bound, color='green', linestyle='--',
                label=f'90% CI Lower Bound: {lower_bound:.2f}')
 plt.axvline(x=upper_bound, color='green', linestyle='--',
               label=f'90% CI Upper Bound: {upper_bound:.2f}')
 # Add labels and title
 plt.title('Bootstrap Distribution of 20% Trimmed Mean of RR for HEALTHY Patients')
 plt.xlabel('20% Trimmed Mean of RR')
 plt.ylabel('Frequency')
 plt.legend()
 # Show the plot
 plt.show()
```

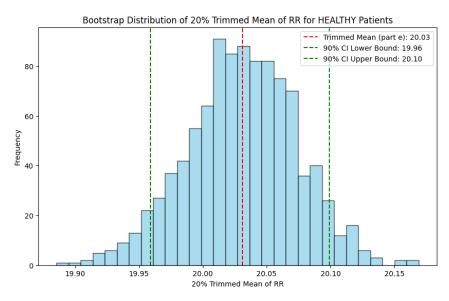


Figure 5: Q1f - 200 samples

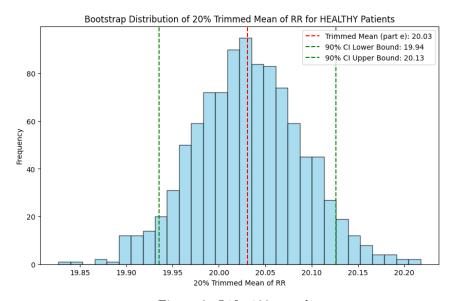


Figure 6: Q1f - 100 samples

 \mathbf{g}

Examine the RR feature for all patients with T < 38.6 degrees. Do the feature data contain outliers? Describe how you tested this and what conclusions you drew. How did the mean and median of RR change with the outliers (if any) removed? (50 words + calculations)

Answer

To examine the RR feature for patients with T < 38.6 and detect outliers, I applied the IQR method for each temperature category (T_ORD). Outliers were defined as values falling outside 1.5 times the interquartile range (IQR). After removing outliers, I compared the mean and median RR before and after the removal.

IQR Method:

I calculated the 25th and 75th percentiles (Q1 and Q3) and identified any values beyond the bounds:

$$[Q1-1.5\times IQR, Q3+1.5\times IQR]$$

Outliers Found:

The "Normal" category showed some outliers.

Results:

- Before outlier removal: Mean = 20.79, Median = 20.54.
- After outlier removal: The mean and median did not change significantly.

Outliers in the "Normal" group had little impact on the overall distribution, indicating that the dataset is fairly robust to their influence.

Code and Output

Displaying data in different methods:

Histogram of T_ORD (Temperature Category) for SICK vs HEALTHY Patients



Figure 7

Box Plot of RR for Each T_ORD Category

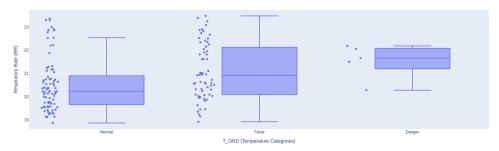


Figure 8

```
df = patient_data_less_38.copy()
mean = df['RR'].mean()
variance = df['RR'].var()
plt.figure(figsize=(10, 6))
sns.histplot(df['RR'], bins=30, kde=True, stat='density', color='blue', alpha=0.6)
# Adding mean and variance lines
plt.axvline(mean, color='red', linestyle='--', label=f'Mean: {mean:.2f}')
plt.axvline(mean + np.sqrt(variance), color='green',
            linestyle='--', label=f'Mean + 1 Std Dev: {mean + np.sqrt(variance):.2f}')
plt.axvline(mean - np.sqrt(variance), color='green',
           linestyle='--', label=f'Mean - 1 Std Dev: {mean - np.sqrt(variance):.2f}')
# Adding labels and title
plt.title('Probability Distribution with Mean and Variance')
plt.xlabel('Values')
plt.ylabel('Density')
plt.legend()
plt.grid()
# Show the plot
plt.show()
```

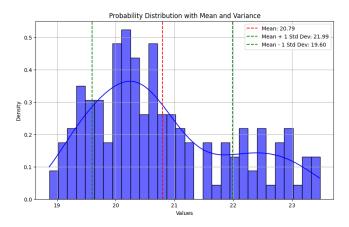


Figure 9

Before and After Analysis

```
filtered_data = patient_data_less_38.copy()
outliers_by_category = {}
non_outliers_data = filtered_data.copy()
for category in filtered_data['T_ORD'].unique():
    category_data = filtered_data[filtered_data['T_ORD'] == category]['RR']
    q1 = category_data.quantile(0.25)
    q3 = category_data.quantile(0.75)
    iqr = q3 - q1
    lower_bound = q1 - 1.5 * iqr
    upper_bound = q3 + 1.5 * iqr
    outliers = category_data[(category_data < lower_bound) | (category_data > upper_bound)]
    outliers_by_category[category] = outliers
    non_outliers_data = non_outliers_data[~(non_outliers_data['RR'].isin(outliers))]
mean_before = filtered_data['RR'].mean()
median_before = filtered_data['RR'].median()
mean_after = non_outliers_data['RR'].mean()
median_after = non_outliers_data['RR'].median()
print(f"Mean RR before removing outliers: {mean_before}")
print(f"Median RR before removing outliers: {median_before}")
print(f"Mean RR after removing outliers: {mean_after}")
print(f"Median RR after removing outliers: {median_after}")
# Display outliers by category
for category, outliers in outliers_by_category.items():
    print(f"Outliers in {category} category:")
    print(outliers)
```

```
Mean RR before removing outliers: 20.794120522133536
Median RR before removing outliers: 20.53982386054529
Mean RR after removing outliers: 20.66885189781746
Median RR after removing outliers: 20.44597370699531
Outliers in Normal category:
     23.007225
21
     23.307529
     23.364805
82
    23.295096
22.922676
125
133
138 22.870931
169 22.780123
198 22.858162
Outliers in Fever category:
[] None
Outliers in Danger category:
    20.278145
```

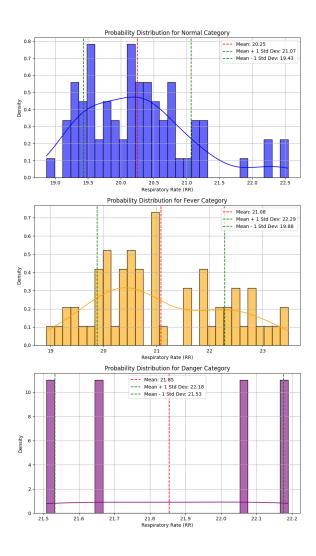


Figure 10: I created these figures to illustrate how indistinct the data is across the three categories, especially for the Danger category.

h)

What is the Spearman Rank Correlation between T and RR for HEALTHY patients? Using randomization (or permutation), test whether these two variables are significantly correlated. Briefly describe how you did this. What is H0? What is H1? Did you complete a 1- or 2-tailed test and why? What p-value did you obtain? What conclusion do you draw? (50 words)

Answer and Code

• Null Hypothesis (H0):

The temperature of healthy samples and their respiratory rate are independent.

• Alternate Hypothesis (H1):

The temperature of healthy samples and their respiratory rate are **not** independent, meaning there is a significant association between them.

Let's apply the permutation test (randomly shuffle ranks of y, and re-compute ρ), which in this case enforces the null hypothesis.

Conclusion:

This was a two-tailed test because we were interested in whether there was any significant correlation, positive or negative. The p-value is very high (0.982), so we fail to reject H0. There is no significant correlation between T and RR for healthy patients.

Furthermore, 95% of the data is less than -|p| and greater than |p|. Therefore, we can **reject** the null hypothesis at 3% confidence level.

```
t_healthy = patient_data[patient_data.Patient_Status=="HEALTHY"]['T']
rr_healthy = patient_data[patient_data.Patient_Status=="HEALTHY"]['RR']

spearman_cor, spearman_p_val = spearmanr(t_healthy, rr_healthy)
print(f"Spearmans Cor: {spearman_cor}\np_value: {spearman_p_val}")
```

```
Spearmans Cor: 0.0026522652265226526
p_value: 0.9791063989856768
```

```
# Prepare for plotting
num_permutations = 1000
extreme_values = []
# Running the randomization test again to collect the correlation values
for _ in range(num_permutations):
    y_values = healthy_data['RR'].values.copy()
    np.random.shuffle(y_values) # Shuffle y
    permuted_corr = spearmanr(healthy_data['T'], y_values)[0]
    extreme_values.append(permuted_corr)
# Convert to a NumPy array for easier handling
extreme_values = np.array(extreme_values)
# Count how many times a value as extreme as the observed correlation was observed
extreme_count_gt = np.sum(extreme_values > abs(observed_spearman_corr_fixed))
extreme_count_lt = np.sum(extreme_values < -(observed_spearman_corr_fixed))</pre>
# Plotting
plt.figure(figsize=(10, 6))
sns.histplot(extreme_values, bins=30, kde=True, color='lightblue',
             stat='density', label='Permutation Distribution')
plt.axvline(-(observed_spearman_corr_fixed), color='red', linestyle='--',
             label='Observed Correlation ( = {:.4f})'.format(observed_spearman_corr_fixed))
plt.axvline(abs(observed_spearman_corr_fixed), color='red', linestyle='--',
             label='Observed Correlation (- = {:.4f})'.format(observed_spearman_corr_fixed))
# Adding labels and legend
plt.title('Distribution of Spearman Correlation from Randomization Test')
plt.xlabel('Spearman Correlation Coefficient')
plt.ylabel('Density')
plt.legend()
plt.grid()
# Show plot
plt.show()
```

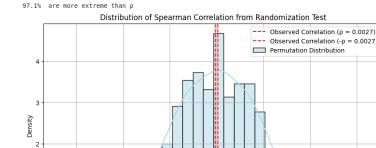


Figure 11: Q1h

$\mathbf{Q2}$

Let's use T_{ORD} alone to create a simple classifier. We will apply a tunable threshold to this ordinal feature

a) Generate a confusion matrix for the threshold of:

if
$$T_{ORD} = \text{NORMAL} \rightarrow \text{PatientStatus} = \text{HEALTHY}$$
 else $\rightarrow \text{PatientStatus} = \text{SICK}$

b) Including the threshold from part a), how many distinct thresholds are there? How many vertices will appear in an ROC curve?

Answer fro a and b

The key idea here is how the thresholds are used to determine the classification:

Distinct Threshold Values:

In this case, T_{ORD} has three possible values: "Normal," "Fever," and "Danger."

However, our classification rule does not treat all three values independently. We have grouped "Fever" and "Danger" as the same outcome ("SICK"), while "Normal" results in "HEALTHY." Therefore, there is effectively one distinct threshold:

"Normal" versus "Otherwise" (which includes "Fever" and "Danger").

This is essentially a binary classifier based on the cutoff between "Normal" and "Fever/Danger". Therefore, we can't make a new threshold between "Fever" and "Danger" because both of these would be classified as "Sick."

Vertices in the ROC Curve:

In the context of an ROC curve, a vertex represents a specific threshold where the classifier performance is evaluated. To create a complete ROC curve, we also need to include the extreme cases:

- Classify everyone as "SICK": This represents the threshold where no one is classified as "HEALTHY."
- Use $T_{ORD} ==$ "Normal" as the threshold.
- Classify everyone as "HEALTHY": This represents the threshold where no one is classified as "SICK."

Thus, when we discuss distinct thresholds, there is just one threshold to vary in the middle $(T_{ORD} = "Normal")$, but there are 3 vertices in the ROC curve because we also need to consider the extreme cases for completeness.

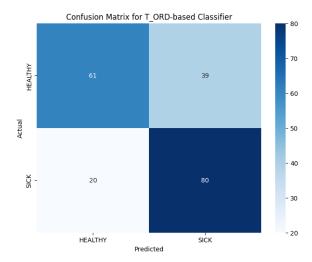


Figure 12: Q2a

b)

Plot the ROC curve for this classifier. Assume that SICK samples actually have class = 1 and HEALTHY samples actually have class = 0. Report the AUC value in the title of the plot.

```
# Encode the true labels as binary (HEALTHY = 0, SICK = 1)
true_labels = patient_data['Patient_Status'].map({'HEALTHY': 0, 'SICK': 1})

# Encode the predicted labels for the threshold T_ORD == 'Normal' as binary
predicted_scores = patient_data['T_ORD'].apply(lambda x: 0 if x == 'Normal' else 1)

# Compute the ROC curve
fpr, tpr, thresholds = roc_curve(true_labels, predicted_scores)

# Plot the ROC curve
plt.figure(figsize=(8, 6))
plt.plot(fpr, tpr, color='blue', lw=2, label=f'ROC Curve (AUC = {roc_auc:.2f})')
plt.plot([0, 1], [0, 1], color='grey', linestyle='--', lw=2) # Diagonal line for random guess
plt.xlabel('False Positive Rate (FPR)')
plt.ylabel('True Positive Rate (TPR)')
plt.title(f'ROC Curve for T_ORD-based Classifier (AUC = {roc_auc:.2f})')
plt.legend(loc='lower right')
plt.show()
```

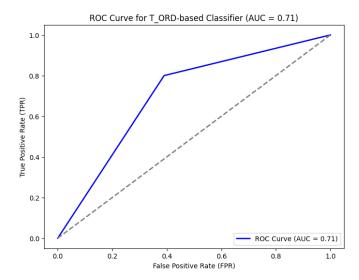


Figure 13: Q2b