Predmet: Pravděpodobnost a statistika 1
Ukol: 6.
Verze: 2.
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Zadani

Budte $X, Y, Z \sim Exp(\lambda)$ nezavisle nahodne veliciny

Jake je rozdeleni $X + Y$?

pomoci konvolunce:
$f_u(u) = \int_{-\infty}^{\infty} f_x(x) f_y(u - x) dx$
$f_u(u) = \int_0^u \lambda e^{-\lambda x} \cdot \lambda e^{-\lambda(u-x)}\ dx$
$f_u(u) = \int_0^u \lambda^2 e^{-\lambda(x-(u-x))}\ dx$
$f_u(u) = \int_0^u \lambda^2 e^{-\lambda u}$
$f_u(u) = \lambda^2 e^{-\lambda u} \int_0^u 1$
$f_u(u) = \underline{\underline{\lambda^2 e^{-\lambda u} u}}$

Jake je rozdeleni $X + Y + Z$?

prevedeme na Z+(X+Y)
pomoci dvojite konvolunce:
$f_v(v) = \int_{-\infty}^{\infty} f_z(z) f_u(v - z) dz$
$f_v(v) = \int_0^v \lambda e^{-\lambda z} \cdot \lambda^2 e^{-\lambda(v-z)} x\ dz$
$f_v(v) = \lambda^3 \int_0^v e^{-\lambda z} e^{-\lambda(v-z)} x\ dz$
$f_v(v) = \lambda^3 \int_0^v e^{-\lambda(z+v-z)} x\ dz$
$f_v(v) = \lambda^3 \int_0^v e^{-\lambda v} x\ dz$
$f_v(v) = \lambda^3 e^{-\lambda v} x \int_0^v 1\ dz$
$f_v(v) = \underline{\underline{\lambda^3 e^{-\lambda v} v z}}$