Predmet: Mataliza 1
Ukol: 7.
Verze: 1.
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Prezdivka: DN

Spolecne zadani

Spoctete limity

zadani

$\lim_{x\to 1} \frac{x^2+2x-1}{x^2-1}$

reseni

Pouzijeme vetu o aritmetice limit pro nasobeni
$\lim_{x\to 1} \frac{1}{x^2-1} * \lim_{x\to 1} \frac{x^2+2x-1}{1}$
$\lim_{x\to 1} \frac{x^2+2x-1}{1} = 2$
$\lim_{x\to 1} \frac{1}{x^2-1}$ nema reseni, proto ji vyresime zleva a zprava
$\lim_{x\to 1^+} \frac{1}{x^2-1} = \infty$
$\lim_{x\to 1^-} \frac{1}{x^2-1} = -\infty$

$\underline{\underline{\lim_{x\to 1^+} \frac{x^2+2x-1}{x^2-1}}} = \infty$
$\underline{\underline{\lim_{x\to 1^1} \frac{x^2+2x-1}{x^2-1}}} = -\infty$

zadani

$\lim_{x\to \pi/4} \frac{\sqrt{\tan x}}{2\sin^2(x)-1}$

reseni

Pouzijeme vetu o aritmetice limit pro nasobeni
$\lim_{x\to \pi/4} \frac{1}{2\sin^2(x)-1}$
$\lim_{x\to \pi/4} \frac{\sqrt{\tan x}}{1}$
$\lim_{x\to \pi/4} \frac{\sqrt{\tan x}}{1} = 1 \lim_{x\to \pi/4} \frac{1}{2\sin^2(x)-1}$ nema reseni, proto ji vyresime zleva a zprava

$\lim_{x\to (\pi/4)^+} \frac{1}{2\sin^2(x)-1} = \frac{1}{2*(.5^+)-1} = \infty$
$\lim_{x\to (\pi/4)^-} \frac{1}{2\sin^2(x)-1} = \frac{1}{2*(.5^-)-1} = -\infty$

$\underline{\underline{\lim_{x\to (\pi/4)^+} \frac{\sqrt{\tan x}}{2\sin^2(x)-1}}} = \infty$
$\underline{\underline{\lim_{x\to (\pi/4)^-} \frac{\sqrt{\tan x}}{2\sin^2(x)-1}}} = -\infty$

zadani

$\lim_{x\to \infty} \frac{\log(1+\sqrt{x}+\sqrt[3]{x})}{\log(1+\sqrt[3]{x}+\sqrt[4]{x})}$

reseni

Pouzijeme Lhopitalovo pravidlo, protoze $\frac{\infty}{\infty}$ je zakazana operace
$\lim_{x\to \infty} \frac{f(x)}{g(x)} = \lim_{x\to \infty} \frac{f'(x)}{g'(x)}$
vypocteme tedy derivace
$\lim_{x\to \infty} \frac{\log(1+\sqrt{x}+\sqrt[3]{x})}{\log(1+\sqrt[3]{x}+\sqrt[4]{x})}' =$

$\lim_{x\to\infty} \frac{\frac{3x^{1/6}+2}{6(x^{7/6}+x^{2/3}+x)}}{\frac{4x^{1/12}+3}{12(x^{1/3}+x^{1/4}+1)x^{3/4}}} =$
$\lim_{x\to\infty} \frac{(3x^{1/6}+2)12x^{3/4}(x^{1/3}+x^{1/4}+1)}{(4x^{1/12}+3)6(x^{7/6}+x^{2/3}+x)} =$
$\lim_{x\to\infty} \frac{(3x^{1/6}+2)12x^{3/4}(x^{1/3}+x^{1/4}+1)}{(4x^{1/12}+3)6(x^{7/6}+x^{2/3}+x)} =$
$\lim_{x\to\infty} \frac{(3x^{2/12}+2)12x^{9/12}(x^{4/12}+x^{3/12}+1)}{(4x^{1/12}+3)6(x^{14/12}+x^{8/12}+x)} =$
odebereme konstanty a male mocniny u scitani, nebot jdeme k nekonecnu
$\lim_{x\to\infty} \frac{(3x^{2/12})12x^{9/12}(x^{4/12})}{(4x^{1/12})6(x^{14/12})} =$
$\lim_{x\to\infty} \frac{3*12*x^{\frac{2+9+4}{12}}}{4*6*x^{\frac{1+14}{12}}} =$
$\lim_{x\to\infty} \frac{3x^{\frac{15}{12}}}{2x^{\frac{15}{12}}} =$
$\underline{\underline{\frac{3}{2}}}$

zadani

Urcete extremy funkce $\sqrt[x]{x}$ definovane na kladnych realnych cislech

reseni

pomoci derivace najdeme kde ma funkce vrchol
$\sqrt[x]{x}' = x^{\frac{1}{x}-2}(1 - \log(x))$
$x^{\frac{1}{x}-2}(1 - \log(x)) = 0$
$x = e$

najdeme limitu u nuly a v nekonecnu
$\lim_{x\to 0^+} \sqrt[x]{x} = 0$
$\lim_{x\to\infty} \sqrt[x]{x} = 1$

limita nabiva maxima $y = e^e$ v bode $x = e$ a
minima $y = 0$ v bode $x = 0$