Predmet: Vyrokova a predikatorova logika

Ukol: 11. Verze: 1.

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axiomy rovnosti

- i axiom reflexivity: x = x
- ii schema axiomu kongruence vzhledem k relacim: $x_1 = y_1, ..., x_n = y_n \rightarrow (R(x_1, ..., x_n) \rightarrow R(y_1, ..., y_n))$, kde n je přirozené číslo a R je n-ární relační symbol.
- iii schema axiomu kongruence vzhledem k funkcim: $x_1 = y_1, ..., x_n = y_n \rightarrow F(x_1, ..., x_n) = F(y_1, ..., y_n)$, kde n je přirozené číslo a F je n-ární funkční symbol.

a)
$$T^* | = x = y \rightarrow y = x$$

$$F(\forall x)(\forall y)(x = y \rightarrow y = x)$$

$$F(c = d \rightarrow d = c) \text{ nove } c, d$$

$$Tc = d$$

$$Fd = c$$

$$T(\forall x)(\forall y)(x = x \land x = y) \rightarrow (f(x, x) \rightarrow f(y, x))$$

$$T(c = c \land c = d) \rightarrow (f(c, c) \rightarrow f(d, c)) \text{ c=x, d=y}$$

$$Fc = c \land c = c$$

$$Tf(c, c) \rightarrow f(d, c)$$

$$Fc = c \qquad Fc = d \qquad Ff(c, c) \qquad Tf(d, c)$$

$$T(\forall x)(x = x) \qquad \otimes \qquad T(\forall x)(x = x) \qquad Td = c$$

$$Tc = c \qquad \qquad |$$

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$$Tc = c \qquad |$$

vsechny vetve jsou sporne, tudiz puvodni tvrzeni plati \Box

$$\begin{array}{c|c} \mathbf{b}) \ T^{\star} | = \left(x = y \wedge y = z \right) \rightarrow x = z \\ F(\forall x)(\forall y)(\forall z)(x = y \wedge y = z \rightarrow x = z) \\ | \\ F(c = d \wedge d = e) \rightarrow c = e \ \text{nove c, d, e} \\ | \\ Tc = d \wedge d = e \\ | \\ Fc = e \\ Tc = d \\ Td = e \\ | \\ T(\forall x)(\forall y)(\forall z)(x = x \wedge y = z) \rightarrow (f(x,y) \rightarrow f(y,z)) \\ | \\ T(c = c \wedge d = e) \rightarrow (f(c,d) \rightarrow f(d,e)) \ \text{c=x, d=y, e=z} \\ | \\ Fc = c \wedge d = e \\ | \\ Fc = c \wedge d = e \\ | \\ Fc = d \rangle \\ | \\ T(\forall x)(x = x) \otimes F(c = d) \\ | \\ Tc = c \\ | \\ | \\ \otimes \\ \otimes \\ \end{array}$$

vsechny vetve jsou sporne, tudiz puvodni tvrzeni plati \Box