1 22.11.

1.1 jak se mela resit minula pisemka

$$\begin{bmatrix} 1 & 2 & 03 & 4 \\ 2 & p & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & p \end{bmatrix}$$

REF kam az to pujde

$$\begin{bmatrix} 1 & 2 & 03 & 4 \\ 0 & p+1 & 3 & 3 \\ 0 & 3 & 2 & 0 \\ 0 & 3 & 0 & p+4 \end{bmatrix}$$

vybereme si jen radky a sloupce co poterbujeme
protoze zbytek uz ma pivoty definovane

$$\begin{bmatrix} p+4 & 3 \\ 3 & p+4 \end{bmatrix}$$

Zjednodusime a vyzkousime vse v Z^5

$$\begin{bmatrix} 1 & 2p+3 \\ 0 & 2p+1-p(2p+3) \end{bmatrix}$$

1.2 jsou linearne nezavisle?

v $\mathbb{R}^4$:
$x_1 = (1,2,0,0)^T$
$x_2 = (2,1,1,3)^T$
$x_3 = (0,1,0,1)^T$
matice pro vypocet:

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} - REF -> \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

tudiz ma dimenzi 3 a tudiz jsou ty vektory linearne nezavisle
jaky vektor jeste muzeme prdat aby byly stale linearne nezavisle?

$$\begin{bmatrix} 1 & 2 & 0 & a \\ 2 & 1 & 1 & b \\ 0 & 1 & 0 & c \\ 0 & 3 & 1 & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & a \\ 0 & 1 & 0 & c \\ 0 & 0 & 1 & b-2a+3c \\ 0 & 0 & 1 & d-3c \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & a \\ 0 & 1 & 0 & c \\ 0 & 0 & 1 & b-2a+3c \\ 0 & 0 & 0 & d-3c-(b-2a+3c) \end{bmatrix}$$

vysledek:

$$2a - b - 6c + d \neq 0$$

1.3 jak vypocist souradnice s jinou bazi

zadani: v $\mathbb{R}^4$:
$B = ((1,-3,7,2)^T, (3,2,1,-4)^T, (0,-1,4,-3)^T, (-2,4,-3,0)^T)$
$x_1 = (2,2,9,-5)^T$
$x_2 = (-7,2,9,-8)^T$
$x_3 = (-2,4,-3,0)^T$

dosazeni do matice:

$$\left[\begin{array}{cccc|ccc} 1 & 3 & 0 & -2 & 2 & -7 & -2 \\ -3 & 2 & -1 & 4 & 2 & 2 & 4 \\ 7 & 1 & 4 & -3 & 9 & 9 & -3 \\ 2 & -4 & -3 & 0 & -5 & -8 & 0 \end{array}\right] RREF \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & -7 \end{array}\right]$$

tudiz nam vzniknou nove souradnice:
$[x_1]_B = (1,1,1,1)^T$
$[x_2]_B = (0,-1,4,2)^T$
$[x_3]_B = (2,-4,0,-7)^T$

2 29.11.

2.1 hledani bazi

V $\mathbb{R}^5$ mejme vektorove podprostory U, V definovane
$U = span((2,1,-2,1,-1)^T, (2,4,-2,-1,1)^T, (4,1,-4,3,-3)^T)$
$V = span((0,1,1,0,-1)^T, (1,2,-1,-2,0)^T, (1,,1-2,-2,1)^T, (1,4,0,-3,0)^T, (2,6,-1,-5,0)^T)$
Najdete baze podprostoru $U, V, U \cap V$ a $U+V$

$$\begin{bmatrix} 2 & 2 & 4 \\ 1 & 4 & 1 \\ -2 & -2 & -4 \\ 1 & -1 & 3 \\ -1 & 1 & -3 \end{bmatrix} REF \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

tudiz dimenze je tri ($dim(U) = 3$)
pozorovani: nektere radky jsou stejne (jen jine znamenko), ty se daji lehce odstranit

v $\mathbb{R}^5$:
$dim(U) = dim(V) = 3$
$dim(U+V) = 5$
pro sjednoceni i soucet se udela REF vseho v jedne matici, s tim ze se tam nemusi davat duplicitni „sloupecky“

2.2 baze (pokracovani z minula)

$B = (x^4 + x^3, x^3 + x^2, x^2 + x, x + 1, 1 + x^4)$
$f(x) = 4x^4 + 4x^3 + x + 3$
$[f(x)]_B = (a, b, c, d, e)^T$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 4 \\ 1 & 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 3 \end{array}\right] RREF \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] = (3,1,-1,2,1)^T$$

2.3 co se bude dit, kdyz u prvniho prikladu zanedbame transpozici

dimenze sice zustane, ale ztratime prehled o tom ktery vektor je ktery radek, coz se nam pri sloupcovem zapisu nestane
v obou pripadech to ulohu nejak resi, ale vysledek se pak musi „spravne precist“

3 20.12.

3.1 reseni pisemky

$f((0,0,1)^T) = (1,2,3)^T$
$f((0,1,1)^T) = (1,0,0)^T$
$f((1,1,1)^T) = (-1,2,3)^T$

mezivypocet:
odectenim privniho vektoru od druheho dostaneme druhy vektor kan. baze
podobne i pro treti vektor...
$f((0,1,0)^T) = (0,-2-,-3)^T$
$f((1,0,0)^T) = (-2,2,3)^T$

vysledek:

$$\begin{bmatrix} -2 & 0 & 1 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix}$$

3.2 priklad na linearni zobrazeni

$f : Z_5^3 \to Z_5^3$
$f((2,4,1)^T) = (2,1,2)^T$
$f((2,3,4)^T) = (0,4,1)^T$
$f((3,0,1)^T) = (4,4,1)^T$

$$\begin{bmatrix} -2 & 0 & 1 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix} = {}_{kan}[f]_B$$

to co mame krat matice prechodu = to co hledame

${}_{kan}[f]_{B\,B}[id]_{kan} = {}_{kan}[f]_{B\,kan}[id]_B^{-1} = {}_{kan}[f]_{kan}$

veta:

${}_{B3}[gf]_{B_1} = {}_{B_3}[g]_{B2\,B_2}[f]_{B1}$

vysledek:

$${}_k[f]_k = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 1 & 3 \\ 0 & 4 & 1 \end{bmatrix}$$

pozorovani:

${}_k[f]_k * [A] = [B] \to {}_k[f]_k = [b] * [A]^{-1}$, kde $[A]$ a $[B]$ jsou matice

3.3 dalsi priklad na linearni zobrazeni

$Z_5^4 : (1, 2, 0, 1)^T, (4, 1, 3, 1)^T, (3, 1, 3, 4)^T, (2, 0, 2, 2)^T$
$(1, 2, 3, 1)^T, (4, 4, 1, 1)^T, (2, 0, 2, 1)^T, (3, 1, 4, 0)^T$

${}_{B2}[id]_{B1} = {}_{B2}[id]_{k\,k}[id]_{B1}$
$= ({}_k[id]_{B2})^{-1}{}_k[id]_{B1}$

3.4 linearni zobrazeni v komplexnich cislech

$f : \mathbb{C} \to \mathbb{C}$
a) $f_1(a + bi) = a - bi$ . . . neplati definice linearniho zobrazeni tudiz neni linearni zobrazeni
b) $f_2(a + bi) = -b + ai$