

Predmet: Mataliza 1
Ukol: 9.
Verze: 1.
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Prezdivka: DN

zadani

$$\int \frac{2x}{1-x^2} dx$$

reseni

pres parcialni zlomky:

$$\frac{2x}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$$

$$2x = A(x+1) + B(1-x)$$

$$A \Rightarrow x=1 \Rightarrow 2 = A(2) \Rightarrow A=1$$

$$B \Rightarrow x=-1 \Rightarrow -2 = B(2) \Rightarrow B=-1$$

pokracujeme v reseni integralu

$$\int \frac{1}{1-x} - \int \frac{1}{1+x} dx$$

$$- \int \frac{1}{x-1} - \int \frac{1}{1+x} dx$$

$$-\log(x-1) - \log(1+x) + c$$

zadani

$$\int \frac{x+1}{x^2+5x+6} dx$$

reseni

$$\int \frac{x+2}{(x+3)(x+2)} - \frac{1}{(x+3)(x+2)} dx$$

$$\int \frac{1}{(x+3)} - \frac{1}{(x+3)(x+2)} dx$$

$$\int \frac{1}{(x+3)} dx - \int \frac{1}{(x+3)(x+2)} dx$$

odbocka k parcialnim zlomkum:

$$\frac{1}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+3)$$

$$A = -1; B = 1$$

$$\int \frac{1}{(x+3)} dx - \int \left(\frac{1}{x+2} - \frac{1}{x+3} \right) dx$$

$$\log(x+3) - (\log(x+2) - \log(x+3)) + c$$

$$2 * \log(x+3) - \log(x+2) + c$$

zadani

$$\int \frac{x^2-2x-2}{x^2+x-2} dx$$

reseni

$$\int \frac{x^2+x-2}{x^2+x-2} - \frac{3x}{x^2+x-2} dx$$

$$\int 1 - \frac{3x}{x^2+x-2} dx$$

$$x - \int \frac{3x}{x^2+x-2} dx$$

$$x - \int \frac{3x}{(x-1)(x+2)} dx$$

odbocka k parcialnim zlomkum:

$$\frac{3x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$3x = A(x+2) + B(x-1)$$

$$A = 1; B = 2$$

$$x - \int \left(\frac{1}{x-1} + \frac{2}{x+2} \right) dx$$

$$x - \int \frac{1}{x-1} - \int \frac{2}{x+2} dx$$

$$\underline{\underline{x - \log(x-1) - 2\log(x+2) + c}}$$

zadani

$$\int \frac{3x+5}{x^2+2x+1} dx$$

reseni

$$\int \frac{1}{x+1} + \frac{2x+4}{(x+1)^2} dx$$

$$\log(x+1) + 2 \int \frac{x+2}{(x+1)^2} dx$$

$$\log(x+1) + 2 \int \left(\frac{x+1}{(x+1)^2} + \frac{1}{(x+1)^2} \right) dx$$

$$\log(x+1) + 2 \int \left(\frac{1}{(x+1)} + \frac{1}{(x+1)^2} \right) dx$$

$$\log(x+1) + 2 \int \left(\frac{1}{(x+1)} \right) + 2 \int \left(\frac{1}{(x+1)^2} \right) dx$$

$$\log(x+1) + 2\log(x+1) + 2 \int \left(\frac{1}{(x+1)^2} \right) dx$$

$$3\log(x+1) + 2 \int \left(\frac{1}{(x+1)^2} \right) dx$$

$$3\log(x+1) + 2\left(-\frac{1}{x+1}\right) + c$$

$$\underline{\underline{3\log(x+1) - \frac{2}{x+1} + c}}$$

zadani

$$\int x^2 \cos x dx$$

reseni

$$\int f dg = fg - \int g df$$

$$f = x^2; df = 2x$$

$$dg = \cos x; g = \sin x$$

$$x^2 \sin x - 2 \int (\sin(x)x)$$

$$f = x; df = 1$$

$$dg = \sin x; g = -\cos x$$

$$x^2 \sin x - 2(-\cos(x)x - \int (-\cos(x)))$$

$$x^2 \sin x - 2(-\cos(x)x - (-\sin(x))) + c$$

$$\underline{\underline{x^2 \sin x + 2 \cos(x)x - 2 \sin(x)}}$$

zadani

$$\int \frac{1}{(x+1)\sqrt{x}} dx$$

reseni

Substitute:

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$dx = 2u du$$

$$\int \frac{1}{(u^2+1)u} dx$$

$$\int \frac{2u}{(u^2+1)u} du$$

$$2 \int \frac{u}{(u^2+1)u} du$$

$$\begin{aligned}
& 2 \int \frac{1}{(u^2+1)} \, du \\
& 2(\tan^{-1}(u)) + c \\
& \underline{\underline{2(\tan^{-1}(\sqrt{x})) + c}}
\end{aligned}$$