Predmet: Mataliza 1

Ukol: 6. Verze: 1.

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Prezdivka: DN

Spolecne zadani

Urcete definicni obory funkci a spoctete jejich derivace

zadani

$$\sqrt{\frac{x-1}{x^2+1}}$$

reseni

Definic
ni obor: $x \ge 1$

$$= \frac{1}{2} \left(\frac{x-1}{x^2+1} \right)^{-\frac{1}{2}} * \left(\frac{x-1}{x^2+1} \right)'$$

Definicni obor:
$$x \ge 1$$

Pro derivaci pouzijeme pravidlo $(g(f))' = g'(f) * f'$
 $= \frac{1}{2} \left(\frac{x-1}{x^2+1}\right)^{-\frac{1}{2}} * \left(\frac{x-1}{x^2+1}\right)'$
pouzijeme pravidlo $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$
 $= \frac{1}{2} \left(\frac{x-1}{x^2+1}\right)^{-\frac{1}{2}} * \frac{(x-1)'(x^2+1)-(x-1)(x^2+1)'}{(x^2+1)^2}$
pouzijeme pravidlo $(f+g)' = f' + g'$
 $= \frac{1}{2} \left(\frac{x-1}{x^2+1}\right)^{-\frac{1}{2}} * \frac{(x^2+1)-2x(x-1)}{(x^2+1)^2}$
 $= \frac{1}{2} \left(\frac{x^2+1}{x-1}\right)^{\frac{1}{2}} * \frac{x^2+1-2x(x-1)}{(x^2+1)^2}$
 $= \frac{\sqrt{x^2+1}}{x-1} * (-x^2+2x+1)$
 $= \frac{\sqrt{x^2+1}}{x-1} * (-x^2+2x+1)}{2(x^2+1)^2}$

$$= \frac{1}{2} \left(\frac{x-1}{x^2+1} \right)^{-2} * \frac{(x^2+1)-2x(x-1)}{(x^2+1)^2}$$

$$= \frac{1}{2} \left(\frac{x^2+1}{x^2+1} \right)^{\frac{1}{2}} x^2 + 1 - 2x(x-1)$$

$$= \frac{1}{2} \left(\frac{x^2 + 1}{x - 1} \right)^2 * \frac{x^2 + 1 - 2x(x - 1)}{(x^2 + 1)^2}$$
$$= \frac{\sqrt{\frac{x^2 + 1}{x - 1}} * (-x^2 + 2x + 1)}{2(x^2 + 1)^2}$$

zadani

 $ln(sin(e^x))$

reseni

Definicni obor: $x \in \mathbb{R}$

Pro derivaci pouzijeme pravidlo (g(f))' = g'(f) * f'

$$(\log(s))' = \frac{1}{s} * s'$$

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$$s = \sin(t); s' = \cos(t) * t'$$

$$t = e^x; t' = e^x$$
$$= \frac{\cos(e^x) * e^x}{(x)}$$

$$=\frac{\cos(e^x)*e^x}{\sin(e^x)}$$

zadani

$$x\cos x + \sin(2x^2)$$

reseni

Definic
ni obor: $x \in \mathbb{R}$

Pro derivaci pouzijeme pravidlo (f+g)'=f'+g'

$$(x\cos(x))' = -x * \sin(x) + \cos x$$

$$(\sin(2x^2))' = 4x * \cos(2x^2)$$

$$(\sin(2x^2))' = 4x * \cos(2x^2)$$

$$x \cos x + \sin(2x^2) = 4x \cos(2x^2) - x \sin(x) + \cos(x)$$

zadani

$$2^{x} + 3^{x}$$

reseni

Definic
ni obor: $x \in \mathbb{R}$ Pro derivaci pouzijeme pravidlo $(f+g)'=f'+g'=2^x\log(2)+3^x\log(3)$

zadani

$$e^{-x^2}$$

reseni

Definicni obor: $x \in \mathbb{R}$ Derivace: = $e^{-x^2} * (-x^2)' = \underbrace{e^{-x^2} * (-2x)}_{=}$

zadani

 $\arctan(\frac{1}{x})$

reseni

Definicni obor: $x \in \mathbb{R} \setminus 0$ Pro derivaci pouzijeme pravidlo (g(f))' = g'(f) * f' $\arctan(y)' = \frac{1}{1+y^2} * y'$ $y = \frac{1}{x}; y' = -\frac{1}{x^2}$ $\arctan(\frac{1}{x}) = \frac{1}{x^2+1}$