Predmet: Vyrokova a predikatorova logika

Ukol: 11. Verze: 2

Autor: David Napravnik

axiomy rovnosti

- i axiom reflexivity: x = x
- ii schema axiomu kongruence vzhledem k relacim: $x_1 = y_1, ..., x_n = y_n \rightarrow (R(x_1, ..., x_n) \rightarrow R(y_1, ..., y_n))$, kde n je přirozené číslo a R je n-ární relační symbol.
- iii schema axiomu kongruence vzhledem k funkcim: $x_1 = y_1,...,x_n = y_n \to F(x_1,...,x_n) = F(y_1,...,y_n)$, kde n je přirozené číslo a F je n-ární funkční symbol.

a)
$$T^* | = x = y \rightarrow y = x$$

$$F(\forall x)(\forall y)(x = y \rightarrow y = x)$$

$$F(c = d \rightarrow d = c) \text{ nove } c, d$$

$$Tc = d$$

$$Fd = c$$

$$T(\forall x)(\forall y)(x = x \land x = y) \rightarrow (R(x, x) \rightarrow R(y, x))$$

$$T(c = c \land c = d) \rightarrow (R(c, c) \rightarrow R(d, c)) c = x, d = y$$

$$Fc = c \land c = c$$

$$TR(c, c) \rightarrow R(d, c)$$

$$Fc = c \qquad Fc = d \qquad FR(c, c) \qquad TR(d, c)$$

$$T(\forall x)(x = x) \qquad \otimes \qquad T(\forall x)(x = x) \qquad Td = c$$

$$Tc = c \qquad \qquad 0$$

$$Tc = c \qquad \qquad 0$$

$$Tc = c \qquad \qquad 0$$

vsechny vetve jsou sporne, tudiz puvodni tvrzeni plati \Box

$$\begin{array}{c|c} \mathbf{b}) \ T^{\star} | = \left(x = y \wedge y = z \right) \rightarrow x = z \\ F(\forall x)(\forall y)(\forall z)(x = y \wedge y = z \rightarrow x = z) \\ | F(c = d \wedge d = e) \rightarrow c = e \text{ nove c, d, e} \\ | Tc = d \wedge d = e \\ | Fc = e \\ | Tc = d \\ | Td = e \\ | \\ T(\forall x)(\forall y)(\forall z)(x = x \wedge y = z) \rightarrow (R(x,y) \rightarrow R(y,z)) \\ | T(c = c \wedge d = e) \rightarrow (R(c,d) \rightarrow R(d,e)) \text{ c=x, d=y, e=z} \\ | Fc = c \wedge d = e \\ | Fc = c \wedge d = e \\ | Fc = c \wedge d = e \\ | Fc = d \wedge d = e \\ | T(\forall x)(x = x) \otimes F(c = d) & Tc = e \\ | & | \\ | Tc = c & Fc = d \\ | & \otimes \\ | \otimes \\ | \end{array}$$

vsechny vetve jsou sporne, tudiz puvodni tvrzeni plati \Box