Predmet: Mataliza 1
Ukol: 3.
Verze: 1.
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zadani

Spoctete limitu: $lim_{n\to\infty}\frac{2^n}{n!}$

reseni

to ze $n!$ roste rychleji nez 2^n je zrejme, staci dokazat, ze jejich podil je limitne nulovy.

Dokazme si nejdrive ze plati:
$lim_{n\to\infty}\frac{n!}{2^n} = \infty$

pro $n \geq 6$ plati: $2^n * n < n!$ (magicke cislo 6 muzeme pouzit, protoze se budeme pohybovat v nekonecnu a $6 < \infty$)
z toho ziskame ze $2^n < \frac{n!}{n}$ pro $n > 6$
pak $\frac{n!}{2^n} > n$ pro $n > 6$
tudiz $lim_{n\to\infty}\frac{n!}{2^n} = \infty$

z toho dostavame ze $\underline{\underline{lim_{n\to\infty}\frac{2^n}{n!} = 0}}$

zadani

Spoctete limitu: $lim_{n\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n-1}\right)$

reseni

$$lim_{n\to\infty}\sqrt{n\left(\sqrt{n+1}-\sqrt{n-1}\right)^2}$$

$$lim_{n\to\infty}n\left(\sqrt{n+1}-\sqrt{n-1}\right)^2$$

$$lim_{n\to\infty}\frac{n\left(\sqrt{n+1}-\sqrt{n-1}\right)^2\left(\sqrt{n+1}+\sqrt{n-1}\right)^2}{\left(\sqrt{n+1}+\sqrt{n-1}\right)^2}$$

$$lim_{n\to\infty}\frac{n(A-B)^2(A+B)^2}{(A+B)^2}; A=\sqrt{n+1}; B=\sqrt{n-1};$$

$$lim_{n\to\infty}\frac{n(A^2+B^2-2AB)(A^2+B^2+2AB)}{(A+B)^2}$$

$$lim_{n\to\infty}\frac{n\left((A^2+B^2)^2-(2AB)^2\right)}{(A+B)^2}$$

$$lim_{n\to\infty}\frac{n(A^4+B^4-2A^2B^2)}{A^2+B^2+2AB}$$

$$lim_{n\to\infty}\frac{n(n^2+1+2n+n^2+1-2n-2(n+1)(n-1))}{n+1+n-1+2\sqrt{(n+1)(n-1)}}$$

$$lim_{n\to\infty}\frac{4n}{2n+2\sqrt{n^2+1-2n}}$$

v $\sqrt{n^2+1-2n}$ ponechame pouze nejvyssi polynom

$$lim_{n\to\infty}\frac{4n}{2n+2\sqrt{n^2}} = lim_{n\to\infty}\frac{4n}{4n} = 1$$

$$\underline{\underline{lim_{n\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n-1}\right) = 1}}$$