Predmet: Linearni algebra 2
Ukol: 2.
Verze: 1.
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Prezdivka: DN

1. zadani

Urcete charakteristicky polynom, spocitejte vlastni cisla a odpovidajici vlastni vektory

reseni A

$(4-\lambda)*(1-\lambda)-(-3*(-6))$
$4-5\lambda+\lambda^2-18$
charakteristicky polynom: $\underline{\underline{\lambda^2-5\lambda-14}}$

vlastni cisla:
$\lambda^2-5\lambda-14=0$
$\underline{\underline{\lambda_1=7}}$; $\underline{\underline{\lambda_2=-2}}$

vlastni vektory:
$\begin{bmatrix} 4-\lambda & -3 \\ -6 & 1-\lambda \end{bmatrix}$
pro λ_1 : $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ vlastni vektor pro $\lambda_1 = \underline{\underline{[-1,1]}}$
pro λ_2 : $\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$ vlastni vektor pro $\lambda_2 = \underline{\underline{[1,2]}}$

reseni B

$(2-\lambda)^2+1$
charakteristicky polynom: $\underline{\underline{\lambda^2-4\lambda+5}}$

vlastni cisla:
$\lambda^2-4\lambda+5=0$
$\underline{\underline{\lambda_1=2+i}}$; $\underline{\underline{\lambda_2=2-i}}$

vlastni vektory:
$\begin{bmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{bmatrix}$
pro λ_1 : $\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix}$ vlastni vektor pro $\lambda_1 = \underline{\underline{[i,1]}}$
pro λ_2 : $\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}$ vlastni vektor pro $\lambda_2 = \underline{\underline{[-i,1]}}$

reseni C

$(1-\lambda)^3-1+3\lambda$
charakteristicky polynom: $\underline{\underline{-\lambda^3+3\lambda^2}}$

vlastni cisla:
$rank\ C=1, rank\ ker\ C=2$
$\lambda_1=\lambda_2=0$
$\lambda^2-4\lambda+5=0$
$\underline{\underline{\lambda_1=2+i}}$; $\underline{\underline{\lambda_2=2-i}}$

vlastni vektory:

$\begin{bmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{bmatrix}$
pro λ_1 : $\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix}$ vlastni vektor pro $\lambda_1 = \underline{\underline{[i, 1]}}$
pro λ_2 : $\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}$ vlastni vektor pro $\lambda_2 = \underline{\underline{[-i, 1]}}$

2. zadani

Najdete $\alpha \in \mathbb{R}$ tak, aby $\lambda = 3$ bylo jedno z vlastnich cisel matice M

reseni

$3 + \lambda_2 + \lambda_3 = track\ M = 9$
$3 * \lambda_2 * \lambda_3 = det\ M = \frac{8\alpha}{3} - 8$
zvolime nahodne jedno vlastni cislo, druhe dopocitame, treti mame zadane
$\lambda_2 = 2$, $\lambda_3 = 4$
po dosazeni do rovnice tri vlastnich cisel dopocitame pres determinant α
$3 * 2 * 4 = \frac{8\alpha}{3} - 8$
$\underline{\underline{\alpha = 6}}$

3. zadani

Najdete nejmensi cislo $\alpha \in \mathbb{R}$ takove, ze matice $A + \beta I_n$ je regularni pro vsechna $\beta > \alpha$
(pozn. predpokladam za matici A se mysli matice z prikladu 1)

reseni

nejdrive najdeme pro ktera β je matice A singularni
$\begin{bmatrix} 4+\beta & -3 \\ -6 & 1+\beta \end{bmatrix}$
$\beta^2 + 5\beta - 14 = 0$
$\beta_1 = -7$; $\beta_2 = 2$
aby byla matice regularni musi platit $\beta > max(\beta_1, \beta_2) = \underline{\underline{\alpha = 2}}$

4. zadani

Matice $A \in \mathbb{R}^{3\times 3}$ ma vlastni cisla $\lambda_1 = -1$, $\lambda_2 = 2$ a $\lambda_3 = 5$
Urcete stopu a determinant matice $(-A^2 + 5I_3)^{-1}$

reseni

$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 25 \end{bmatrix}$
$(-A^2 + 5I_3)^{-1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -20 \end{bmatrix}$
$track\ A = 4 + 1 - 20 \underline{\underline{= -15}}$
$determinant\ A = 4 * 1 * (-20) \underline{\underline{= -80}}$