Image Restoration

Modelling the image degradations, inverting the model

Acquired image is a degraded version of the original scene

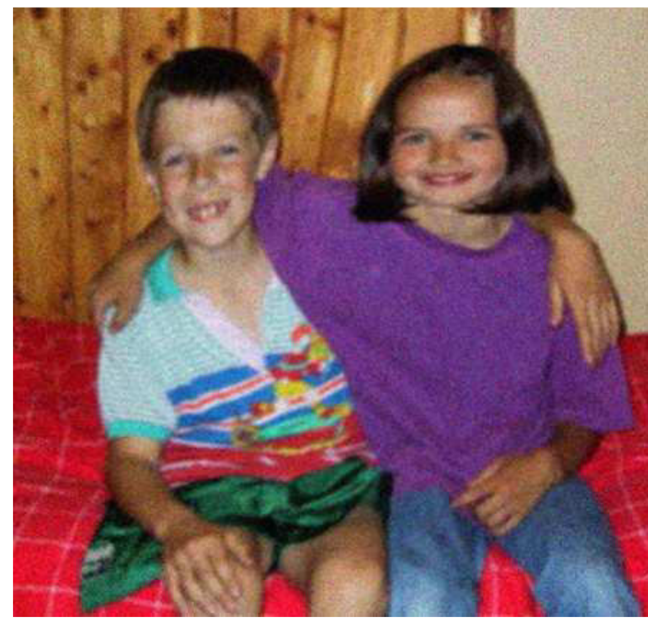
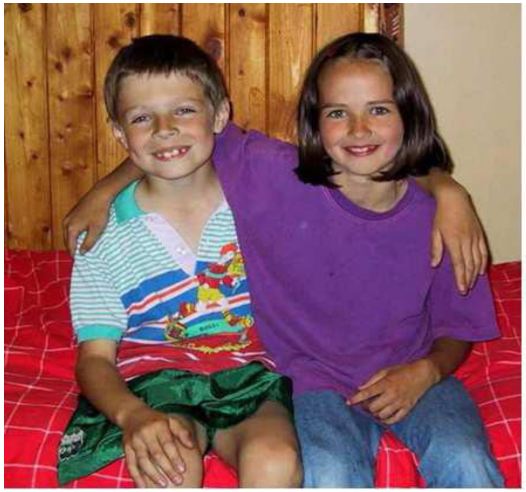
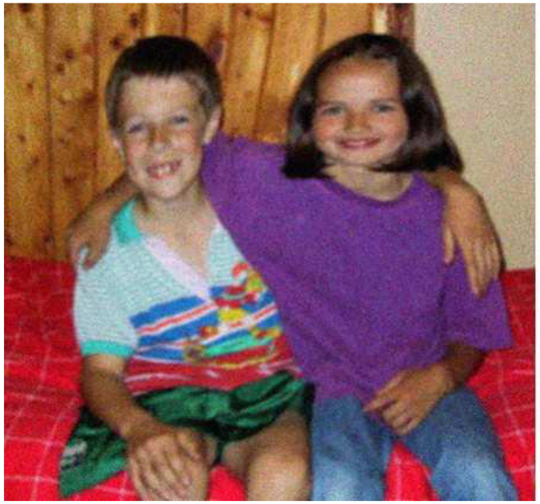


Image degradation model





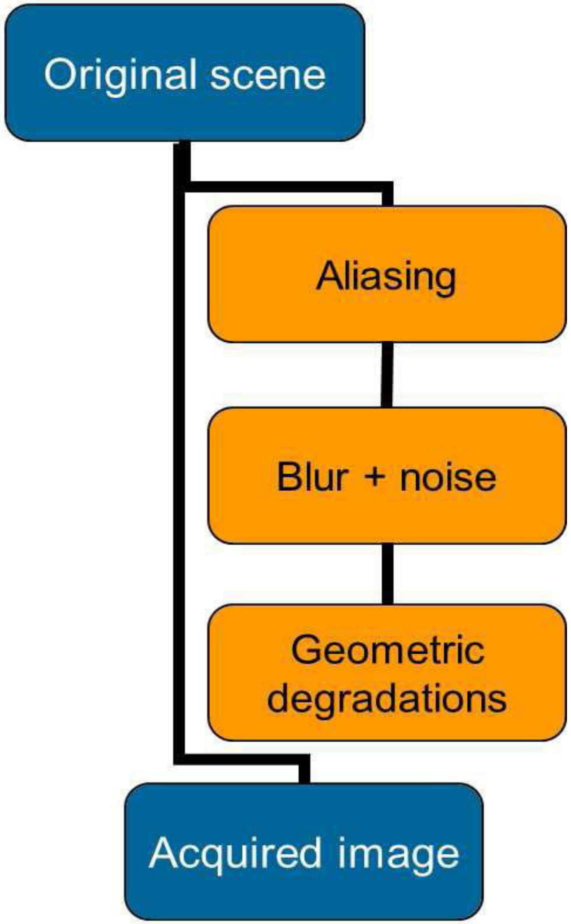


Image acquisition model

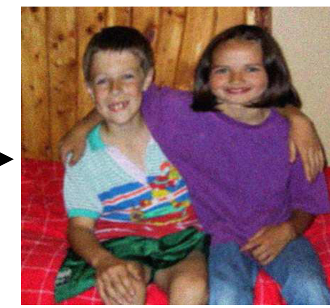
original scene

degradations

$$\mathcal{D} = QS\mathcal{D}_{\mathcal{I}}\mathcal{D}_{\mathcal{G}}$$

noise

+



acquired image

$$\mathcal{D}_{\mathcal{G}}(f)(x,y) = f(\tau(x,y))$$

$$\mathcal{D}_{\mathcal{I}}(f)(x,y) = \int_{\Lambda}\int_{T}\int\int h(x,y,a,b,\lambda,t)f(a,b)\,da\,db\,d\lambda\,dt$$

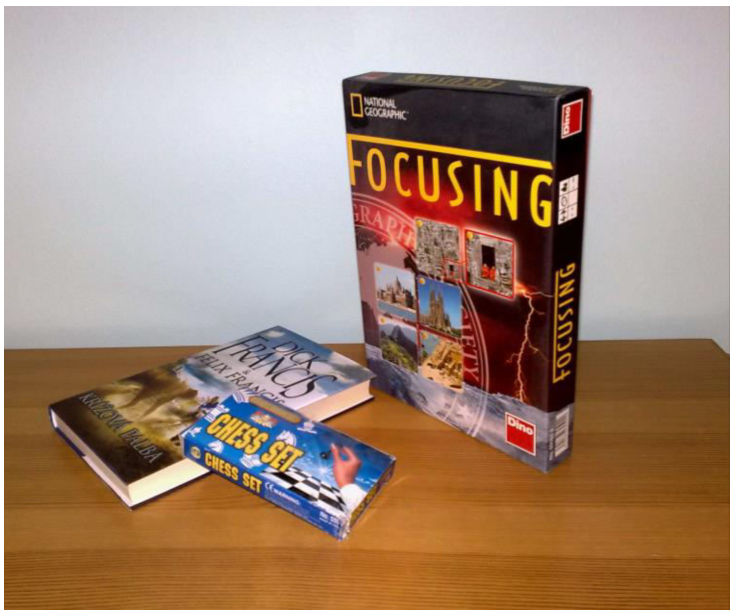
Sources, models and appearance of individual degradations

Geometric deformation

$$\mathcal{D}_{\mathcal{G}}(f)(x,y) = f(\tau(x,y))$$

Image spatial transformation/warping
Invertible in most cases

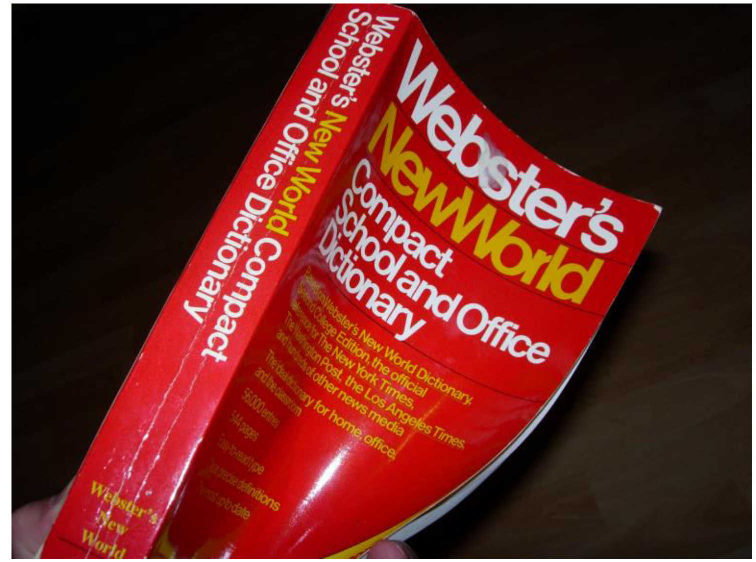
Non-ideal view, planar objects → projective geometry



$$u = \frac{a_0 + a_1 x + a_2 y}{1 + c_1 x + c_2 y}$$

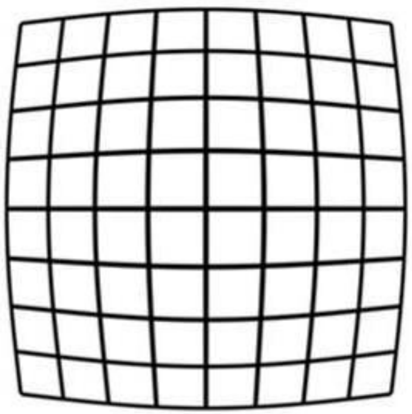
$$v = \frac{b_0 + b_1 x + b_2 y}{1 + c_1 x + c_2 y}$$

2D projection of curved surface → nonlinear models

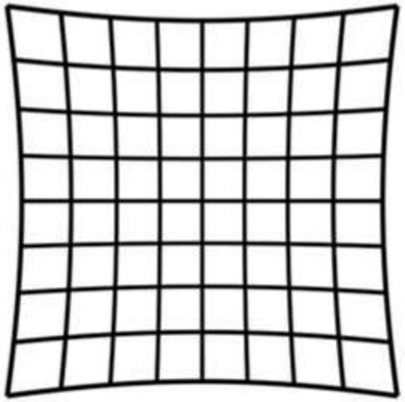


$$\mathcal{D}_{\mathcal{G}}(f)(x,y) = f(\tau(x,y))$$

Lens distortion



Barrel

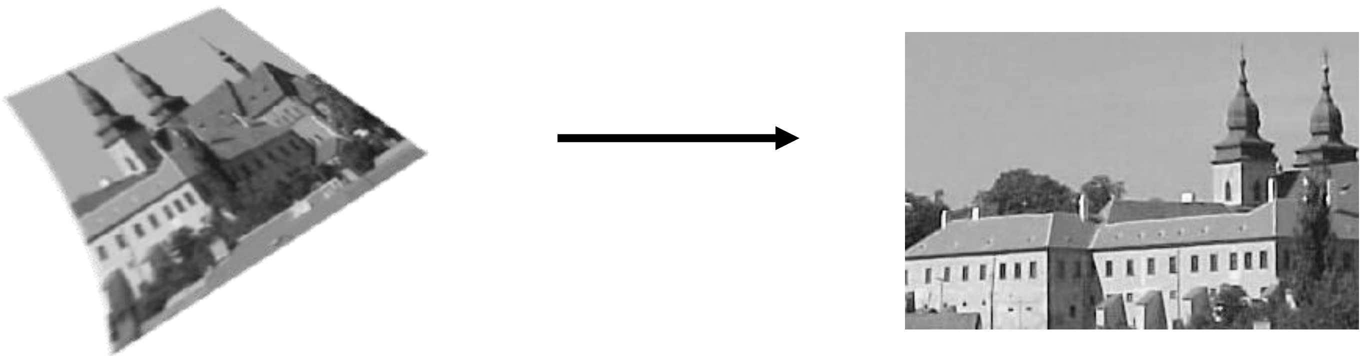


Pincushion

Non-standard lenses/cameras (omnivision, fish-eye)

Dealing with geometry

- Modelling and inverting *D*



- Comparing two images of the same scene regardless of *D*

Image Registration

Overlaying two or more images of the same scene

Image registration

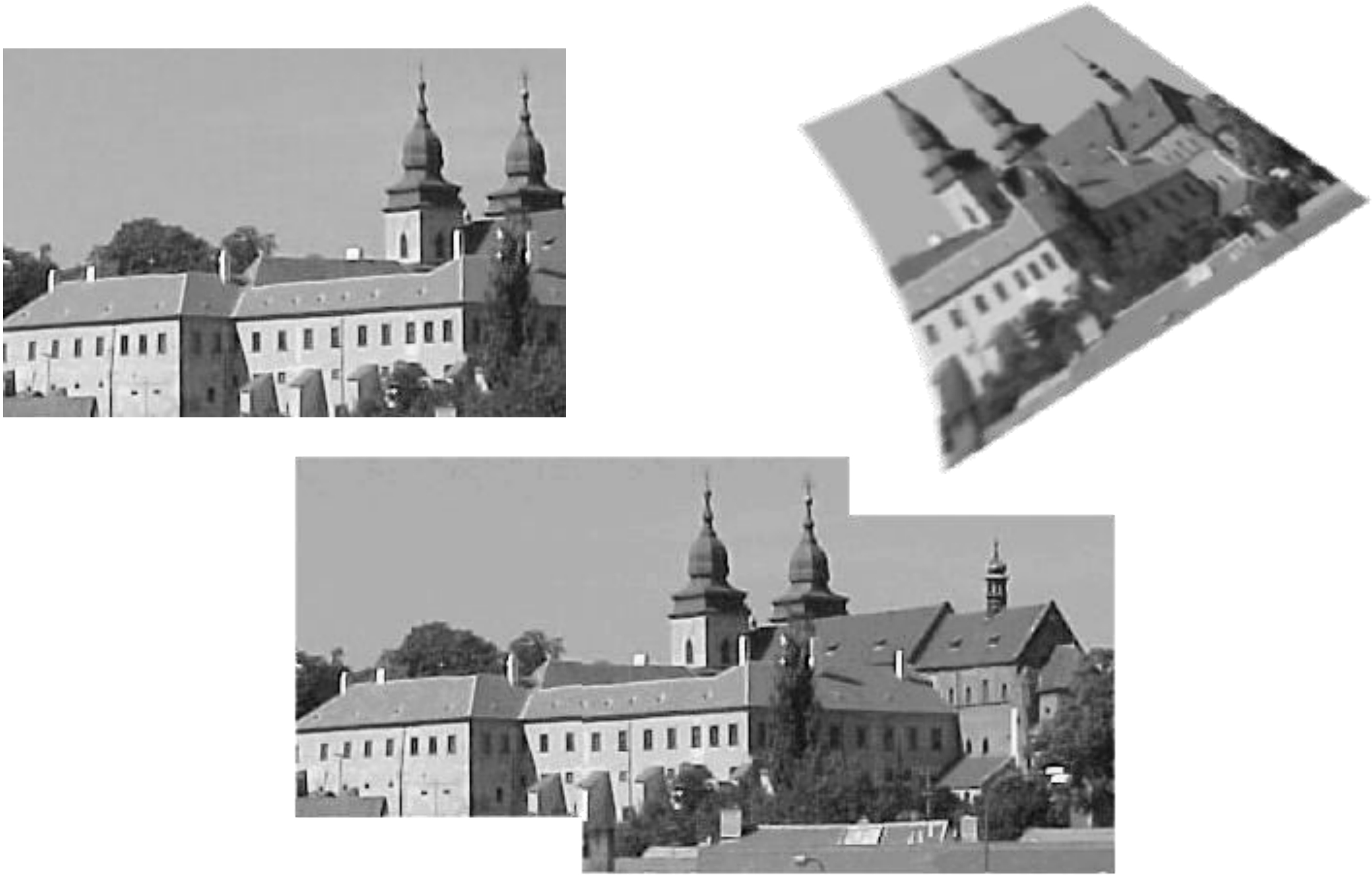


IMAGE REGISTRATION CATEGORIES

1. Different viewpoints - multiview

2. Different times - multitemporal

3. Different modalities - multimodal

4. Scene to model registration

IMAGE REGISTRATION METHODOLOGY

Four basic steps of image registration

1. Control point selection

1. Control point selection

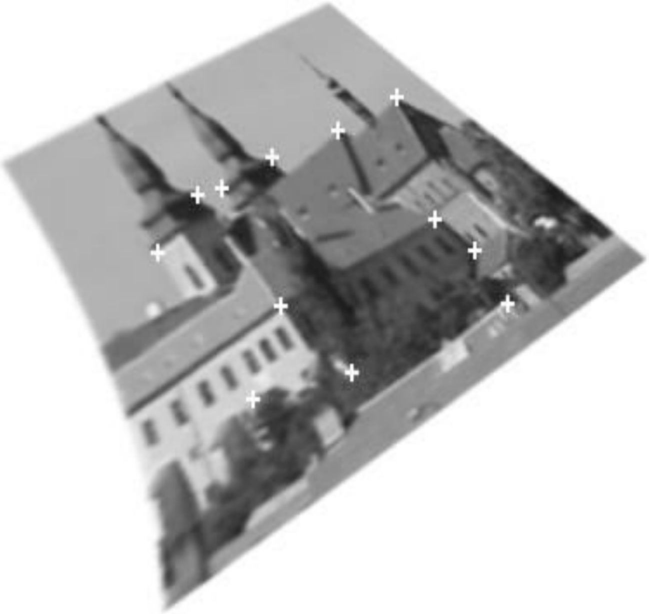


IMAGE REGISTRATION METHODOLOGY

Four basic steps of image registration

1. Control point selection

2. Control point matching

2. Control point matching

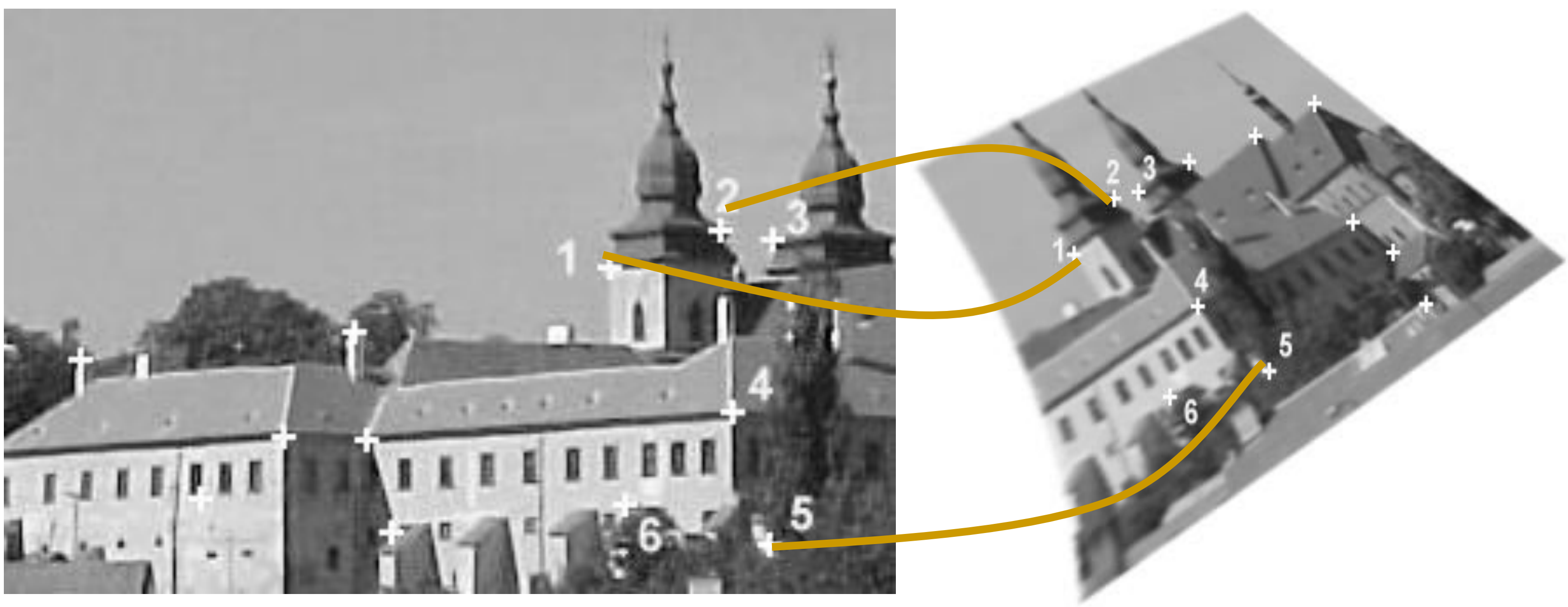


IMAGE REGISTRATION METHODOLOGY

Four basic steps of image registration

1. Control point selection
2. Control point matching
3. Transform model estimation

3. Mapping function design

$$u = f(x,y)$$
$$v = g(x,y)$$

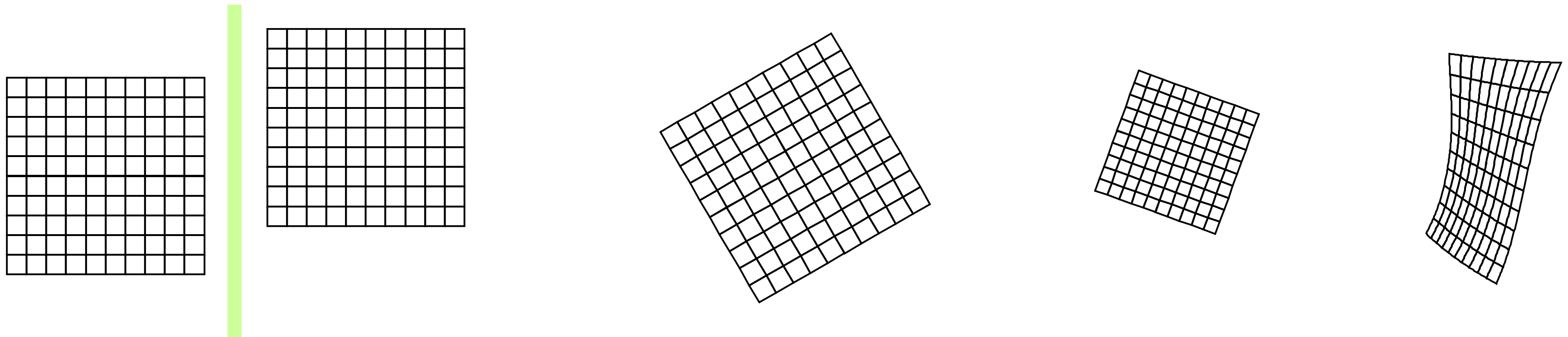


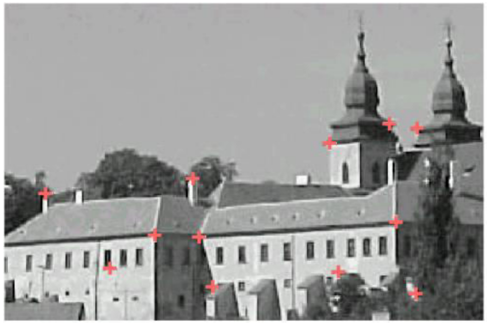
IMAGE REGISTRATION METHODOLOGY

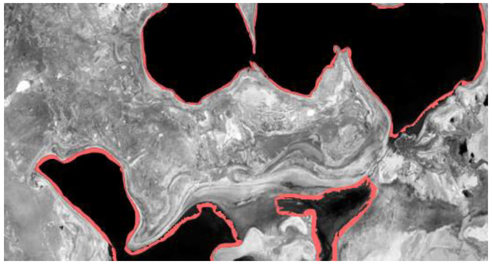
Four basic steps of image registration

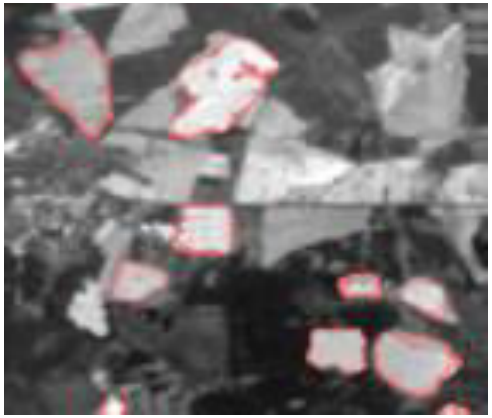
1. Control point selection
2. Control point matching
3. Transform model estimation
4. Image resampling and transformation

4. Image resampling and transformation

CONTROL POINT SELECTION

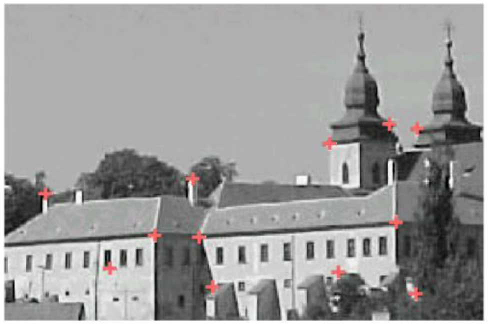


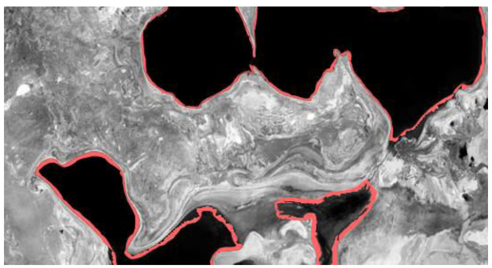


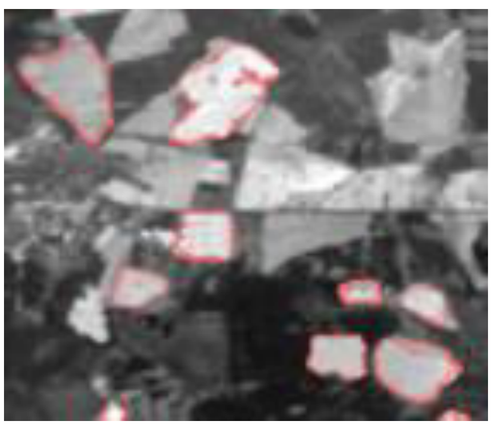


- distinctive points
- corners
- lines
- closed-boundary regions
- virtual invariant regions
- window centers

DESIRABLE PROPERTIES OF CONTROL POINTS







Distinctive and detectable objects

Physical interpretability

Frequently spread over the image

Enough common elements in all images

Robust to degradations

Control point matching

Signal-based methods

Similarity measures calculated directly from the image graylevels

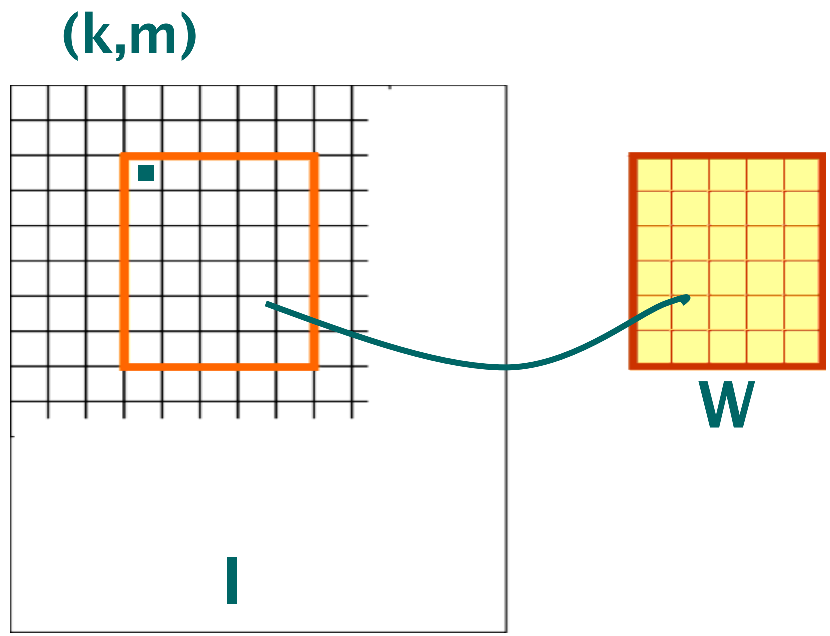
Examples - Image correlation, image differences, phase correlation, mutual information, …

Feature-based methods

Symbolic description of the features

Matching in the feature space (classification)

Image correlation



$$C(k,m) = \frac{\sum (I_{k,m} - \mathrm{mean}(I_{k,m})) \cdot (W - \mathrm{mean}(W))}{\sqrt{\sum (I_{k,m} - \mathrm{mean}(I_{k,m}))^2} \cdot \sqrt{\sum (W - \mathrm{mean}(W))^2}}$$

Image correlation

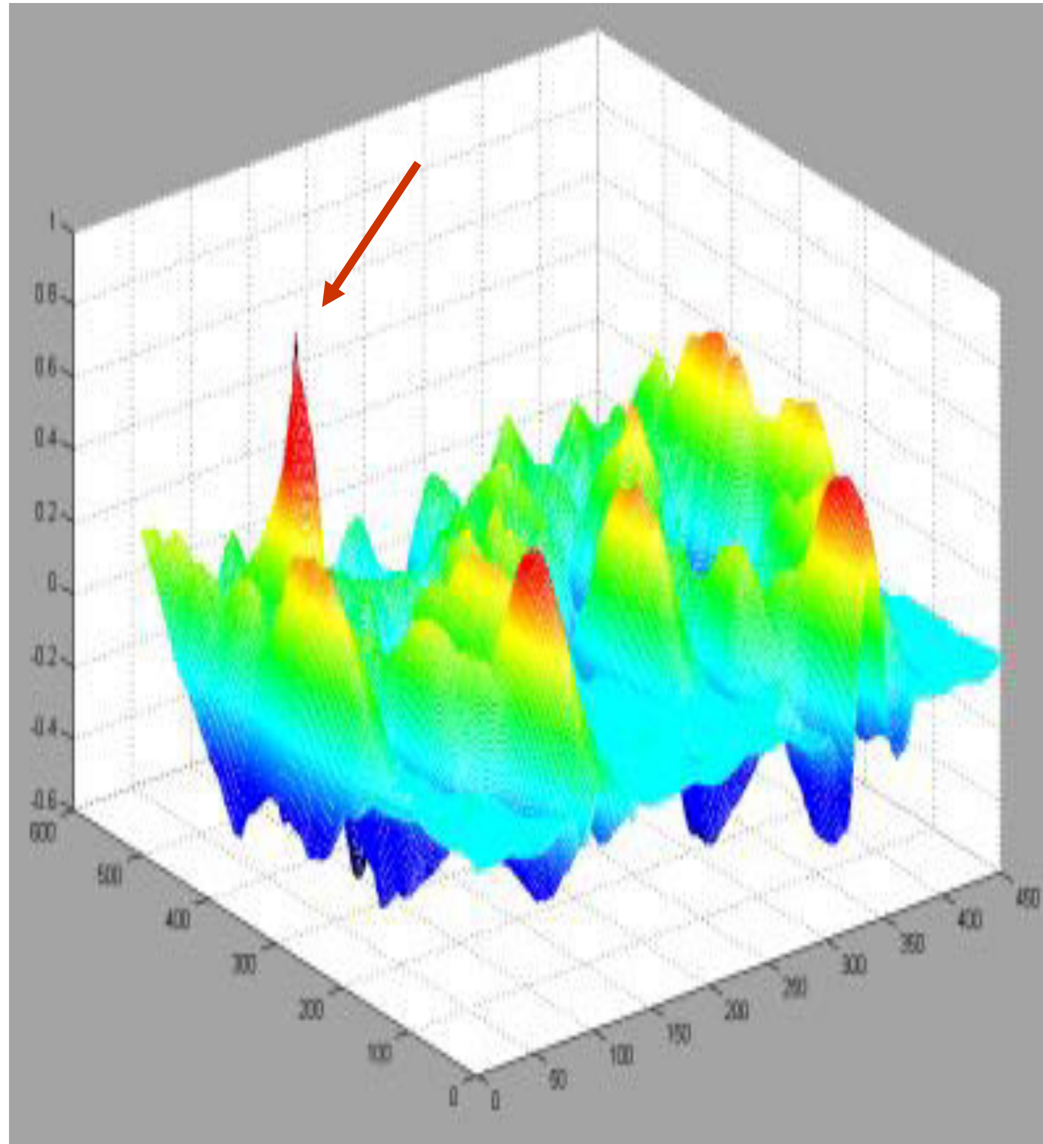
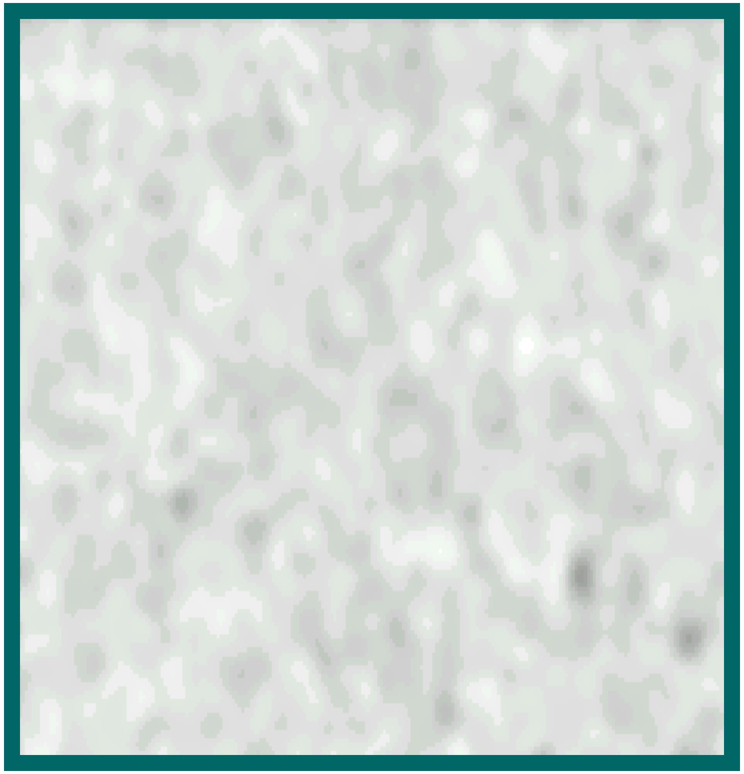


Image correlation



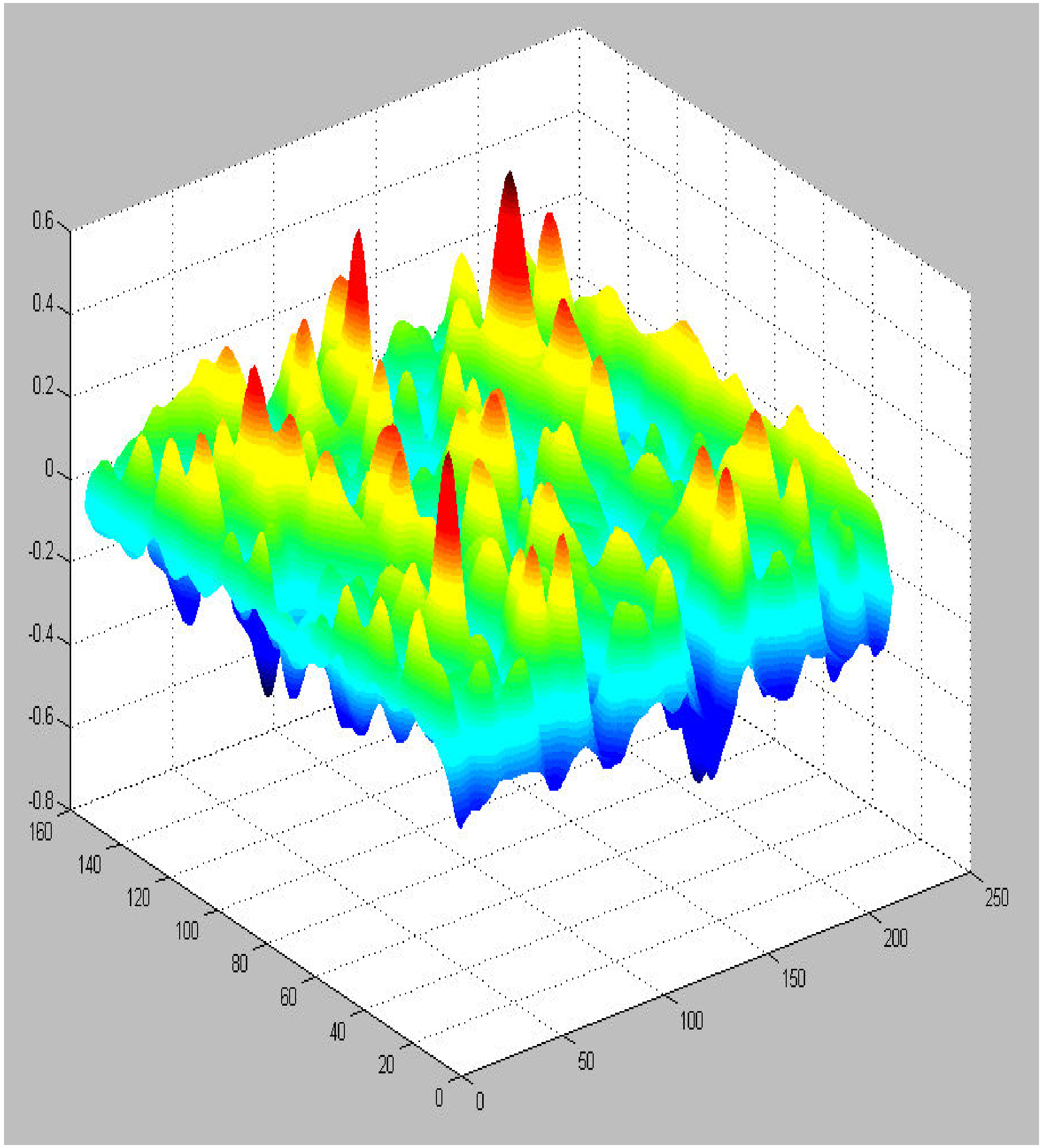
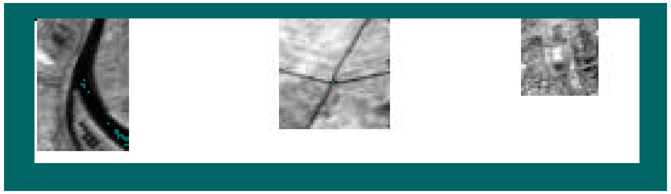
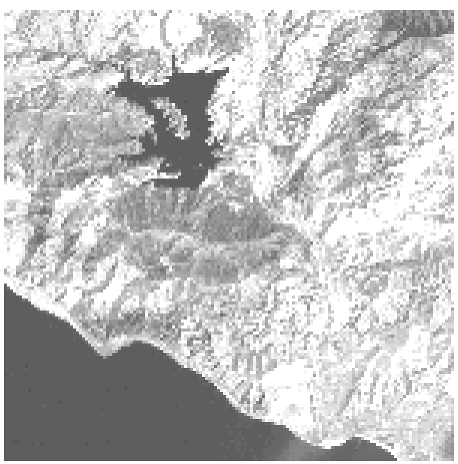


Image correlation – template matching



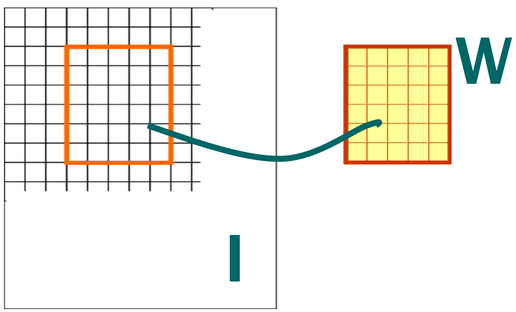
Band-to-band registration

5



7

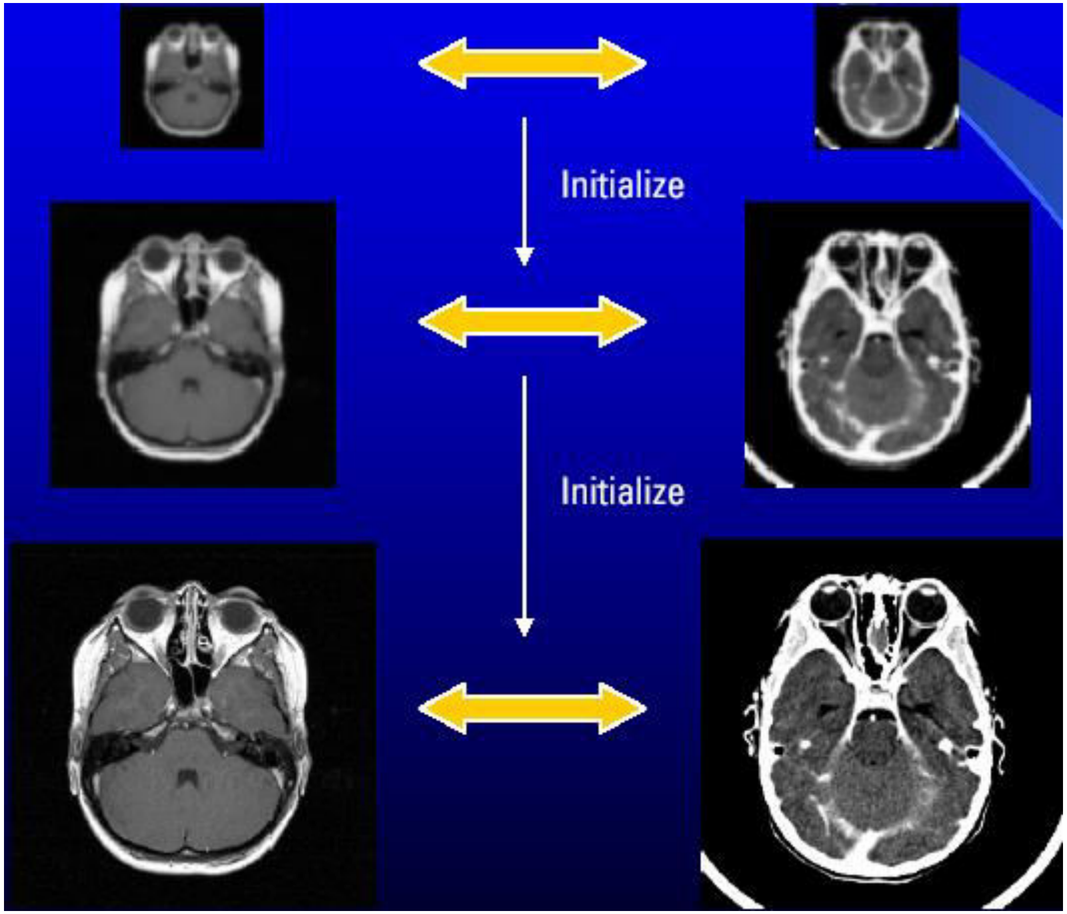
Correlation-like methods



- edge correlation, vector correlation
- correlation in F domain (phase corr)
- other similarity measures (L1 norm, Mutual information)
- extension to more complex transformations (rotation, scaling)
- extension to 3-D
- subpixel accuracy
- speed-up techniques (optimization algorithms, pyramidal representation, real-time HW implementation)

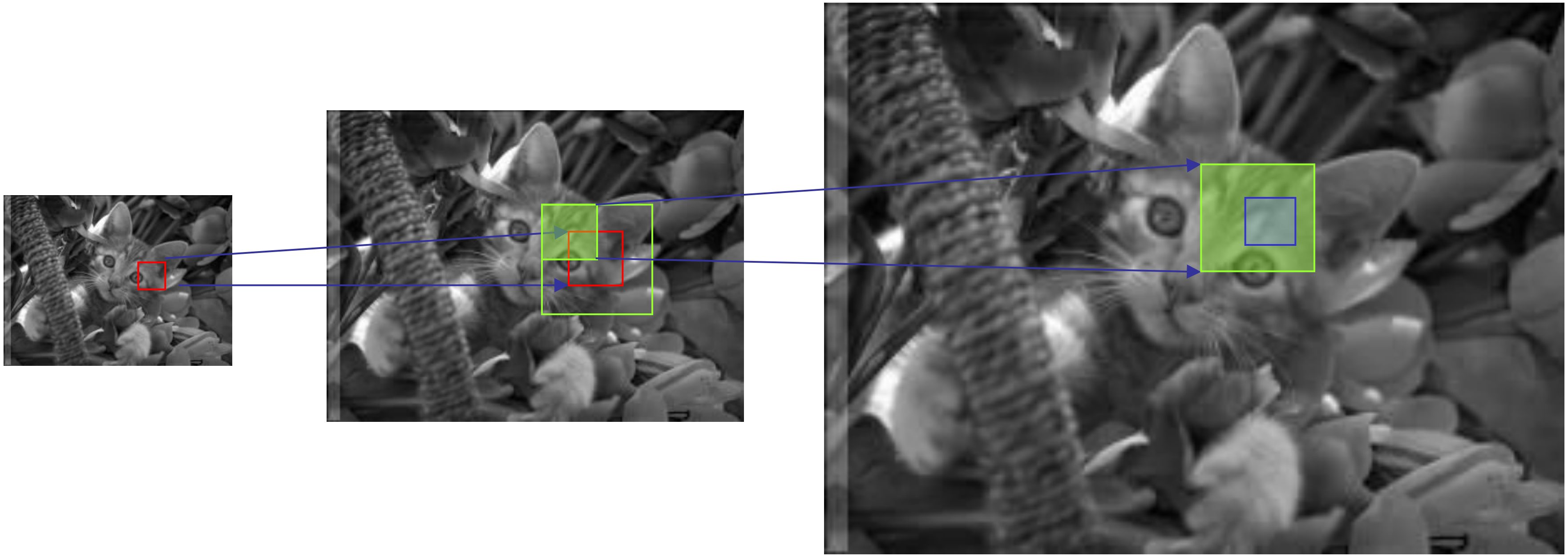
Pyramidal representation

Processing from coarse to fine level



FEATURE MATCHING PYRAMIDAL REPRESENTATION

processing from coarse to fine level

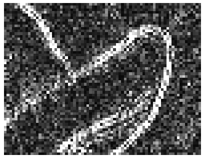


wavelet transform

Phase correlation

Equivalent to standard correlation of “whitened” images (similar to correlation of edges).

Does not depend on actual image colors.



Phase correlation

Fourier Shift Theorem:

If $f(x)$ is shifted by a to $f(x\text{-}a)$ then the FT magnitude stays constant while the phase is shifted by $-2\pi au$.

Shift parameter can be detected by a comparison of both spectra and by Inverse Fourier Transform.

Phase correlation

Cross-power spectrum

$$\frac{W \cdot F^*}{|W \cdot F|} = e^{-2\pi i\,(ua + vb)}$$

Image *f*

Window *w*

F - Fourier transform of *f*

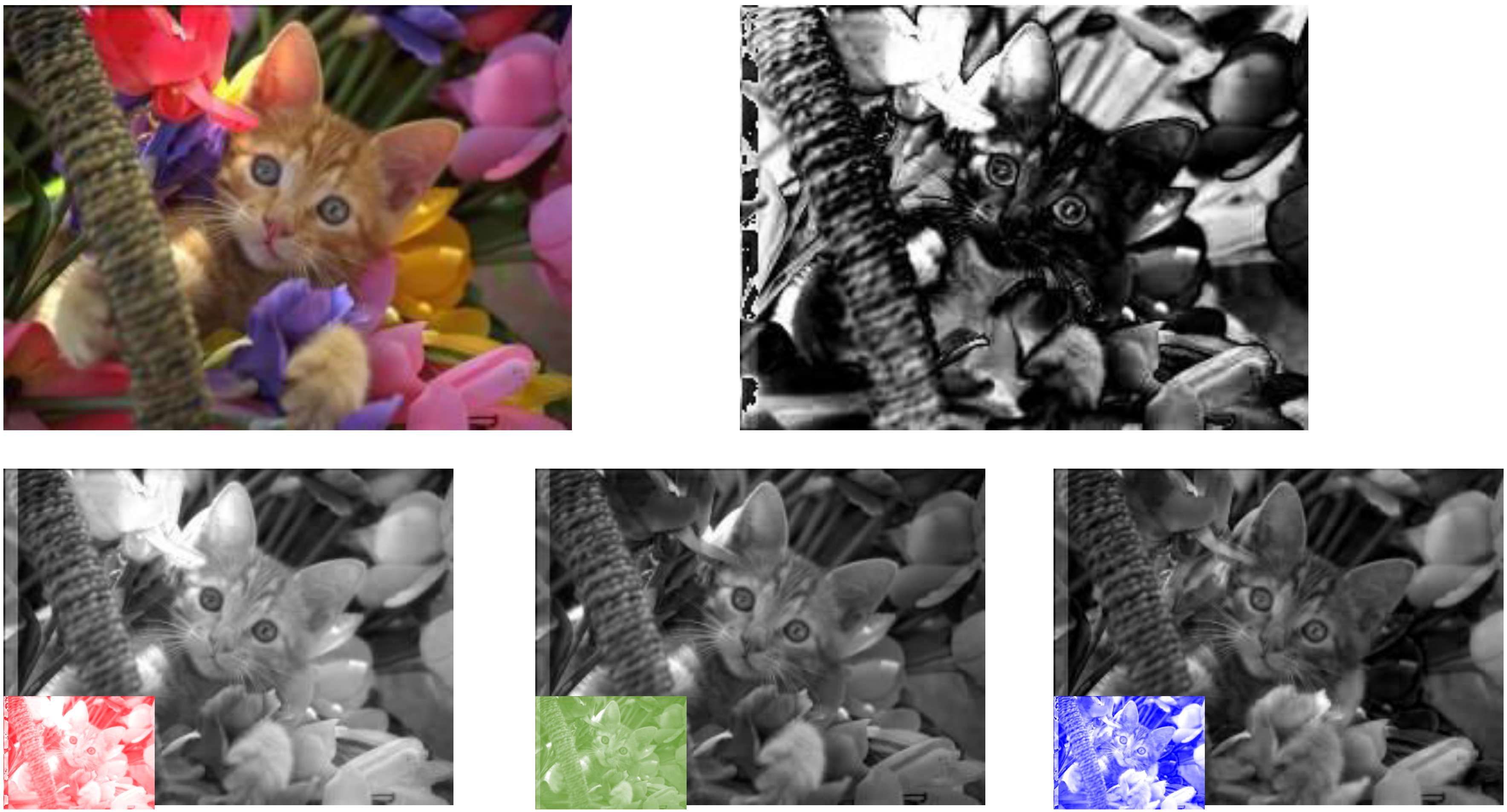
W - Fourier transform of *w*

a,b - unknown shift parameters

* - complex conjugate

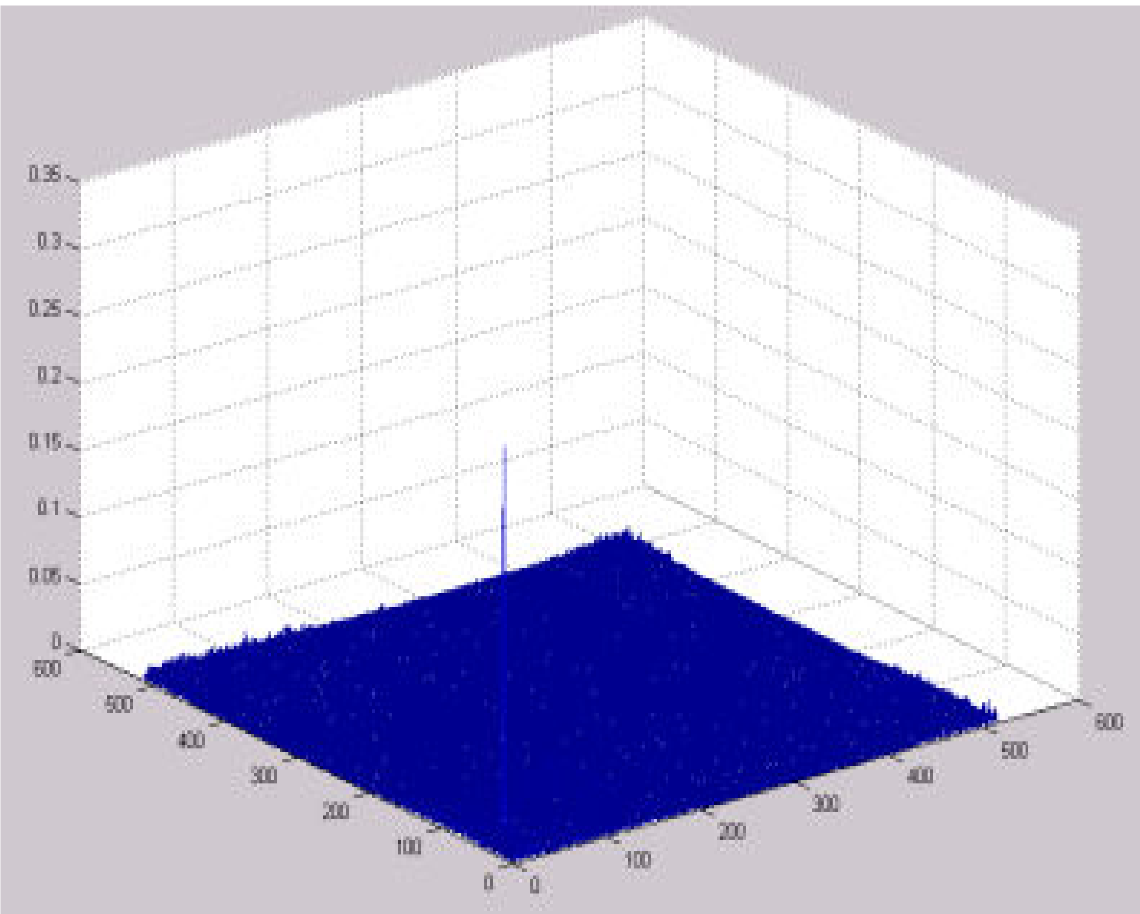
$$\mathrm{IFT}\left(e^{-2\pi i\,(ua + vb)}\right) = \delta(x-a, y-b)$$

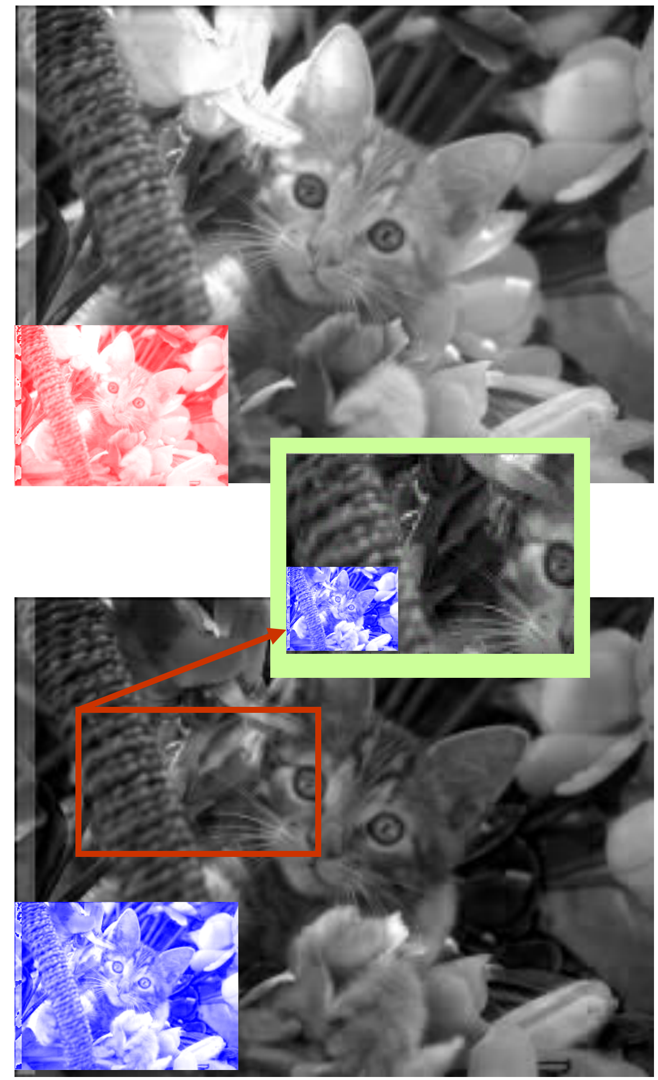
- **Multimodal registration - experiment**
- **Comparison of image correlation and phase correlation**



Channel to channel registration

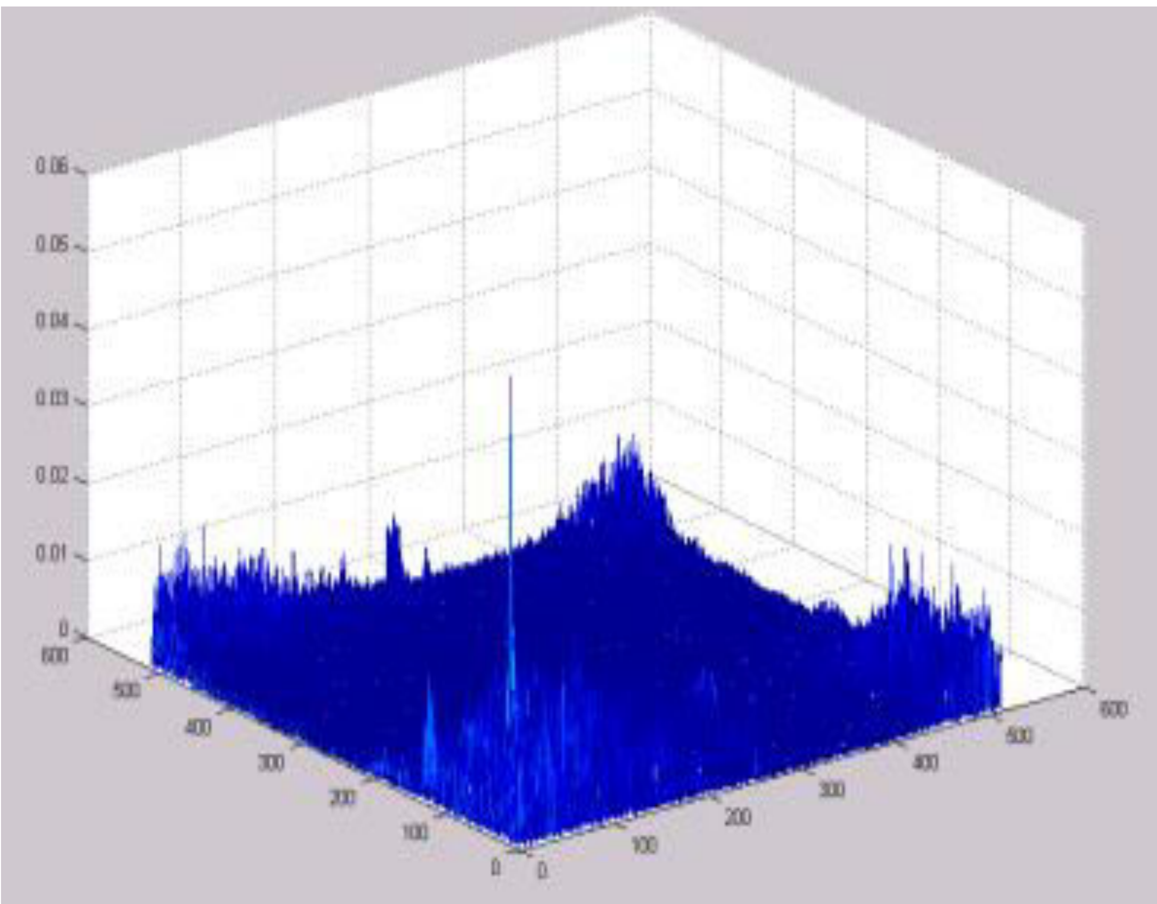
phase - correlation R - B



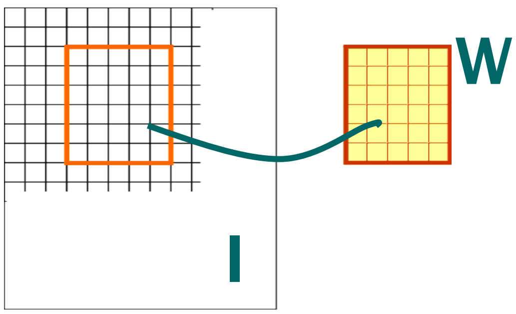


Multimodal registration

phase - correlation R-R

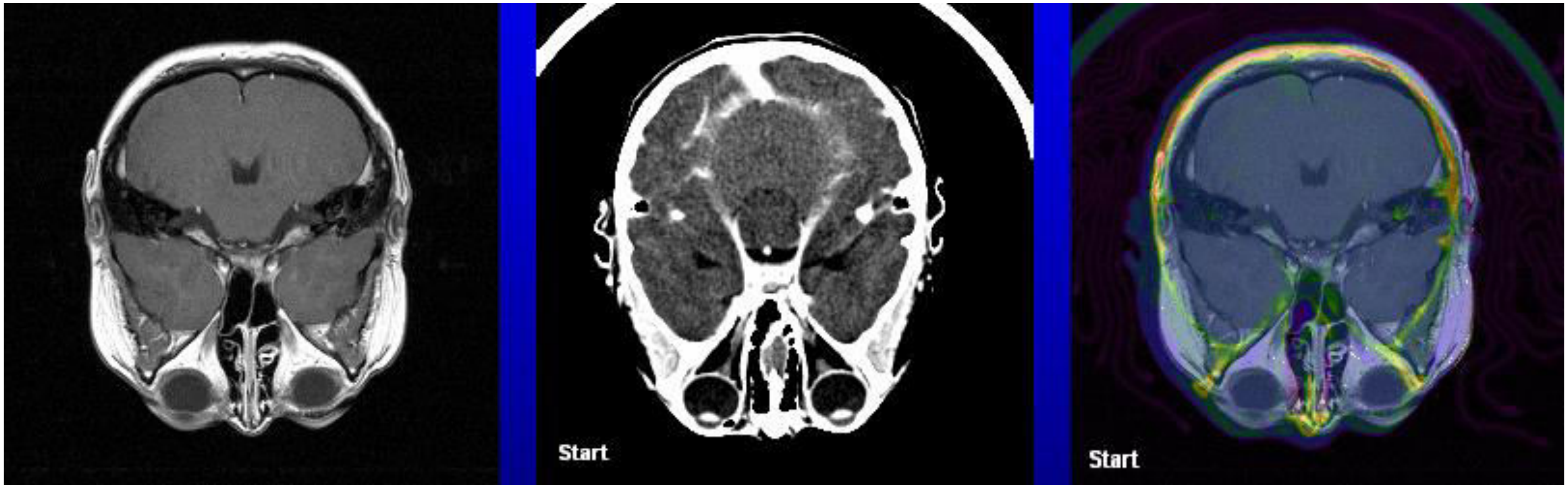


Mutual information method



Statistical measure of the dependence between two images

$$MI(f,g) = H(f) + H(g) - H(f,g)$$



MUTUAL INFORMATION

Entropy

$$H(X) = -\sum_{x} p(x) \log p(x)$$

Joint entropy

$$H(X,Y) = -\sum_{x} \sum_{y} p(x,y) \log p(x,y)$$

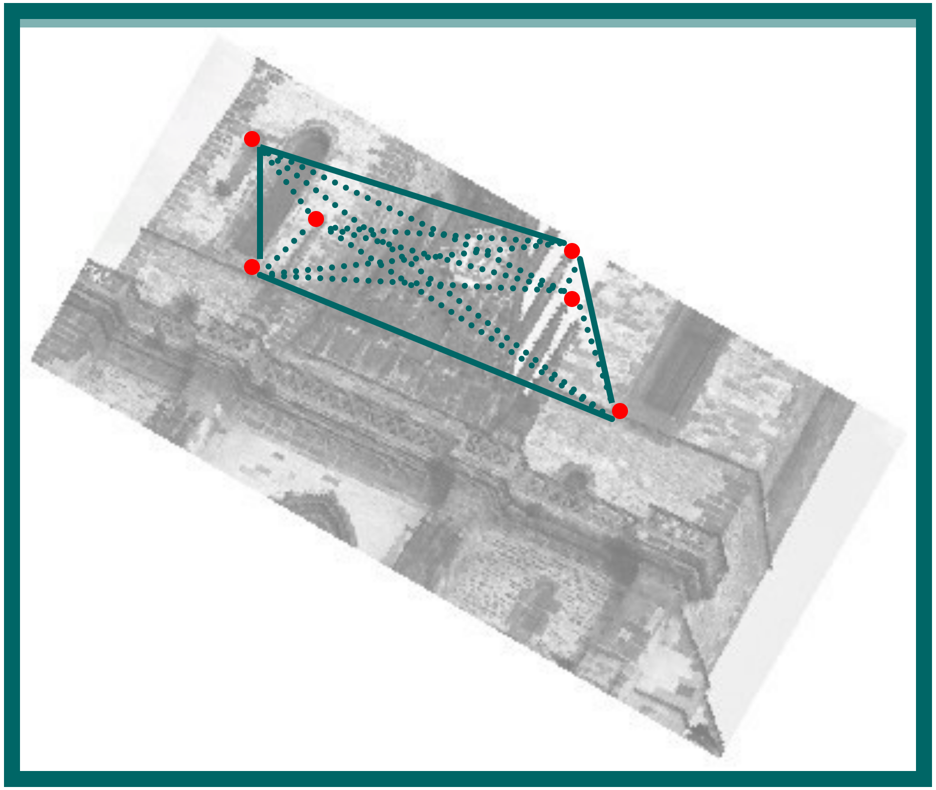
Mutual infomation

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

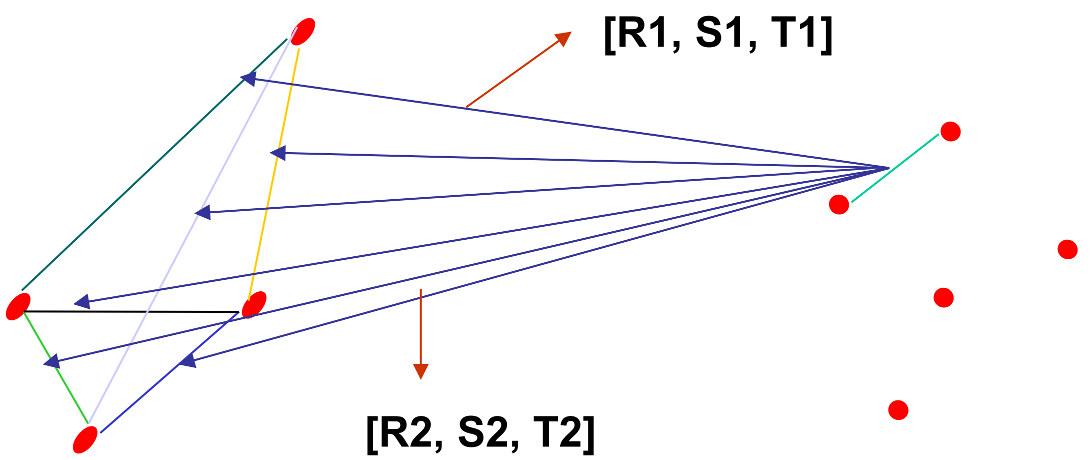
Feature-based methods

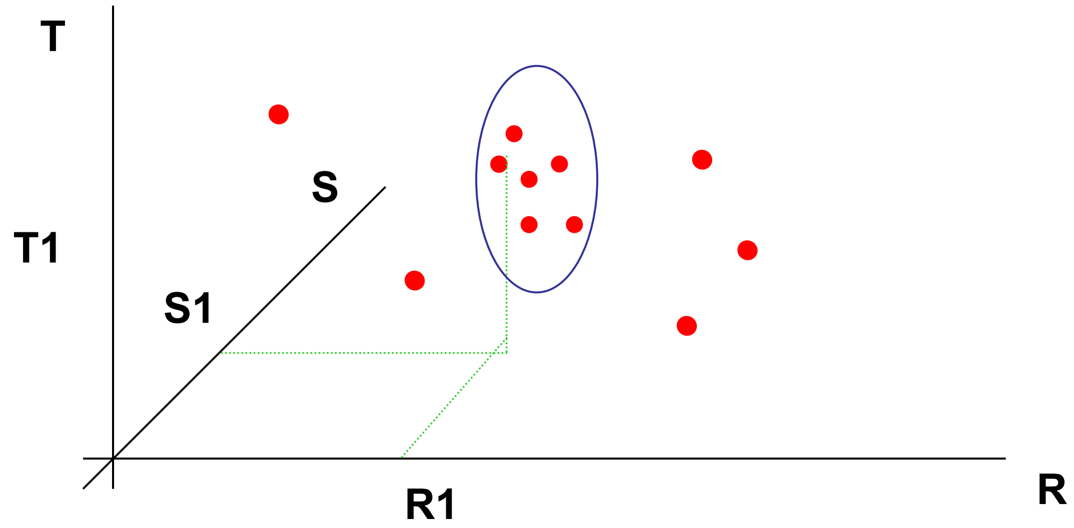
- Combinatorial matching (no feature description). Graph matching, parameter clustering. Global information only is used.
- Matching in the feature space (pattern classification). Local information only is used.
- Hybrid matching (combination of both to get higher robustness)

Combinatorial matching

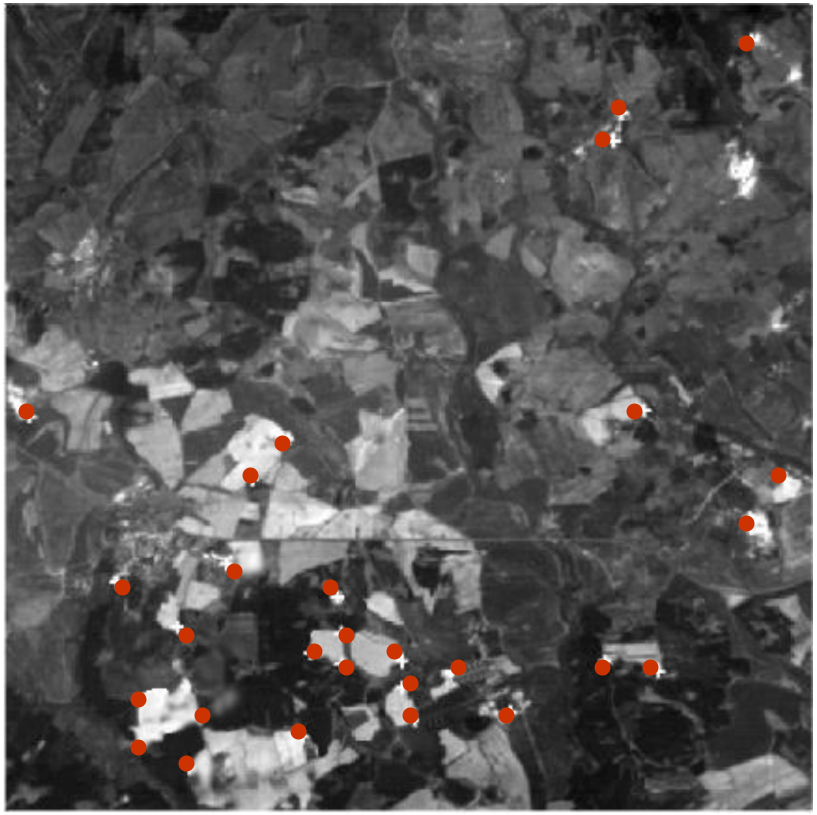


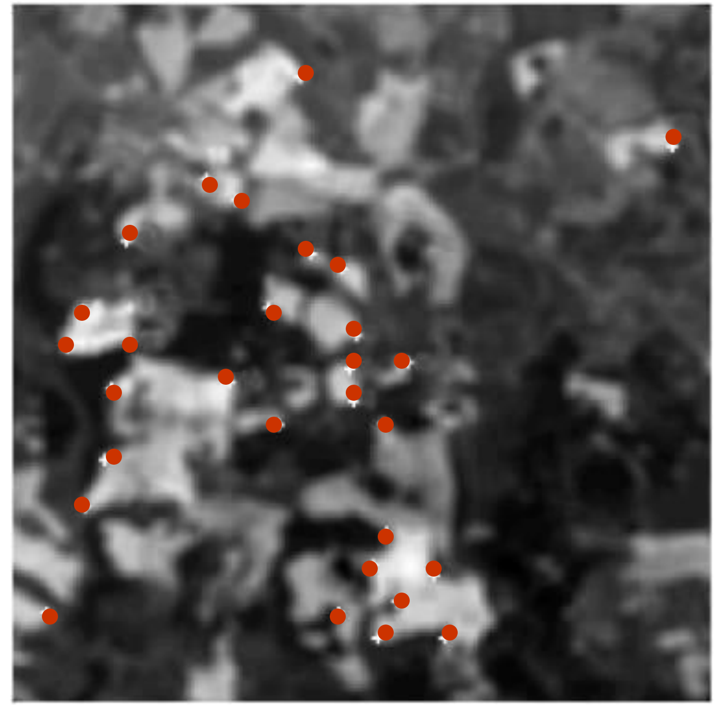
PARAMETER CLUSTERING





Matching in the feature space

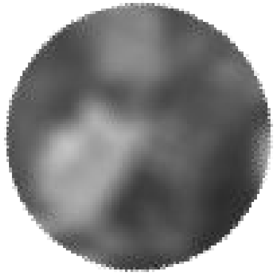




$$\min_{k,m} \ \text{distance}\left((v1_k, v2_k, v3_k, \ldots), (\overline{v1}_m, \overline{v2}_m, \overline{v3}_m, \ldots) \right)$$

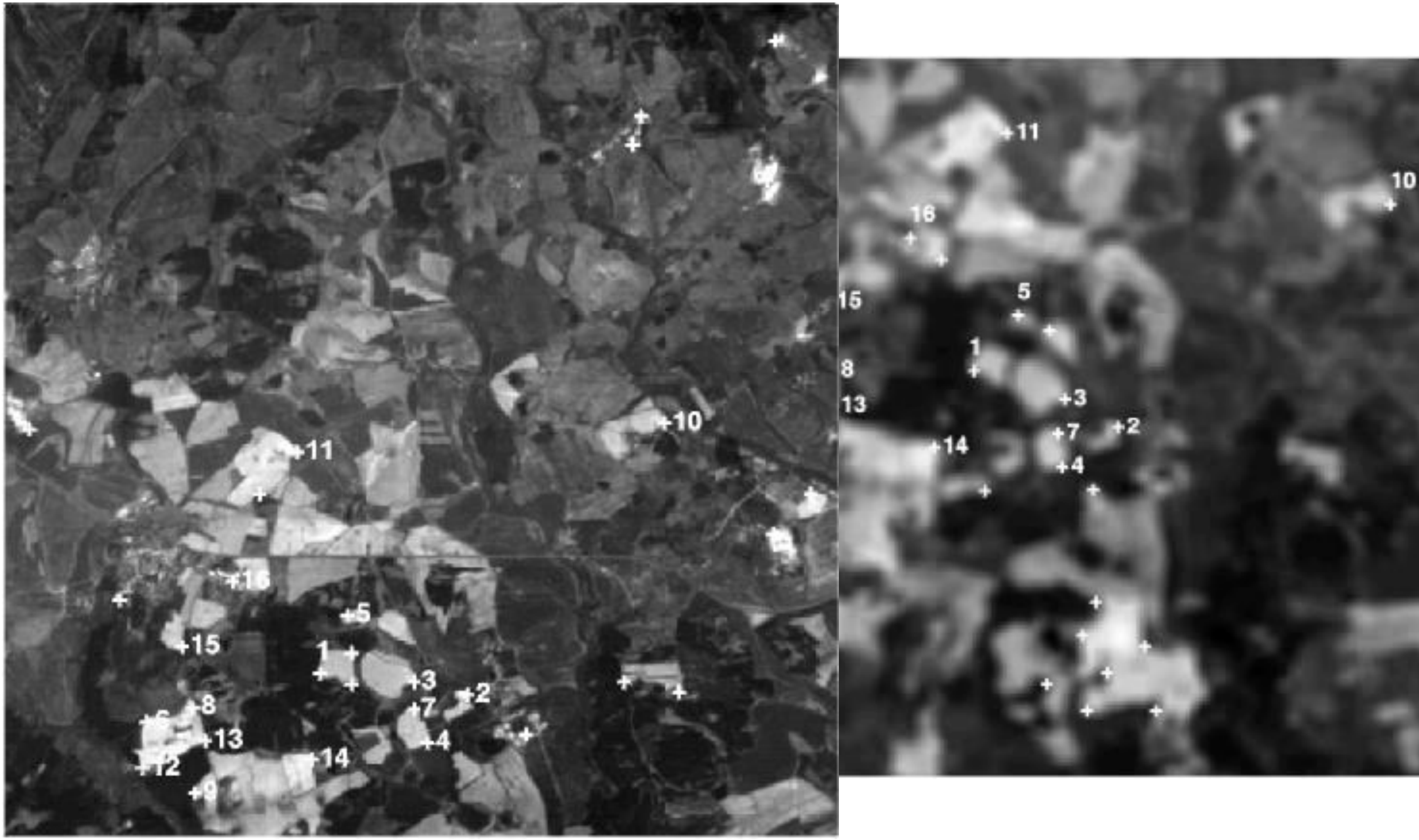
Features for CP description

- calculated over a circular neighborhood of each CPC
- invariant to all assumed image degradations

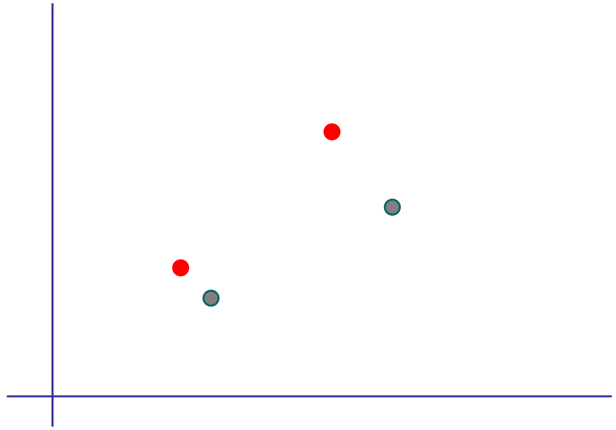


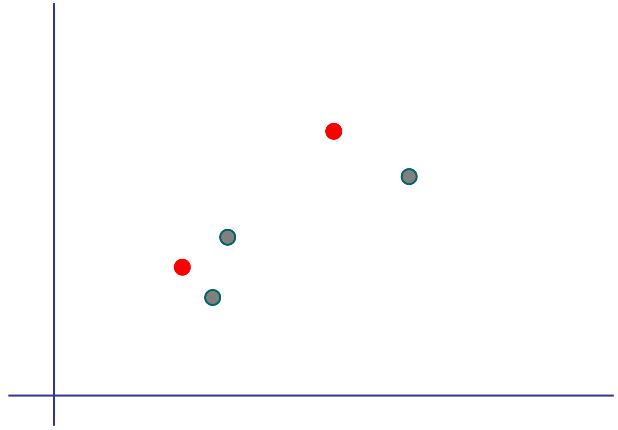
- discriminative
- robust to noise

Matching result



ROBUST MATCHING IN THE FEATURE SPACE

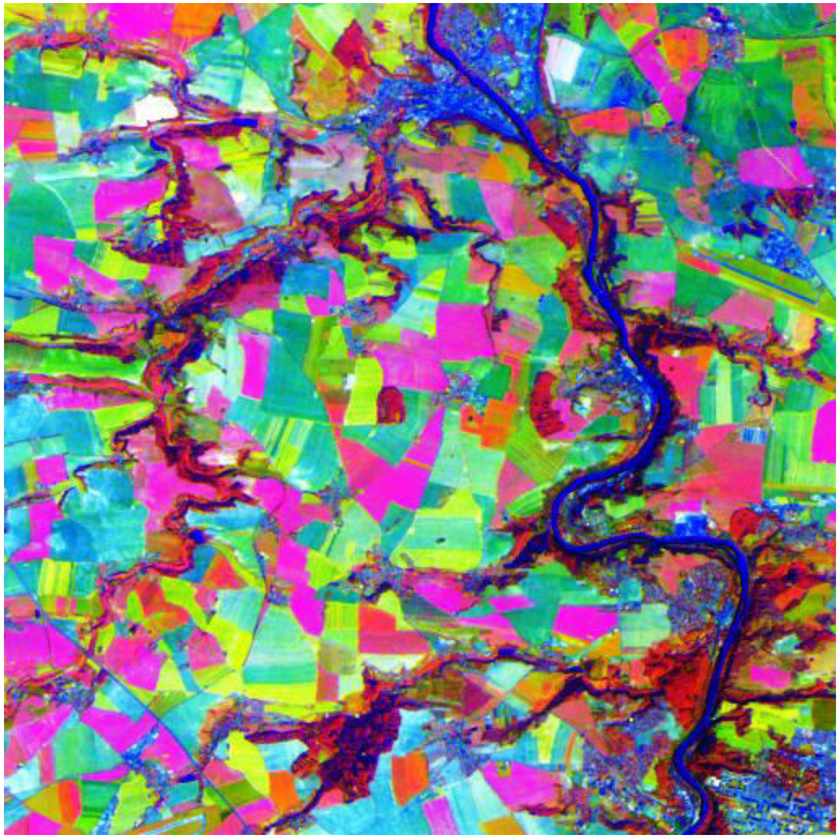




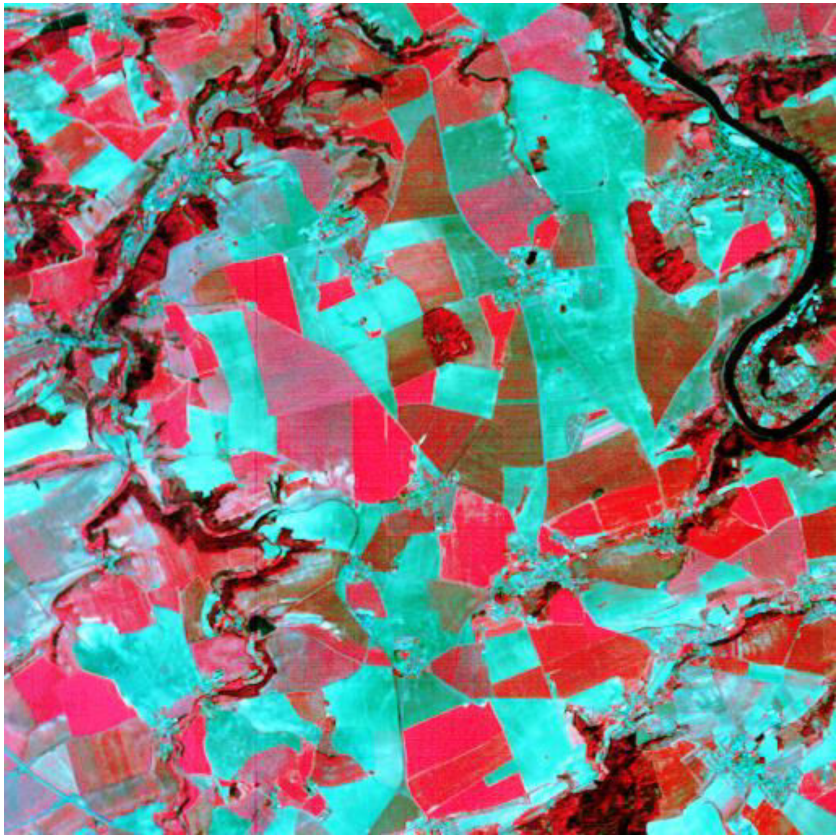
Hybrid matching

- **Step 1: Matching few CP's by invariant features**
- **Step 2: Matching the rest of the CP's in the image space**

Hybrid matching

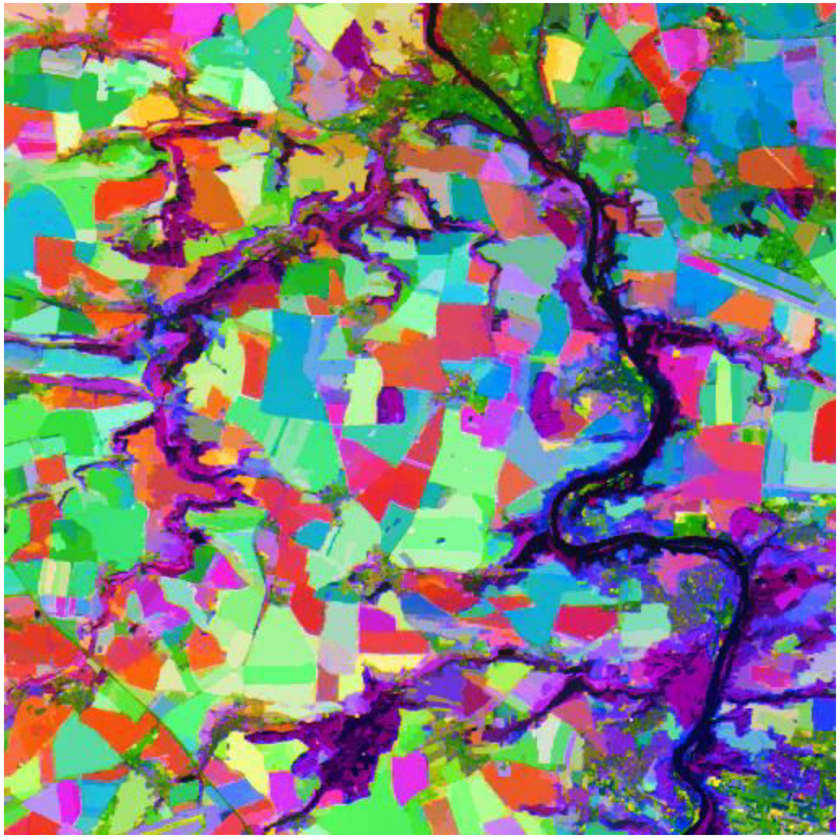


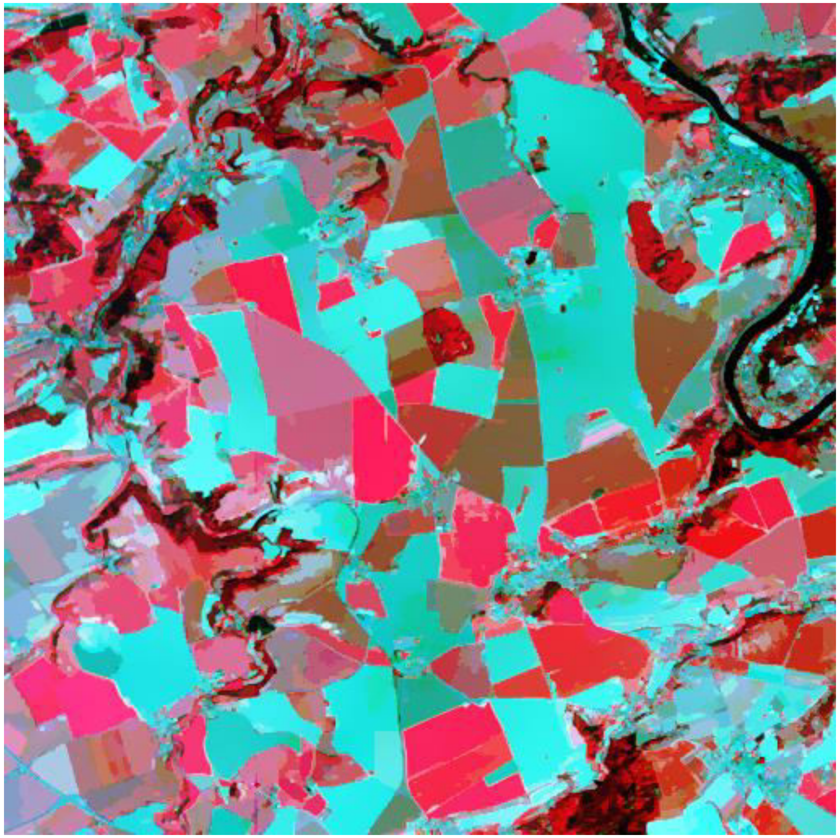
Landsat



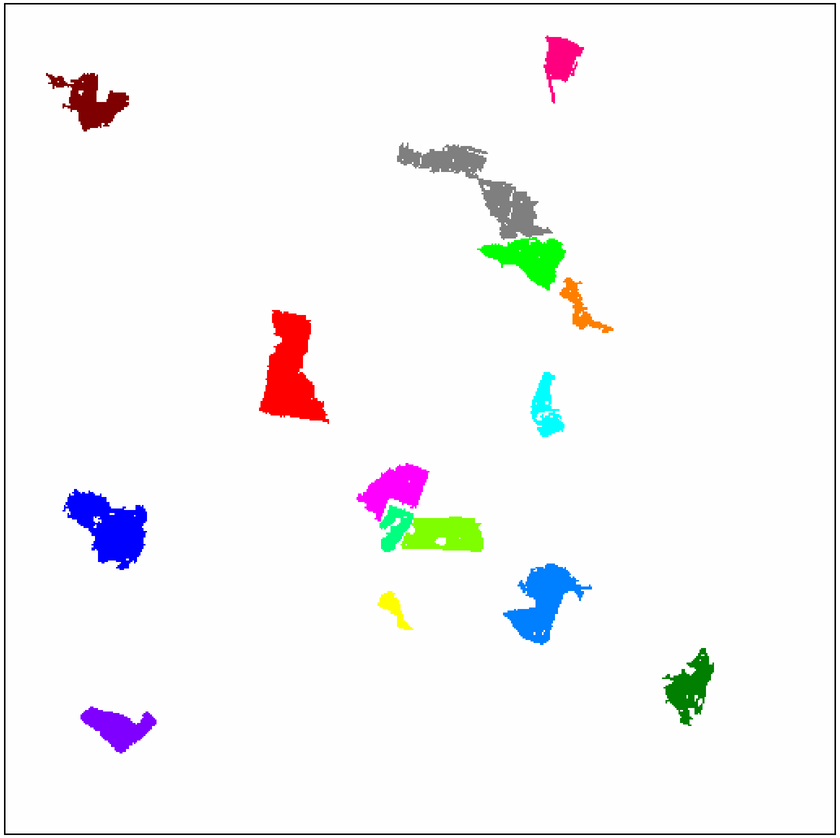
Spot

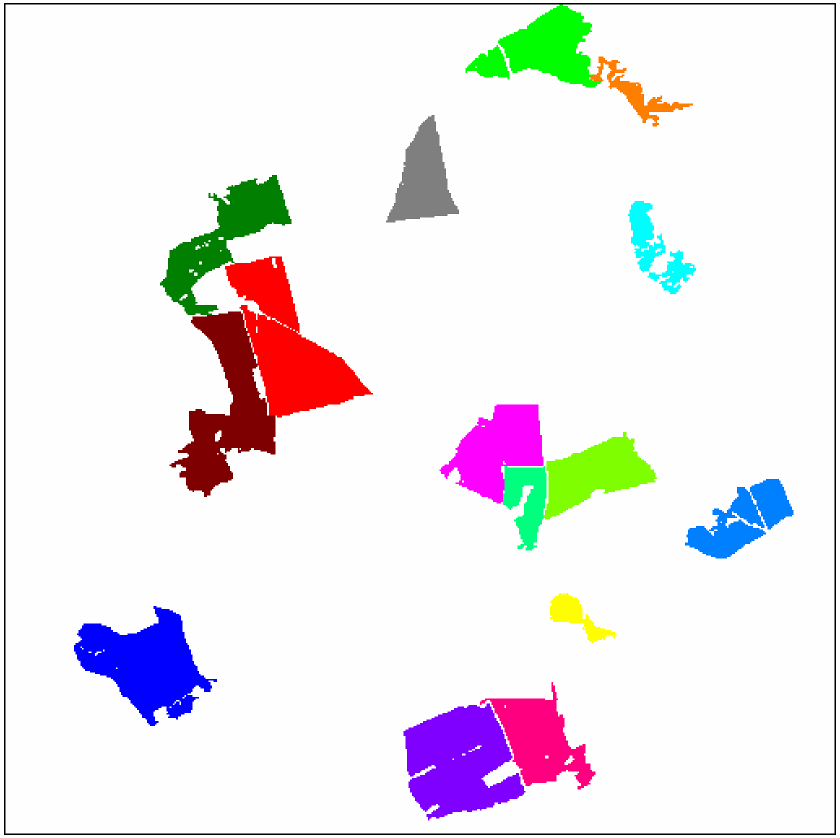
Segmentation by Mumford - Shah method



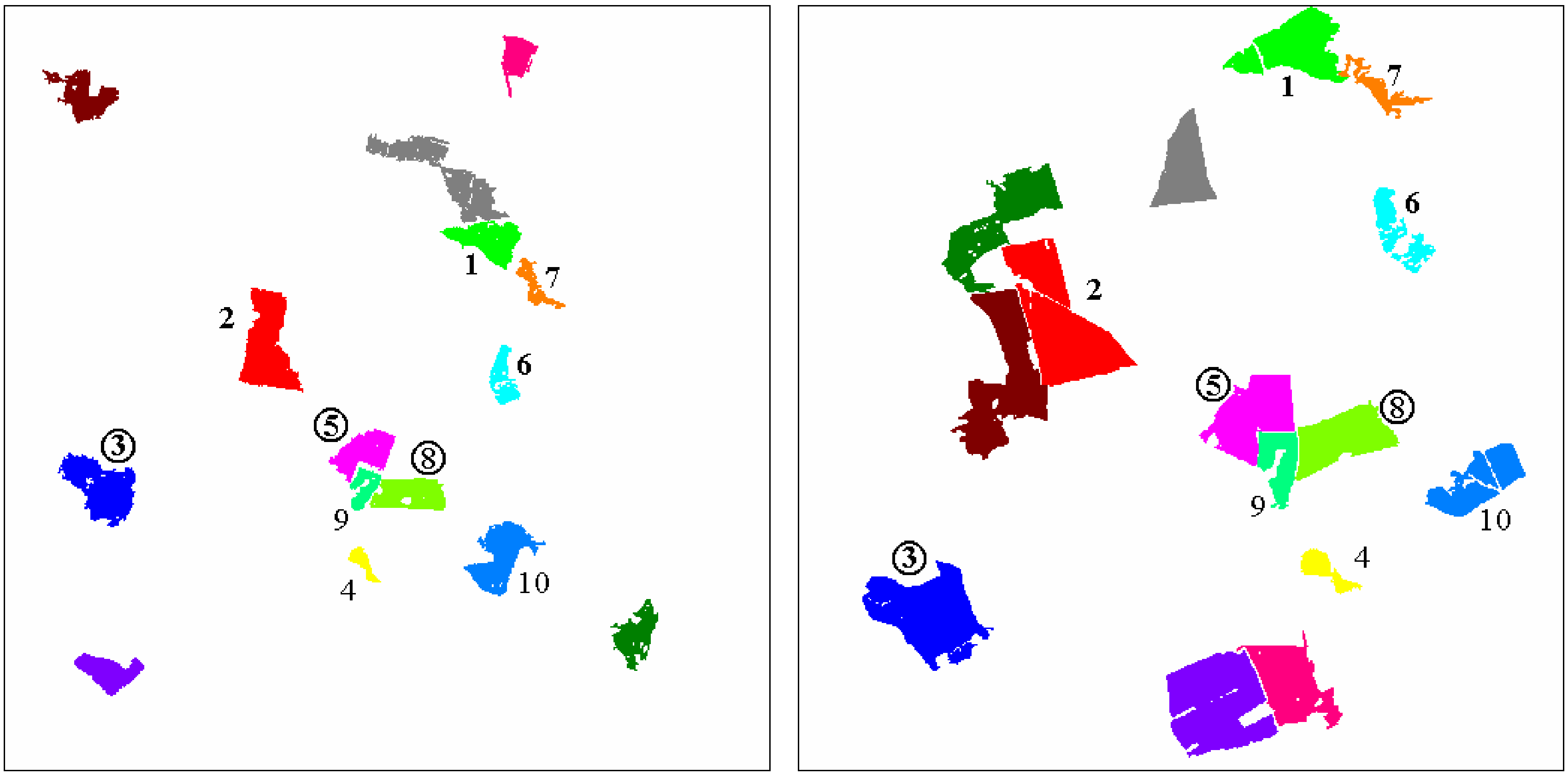


Selected regions





Matching pairs



Mapping function design

- Global functions

  Similarity, affine, projective transform

  Low-order polynomials

Mapping function design

- Global functions

  Similarity, affine, projective transform

  Low-order polynomials

- Local functions

  Piecewise affine, piecewise cubic

  Thin-plate splines

  Radial basis functions

Global mapping functions

- **Affine transform**

$$x' = a_0 + a_1 x + a_2 y$$
$$y' = b_0 + b_1 x + b_2 y$$

Similarity transform – Least square fit

translation [Δx, Δy], rotation φ, uniform scaling s

$$x' = s\,(x * \cos\varphi - y * \sin\varphi) + \Delta x$$
$$y' = s\,(x * \sin\varphi + y * \cos\varphi) + \Delta y$$

$$s\cos\varphi = a, \qquad s\sin\varphi = b$$

$$\min\left(\Sigma_{i=1}\{[x_i' - (ax_i - by_i) - \Delta x]^2 + [y_i' - (bx_i + ay_i) - \Delta y]^2\}\right)$$

$$\begin{vmatrix} \Sigma(x_i^2 + y_i^2) & 0 & \Sigma x_i & \Sigma y_i \\ 0 & \Sigma(x_i^2 + y_i^2) & -\Sigma y_i & \Sigma x_i \\ \Sigma x_i & -\Sigma y_i & N & 0 \\ \Sigma y_i & \Sigma x_i & 0 & N \end{vmatrix} \bullet \begin{vmatrix} a \\ b \\ \Delta x \\ \Delta y \end{vmatrix} = \begin{vmatrix} \Sigma(x_i' x_i - y_i' y_i) \\ \Sigma(y_i' x_i - x_i' y_i) \\ \Sigma x_i' \\ \Sigma y_i' \end{vmatrix}$$

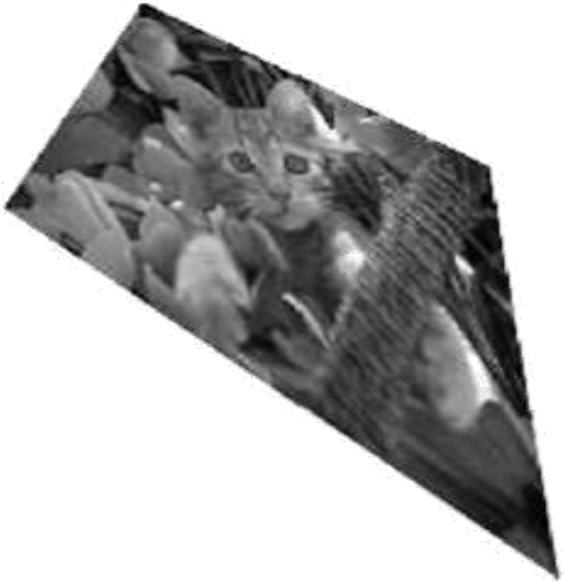
Global mapping functions

- **Affine transform**

$$x' = a_0 + a_1 x + a_2 y$$
$$y' = b_0 + b_1 x + b_2 y$$

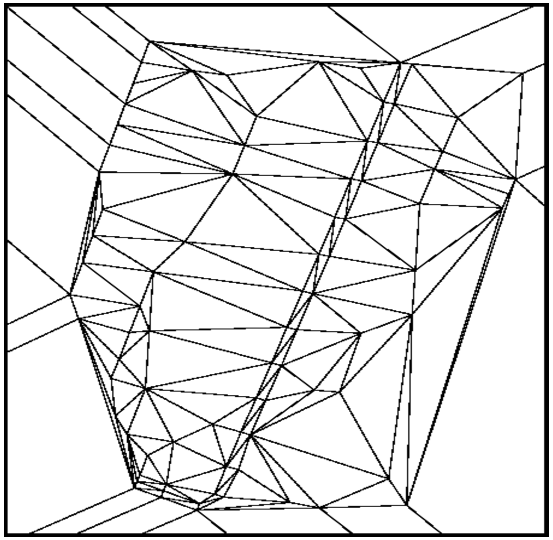
- **Projective transform**

$$x' = (a_0 + a_1 x + a_2 y) / (1 + c_1 x + c_2 y)$$
$$y' = (b_0 + b_1 x + b_2 y) / (1 + c_1 x + c_2 y)$$



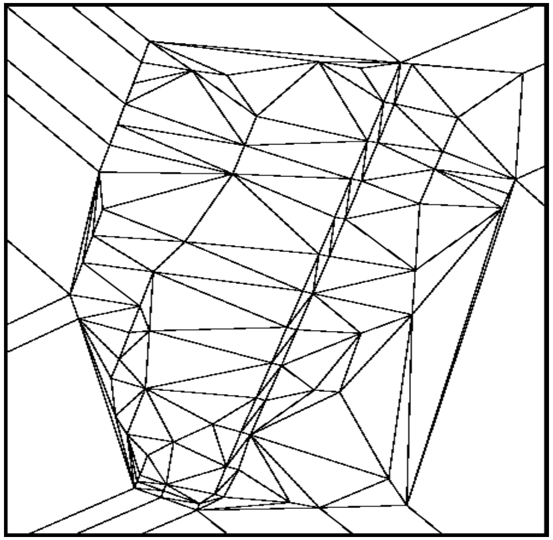
Local mapping functions

- Piecewise affine or cubic



Local mapping functions

- Piecewise affine or cubic



- Thin-Plate Splines (TPS)

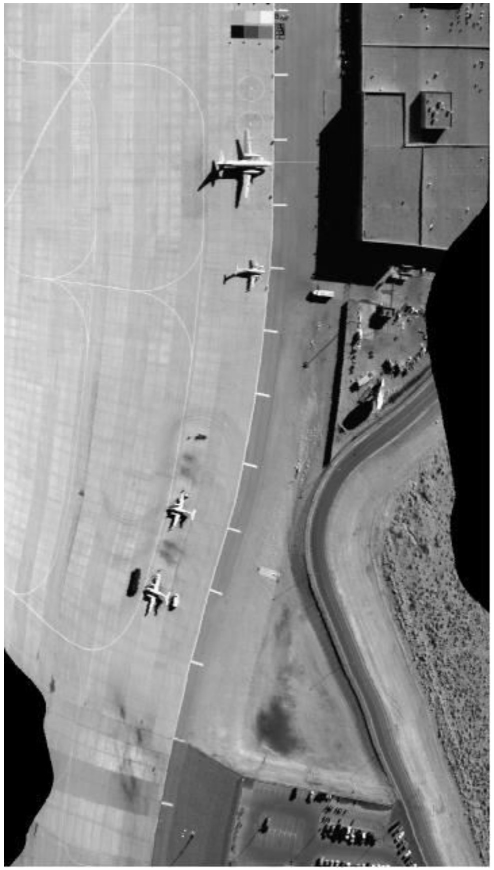
$$\alpha_1 + \alpha_2 x + \alpha_3 y + \sum_{i=1}^{N} a_i g_i(\|x - x_i, y - y_i\|),$$

$$g_i(t) = t^2 \log t.$$

Airport images

Reference

Sensed (simulation)

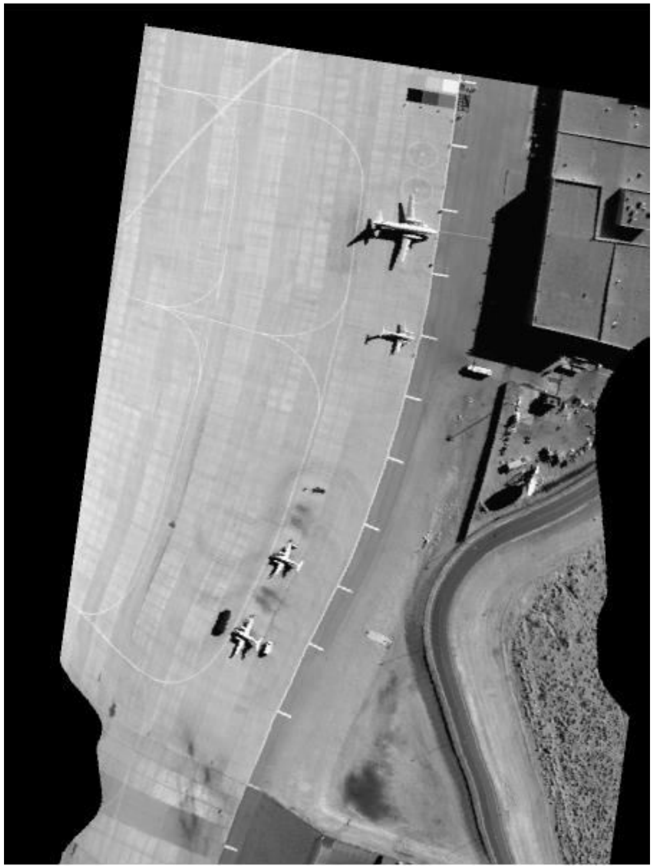


From D. N. Fogel et al., UCSB

Airport images

Reference

Affine mapping



Airport images

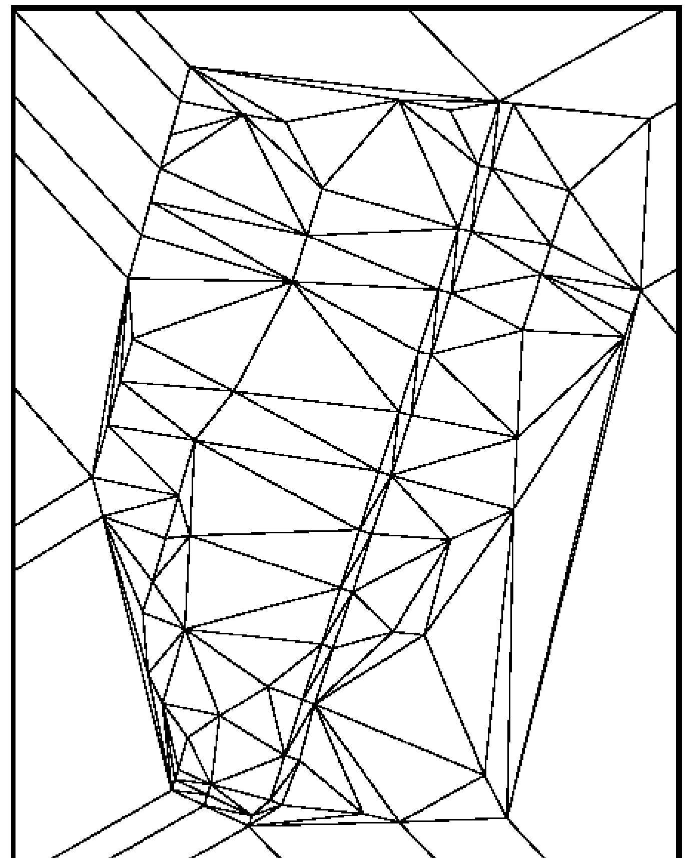
Reference

Cubic mapping

Airport images

Reference

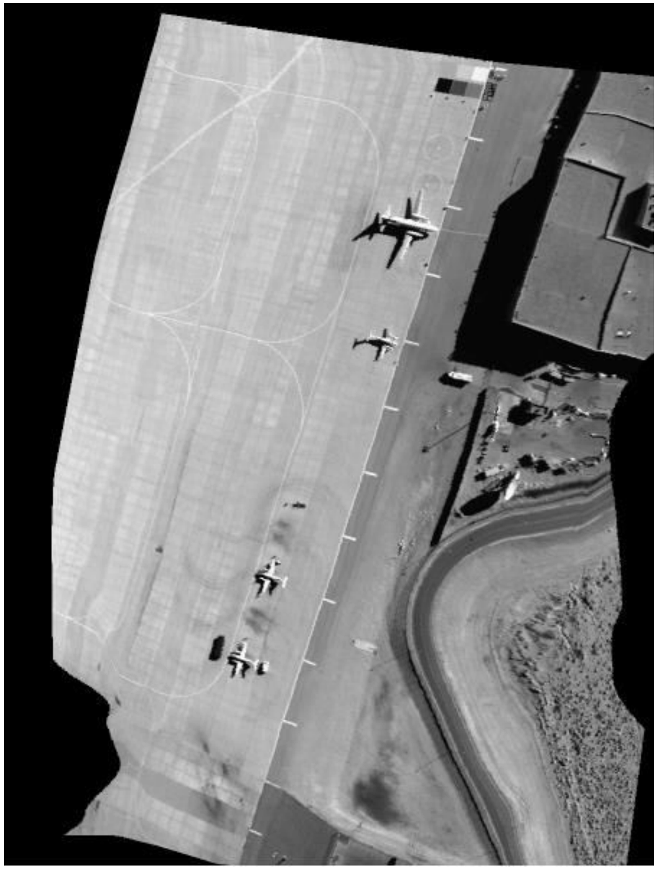
Piecewise affine



Airport images

Reference

Piecewise affine



Airport images

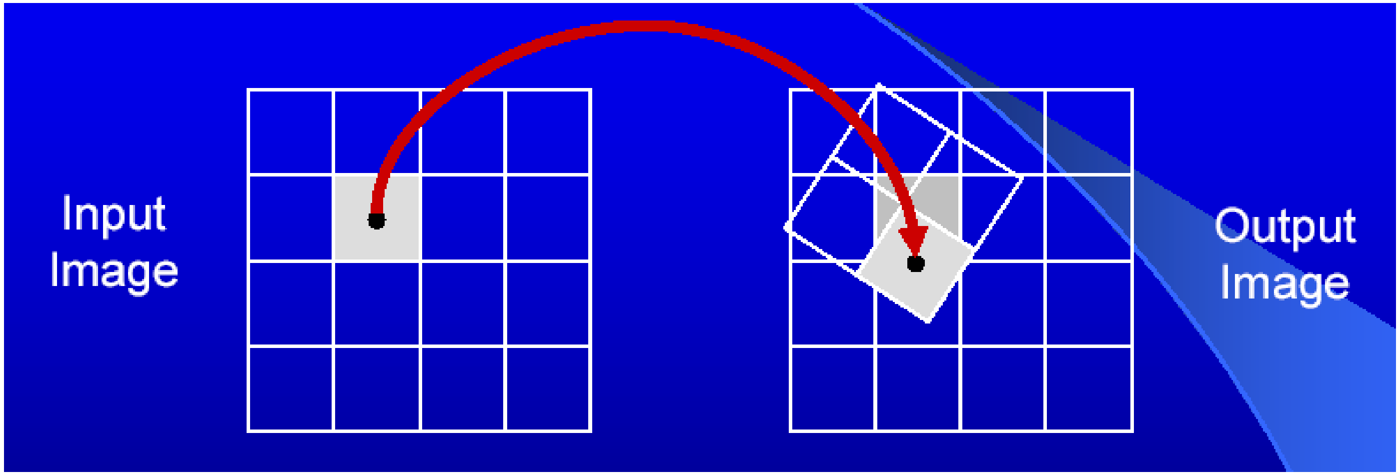
Reference

Thin-plate splines

Image resampling

Image resampling

Forward x backward methods



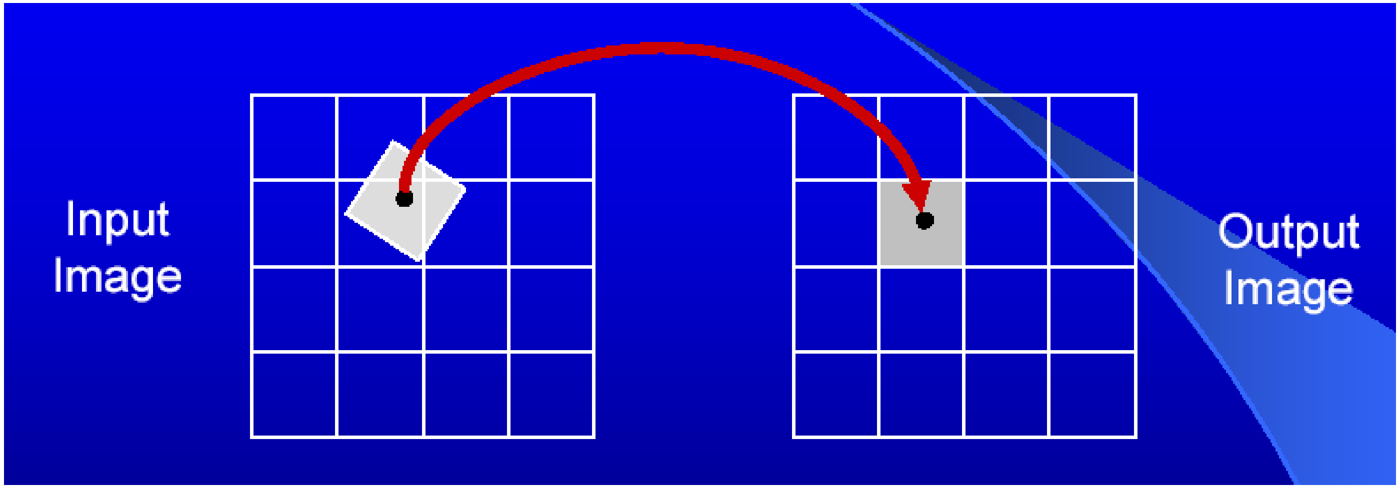


IMAGE RESAMPLING AND TRANSFORMATION

Interpolation **nearest neighbor**

bilinear

bicubic

Implementation **1-D convolution**

$$f(x_o,k) = \Sigma\, d(I,k).c(i-x_0)$$

$$f(x_0, y_0) = \Sigma\, f(x_0, j).c(j-y_0)$$

ideal $c(x) = k.sinc(kx)$

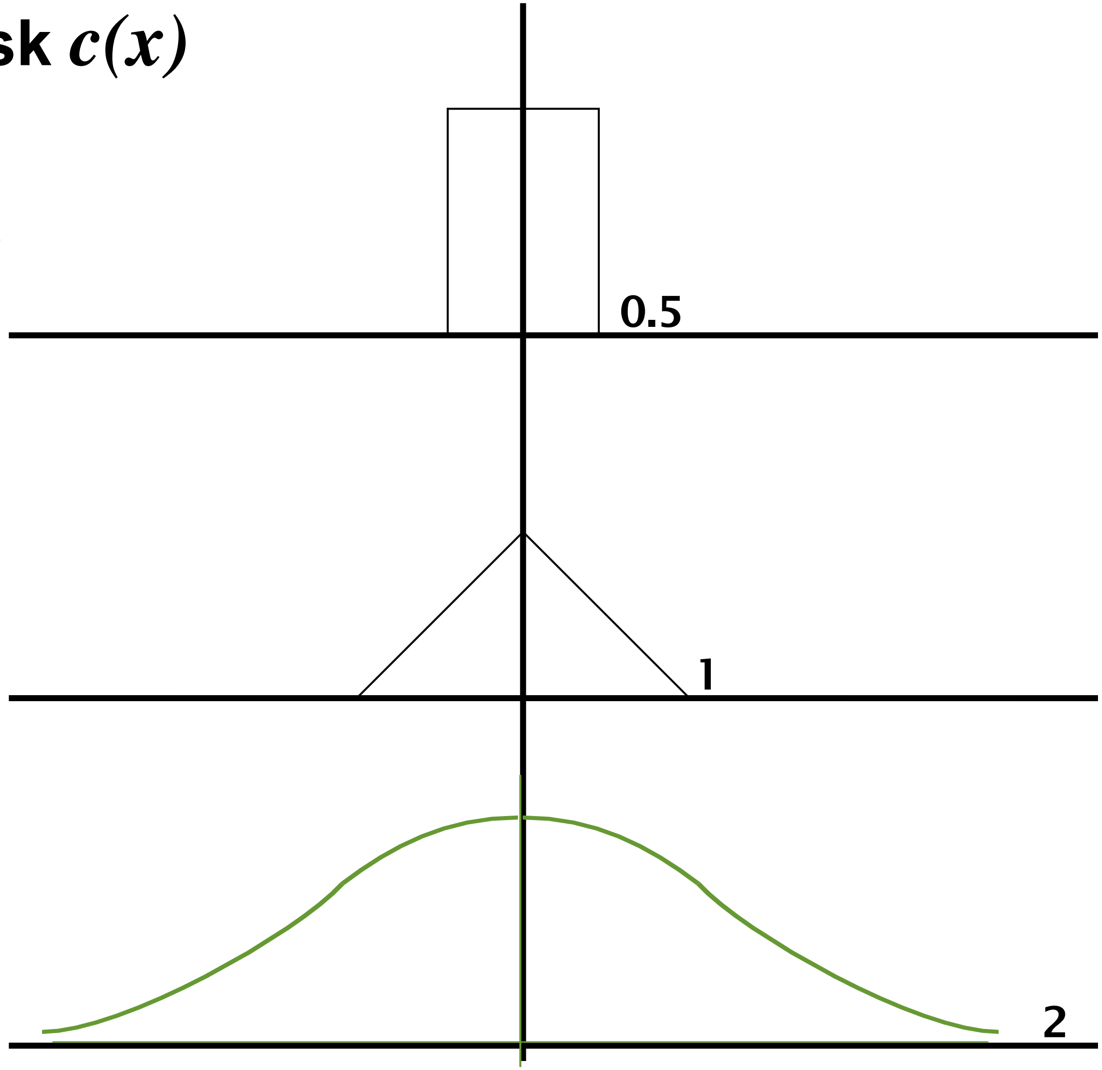
IMAGE RESAMPLING AND TRANSFORMATION

Interpolation mask $c(x)$

nearest neighbor

linear

smooth cubic



Interpolation methods

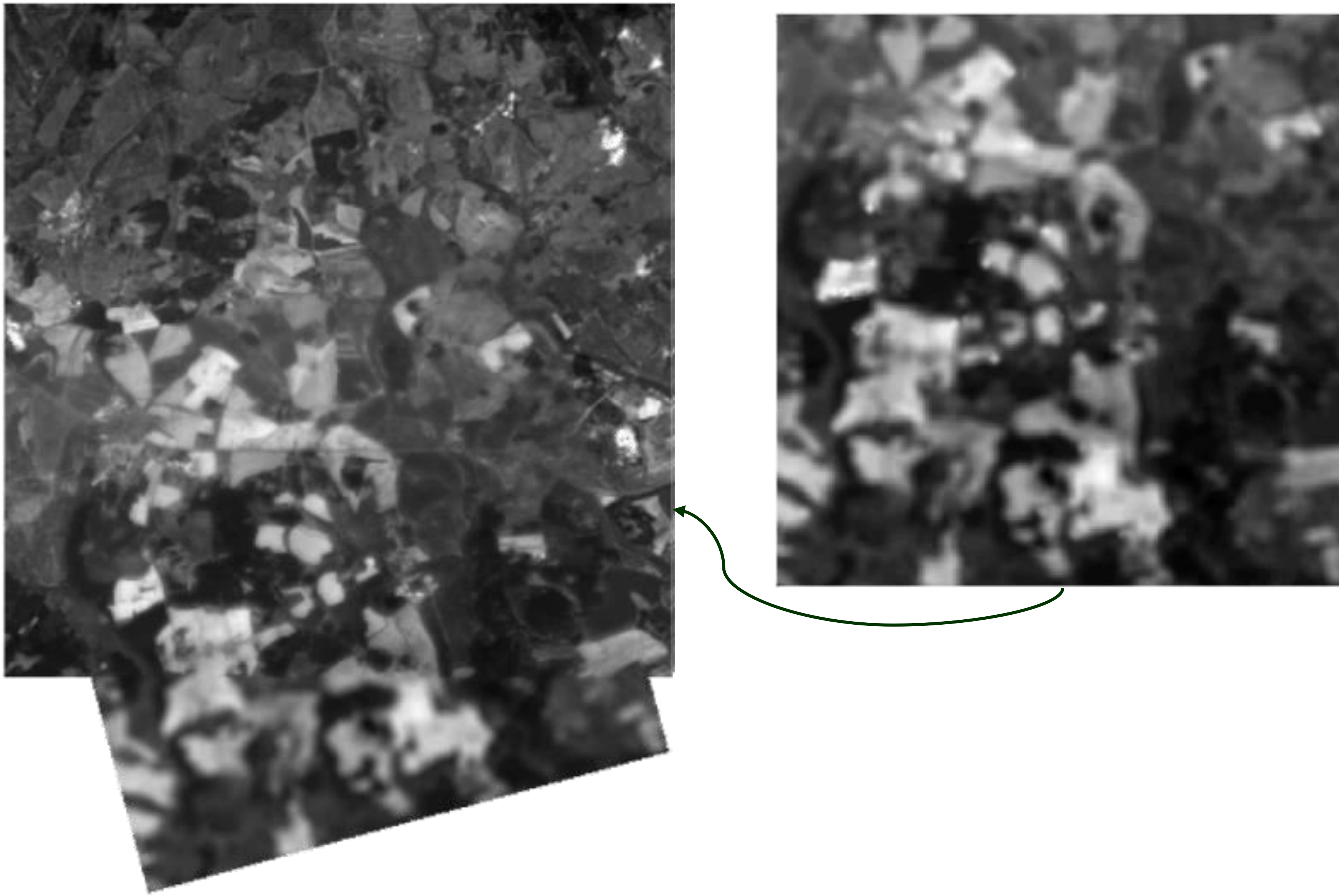
Original

Nearest neighbor

Bilinear

Bicubic

Registered images

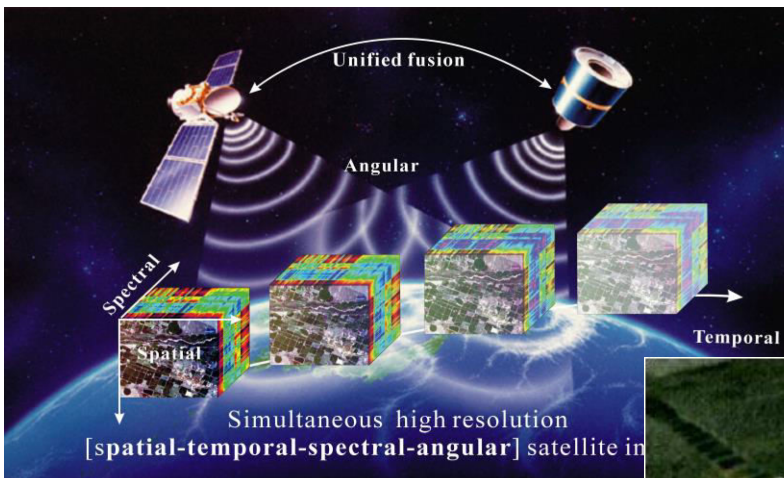


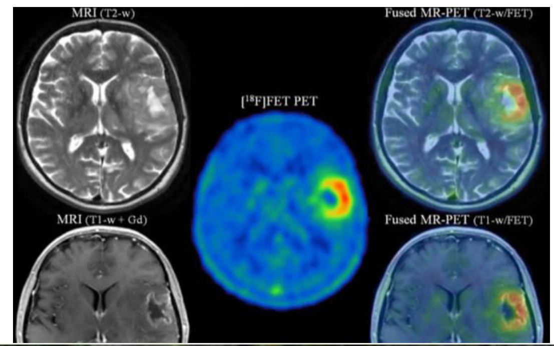
Applications

- Remote sensing
- Medicine
- Art history
- Robot vision
- Image warping
- Image fusion

Image Fusion

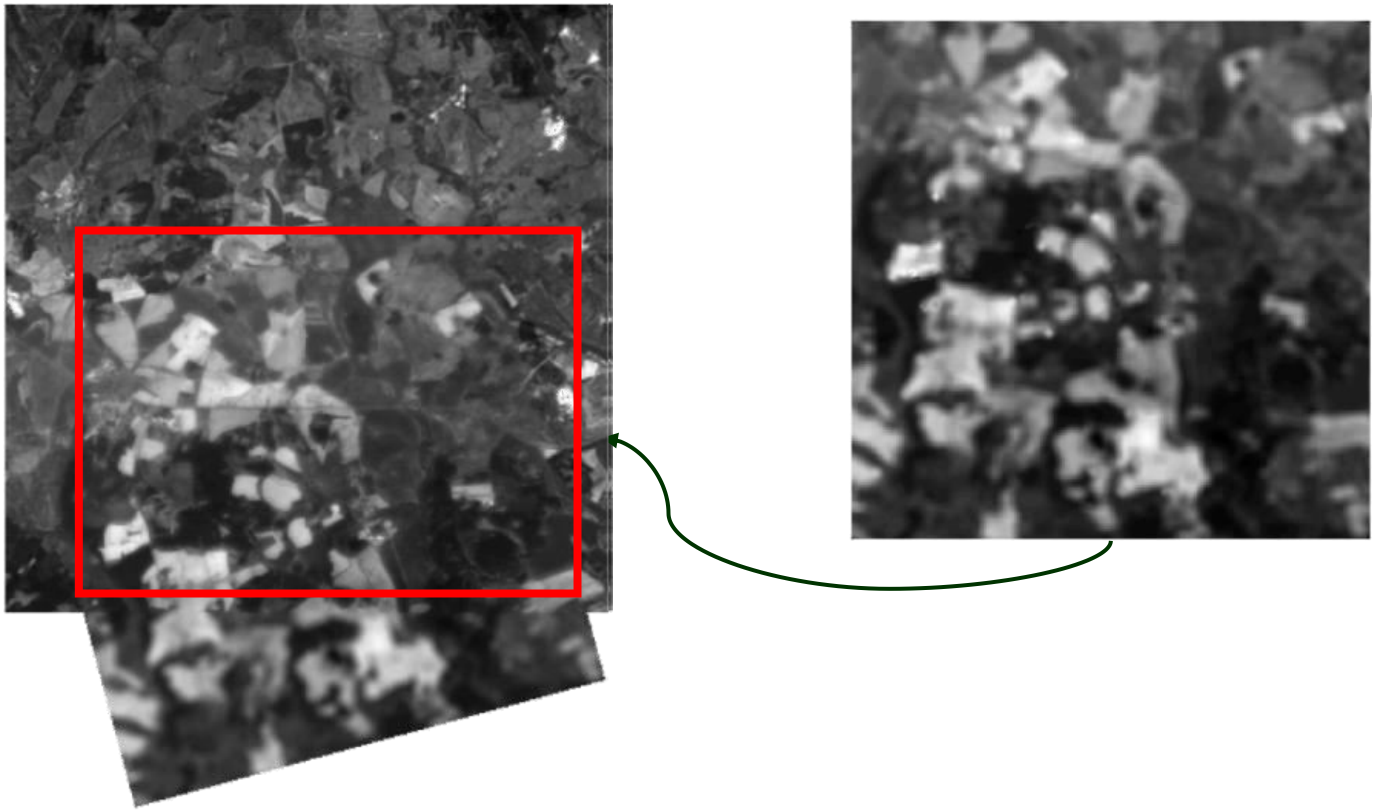
Combining several images of the same scene into a single image of a higher quality





Two stages of image fusion

1. Image-to-image registration

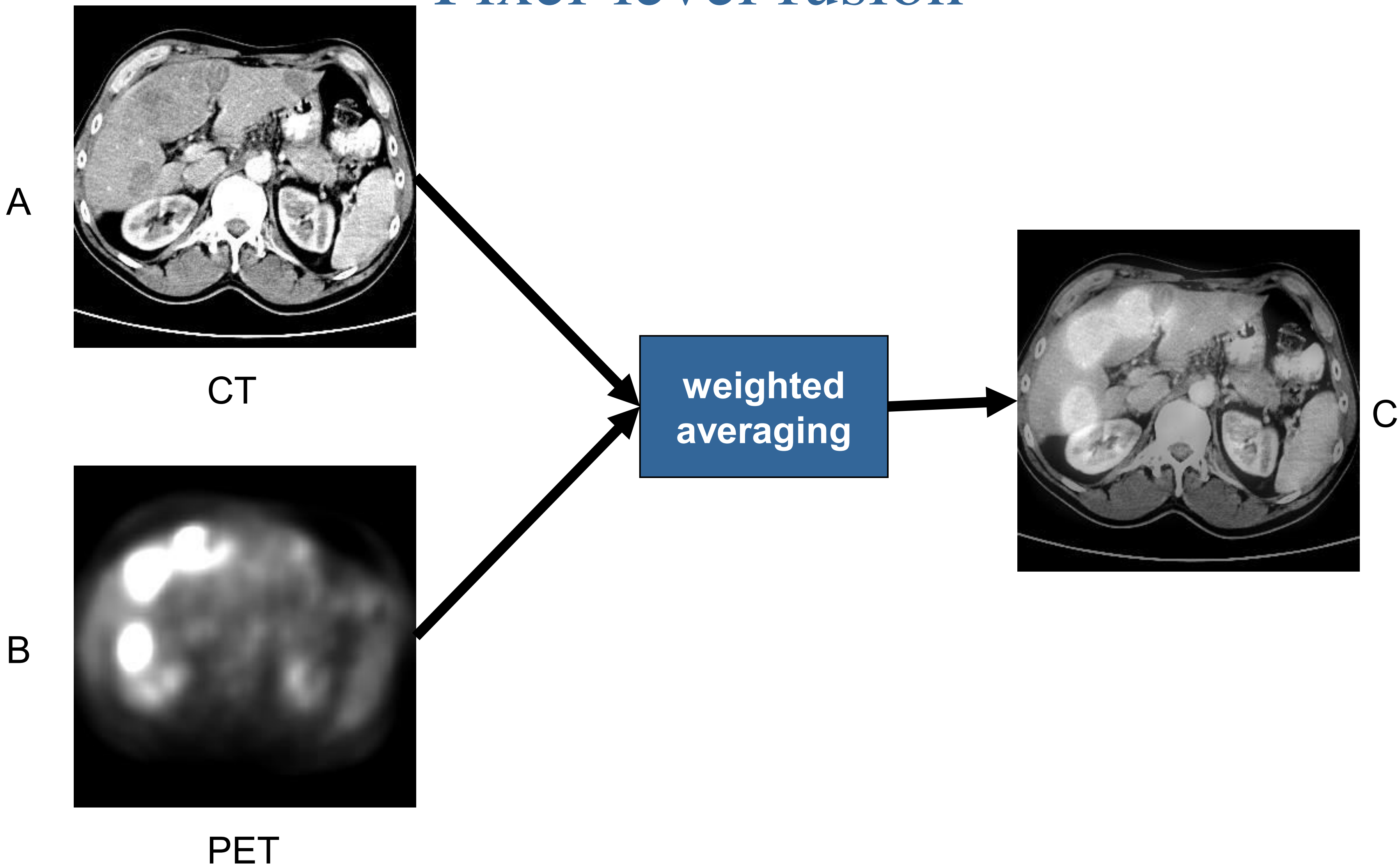


2. The fusion itself

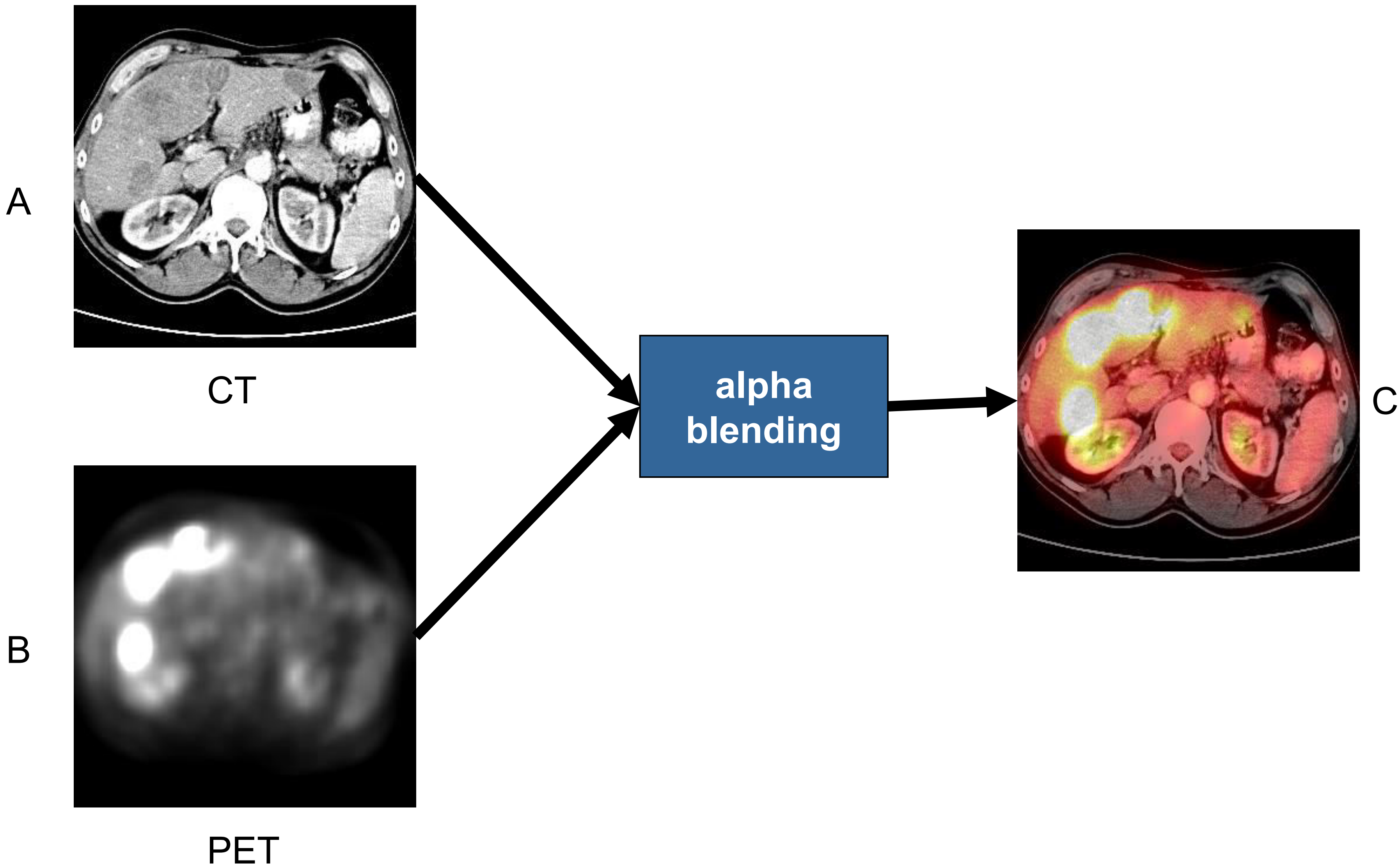
Fusion levels

- **Pixel level**
- **Feature level**
- **Decision level**

Pixel-level fusion



Pixel-level fusion



Digital subtraction angiography

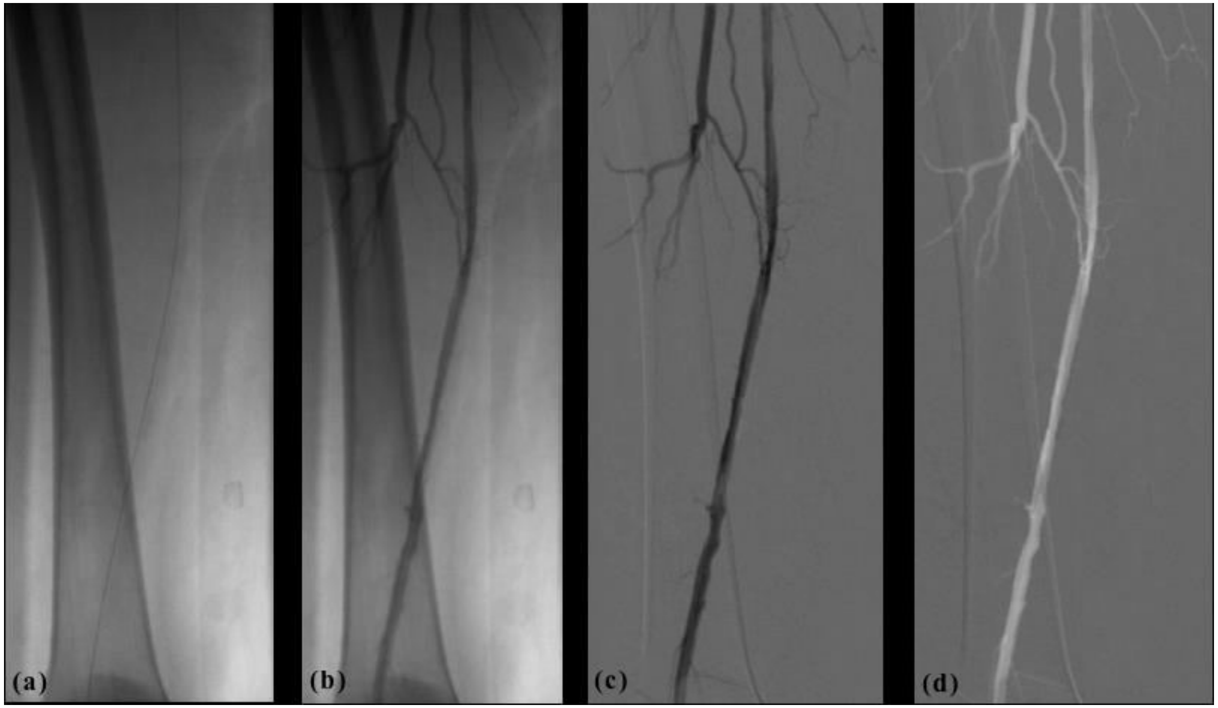


IMAGE REGISTRATION: ACCURACY EVALUATION

Localization error	**- displacement of features**
	- due to detection method
Matching error	**- false matches**
	- ensured by robust matching (hybrid)
	- consistency check, cross-validation
Alignment error	**- difference between model and reality**
	- test point error

Thank you !

Any questions ?