Image restoration

Acquired image is a degraded version of the original scene

Image degradation model

Image acquisition model

original scene

degradations

$$\mathcal{D} = \mathcal{Q}\mathcal{S}\mathcal{D}_{\mathcal{I}}\mathcal{D}_{\mathcal{G}}$$

noise

acquired image

$$\mathcal{D}_{\mathcal{G}}(f)(x,y) = f(\tau(x,y))$$

$$\mathcal{D}_{\mathcal{I}}(f)(x,y) = \int_{\Lambda}\int_{T}\int\int h(x,y,a,b,\lambda,t)f(a,b)dadbd\lambda dt$$

Sources, models and appearance of individual degradations

Blur

Image smoothing, low-pass filter
Suppression of high frequencies

$$\mathcal{D}_{\mathcal{I}}(f)(x,y) = \int_{\Lambda}\int_{T}\int\int h(x,y,a,b,\lambda,t)f(a,b)dadbd\lambda dt$$

Blur examples

Typical blur sources

Camera shake/motion

Typical blur sources

Scene/object motion

Typical blur sources

Wrong focus

Karen Walrond

Typical blur sources

Medium turbulence

General space-variant blur model

$$\mathbf{z}(x,y) = \int_{\Omega} \mathbf{u}(x-s, y-t)\mathbf{h}(s,t; x-s, y-t)\, dsdt + \mathbf{n}(x,y)$$

Simplified space-invariant blur model

- *Flat scene*

- *Constant*
- *motion*

$$z(x) = (h * u)(x) + n(x)$$

h(x) is the PSF of the camera

Understanding PSF

- PSF is the response to
- an ideal point source

$$z(x) = (h * u)(x) + n(x)$$

- *camera shake/motion*

- *out-of-focus*

- *turbulence*

Recalling convolution

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s,t) g(x-s, y-t) ds dt$$

Image restoration

"Inverting" the degradations $g = \mathcal{D}(f) + n$

$\xrightarrow{?}$

Looking for f^ such that $\varrho(f, \hat{f})$ is minimized

Noise-free case

$$\hat{f} = \mathcal{D}^{-1}(g) = \mathcal{D}_{\mathcal{G}}{}^{-1}\mathcal{D}_{\mathcal{I}}{}^{-1}\mathcal{S}^{-1}\mathcal{Q}^{-1}(g)$$

Noisy case

$$g = \mathcal{D}(f) + n$$

Noise makes the problem ill-conditioned and difficult to handle

$$\hat{f} \neq \mathcal{D}^{-1}(g)$$

Denoising and/or regularization techniques are required.

Historical remark

- Image restoration has been a very traditional area of image processing

- A. Rosenfeld: *Picture Processing by Computer*, Academic Press, 1969

- 2018 - 200 000 000 search results by Google
- - 3 000 000 results at Google Scholar

Why is image restoration difficult?

- The problem is ill-conditioned and/or ill-posed

$$z(x) = (h * u)(x) + n(x)$$

- Too many unknowns
- Even if the PSF is known, the noise makes the task ill-conditioned and requires a special care (regularization)

Why is image restoration difficult?

- Even if we had a perfect deconvolution algorithm and no noise was present, there would be still a solution ambiguity

$$z(x) = ((h_1 * h_2 * \cdots * h_L) * u)(x)$$

Understanding deconvolution

What shall we do?

- We need to choose the correct trajectory in the image space and the point where to stop
- This can be supported by incorporating our preferences/priors/constraints into the restoration algorithms
- Both the original image *f* and the PSF can be constrained

Restoration categories

- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

Intuitive solution to the inverse problem

- No noise, PSF known – Fourier transform

$$G = F \cdot H$$

Inverse filter in Fourier domain

Inverse filter in Fourier domain

$$R(u,v) = \frac{1}{H(u,v)}$$

Intuitive solution to the inverse problem

- ... does not work on images with noise

$$G = F \cdot H + N$$

$$F = \frac{G}{H} + \frac{N}{H}$$

- ... does not work on images with noise

Wiener filter

$$E(\|f' - f\|^2) \to min$$

- Minimization on the set of linear filters F’ = RG

Wiener filter

$$R(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}$$

- $S_n(u,v)/S_f(u,v) \approx SNR^{-1}$

Wiener filter

The role of the SNR value

The role of the PSF knowledge

Image restoration flowchart

Blurred image

Image restoration flowchart

Image restoration flowchart

Image restoration flowchart

$$h * u = \int u(x-s, y-t)h(s,t)dsdt$$

Images from the Hubble Space Telescope

S82E5937 1997:02:19 07:06:57

Images from the Hubble Space Telescope

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Hubble Space Telescope after COSTAR

Wide Field Planetary Camera 1

Wide Field Planetary Camera 2

High-speed camera

Sampled trajectory (secondary

Interpolated trajectory

Estimated PSF

PSF estimation in smartphones

- Camera shake blur
- Dominant motion - rotation

PSF estimation in smartphones

- Using accelerometers and/or gyroscopes
- Rotation and translation of the phone

Acquired blurred image

PSF estimation

Wiener filtering

Wiener filtering in Samsung smartphone

Restoration categories

- PSF is completely known
- PSF is constant and of a known parametric shape
- PSF is constant and unknown
- PSF is variable and unknown

Common point-spread functions

Motion blur: 1-D rectangular pulse, FT = sinc(u)

Out-of-focus blur:
Cylinder, FT = B(r)/r

Atmospheric turbulence:
Gaussian G(d),
FT = Gaussian G(1/d)

Spectrum of a degraded image

Motion blur

Out-of-focus blur

Atmospheric turbulence blur

Motion blur restoration by Wiener filter

Original

Motion blurred

Spectrum

Wrong velocity

Wrong direction

Correct PSF

Motion blur restoration by Wiener filter

Blur + noise

Spectrum

Inverse filtering

Constant SNR (good)

Constant SNR (wrong)

Actual SNR

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Licence plate recognition

Restoration categories

- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

Blind deconvolution

$$z(x) = (h * u)(x) + n(x)$$

- almost impossible to resolve
- solution ambiguity

$$z(x) = ((h_1 * h_2 * \cdots * h_L) * u)(x) + n(x)$$

Alternating Minimization

$$\min_{u,h} E(u,h) = \min_{u,h} \frac{1}{2}\|h * u - z\|^2 + \lambda Q(u) + \gamma R(h)$$

- Alternating Minimization

  1. *u-step:* $\tilde{u} = \arg\min_{u} E(u, \tilde{h})$

  2. *h-step:* $\tilde{h} = \arg\min_{h} E(\tilde{u}, h)$

  3. *repeat 1 and 2.*

Example of blind deconvolution

Blurred image
$z(x)$

Reconstructed image
$\tilde{u}(x)$

Image blind deconvolution

Restoration categories

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Space-variant PSF

Space-variant PSF

$$\mathbf{z}(x,y) = \int_{\Omega} \mathbf{u}(x-s, y-t)\mathbf{h}(s,t; x-s, y-t)\, dsdt + \mathbf{n}(x,y)$$

Multichannel image restoration

- Assumptions:
- Several input images of the same scene are available
- They are blurred by convolution with different convolution kernels
- The original scene does not change during the acquisitions

Multichannel acquisition model

$$[u * h_k](x, y) + n_k(x, y) = z_k(x, y)$$

MC Blind Deconvolution

- System of integral equations

  (ill-posed, underdetermined)

$$z_k(x) = (h_k * u)(x) + n_k(x)$$

- Energy minimization problem (well-posed)

$$E(u, \{h_i\}) = \frac{1}{2} \sum_{i=1}^{K} \|h_i * u - z_i\|^2 + \lambda Q(u) + \gamma R(\{h_i\})$$

Out-of-focus Camera

reconstructed

in focus

Astronomical images

Degraded images

Reconstructed image

PSF estimation

Misregistration of the channels

... leads to artefacts if not handled properly

Incorporating a between-image shift

original image PSF degraded image

$$[u * h_k](\tau_k(x,y)) + n_k(x,y) = z_k(x,y)$$

$$[u * g_k](x,y) + n_k(x,y) = z_k(x,y)$$

Simulated blurring

misalignment

misalignment compensation

reconstructed image

Long-time exposure I

(a) Blurred input images, 1024×768 pixels

(b) Deconvolution from only first image

(c) Result of multi-image deconvolution

Out of focus blur

Super-resolution imaging

Method: Multiple acquisitions of the same scene with subpixel shift

Aliasing

- The loss of the high frequencies (details) due to an insufficient resolution of the camera

To suppress aliasing, apply multiple acquisitions with sub-pixel shifts

Multiple acquisitions with shifts

Multiple acquisitions with shifts

Multiple acquisitions with shifts

Multiple acquisitions with shifts

The image resolution has increased
This process is called super-resolution

Realistic superresolution

SR must include also de-blurring

Realistic superresolution

$$E(u, \{g_i\}) = \frac{1}{2}\sum_{i=1}^{K} \|D(g_i * u) - z_i\|^2 + \lambda Q(u) + \gamma R(\{g_i\})$$