

TUTORIAL 6 Characteristic

PAGE No.

DATE

Q1

a) $P(H) = \lambda$

$\therefore P(T) = 1 - \lambda$

\therefore for first H at K^{th}

$$(1-\lambda)^{K-1}(\lambda)$$

b) M : no. of times for first head

$$S = E[M]$$

$$S = \lambda \times 1 + (1-\lambda) \times (S+1)$$

$$\therefore S = \lambda + S + 1 - \lambda S - \lambda$$

$$\therefore S = \frac{1}{\lambda}$$

$$Q2 \quad \text{Var}(X) = E[(X - E(X))^2]$$

$$\text{TP: } \text{Var}(X) = E(X^2) - E(X)^2$$

$$\text{Var}(X) = E(X - E(X))^2$$

$$= E[X^2 - 2XE(X) + E(X)^2]$$

$$= E[X^2] - 2E[XE(X)] + E[X]^2$$

$$= E(X^2) - 2E(X)^2 + E(X)^2$$

$$= E(X^2) - E(X)^2$$

$$b) \quad E(X) = 0 \quad \& \quad E(X^2) = 1$$

$$i) \quad \text{Var of } x = ?$$

$$ii) \quad \text{If } Y = a + bX, \text{ Var}(Y) = ?$$

$$1) \text{Var}(x) = E(x^2) - [E(x)]^2$$

$$= 1 - 0$$

$$= 1$$

$$2) E(Y^2) = E[(a+bx)^2]$$

$$= E(a^2 + 2abx + b^2x^2)$$

$$= a^2 + 2abE(x) + b^2E(x^2)$$

$$= a^2 + b^2$$

$$E(Y) = E(a+bx) = a + bE(x)$$

$$= a + b(0)$$

$$\text{Var}(Y) = a^2 + b^2 - a^2$$

$$= b^2$$

Q3 $P(A)$: Aku predicts

$P(B)$: benar atau

$$\begin{aligned}
 a) \quad P(B) &= P(B, A) + P(B, \sim A) \\
 &= P(B/A)P(A) + P(B/\sim A)P(\sim A) \\
 &= 0.99 \times 10^{-5} + (1 - 0.99 \times 10^{-5}) \times 1 \times 10^{-5} \\
 &= 1.99 \times 10^{-5}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad P(A/B) &= \frac{P(A, B)}{P(B)} = \frac{P(A/B)P(A)}{P(B)} \\
 &= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}
 \end{aligned}$$

$$\therefore P(A/B) = 0.497$$