

Ex 1

$$\text{Var}(x) = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$= \langle (\sum_i w_i s_i)^2 \rangle - \langle \sum_i w_i s_i \rangle^2$$

$$= \langle (\sum_i w_i s_i) (\sum_j w_j s_j) \rangle - (\sum_i w_i \langle s_i \rangle) (\sum_j w_j \langle s_j \rangle)$$

$$= \sum_{i,j} w_i w_j \langle s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle)$$

$$= \sum_{\substack{i,j \\ i \neq j}} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle)$$

$$= \sum_i w_i^2 (\langle s_i s_i \rangle - \langle s_i \rangle^2) + \sum_{i,j: i \neq j} w_i w_j (\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle)$$

$$[i \neq j \Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0]$$

$$\text{Var}(s_i) = 1$$

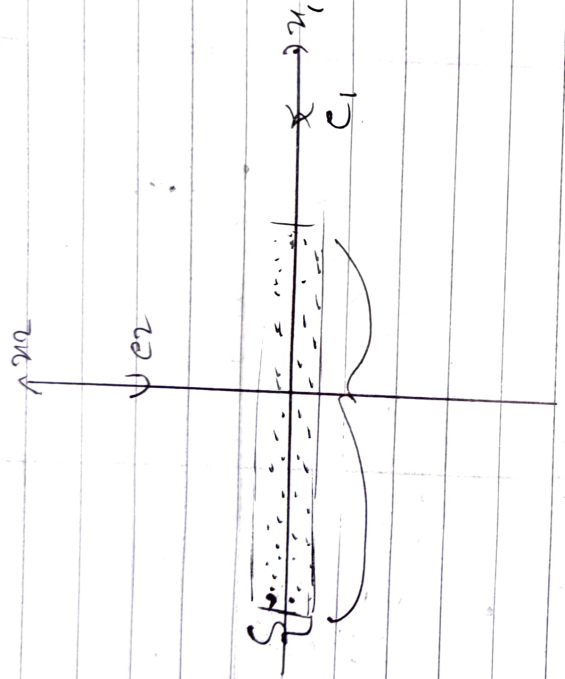
$$\therefore \text{Var}(x) = \sum_i w_i^2$$

for Unit Var

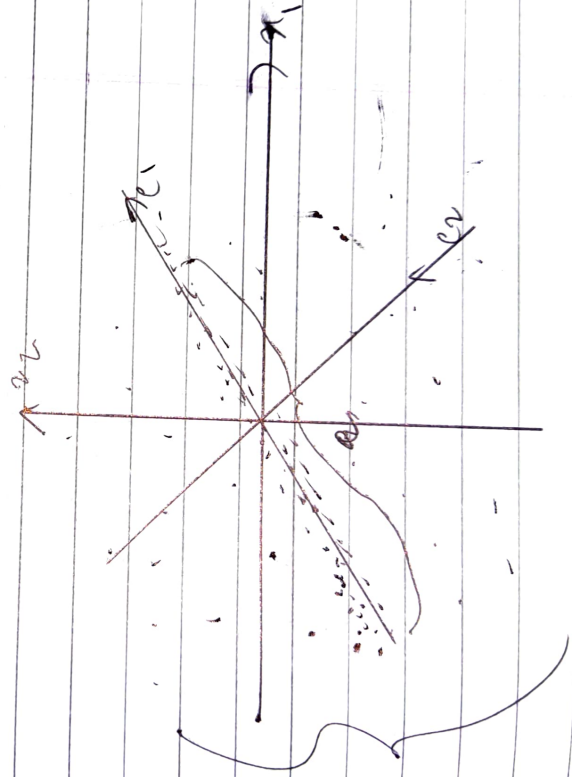
$$\text{Var}(x) = 1$$

$$\therefore \sum_i w_i^2 = 1 \quad \rightarrow \text{the constraint}$$

Ex 12



a



b

c)



not possible to find independent component