

LOJİSTİK REGRESYON SINIFLAMA

- $X = [1, 2, 3, 4, 5, 6]$
- $y = [0, 0, 0, 1, 1, 1]$

Modelimiz:

$$p(x) = \frac{1}{1 + e^{-(wx+b)}}$$

Kayıp fonksiyonu (binary cross-entropy):

$$J(w, b) = -\frac{1}{N} \sum_{i=1}^N \left[y_i \log(p(x_i)) + (1 - y_i) \log(1 - p(x_i)) \right]$$

ve gradyanlar:

$$\frac{\partial J}{\partial w} = \frac{1}{N} \sum_{i=1}^N (p(x_i) - y_i) x_i, \quad \frac{\partial J}{\partial b} = \frac{1}{N} \sum_{i=1}^N (p(x_i) - y_i)$$

Öğrenme oranı $\alpha = 0.1$ olarak alınmıştır.

İterasyon 1

Başlangıç Parametreleri:

$$w_0 = 0, \quad b_0 = 0$$

Başlangıçta her x için:

$$z_i = 0 \quad \Rightarrow \quad p(x_i) = \frac{1}{1 + e^0} = 0.5$$

Hesaplamalar

x_i	y_i	$p(x_i) = 0.5$	$p(x_i) - y_i$	$(p(x_i) - y_i)x_i$
1	0	0.5	0.5	$0.5 \times 1 = 0.5$
2	0	0.5	0.5	$0.5 \times 2 = 1.0$
3	0	0.5	0.5	$0.5 \times 3 = 1.5$
4	1	0.5	-0.5	$-0.5 \times 4 = -2.0$
5	1	0.5	-0.5	$-0.5 \times 5 = -2.5$
6	1	0.5	-0.5	$-0.5 \times 6 = -3.0$

- Gradyan w :

$$\frac{\partial J}{\partial w} = \frac{1}{6}(0.5 + 1.0 + 1.5 - 2.0 - 2.5 - 3.0) = \frac{-4.5}{6} \approx -0.75$$

- Gradyan b :

$$\frac{\partial J}{\partial b} = \frac{1}{6}(0.5 + 0.5 + 0.5 - 0.5 - 0.5 - 0.5) = 0$$

Parametre Güncellemesi

$$w_1 = w_0 - \alpha \cdot \frac{\partial J}{\partial w} = 0 - 0.1 \times (-0.75) = 0.075$$

$$b_1 = b_0 - \alpha \cdot \frac{\partial J}{\partial b} = 0 - 0.1 \times 0 = 0$$

Sonuç (İterasyon 1):

$$w_1 = 0.075, \quad b_1 = 0$$

İterasyon 2

Artık $w_1 = 0.075$ ve $b_1 = 0$ ile yeni tahminleri hesaplıyoruz:

$$p(x) = \frac{1}{1 + e^{-0.075x}}$$

Hesaplanan $p(x)$ Değerleri

- $x = 1$: $z = 0.075$, $p(1) \approx \frac{1}{1 + e^{-0.075}} \approx 0.519$
- $x = 2$: $z = 0.15$, $p(2) \approx 0.537$
- $x = 3$: $z = 0.225$, $p(3) \approx 0.556$
- $x = 4$: $z = 0.3$, $p(4) \approx 0.574$
- $x = 5$: $z = 0.375$, $p(5) \approx 0.593$
- $x = 6$: $z = 0.45$, $p(6) \approx 0.610$

Hata ve Gradyan Hesaplamaları

x_i	y_i	$p(x_i)$	$p(x_i) - y_i$	$(p(x_i) - y_i)x_i$
1	0	0.519	0.519	$0.519 \times 1 = 0.519$
2	0	0.537	0.537	$0.537 \times 2 = 1.074$
3	0	0.556	0.556	$0.556 \times 3 = 1.668$
4	1	0.574	-0.426	$-0.426 \times 4 = -1.704$
5	1	0.593	-0.407	$-0.407 \times 5 = -2.035$
6	1	0.610	-0.390	$-0.390 \times 6 = -2.340$

- Gradyan w :

$$\frac{\partial J}{\partial w} = \frac{1}{6} \left[0.519 + 1.074 + 1.668 - 1.704 - 2.035 - 2.340 \right] \approx \frac{-2.817}{6} \approx -0.470$$

- Gradyan b :

$$\frac{\partial J}{\partial b} = \frac{1}{6} \left[0.519 + 0.537 + 0.556 - \underset{\downarrow}{0.426} - 0.407 - 0.390 \right] \approx \frac{0.389}{6} \approx 0.065$$

Parametre Güncellemesi

$$w_2 = w_1 - \alpha \cdot (-0.470) = 0.075 + 0.047 = 0.122 \quad (\text{yaklaşık})$$

$$b_2 = b_1 - \alpha \cdot (0.065) = 0 - 0.0065 = -0.0065$$

Sonuç (İterasyon 2):

$$w_2 \approx 0.122, \quad b_2 \approx -0.0065$$

İterasyon 3

Yeni parametrelerle $w_2 = 0.122$ ve $b_2 = -0.0065$ kullanarak:

$$p(x) = \frac{1}{1 + e^{-(0.122x - 0.0065)}}$$

Hesaplanan $p(x)$ Değerleri

- $x = 1 : z = 0.122 \times 1 - 0.0065 \approx 0.1155, \quad p(1) \approx 0.529$
- $x = 2 : z = 0.244 - 0.0065 \approx 0.2375, \quad p(2) \approx 0.560$
- $x = 3 : z = 0.366 - 0.0065 \approx 0.3595, \quad p(3) \approx 0.589$
- $x = 4 : z = 0.488 - 0.0065 \approx 0.4815, \quad p(4) \approx 0.618$
- $x = 5 : z = 0.610 - 0.0065 \approx 0.6035, \quad p(5) \approx 0.646$
- $x = 6 : z = 0.732 - 0.0065 \approx 0.7255, \quad p(6) \approx 0.673$

Hata ve Gradyan Hesaplamaları

x_i	y_i	$p(x_i)$	$p(x_i) - y_i$	$(p(x_i) - y_i)x_i$
1	0	0.529	0.529	$0.529 \times 1 = 0.529$
2	0	0.560	0.560	$0.560 \times 2 = 1.120$
3	0	0.589	0.589	$0.589 \times 3 = 1.767$
4	1	0.618	-0.382	$-0.382 \times 4 = -1.528$
5	1	0.646	-0.354	$-0.354 \times 5 = -1.770$
6	1	0.673	-0.327	$-0.327 \times 6 = -1.962$

- Gradyan w :

$$\frac{\partial J}{\partial w} = \frac{1}{6} [0.529 + 1.120 + 1.767 - 1.528 - 1.770 - 1.962] \approx \frac{-1.846}{6} \approx -0.308$$

- Gradyan b :

$$\frac{\partial J}{\partial b} = \frac{1}{6} [0.529 + 0.560 + 0.589 - 0.382 - 0.354 - 0.327] \approx \frac{0.614}{6} \approx 0.102$$

Parametre Güncellemesi

$$w_3 = w_2 - \alpha \cdot (-0.308) = 0.122 + 0.0308 \approx 0.153$$

$$b_3 = b_2 - \alpha \cdot (0.102) = -0.0065 - 0.0102 \approx -0.0167$$

Sonuç (iterasyon 3):

$$w_3 \approx 0.153, \quad b_3 \approx -0.0167$$