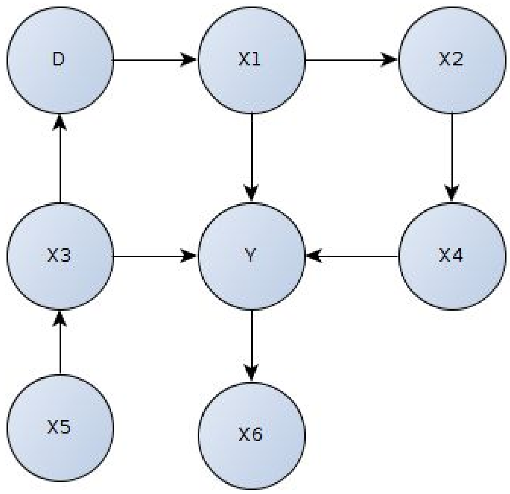
For each graph, find a set that d-separates all paths between D and Y, and a set that satisfies the back-door criterion for the effect of D on Y.

1. 

D-separation:

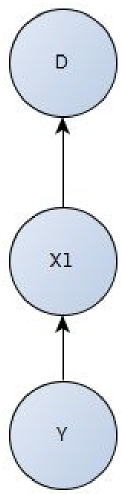
Start by enumerating the paths

a) D → X1 → X2 → X4 → Y
b) D ← X3 → Y
c) D → X1 → Y

Now, we need to block all paths (consult the defintion of d-separation). The first is blocked by any of X1, X2, or X4 (they are all middle nodes of 3-node chains). The second is blocked by X3 (the middle node of a fork). The third is blocked by X1 (the middle node of a chain). If we make Z = {X1, X3}, we block all three paths.

Back-door criterion

Of the paths above, we only want to block back-door paths: ones with arrows pointing toward D. Of the three, only (b) is a back-door path. Z={x3} satisfies the back-door criterion, since it blocks back-door paths, and isn't a descendant of the causal state, D.



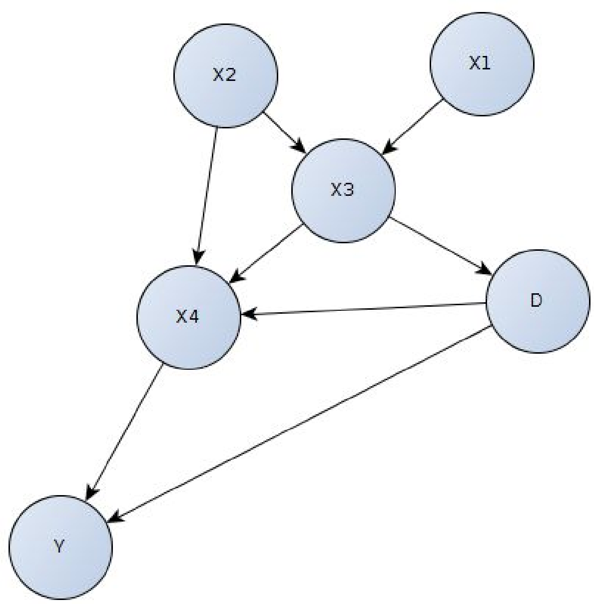
2.

D-separation:

There's only one path here, D ← X1 ← Y. Conditioning on Z={X1} will block this path.

Back-door criterion

We need to block all back-door paths. The one path has an arrow into D, so we should block that path. Z={X1} satisfies the back-door criterion.



3.

D-separation

There are a few paths here:

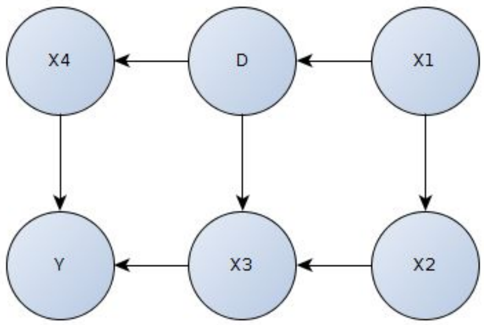
a) D ← X3 ← X2 → X4 → Y
b) D ← X3 → X4 → Y
c) D → X4 → Y
d) D → Y

We can block all except (d), since it's a direct effect. There are no nodes between D and

Y we can condition on to block. In this case, we can't D-separate all paths between D and Y.

Back-door criterion

Of the paths we found before, only (b) has an arrow into D. We can condition on X3 or X4 (or both) to block this path. Z={X3} satisfies the back-door criterion, but we can't actually use X4. X4 is a descendant of the causal state. While it would block the back-door paths, it would also block the chain D → X4 → Y, so Z={X4} and Z={X3, X4} don't satisfy the back-door criterion.

4. 

D-separation

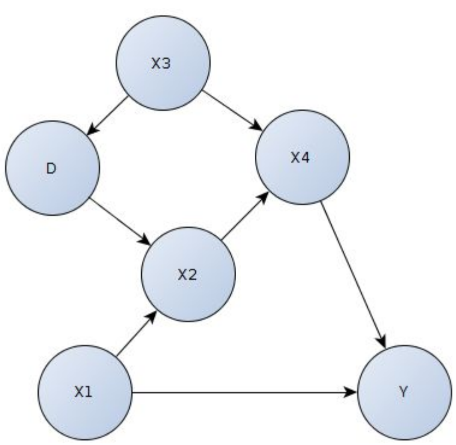
The paths are

a) D → X3 → Y
b) D → X4 → Y
c) D ← X1 → X2 → X3 → Y

We can block the first with X3, the second with X4, and the third with any of X1, X2, and X3. We can d-separate all paths with Z={X3, X4}

Back-door criterion

Only path c is a back-door path. We can block it with X1, X2, or X3, but we have to be Careful. X3 is a descendant of the causal state, so we don't want to condition on it. We're left with just X1 or X2. Z={X1} satisfies the back-door criterion.

5. 

D-separation:

The paths are

a) D ← X3 → X4 → Y
b) D ← X3 → X4 ← X2 ← X1 → Y
c) D → X2 ← X1 → Y
d) D → X2 → X4 → Y

The first is blocked by X3 or X4. The second (b) is blocked by the empty set (there's a collider at x4), X2, X1, or X3. The third path is blocked by the empty set (a collider at x2), or X1. The last path is blocked by X2 or X4. You have to be careful here: If we block the last path with X4, notice that we'd unblock the second path by "opening" the collider. The way to be careful is to define a set Z, and then check it against every path. Let's try Z = {X2}. The first a second paths are blocked by colliders and X2. The third was blocked but is opened by X2. X2 blocks the last path. Let's try adding X1 to re-block the third path. Now, Z={X1, X2}. The first and second paths are still blocked. The third path is now blocked by X1, even though we've opened the collider at X2. The last path is blocked by X2.

Back-door criterion

Paths (a) and (b) are back-door paths. Conditioning on Z={X3} will block both and no others, so satisfies the back-door criterion.