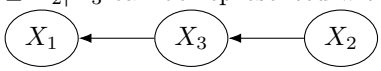
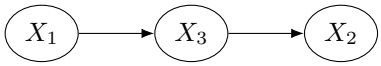
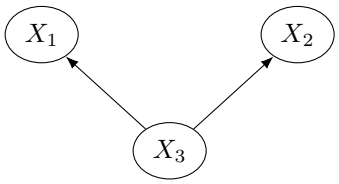
Eric Boxer UNI ecb2198 Homework 2

1

$X_1 \perp\!\!\!\perp X_2 | X_3$ can be represented with any of the following graphs:







Since $X3$ d-separates $X1$ and $X2 \implies X1 \perp\!\!\!\perp X2 | X3$ and we know that the only conditional independence is between $X1, X2$

2

(a) $Z = \varnothing$
Paths are: $X1 \to X2 \leftarrow X6 \to X7$ Blocked by collider $X2$
$X1 \to X2 \to X4 \to X5 \leftarrow X7$ Blocked by collider $X5$
$X1 \to X3 \to X4 \to X5 \leftarrow X7$ Blocked by collider $X5$
$X1 \to X3 \to X4 \leftarrow X2 \leftarrow X6 \to X7$ Blocked by collider $X4$
(b) $Z = \{X4, X6\}$
Paths are: $X2 \to X4 \to X5$ Blocked by $X4 \in Z$
$X2 \leftarrow X1 \to X3 \to X4 \to X5$ Blocked by $X4 \in Z$
$X2 \leftarrow X6 \to X7 \to X5$ Blocked by $X6 \in Z$
(c) $Z = \{X2, X3, X6\}$
$X1 \to X3 \to X4 \to X5$ Blocked by $X3 \in Z$
$X1 \to X3 \to X4 \leftarrow X2 \leftarrow X6 \to X7 \to X5$ Blocked by $X3 \in Z$
$X1 \to X2 \to X4 \to X5$ Blocked by $X2 \in Z$
$X1 \to X2 \leftarrow X6 \to X7 \to X5$ Collider $X2$ is unblocked by including, but the path is blocked by $X6 \in Z$

3

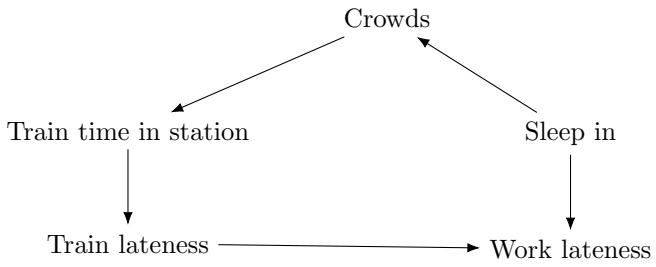
(a) $Z = \varnothing$ All paths enumerated above are blocked back-door paths so we do not need to condition on anything.
(b) $Z = \{X1, X6\}$ Of the paths above, the first is a direct causal path and must not be blocked. The other two are back-door paths and are blocked by $X1$ and $X6$, respectively.

4

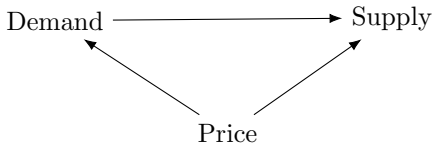
D-separation blocks every path between two nodes and is commutative. In other words, if X is d-separated with Y then Y is d-separated with X, since every path from X to Y is a path from Y to X and vice-versa. In contrast, the back-door criterion leaves causal paths unblocked, see the difference in sets Z that d-separate $X2$ and $X5$ from the set satisfying the BDC. The set satisfying the BDC does not d-separate the two nodes, because we have left the direct causal path unblocked. Also, the BDC for X on Y is not commutative. For example, the set Z satisfying the BDC for $X2$ on $X5$ does not satisfy the BDC for $X5$ on $X2$ because it leaves back-door path $X5 \leftarrow X4 \leftarrow X2$ unblocked.

5

(a)



(b)



Price has a causal effect on demand. Similarly, price has a causal effect on supply and as a result price creates a statistical dependence between demand and supply. Given some price p, an increase in demand will result in an accompanying increase in supply. Demand and supply are also indirectly correlated through collider supply.