

Maximum Likelihood Estimation (MLE)

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14 Nov 2016

Binomial Probability Estimation

See [Newsom 2016-CDA Handout-4](#)

$$P(Y = k; n, \pi) = \binom{n}{k} \pi^k (1 - \pi)^{n-k}$$

For MLE, the first term on the right side of the *binomial coefficient* $\binom{n}{k}$ is ignored, as it does not inform the estimation of π , which is what MLE is interested in finding. The rest of the right side of the *binomial estimation* is the “kernel” in MLE¹.

¹ Agresti, *Catagorical Data Analysis*; Eliason, *Maximum Likelihood Estimation*; Fisher, “Contributions to Mathematical Statistics.”; Myung, “Tutorial on Maximum Likelihood Estimation.”

[R-Code Source](#)

eq.pbn(): Binomial Probability Equation

h.bn(): Plot a relative frequency histogram of the binomial distribution

```
R _____

par(family = "ETBembo")
eq.pbn <- expression(paste("P(", Y == k, "; n,", pi, ") = ",
                           bgroup("(", atop(n, x), ")"), pi^k,
                           " (", 1 - pi^{n - k}, ")"))

x5 <- for (n in 0:5) choose(n, k = 0:n) ## n = 10 ##
x10 <- seq(1:10) ## n = 10 ##
x100 <- seq(1:100) ## n = 100 ##

ppi <- 0.5

nk <- c(5, 10, 100)

x1 <- matrix(c(0:5, dbinom(0:5, 5, 0.5)), ncol = 2,
            dimnames = list(NULL, c("n", "P")))

h.bn <- function(n, p, ...) {
  k <- 0:n
  p <- dbinom(k, n, p)
  names(p) <- as.character(0:n)
}

h.bn(5, 0.5)
```

R

```
clinton <- c(22, 40, 11, 33, 27, 30, 25, 25, 20, 19, 44, 27,
            28, 30, 34, 24, 28, 29, 31, 19, 24, 29, 33, 32, 25)
k <- clinton
n <- 50
p <- k/n
```

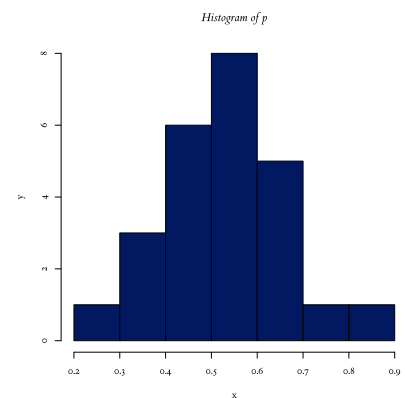
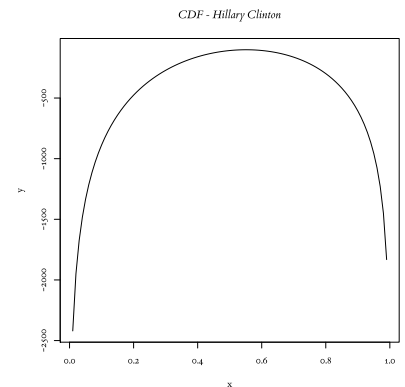
```
p
```

```
0.44, 0.8, 0.22, 0.66, 0.54, 0.6, 0.5, 0.5, 0.4, 0.38, 0.88, 0.54, 0.56, 0.6,
0.68, 0.48, 0.56, 0.58, 0.62, 0.38, 0.48, 0.58, 0.66, 0.64 and 0.5
```

```
ll <- function(p) sum(dbinom(k, n, p, log = TRUE))
p.sq <- seq(0.01, 0.99, 0.01)
p.sql <- sapply(p.sq, ll)
```

```
par(family = "ETBembo", pch = 20);
plot(p.sq, p.sql, type = "l", xlab = "x", ylab = "y",
     main = "CDF - Hillary Clinton")
```

```
hist(p, freq = TRUE, xlab = "x", ylab = "y", col = mypal[16])
```



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² **Note:** This document was created using R-v3.3.2 R Core Team, R, and the following R-packages: *base-v3.3*. R Core Team, R, *bibtex-v0.4*. Francois, *Bibtex*, *dplyr-v0.5*. Wickham and Francois, *Dplyr*, *DT-v0.2*. Xie, *DT*, *extrafont-v0.17*. Chang, *Extrafont*, *ggplot2-v2.1*. Wickham, *Ggplot2*, *knitcitations-v1.0*. Boettiger, *knitcitations*, *knitr-v1.14*. Xie, *Dynamic Documents with R and Knitr*, *pander-v0.6*. Daroczi and Tsegelskyi, *Pander*, *papaja-v0.1*. Aust and Barth, *Papaja*, *plyr-v1.8*. Wickham, "The Split-Apply-Combine Strategy for Data Analysis.", *rmarkdown-v1.1*. Allaire et al., *rmarkdown*, *scales-v0.4*. Wickham, *Scales*, *tidyr-v0.6*. Wickham, *Tidyr*, *ggthemes-v3.2*. Arnold, *Ggthemes*, *gtable-v0.2*. Wickham, *Gtable*, *kableExtra-v0.0*. Zhu, *KableExtra*, *tuftes-v0.2*. Xie and Allaire, *Tuftes*, *devtools-v1.12*. Wickham and Chang, *Devtools*, *highlight-v0.4*. Francois, *Highlight*, *sysfonts-v0.5*. Qiu and others, *Sysfonts*, and *showtext-v0.4*. Qiu, *Showtext*

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