T4,

а) $\xi \sim \frac{\theta}{2}\{(-1;1)\}\setminus 0\} + \frac{1-\theta}{2}\{0;2)$

$\alpha_1 = M[\xi] = 1-\theta$

$\alpha_2 = M[\xi^2] = \frac{\theta}{2}\int_{-1}^{1} x\,dx + \frac{1-\theta}{2}2^2 =$

$= 2 - \frac{5}{3}\theta$

$\mu_2 = \alpha_2 - \alpha_1^2 = 2 - \frac{5}{3}\theta - \theta^2 + 2\theta - 1 = 1 + \frac{\theta}{3} - \theta^2$

ОММ

б) α_1 $1-\theta = \overline{x} \Rightarrow \tilde{\theta}_{ММ} = 1-\overline{x}$

$M[\tilde{\theta}] = M[1-\overline{x}] = 1 - M[\xi] = 1+\theta+1 = \theta$

несм.

$D(\tilde{\theta}) = \frac{1}{n}D[\xi] \xrightarrow[n\to\infty]{} 0$ сост.

а) ОМП а

$$L = b\left(\frac{\theta}{2}\right)^{n-m_1-m_2}\left(\frac{1-\theta}{2}\right)^{m_1+m_2}$$

$$\ln L = (n-m_1-m_2)\ln\frac{\theta}{2} + (m_1+m_2)\ln\frac{1-\theta}{2}$$

$$(\ln L)' = \frac{n\theta - n + m_1 + m_2}{\theta(\theta-1)} = 0 \Rightarrow \underbrace{\theta = 1-\nu_1-\nu_2}_{\text{Оценка}}$$

$$(\ln L)'' = \frac{m_1+m_2-n}{\theta^2} - \frac{m_1+m_2}{(\theta-1)^2} = \{\ldots\}$$

$$= \frac{(\nu_1+\nu_2)(\nu_1+\nu_2-1)}{n\theta^2(\theta-1)^2} < 0 \Rightarrow max$$

$$\tilde{\theta}_{МП} = 1-\nu_1-\nu_2$$

б)

$$M[\tilde{\theta}] = 1 - \frac{1-\theta}{2} - \frac{1-\theta}{2} = \theta$$

несм.

$$D[\tilde{\theta}] = D[\nu_1+\nu_2] =$$

$$= \frac{4\left(\frac{1-\theta}{2}\right)\left(1-\frac{1-\theta}{2}\right)}{n} \xrightarrow[n\to\infty]{} 0$$

сост.

[5]

$\xi \sim R(\theta; 2\theta)$

$g(x,\theta) = \frac{1}{\theta} I[\theta; 2\theta]$

a) Оценка методом моментов

$\alpha_1 = \int_{-\infty}^{+\infty} x g(x) dx = \int_{\theta}^{2\theta} x\, dx = \frac{3}{2}\theta = \overline{X}$

$\tilde{\theta}_{мм} = \frac{2}{3}\overline{X}$

Оценка методом макс. правд.

$L = \frac{1}{\theta^n} I\{x_i \in [\theta; 2\theta]\}$

$\theta \geq \frac{x_{max}}{2} \Rightarrow \tilde{\theta}_{мп} = \frac{x_{max}}{2}$

b) - ОММ

1) несм.

$M[\tilde{\theta}] = M[\frac{2}{3}\overline{x}] = \frac{2}{3} M\xi = \frac{2}{3} \cdot \frac{3}{2}\theta = \theta$

сост.

2) $D[\tilde{\theta}] = D[\frac{2}{6}\overline{X}] = \frac{4}{9} D[\overline{X}] =$

$\frac{4 D\xi}{9 \cdot n} =$

$$M\xi^2 = \int_0^{2\theta} x^2 \frac{1}{\theta} dx = \frac{1}{\theta}\frac{x^3}{3}\Big|_\theta^{2\theta} = \frac{7}{3}\theta^2$$

$$D[\tilde{\theta}] = \frac{4}{9n}\left[\frac{7}{3}\theta^2 - \frac{9}{4}\theta^2\right] = \frac{4\theta^2}{12\cdot 9n} \xrightarrow[n\to\infty]{} 0$$

$\tilde{\theta}_{ММ}$ – несм. и сост.

– ОМП

$$M[\tilde{\theta}] = M\left[\frac{x_{max}}{2}\right] = \frac{1}{2}\int_\theta^{2\theta} \frac{n}{\theta}\left(\frac{x}{\theta}-1\right)^{n-1} x\,dx =$$

$$= \frac{2\theta n + \theta}{2(n+1)} = \theta\frac{2n+1}{2(n+1)}$$

$$\tilde{\theta}^* = \frac{2(n+1)}{2n+1}\tilde{\theta} = \frac{2(n+1)}{2n+1}\frac{x_{max}}{2} = \frac{n+1}{2n+1}x_{max}$$

$M[\tilde{\theta}^*] = \theta$ – несм.

$$D[\tilde{\theta}^*] = D\left[\frac{2(n+1)}{2n+1}\frac{x_{max}}{2}\right] = \left(\frac{n+1}{2n+1}\right)^2 D[x_{max}] =$$

$$= \left(\frac{n+1}{2n+1}\right)^2\left(M[x_{max}^2] - M^2[x_{max}]\left(\frac{2n+1}{n+1}\right)^2\right) =$$

$$= \left(\frac{n+1}{2n+1}\right)^2 \frac{4n^2+8n+2}{(n+1)(n+2)}\theta^2 - \theta^2 \xrightarrow[n\to\infty]{} 0$$

$\tilde{\theta}^*_{МП}$ – несм. и сост.

c) $D[\tilde{\theta}_{MM}] = \frac{\theta^2}{27n}$

$\vee$

$D[\tilde{\theta}_{MП}] = \frac{n\,\theta^2}{(2n+1)^2(n+2)}$

θ_2^* - эффект. оценка

d) $\frac{x_i}{\theta} \in [1; 2]$

$\Phi(x_{max}) - F((x))^n = \left(\int_1^x dx\right)^n = (x-1)^n$

$\sqrt[n]{0,025} + 1 < x < \sqrt[n]{0,975} + 1$

с лекции (и это тоже)

$\frac{x_{max}}{\sqrt[n]{0,025}+1} > \theta > \frac{x_{max}}{\sqrt[n]{0,975}+1}$

16)

a) L

$\ln L$

$\frac{\partial L}{\partial \theta}$

$\frac{n}{\theta - 1}$ =

$\tilde{\theta} = \Sigma$

b) g

$\frac{\partial}{\partial \theta}\int$

$\int \frac{\partial}{\partial \theta}($

$\int_1^{\hat{x}} \frac{\theta - 1}{x^\theta}$

$g(\tilde{\theta})$

№6) $p(x) = \begin{cases} \dfrac{\theta-1}{x^\theta}, & x \geq 1 \\ 0, & x < 1 \end{cases}, \theta > 1$

a) $L = \dfrac{(\theta-1)^n}{x_1^\theta \cdot \ldots \cdot x_n^\theta}$

$\ln L = n \ln(\theta-1) - \theta \sum \ln x_i$

$\dfrac{\partial L}{\partial \theta} = \dfrac{n}{\theta-1} - \sum \ln x_i \,; \quad \dfrac{\partial L}{\partial \theta} = 0$

$\dfrac{n}{\theta-1} = \sum \ln x_i$

$\tilde{\theta} = \dfrac{n}{\sum \ln x_i} + 1 \,; \quad \dfrac{\partial^2 \ln L}{\partial \theta^2} = -\dfrac{n}{(\theta-1)^2} < 0 \Rightarrow$

$\Rightarrow$ max

в) дов. инт. медианы

$\dfrac{\partial}{\partial \theta} \int \dfrac{\theta-1}{x^\theta} dx = x^{1-\theta} \ln x$

$\int \dfrac{\partial}{\partial \theta} \left(\dfrac{\theta-1}{x^\theta} \right) dx = x^{1-\theta} \ln x$

$\Rightarrow$ сильная регул.

$\int_1^{\hat{x}} \dfrac{\theta-1}{x^\theta} dx = -\dfrac{1}{\hat{x}^{\theta-1}} + 1 = \dfrac{1}{2} \,; \quad \hat{x} = \sqrt[\theta-1]{2}$

$g(\tilde{\theta}) = 2^{\frac{1}{\theta-1}}$

$$\sqrt{n} \cdot \frac{g(\tilde{\theta}) - g(\theta)}{\sigma(\theta)} \rightsquigarrow N(0;1)$$

$$\sigma(\tilde{\theta}) = \sqrt{\nabla^T g(\tilde{\theta})\, I^{-1}(\tilde{\theta})\, \nabla g(\tilde{\theta})}$$

$$I(\tilde{\theta}) = M\left[\left(\frac{\partial \ln p}{\partial \theta}\right)^2\right] =$$

$$= M\left[\left(\frac{1}{\theta-1} - \ln x\right)^2\right] = \int_1^{+\infty}\left(\frac{1}{\theta-1} - \ln x\right)^2 \cdot$$

$$\cdot\, g(x,\theta)\, dx = \int_1^{+\infty}\left(\frac{1}{\theta-1} - \ln x\right)\frac{\theta-1}{x^\theta}\, dx =$$

$$= \frac{1}{(\theta-1)^2} \quad \in C[\theta; +\infty)$$

$$\nabla g(\tilde{\theta}) = -\frac{\ln 2 \cdot 2^{\frac{1}{\theta-1}}}{(\theta-1)^2}$$

$$\sigma(\tilde{\theta}) = \frac{-\ln 2 \cdot 2^{\frac{1}{\theta-1}}}{\theta-1}$$

$$\frac{1{,}96\, \sigma(\tilde{\theta})}{\sqrt{n}} + g(\tilde{\theta}) < g(\theta) < -\frac{1{,}96\, \sigma(\tilde{\theta})}{\sqrt{n}} + g(\tilde{\theta})$$

e) $\sqrt{n}\ \dfrac{\tilde{\theta}-\theta}{\sigma(\theta)} \rightsquigarrow N(0,1)$

$\sigma(\tilde{\theta}) = \theta-1$

$\sqrt{n}\ \dfrac{\tilde{\theta}-\theta}{\theta-1} \rightsquigarrow N(0,1)$

$-\dfrac{1{,}96(\theta-1)}{\sqrt{n}} + 1 + \dfrac{n}{\sum_i \ln x_i} < \theta < \dfrac{1{,}96(\theta-1)}{\sqrt{n}} + 1 + \dfrac{n}{\sum_i \ln x_i}$