T11,

$H_0: \xi \sim p_0(x) = 1 \{(0,1)\}$

$H_1: \xi \sim p_1(x) = \frac{e}{e-1} e^{-x} \{(0,1)\}$

a) $n = 1$

$$\rho = \frac{L_1}{L_0} = \frac{e}{e-1} e^{-x} \geqslant C \Rightarrow e^{-x} \geqslant B$$

$$x \leq A$$

$$P(x \leq A | H_0) = \alpha \Rightarrow \int_0^A dx = A = \alpha$$

$G: \; x \leq \alpha \quad \alpha_1 = \alpha$

$$W = P(x \leq A | H_1) = \int_0^{\alpha} \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}(1 - e^{-\alpha})$$

$$\alpha_2 = 1 - W$$

б) $n = 2$

$$\rho = \left(\frac{e}{e-1}\right)^2 e^{-(x_1 + x_2)} \geqslant C \Rightarrow e^{-(x_1+x_2)} \geqslant B$$

$$x_1 + x_2 \leq A$$

$$P(x_1 + x_0 \leq A | H_0) = \alpha$$

$$\iint\limits_{x_1 + x_2 \leq A} dx_1 dx_2 = \frac{A^2}{2} = \alpha \quad \Rightarrow \quad A = \sqrt{2\alpha}$$

$G: x_1 + x_2 \leq \sqrt{2\alpha}, \quad \alpha_1 = \alpha$

$$W = P(x_1 + x_2 \leq A \mid H_1) = \iint_{x_1+x_2 \leq A} \left(\frac{e}{e-1}\right)^2 e^{-(x_1+x_2)} dx_1 dx_2 =$$

$$= \left(\frac{e}{e-1}\right)^2 (1 - e^{-A} - Ae^{-A}) = \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-\sqrt{2\alpha}} - Ae^{-\sqrt{2\alpha}}\right)$$

$\alpha_2 = 1 - W$

c) $$P = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{P_1(x_i)}{P_0(x_i)} \geq C$$

$$\ln P = \sum_{i=1}^{n} \ln \frac{P_1(x_i)}{P_0(x_i)} \geq \ln C$$

$$\frac{\sum_{i=1}^{n} -''- \; - n M[-''-]}{\sqrt{n D[-''-]}} \sim N(0, 1)$$

$G: P(\ln l \geq \ln C \mid H_0) = \alpha$

$H_0: -''- = \ln \left(\frac{e}{e-1} e^{-x}\right) = \ln \frac{e}{e-1} - x$

$\ln l = n \ln \frac{e}{e-1} - \sum_{i=1}^{n} x_i \geq \ln C$

$G: \sum_{i=1}^{n} x_i \leq A$

$P(\sum x_i < A / H_0) = \alpha, \quad M[x] = \frac{1}{2} \quad D[x] = \frac{1}{12}$

$$P\left(\frac{\sum x_i - n M[x]}{\sqrt{n D[x]}} \leq \frac{A - n M[x]}{\sqrt{n D[x]}} \;\Big|\; H_0\right) = \alpha$$

$$\frac{A-\frac{n}{2}}{\sqrt{\frac{n}{12}}}=u_\alpha \Rightarrow A=\frac{n}{2}+u_\alpha\sqrt{\frac{n}{12}},$$

$$u_\alpha: \int\limits_{-\infty}^{u_\alpha}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx=\alpha$$

$$G: \sum_{i=1}^{n} x_i \le \frac{n}{2}+u_\alpha\sqrt{\frac{n}{12}}, \quad \alpha_1=\alpha$$

$$W=P(\sum x_i \le A \mid H_1)=P\left(\frac{\sum x_i - nM[x]}{\sqrt{nD[x]}} \le\right.$$

$$\left.\le \frac{A-nM[x]}{\sqrt{nD[x]}} \middle| H_1\right)$$

$$M[x]=\int\limits_0^1 \frac{xe}{e-1}e^{-x}dx=\frac{e-2}{e-1}$$

$$M[x^2]=\frac{2e-5}{e-1} \quad D[x]=\frac{e^2-3e+1}{(e-1)^2}$$

$$G: \sum x_i \le \frac{n}{2}+u_\alpha\sqrt{\frac{n}{12}}$$

$$W=\int\limits_{-\infty}^{B}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx \to ?$$

$$B=\frac{A-nM[x]}{\sqrt{nD[x]}}=\frac{\frac{n}{2}+u_\alpha\sqrt{\frac{n}{12}}-n\frac{e-2}{e-1}}{n\frac{e^2-3e+1}{(e-1)^2}} \xrightarrow{n\to\infty} \infty$$

$$W=\int\limits_{-\infty}^{+\infty}\dots dx \xrightarrow{n\to\infty} 1$$

T12,

$\alpha = 0,2$

$H_0: \xi \sim p_0(x) = \frac{1}{4}\{1\} + \frac{1}{4}\{2\} + \frac{1}{6}\{3\} + \frac{1}{3}\{4\}$

$H_1: \xi \sim p_1(x) = \frac{1}{4}\{1\} + \frac{1}{4}\{2\} + \frac{1}{4}\{3\} + \frac{1}{4}\{4\}$

$n = 2$

$$\rho = \frac{S_1(x_1)\, S_1(x_2)}{S_0(x_1)\, S_0(x_2)}$$

ρ:	1	2	3	4
1	1	1	3/2	3/4
2	1	1	3/2	3/4
3	3/2	3/2	9/4	9/8
4	3/4	3/4	9/8	9/16

H_0	1	2	3	4
1	1/16	1/16	1/24	1/12
2	1/16	1/16	1/24	1/12
3	1/24	1/24	1/36	1/18
4	1/12	1/12	1/18	1/9

H_1	1	2	3	4
1	1/16	1/16	1/16	1/16
2	1/16	1/16	1/16	1/16
3	1/16	1/16	1/16	1/16
4	1/16	1/16	1/16	1/16

$\rho \geq c$, $c = \frac{3}{2}$

$\alpha_1 = \frac{7}{36}$, $\alpha_2 = \frac{11}{16}$

$W = \frac{5}{16}$