Introduction to the `robGarch` package (Version 0.1.0)

Echo Liu*

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Contents

*Mentors: R. Douglas Martin, Dan Hanson, and Alexios Galanos

1 Introduction

AutoRegressive Conditional Heteroscedasticity (ARCH) was introduced by Engle (1982) and extended to Generalized Autoregressive Conditional Heteroscedasticity (Garch) models by Bollerslev (1987), the latter proven very popular in many applications as they provide a rich statistical framework for non-constant variance and their ability to capture observed empirical phenomena such as volatility clustering. Chou (1988) showed the ability of Garch to estimate risk premium valuation of a stock. In Duan and Jin-Chuan (1995), Garch is applied to pricing options, where volatility plays a crucial role in the Black-Scholes option pricing model, thereby adding a new perspective in comparison to stochastic volatility model. Bias caused by outliers has been studied in some literature. Huber (1981) established two properties for a good robust model fitting method. The first is that a robust estimate needs to be efficient and should be efficient in the sense of having a variance that is not much larger than that of a maximum likelihood estimate typically based on a normal distribution. The second property, introduced by later researchers is that replacing a small fraction of the data by outliers should not cause big difference as measured by estimator bias. Mendes (2000) showed how outliers create asymptotic bias in Quasi Maximum Likelihood Estimate of the Garch parameters. Muler and Yohai (2008)[1] showed that QML parameter estimation based on normal likelihood is very sensitive to outliers in financial returns, and even a single outlier can have a huge influence on the QML parameter estimates. Thus, it is very crucial to build accurate robust parameter estimation methods.

The robGarch package aims to provide two main methods for modelling robust Garch processes, modified M-Estimate method providing M model family and bounded M-Estimate method providing BM model family, addressing the issue of robustness toward additive outliers, rather than innovations outliers. The package also contains quasi maximum likelihood (QML) method as an option for the user. It currently contains fitting and plotting ready for users to use.

This document discusses the details of the included models and how they are implemented in the package with some examples.

The robGarch package can be installed as following:

```
devtools::install_github("EchoRLiu/robGarch")
library(robGarch)
```

and the development version on github (https://github.com/EchoRLiu/robGarch) with examples and and demos.

[1] We reference the paper as MY2008 in the following content

2 Model Specification

This section discusses the model specification in the modelling process. This is done via specifying `methods` and `fixed_pars` parameters by the user for the main fitting function `rGarch`,

```
args(rGarch)

## function (data, methods = c("bounded MEst", "modified MEst",
##     "QML"), fixed_pars = c(0.85, 3), optimizer = c("Rsolnp",
##     "nloptr", "nlminb"), optimizer_control = list(), stdErr_method = c("numDeriv"
##     "optim", "sandwich"))
## NULL
```

The following sub-sections outline the background and details of the methods implemented in `robGarch`.

2.1 BM Models

Robust Garch is first introduced by Muler and Yohai (2008) with the foundation work of Huber (1981). Defined as $x_t = \sigma_t z_t$, where z_t is i.i.d random variables with continuous density f such that $E(z_t) = 0$, $Var(z_t) = 1$, and where the conditional variance σ_t^2 are given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{1}$$

the robust parameters estimation is based on M-estimate with more flavors.

The main structure of parameter estimation is

$$\hat{\gamma_T}^B = \begin{cases} \hat{\gamma_{1,T}}, & M_T(\hat{\gamma_{1,T}}) \leq M_{Tk}^*(\hat{\gamma_{2,T}}) \\ \hat{\gamma_{2,T}}, & M_T(\hat{\gamma_{1,T}}) > M_{Tk}^*(\hat{\gamma_{2,T}}) \end{cases} \tag{2}$$

where

$$\begin{aligned} \hat{\gamma_{1,T}} &= arg \min_{\mathbf{c}} M_T(\mathbf{c}) \\ &= arg \min_{\alpha_i, \beta_j} \frac{1}{T-p} \sum_{t=p+1}^{T} \rho(log(x_t^2) - log(\alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2)) \end{aligned} \tag{3}$$

and

$$\begin{aligned}\hat{\gamma_{2,T}} &= arg\min_{\mathbf{c}} M^*_{Tk}(\mathbf{c}) \\ &= arg\min_{\alpha_i,\beta_j} \frac{1}{T-p}\sum_{t=p+1}^{T}\rho(log(x_t^2) \\ &- log(\alpha_0 + \sum_{i=1}^{p}\alpha_i\sigma^2_{t-i,k}(\alpha_i,\beta_j)r_k(\frac{x^2_{t-i}}{\sigma^2_{t-i,k}}) + \sum_{j=1}^{q}\beta_j\sigma^2_{t-j}))\end{aligned} \tag{4}$$

where

$$r_k(u) = \begin{cases} u, & u \le k \\ k, & u > k \end{cases} \tag{5}$$

The use of Equation 5 is to restrict the propagation of the outlier effect, and thus small `k` can provide a robust model. The different optimization under different cases makes sure that the model has robustness with outliers and maintains consistency when the series follows a Garch without outliers.

BM models are also controlled by $\rho = m_1(\rho_0)$, where $\rho_0 = -log(\frac{1}{\sqrt{2\pi}}e^{-(e^w-w)/2}))$, $w = log(z_t^2)$, and m_1 is a non-decreasing, bounded function, defined as

$$m_1(x) = \begin{cases} x, & x \le 4 \\ P(x), & 4 < x \le 4.34.15, x > 4.3 \end{cases} \tag{6}$$

and

$$\begin{aligned}P(x) = \frac{2}{(b-a)^3}(\frac{1}{4}(x^4-a^4) - \frac{1}{3}(2a+b)(x^3-a^3) + \frac{1}{2}(a^2+2ab)(x^2-a^2)) \\ - \frac{2a^2b}{(b-a)^3}(x-a) - \frac{1}{3(b-a)^2}(x-a)^3 + x, a = 4, b = 4.3\end{aligned} \tag{7}$$

satisfying constraints $P(a) = a, P'(a) = 1, P'(b) = P''(a) = P''(b) = 0$. To have different level of robustness, BM model can use $\rho = div * m_1(\rho_0/div)$, where $0 < div <= 1$. The smaller div is, the larger ρ_0/div would be, thus obtaining more control and gaining more robustness.

To use BM models, the user needs to choose **"`bounded MEst`"** as `methods`, and to control the robustness, `fixed_pars = c(div, k)` (default c(0.8, 3.0)) need to be specified.

2.2 M Models

M models are similar to BM models without the restriction on the propagation of the outlier effect. If k in Equation 5 is large, then $\sum_{i=1}^{p}\alpha_i\sigma^2_{t-i,k}(\alpha_i,\beta_j)r_k(\frac{x^2_{t-i}}{\sigma^2_{t-i,k}})$ becomes the normal form

$\sum_{i=1}^{p} \alpha_i x_{t-i}^2$, and thus lose the robustness of BM models. When the `methods` parameter of `rGarch` is specified as "`modified MEst`", only `div` (default = 0.8) controls the robustness of M models. To use M models, the user needs to specify the `methods` as "`modified MEst`" and `fixed_pars = div`.

2.3 QML

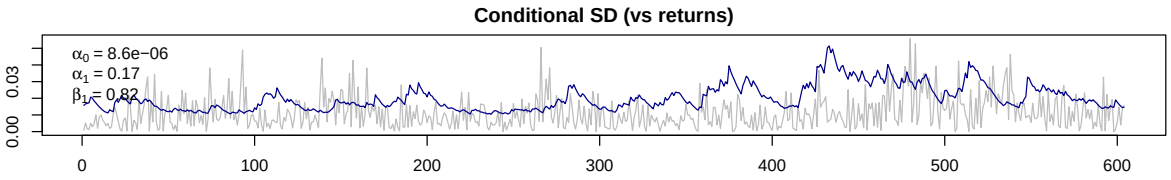
QML is obtained without any restriction or control over the outlier effect. Thus it is a simple M-Estimate. The user only needs to choose the "`QML`" as `methods` to specify the model.

3 Fitting

With the `methods` and `fixed_pars` parameters specified, the `rGarch` function can take in the `data`, and output the parameter estimations. The following is an example using the internal data in `robGarch` package.

```
data(rtn)
fit <- rGarch(rtn[1:604], methods="bounded MEst", fixed_pars=c(0.8, 3.0))

##
## Iter: 1 fn: 2.7158  Pars:  0.04251 0.17465 0.82392 0.10650
## Iter: 2 fn: 2.7158  Pars:  0.04250 0.17463 0.82394 0.10646
## solnp--> Completed in 2 iterations
##      alpha_0      alpha_1       beta_1
## 8.647505e-06 1.746338e-01 8.239391e-01

summary(fit)

## Model:  bounded MEst
## with div =  0.8 , k =  3
## Observations:  604
##
## Result:
##                      alpha_0   alpha_1       beta_1
## fitted_pars    8.647505e-06 0.1746338 8.239391e-01
## standard_error 2.002408e-01 0.3144657 1.335392e-01
## t_value        2.122592e-01 0.5553348 6.170017e+00
## p_value        8.319048e-01 0.5786657 6.828247e-10
```

```
##
## Log-likelihood:  2.715769
##
## Optimizer:  Rsolnp
##
##
## Time elapsed:  0.8221269
## Convergence Message:  0

plot(fit)
```



The following sub-sections provide the results of testing `robGarch` package function `rGarch` and comparing the results with those obtained using the `rugarch` package function `ugarchfit` for the following five data sets:

- SP500 daily returns from February 1, 2000 through June 30, 2002, a total of n = 604 observations
- 5 artificially generated Garch11 returns, a total of n = 500 observations, using the five models in Table 1 that were provided by Doug Martin

	μ_e	σ	α_0	α_1	β_1	$\alpha_1 + \beta_1$
1	0.01	0.07	8.76e-04	0.135	0.686	0.821
2	0.01	0.07	3.90e-04	0.144	0.776	0.920
3	0.01	0.07	2.43e-04	0.117	0.833	0.950
4	0.01	0.07	2.45e-04	0.103	0.867	0.971
5	0.01	0.07	1.01e-4	0.083	0.909	0.992

Table 1: Garch(1,1) Parameter Values Used for robGarch Test

We are testing the following three Garch11 fitting methods from the `robGarch` package:

1. The QML method: quasi-maximum likelihood method (MLE for normal distribution Garch11 innovations)

2. M1 model: a bounded loss function M-estimator, as described in MY2008, with tuning parameter $c = 1.0$

3. BM1 model: M1 with robust filter added, with $c = 1.0$ and filter tuning parameter $k = 5.02$

In further testing we will also include M2 and BM2, which are more robust versions of M and BM, obtained by using smaller values of c and k.

3.1 Test Result with SP500 Returns

For the SP500 returns we also compare our `robGarch` results with those in MY2008, and refer to their results as the MY results. Table 2 below contains the results of testing our QML, M1, BM1 methods, along witth the MY results labeled QML-MY, BM1-MY (MY results did not provide M1 result), in comparison with the `rugarch` results (in the first row) using the SP500 data.

The main conclusion of Table 2 for our estimators QML, M1 and BM1 is that the point estimates seem reasonable. We are currently working on the standard error, t stats and p stats, which would be available for the users in the future.

Table 2: Analysis for Garch(1,1) models with SP500 data (02/01/2000-06/30/2002)

	α_0	α_1	β_1	$\alpha_1 + \beta_1$
rugarch	1.1e-05	0.110	0.84	0.95
QML	9.0e-06	0.095	0.87	0.96
QML-MY	3.9e-06	0.110	0.86	0.97
M1	7.2e-06	0.087	0.88	0.96
BM1	7.2e-06	0.087	0.88	0.96
BM1-MY	4.5e-06	0.100	0.84	0.94

Comparing 2 with 1, we can notice that the BM1 captains the main dynamics of the SP500 series, but has smaller lagging, which shows BM model is more robust over outliers.

```
library(rugarch)
spec <- ugarchspec(mean.model = list(armaOrder = c(0,0), include.mean = FALSE))
mod_rugarch <- ugarchfit(spec, rtn[1:604])
plot(mod_rugarch, which=3)
```

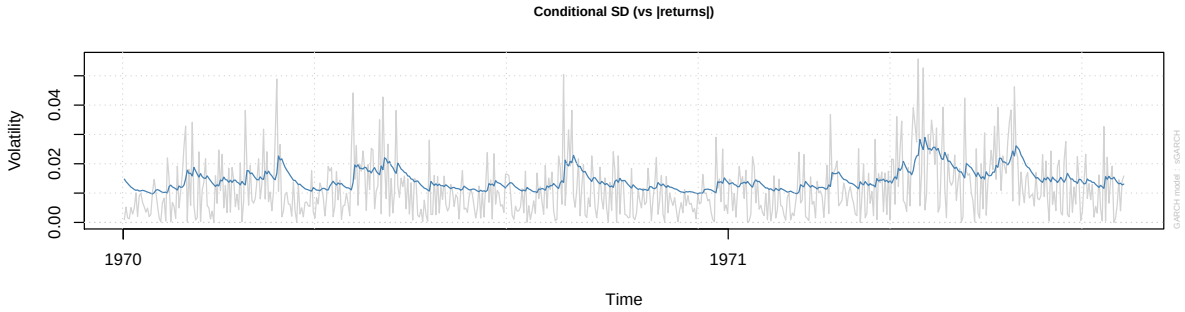


Figure 1: Conditional SD vs |returns| of SP500 with rugarch

```
fit_BM1 <- rGarch(rtn[1:604], methods = "bounded MEst", fixed_pars = c(1.0, 5.02))

##
## Iter: 1 fn: 2.3083  Pars:  0.03558 0.08653 0.87511 0.08805
## Iter: 2 fn: 2.3083  Pars:  0.03558 0.08654 0.87511 0.08806
## solnp--> Completed in 2 iterations
##      alpha_0      alpha_1       beta_1
## 7.239803e-06 8.653593e-02 8.751077e-01

plot(fit_BM1, main = "Conditional SD (vs |returns|) of SP500 with BM1 - robGarch")
```

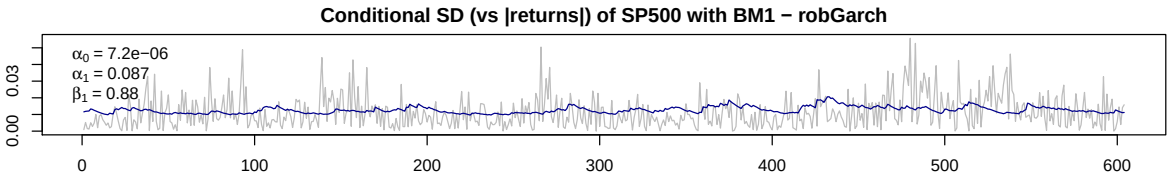


Figure 2: Conditional SD vs |returns| of SP500 with robGarch BM1

3.2 Test Result with Simulated Garch Series

In Table 3 through 7, for returns generated by the five Garch 11 models in Table 1, we present the `rugarch` results along with the results of our QML, M1 and BM1 methods. We note that the parameter estimates of QML are reasonably close to those of `rugarch` in Tables 3-7, which

shows the parameter estimation of `robGarch` package is correct.

Table 3: Garch(1,1) model fits for returns simulated with first model, $\alpha_1 + \beta_1 = 0.821$)

	α_0	α_1	β_1	$\alpha_1 + \beta_1$
true parameters	0.00088	0.14	0.69	0.82
rugarch	0.00072	0.19	0.65	0.84
QML	0.00072	0.19	0.65	0.84
M1	0.00075	0.11	0.69	0.81
BM1	0.00077	0.14	0.67	0.81

Similar to SP500 test results, from Figure 3 to Figure 12, the conditional standard deviation is more robust in BM1 compared to rugarch.

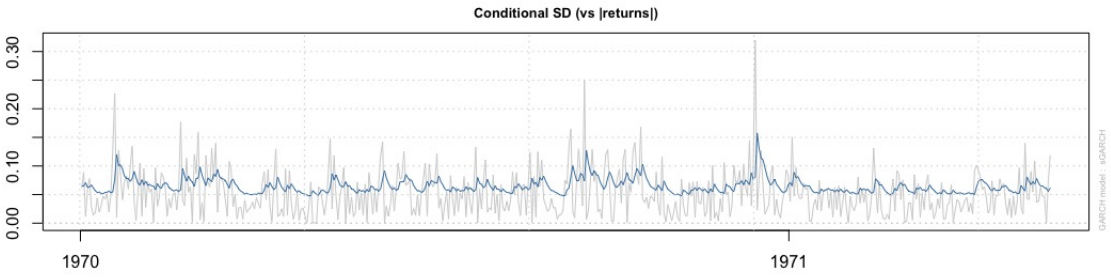


Figure 3: Conditional SD vs |returns| of first model with rugarch

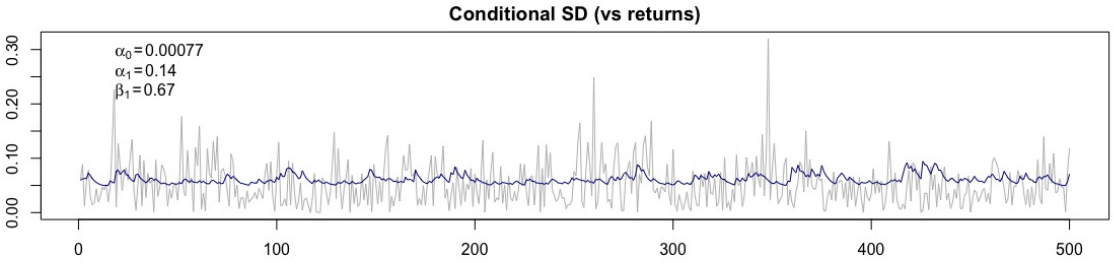


Figure 4: Conditional SD vs |returns| of first model with robGarch BM1

Table 4: Garch(1,1) model fits for returns simulated with second model, $\alpha_1 + \beta_1 = 0.920$

	α_0	α_1	β_1	$\alpha_1 + \beta_1$
true parameters	0.00039	0.14	0.78	0.92
rugarch	0.00082	0.12	0.66	0.78
QML	0.00068	0.11	0.72	0.82
M1	0.00075	0.12	0.70	0.82
BM1	0.00075	0.12	0.69	0.82

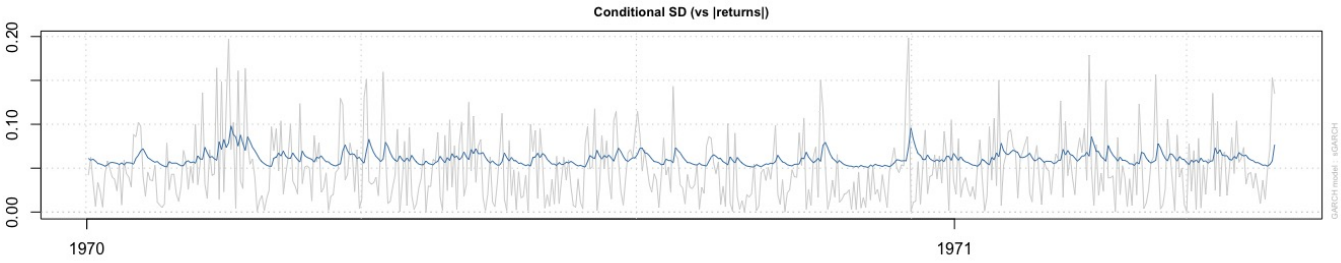


Figure 5: Conditional SD vs |returns| of second model with rugarch

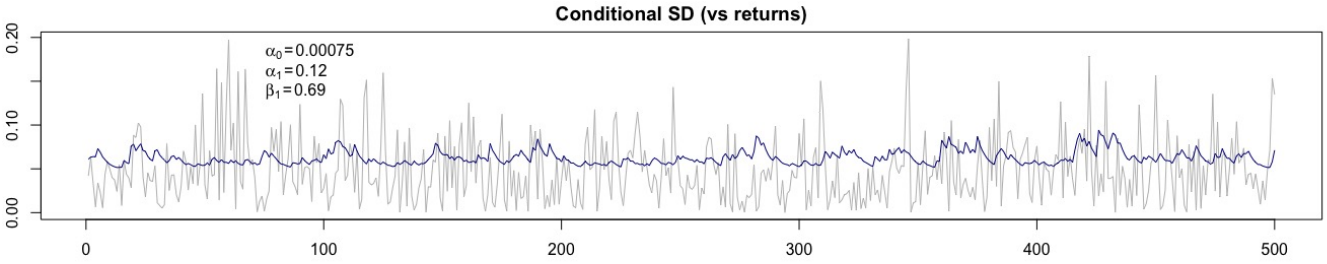


Figure 6: Conditional SD vs |returns| of second model with robGarch BM1

Table 5: Garch(1,1) model fits for returns simulated with third model, $\alpha_1 + \beta_1 = 0.950$

	α_0	α_1	β_1	$\alpha_1 + \beta_1$
true parameters	0.00024	0.120	0.83	0.95
rugarch	0.00058	0.082	0.76	0.84
QML	0.00037	0.061	0.84	0.90
M1	0.00038	0.067	0.83	0.90
BM1	0.00037	0.071	0.84	0.91

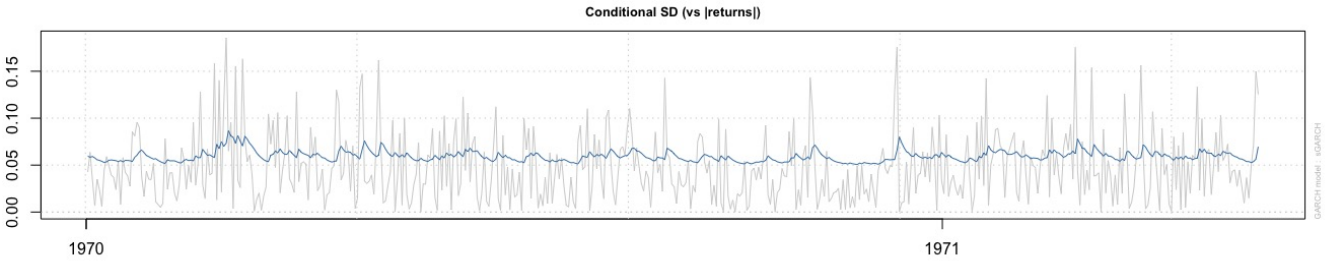


Figure 7: Conditional SD vs |returns| of third model with rugarch

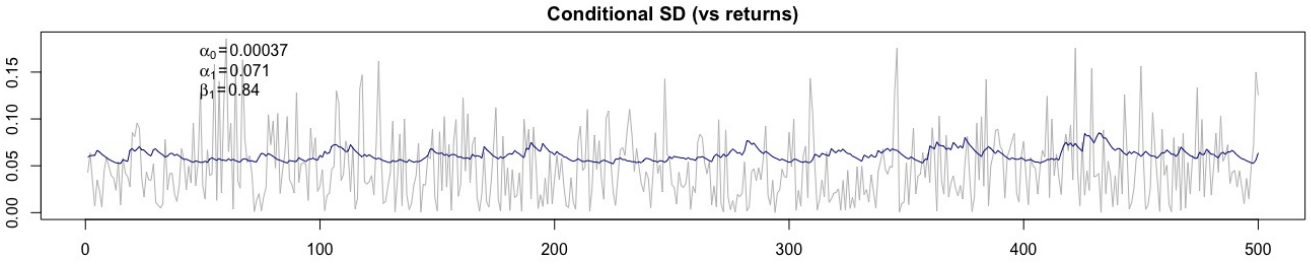


Figure 8: Conditional SD vs |returns| of third model with robGarch BM1

Table 6: Garch(1,1) model fits for returns simulated with fourth model, $\alpha_1 + \beta_1 = 0.971$

	α_0	α_1	β_1	$\alpha_1 + \beta_1$
true parameters	0.00024	0.100	0.87	0.97
rugarch	0.00051	0.073	0.85	0.93
QML	0.00057	0.078	0.84	0.92
M1	0.00058	0.087	0.83	0.92
BM1	0.00058	0.087	0.83	0.92

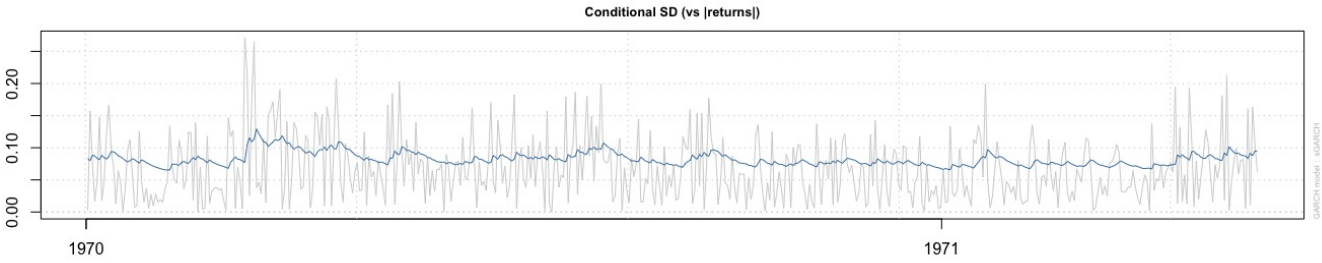


Figure 9: Conditional SD vs |returns| of fourth model with rugarch

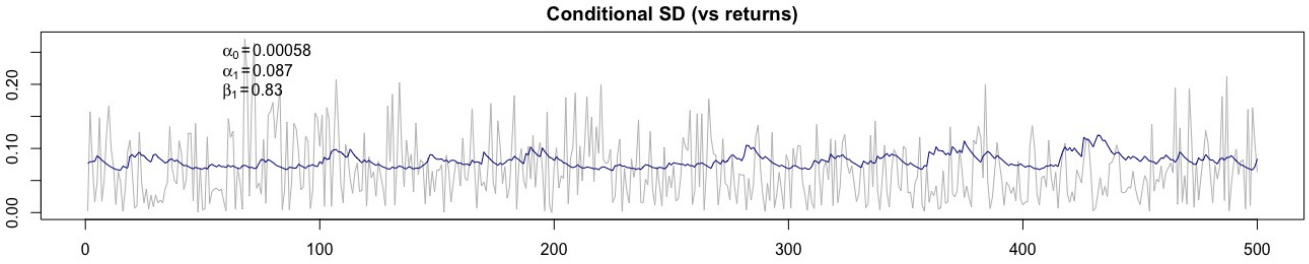


Figure 10: Conditional SD vs |returns| of fourth model with robGarch BM1

Table 7: Garch(1,1) model fits for returns simulated with fifth model, $\alpha_1 + \beta_1 = 0.992$

	α_0	α_1	β_1	$\alpha_1 + \beta_1$
true parameters	1.0e-04	0.083	0.91	0.99
rugarch	9.3e-05	0.041	0.95	0.99
QML	1.0e-04	0.043	0.95	0.99
M1	1.1e-04	0.048	0.94	0.99
BM1	1.1e-04	0.048	0.94	0.99

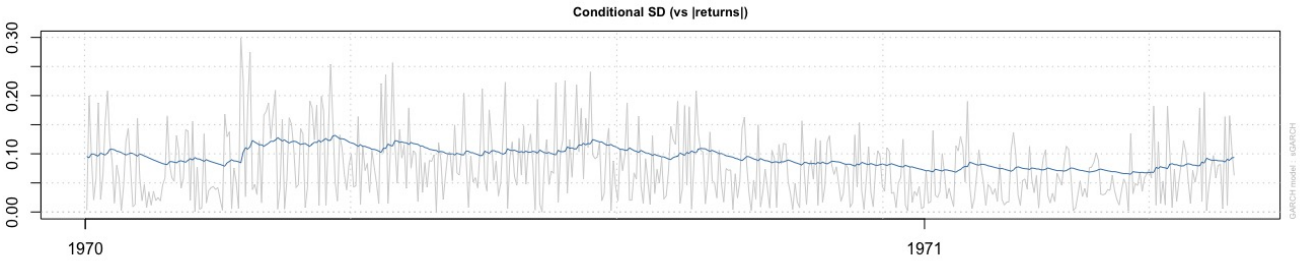


Figure 11: Conditional SD vs |returns| of fifth model with rugarch

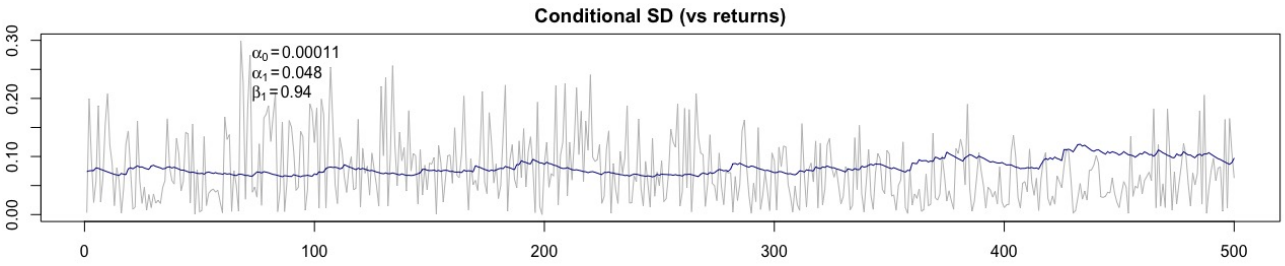


Figure 12: Conditional SD vs |returns| of fifth model with robGarch BM1

4 Future Development

Any future development will be released in the github page. A few key features will be added to the package in September 2020:

- Statistics tests such as std_error, t_value, p_value for Garch parameters
- Code debug on model filter for M model and QML
- More optimization choices
- Extension to robust Garch(p, q)

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