Introduction to the robGarch package (Version 0.0.1)

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1 Introduction

AutoRegressive Conditional Heteroscedasticity (ARCH) was introduced by Engle (1982) and extended to Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models by Bollerslev (1987), the latter proven very popular in many applications as they provide a rich statistical framework for non-constant variance and their ability to capture observed empirical phenomena such as volatility clustering. Chou (1988) showed the ability of GARCH to estimate risk premium valuation of a stock. In Duan and Jin-Chuan (1995), GARCH is applied to pricing options, where volatility plays a crucial role in the Black-Scholes option pricing model, thereby adding a new perspective in comparison to stochastic volatility model. Bias caused by outliers has been studied in some literature. Huber (1981) established two properties for a good robust model fitting method. The first is that a robust estimate needs to be efficient and should be efficient in the sense of having a variance that is not much larger than that of a maximum likelihood estimate typically based on a normal distribution. The second property, introduced by later researchers is that replacing a small fraction of the data by outliers should not cause big difference as measured by estimator bias. Mendes (2000) showed how outliers create asymptotic bias in Quasi Maximum Likelihood Estimate of the GARCH parameters. Muler and Yohai (2008) showed that QML parameter estimation based on normal likelihood is very sensitive to outliers in financial returns, and even a single outlier can have a huge influence on the QML parameter estimates.

The robGarch package aims to provide a method for modelling robust GARCH processes, addressing the issue of robustness toward additive outliers, rather than innovations outliers. It contains fitting, forecasting, simulation with diagnostic tools including plots and tests. This document discusses the details of the included models and how they are implemented in the package with some examples.

The robGarch package can be installed as following:

```
devtools::install_github("EchoRLiu/robGarch")
library(robGarch)
```

and the development version on github (https://github.com/EchoRLiu/robGarch) with examples and and demons.

2 Robust Garch Model

2.1 robust Garch

Robust Garch is first introduced by Muler and Yohai (2008) with the foundation work of Huber (1981). The following content focuses on how the two robust GARCH model in Muler and Yohai (2008) work.

Defined as $x_t = \sigma_t z_t$, where z_t is i.i.d random variables with continuous density f such that $E(z_t) = 0$, $Var(z_t) = 1$, and where the conditional variance σ_t^2 are given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{1}$$

the robust parameters estimation is based on M-estimate with more flavors. Two special things are done for BM1 and BM2 models: (1) for the BM1, the M-estimates has bounded ρ' and unbounded ρ, such as $\rho = m_1(\rho_0)$, where ρ_0 is defined as follows $\rho_0 = -log(\frac{1}{\sqrt{2\pi}}e^{-(e^w - w)/2}))$, where $w = log(z_t^2$ and m_1 is a non-decreasing, bounded function

$$m_1(x) = \begin{cases} x, & x \leq 4 \\ P(x), & 4 < x \leq 4.34.15, x > 4.3 \end{cases} \tag{2}$$

$$P(x) = \frac{2}{(b-a)^3}(\frac{1}{4}(x^4 - a^4) - \frac{1}{3}(2a+b)(x^3 - a^3) + \frac{1}{2}(a^2 + 2ab)(x^2 - a^2)) \\ - \frac{2a^2 b}{(b-a)^3}(x-a) - \frac{1}{3(b-a)^2}(x-a)^3 + x, a = 4, b = 4.3 \tag{3}$$

which satisfies constraints $P(a) = a, P'(a) = 1, P'(b) = P''(a) = P''(b) = 0$ this model is robust when z_t is heavy-tail distribution and may be affected by additive outliers, in the Monte-Carlo section, $k = 5.02$ for the testing; (2) for BM2, ρ is bounded as well, thus more robust, defined as

$$m_2(\rho_0) = 0.8 m_1(\rho_0/0.8) \tag{4}$$

and k is chosen to be 3 for the testing. However, BM2 unfortunately would still be influenced by large outliers.

The main structure of parameter estimation is

$$\hat{\gamma_T}^B = \begin{cases} \hat{\gamma_{1,T}}, & M_T(\hat{\gamma_{1,T}}) \le M^*_{Tk}(\hat{\gamma_{2,T}}) \\ \hat{\gamma_{2,T}}, & M_T(\hat{\gamma_{1,T}}) > M^*_{Tk}(\hat{\gamma_{2,T}}) \end{cases} \tag{5}$$

where

$$\begin{aligned} \hat{\gamma_{1,T}} &= arg \min_{\mathbf{c}} M_T(\mathbf{c}) \\ &= arg \min_{\alpha_i,\beta_j} \frac{1}{T-p} \sum_{t=p+1}^{T} \rho(log(x_t^2) - log(\alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2)) \end{aligned} \tag{6}$$

and

$$\begin{aligned} \hat{\gamma_{2,T}} &= arg \min_{\mathbf{c}} M^*_{Tk}(\mathbf{c}) \\ &= arg \min_{\alpha_i,\beta_j} \frac{1}{T-p} \sum_{t=p+1}^{T} \rho(log(x_t^2) \\ &- log(\alpha_0 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i,k}^2(\alpha_i,\beta_j) r_k(\frac{x_{t-i}^2}{\sigma_{t-i,k}^2}) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2)) \end{aligned} \tag{7}$$

where

$$r_k(u) = \begin{cases} u, & u \le k \\ k, & u > k \end{cases} \tag{8}$$

the use of function $r_k(u)$ is to restrict the propagation of the outlier effect. If k is large, then $\sum_{i=1}^{p} \alpha_i \sigma_{t-i,k}^2(\alpha_i,\beta_j) r_k(\frac{x_{t-i}^2}{\sigma_{t-i,k}^2})$ becomes the normal form $\sum_{i=1}^{p} \alpha_i x_{t-i}^2$. The different optimization under different cases makes sure that the model has robustness with outliers and maintains consistency when the series follows a GARCH without outliers.

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