

**Stockton University**

**APPLIED PROBABILITY AND STATISTICS: FINAL PROJECT**

Dante Anzalone

Stockton University

3327: Probability and Applied Statistics

Byron Hoy

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GitHub Link

<https://github.com/EchoingTime/ProbabilityAndStatistics>

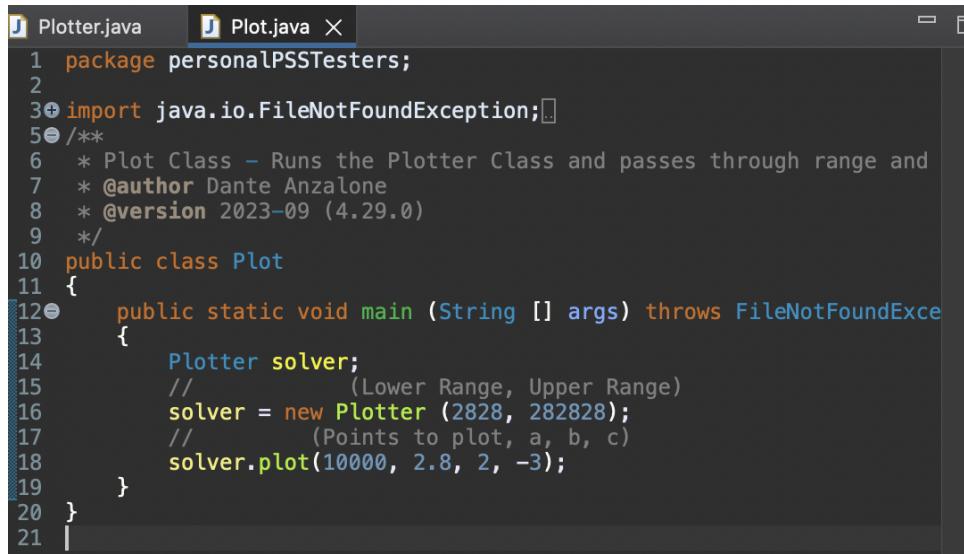
Contains the programs discussed in this paper

PART I - Personal Plotter, Salter, and Smoother

Introduction to the PSS Programs

Applied Probability and Statistics course project two, this report focuses on the plotter, salter, and smoother programs. The Plotter.java class utilizes the quadratic equation formula,  $y = ax^2 + bx + c$ , to produce a set amount of y-coordinates in a determined range provided via the user, manually, in the Plot.java tester class. The x-coordinates are gathered by utilizing  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , checking if the value is equal or greater than 0, to see if it is a real solution, and then finding two x-coordinates via equations, obtaining both points, then, using BufferedWriter class, writing into a CSV, comma-separated values, a text file called personalPlotsXY.csv. The class will allow a user to collect sample outputs and plot each by uploading the file into either Google Sheets or Excel into a chart of their choice. For this program and the remaining two yet to be mentioned, there will be graph figures later in the paper demonstrating their usage and how changing the configurations could impact the graph. The Salter.java class program will utilize the personalPlotsXY.csv file using the BufferedReader class, created in the previous Plotter.java class. This new class will loop through each y-coordinate value and either add or subtract a random value from it, which is also determined by the user. The user can provide the random number via a range. For example, a negative range from negative ten to positive ten would gather random values through the bounds and add or subtract them to a specific y-coordinate. The class allows the user to have a significant number of data points if they desire, and salt the values and write them into a new file called personalSaltedXY.csv. Users can manually choose the ranges within the Salt.java tester class. Finally is the Smoother.java class, which will accept the salted file, personalSaltedXY.csv, loop through each y-coordinate, and gather an average of the points around it. The user also can provide the range. The average result will replace that specific y-coordinate, therefore smoothing the data. Another method, containing shorter lines of code, is creating the plotter, salter, and smoother via MatLab. MatLab is another programming and computing platform that allows users to use its methods to easily create, using Linear Algebra rules, equations, scripts similar to classes, and plot graphs. Then there is the usage of outside Jar files found via the internet, such as Apache Common Math, which contains classes and methods for smoothing and salting data, and JFreeCharts to create a graph containing data points, like MatLab. These additional methods are other shortcuts to plotting than writing the whole program themselves.

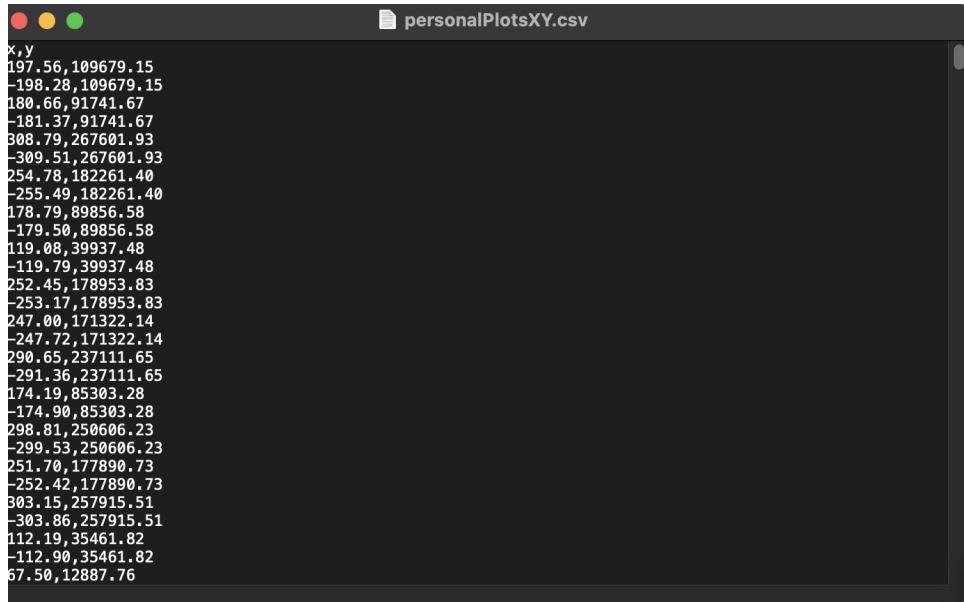
Plotter



```
1 package personalPSSTesters;
2
3 import java.io.FileNotFoundException;
4 /**
5  * Plot Class - Runs the Plotter Class and passes through range and
6  * @author Dante Anzalone
7  * @version 2023-09 (4.29.0)
8  */
9
10 public class Plot
11 {
12     public static void main (String [] args) throws FileNotFoundException
13     {
14         Plotter solver;
15         // (Lower Range, Upper Range)
16         solver = new Plotter (2828, 282828);
17         // (Points to plot, a, b, c)
18         solver.plot(10000, 2.8, 2, -3);
19     }
20 }
21
```

Figure 1: Screenshot of Configuration

The lower range is 2,828, and the upper is 282,828, so randomly generated points through these two ranges will be saved into a CSV file and later plotted with 10,000 points. Coefficients are labeled a, b, and c via comments in the code.



x,y
197.56,109679.15
-198.28,109679.15
180.66,91741.67
-181.37,91741.67
388.79,267601.93
-309.51,267601.93
254.78,182261.40
-255.49,182261.40
178.79,89856.58
-179.50,89856.58
119.00,39937.48
-119.79,39937.48
252.45,178953.83
-253.17,178953.83
247.00,171322.14
-247.72,171322.14
290.65,237111.65
-291.36,237111.65
174.19,85303.28
-174.90,85303.28
298.81,250606.23
-299.53,250606.23
251.70,177890.73
-252.42,177890.73
303.15,257915.51
-303.86,257915.51
112.19,35461.82
-112.90,35461.82
67.50,12887.76

Figure 1.1: CSV File

The image shows a generated CSV file after the program runs. Users can upload the data into Google Sheets.

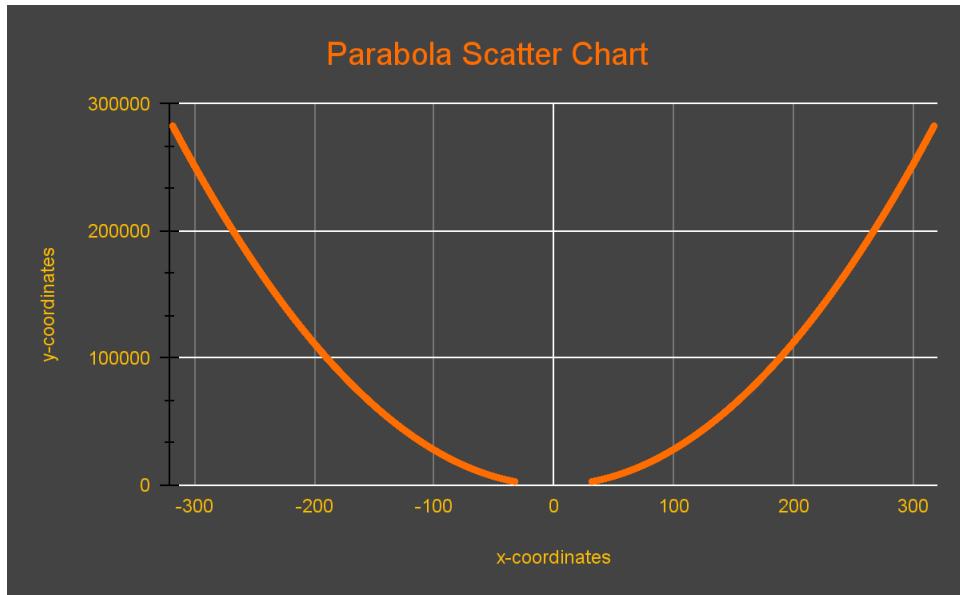


Figure 1.2: The Parabola

With 10,000 points, ranges, and coefficients in the quadratic formula. Result of the program's output to a CSV file and from the CSV file to Google Sheets.

Salter

```
1 package personalPSSTesters;
2
3+ import java.io.FileNotFoundException;[]
5+ /**
6 * Salt Class - Runs the Salter class and passed through a range to alter the y values
7 * in a CSV file by
8 * @author Dante Anzalone
9 * @version 2023-09 (4.29.0)
10 */
11 public class Salt
12 {
13+     public static void main (String [] args) throws FileNotFoundException
14     {
15         Salter solve;
16         solve = new Salter ();
17
18         solve.salting(-28228.28, 28228.28);
19     }
20 }
21
```

Figure 2: Screenshot of Configuration

Using Figure 1.1's CSV file. Using a range of -28,228.28 through 28,228.28 to alter y-coordinates by.

x,y
197.56,108391.65
-198.28,114577.34
180.66,78056.44
-181.37,101054.76
308.79,240119.36
-309.51,295371.72
254.78,194295.10
-255.49,206477.35
178.79,81255.27
-179.50,76571.45
119.08,35096.34
-119.79,48610.01
252.45,181224.56
-253.17,160580.66
247.00,147608.80
-247.72,149211.92
290.65,226049.66
-291.36,233615.50
174.19,66088.50
-174.90,57886.80
298.81,247807.65
-299.53,267157.66
251.70,151289.50
-252.42,183992.00
303.15,246072.53
-303.86,285282.48
112.19,60080.02
-112.90,7727.29
67.50,33276.93

Figure 2.1: Salted CSV File

The y-coordinates are now salted, as compared to Figure 1.1.

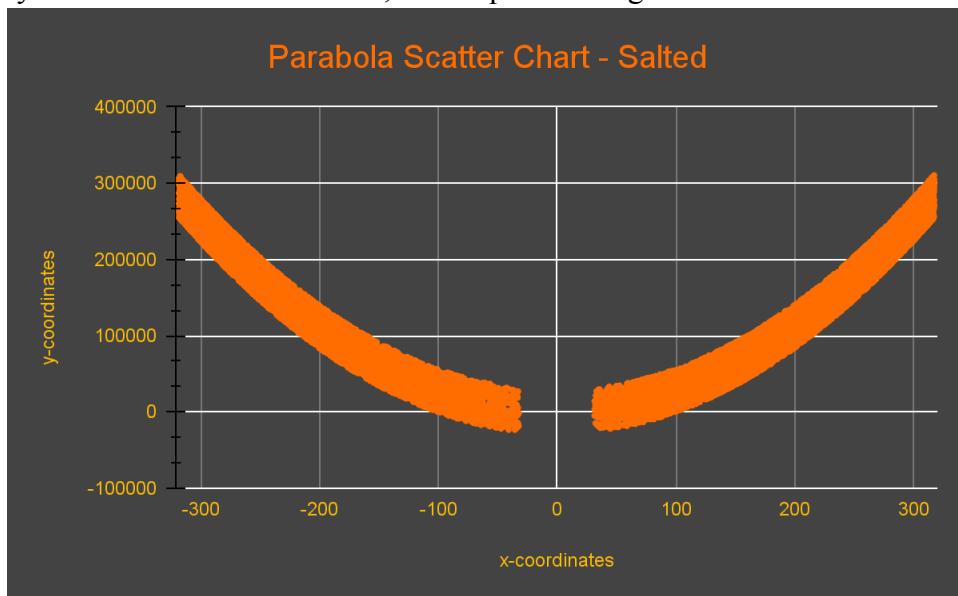
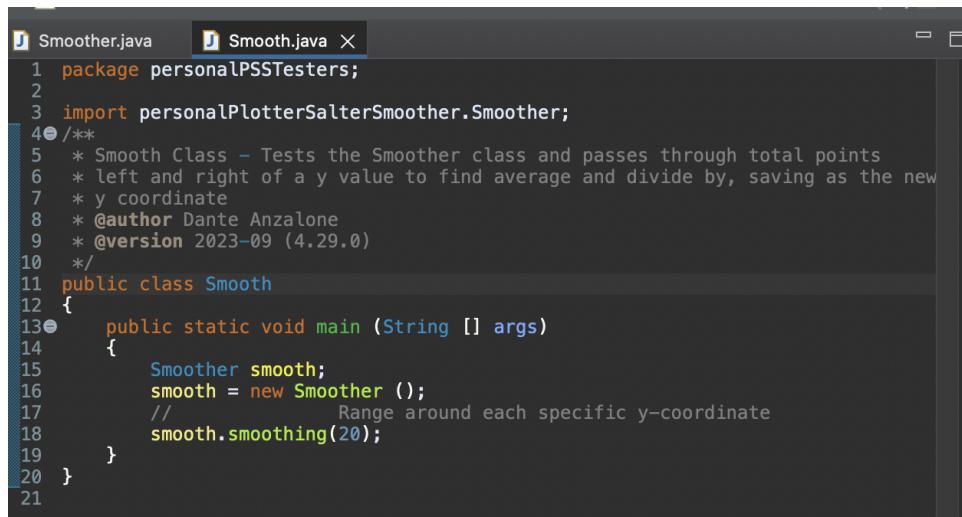


Figure 2.2: The Parabola - Salted

Compared to Figure 1.2, both graphs have the same format, a size 2 for the points. The salted chart shows a rise in the y-coordinate interception and a thicker point difference. The program altered the y-coordinates while keeping the x-coordinates the same.

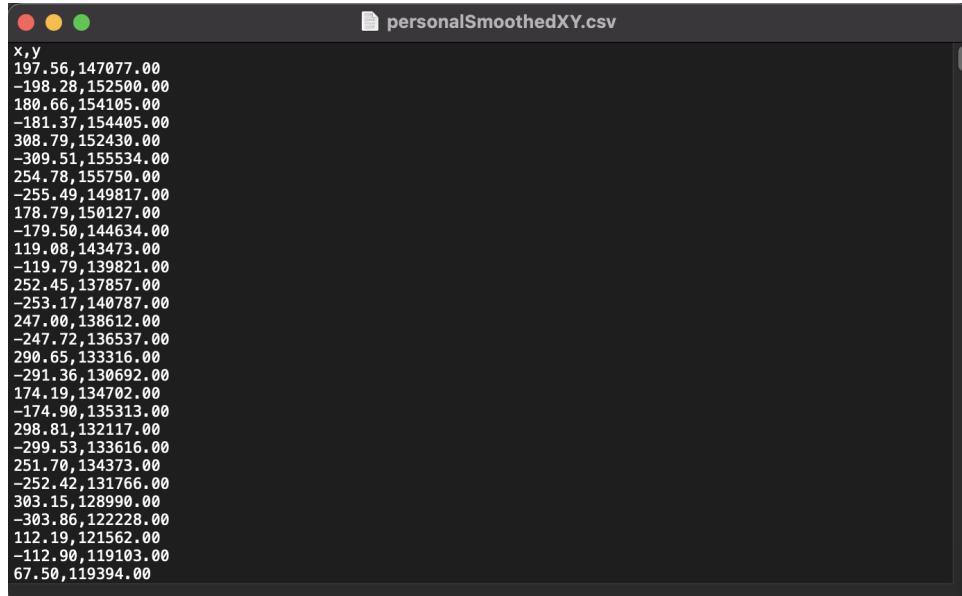
### Smoother



```
1 package personalPSSTesters;
2
3 import personalPlotterSalterSmoother.Smoother;
4 /**
5  * Smooth Class - Tests the Smoother class and passes through total points
6  * left and right of a y value to find average and divide by, saving as the new
7  * y coordinate
8  * @author Dante Anzalone
9  * @version 2023-09 (4.29.0)
10 */
11 public class Smooth
12 {
13     public static void main (String [] args)
14     {
15         Smoother smooth;
16         smooth = new Smoother ();
17         // Range around each specific y-coordinate
18         smooth.smoothing(20);
19     }
20 }
21
```

Figure 3: Screenshot of Configuration

The range to smooth each specific y-coordinate by the point range around it is 20 points, using the CSV file from Figure 2.1.



x,y
197.56,147077.00
-198.28,152500.00
180.66,154105.00
-181.37,154405.00
308.79,152430.00
-309.51,155534.00
254.78,155750.00
-255.49,149817.00
178.79,150127.00
-179.50,144634.00
119.08,143473.00
-119.79,139821.00
252.45,137857.00
-253.17,140787.00
247.00,138612.00
-247.72,136537.00
290.65,133316.00
-291.36,130692.00
174.19,134702.00
-174.90,135313.00
298.81,132117.00
-299.53,133616.00
251.70,134373.00
-252.42,131766.00
303.15,128990.00
-303.86,122228.00
112.19,121562.00
-112.90,119103.00
67.50,119394.00

Figure 3.1: Smoothed CSV File

The y-coordinates are now smoothed, as compared to Figure 2.1.

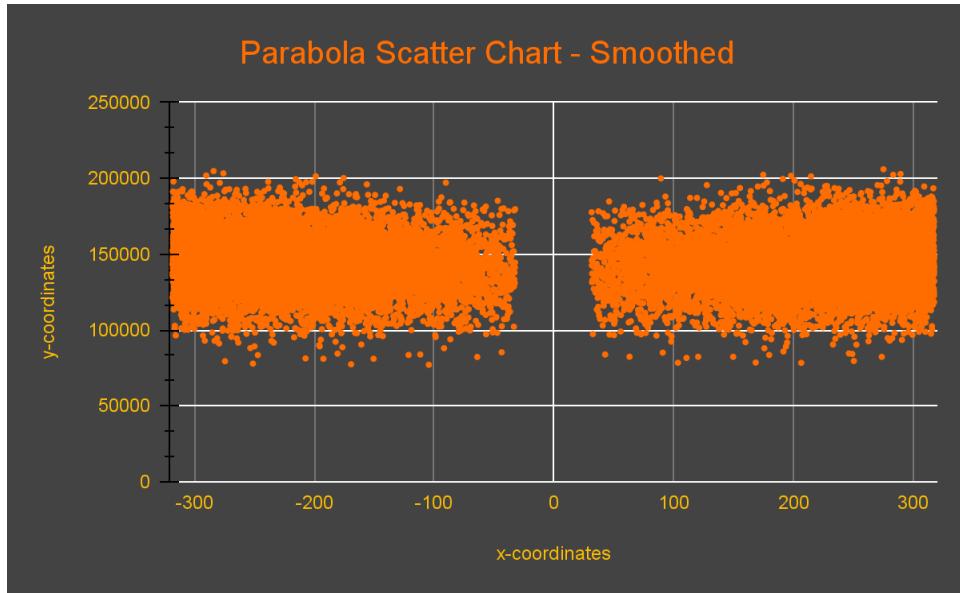


Figure 3.2: The Parabola - Smoothed  
The smoothed data from Figure 2.1 salted data.

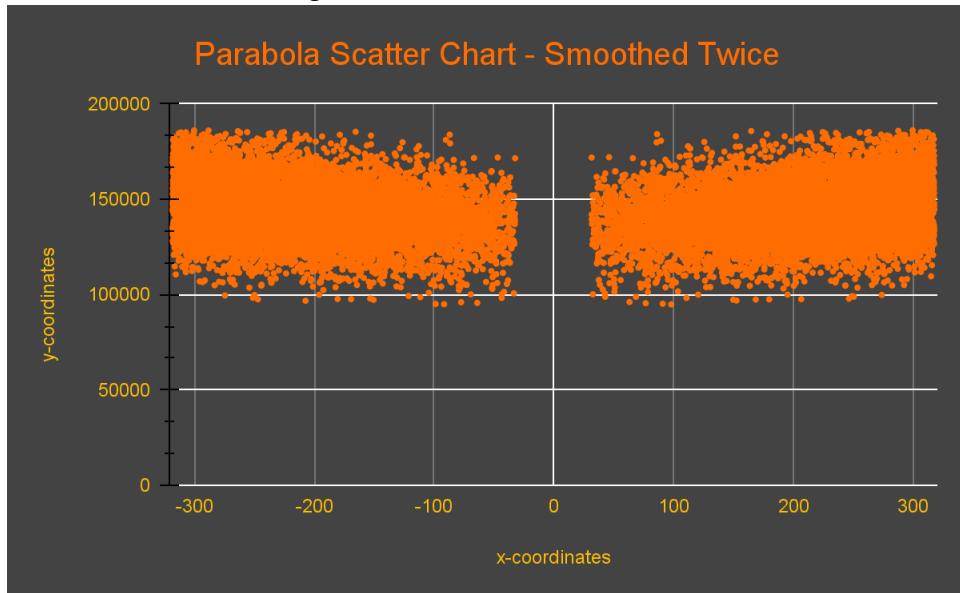


Figure 3.3: The Parabola - Smoothed Again  
Using the smoothed file to smooth once again with the same program parameters.

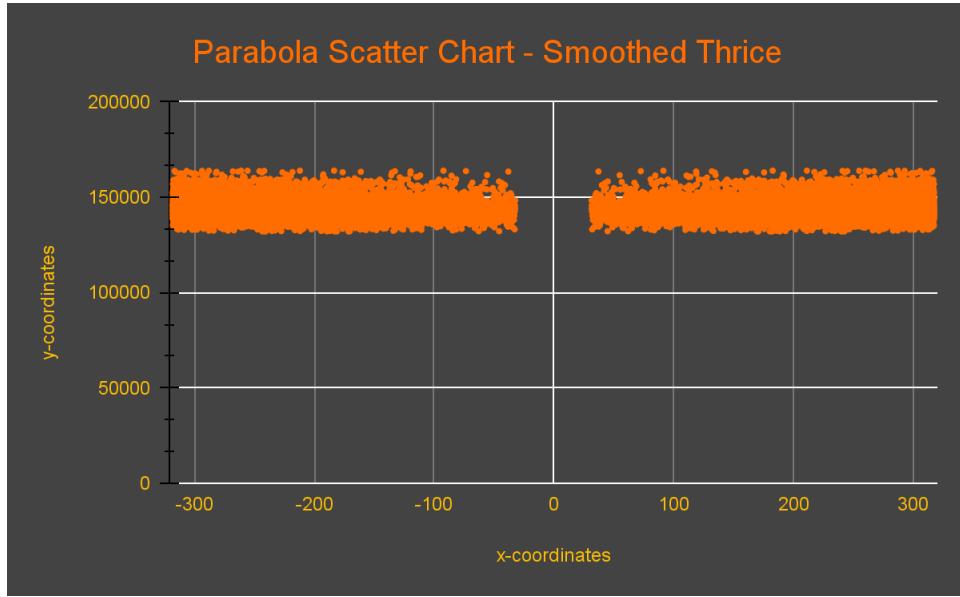


Figure 3.4: The Parabola - Smoothed Thrice  
Smoothed the smoothed file that was smoothed with the range set to 250.

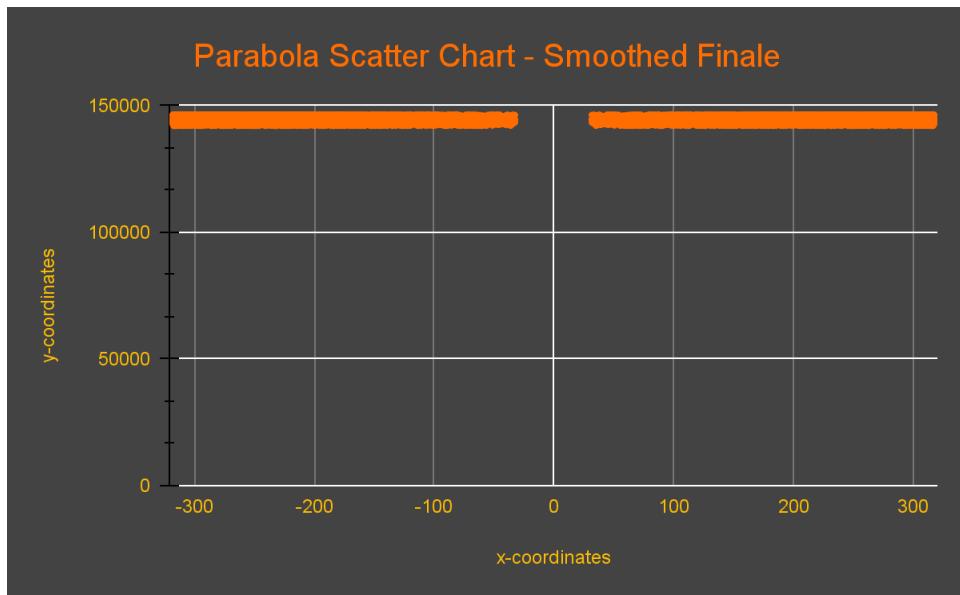
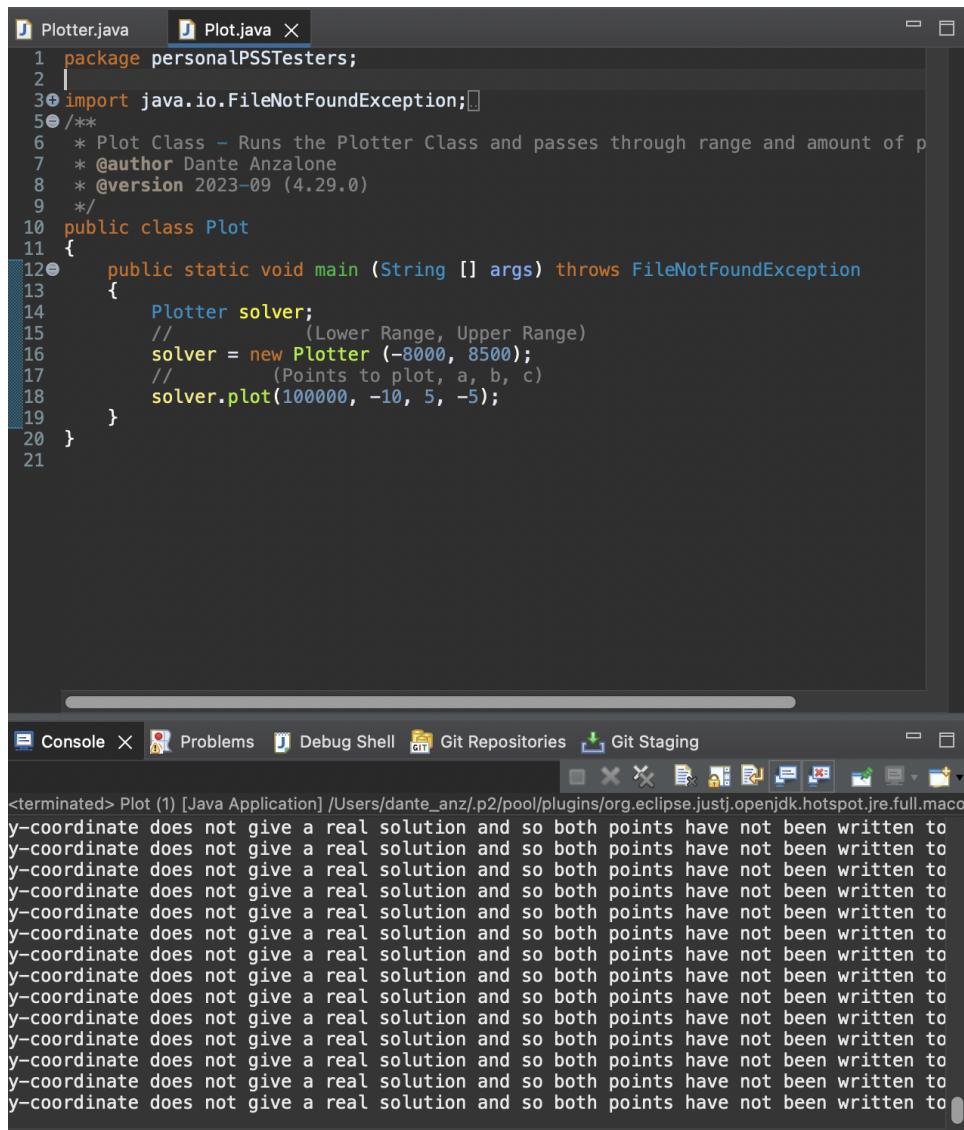


Figure 3.5: The Parabola - Smoothed Finale  
Smoothed the smoothed file that was a smoothed file smoothed and then smoothed one final time with a range set to 2,500.

Plotter - Second



The screenshot shows the Eclipse IDE interface with two tabs open: "Plotter.java" and "Plot.java X". The "Plotter.java" tab contains the following Java code:

```
1 package personalPSSTesters;
2
3 import java.io.FileNotFoundException;
4 /**
5  * Plot Class - Runs the Plotter Class and passes through range and amount of points
6  * @author Dante Anzalone
7  * @version 2023-09 (4.29.0)
8  */
9
10 public class Plot
11 {
12     public static void main (String [] args) throws FileNotFoundException
13     {
14         Plotter solver;
15         // (Lower Range, Upper Range)
16         solver = new Plotter (-8000, 8500);
17         // (Points to plot, a, b, c)
18         solver.plot(100000, -10, 5, -5);
19     }
20 }
```

The "Console" tab at the bottom shows the following output:

```
<terminated> Plot (1) [Java Application] /Users/dante_anz.p2/pool/plugins/org.eclipse.justj.openjdk.hotspot.jre.full.macosx-x86_64
y-coordinate does not give a real solution and so both points have not been written to
y-coordinate does not give a real solution and so both points have not been written to
y-coordinate does not give a real solution and so both points have not been written to
y-coordinate does not give a real solution and so both points have not been written to
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y-coordinate does not give a real solution and so both points have not been written to
y-coordinate does not give a real solution and so both points have not been written to
y-coordinate does not give a real solution and so both points have not been written to
y-coordinate does not give a real solution and so both points have not been written to
```

Figure 4: Screenshot of Configuration

Added more points, altered coefficients, and lowered the range of points. Some points did not give a solution once plugged into the equation, and the program did not write the points to the CSV file.

x,y
-12.12,-1533.54
12.62,-1533.54
-14.77,-2258.95
15.27,-2258.95
-14.80,-2269.73
15.30,-2269.73
-24.66,-6207.38
25.16,-6207.38
-25.11,-6434.55
25.61,-6434.55
-21.68,-4814.33
22.18,-4814.33
-6.95,-523.16
7.45,-523.16
-2.68,-90.22
3.18,-90.22
-26.23,-7014.79
26.73,-7014.79
-23.68,-5728.68
24.18,-5728.68
-22.15,-5022.20
22.65,-5022.20
-26.88,-7363.23
27.38,-7363.23
-10.83,-1230.99
11.33,-1230.99
-8.84,-830.00
9.34,-830.00
-5.09,-290.04

Figure 4.1: CSV File

Still an ample amount of points.

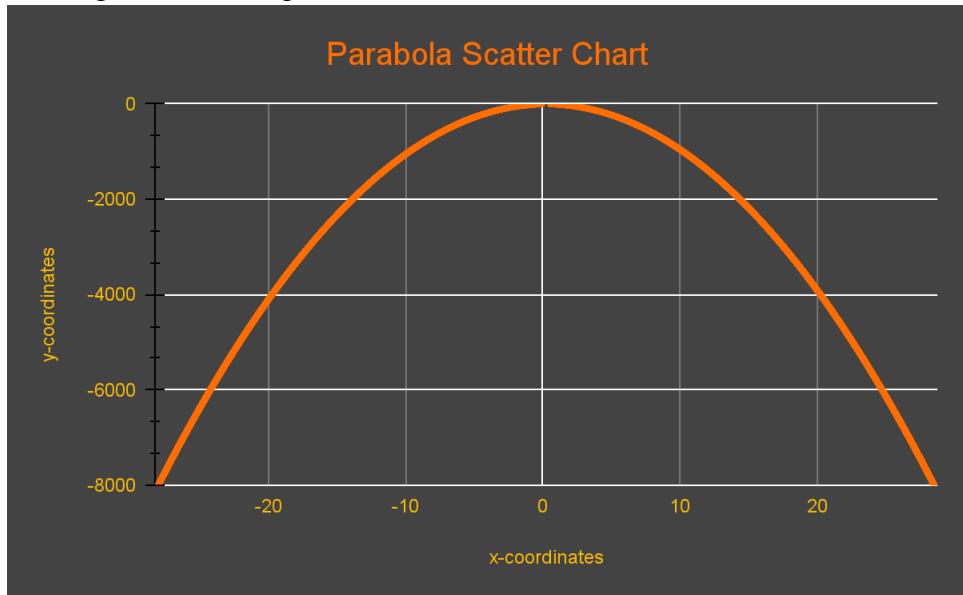
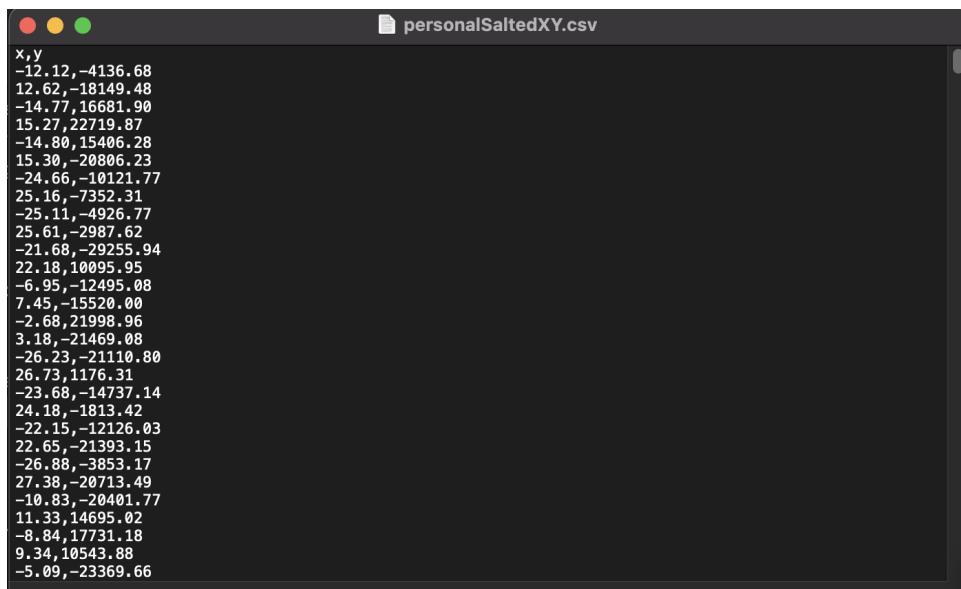


Figure 4.2: The Parabola

Upside Down compared to Figure 1.2.

Salter - Second



```
x,y
-12.12,-4136.68
12.62,-18149.48
-14.77,16681.90
15.27,22719.87
-14.80,15406.28
15.30,-20806.23
-24.66,-10121.77
25.16,-7352.31
-25.11,-4926.77
25.61,-2987.62
-21.68,-29255.94
22.18,10095.95
-6.95,-12495.08
7.45,-15520.00
-2.68,21998.96
3.18,-21469.08
-26.23,-21110.80
26.73,1176.31
-23.68,-14737.14
24.18,-1813.42
-22.15,-12126.03
22.65,-21393.15
-26.88,-3853.17
27.38,-20713.49
-10.83,-20401.77
11.33,14695.02
-8.84,17731.18
9.34,10543.88
-5.09,-23369.66
```

Figure 5: Salted CSV File

It has the same configurations as Figure 2, but now see what impact it has on a data range that was smaller to begin with. Compared with Figure 4.1, the y-coordinates vary greatly. A smaller salt configuration would not have such an impact on the overall graph.

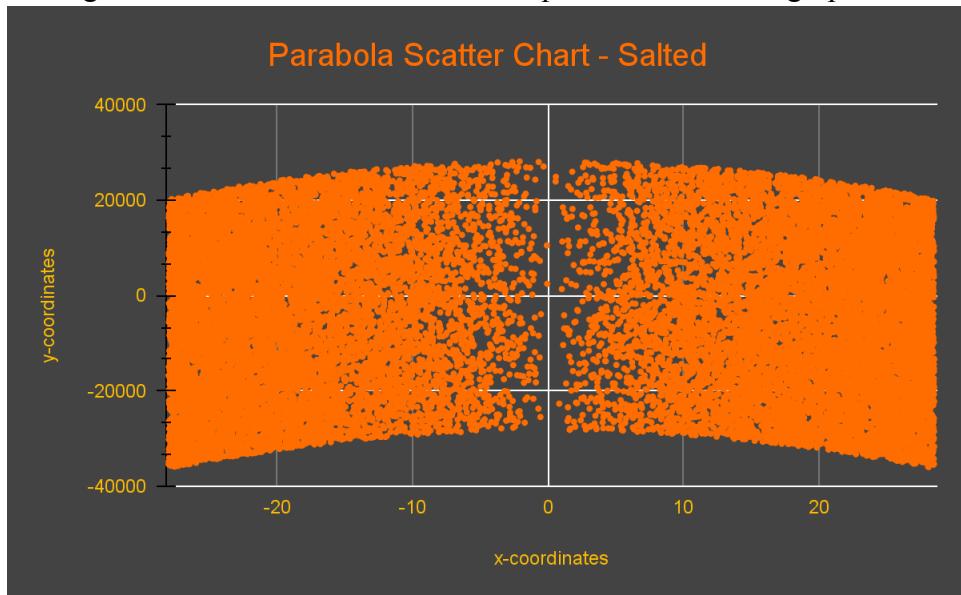


Figure 5.1: The Parabola - Salted  
Greater impact to the original data from Figure 4.1.

Smoother - Second

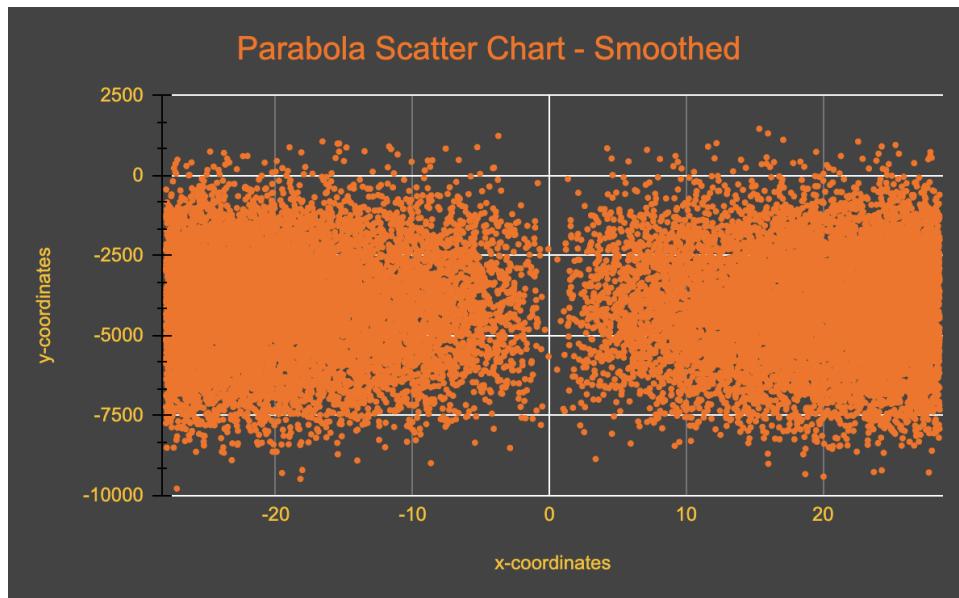


Figure 5: The Parabola - Smoothed

The points smoothed by 1000. Figure 6 will show a plotter with fewer points and a salter and smoother one that does not alter the data by too much.

Final Plotter, Salter, and Smoother

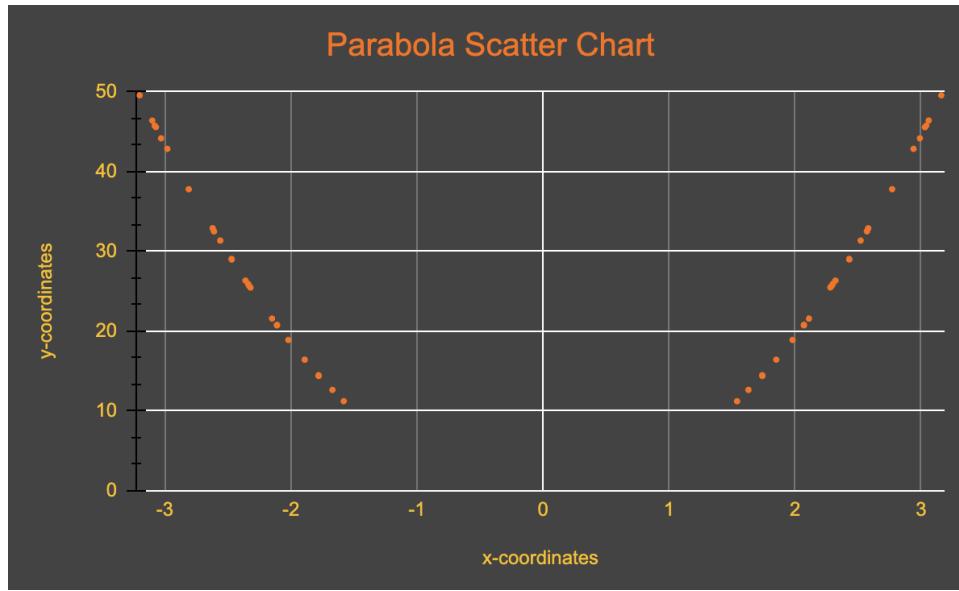


Figure 6: The Parabola

Graph contains fewer points to give a better look at individual points forming a parabola.

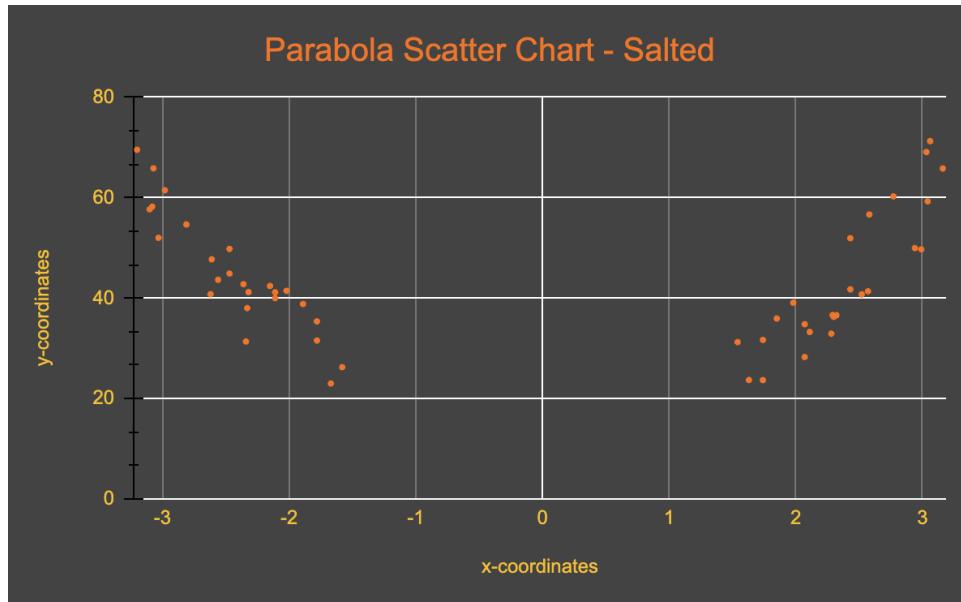


Figure 6.1: The Parabola - Salted

Figure 6 is salted.

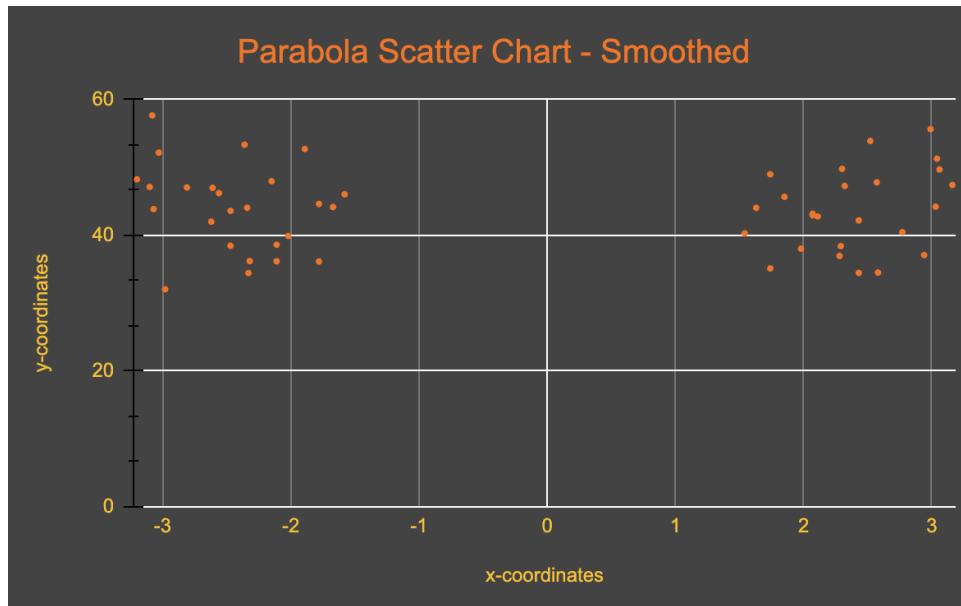


Figure 6.2: The Parabola - Smoothed

Figure 6.1 is smoothed.

## Improvements

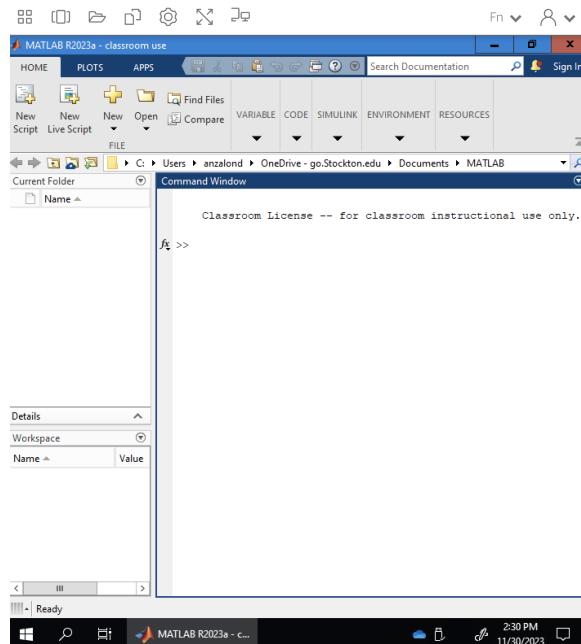
Inside the Smoother.java class, variables swapped from integer to double in the private average method. Overall, the plotter and salter programs work as intended. Perhaps adding data structure for the points and allowing the file to be a parameter to pass to the classes would improve the programs. The smoother does not seem too efficient and could be faulty. It smoothes the data into a single line instead of the original graph. After completing Part II, the MatLab smooth method shows the correct form. It pushes the data into a flat line but should make the points into a closer value-range parabola.

## PART II - Plotter, Salter, and Smoother in MatLab

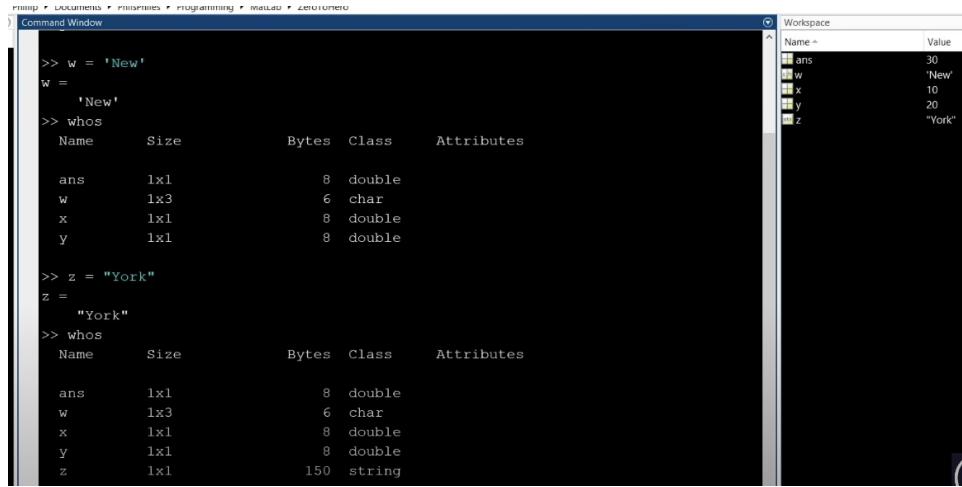
MatLab is a programming language and software suite used for data analysis, scientific Computing and visualizations and widely used in Academia and Industry (“MATLAB Crash Course for Beginners”).

### MatLab Introduction and Process - YouTube Tutorial

For Part II, the “MATLAB Crash Course for Beginners” was utilized to learn about MatLab. When opening the MatLab program, it contains an integrated development environment. Users must use the command window to run commands, like coding. Stockton students can use MatLab from home by running MatLab via Stockton University’s appstream service. The "clc" command clears the command window, and "clearvars" clears the variables in the workspace. The command "whos" brings up the variables. This section will paraphrase the steps in the YouTube video into word format via document.



MatLab via Stockton’s appstream.



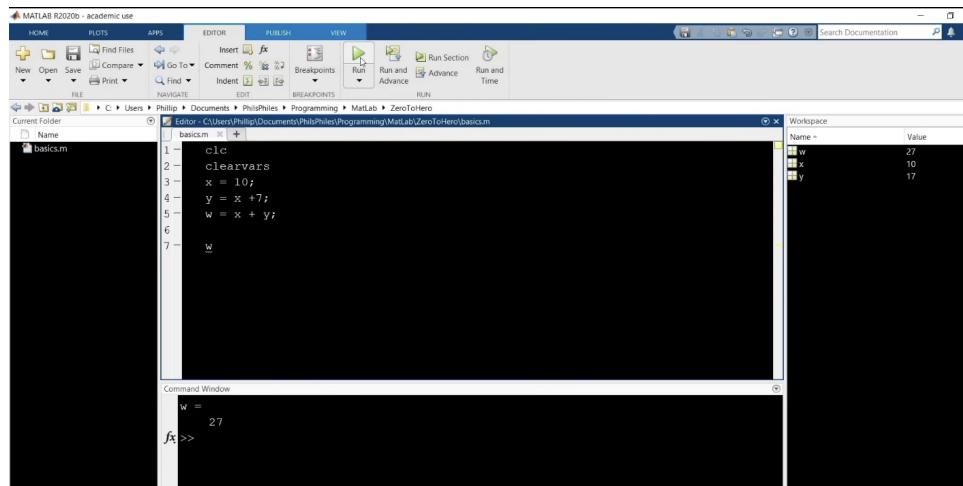
```

>> w = 'New'
w =
    'New'
>> whos
  Name      Size            Bytes  Class     Attributes
  ans        1x1              8  double
  w          1x3              6  char
  x          1x1              8  double
  y          1x1              8  double

>> z = "York"
z =
    "York"
>> whos
  Name      Size            Bytes  Class     Attributes
  ans        1x1              8  double
  w          1x3              6  char
  x          1x1              8  double
  y          1x1              8  double
  z          1x1           150  string

```

"`w = 'New'`" creates a char variable, "`z = "York"`" creates a string variable, and "`x = 8`" creates a double variable. The YouTube video mentions that these lines cover almost anything a user needs to do for basic MatLab. MatLab requires semicolons at the end of a piece of expression to suppress output. "`x = 9 + 10;`" still processes the command but does not print the output. Use commas to separate statements and to have two commands on one line. You can do operations like any programming IDE, like addition, subtraction, division, etc. The program can also do compound expressions "`sqrt(8 * 9)`" or "`x = 8 * 9`" and "`sqrt(x)`". Remember the whos command to display all the variables. Also, users can double-click variables in the workspace to view them in further detail.



The screenshot shows the MATLAB R2020b interface. The top menu bar includes HOME, PLOTS, APPS, EDITOR, PUBLISH, and VIEW. The EDITOR tab is active, showing the file `basics.m` with the following code:

```

1 - clc
2 - clearvars
3 - x = 10;
4 - y = x +7;
5 - w = x + y;
6 -
7 - w

```

The Command Window below shows the output of running the script:

```

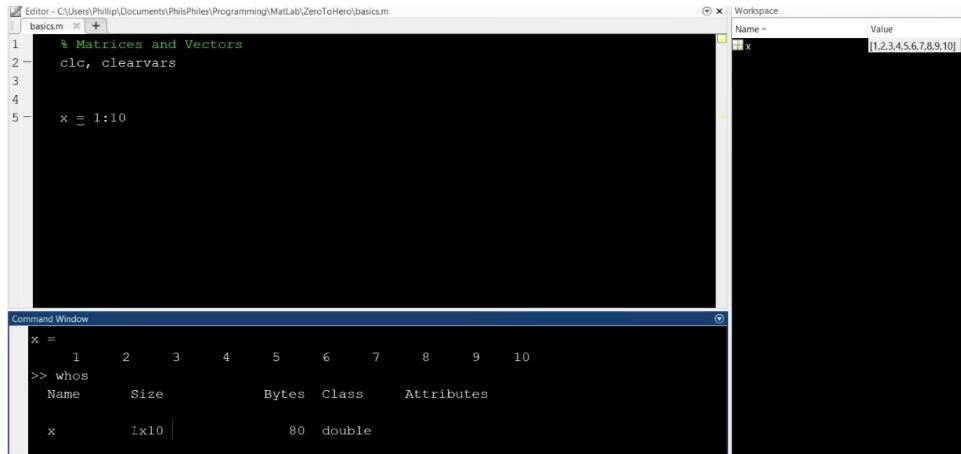
w =
    27
fx>>

```

The Workspace browser on the right lists the variables:

Name	Value
ans	30
w	'New'
x	10
y	20
z	"York"

These scripts are similar to classes and their methods, and control enters to run a script the fast way. Scripts in the toolbar allow users to type several commands and run them all at once. Write "`clc, clearvars`" to clear the command window and variables. Users can create a header for scripts by using %, which allows commenting and helps people when returning to the code. MatLab uses Vectors and Arrays interchangeably. "`x = 1:10`" a colon is the term for through.



The screenshot shows the MATLAB interface with three windows:

- Editor - basics.m**: Displays the MATLAB script code:
 

```

1 % Matrices and Vectors
2 clc, clearvars
3
4
5 x = 1:10

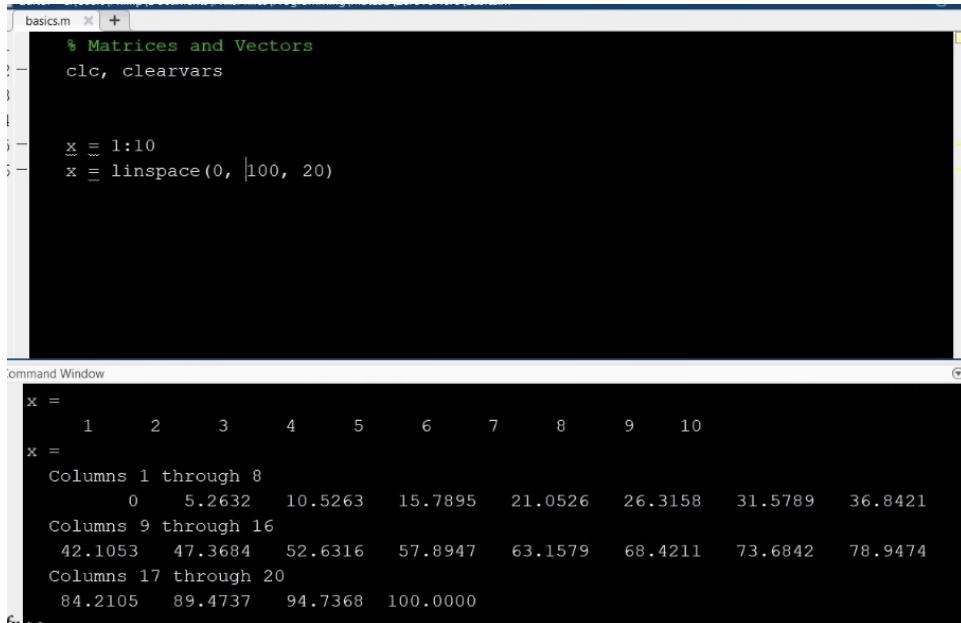
```
- Workspace**: Shows the variable **x** with a value of **[1,2,3,4,5,6,7,8,9,10]**.
- Command Window**: Displays the output of the script:
 

```

x =
     1     2     3     4     5     6     7     8     9     10
>> whos
  Name      Size            Bytes  Class       Attributes
  x            1x10          80  double

```

The picture shows a horizontal array. Users can use transpose, a single apostrophe, to make it into a vertical array. Use " $x = linspace(20, 50)$ " to create 100 values through 20 to 50. Customize the amount by doing " $x = linspace(20, 50, 20)$ ". These are built-in tools that MatLab allows users to use freely. Functions take arguments within the parentheses; to separate each argument, use commas.



The screenshot shows the MATLAB interface with three windows:

- Editor - basics.m**: Displays the MATLAB script code:
 

```

1 % Matrices and Vectors
2 clc, clearvars
3
4
5 x = 1:10
6 x = linspace(0, 100, 20)

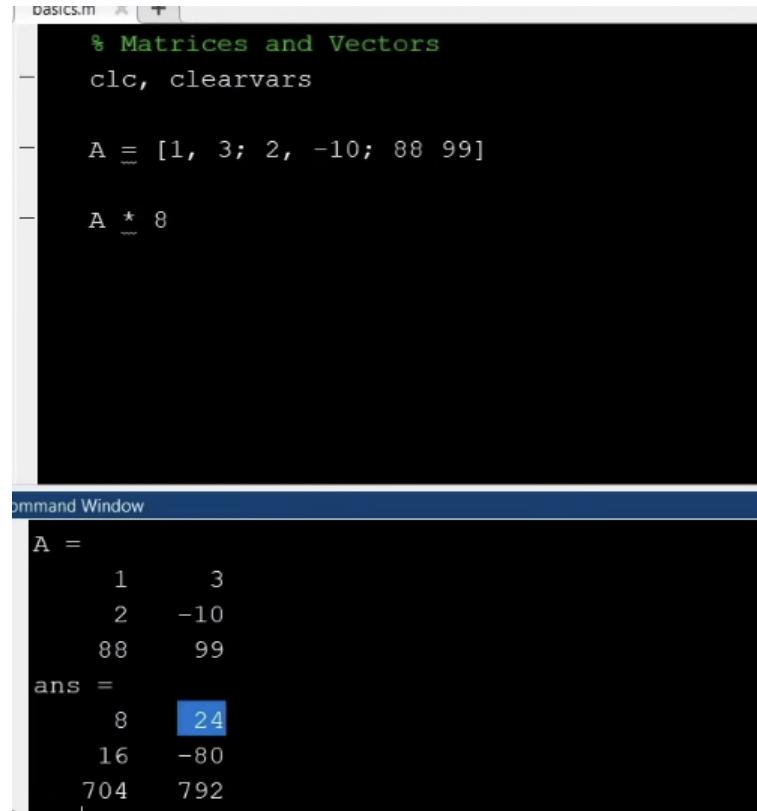
```
- Command Window**: Displays the output of the script:
 

```

x =
     1     2     3     4     5     6     7     8     9     10
x =
  Columns 1 through 8
    0      5.2632   10.5263   15.7895   21.0526   26.3158   31.5789   36.8421
  Columns 9 through 16
    42.1053   47.3684   52.6316   57.8947   63.1579   68.4211   73.6842   78.9474
  Columns 17 through 20
    84.2105   89.4737   94.7368   100.0000

```

MatLab allows users to create arrays manually by doing " $y = [12 50 - 8 - 100]$ ". Spaces are the only separators needed, but users can use commas to keep it similar to other IDEs like Eclipse. " $A = [1 3; 2 - 10]$ " to create two rows and two columns of the array. Semicolons will produce new rows. Assume you want to do linear algebra and follow its rules. Saying " $A + 2$ " or " $A * 8$ " would add/multiply a scalar value to each variable inside the array.



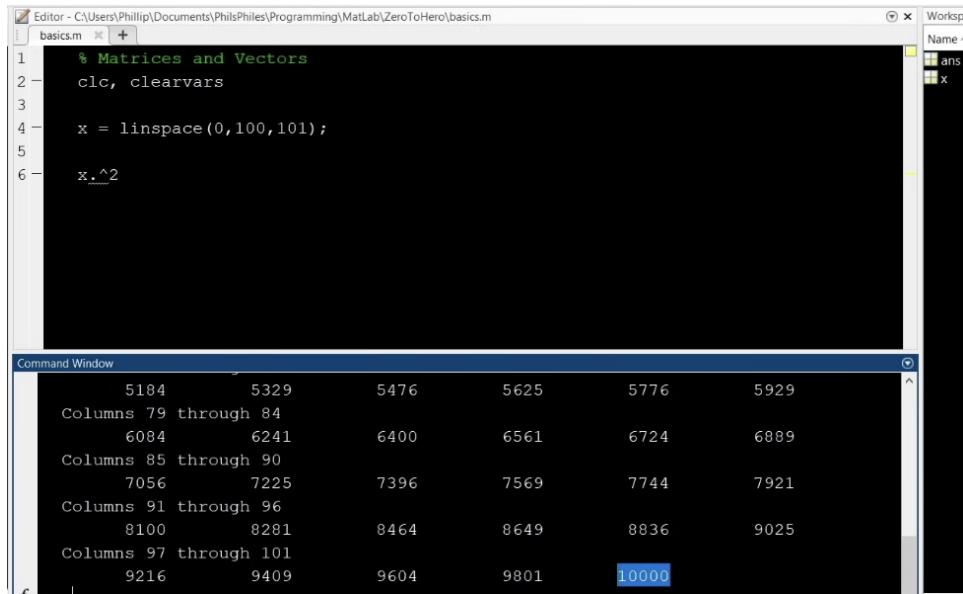
The screenshot shows the MATLAB environment. The top window is the 'Editor' showing the code:

```
basics.m
1 % Matrices and Vectors
2 clc, clearvars
3
4 A = [1, 3; 2, -10; 88 99]
5
6 A * 8
```

The bottom window is the 'Command Window' showing the output:

```
A =
    1     3
    2   -10
   88    99
ans =
    8    24
   16   -80
  704   792
```

To do element-wise operations, use a dot ". ".



The screenshot shows the MATLAB environment. The top window is the 'Editor' showing the code:

```
Editor - C:\Users\Philip\Documents\Phil's Files\Programming\MatLab\ZeroToHero\basics.m
1 % Matrices and Vectors
2 clc, clearvars
3
4 x = linspace(0,100,101);
5
6 x.^2
```

The bottom window is the 'Command Window' showing the output of element-wise squaring:

```
5184      5329      5476      5625      5776      5929
Columns 79 through 84
  6084      6241      6400      6561      6724      6889
Columns 85 through 90
  7056      7225      7396      7569      7744      7921
Columns 91 through 96
  8100      8281      8464      8649      8836      9025
Columns 97 through 101
  9216      9409      9604      9801      10000
```

This picture shows the demonstration of element-wise operations. It will go through every x value and square it. To grab a value from an array, use the index. " $A(2, 2)$ " to get the index at the second row, second column. Users can also add two specific elements at indexes together. Use " $A(1)$ " to get the first index, and unlike other programming IDEs, the index does not start at 0.

The screenshot shows the MATLAB Editor and Command Window. The Editor window displays a script named 'basics.m' with the following code:

```
% Matrices and Vectors
clc, clearvars
A = [1 2 4; 4 5 6];
A(1,1) = 100;
A(2,1:end)
```

The Command Window below shows the output of the script:

```
ans =
    4     5     6
```

This example gives an array that has two rows. To get just the second row, do " $A(2, 1: end)$ ". " $A(2, 1: 2)$ " would get 4 and 5.

The YouTube video goes through an example of working on equations. To generate a specific amount of x values and then calculate the y values correlating to those x values.

- A) What is the maximum value of the following equation on the range  $0 < x < 5$

$$y = -(x - 3)^2 + 10$$

- B) What is the minimum of the function?

- C) At what x-value does the maximum y-value occur?

- D) What is  $y(20.7)$ ?

Math in MatLab is Matrix Algebra, so go back to element-wise operation!

The screenshot shows the MATLAB Editor window with a script named 'basics.m' containing the following code:

```

1 % Example Problem
2 clc, clearvars, close all
3
4
5 x = linspace(0,5);
6 y = -(x-3).^2

```

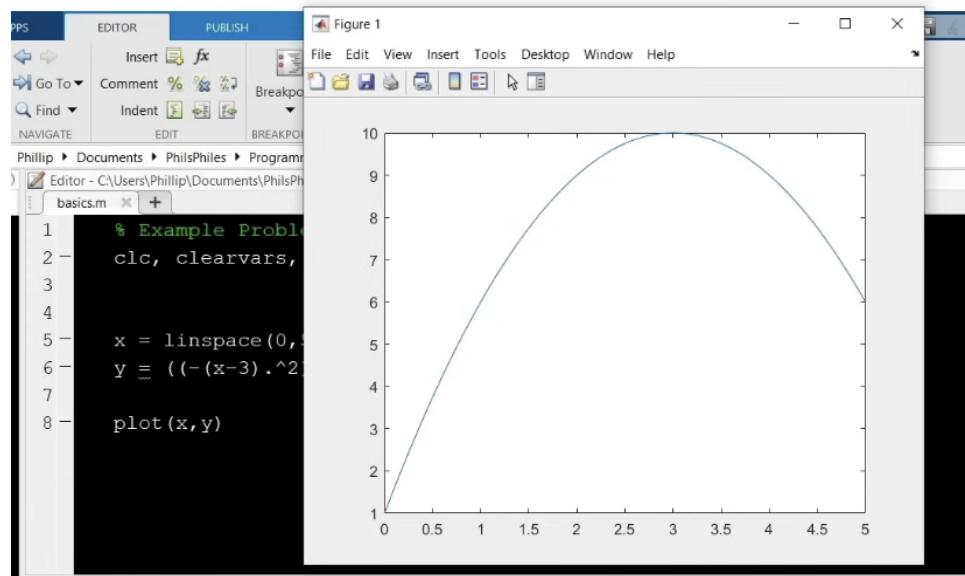
Below the editor is the Command Window, which displays the following error message:

```

Error using ^ (line 51)
Incorrect dimensions for raising a matrix to a power. Check that the matrix is
square and the power is a scalar. To perform elementwise matrix powers, use '.^'.
Error in basics (line 6)
y = -(x-3)^2

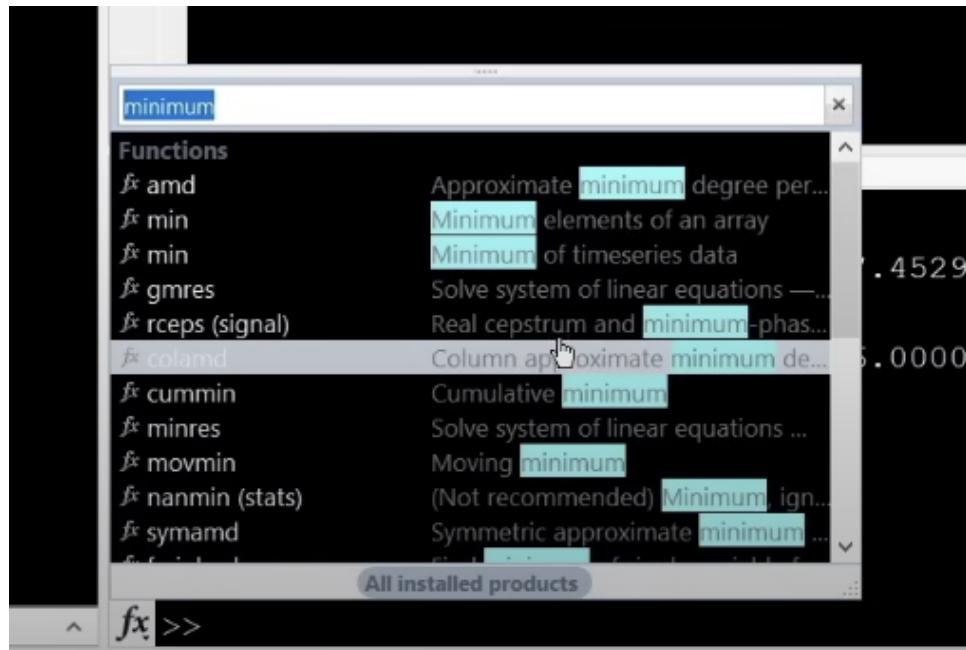
```

Command Window shows the previous command “ $y = -1(x-3)^2$ ”, without the dot. Add ten at the end, and use parentheses to ensure the order of operations. “ $y = (-1(x-3).^2) + 10$ ”. To plot, use the plot command to plot it for a visual of it!



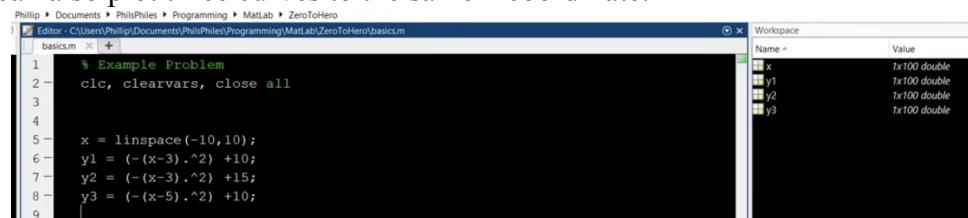
Use "plot(x,y, ' \* ') to show individual calculations of each point. Include a "close all" with clc and clearvars to clear the graph and create an updated one.

MatLab provides ways to get help for searching for functions at the bottom of the Command Window.



Do “`help “function name”`” in the command window to see documentation on any function that the user wants “`doc “function name”`” gets the documentation version but on the web. The Internet is the best resource, as the YouTube video states.

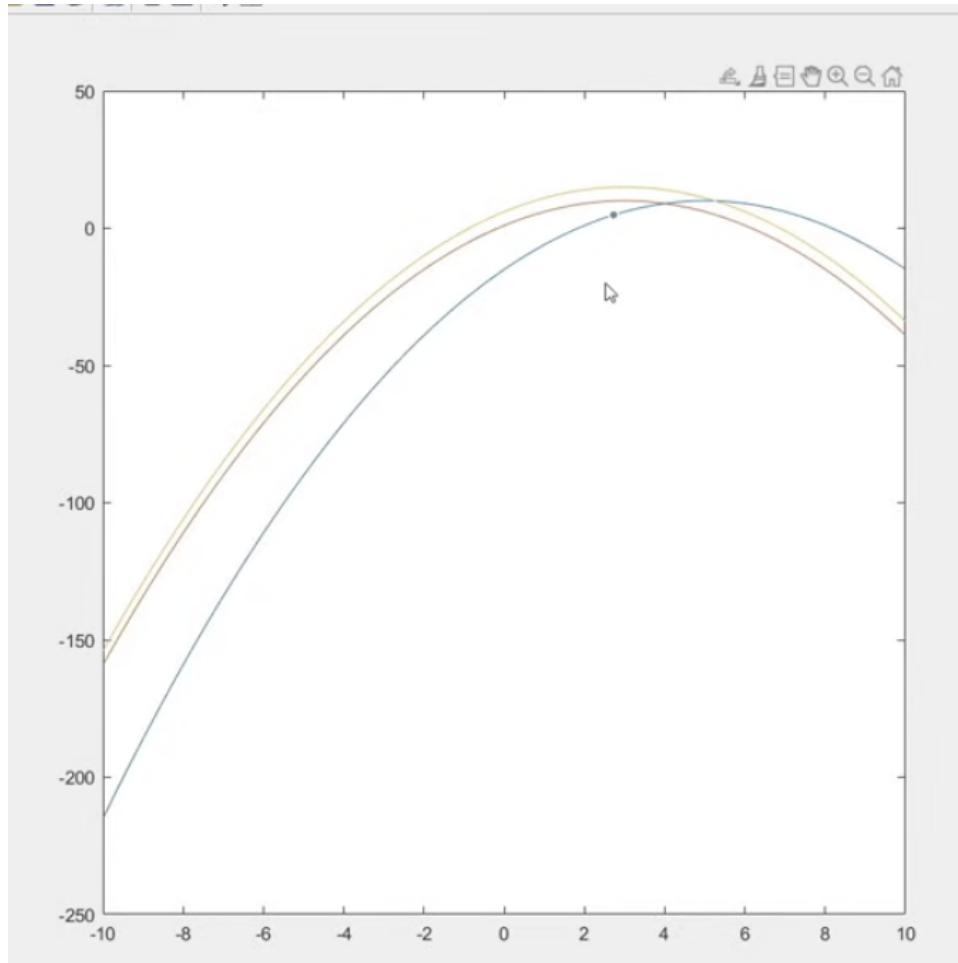
Users can also plot three curves to the same x coordinate.



Use the `hold on` command to plot other curves on the same graph.

A screenshot of the MATLAB Command Window. The user has entered the following commands:

```
>> hold on
>> plot(x,y1)
>> hold on
>> plot(x,y2)
fx >>
```



Now, to customize the plot! The picture below shows some labeling and holding the plot in figure 1 in the video.

```
figure(1)
plot(x,y1,'*')
xlabel('x'), ylabel('y'), title('Y vs. X - Problem A')
grid on
```

Here are some extra features using the help command, and add the command within the '\*' replacing \*. To fill the symbols in, add 'MarketFaceColor', 'm', 'MarketSize', 10 after '\*'. There is also MarketEdgeColor and other special customizations. To add lines, put – before the \* inside '\*'.

b	blue	.	point	-	solid
g	green	o	circle	:	dotted
r	red	x	x-mark	-.	dashdot
c	cyan	+	plus	--	dashed
m	magenta	*	star	(none)	no line
y	yellow	s	square		
k	black	d	diamond		
w	white	v	triangle (down)		
		^	triangle (up)		
		<	triangle (left)		
		>	triangle (right)		
		p	pentagram		
		h	hexagram		

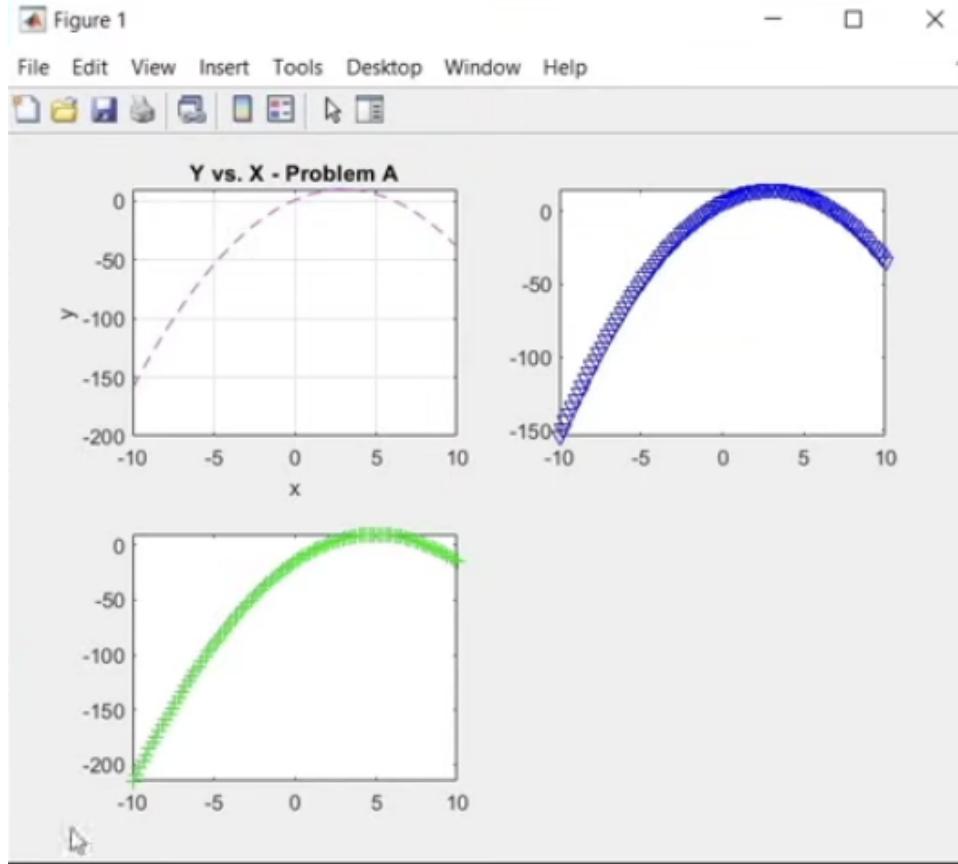
Adding a legend to show users what the graph is about.

```
legend('Y1','Y2','Y3')
```

It goes in order. The first thing plotted goes with the first thing named. MatLab allows users to copy the figure or save it in the toolbar.

xLim([0, 2]) the limits if a user wanted to change the x-axis, and yLim([0, 100]) to change the y-axis.

The subplot creates matrices of graphs, subplot(2,2,1), subplot (2,2,2), and subplot (2,2,3). Users need to add other labels separately. A one-by-three would be better for a side by side.



For a plotter, salter, and smoother created in MatLab, the YouTube tutorial at this point should be adequate to get the work done. The YouTube timestamp to continue into logic, loops, and other functions starts at 1:05:20. Link via reference page.

### Creating and Using the Plotter

```

Editor - Plotter.m
Plotter.m + Variables - x2

1 % Plotter, will plot the quadratic equation
2 % Dante Anzalone's Plotter Script
3 clc, clearvars, close all
4 y = linspace(10,50); % Generates 100 y-coordinates range from 10 through 50
5 a = 5; % Shape and direction of a parabola (negative for a downward
6 % parabola and positive for an upward parabola)
7 b = .2; % Position of the vertex and the slope of the parabola
8 c = -1; % y-intercept of the parabola
9
10 z = (b * b) - (4*a*c); % b^2 - 4ac
11 z = z + (4 * a * c) + ((4 * a).*y); % b^2 - 4ac + 4ay for all y values
12 z = sqrt(z); % squares z
13 x = (-b + z) / (2 * a); % finds value of x one, finishing quadratic equation
14 x2 = (-b - z) / (2 * a); % finds value of x2, finishing quadratic equation 2
15
16 % Creating the plot!
17
18 figure(1)
19 plot(x, y, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 4)
20 xlabel('x-Coordinates'), ylabel('y-Coordinates')
21 title(['MatLab Plotter - Quadratic Equation'])
22 grid on
23 hold on
24 plot(x2, y, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 4)

```

Figure 7.1: Code for the Plotter in MatLab

Using the MatLab tutorial here, compared to the code from part I, fewer lines of command code.

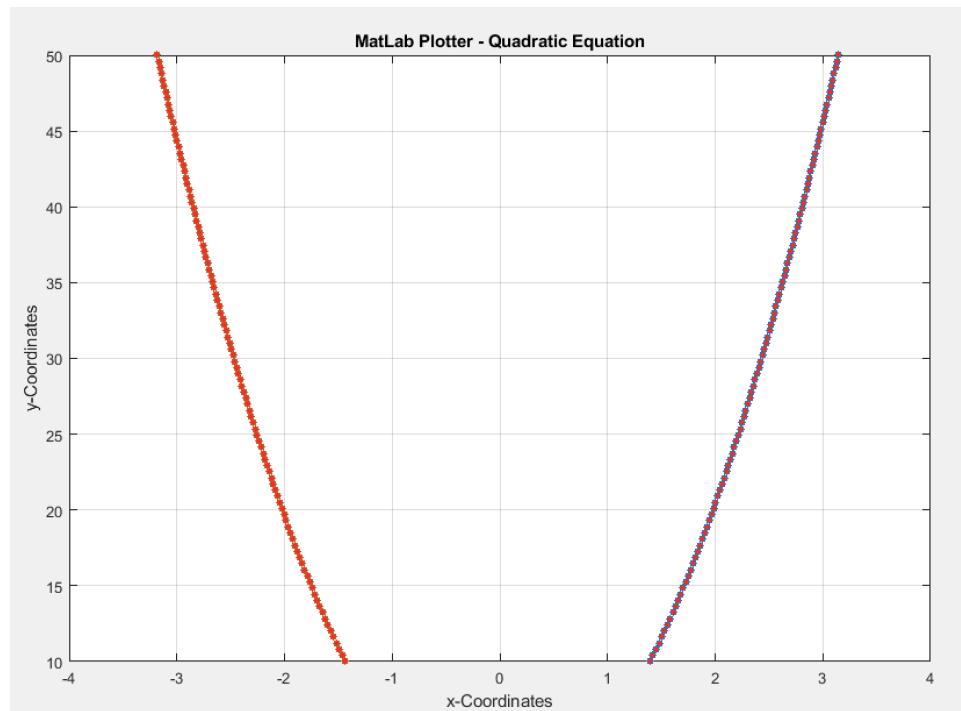
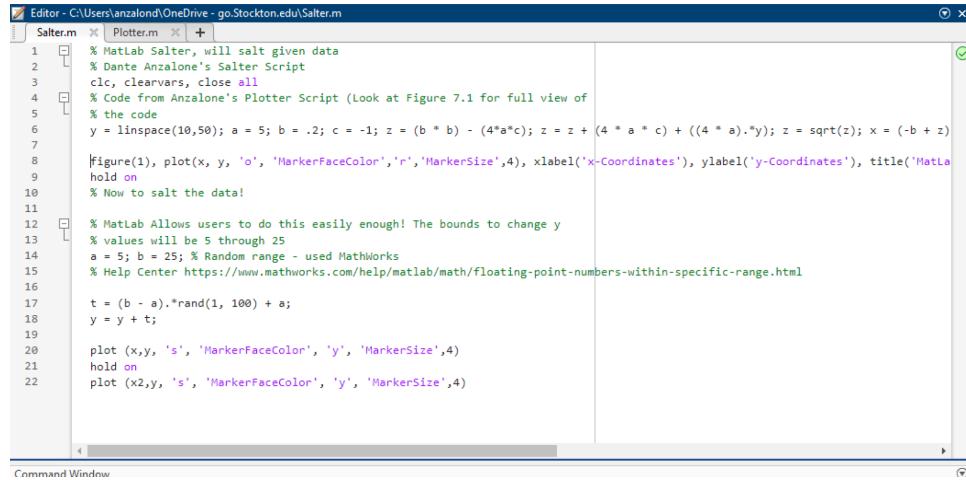


Figure 7.2: MatLab Line Plot

The left red line is the negative x-coordinates of the parabola, while the purple side is the positive x-values. Looks similar to Figure 1.2's parabola.

### Creating and Using the Salter

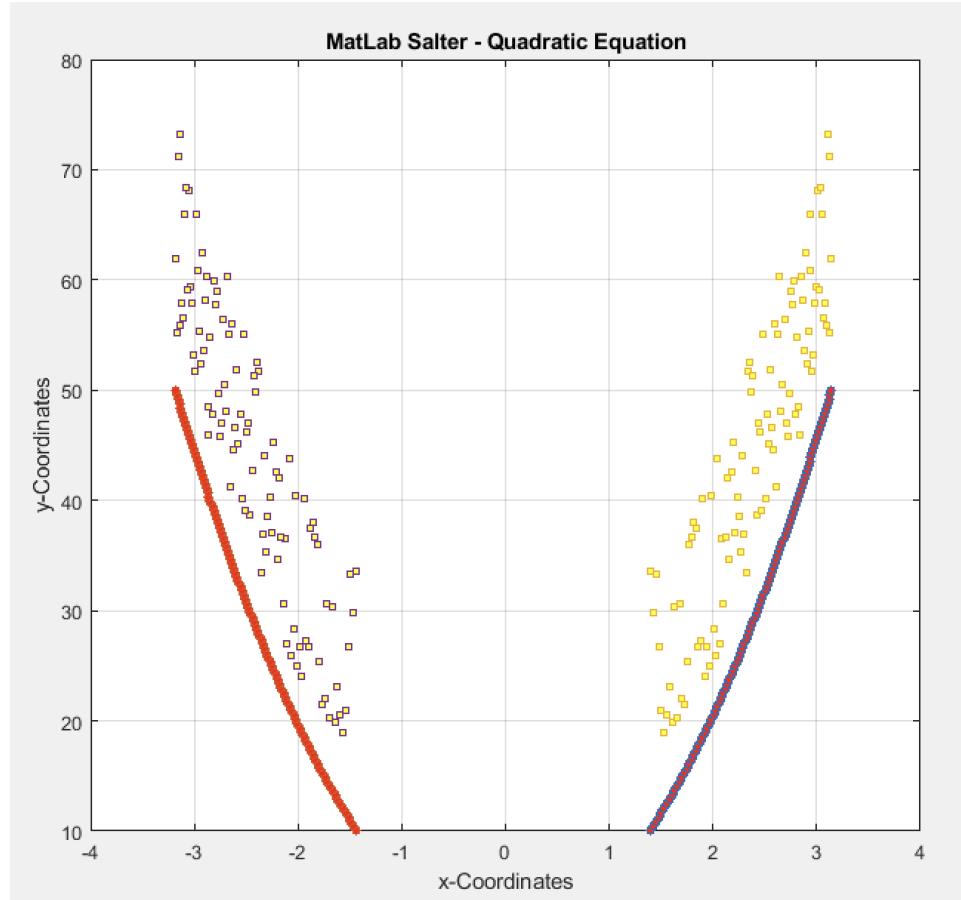


The screenshot shows the MATLAB Editor window titled "Editor - C:\Users\anzalond\OneDrive - go.Stockton.edu\Salter.m". The code in the editor is as follows:

```
Editor - C:\Users\anzalond\OneDrive - go.Stockton.edu\Salter.m
Salter.m  Plotter.m  +
1 % Matlab Salter, will salt given data
2 % Dante Anzalone's Salter Script
3 clc, clearvars, close all
4 % Code from Anzalone's Plotter Script (Look at Figure 7.1 for full view of
5 % the code
6 y = linspace(10,50); a = 5; b = .2; c = -1; z = (b * b) - (4*a*c); z = z + (4 * a * c) + ((4 * a).*y); z = sqrt(z); x = (-b + z)
7 figure(1), plot(x, y, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 4), xlabel('x-Coordinates'), ylabel('y-Coordinates'), title('MatLa
8 hold on
9 % Now to salt the data!
10
11 % Matlab Allows users to do this easily enough! The bounds to change y
12 % values will be 5 through 25
13 a = 5; b = 25; % Random range - used MathWorks
14 % Help Center https://www.mathworks.com/help/matlab/math/floating-point-numbers-within-specific-range.html
15
16 t = (b - a).*rand(1, 100) + a;
17 y = y + t;
18
19 plot (x,y, 's', 'MarkerFaceColor', 'y', 'MarkerSize',4)
20 hold on
21 plot (x2,y, 's', 'MarkerFaceColor', 'y', 'MarkerSize',4)
```

Figure 7.3: Code for the Salter in MatLab

Reused the Plotter commands, pushed into longer lines, with the new commands creating the random ranges, values between 5 through 25, to be added to the y array.



*Figure 7.4: MatLab Line Plot - Salted*

It is easier to compare with Figure 6.1's salted parabola as it has less data points. The y-values salted!

#### Creating and Using the Smoother

*Figure 7.5: Code for the Smoother in MatLab*

```

Editor - C:\Users\anzalond\OneDrive - go.Stockton.edu\Smoothen.m
Salter.m | Plotter.m | Smoothen.m | +
1 % MatLab Smoother, will smooth data
2 % Dante Anzalone's Smoother Script (Figure 7.1 for below's Original Plotter Data
3 clc, clearvars, close all
4
5 % Initial Plotter Commands
6 y = linspace(10,50); a = .5; b = .2; c = -1; z = (b * b) - (4*a*c); z = z + (4 * a * y) + ((4 * a).*y); z = sqrt(z); x = (-b + z) /
7 % Salter Commands
8 figure(1), a = .5; b = .25; t = (b - a).*rand(1, 100) + a; y = y + t; plot (x,y, 's', 'MarkerFaceColor', 'r', 'MarkerSize',4), hold
9
10 % Now to Smooth the data!
11 % MatLab provides a smoothing method of its own, cutting back the work and
12 % lines needed!
13 % Help Center https://www.mathworks.com/help/matlab/ref/smoothdata.html
14
15 y2 = smoothdata(y, "gaussian", 10)
16
17 plot (x,y2, "*", 'MarkerFaceColor', 'k', 'MarkerSize',5), hold on, plot (x2,y2, "*", 'MarkerFaceColor', 'k', 'MarkerSize',5)

```

MatLab seems to be efficient! Allowing users access to methods to easily create data, plot it, salt it, and smooth it.

Figure 7.6: MatLab Line Plot - Smoothed

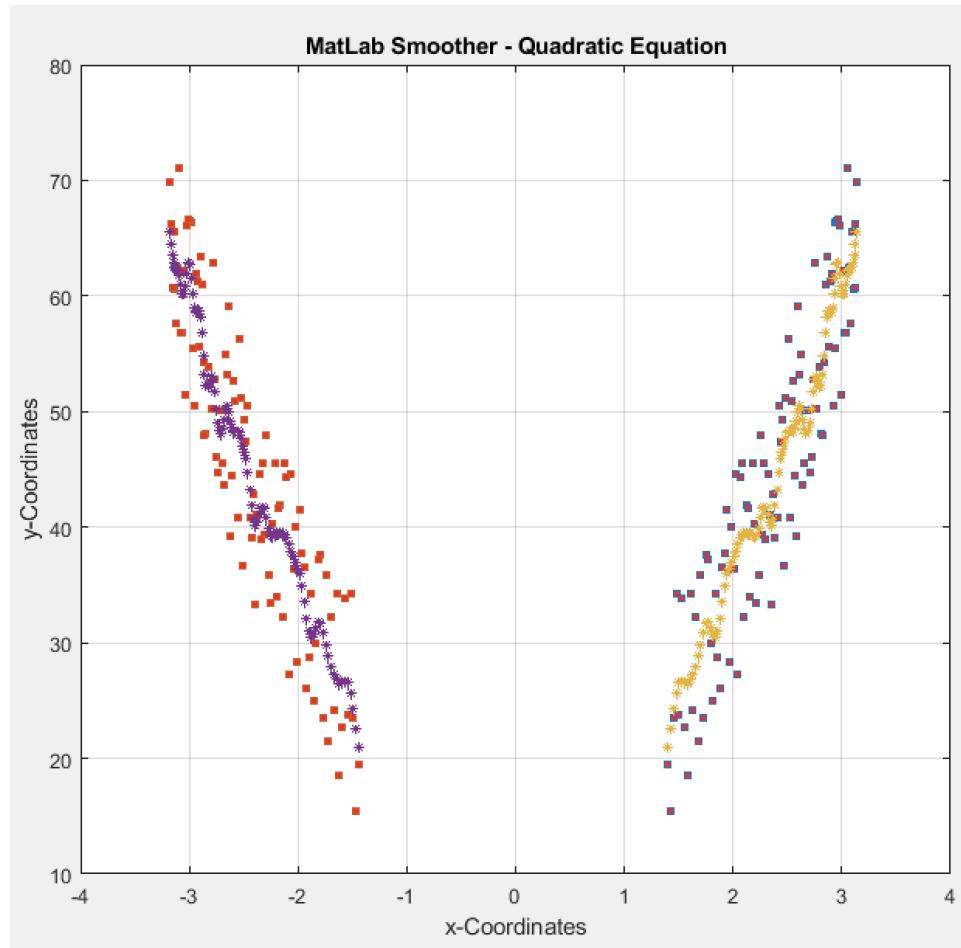


Figure 3.5's smoothed data smoothes it into a flat line, which should not occur. MatLab's smooth method shows the values closer to its initial parabola, as seen in Figure 7.2.

### PART III - Plotter, Salter, and Smoother Shortcut Using Jar Files

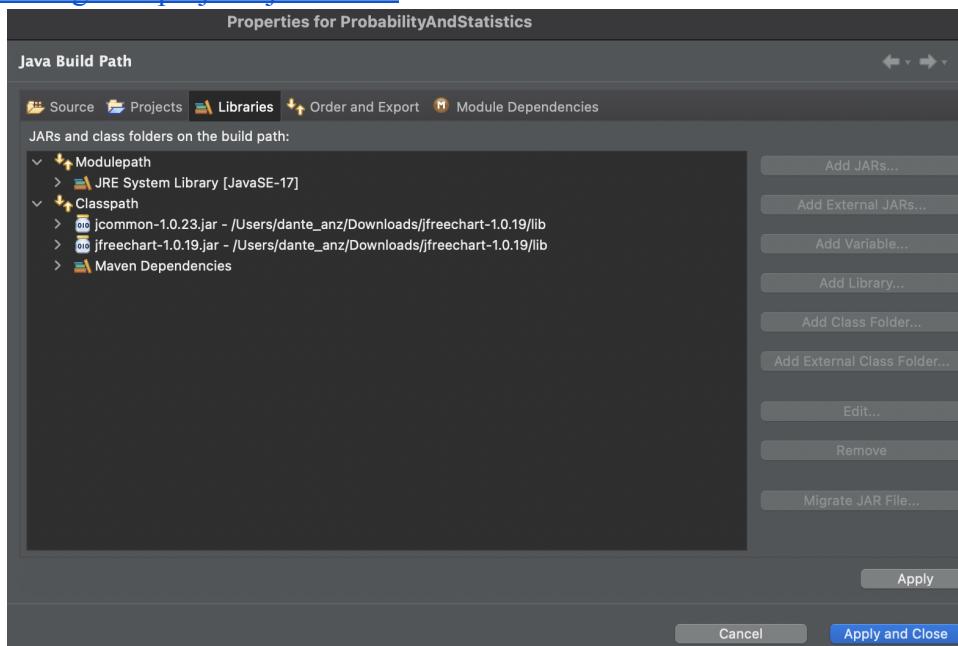
#### Introduction and Process to Utilizing JFreeChart and Apache Library

A skill set for programmers is having the ability to use other functional libraries to shortcut tasks (Hoy). In part three of the paper, grabbing a copy of a jar file containing a different

repository's code and importing such software, JFreeChart, and Apache Stats Library, will allow access to the Apache Math Library and JFreeChart. These libraries let users plot, salt, and smooth data without needing personal classes. JFreeChart will create graphs. With the help of the Apache Math Library, users can manipulate the data by smoothing and salting y-coordinates.

### Process to Utilizing JFreeChart

Through the help of JavatPoint.com, via articles “Jfreechart Installation - Javatpoint” and “JFreeChart Scatter Chart - Javatpoint,” users can easily add JFreeChart and JCommon, needed to work with JFreeChart, to their IDEs. For Eclipse, users have to add both these jar files to the ClassPath in their projects. The location of the jar files can be found via this link: <https://sourceforge.net/projects/jfreechart/>.



Users will have to add the jar files of both jcommon-1.0.23.jar and jfreechart-1.0.19.jar to → Properties for “Project Name” → Java Build Path → Libraries → Click Classpath → Click the box that should be blue on the left hand side, called “Add External JARS...” and find the jar files downloaded via SourceForge.

After correctly following the first article’s installation process, the article “JFreeChart Scatter Chart - Javatpoint,” helped to provide the code to allow the transfer of data points from the earlier CSV file, personalPlotterXY.csv, and create a Scatter Plot with the coordinates!

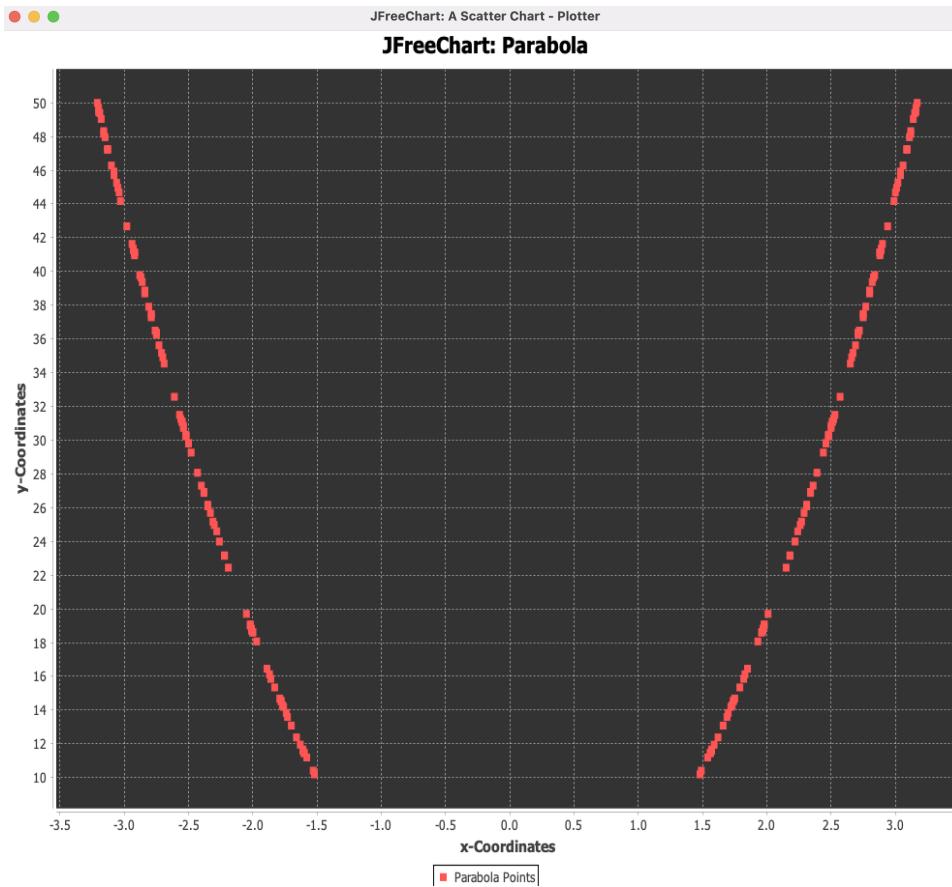


Figure 8.1: JFreeChart Scatter Plot

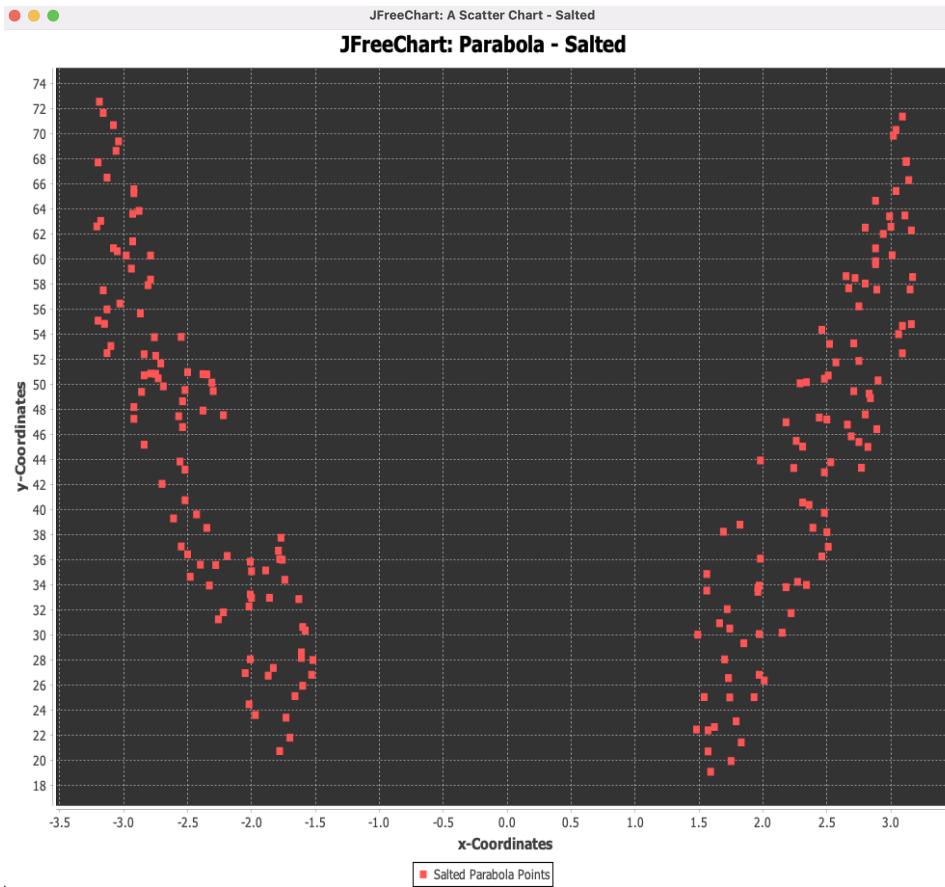


Figure 8.2: JFreeChart Scatter Plot - Salted

In this salter class, users must use a similar method to the personal salter. Due to Apache lacking a salting method, users need to reuse their salter code. Apache does have a method for hashing a password and applying salt to it, but this is not useful for coordinates. However, with JFreeChart, use the previous Plotter class for JFreeChart to graph Figure 8.2. Still short, cutting some time! Now, Apache does have a smoothing method, which will help short-cutting the process even more.

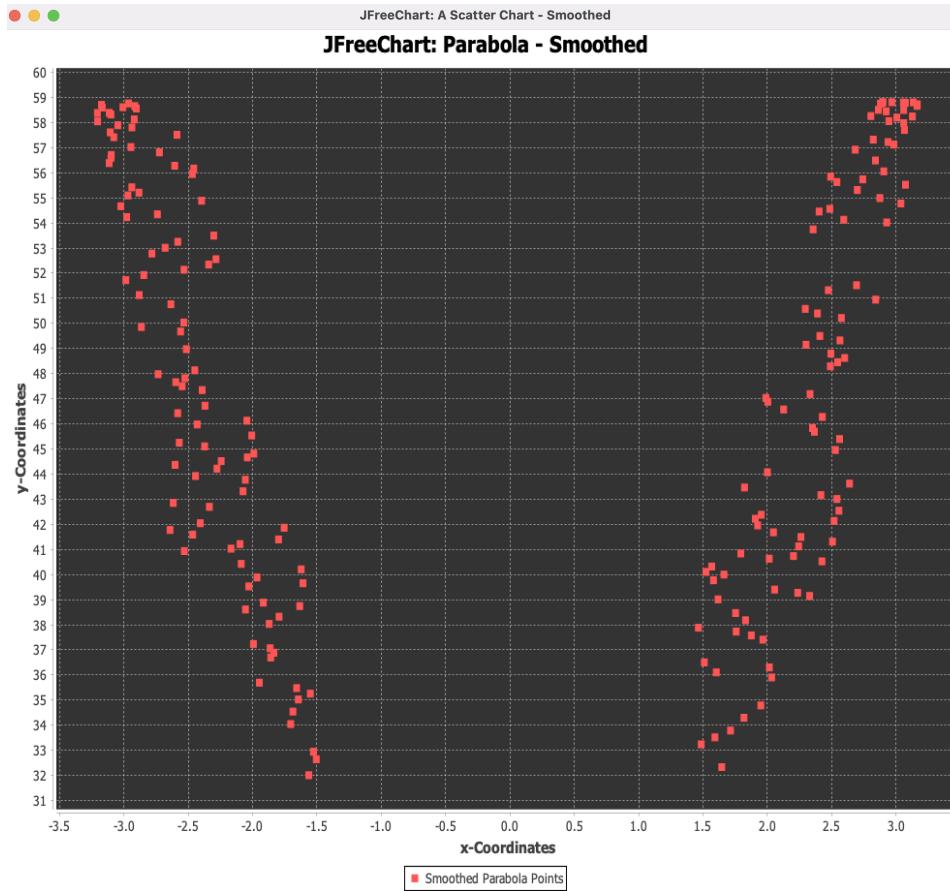


Figure 8.3: JFreeChart Scatter Plot - Smoothed

Smoothes the salted graph in Figure 8.2 and follows the smoother code from personal smoother. It seems to follow the same fault, smoothing the points into each other at the top. The JFreeChart works and short cuts the graphing portion.

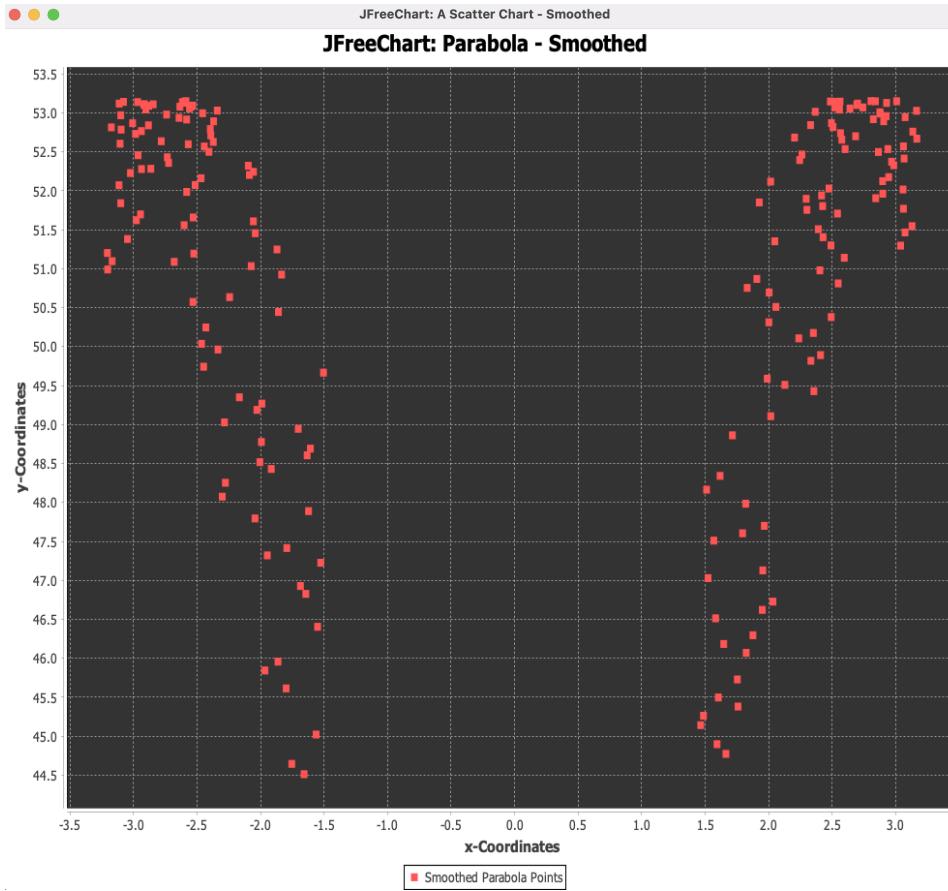


Figure 8.4: Smoothed Attempt With Apache

Attempts to smooth the salted graph in Figure 8.2 and follows the smoother code provided via apache, running the code, “ $yA[i] = \text{StatUtils.mean}(yA)$ ” inside a for loop. It seems to follow the same fault, smoothing the points into each other at the top. The JFreeChart works and short cuts the graphing portion.

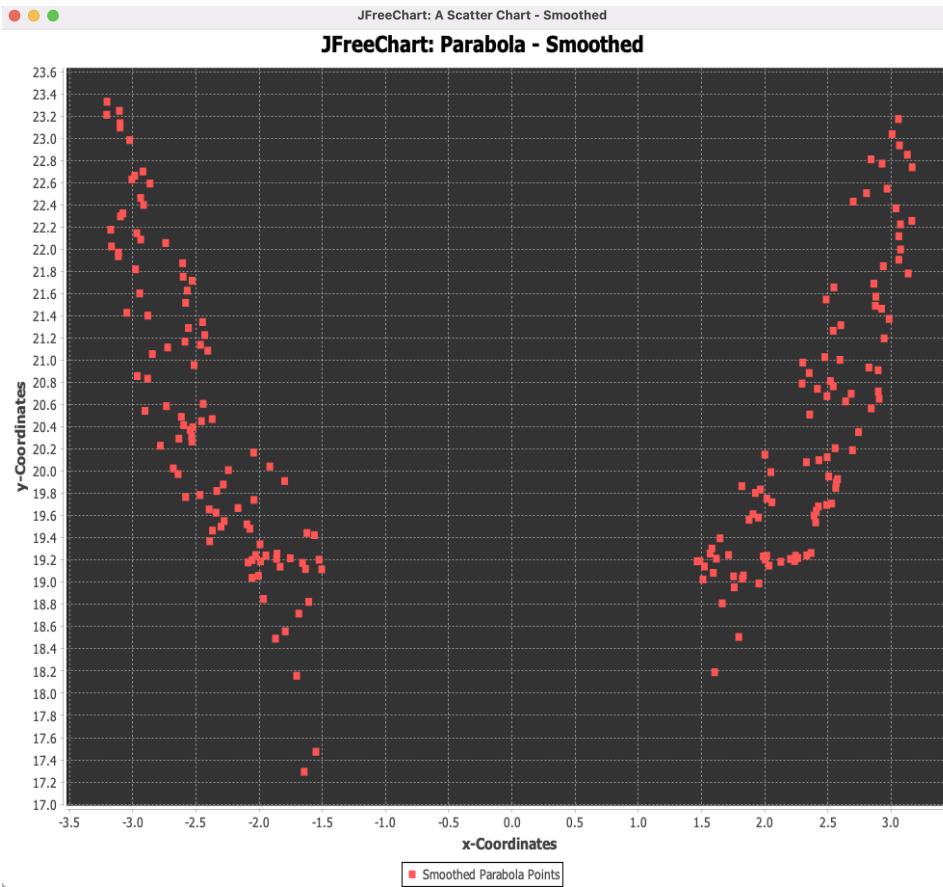


Figure 8.5: Second Smoothed Attempt With Apache

Second attempt to smooth the salted graph in Figure 8.2 and follows the smoother code provided via apache, running the code, “average = StatUtils.mean(yA, begin, length - begin + 1);” and “yA[i] = average;” inside a for loop. It seems to work better than the previous attempts, smoothing the points into each other throughout the graph. The JFreeChart works and short cuts the graphing portion.

PART IV - Updated Formula Sheet

Standard Deviation $\sigma = \sqrt{\sigma^2}$	Multiplicative Law of Probability
Variance $\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2$	$P(A \cap B) = P(A)P(B A) = P(B)P(A B)$
Permutation $nPr = \frac{n!}{(n-r)!}$	General Addition Rule
Multinomial Coefficients $N = \frac{n!}{n_1!n_2!\dots n_k!}$	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Combination $\binom{n}{r} = \frac{n!}{r!(n-r)!}$	Bayes Theorem
Conditional Probability $P(A B) = \frac{P(A \cap B)}{P(B)}$	$P(A B) = \frac{P(A B)P(B)}{P(A)}$ w/ $P(A) > 0$ and $P(B) > 0$
Independent $P(A B) = P(A)$	Theorem of Total Probability if $0 < P(B) < 1$
$P(B A) = P(B)$	$P(B A) = \frac{P(A B)P(B)}{P(A B)P(B) + P(A B)P(B)}$
$P(A \cap B) = P(A)P(B)$	Discrete Random Variables Expectations and Variance
Probability Mass Function $P(X = x)$	$E[Y] = \sum_{y \in Y} yP(y)$   $V[Y] = E[(Y - \mu)^2]$   $\mu = E[Y]$
Binomial Distribution $P_x = \binom{n}{x} p^x q^{n-x}$	Standard Deviation of Y is $\sqrt{V[Y]}$
Geometric Distribution	Extra Formulas (Derived Geometric Distribution)
$P(X = x) = q^{x-1} p$	$P(X \leq n) = 1 - (1 - p)^n$
$E(Y) = \frac{1}{p}$ and $V(Y) = \frac{(1-p)}{p^2}$	$P(X < n) = 1 - (1 - p)^{n-1}$
Hypergeometric Distribution	$P(X \geq n) = (1 - p)^{n-1}$
$n_A = \binom{r}{y} \binom{N-r}{n-y}$	$P(X > n) = (1 - p)^n$
$\mu = E(Y) = \frac{nr}{N}$ and	Negative Binomial Probability Distribution
$\sigma^2 = V(Y) = n(\frac{r}{N})(\frac{N-r}{N})(\frac{N-n}{N-1})$	$p(y) = \binom{y-1}{r-1} p^r q^{y-r}$ and $\mu = \frac{r}{p}$ and $\sigma^2 = \frac{r(1-p)}{p^2}$
Poisson Distribution	Tchebysheff's Theorem $1 - \frac{1}{k^2}$
$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$ and $\mu = \lambda$ and $\sigma^2 = \lambda$	$k = \frac{\text{the "within" number}}{\text{the standard deviation}}$ and $k$ must be $> 1$
Cumulative Distribution Function	Distribution Function for a Continuous Random Variable
$F(y) = P(Y \leq y)$ for $-\infty < y < \infty$	$f(y) = \frac{dF(y)}{dy} = F'(y)$
Theorem if Random Variable has Density	Expected Value of a Continuous Random Variable Y
Function $f(y)$ and $a < b$	$E(Y) = \int_{-\infty}^{\infty} yf(y)dy$ , if integral exists
$P(a \leq Y \leq b) = \int_a^b f(y)dy$	Continuous Uniform Probability Distribution
If $g(Y)$ is a function of Y	$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 \leq y \leq \theta_2 \\ 0, & \text{elsewhere} \end{cases}$

$E[g(Y)] = \int_{-\infty}^{\infty} g(y)f(y)dy$	Random Variable Uniformly Distributed
Normal Probability Distribution if $\sigma > 0$	$ \mu = E(Y) = \frac{\theta_1 + \theta_2}{2}$ and $\sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$
$f(y) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(y-\mu)^2/(2\sigma^2)}$ , $-\infty < y < \infty$	Gamma Probability Distribution
$E(Y) = \mu$ and $V(Y) = \sigma^2$	$ f(y) = \left\{ \begin{array}{ll} \frac{y^{\alpha-1} e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, & 0 \leq y < \infty \\ 0, & \text{elsewhere} \end{array} \right.$
Beta Probability Distribution	$  \text{where } \Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y} dy$
$f(y) = \left\{ \begin{array}{ll} \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha, \beta)}, & 0 \leq y < 1 \text{ and } 0 \\ 0, & \text{elsewhere, where} \end{array} \right.$	$ \mu = E(Y) = \alpha\beta$ and $\sigma^2 = V(Y) = \alpha\beta^2$
$B(\alpha, \beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1} dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$	$  \text{Exponential Distribution}$
$\mu = E(Y) = \frac{\alpha}{\alpha+\beta}$ and	$ \mu = E(Y) = \beta$ and $\sigma^2 = V(Y) = \beta^2$
$\sigma^2 = V(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$  \text{Joint (bivariate) probability function}$
Continuous Random Variables with joint	$  p(x, y) = P(X = x, Y = y), -\infty < x < \infty, -\infty < y < \infty$
$F(x, y) = \int_{-\infty}^x \int_{-\infty}^y f(t_1, t_2) dt_2 dt_1$	For any random variables X and Y, it is
Marginal Probability Functions	$  F(x, y) = P(X \leq x, Y \leq y), -\infty < x < \infty, -\infty < y < \infty$
$p_1(x) = \sum_{\text{all } y} p(x, y), p_2(y) = \sum_{\text{all } x} p(x, y)$	Marginal Density Functions
Conditional Discrete Probability Function	$  f_1(x) = \int_{-\infty}^{\infty} f(x, y) dy$ and $f_2(y) = \int_{-\infty}^{\infty} f(x, y) dx$
$p(x   y) = \frac{p(x, y)}{p(y)}$	Conditional Density
Conditional Distribution Function	$  f(x   y) = \frac{f(x, y)}{f(y)}$ or $f(y   x) = \frac{f(x, y)}{f(x)}$
$F(x   y) = P(X \leq x   Y = y)$	Conditional Density Function
Discrete Random Variables with joint	$  F(x y)f(y) = \int_{-\infty}^x f(t_1, y) dt_1$ or $F(x y) = \int_{-\infty}^x \frac{f(t_1, y)}{f(y)} dt_1$
and marginal probability functions	Independent Random Variables if and only if
independence if any only if	$F(x, y) = F(x)F(y)$ for every pair of real numbers
$p(x, y) = p(x)p(y)$	Continuous Random Variables independent if and only if
	$f(x, y) = f(x)f(y)$

## PART V - Overview of Normal, Gamma, and Beta Distribution

Note that this section relies heavily on “Mathematical Statistics with Applications, 7th Edition,” by Wackerly, for the statistics of Normal, Gamma, and Beta Distribution alongside other resources explaining each distribution’s purpose and applications.

Normal Probability Distribution, like Gamma and Beta Distributions, focuses on continuous variables. Normal Probability Distribution widely used in the continuous probability distribution. It has a bell shape. Here is its definition: A random variable  $Y$  has a normal probability distribution if and only if, for  $\sigma > 0$  and  $-\infty < \mu < \infty$ , the density function of  $Y$  is  $f(y) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(y-\mu)^2/(2\sigma^2)}$ ,  $-\infty < y < \infty$ . The normal density function contains two parameters, and which is average and standard deviation. For the theorem, if  $Y$  is a normally distributed random variable with parameters  $\mu$  and  $\sigma$ , then  $E(Y) = \mu$  and  $V(Y) = \sigma^2$ , the expected and variance. The theorem will imply that the parameter of the average symbol locates the distribution’s center and that the standard deviation measures its spread. The book supplies a graph of the function, an upside down U slope, which is like a hill starting from the negative x-axis and gradually slopes to a peak and then slopes down like a rollercoaster. It can be found on page 179 of the textbook (Wackerly et al. 30 through 290). To find the areas under the function, which corresponds to  $P(a \leq Y \leq b)$ , will require an integral evaluation of the

following:  $\int_a^b \frac{1}{\sigma\sqrt{2\pi}}e^{-(y-\mu)^2/(2\sigma^2)} dy$ . Evaluation requires numerical integration techniques due to

lacking a closed-form expression. There are infinitely more normal distributions due to  $\mu$  taking a finite value and  $\sigma$  any positive finite value. Using software can help provide correct answers to the number of decimal places of normally distributed random variables. The text also states that the normal density function is symmetric around the value  $\mu$ , meaning tabulated areas on only one side of the mean. These areas are on the right of points  $z$ , the distance from the mean, and measured in standard deviations. By transforming the normal random variable  $Y$  to a standard normal random variable  $Z$ , using  $Z = \frac{Y-\mu}{\sigma}$ , can allow the computation of probabilities. The  $Z$  will locate a point measured from the mean of a normal random variable. However, the mean value of  $Z$  equals 0, and its standard deviation equals one. To sum up this distribution, its purpose is to find the probability of observations in a distribution falling above or below a given value, the probability that a sample mean significantly differs from a known population mean, and to compare scores on different distributions with different means and standard deviations (Bhandari). According to LinkedIn, it is widely used in business for quality control by measuring the mean and standard deviation of the process, setting control limits to ensure that the process stays within acceptable bounds, sales forecasting, financial analysis, customer behavior analysis, and employee performance evaluation. It is a powerful tool for business analysis and decision-making (“The Normal Distribution and Its Applications in Quality Control and Process Improvement”).

Gamma Probability Distribution is for nonnegative random variables that yield skewed distributions to the right. Most of their locations are under the density function near the origin, and the function drops as  $y$  increases. The length of time between malfunctions for aircraft engines has a skewed frequency distribution, and so do the arrivals at a supermarket checkout queue. Another example in the text is the time to complete a maintenance checkup for an automobile. The populations associated with these random variables frequently possess density

functions modeled adequately by a gamma density function (Wackerly et al. 30 through 290). The definition for this distribution is the following: A random variable  $Y$  is said to have a gamma distribution with parameters  $\alpha > 0$  and  $\beta > 0$  if and only if the density function of  $Y$  is

$$f(y) = \left\{ \frac{y^{\alpha-1} e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, 0 \leq y < \infty \text{ and } 0 \text{ elsewhere, where } \Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy. \right. \text{The quantity } \Gamma(\alpha) \text{ is}$$

known as the gamma function and direct integration verifies that  $\Gamma(1) = 1$ . Page 185 of the text provides graphs of the Gamma density function and skewed probability density function.  $\alpha$  can be called the shape parameter associated with a gamma distribution while the parameter  $\beta$  is the scale parameter because multiplying a gamma-distributed random variable by a positive constant produces a random variable with a same value of  $\alpha$ . It is also impossible to get the areas under the gamma density function by direct integration, except if  $\alpha = 1$ . Using statistical software can easily compute probabilities associated with this distribution of random variables. The theorem of gamma is the following: If  $Y$  has a gamma distribution with parameters  $\alpha$  and  $\beta$ , then

$\mu = E(Y) = \alpha\beta$  and  $\sigma^2 = V(Y) = \alpha\beta^2$ . Page 187 in the text provides a proof for this, but for the course, let's assume it is correct. Gamma-distributed random variables have two special cases, with the following definition for one case: Let  $v$  be a positive integer. A random variable  $Y$  is said to have a chi-square distribution with  $v$  degrees of freedom if and only if  $Y$  is a gamma-distributed random variable with parameters  $\alpha = v/2$  and  $\beta = 2$ . The random variable with the chi-square distributed is also called a chi-square ( $X^2$ ) random variable, often in statistical theory. Calling  $v$  the degrees of freedom of the  $X^2$  distribution is one of the major ways of generating a random variable. It also has a theorem too, if  $Y$  is a chi-square random variable with  $v$  degrees of freedom, then  $\mu = E(Y) = v$  and  $\sigma^2 = V(Y) = 2v$ . The exponential density function is when the gamma density function  $\alpha = 1$ . The second case definition is as follows: a random variable  $Y$  is said to have an exponential distribution with parameter  $\beta > 0$  if and only if the density function of  $Y$  is  $f(y) = \left\{ \frac{1}{\beta} e^{-y/\beta}, 0 \leq y < \infty, \text{ and elsewhere } 0. \right.$  This is useful for modeling the length of life of electronic components. Lastly, the theorem for this last special case is as follows: If  $Y$  is an exponential random variable with parameter  $\beta$ , then  $\mu = E(Y) = \beta$  and  $\sigma^2 = V(Y) = \beta^2$ . To sum up this distribution, its purpose is to predict the wait time until future events occur (Team). Its applications are within engineering models. "For example, time to failure of equipment and load levels for telecommunication services, meteorology rainfall, and business insurance claims and loan defaults, where the variables are always positive and the results are skewed" ("Exam 2020-21 - Get Direct Link to Download Mains Admit Card").

Finally, is the Beta Probability Distribution, and the beta density function is a two-parameter density function defined over the closed interval  $0 \leq y \leq 1$ . It is used as a model for proportions, such as impurities in a chemical product or the proportion of time it takes a machine to be repaired. The definition is as follows: A random variable  $Y$  is said to have a beta probability distribution with parameters  $\alpha > 0$  and  $\beta > 0$  if and only if the density function of  $Y$  is  $f(y) = \left\{ \frac{y^{\alpha-1} (1-y)^{\beta-1}}{B(\alpha, \beta)}, 0 \leq y \leq 1, \text{ and elsewhere } 0, \text{ where } \right.$

$$B(\alpha, \beta) = \int_0^1 y^{\alpha-1} (1-y)^{\beta-1} dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}. \text{ The graphs for these functions assume widely}$$

differing shapes for various values of the two parameters. The cumulative distribution function for the beta random variable is called incomplete beta function and denoted by

$F(y) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha, \beta)} dt = I_y(\alpha, \beta)$ . Graph examples can be found on page 195 of the text. To summarize, the purpose of this distribution is to model the uncertainty about the probability of success of a random experiment, and in project management, a three-point technique is used that recognise the uncertainty in the estimation of the project time, and its applications include Bayesian hypothesis testing, the rule of succession, task duration modeling, and project planning control systems like CPM and PERT (“Beta Distribution - Definition, Formulas, Properties, Applications”).

## PART VI - Lucid Dreaming Data Set

### Statistics and Questions on Lucid Dreaming

Whenever a person falls asleep and realizes, once in a dream, that they are dreaming and can interact and change their environment, they have entered a unique dream state called lucid dreaming. This awareness is remarkable. The concept does not have much research, but it is fantastic. Using the “Lucid Dreaming: Large Database Analysis” by Dr. Patrick Bourke and the statistics textbook “Mathematical Statistics with Applications” by Dennis D. Wackerly, we can solve questions regarding the dreaming statistics concerning time to acquire lucidity, age, gender, if a user has experienced a lucid dream before, and if the dream was lucid (Bourke). Obtaining data via an app that allows users to enter their birth date and gender, categorize their dreams as lucid or not, and whether they have had a lucid dream before. The data consists of 11,762 users! Another research article, “Is There a Link Between Frequency of Dreams, Lucid Dreams, and Subjective Sleep Quality?” was also used for other statistics, which used a French student population with a student population of ( $n = 274$ ) and a general population sample of ( $n = 681$ ), to access sleep quality and dream experience frequencies. Lastly, another article, “Benefits and concerns of seeking and experiencing lucid dreams:

Benefits are tied to successful induction and dream control,” focused on Reddit posts, their frequency, and the negative and positive themes associated with lucid dreaming along with lucidity and dream control. Part VI will use statistical equations to illustrate lucid dreaming data.

Beginning with Dr. Patrick Bourke’s data, his results indicated that the probability, out of 11,762 users, that a dream was recorded as non-lucid was 94%. The probability of having a dream recorded as lucid was 6%. This reveals a shocking finding, that lucid dreaming is difficult to achieve and is not as common. So, with 11,762 users, with 75% being males and 25% as females, and 94% having dreams that were not lucid and 6% as lucid, a table can be formed. Bourke’s data indicates that out of the 11,762 dreamers, roughly 8,821 are males and 2,940 are females. Out of 11,762 dreamers, roughly 706 dreamers had a lucid dream and 11,056 dreamers did not have a lucid dream, which could have been a normal dream or nightmare.

Dreams: Lucid or Non-Lucid		Male or Female Dreamer	
Lucid	.06	Male	.75
Non-Lucid	.94	Female	.25

Figure 9.0: Table of Lucidity and Gender

So, what is the probability that a dreamer had a lucid dream and was a male? Given the probability of event A being .06 or 6% and event B being .75 or 75%, the result would be a conditional probability of 6% that a male dreamer has a lucid dream. It is difficult to determine whether gender is independent of whether one has a lucid dream. Although research conducted, including 2,155 dream reports, indicated that women, upon falling asleep, often immediately entered a lucid dream 50% more than men, with spontaneous lucid dreaming 50% more frequently and can remain in these dreams longer, both genders can achieve their goals in lucid dreams at an equal rate (Staff). Is being a female dependent on having a lucid dream or not? Well, that question requires more data and research, but according to Staff, frequency depends on

gender! For the initial question, the data includes having a lucid dream and the percentage of females, but the events are not dependent on each other. However, having a lucid dream depends on the sleep one has per night, which is roughly 7-9 hours. Quality sleep is the key to lucid dreaming since REM sleep occurs in the final hours of the night, and it would be hard to lucid dream without a healthy amount of sleep (Brandwein). With the initial question, how would the multiplicative law of probability help solve the problem? The probability of being a male and having a lucid dream would be 0.045. How about the general addition rule? Applying the general addition rule would result in being a male or having a lucid dream with a probability of .765. If having a lucid dream, then it was a female, or if not having a lucid dream, the dreamer was still a male.

In his experiment, ten people signed onto the app to input whether they had a dream. How many ways can the outcome look for either dreaming or not dreaming? Well, utilizing permutations via the formula sheet in part IV, the problem would be  $P(10, 2) = \frac{10!}{(10-2)!} = 90$  different outcomes if ten people had a lucid or regular dream. Now, what are the number of ways of selecting only two dreamers who had a lucid dream for future studying and research out of the total 706 dreamers who had a lucid dream? For a researcher interested in future studies regarding lucid dreamers, utilizing the combination formula, the problem would be

$$C(706, 2) = \frac{706!}{(2!(706-2)!)} = 248,865 \text{ combinations of selecting two lucid dreamers out of the } 706 \text{ dreamers.}$$

The result is helpful if the two lucky dreamers got a hefty compensation for participation at a sleeping facility! What is the probability that an app user who never had a lucid dream before using the app then had a lucid dream? Utilizing Bayes' Theorem, event A's probability is 56%, that is, out of the total users sampled, 56% never had a lucid dream before starting the app, and Event B's probability of 6%, having a lucid dream according to the data, gives the equation  $P(.56|.06) = \frac{P(.56|06)P(.06)}{P(.56)} = 56\%$ .

Wrapping up with Dr. Bourke's experiment, the doctor wished out of 8,821 males and 2,940 females to select one from each group. How would the probability of mass function be used to solve this issue of selecting one from each for equality? The doctor can get two dreamers from 11,762 in 69,166,441 ways, which will be the sample points and equally likely to occur for equality with randomness. The number of ways of selecting Y = 0 men is  $\binom{8821}{0} \binom{2940}{2} = 0.0625$ , Y = 1 is 0.3749, and Y = 2 is 0.5624. The odds of equality of having both genders picked is a probability of 37.49%, but there is a higher probability of picking two men. While for two women, it has the lowest probability of 6.25%.

	Student n = 274		General pop. n = 681	
	Counts	Percentage	Counts	Percentage
<b>Sleep quality</b>				
Score = 0	9	3,28	52	7,64
Score = 1	128	46,72	322	47,28
Score = 2	113	41,24	251	36,86
Score = 3	24	8,76	56	8,22
<b>Sleep latency</b>				
Score = 0	51	18,61	149	21,88
Score = 1	74	27,01	225	33,04
Score = 2	76	27,74	183	26,87
Score = 3	73	26,64	124	18,21
<b>Sleep duration</b>				
Score = 0	95	34,67	249	36,56
Score = 1	92	33,58	214	31,42
Score = 2	57	20,8	136	19,97
Score = 3	30	10,95	82	12,04
<b>Sleep efficiency</b>				
Score = 0	179	65,33	455	66,81
Score = 1	60	21,9	98	14,39
Score = 2	17	6,2	75	11,01
Score = 3	18	6,57	53	7,78
<b>Sleep disturbance</b>				
Score = 0	12	4,38	38	5,58
Score = 1	208	75,91	469	68,87
Score = 2	49	17,88	160	23,49
Score = 3	5	1,82	14	2,06
<b>Medication</b>				
Score = 0	240	87,59	593	87,08
Score = 1	9	3,28	33	4,85
Score = 2	13	4,74	14	2,06
Score = 3	12	4,38	41	6,02
<b>Daytime dysfunction</b>				
Score = 0	28	10,22	101	14,83
Score = 1	97	35,4	278	40,82
Score = 2	107	39,05	233	34,21
Score = 3	42	15,33	69	10,13
<b>PSQI Score, Mean (SD) n for score &gt;5</b>	8.67 (3.41) n = 225		8.33 (3.46) n = 537	

Figure 9.1: Table Obtained via Ribeiro's Research Article

Here is a summarized answer to questions about sleep quality, a table with descriptive data for the PSQI across the two groups, student and general population (Ribeiro et al.).

	Student <i>n</i> = 274		General pop. <i>n</i> = 681	
	Counts	Percentage	Counts	Percentage
<b>Dreaming</b>				
Less than once a month	23	8,39	163	23,94
Once a month	30	10,95	72	10,57
Two or three times a month	28	10,22	100	14,68
Once a week	55	20,07	117	17,18
Two or three times a week	83	30,29	143	21
Four or more times a week	55	20,07	86	12,63
<b>Lucid dreaming</b>				
Never	99	36,13	344	50,51
Less than once a year	43	15,69	80	11,75
About once a year	25	9,12	51	7,49
About 2 to 4 times a year	48	17,52	98	14,39
About once a month	25	9,12	43	6,31
About 2 to 3 times a month	17	6,2	36	5,29
About once a week	12	4,38	12	1,76
Several times a week	5	1,82	17	2,5
<b>Dream with awareness</b>				
Never	55	20,07	185	27,17
Once	25	9,12	70	10,28
Less than once a year	45	16,42	107	15,71
Many times a year	63	22,99	107	15,71
Many times a month	26	9,49	51	7,49
many times a week	60	21,9	161	23,64
<b>Dream with control</b>				
Never	391	44,53	122	57,42
Once	45	8,39	23	6,61
Less than once a year	83	13,5	37	12,19
Many times a year	55	14,6	40	8,08
Many times a month	25	4,01	11	3,67
Many times a week	82	14,96	41	12,04

Figure 9.2: Another Table Obtained via Riberio's Research Article

Here is another table from the same article that shows the descriptive data for all dream-related experiences frequencies.

Consider Figure 9.2: regarding the student population, 175 students successfully have a lucid dream while 99 fail to have a lucid dream. Selecting ten random students from the population, and there is a 36.13% chance of failing to have a lucid dream, what is the probability that at least nine of ten attempting to lucid dream achieve it? Let  $Y$  denote the number of people who have lucid dreams and the probability that a single person achieves lucid dreaming is  $p =$

0.6387. The number of total trials is  $n = 10$ , and the probability of exactly nine successful lucid dreaming attempts is  $P(Y = 9) = p(9) = \binom{10}{9} (.6387)^9 (.3613) = 0.06391$ . The formula used to solve this question was Binomial Distribution, as it finds the probability of an event succeeding or failing, and in this case, a dream succeeding in achieving lucid dreaming! What if someone wanted to count the number of trials carried out until the first successful dream with awareness? Within the student population in Figure 9.2, what is the probability that the 8th student is the first student that fits the category of having a dream with awareness many times a week? The probability of having a dream many times a week is  $p = 0.219$  and  $q = 1 - 0.219 = 0.781$ , the probability of having any other option listed in Figure 9.2 Dream with awareness category. According to the Geometric Distribution equation, solving this question is

$0.0781^7 * 0.219 = 0.039$ , and so there is a 4% probability that the first student that dreams with awareness many times a week is on the 8th interview. Now, say someone wanted to know, from the data table, how many students are skilled in controlling their dreams. However, in Figure 9.2, the student and general population are swapped for the dream with the control category. Anyway, consider many times a week as adept at controlling their dreams, and 41 students selected this choice or 12.04%. A psychologist, hoping to get these experts, is forced to pick from all 274 students. What is the probability that the 41 selected include half of the 41 adept dreamers, 20, with control in the group of 274 students? Should the psychologist go through with it? Hypergeometric Distribution allows the psychologist to consider their choices and the likelihood of getting some of these students that the psychologist considers experts in dream control! For the equation,  $N = 274$ ,  $n = 20$ , and  $r = 41$ . The psychologist seeks the probability that  $Y = 41$ . Unfortunately, after plugging the numbers into the equation,  $p(41) = \binom{41}{41} \binom{274-41}{20-41} = 0$  is the probability! The psychologist would have to decrease the

number of successes in  $r$  to 5, which is trying for only 5 of the adepts in the selected sample size. The new equation would include  $N = 274$ ,  $n = 20$ , and  $r = 5$  with  $Y = 41$ . The new  $P(Y = 5) = \text{roughly } 0.102$ , or 10.2% probability, the psychologist can work with some of the adepts.

Assume that there are independent student interviews and that the probability of no sleep disturbance, Figure 9.1, is 4.38%. Find the probability that the 5th student interviewed with a sleep disturbance score of 0 is on the 10th interview. Doing ten student interviews of 274 students, what are the odds that there are five students interviewed with a sleep disturbance score of 0 within ten interviews? The odds of no sleep disturbance is 4.38%. Utilizing the Negative Binomial Probability Distribution, the variables for the equation are  $p = 0.0438$ ,  $q = 0.9562$ ,  $r = 10$ , and  $y = 5$ . After plugging the variables into the equation, in part IV, the probability of five students who score 0 on sleep disturbance within ten interviews of 274 students is 0.00002.

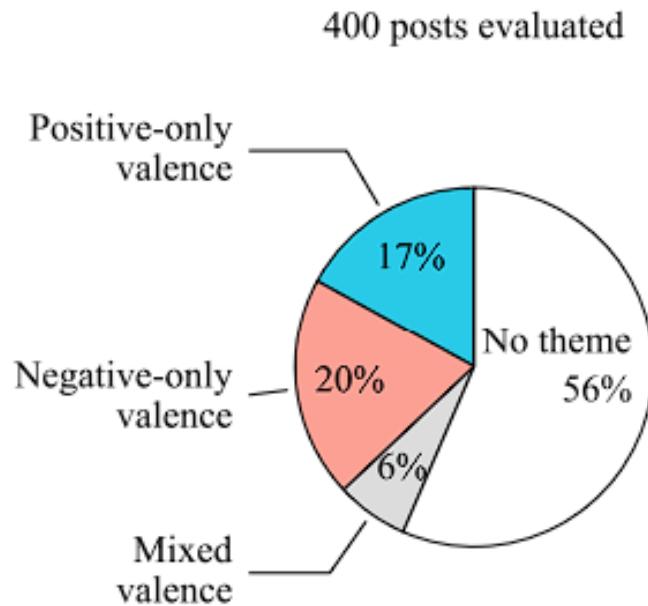


Figure 9.3: A Pie Graph Obtained via Mallett's et al. Research Article

The pie graph is the relative post frequencies aggregated by valence, and to the total sample, posts can contain several themes (Mallett et al.). These were posts via Reddit, with positive-only posts containing themes about lucid dreaming creativity, insight, dream enhancement, nightmare resolution, positive waking mood, and rehearsal. Negative themes include lucid dysphoria, poor sleep, reality confusion, sleep paralysis, and unwanted lucid dreams.

	Theme	Brief definition
Positive	Creativity / insight	Any explicit reference to a creative action in the dream or a realization made while dreaming that has an outcome on waking life.
	Dream enhancement	Any reference to dreams being enhanced by lucidity, including but not limited to pleasure-seeking, excitement, and vividness.
	Nightmare resolution	Any time an explicit connection is made between lucid dreaming and the reduction of bad dream content.
	Positive waking mood	Any reference to lucidity resulting in a positive mood during the day, especially upon awakening in the morning.
	Rehearsal	Any reference to the use of lucid dreams as a virtual practice ground for a waking activity.
Negative	Lucid dysphoria	Any reference to a dysphoric/bad dream or nightmare where the dreamer is lucid and unable to do anything positive about it.
	Poor sleep	Any reference to poor or degraded sleep quality, including that as a result of induction methods and poor sleep leading to lucidity.
	Reality confusion	Any expression of being wrong about one's state of consciousness (excluding typical non-lucid dream misperceptions and sleep paralysis). This includes wake being dream-like and false awakenings.
	Sleep paralysis	Any reference to sleep paralysis or related phenomena (e.g., visual or auditory hallucinations, a feeling of weight on the chest).
	Unwanted lucid dreams	Any mention of a user having naturally occurring lucid dreams and not being able to make them stop despite wanting to.

Figure 9.4: A Table Obtained via Mallett's et al. Research Article

Figure 9.4 shows the various themes and brief definitions for each. Figure 9.3 has a pie graph containing 400 Reddit posts: 68 posts were positive, 80 were negative, 24 were mixed, and the remaining 224 posts lacked a theme. Consider, hypothetically, that 25 posts are posted on average every hour regarding lucid dreaming. 4.5 posts are positive, five are negative, 1.5 mixed, and 14 lack a theme. What is the probability that a user posts, within a minute, at least one post regarding lucid dreaming? Using the Poisson Distribution, first find  $\lambda = 25/60 = 0.417$ .

Then,  $P(Y \geq 1) = 1 - p(Y = 0) = 1 - e^{-0.417} = 0.341$  chance a user posts within one minute at least one post. Suppose there is a lucid dreaming group of 14 people, with a mean age of 22.64 with a standard deviation of 5.46 ("Table 1 Demographic, Behavioral and Questionnaire Data for the Frequent Lucid Dream Group and Control Group."). Find the percentages of values that lie within the age group 12 and 32. With Tchebysheff's Theorem, find k, which is the number of standard deviations. Finding k:  $22.64 + k5.46 = 32$ , after solving for k,  $k = 1.714$ . Plugging into the equation  $1 - \frac{1}{k^2} = 0.66$ , implying that 66% of the age range is between 12 and 32 years old.

So far, statistics has allowed the explanation of lucid dreaming data in a more in-depth way, using various functions, probabilities, and distributions. Part VI discussed the data using text chapters two and three, Probability and Discrete Random Variables and Their Probability Distributions, respectively. The final two chapters, four, Continuous Variables and Their Probability Distributions, and five, Multivariate Probability Distributions, will help further explain statistics in unique ways. Note that some of the questions asked so far were hypothetical, while using other data via data provided via several sources, but some data was independent rather than dependent. Applying these statistical methods to different data sets, and when solving complex questions, can provide an adequate in-depth explanation and connections. Part V went

over sections five, six, and seven in chapter four and therefore skipped, but discusses the rest here, except section 11. Part VI incorporates most sections in chapter five, too.

Once again, let  $Y$  denote the number of people having lucid dreams, and the probability that a single person achieves lucid dreaming is  $p = 0.6387$ , but for this example, the number of total trials is  $n = 2$ . As mentioned before, this is a binomial distribution. However, applying probability distribution for a continuous random variable and the distribution function  $Y$ : attempt to find  $F(y)$ . The probability function for  $Y$  is given by

$$p(y) = \binom{2}{y} (0.6387)^y (0.6387)^{2-y}, y = 0, 1, 2. \text{ The solutions are}$$

$p(0) = 0.4079$ ,  $p(1) = 0.8159$ , and  $p(2) = 0.4079$ . Consider how, in the sleep laboratory, a person is uniformly late to sleeping on time between 10 to 30 minutes. Remember, sleeping on time and getting 7 to 9 hours of sleep increases the potential of lucid dreaming, and so, being late impacts total sleep time and could get you removed from the experiment, affecting your compensation! Say you are running late, and today being over 16 minutes will guarantee lateness, what is the probability of you being late to your appointment and sleep schedule? Well, the upper bounds is 30 minutes, and 16 is between 10 to 30 minutes. The question to solve would be the following:  $P(16 \leq X \leq 30)$ , and the answer, using the uniform distribution equation, would be  $\frac{30-16}{30-10} = 0.7$ , meaning there is a 70% chance of being late for that particular day.

Should have gotten there earlier! Now, consider the given function  $f(x) = cx$ ,  $7 \leq x \leq 9$ . Before solving  $P(7 \leq X \leq 9)$ , first find the value of  $c$  for which  $f(x)$  is a valid density function. Using the equation provided in the formula section,

$$\int_7^9 cx dx = \frac{cx^2}{2} \Big|_7^9 = \frac{c81}{2} - \frac{c49}{2} = (\frac{32}{2})c. \text{ Since } (\frac{32}{2})c = 1, c = \frac{2}{32}.$$

The second portion of the

question is asking for the probability of sleeping between the interval 7 and 9 hours when given a density function. Remember, this is ideal for lucid dreaming and overall health and so when given a sleeping window between 7 and 9 hours, the success increases! Let's solve:

$$P(7 \leq X \leq 9) = \int_7^9 f(x) dx = \frac{2}{32} \int_7^9 x dx = (\frac{2}{32}) \frac{x^2}{2} \Big|_7^9 = 1.$$

Good news, there is a guaranteed

chance of sleeping for the required amount of time when giving a sleeping window between 7 through 9 hours. Of course, this accounts for a fair sleeping environment without potential health issues. Finally, consider the time, in minutes, it takes to write down the dream into a dream journal by the continuous random variable  $X$ , with probability density function

$f(x) = \{\frac{1}{128}x, 0 \leq X \leq 15, \text{ and } 0, \text{ elsewhere.}$  Find the expected amount of time it takes for one to write their dream in a dream journal. Note: 0 means that they did not remember their dream and therefore had nothing to write down. Solving:

$$\int_0^{15} x \frac{1}{128} x dx = \frac{1}{128} \int_0^{15} x^2 dx = \frac{1}{128} [\frac{x^3}{3}]_0^{15} = 8.79 \text{ minutes expected to write their dream in}$$

their dream journal.

While dreaming, the parts that remain active inside the brain are the medial prefrontal cortex, posterior cingulate cortex, and the thalamus, a system of interconnected regions (“Nightmares and the Brain”). Suppose, hypothetically, there is increased activity inside the brain region along with one of the areas of the brain previously mentioned, so two regions within the brain. Scientists are mainly concerned with the lucid dreaming portion, but equal areas are

considered and what they look at could be in either region with X and Y coordinates being the location of interest. Here is a bivariate analogue of the univariate uniform density function:

$f(x, y) = \{1, 0 \leq X \leq 1, 0 \leq Y \leq 1\}$ . Find  $F(6, 8)$ , the hypothetical volume of interest. The

$$\text{equation will be written as: } \int_{-\infty}^{8} \int_{-\infty}^{6} f(x, y) dx dy = \int_0^8 \int_0^6 1 dx dy = \int_0^8 [y]_0^6 dy = \int_0^8 6 dy = 48,$$

which is the desired probability volume the scientists are looking for inside the anterior prefrontal cortex. This area is where scientists have observed an increased activity, along with the medial and lateral parietal cortex, including the supramarginal and angular gyrus and inferior and middle temporal gyrus during rapid-eye-movement, or REM, sleep contrasted with non-lucid REM sleep (Baird et al.). Now, consider scientists wanting to zone in on some unusual activity, find  $P(2 \leq X \leq 7, 0 \leq Y \leq 9)$ . Plugging into the equation,

$$\int_0^9 \int_{0.2}^7 f(x, y) dx dy = \int_0^9 \int_{0.2}^7 1 dx dy = \int_0^9 [y]_0^7 dy = \int_0^9 5 dy = 45, \text{ the volume under the density}$$

function,  $f(x, y)$  and above the brain region is where this unusual activity is coming from, which scientists figure to be the prefrontal cortex responsible for lucid dreaming.

Y: Sleep Duration Scores	X: A student		
	0	1	2
0	.3467	.3467	.3467
1	.3358	.3358	.3358
2	.2080	.2080	.2080
3	.1095	.1095	.1095

Figure 9.5: A Table Showing a Student and Their Sleep Duration

Here is a table obtained via Riberio's research article in Figure 9.1. Student 0, the first student, has an equal probability as the other students to have a sleep duration score of 0, and the same applies to the students and the probability for other sleep duration scores. Find  $F(0, 2)$  using the joint distribution function  $F(x, y) = P(X \leq x, Y \leq y)$  and

$$F(x, y) = \sum_{t_1 \leq x} \sum_{t_2 \leq y} p(t_1, t_2). \text{ So, } P(X \leq 0, Y \leq 2) = P(0, 0) + P(0, 1) + P(0, 2) = .8905,$$

or 89.05% probability of student one having a sleep duration score from 0 through 2.

Consider  $f(x, y) = \begin{cases} x, & 0 \leq x \leq 3, 0 \leq y \leq 3 \\ 0, & \text{elsewhere} \end{cases}$ , x being student one and y being student two, with the intervals being sleep efficiency scores. Find the marginal density function. If  $0 \leq x \leq 3$ ,

$$f(x) = \int_0^3 x dy = x(y)]_0^3 = 3x, \text{ and if } 0 \leq y \leq 3, f(y) = \int_0^3 x dx = x^2)]_0^3 = 9.$$

Consider this equation,  $f(x, y) = \begin{cases} \frac{3}{4}, & 0 \leq x \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$ . This represents X and Y having a joint density. Background information: The sale of a random amount of premium lucid dreaming

supplements, Y, at the beginning of a given Amazon day and it dispenses a random amount X during the day. The supplements are not resupplied during the day, therefore  $X \leq Y$ .  $(x, y)$  are uniformly distributed, so find the conditional density of X given  $Y = y$ . Evaluate the probability that less than  $\frac{3}{4}$ th of the supple will be sold, given that the company has 1.5 pounds of the

premium top edition supplement. The marginal density of Y is given by  $f(y) = \int_{-\infty}^{\infty} f(x, y) dx$ .

$$\int_{-\infty}^y (\frac{3}{4}) dx = (\frac{3}{4})y, 0 \leq y \leq 2,$$

So,  $f(y) = \begin{cases} \frac{3}{4}, & 0 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$ . Using definition 5.7 in the textbook,

$$f(x|y) = \frac{f(x,y)}{f(y)} = \frac{\frac{3}{4}}{(\frac{3}{4})(y)} = \frac{1}{y}, 0 \leq x \leq y. \text{ The probability of interest is}$$

$$P(X \leq \frac{3}{4} | Y = 1.5) = \int_{-\infty}^{\frac{3}{4}} f(x|y=1.5) dx = \int_0^{\frac{3}{4}} \frac{1}{1.5} dx = \frac{\frac{3}{4}}{1.5} = \frac{1}{2} \text{ is the probability that less}$$

than  $\frac{3}{4}$ th of the supple will be sold given 1.5 pounds of the supplement.

## PART VII - Stocks Program and Graphs

### Abstract

Collecting Stock Data: Went on Yahoo Finance and found and collected weekly Amazon Historical Data via a csv file. Figures 10.0 through 10.4 show various graphs of data. After graphing the data, the data was loaded into ArrayLists in the StockDataLoader.java program. The same program calculates the relative strength index, creates a moving average trend line, and uses these, along with the trading algorithm incorporating heuristics, to determine when to buy and sell a stock.

### Results

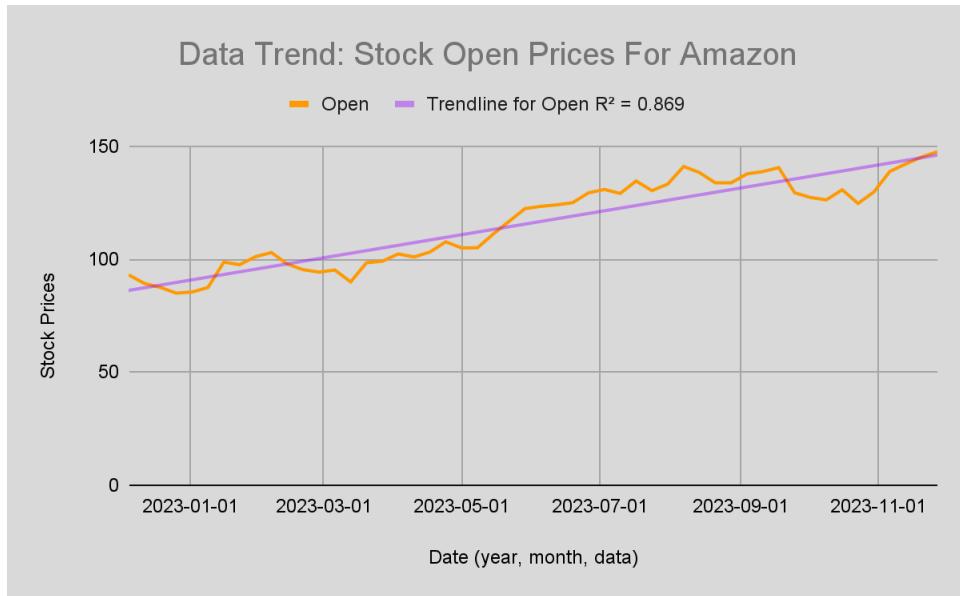


Figure 10.0: Graph Demonstrating Amazon Open Stock Prices Weekly

Trendline shows a gradual increase in stock open prices for amazon ranging from January 1st of 2023 to November 1st of 2023.

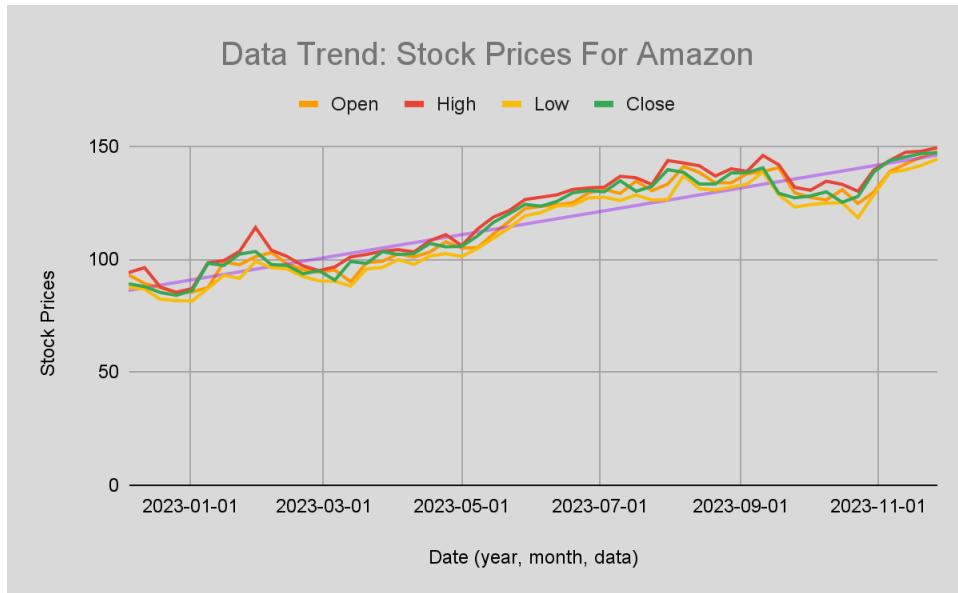


Figure 10.1: Graph Of Weekly Open, High, Low, and Close Stock Prices

As expected, high prices have some peaks throughout the graph while low prices show the greatest dip in price. Close prices are stable and within the high and lows, while open price follows closely to the close lines. Useful for investors, the graph indicates a gradual increase.

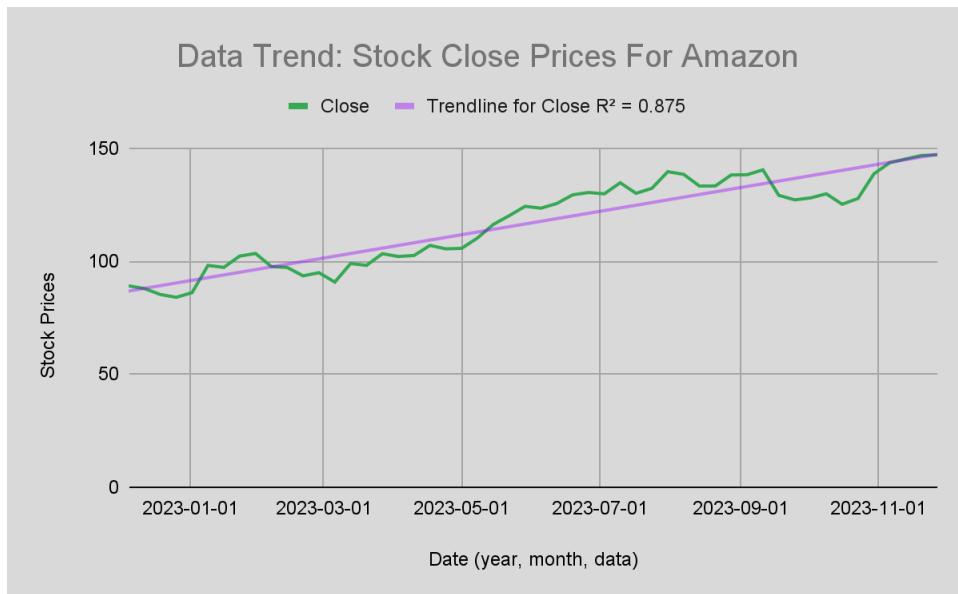


Figure 10.2: Graph Demonstrating Amazon Close Stock Prices Weekly  
Similar to Figure 10.0 and the open prices, there is a gradual increase.

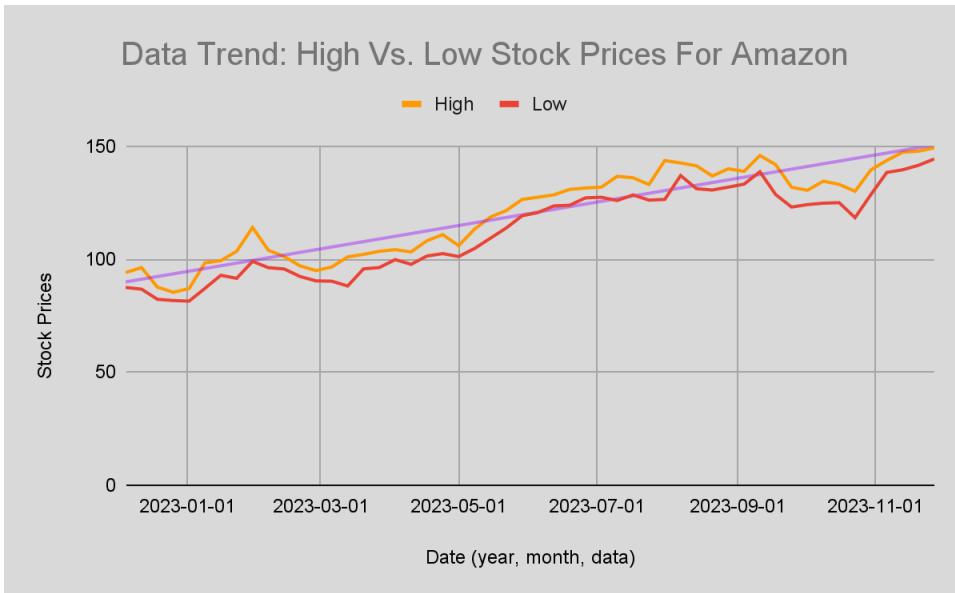


Figure 10.3: Graph Demonstrating Amazon High vs. Low Stock Prices Weekly  
Shows the relationship of the high and low stock prices, which shows the peaks and dips, respectively.

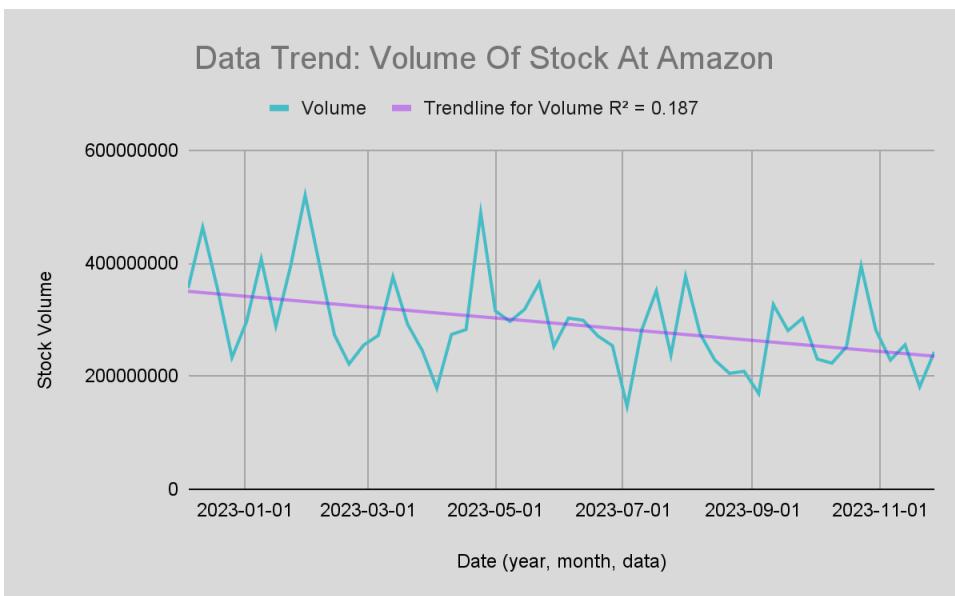


Figure 10.4: Graph Demonstrating Amazon Stock Volume Weekly  
The volume of stock at amazon seems to be at a gradual decrease, with massive spikes and dips shrinking as time goes on.

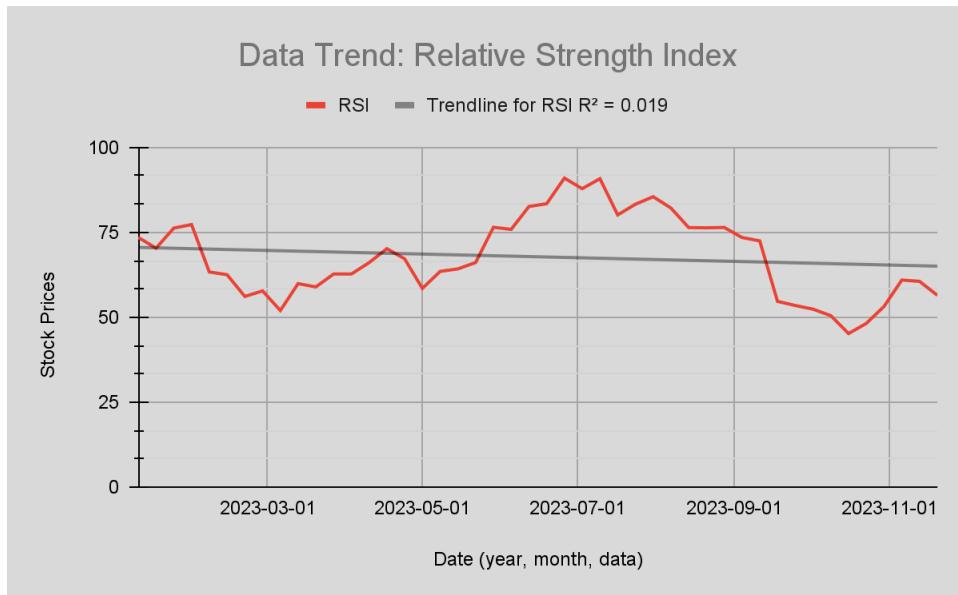


Figure 10.5: Relative Strength Index Graph

The following graph shows the RSI from January 9th to the most recent being 2023-11-20. For the Amazon stock within the data range, the RSI steadily decreased between February and the early days of March. From there, it steadily increased to a few days before the beginning of May, where it experienced a sharp decrease. From May to July, the RSI increased steadily. From July to mid-September, the RSI has been steadily decreasing, then around September 10th, the RSI dropped below 50, where it steadily increased again but started to trend downward again in mid-November.

The screenshot shows the Matlab interface. The Editor window at the top contains a script named 'Smoother.m' with the following code:

```
% Smoothing RSI
% Author: Dante Anzalone
clc, clearvars
file = [100.00 56.44 60.55 60.93 53.17 48.17 45.18 50.46 52.36 53.45 54.66 72.46 73.46 76.
% Obtained RSI numbers, now will smooth
% movmean(file, 15)
file = smoothdata(file, "gaussian", 15)
```

The Command Window below displays the output of the script:

```
file =
Columns 1 through 9
66.9391 64.0786 61.2600 58.6105 56.4542 54.9578 54.3689 54.8253 56.1714
Columns 10 through 18
```

At the bottom of the window, there are several tabs: Zoom: 100%, UTF-8, CRLF, script, Ln 7, Col 40.

Figure 10.6: Matlab Smoothed RSI Results

Obtaining RSI data points from rsi.txt via StockDataLoader.java program, users can easily copy and paste the points into MatLab and get the smoothed results.

Here are the results: file = **66.9391**, 64.0786, 61.2600, 58.6105, 56.4542, 54.9578, 54.3689, 54.8253, 56.1714, 58.7574, 61.9699, 65.4999, 69.0365, 72.3836, 75.3327, 77.8600, 79.9682, 81.7276, 83.1027, 84.0229, 84.4301, 84.2097, 83.2888, 81.6125, 79.3307, 76.6630, 73.8746, 71.2427, 68.9623, 67.2182, 65.9772, 65.0802, 64.3360, 63.5478, 62.6312, 61.6526, 60.8090, 60.2771, 60.2575, 60.8660, 61.7494, 62.5101, 62.5909, 61.2210, 58.6728, 54.3082, 48.8774, **43.2569**, **38.5025**, **35.2966**, **33.9755**, **34.3510**. Bold points are not included in the graphs.

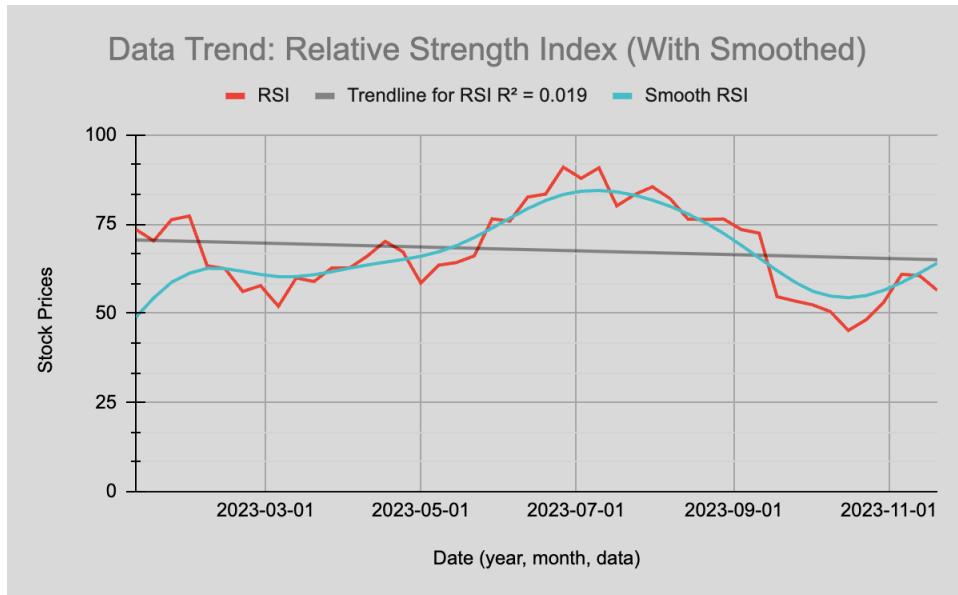
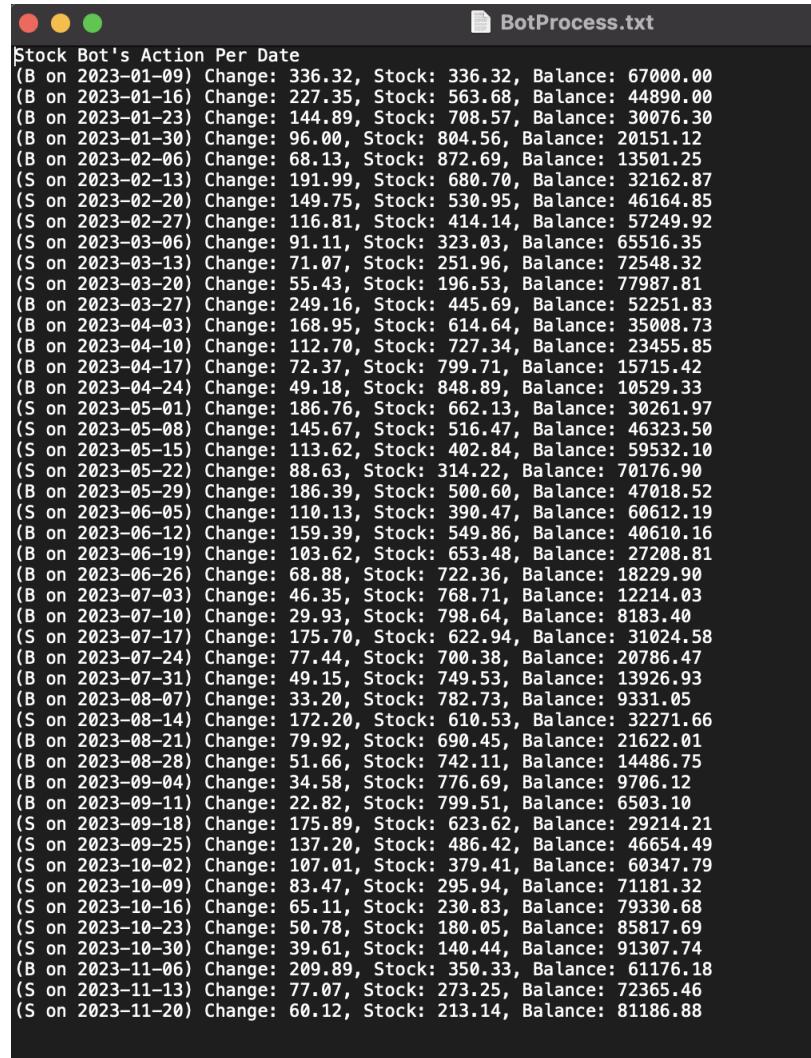


Figure 10.7: Relative Strength Index With Moving Line Graph

Once smoothing the RSI from the algorithm provided via MatLab, creating a moving average trend line in excel was straightforward. It is one of the statistics used to help traders determine good trades (Hoy).

This specific stock bot will sell stocks when the moving average is above the RSI and will buy when the RSI is below the average. Starting with \$100,000 as its balance, and with the help of the Relative Strength Index and Moving Average, will make a decision on each data point at a time. The bot will track its total net worth after each data point, whether it bought shares, sold shares, or did nothing. Since the bot is feeling extra confident with its \$100,000, it will buy shares with .33% of its balance when it is time to buy stocks, and then sell .22 when it feels the time is right. This could be a risky move and could show potential investors to not follow its footsteps, or luck may be on the bot's side, or even the power of statistics. However, the bot will continue buying shares if the moving average is below the RSI. With more time to work on the stock bot, the bot should stop buying shares once the RSI starts to rise again. Also, with more expertise on the stock market, someone could apply their knowledge and make the stock bot more advanced. This program shows the blueprints and the idea that it is a possibility.



The screenshot shows a terminal window with a dark background and light-colored text. At the top, there are three colored dots (red, yellow, green) followed by the file name "BotProcess.txt". The file content is a log titled "Stock Bot's Action Per Date" which tracks daily changes in stock purchases and sales. The log includes columns for date, action type (B for buy, S for sell), change in stock value, and balance. The log spans from January 9, 2023, to November 20, 2023.

```
Stock Bot's Action Per Date
(B on 2023-01-09) Change: 336.32, Stock: 336.32, Balance: 67000.00
(B on 2023-01-16) Change: 227.35, Stock: 563.68, Balance: 44890.00
(B on 2023-01-23) Change: 144.89, Stock: 708.57, Balance: 30076.30
(B on 2023-01-30) Change: 96.00, Stock: 804.56, Balance: 20151.12
(B on 2023-02-06) Change: 68.13, Stock: 872.69, Balance: 13501.25
(S on 2023-02-13) Change: 191.99, Stock: 680.70, Balance: 32162.87
(S on 2023-02-20) Change: 149.75, Stock: 530.95, Balance: 46164.85
(S on 2023-02-27) Change: 116.81, Stock: 414.14, Balance: 57249.92
(S on 2023-03-06) Change: 91.11, Stock: 323.03, Balance: 65516.35
(S on 2023-03-13) Change: 71.07, Stock: 251.96, Balance: 72548.32
(S on 2023-03-20) Change: 55.43, Stock: 196.53, Balance: 77987.81
(B on 2023-03-27) Change: 249.16, Stock: 445.69, Balance: 52251.83
(B on 2023-04-03) Change: 168.95, Stock: 614.64, Balance: 35008.73
(B on 2023-04-10) Change: 112.70, Stock: 727.34, Balance: 23455.85
(B on 2023-04-17) Change: 72.37, Stock: 799.71, Balance: 15715.42
(B on 2023-04-24) Change: 49.18, Stock: 848.89, Balance: 10529.33
(S on 2023-05-01) Change: 186.76, Stock: 662.13, Balance: 30261.97
(S on 2023-05-08) Change: 145.67, Stock: 516.47, Balance: 46323.50
(S on 2023-05-15) Change: 113.62, Stock: 402.84, Balance: 59532.10
(S on 2023-05-22) Change: 88.63, Stock: 314.22, Balance: 70176.90
(B on 2023-05-29) Change: 186.39, Stock: 500.60, Balance: 47018.52
(S on 2023-06-05) Change: 110.13, Stock: 390.47, Balance: 60612.19
(B on 2023-06-12) Change: 159.39, Stock: 549.86, Balance: 40610.16
(B on 2023-06-19) Change: 103.62, Stock: 653.48, Balance: 27208.81
(B on 2023-06-26) Change: 68.88, Stock: 722.36, Balance: 18229.90
(B on 2023-07-03) Change: 46.35, Stock: 768.71, Balance: 12214.03
(B on 2023-07-10) Change: 29.93, Stock: 798.64, Balance: 8183.40
(S on 2023-07-17) Change: 175.70, Stock: 622.94, Balance: 31024.58
(B on 2023-07-24) Change: 77.44, Stock: 700.38, Balance: 20786.47
(B on 2023-07-31) Change: 49.15, Stock: 749.53, Balance: 13926.93
(B on 2023-08-07) Change: 33.20, Stock: 782.73, Balance: 9331.05
(S on 2023-08-14) Change: 172.20, Stock: 610.53, Balance: 32271.66
(B on 2023-08-21) Change: 79.92, Stock: 690.45, Balance: 21622.01
(B on 2023-08-28) Change: 51.66, Stock: 742.11, Balance: 14486.75
(B on 2023-09-04) Change: 34.58, Stock: 776.69, Balance: 9706.12
(B on 2023-09-11) Change: 22.82, Stock: 799.51, Balance: 6503.10
(S on 2023-09-18) Change: 175.89, Stock: 623.62, Balance: 29214.21
(S on 2023-09-25) Change: 137.20, Stock: 486.42, Balance: 46654.49
(S on 2023-10-02) Change: 107.01, Stock: 379.41, Balance: 60347.79
(S on 2023-10-09) Change: 83.47, Stock: 295.94, Balance: 71181.32
(S on 2023-10-16) Change: 65.11, Stock: 230.83, Balance: 79330.68
(S on 2023-10-23) Change: 50.78, Stock: 180.05, Balance: 85817.69
(S on 2023-10-30) Change: 39.61, Stock: 140.44, Balance: 91307.74
(B on 2023-11-06) Change: 209.89, Stock: 350.33, Balance: 61176.18
(S on 2023-11-13) Change: 77.07, Stock: 273.25, Balance: 72365.46
(S on 2023-11-20) Change: 60.12, Stock: 213.14, Balance: 81186.88
```

Figure 10.8: Text File of Basic Stock Bot

“B” represents that the stock bot bought stocks and “S” that it sold.

Date	Net Worth
2023-01-09	100000.00
2023-01-16	99707.40
2023-01-23	102520.14
2023-01-30	103334.99
2023-02-06	98684.61
2023-02-13	98326.81
2023-02-20	95808.22
2023-02-27	96551.54
2023-03-06	94824.59
2023-03-13	97479.87
2023-03-20	97273.27
2023-03-27	98287.36
2023-04-03	97739.16
2023-04-10	98015.75
2023-04-17	101252.42
2023-04-24	100044.86
2023-05-01	100223.13
2023-05-08	103268.95
2023-05-15	106362.57
2023-05-22	107917.55
2023-05-29	109218.41
2023-06-05	108807.91
2023-06-12	109612.28
2023-06-19	111723.75
2023-06-26	112396.83
2023-07-03	111977.87
2023-07-10	115744.57
2023-07-17	112006.92
2023-07-24	113383.62
2023-07-31	118538.41
2023-08-07	117668.96
2023-08-14	113606.58
2023-08-21	113631.01
2023-08-28	116986.58
2023-09-04	117068.21
2023-09-11	118745.86
2023-09-18	109735.42
2023-09-25	108488.19
2023-10-02	108896.79
2023-10-09	109591.10
2023-10-16	108223.87
2023-10-23	108817.10
2023-10-30	110772.43
2023-11-06	111469.00
2023-11-13	112036.53
2023-11-20	112462.81

Figure 10.9: CSV File of Stock Bot's Net Worth

Perhaps it was luck or the power of statistics or the stock itself, or a little of each, but the basic stock bot made a profit of \$12,462 between January 9th of 2023 to November 20th of 2023!

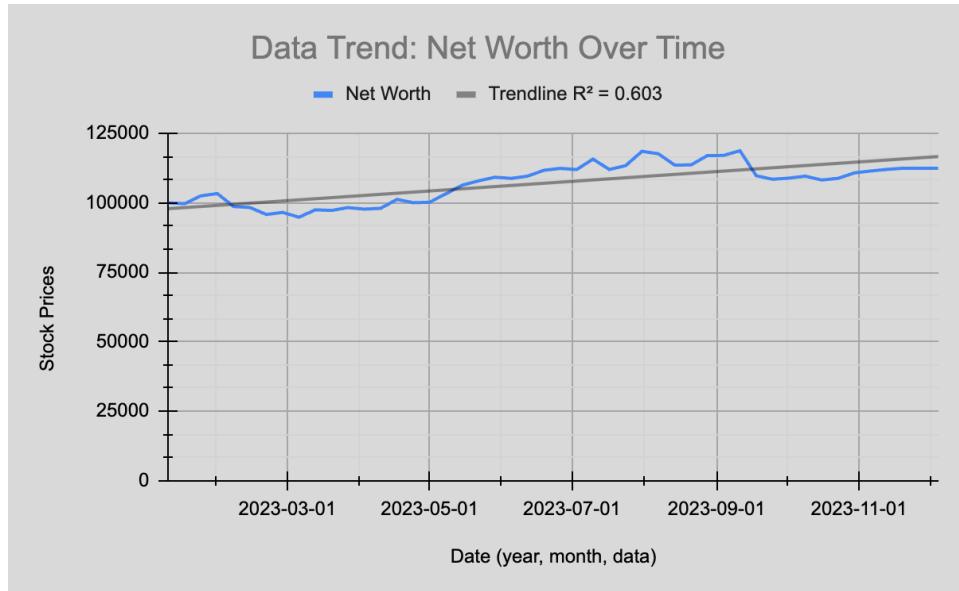


Figure 10.10: Net Worth Over Time Graph

The stock bot did a decent job staying above the initial balance of \$100,000. Now, we will test the stock bot for Amazon's daily stock prices!

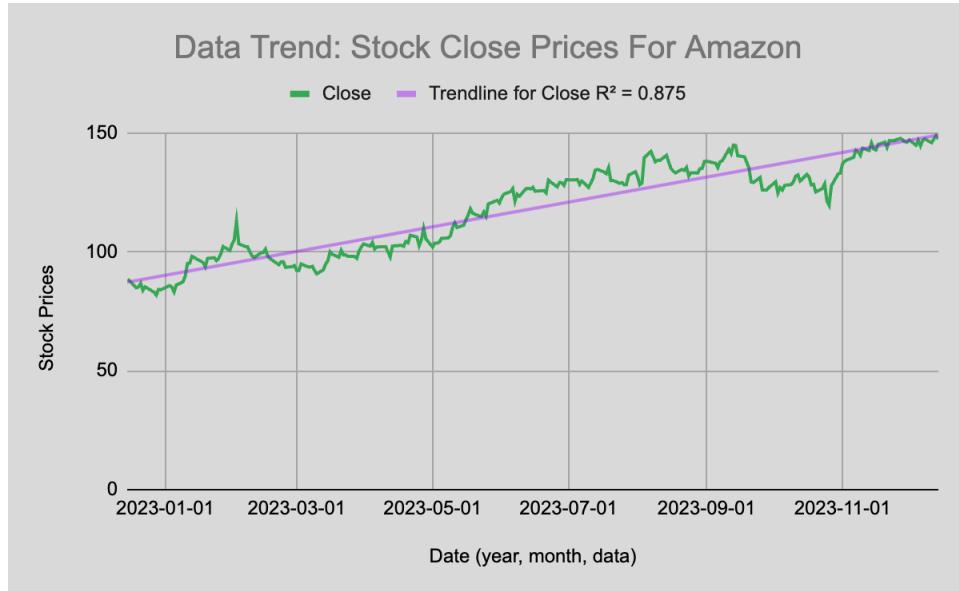


Figure 11.0: Graph Demonstrating Amazon Close Stock Prices Daily

With daily closing prices for Amazon, it is easier to see the various spikes throughout the year. The file AMZNDaily.csv contains the data for Figure 11.0.

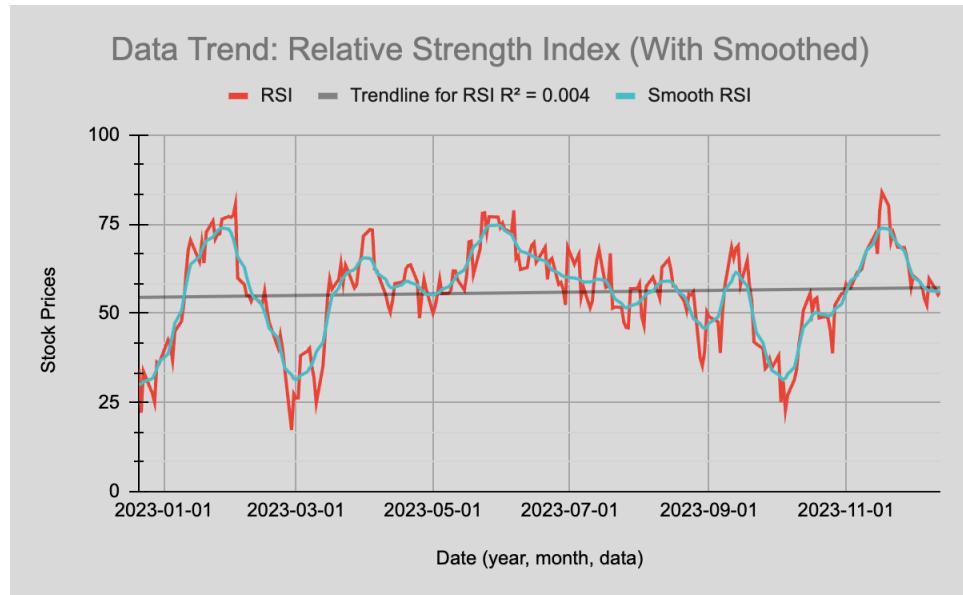
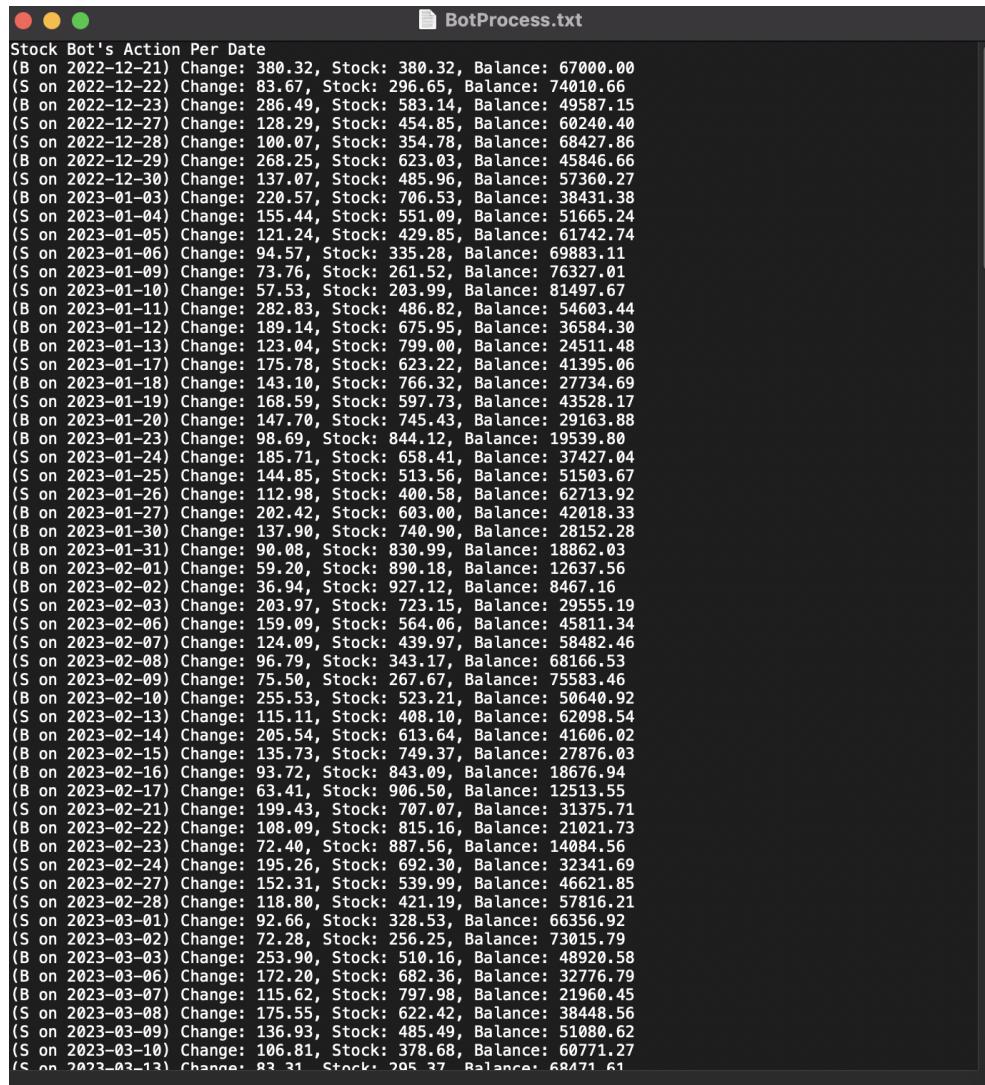


Figure 11.1: Relative Strength Index With Moving Average (Daily)  
Compared to Figure 10.7, this graph provides a better moving average!



The screenshot shows a terminal window with a dark background and light-colored text. At the top, there are three colored dots (red, yellow, green) followed by the file name "BotProcess.txt". The text within the window is a log of daily stock transactions from December 2022 to March 2023. Each entry consists of a trade type (B or S), date, change in stock value, current stock value, and current balance. The log starts with a header "Stock Bot's Action Per Date" and continues with approximately 100 entries. The final entry shows a balance of 68171.61.

```
Stock Bot's Action Per Date
(B on 2022-12-21) Change: 380.32, Stock: 380.32, Balance: 67000.00
(S on 2022-12-22) Change: 83.67, Stock: 296.65, Balance: 74010.66
(B on 2022-12-23) Change: 286.49, Stock: 583.14, Balance: 49587.15
(S on 2022-12-27) Change: 128.29, Stock: 454.85, Balance: 60240.40
(S on 2022-12-28) Change: 100.07, Stock: 354.78, Balance: 68427.86
(B on 2022-12-29) Change: 268.25, Stock: 623.03, Balance: 45846.66
(S on 2022-12-30) Change: 137.07, Stock: 485.96, Balance: 57360.27
(B on 2023-01-03) Change: 220.57, Stock: 706.53, Balance: 38431.38
(S on 2023-01-04) Change: 155.44, Stock: 551.09, Balance: 51665.24
(S on 2023-01-05) Change: 121.24, Stock: 429.85, Balance: 61742.74
(S on 2023-01-06) Change: 94.57, Stock: 335.28, Balance: 69883.11
(S on 2023-01-09) Change: 73.76, Stock: 261.52, Balance: 76327.01
(S on 2023-01-10) Change: 57.53, Stock: 203.99, Balance: 81497.67
(B on 2023-01-11) Change: 282.83, Stock: 486.82, Balance: 54603.44
(B on 2023-01-12) Change: 189.14, Stock: 675.95, Balance: 36584.30
(B on 2023-01-13) Change: 123.04, Stock: 799.00, Balance: 24511.48
(S on 2023-01-17) Change: 175.78, Stock: 623.22, Balance: 41395.06
(B on 2023-01-18) Change: 143.10, Stock: 766.32, Balance: 27734.69
(S on 2023-01-19) Change: 168.59, Stock: 597.73, Balance: 43528.17
(B on 2023-01-20) Change: 147.70, Stock: 745.43, Balance: 29163.88
(B on 2023-01-23) Change: 98.69, Stock: 844.12, Balance: 19539.80
(S on 2023-01-24) Change: 185.71, Stock: 658.41, Balance: 37427.04
(S on 2023-01-25) Change: 144.85, Stock: 513.56, Balance: 51503.67
(S on 2023-01-26) Change: 112.98, Stock: 400.58, Balance: 62713.92
(B on 2023-01-27) Change: 202.42, Stock: 603.00, Balance: 42018.33
(B on 2023-01-30) Change: 137.90, Stock: 740.90, Balance: 28152.28
(B on 2023-01-31) Change: 90.08, Stock: 830.99, Balance: 18862.03
(B on 2023-02-01) Change: 59.20, Stock: 890.18, Balance: 12637.56
(B on 2023-02-02) Change: 36.94, Stock: 927.12, Balance: 8467.16
(S on 2023-02-03) Change: 203.97, Stock: 723.15, Balance: 29555.19
(S on 2023-02-06) Change: 159.09, Stock: 564.06, Balance: 45811.34
(S on 2023-02-07) Change: 124.09, Stock: 439.97, Balance: 58482.46
(S on 2023-02-08) Change: 96.79, Stock: 343.17, Balance: 68166.53
(S on 2023-02-09) Change: 75.50, Stock: 267.67, Balance: 75583.46
(B on 2023-02-10) Change: 255.53, Stock: 523.21, Balance: 50640.92
(S on 2023-02-13) Change: 115.11, Stock: 408.10, Balance: 62098.54
(B on 2023-02-14) Change: 205.54, Stock: 613.64, Balance: 41606.02
(B on 2023-02-15) Change: 135.73, Stock: 749.37, Balance: 27876.03
(B on 2023-02-16) Change: 93.72, Stock: 843.09, Balance: 18676.94
(B on 2023-02-17) Change: 63.41, Stock: 906.50, Balance: 12513.55
(S on 2023-02-21) Change: 199.43, Stock: 707.07, Balance: 31375.71
(B on 2023-02-22) Change: 108.09, Stock: 815.16, Balance: 21021.73
(B on 2023-02-23) Change: 72.40, Stock: 887.56, Balance: 14084.56
(S on 2023-02-24) Change: 195.26, Stock: 692.30, Balance: 32341.69
(S on 2023-02-27) Change: 152.31, Stock: 539.99, Balance: 46621.85
(S on 2023-02-28) Change: 118.80, Stock: 421.19, Balance: 57816.21
(S on 2023-03-01) Change: 92.66, Stock: 328.53, Balance: 66356.92
(S on 2023-03-02) Change: 72.28, Stock: 256.25, Balance: 73015.79
(B on 2023-03-03) Change: 253.90, Stock: 510.16, Balance: 48920.58
(B on 2023-03-06) Change: 172.20, Stock: 682.36, Balance: 32776.79
(B on 2023-03-07) Change: 115.62, Stock: 797.98, Balance: 21960.45
(S on 2023-03-08) Change: 175.55, Stock: 622.42, Balance: 38448.56
(S on 2023-03-09) Change: 136.93, Stock: 485.49, Balance: 51080.62
(S on 2023-03-10) Change: 106.81, Stock: 378.68, Balance: 60771.27
(S on 2023-03-13) Change: 83.31, Stock: 205.37, Balance: 68171.61
```

Figure 11.2: CSV of the Bot Process (Daily)

After swapping out files in the stock bot program, and manually doing some MatLab work and downloading csv files via Google Sheets, this is the daily Bot Process! Graphing it will show a better idea of what it looks like.

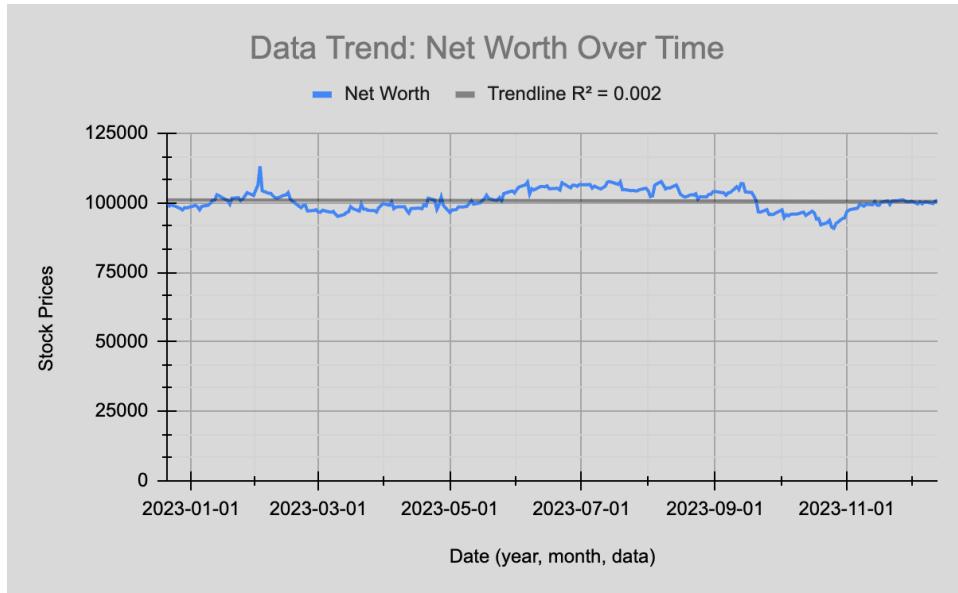


Figure 11.3: Net Worth Over Time Graph (Daily)

The daily net worth over time did experience some jumps in profit in February, but then the stock bot went negative from March through the ending of May. In June through the few days of September, the stock bot maintained a profit, but then dropped until breaking even around November. Overall, not bad for a basic stock bot. Other Heuristics such as mean, variance, and standard deviation, or calculating the momentum of the prices going up and down, may have given the bot a better indication of when to buy and sell stocks. Also, perhaps increasing or decreasing the percentage of purchasing and selling stocks may have had a positive or negative impact. For the initial test of weekly prices, lowering the selling of stocks did boost profits.

PART VIII - New Statistic Library

```
lic static void main (String [] args)
{
    UniformDistribution uniform;
    uniform = new UniformDistribution ();

    System.out.printf("The Uniform Probability of the given parameters is: %s%n", uniform.probability(2, 10, 7, 10));
    System.out.printf("The Uniform Expected of the given parameters is: %s%n", uniform.expected(2, 10));
    System.out.printf("The Uniform Standard Deviation of the given parameters is: %s%n", uniform.standardDeviation(2, 10));
}
```

Figure 12.0: Configuration for UniformDistribution.java Class

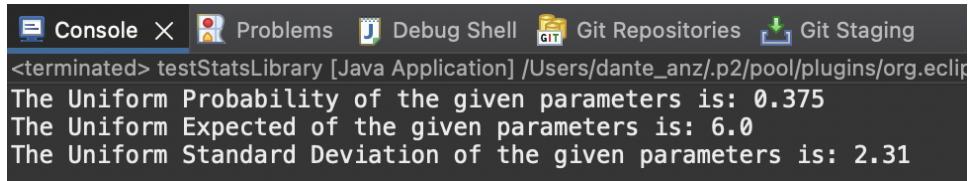


Figure 12.1: Output from UniformDistribution.java Class

## References

- Baird, Benjamin, et al. "The Cognitive Neuroscience of Lucid Dreaming." *Neuroscience & Biobehavioral Reviews*, vol. 100, May 2019, pp. 305–323,  
[www.sciencedirect.com/science/article/pii/S0149763418303361](http://www.sciencedirect.com/science/article/pii/S0149763418303361),  
<https://doi.org/10.1016/j.neubiorev.2019.03.008>.
- "Beta Distribution - Definition, Formulas, Properties, Applications." *BYJUS*,  
[byjus.com/math/beta-distribution/#:~:text=The%20most%20common%20use%20of](http://byjus.com/math/beta-distribution/#:~:text=The%20most%20common%20use%20of).
- Bhandari, Pritha. "Normal Distribution | Examples, Formulas, & Uses." *Scribbr*, 23 Oct. 2020,  
[www.scribbr.com/statistics/normal-distribution/#:~:text=To%20find%20the%20probability%20of](http://www.scribbr.com/statistics/normal-distribution/#:~:text=To%20find%20the%20probability%20of).
- Bourke, Patrick. *School of Psychology Lucid Dreaming: Large Data Base Analysis. Key Points*. 2018.
- Brandwein, Sharon. "Lucid Dreams: What They Are and How to Experience Them | Sleepopolis." *Sleepopolis*,  
[sleepopolis.com/education/lucid-dreams/#:~:text=Prioritize%207%2D9%20Hours%20of](http://sleepopolis.com/education/lucid-dreams/#:~:text=Prioritize%207%2D9%20Hours%20of).
- "Exam 2020-21 - Get Direct Link to Download Mains Admit Card." *Testbook*,  
[testbook.com/math/gamma-distribution/#:~:text=Gamma%20distributions%20are%20common%20in](http://testbook.com/math/gamma-distribution/#:~:text=Gamma%20distributions%20are%20common%20in).
- "Java: Read a CSV File into an Array." *Www.youtube.com*,  
[www.youtube.com/watch?v=-Aud0cDh-J8&ab\\_channel=AlexLee](http://www.youtube.com/watch?v=-Aud0cDh-J8&ab_channel=AlexLee).
- "Jfreechart Installation - Javatpoint." *Www.javatpoint.com*,  
[www.javatpoint.com/jfreechart-installation](http://www.javatpoint.com/jfreechart-installation).
- "JFreeChart Scatter Chart - Javatpoint." *Www.javatpoint.com*,  
[www.javatpoint.com/jfreechart-scatter-chart](http://www.javatpoint.com/jfreechart-scatter-chart).
- Mallett, Remington, et al. "Benefits and Concerns of Seeking and Experiencing Lucid Dreams: Benefits Are Tied to Successful Induction and Dream Control." *Oxford Academic*, 7 Sept. 2022, [academic.oup.com/sleepadvances/article/3/1/zpac027/6693839](http://academic.oup.com/sleepadvances/article/3/1/zpac027/6693839).
- "MATLAB Crash Course for Beginners." *Www.youtube.com*,  
[www.youtube.com/watch?v=7f50sQYjNRA&ab\\_channel=freeCodeCamp.org](http://www.youtube.com/watch?v=7f50sQYjNRA&ab_channel=freeCodeCamp.org).
- "Nightmares and the Brain." *Hms.harvard.edu*,  
[hms.harvard.edu/news-events/publications-archive/brain/nightmares-brain/#:~:text=Dreams%20are%20part%20of%20the](http://hms.harvard.edu/news-events/publications-archive/brain/nightmares-brain/#:~:text=Dreams%20are%20part%20of%20the).
- Ribeiro, Nicolas , et al. "Is There a Link between Frequency of Dreams, Lucid Dreams, and Subjective Sleep Quality?" 25 June 2020.
- "RSI Calculation - Macroption." *Www.macropoint.com*, [www.macropoint.com/rsi-calculation/](http://www.macropoint.com/rsi-calculation/).
- Staff, Sleep Review. "Women Hold Significant Advantage in Mastering Lucid Dreaming." *Sleep Review*, 26 Oct. 2023,  
[sleepreviewmag.com/sleep-health/sleep-whole-body/brain/women-significant-advantage-men-mastering-lucid-dreaming/#:~:text=The%20statistical%20analysis%20showed%20that](http://sleepreviewmag.com/sleep-health/sleep-whole-body/brain/women-significant-advantage-men-mastering-lucid-dreaming/#:~:text=The%20statistical%20analysis%20showed%20that).
- "StatUtils (Apache Commons Math 4.0-Beta1)." *Commons.apache.org*,  
[commons.apache.org/proper/commons-math/commons-math-docs/apidocs/org/apache/commons/math4/legacy/stat/StatUtils.html](http://commons.apache.org/proper/commons-math/commons-math-docs/apidocs/org/apache/commons/math4/legacy/stat/StatUtils.html).
- "Storing Csv File Contents into Multiple Arrays." *Stack Overflow*,  
[stackoverflow.com/questions/33839008/storing-csv-file-contents-into-multiple-arrays](http://stackoverflow.com/questions/33839008/storing-csv-file-contents-into-multiple-arrays).

“Table 1 Demographic, Behavioral and Questionnaire Data for the Frequent Lucid Dream Group and Control Group.” *Www.nature.com*,  
[www.nature.com/articles/s41598-018-36190-w/tables/1](http://www.nature.com/articles/s41598-018-36190-w/tables/1). Accessed 10 Dec. 2023.

Team, Great Learning. “Gamma Distribution Explained | What Is Gamma Distribution?” *Great Learning Blog: Free Resources What Matters to Shape Your Career!*, 25 June 2020,  
[www.mygreatlearning.com/blog/gamma-distribution/#:~:text=events%20are%20relevant](http://www.mygreatlearning.com/blog/gamma-distribution/#:~:text=events%20are%20relevant).

“The Normal Distribution and Its Applications in Quality Control and Process Improvement.”  
*Www.linkedin.com*,  
[www.linkedin.com/pulse/normal-distribution-its-applications-quality-control-process-shaikh#:~:text=The%20normal%20distribution%20is%20widely%20used%20in%20businesses%20for%20various](http://www.linkedin.com/pulse/normal-distribution-its-applications-quality-control-process-shaikh#:~:text=The%20normal%20distribution%20is%20widely%20used%20in%20businesses%20for%20various).

Wackerly, Dennis D., et al. *Mathematical Statistics with Applications*. 1973. 7th ed., Thomson Brooks/Cole, 1 Jan. 2008, pp. 30 through 290.