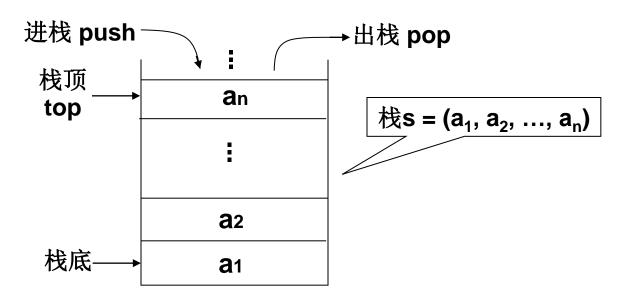
Ch3 栈和队列

■ 栈(stack)的定义和特点

定义 限定仅在表尾进行插入或删除运算的线性表, 表尾 -- 栈顶 (top), 表头 -- 栈底(bottom), 不含数据元素的空表称空栈 特点 后进先出(Last-In-First-Out / LIFO) 或新进后出(FILO)

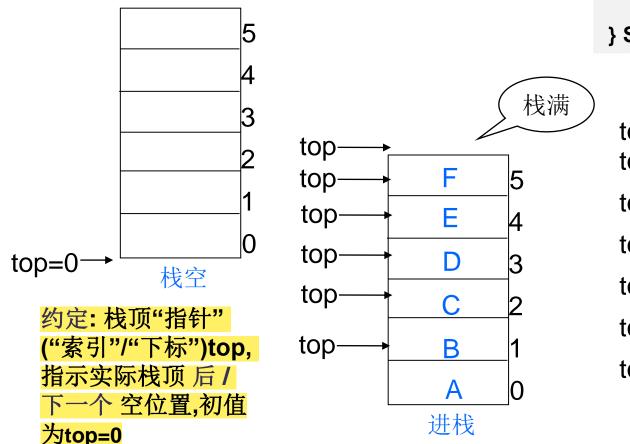


Applications of stack: (生活中, 厨房里叠在一起的盘子)

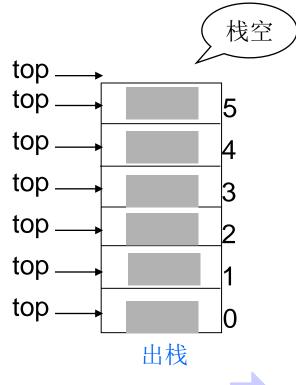
- Balancing of symbols
- Redo-undo features at many places like editors, photoshop
- Forward and backward feature in web browsers
- Used in many algorithms like Tower of Hanoi, tree traversals

栈的存储结构 -- 顺序结构和链式结构 顺序结构的栈

数据结构的表示: 用一维数组实现 ▶



#define MaxSize 6
typedef char StackElem;
typedef struct seqstack {
 int top; //指示栈顶位置
 StackElem elem[MaxSize];
} SeqStack;



top == MaxSize: 栈满, 若此时入栈, 则上溢(overflow)

top == 0: 栈空, 若此时出栈, 则下溢(underflow)

栈的存储结构 -- 顺序结构和链式结构 顺序结构的栈

几个运算及其算法描述 P46-47

```
//create an empty stack
void InitialStack(SeqStack *s)
{ s->top = 0; }

//StackEmpty: returns non-zero
//if the stack is empty
Boolean StackEmpty(SeqStack *s)
{ return s->top <= 0; }</pre>
```

```
Boolean StackFull(SeqStack *s) { return s->top >= MaxSize; }
```

```
//push an item onto the stack
void Push(StackElem item, SeqStack *s)
{ if (StackFull(s))
        Error("Stack is full");
    else s->elem[s->top++] = item; }

//return and remove the item that
//was inserted most recently
StackElem Pop(SeqStack *s)
{ if (StackEmpty(s)) Error("Stack is empty");
    else return s->elem[--s->top]; }
```

若约定栈顶"指针"top 指示实际栈顶的位置, 则其初值为top=–1. 上 述基本运算需作适当修 改: →

```
初始化: s->top = -1;
判栈空: return s->top <= -1;
判栈满: return s->top >= MaxSize -1;
入栈: s->elem[++ s->top] = item;
出栈:s->elem[s->top--]; 取栈顶元素:s->elem[s->top];
```

栈的存储结构 -- 顺序结构和链式结构 链式结构的栈

数据结构的表示 用单链表实现 → typedef struct node {
 StackElem elem;
 struct node *next;
 } Stack, *LinkedStack;

在存储空间足够大的情况下,链栈一般不考虑溢出现象. 考虑到计算的方便性,链栈中一般无需设置头结点

链栈的初始化

```
void InitialStack(LinkedStack top)
{ top = NULL; }
```

判断链栈是否空栈

```
boolean StackEmpty(LinkedStack top)
{ return top == NULL; }
```

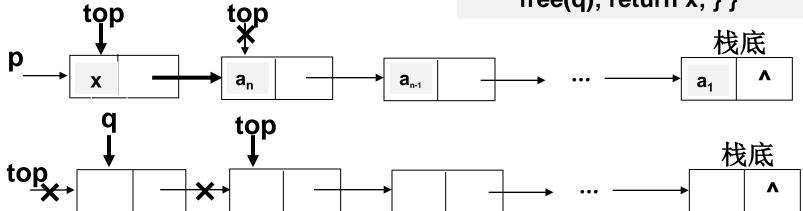
栈的存储结构 -- 顺序结构和链式结构 链式结构的栈 用单链表实现

链栈入栈运算

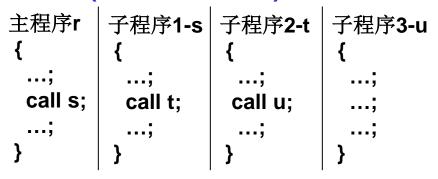
void Push(LinkedStack top, StackElem x) { p = (Stack *)malloc(sizeof(Stack)); p->elem = x; p->next = top; top = p; }

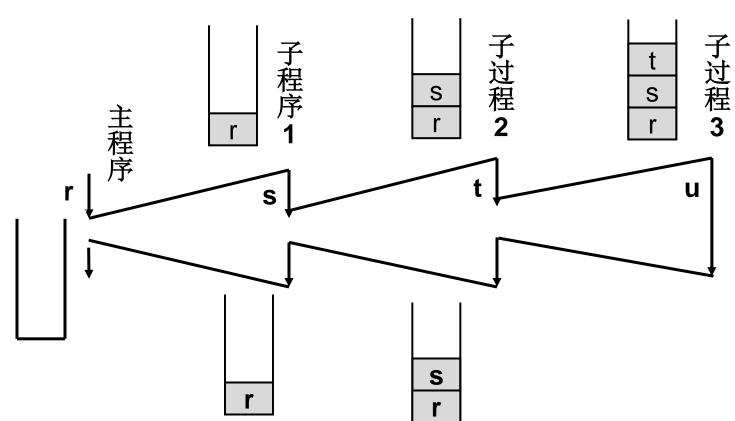
链栈出栈运算

```
StackElem Pop(LinkedStack top) {
   if ( StackEmpty(top) )
      Error("Stack is empty");
   else {
      x = top -> elem;
      q = top; top = top->next;
      free(q); return x; } }
```



1. 子程序(过程/函数/方法)的嵌套调用-- Subprogram calls

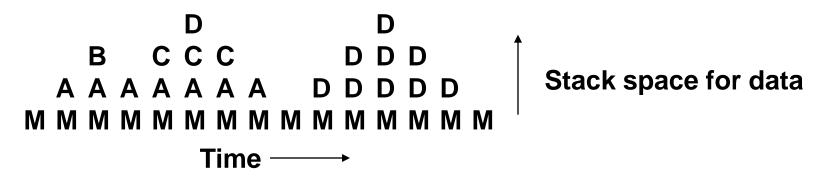


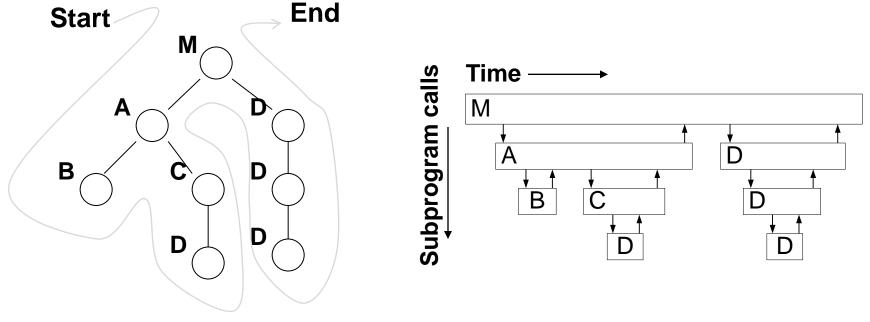




- 栈的应用举例
 - 1. 子程序(过程/函数/方法)的嵌套调用-- Subprogram calls

又如: Stack frames for subprogram calls





Tree of subprogram calls

- 栈的应用举例
 - 2. 递归函数及其实现 → 即递归过程R包含另一过程D, 而D又调用R 递归函数: 函数直接 或 间接调用自身的过程 / function that calls itself.
 - 一个例子 -- 递归函数的执行情况分析

```
void print(int w)
{ int i;
    if ( w != 0 )
        { print(w-1);
            for( i=1; i<=w; ++i ) printf("%3d,",w);
            printf("/n");
        }
}</pre>
```

设w=3,则 运行结果: 1, 2,2, 3,3,3,



■ 栈的应用举例 \mathbf{W} \mathbf{W} 2. 递归函数及其实现 0 递归函数调用执行过程: \mathbf{W} 2 **print(0)**; 返回 (4)输出: 1 \mathbf{W} 主程序 print(1) 3 (3)输出: 2,2 print(2); w=3;(2)输出: 3,3,3 void print(int w){ print(w); int i; if (w!=0)**(1)** { print(w-1); for(i=1; i<=w; ++i) printf("%3d,",w); printf("/n"); } } 结束 top **(4)** 0 top **(3) (3)** top 2 top **(2) (2) (2)** 3 3 3 **(1)** 3 **(1)** (1)(1)



2. 递归函数及其实现

又如 The factorial function:
$$n! = n \times (n-1) \times \cdots \times 1$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{if } n > 0 \end{cases}$$

$$4! = 4 \times 3! = 4 \times (3 \times 2!)$$

$$= 4 \times (3 \times (2 \times 1!))$$

$$= 4 \times (3 \times (2 \times (1 \times 0!)))$$

$$= 4 \times (3 \times (2 \times 1))$$

$$= 4 \times (3 \times 2!) = 4 \times 6 = 24$$
int Factorial(int n)
$$\{ \text{ if } (n == 0) \\ \text{ return } 1; \\ \text{ else } \\ \text{ return } n * \text{ Factorial}(n-1); \}$$

COMMENTS Every recursive process consists of two parts:

- 1. A base case that is processed without recursion; //边界条件
- 2. A general method that reduces a particular case to one or more of the smaller cases, thereby making progress toward eventually reducing the problem all the way to the base case. //收敛: 使问题向边界条件转化的规则

That is find a stopping rule and the reduction step (i.e. define the recursion).

2. 递归函数及其实现

又如 The factorial function: $n! = n \times (n-1) \times \cdots \times 1$ $n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{if } n > 0 \end{cases}$ $4! = 4 \times 3! = 4 \times (3 \times 2!)$ $n! = \begin{cases} 1 & \text{int Factorial(int n)} \\ \text{if } (n == 0) \\ \text{return 1;} \end{cases}$

$$4! = 4 \times 3! = 4 \times (3 \times 2!)$$

= $4 \times (3 \times (2 \times 1!))$
= $4 \times (3 \times (2 \times (1 \times 0!)))$
= $4 \times (3 \times (2 \times (1 \times 1)))$

$$=4\times(3\times(2\times1))$$

$$= 4 \times (3 \times 2) = 4 \times 6 = 24$$

int Factorial(int n)
{ if (n == 0)
 return 1;
 else
 return n * Factorial(n-1); }

如何评估递归描述形式的算法的时间复杂性:

■ T(n) is time used for input n:

$$T(n) = \{ 1 & \text{if n is 0} \\ 1 + T(n-1) & \text{otherwise } \}$$

Repeated substitution expands recurrence:

$$T(n) = 1 + T(n-1)$$

= 1 + (1 + T(n-2))
= 1 + (1 + (1 + T(n-3)))

. . .

Generalize pattern:

$$= i*1 + T(n - i)$$

Solve for i = n:

2. 递归函数及其实现

再如 梵·塔问题(Towers of Hanoi Problem)

Move of 64 disks from tower 1 to tower 3 using tower 2 as temporary storage.

Moving rules:

- Move only one disk at a time
- Never place a larger disk on a smaller one

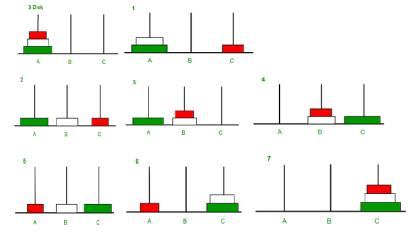
Trace for Disks = 2:

Move disk 1 from 1 to 2

Move disk 2 from 1 to 3

Move disk 1 from 2 to 3

Trace for Disks = 3:



The pattern is:

2. 递归函数及其实现

再如 梵·塔问题(Towers of Hanoi Problem)

```
#define Disks 64 //Number of disks on the first towers. P55/A.3.5
void main() { Move(Disks, 1, 3, 2); }
void Move(int count, int start, int finish, int temp)
{ if (count == 1) {
     printf("Move disk 1 from %d to %d.\n", start, finish);
     return; }
  Move(count - 1, start, temp, finish);
  printf("Move disk %d from %d to %d.\n", count, start, finish);
  Move(count – 1, temp, finish, start); }
T(n) is time used for input n: T(n) = { 1
                                                        if n = 1
                                           1 + 2T(n-1) otherwise }
Repeated substitution expands recurrence:
         T(n) = 1 + 2T(n-1) = 1 + (2 + 4T(n-2)) = 1 + (2 + (4 + 8T(n-3))) = ...
• Generalize pattern: = 1 + 2 + 4 + 8 + 16 + ... + 2^{i-1} + 2^{i}T(n - i) = 2^{i} - 1 + 2^{i}T(n - i)
• Solve for n - i = 0 (i = n): = 2^n - 1 + 2^n T(0) = 2^n - 1 -> O(2^n)
n=64个disks共移动次数:
\Sigma = 1 + 2 + 4 + \cdots + 2^{63} = 2^{0} + 2^{1} + 2^{2} + \cdots + 2^{63} = 2^{64} - 1
  = 2^{n} - 1
```

```
2. 递归函数及其实现
  Comparisons between Recursion(递归) and Iteration(迭代/递推)
  Fibonacci Numbers:
                             F_0 = 0,
                             F_1 = 1,
                             F_n = F_{n-1} + F_{n-2} for n \ge 2
 // Fibonacci: recursive version.
 int Fibonacci(int n)
 { if(n <= 0) return 0;
    else if (n == 1) return 1;
         else return Fibonacci(n-1) + Fibonacci(n-2); }
 // Fibonacci: iterative version.
 int Fibonacci(int n)
 { int i, twoback, oneback, current;
    if(n \le 0) return 0;
    else if (n == 1) return 1;
         else { twoback = 0; oneback =1;
               for (i = 2; i \le n; i++)
                   current = twoback + oneback;
                   twoback = oneback; oneback = current; }
                return current; } }
```

2. 递归函数及其实现

Recursion	Iteration
an implicit stack sometimes duplicate tasks	sometimes an explicit stack much more complicated and
转子(递归调用)语句的时间开销	harder to understand

Conclusions (be at issue)

It's possible to take any recursive program into nonrecursive form. But the only reason for removal recursion is if you are forced to program in a language that does not support recursion.

- 栈的应用举例
 - 2. 递归函数及其实现

递归小结 -- Thinking recursively

- Recursive decomposition is the hard part
 - Find recursive sub-structure solve problems using result from smaller subproblem(s)
 - · Identify base case simple possible case, directly solvable, recursion advances to it
- Common patterns
 - Handle first and/or last, recur on remaining
 - divide in half, recur on one/both halves
 - make a choice among options, recur on updated state
- Placement of recursive call(s)
 - · Recur-then-process vs. process-then-recur

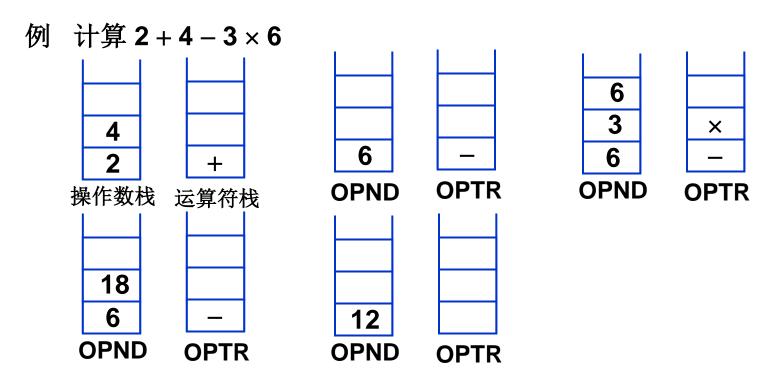
递归算法适用的一般场合:

- 数据的定义形式按递归定义, 如 计算裴波那契数列 等;
- •数据之间的关系按递归定义,如树的遍历,图的搜索等;
- •问题解法按递归算法实现,如 分治法,回溯法 等.

- 栈的应用举例
 - 3. (中缀)表达式的计算
 - P52-53 · 操作数(operand)和运算符(operator)
 - •运算规则(关键是运算符的优先级的考虑: P53/表3.1)
 - · 算法的基本思想: 设有两个栈 OPND & OPTR

依次读表达式中的每个字符, 若是操作数则进OPND栈; 若是操作符, 则和OPTR栈的栈顶运算符比较优先级后作相应操作, 直至表达式空为止.

算法描述: P53-54/A.3.4

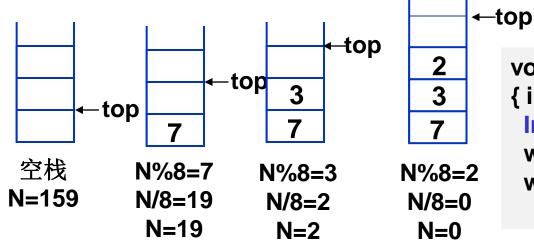


4. 其它应用

数制转换、地图着色等

例 把十进制的数(N=159)转换成八进制(B=8)表示的数:

$$N = 159_{10} = 2 \cdot 8^2 + 3 \cdot 8^1 + 7 \cdot 8^0 = 237_8 = \sum_{i=0}^{\lfloor \log_8 N \rfloor} b_i \cdot B^i$$
 其中: $0 \le b_i \le B-1$ 令 $j = \lfloor \log_8 N \rfloor$,则 $N = b_j \cdot B^j + b_{j-1} \cdot B^{j-1} + \cdots + b_1 \cdot B + b_0$ $= (b_j \cdot B^{j-1} + b_{j-1} \cdot B^{j-2} + \cdots + b_2 \cdot B + b_1) \cdot B + b_0$ 上式中的 b_0 为N整除B所得的余数,括号中的和式恰为N整除B所得的命。由此看出其本质是递归运算: $M = (N \text{ div } B) \times B + N \text{ mod } B$



void MultiBaseOutput(int N, int B)
{ int i; Stack *s;
 InitStack(s);
 while (N) { Push(s, N%B); N = N/B; }
 while (!StackEmpty(s))
 printf(Pop(s)); } //O(log_BN)

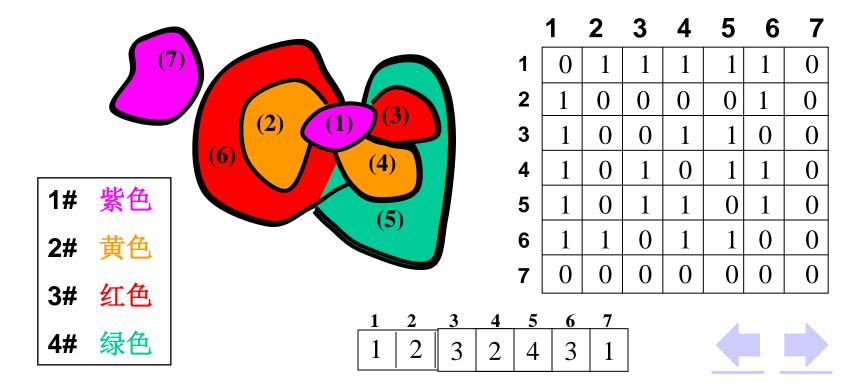
4. 其它应用

数制转换、地图着色等

例 地图四染色问题: 用四种不同的颜色给地图的各个图块填色,

要求相邻的图块所填的颜色不同.

思路: 采用解决"回溯"问题的算法,即一个问题由若干步组成, 在每一步的处理中都有若干种选择可能,整个问题最终 由每一步的恰当选择而集成.通过运用栈的元素"后进先出" 的特点来具体实现回溯.



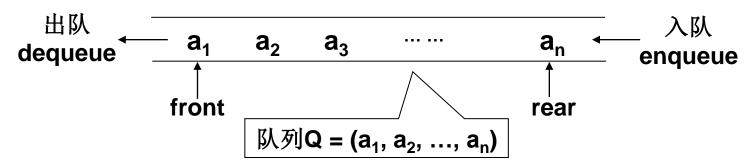
Exercises (After class):

- 1. 字符a, b, c, d依次通过一个栈, 按出栈的先后次序组成字符串, 至多可以组成多少个不同的字符串? 并分别写出它们.
- 2. 若栈的数据元素存放在一维数组S[1..Max]中, 栈顶指示top的初值为0, 若top<Max, 写出将数据元素x入栈的算法关键语句.
- 3. 画出计算表达式 (3+3)/2×5-6 时操作数栈和操作符栈的变化情况.
- 4. 利用栈的基本运算写出求解下列问题的算法(回文游戏): read one line of input and write it backward.
- 5. Euclid's algorithm: The greatest common divisor(GCD) of two positive integers is the largest integer that divides both of them. E.g.GCD(216,192)=24.
 - 1) Write a recursive function GCD(x,y:integer): if y=0,then the GCD of x and y is x; otherwise the GCD of x and y is the same as the GCD of y and x%y.
 - 2) Rewrite the function in iterative form.

■ 队列(queue)的定义及特点

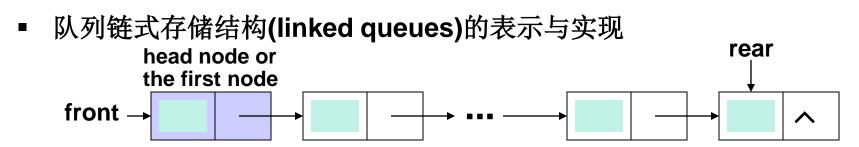
定义 队列是限定只能在表的一端进行插入运算,在表的另一端进行删除的线性表.其中,允许插入的一端称之为队尾(rear),允许删除的一端称之为队头(front)

特点 先进先出(FIFO)



The property of Queue makes it useful in following kind of scenarios:

- When a resource is shared among multiple consumers. Examples include Call Center, CPU scheduling, etc. [*Priority Queues*]
- When data is transferred asynchronously (data not necessarily received at same rate as sent) between two processes.
 Examples include IO Buffers, etc.



■ 队列链式存储结构(linked queues)的表示与实现

链式结构的队列 用单链表表示→

typedef char QueueElem;
typedef struct queuenode {
 QueueElem elem;
 struct queuenode *next;
} QueueNode;
typedef struct queue {
 QueueNode *front;
 QueueNode *rear;
} Queue;

空队列

队头结点

X

x入队

运算/操作 (不带头结点):Queue *q;

*q

*a

q->front

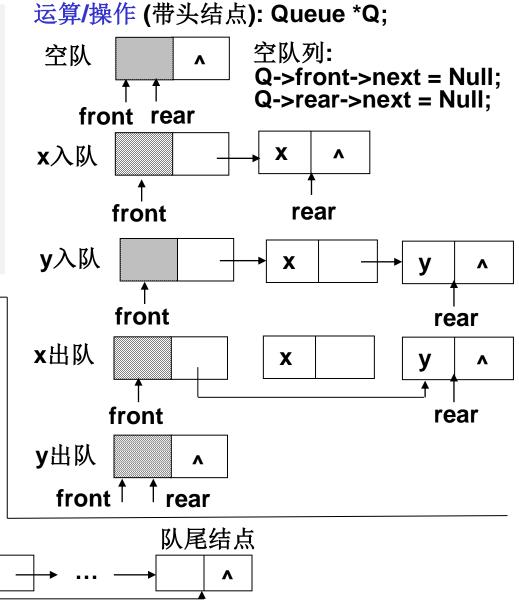
q->rear

q->front

q->front

q->rear

q->rear



■ 队列链式存储结构(linked queues)的表示与实现 算法描述 P61-62

```
//create an empty queue
void InitialQueue(Queue *q)
{ q->front = q->rear = Null; } //P62 Q->front->next = Q->rear->next = Null
                             //because Q points to the head node
//is the queue empty?
boolean QueueEmpty(Queue *q)
{ return q->front == Null || q->rear == Null; }
//Q->front == Q->rear || Q->front->next == Null
//get the front item of the queue
QueueElem QueueFront (Queue *q)
{ if (QueueEmpty(q))
   Error("The queue is empty.");
 else
   return q->front->elem; //Q->front->next->elem
}
```

■ 队列链式存储结构(linked queues)的表示与实现 算法描述 P61-62

```
//insert an item onto queue
void EnQueue(QueueNode *p, Queue *q)
{ if (!p) Error("Attempt to append a nonexistent node to the queue.");
 else if (QueueEmpty(q))
        q->front = q->rear = p; //set both front and rear to p
                              //若带头结点: Q->front->next = Q->rear = p;
     else { //place p after previous rear of the queue
        q->rear->next = p;
        q->rear = p; } }
//return and remove the item that was inserted least recently
QueueElem DeQueue(Queue *q)
{ if (QueueEmpty(q))
   Error("Attempt to delete a node from an empty queue.");
 else { p = q - front; //P62 p = Q - front - next}
       x = p \rightarrow elem;
       q->front = p->next; //Q->front->next = p->next;
       if (q->rear == p) //当队列中只有一个结点时, 出队后队列为空. Q->rear == p.
          q->rear = Null; //所以队尾指针置为空. Q->rear = Q->front(指向头结点)
       free(p); return x; } }
```

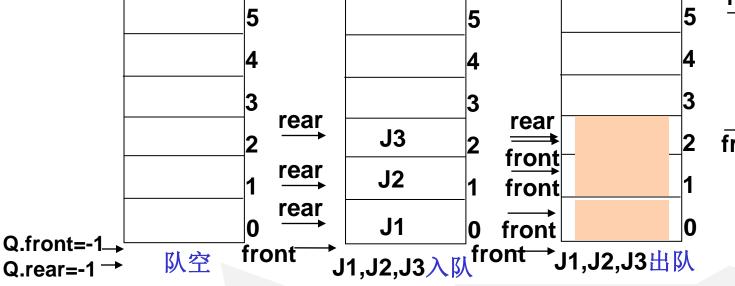
队列顺序存储结构(contiguous queues)的表示与实现

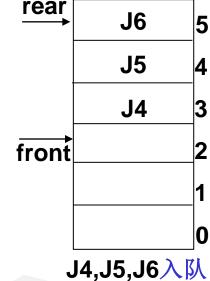
一维数组→

#define MaxQueue 6 typedef char QueueElem; typedef struct queue { int front; int rear; QueueElem elem[MaxQueue]; } Queue;



实现 设Queue Q;





约定:

- Q.rear 指示队尾元素
- Q.front 指示队头元素前一位置 则初值 Q.front = Q.rear = -1

设Queue Q; 则

队列满条件: Q.rear == MaxQueue-1

入队列: Q.elem[++rear] = x

空队列条件: Q.front == Q.rear

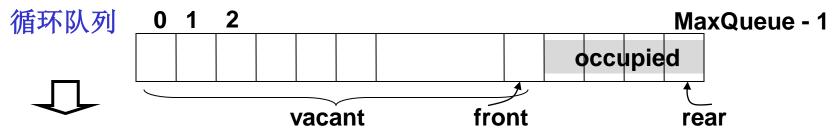
出队列: x = Q.elem[++front]



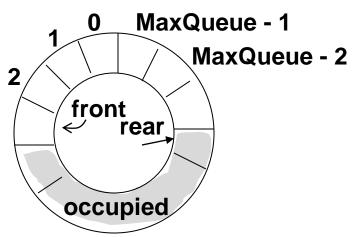
溢

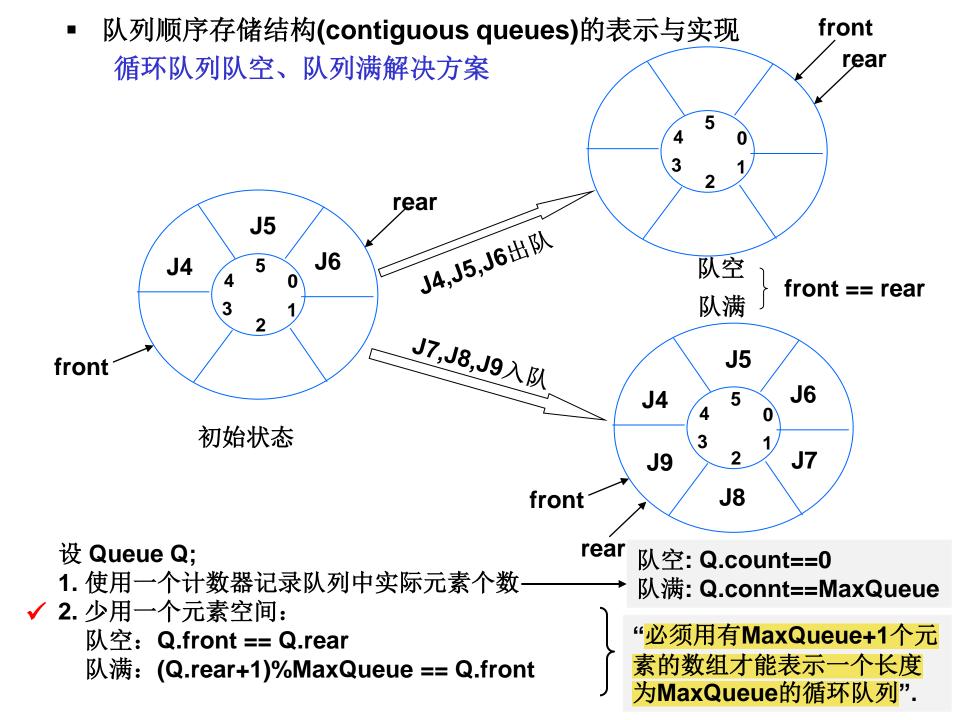
出

■ 队列顺序存储结构(contiguous queues)的表示与实现



Implementation of circular arrays, 即当rear指示数组上界MaxQueue-1时,则:





■ 队列顺序存储结构(contiguous queues)的表示与实现 队列算法的描述---基于循环队列 P64~65

```
void InitialQueue(Queue *q) { q->front = q->rear = -1; }
boolean QueueEmpty(Queue *q) { return q->front == q->rear; }
boolean QueueFull(Queue *q) { return (q->rear+1)%MaxQueue == q->front; }
void EnQueue(QueueElem x, Queue *q)
{ if (QueueFull(q)) Error("Attempt to add an element to a full queue.");
 else { q->rear = (q->rear + 1)%MaxQueue; q->elem[q->rear] = x; } }
QueueElem DeQueue(Queue *q)
{ if (QueueEmpty(q)) Error("Attempt to delete an element from a empty queue.");
 else { q->front = (q->front + 1)%MaxQueue; return q->elem[q->front]; } }
QueueElem QueueFront (Queue *q) //get the front element of the queue
{ if (QueueEmpty(q)) Error("The queue is empty.");
 else return q->elem[q->front + 1]; }
```

若约定队列的rear指示 队尾元素的下一个空位 置, front 指示队头元素, 则这些算法应如何调整?

```
初始化: q->front = 0; q->rear = 0; 判队空、队满: 不变化;

入队: q->elem[q->rear] = x; q->rear = (q->rear+ 1)%MaxQueue;

出队: x = q->elem[q->front]; q->front=(q->front+1)%MaxQueue;

return x;

取队首元素: q->elem[q->front];
```

■ 队列的应用

The Problem of Dancing Partner: 假设在周末舞会上, 男士们和女士们进入舞厅时各自排成一队. 跳舞开始时, 依次从男队和女队的队头上各出一人配成舞伴. 若两队初始人数不同, 则较长的那一队中未配对者需等待下一轮舞曲. 现要求写一算法模拟上述舞伴配对问题.

问题分析及算法思想:该问题具有典型的先进先出的特征,可用队列作为算法的数据结构.在算法中假设男士和女士的记录存放在一个数组中作为输入,①扫描该数组中各元素并根据其性别分别进入男队和女队;②依次将两队列当前的队头元素出队配成舞伴,直至某队列空为止;③若某队仍有等待配对者,算法将输出该队列的队头等待者的名字,他/她将是下一轮舞曲开始时第一个可获得舞伴的人.

```
typedef struc person {
    char *name;
    char sex; // F --- female, M --- male
} Person;
```

typedef Person QueueElem; // 将队列中元素的数据类型改为Person

■ 队列的应用

```
void DancingPartners(Person dancer[], int num) {
   Person p;
   Queue *Fdancers, *Mdancers; //定义两个循环队列
   InitQueue(*Fdancers); InitQueue(*Mdancers); //队列初始化
   for ( i = 0; i < num; i++ ) //入队列
      if ( dancer[i].sex == 'F') EnQueue(dancer[i], Fdancers);
      else EnQueue(dancer[i], Mdancers);
   while ( !QueueEmpty(*Fdancers) && !QueueEmpty(*Mdancers) ) {
      p = DeQueue(*Fdancers); printf("%s ", p.name);
      p = DeQueue(*Mdancers); printf("%s\n", p.name); }
   if (!QueueEmpty(*Fdancers)) { //输出女队剩下的队首女士名字
      p = QueueFront(*Fdancers);
      printf("%s will be the first to get a partner.\n", p.name); }
   else if (!QueueEmpty(*Mdancers)) { //输出男队剩下的队首男士名字
         p = QueueFront(*Mdancers);
         printf("%s will be the first to get a partner.\n", p.name); }
队列的其它应用: 另见实习题
```



Exercises (After class):

- 1. Write the C function QueueSize() for a linked queue that returns the number of items in it.
- 2. 数组Q[1..n]表示一个循环队列,设f的值为队列中第一个元素的位置,r的值为队列中实际队尾的位置加1,并假定队列中最多只有n-1个元素,则计算队列中元素个数的公式是什么?
- 3. 设栈 S = (1,2,3,4,5,6,7),其中7为栈顶元素.写出调用algo(&S)后栈的状态.

```
void algo(Stack *S)
{ int i=0;
   Queue Q; Stack T;
   InitialQueue(&Q); InitialStack(&T);
   while (!StackEmpty(S))
      if ( ( i = !i ) != 0 ) Push(&T, Pop(&S)); else EnQueue(&Q, Pop(&S));
   while (!QueueEmpty(Q)) Push(&S, DeQueue(&Q));
   while (!StackEmpty(T)) Push(&S, Pop(&T)); }
```

4. 假设有一循环队列中只设rear和queuelen来分别指示队尾元素的位置和队列中实际元素的个数. 试给出判断此循环队列的队空、队满条件,并写出相应的入队和出队算法.