

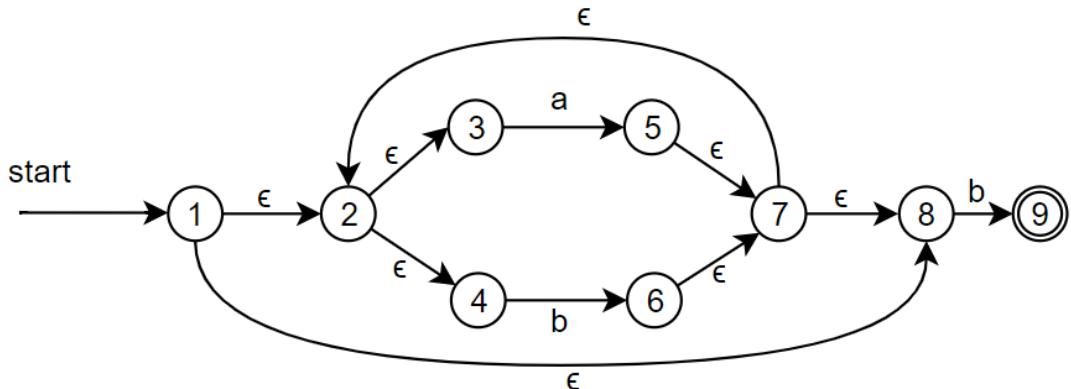
Required Exercises

Exercise 1

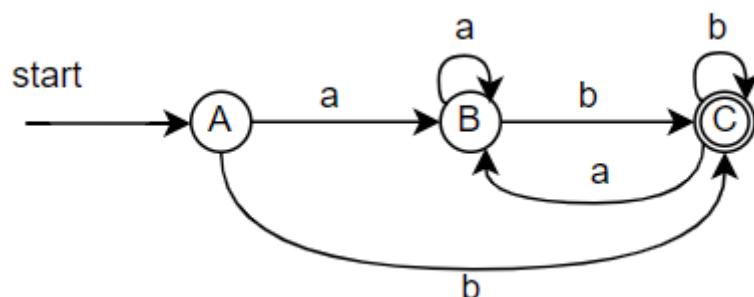
Design NFAs and DFAs to recognize each of the following regular languages:

1. $L((a|b)^*b)$ [10 points]
 2. $L(((\epsilon|a)^*b)^*)$ [10 points]
 3. $L((a|b)^*a(a|b)(a|b))$ [10 points]
 4. $L(a^*ba^*ba^*ba^*)$ [10 points]
1. $L((a|b)^*b)$

NFA:

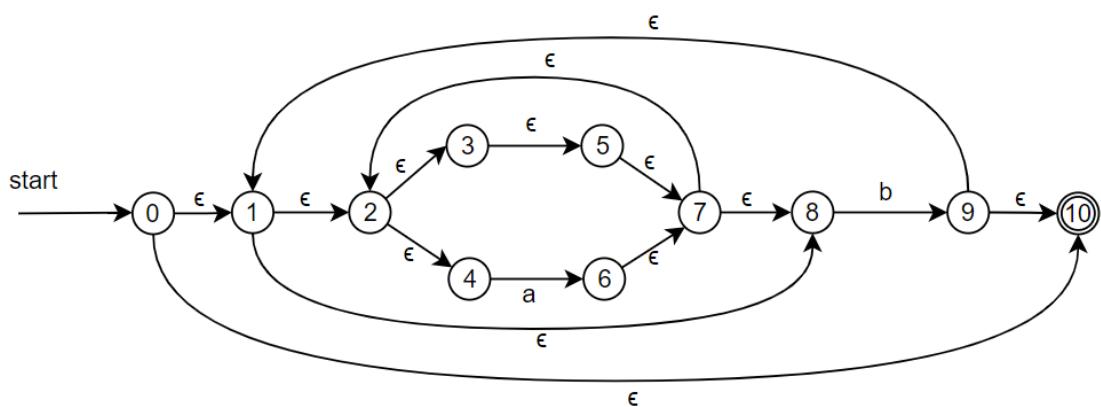


DFA:

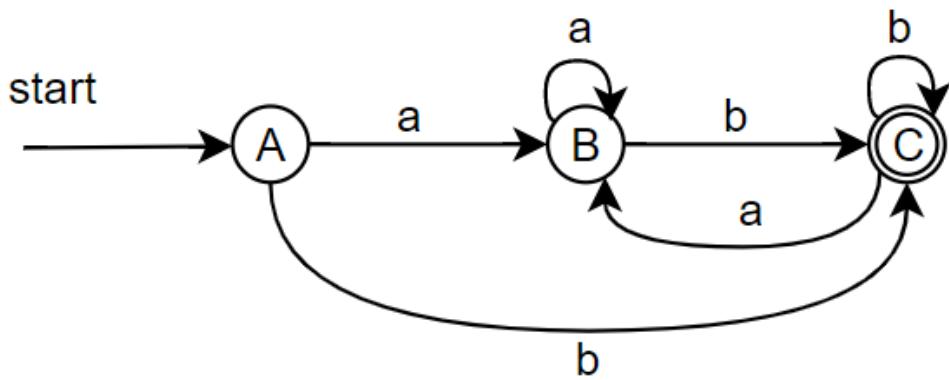


2. $L(((\epsilon|a)^*b)^*)$

NFA:

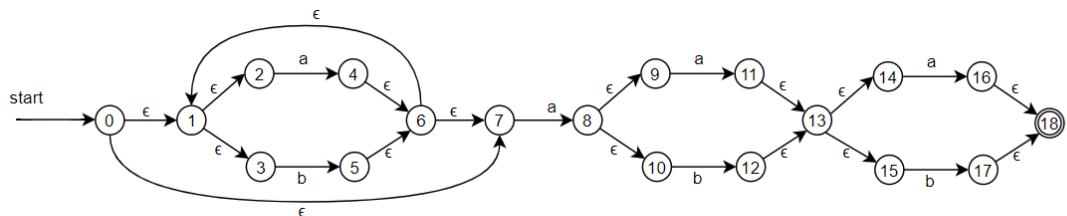


DFA:

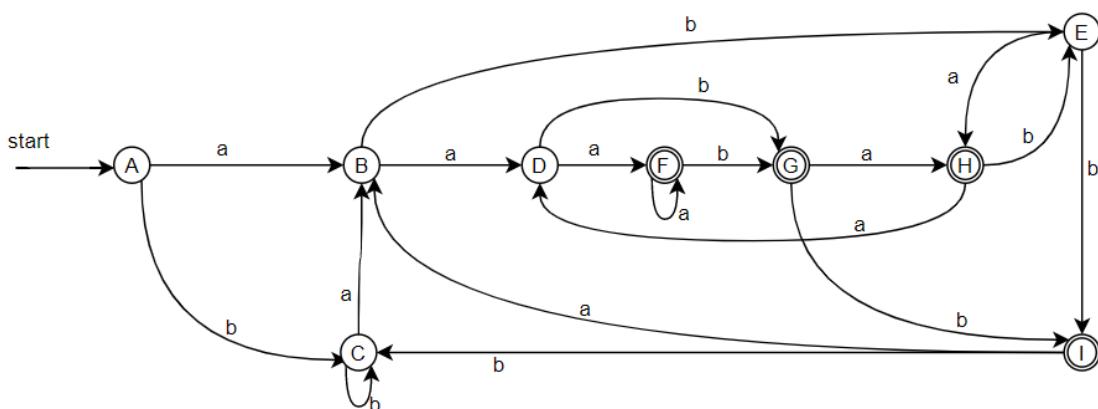


3. $L((a|b)^*a(a|b)(a|b))$

NFA:

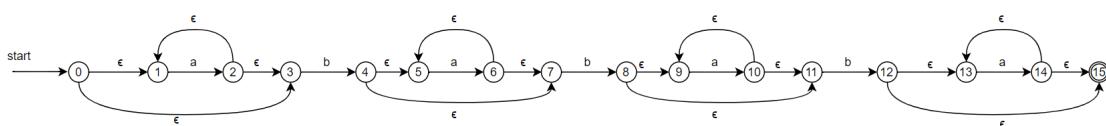


DFA:

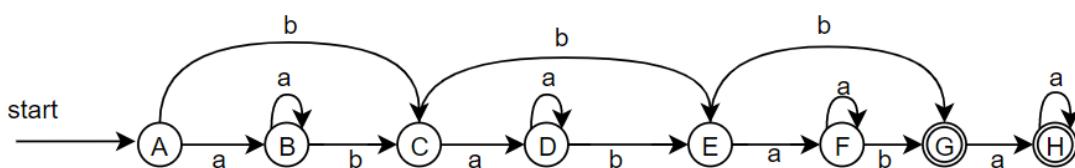


4. $L(a^*ba^*ba^*ba^*)$

NFA:



DFA:



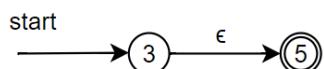
Exercise 2

Convert the following regular expressions to NFAs using the Thompson's Construction Algorithm (Algorithm 3.23 in the dragon book). Please put down the detailed steps.

1. $((\epsilon|a)^*b)^*$ [10 points]
2. $(a|b)^*a(a|b)(a|b)$ [10 points]
3. $a^*ba^*ba^*ba^*$ [10 points]

1. $((\epsilon|a)^*b)^*$

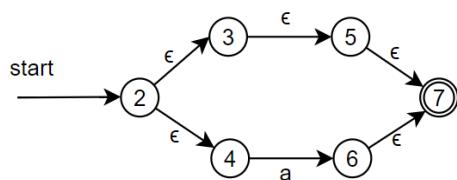
NFA for the first ϵ :



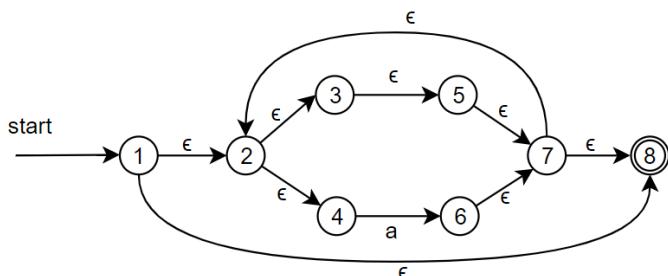
NFA for the first a:



NFA for $(\epsilon|a)$:



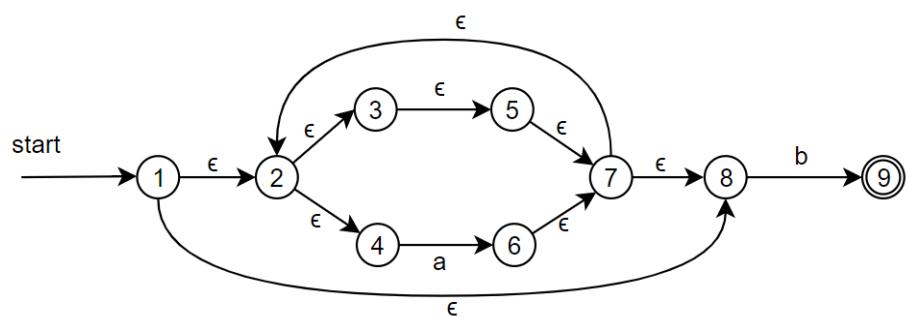
NFA for $(\epsilon|a)^*$:



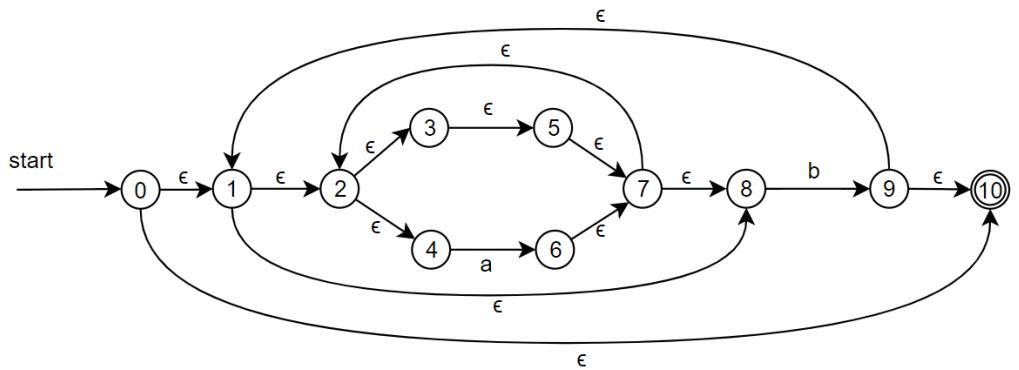
NFA for b:



NFA for $((\epsilon|a)^*b)$:



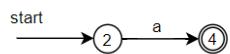
NFA for $((\epsilon \mid a)^* b)^*$:



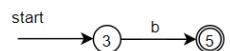
在线验证

2. $(a \mid b)^* a (a \mid b) (a \mid b)$

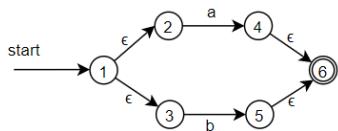
NFA for a:



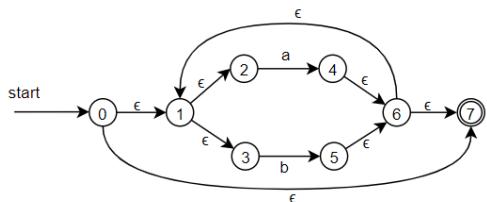
NFA for b:



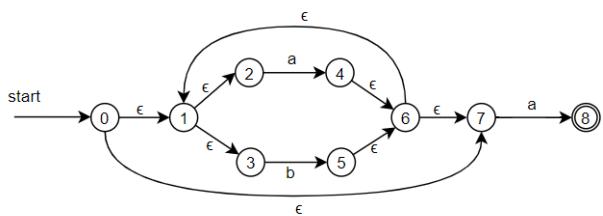
NFA for $(a \mid b)$:



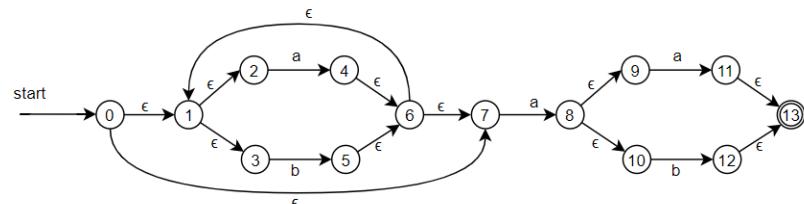
NFA for $(a \mid b)^*$:



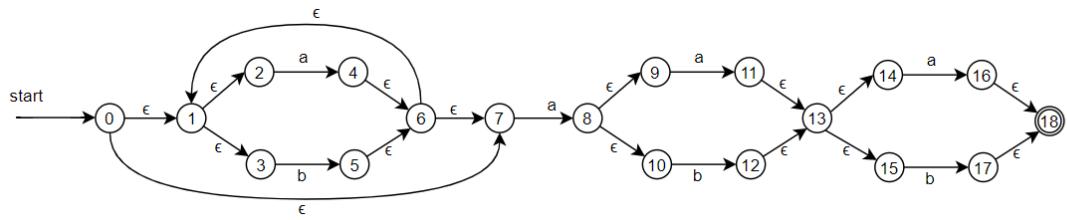
NFA for $(a \mid b)^* a$:



NFA for $(a \mid b)^* a (a \mid b)$:



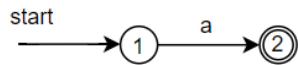
NFA for $(a|b)^*a(a|b)(a|b)$:



[在线验证](#)

3. $a^*ba^*ba^*ba^*$

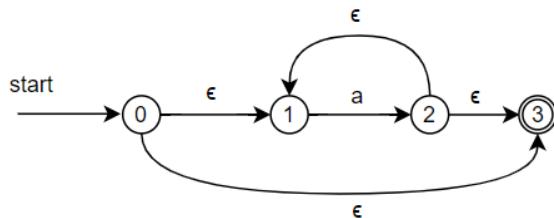
NFA for a:



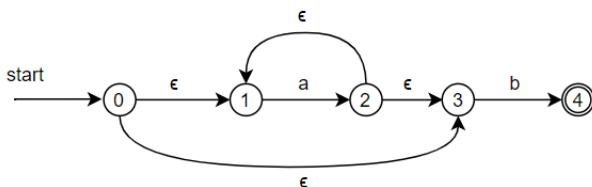
NFA for b:



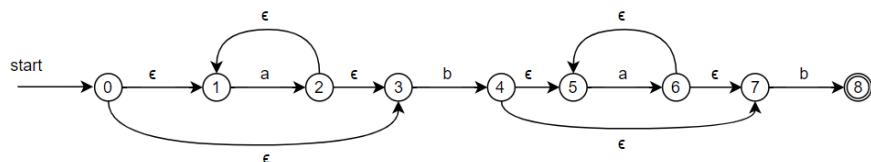
NFA for a^* :



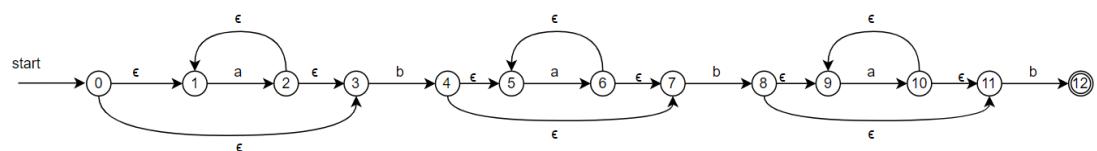
NFA for a^*b :



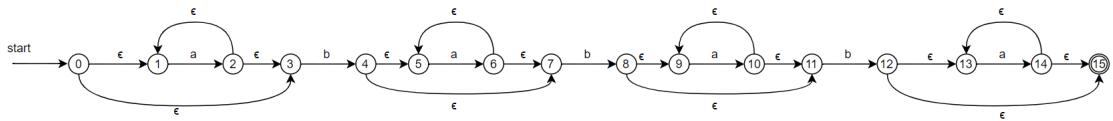
NFA for a^*ba^*b :



NFA for $a^*ba^*ba^*b$:



NFA for $a^*ba^*ba^*ba^*$:



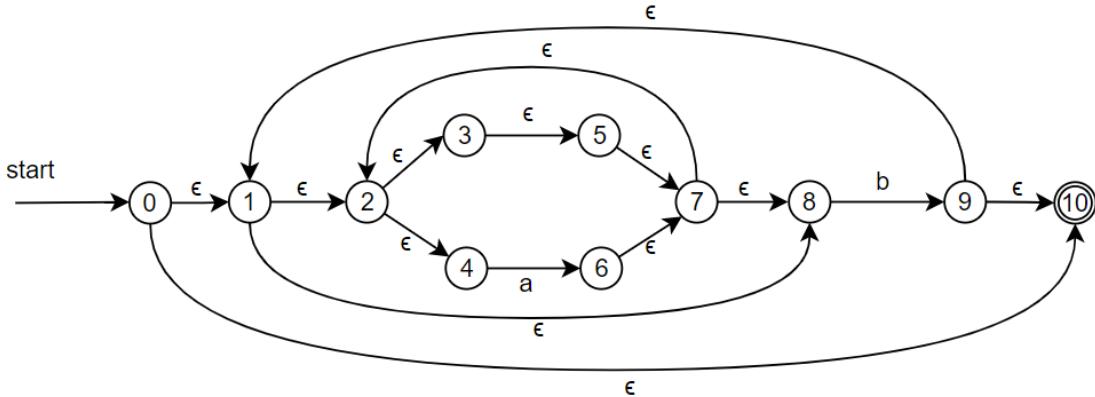
[在线验证](#)

Exercise 3

Convert the NFAs in Exercise 2 to DFAs using the Subset Construction Algorithm (Algorithm 3.20 in the dragon book). Please put down the detailed steps. [30 points in total; 10 points for each correct conversion]

1. $((\epsilon|a)^*b)^*$

NFA:



$$\epsilon\text{-closure}(\{0\}) = \{0, 1, 10, 2, 3, 4, 5, 7, 8\}$$

$$A = \{0, 1, 10, 2, 3, 4, 5, 7, 8\}$$

$$D\text{tran}[A, a] = \epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{6\}) = \{6, 7, 2, 8, 3, 4, 5\}$$

$$B = \{6, 7, 2, 8, 3, 4, 5\}$$

$$D\text{tran}[A, b] = \epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{9\}) = \{9, 10, 1, 2, 3, 4, 5, 7, 8\}$$

$$C = \{9, 10, 1, 2, 3, 4, 5, 7, 8\}$$

$$D\text{tran}[B, a] = \epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{6\}) = \{6, 7, 2, 8, 3, 4, 5\} = B$$

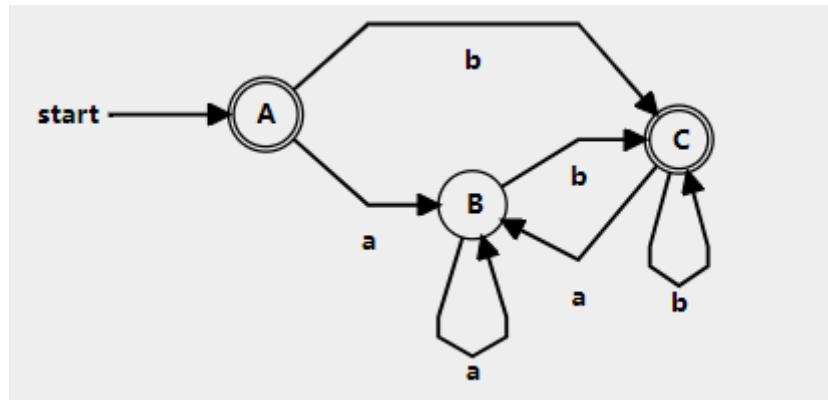
$$D\text{tran}[B, b] = \epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{9\}) = \{9, 10, 1, 2, 3, 4, 5, 7, 8\} = C$$

$$D\text{tran}[C, a] = \epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{6\}) = \{6, 7, 2, 8, 3, 4, 5\} = B$$

$$D\text{tran}[C, b] = \epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{9\}) = \{9, 10, 1, 2, 3, 4, 5, 7, 8\} = C$$

NFA State	DFA State	a	b
{0, 1, 10, 2, 3, 4, 5, 7, 8}	A	B	C
{6, 7, 2, 8, 3, 4, 5}	B	B	C
{9, 10, 1, 2, 3, 4, 5, 7, 8}	C	B	C

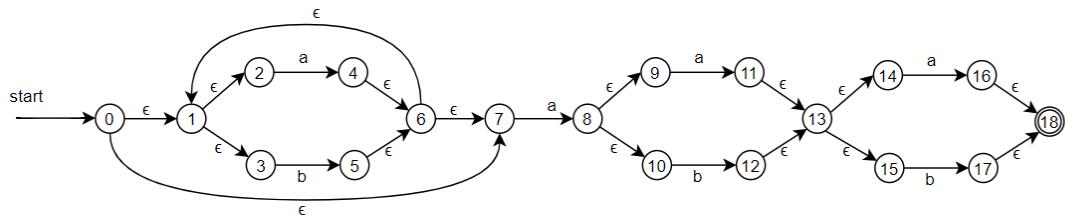
DFA:



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2. $(a|b)^*a(a|b)(a|b)$

NFA:



$$\epsilon\text{-closure}(\{0\}) = \{0, 1, 2, 3, 7\}$$

$$A = \{0, 1, 2, 3, 7\}$$

$$D\text{tran}[A, a] = \epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{4, 8\}) = \{4, 8, 6, 1, 7, 2, 3, 9, 10\}$$

$$B = \{4, 8, 6, 1, 7, 2, 3, 9, 10\}$$

$$D\text{tran}[A, b] = \epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6, 7, 1, 2, 3\}$$

$$C = \{5, 6, 7, 1, 2, 3\}$$

$$D\text{tran}[B, a] = \epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{4, 8, 11\}) = \{4, 8, 11, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15\} = D$$

$$D\text{tran}[B, b] = \epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{5, 12\}) = \{5, 12, 6, 7, 1, 2, 3, 13, 14, 15\} = E$$

$$D\text{tran}[C, a] = \epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{4, 8\}) = \{4, 8, 6, 1, 7, 2, 3, 9, 10\} = B$$

$$D\text{tran}[C, b] = \epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6, 7, 1, 2, 3\} = C$$

$$D\text{tran}[D, a] = \epsilon\text{-closure}(\text{move}(D, a)) = \epsilon\text{-closure}(\{4, 8, 11, 16\}) = \{4, 8, 11, 16, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15, 18\} = F$$

$$D\text{tran}[D, b] = \epsilon\text{-closure}(\text{move}(D, b)) = \epsilon\text{-closure}(\{5, 12, 17\}) = \{5, 12, 17, 6, 7, 1, 2, 3, 13, 14, 15, 18\} = G$$

$$D\text{tran}[E, a] = \epsilon\text{-closure}(\text{move}(E, a)) = \epsilon\text{-closure}(\{4, 8, 16\}) = \{4, 8, 16, 6, 1, 7, 2, 3, 9, 10, 18\} = H$$

$$D\text{tran}[E, b] = \epsilon\text{-closure}(\text{move}(E, b)) = \epsilon\text{-closure}(\{5, 17\}) = \{5, 17, 6, 7, 1, 2, 3, 18\} = I$$

$$D\text{tran}[F, a] = \epsilon\text{-closure}(\text{move}(F, a)) = \epsilon\text{-closure}(\{4, 8, 11, 16\}) = \{4, 8, 11, 16, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15, 18\} = F$$

$$D\text{tran}[F, b] = \epsilon\text{-closure}(\text{move}(F, b)) = \epsilon\text{-closure}(\{5, 12, 17\}) = \{5, 12, 17, 6, 7, 1, 2, 3, 13, 14, 15, 18\} = G$$

$$D\text{tran}[G, a] = \epsilon\text{-closure}(\text{move}(G, a)) = \epsilon\text{-closure}(\{4, 8, 16\}) = \{4, 8, 16, 6, 1, 7, 2, 3, 9, 10, 18\} = H$$

$$D\text{tran}[G, b] = \epsilon\text{-closure}(\text{move}(G, b)) = \epsilon\text{-closure}(\{5, 17\}) = \{5, 17, 6, 7, 1, 2, 3, 18\} = I$$

$D_{\text{tran}}[H, a] = \epsilon\text{-closure}(\text{move}(H, a)) = \epsilon\text{-closure}(\{4, 8, 11\}) = \{4, 8, 11, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15\} = D$

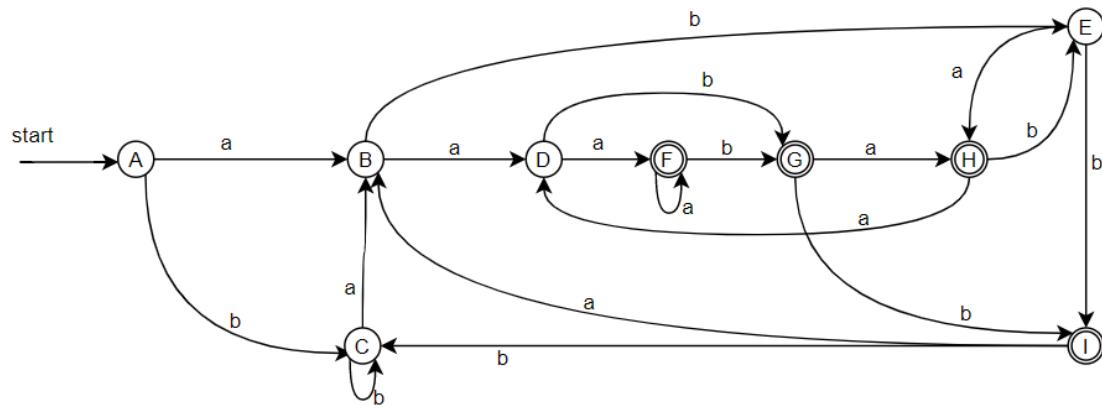
$D_{\text{tran}}[H, b] = \epsilon\text{-closure}(\text{move}(H, b)) = \epsilon\text{-closure}(\{5, 12\}) = \{5, 12, 6, 7, 1, 2, 3, 13, 14, 15\} = E$

$D_{\text{tran}}[I, a] = \epsilon\text{-closure}(\text{move}(I, a)) = \epsilon\text{-closure}(\{4, 8\}) = \{4, 8, 6, 1, 7, 2, 3, 9, 10\} = B$

$D_{\text{tran}}[I, b] = \epsilon\text{-closure}(\text{move}(I, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6, 7, 1, 2, 3\} = C$

NFA State	DFA State	a	b
{0, 1, 2, 3, 7}	A	B	C
{1, 2, 3, 4, 6, 7, 8, 9, 10}	B	D	E
{1, 2, 3, 5, 6, 7}	C	B	C
{1, 2, 3, 4, 6, 7, 8, 9, 10, 13, 14, 15}	D	F	G
{1, 2, 3, 5, 6, 7, 12, 13, 14, 15}	E	H	I
{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 18}	F	F	G
{1, 2, 3, 5, 6, 7, 12, 13, 14, 15, 17, 18}	G	H	I
{1, 2, 3, 4, 6, 7, 8, 9, 10, 16, 18}	H	D	E
{1, 2, 3, 5, 6, 7, 17, 18}	I	B	C

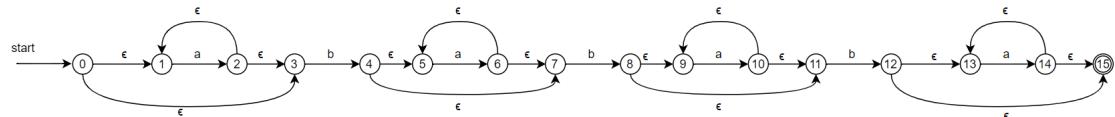
DFA:



[在线验证](#)

3. $a^*ba^*ba^*ba^*$

NFA:



$\epsilon\text{-closure}(\{0\}) = \{0, 1, 3\}$

$A = \{0, 1, 3\}$

$D_{\text{tran}}[A, a] = \epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{2\}) = \{1, 2, 3\}$

$B = \{1, 2, 3\}$

$D_{\text{tran}}[A, b] = \epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{4\}) = \{4, 5, 7\}$

$C = \{4, 5, 7\}$

$D_{\text{tran}}[B, a] = \epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{2\}) = \{1, 2, 3\} = B$

$D_{\text{tran}}[B, b] = \epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{4\}) = \{4, 5, 7\} = C$

$D_{\text{tran}}[C, a] = \epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{6\}) = \{6, 5, 7\} = D$

$D_{\text{tran}}[C, b] = \epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{8\}) = \{8, 9, 11\} = E$

$D_{\text{tran}}[D, a] = \epsilon\text{-closure}(\text{move}(D, a)) = \epsilon\text{-closure}(\{6\}) = \{6, 5, 7\} = D$

$D_{\text{tran}}[D, b] = \epsilon\text{-closure}(\text{move}(D, b)) = \epsilon\text{-closure}(\{8\}) = \{8, 9, 11\} = E$

$D_{\text{tran}}[E, a] = \epsilon\text{-closure}(\text{move}(E, a)) = \epsilon\text{-closure}(\{10\}) = \{10, 9, 11\} = F$

$D_{\text{tran}}[E, b] = \epsilon\text{-closure}(\text{move}(E, b)) = \epsilon\text{-closure}(\{12\}) = \{12, 13, 15\} = G$

$D_{\text{tran}}[F, a] = \epsilon\text{-closure}(\text{move}(F, a)) = \epsilon\text{-closure}(\{10\}) = \{10, 9, 11\} = F$

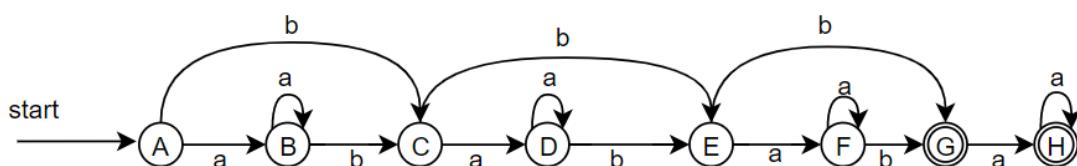
$D_{\text{tran}}[F, b] = \epsilon\text{-closure}(\text{move}(F, b)) = \epsilon\text{-closure}(\{12\}) = \{12, 13, 15\} = G$

$D_{\text{tran}}[G, a] = \epsilon\text{-closure}(\text{move}(G, a)) = \epsilon\text{-closure}(\{14\}) = \{13, 14, 15\} = H$

$D_{\text{tran}}[H, a] = \epsilon\text{-closure}(\text{move}(H, a)) = \epsilon\text{-closure}(\{14\}) = \{13, 14, 15\} = H$

NFA State	DFA State	a	b
{0, 1, 3}	A	B	C
{1, 2, 3}	B	B	C
{4, 5, 7}	C	D	E
{6, 5, 7}	D	D	E
{8, 9, 11}	E	F	G
{10, 9, 11}	F	F	G
{12, 13, 15}	G	H	
{13, 14, 15}	H	H	

DFA:



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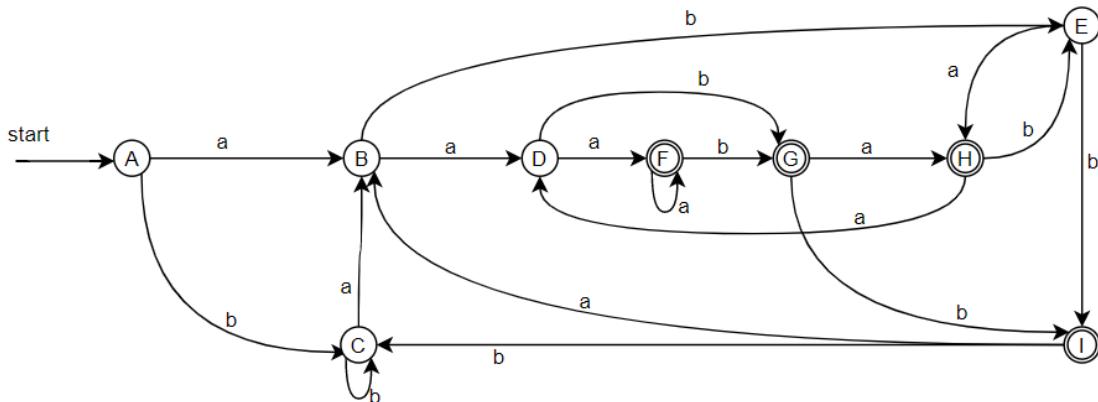
Optional Exercises

Exercise 1

Exercise 1: Minimize the number of states of the DFAs you have built for regular expressions 2 and 3 in Exercise 2 using the State-Minimization Algorithm (Algorithm 3.39 in the dragon book). Please put down the detailed steps. [10 points for each correct minimization process]

1. $(a|b)^*a(a|b)(a|b)$

初始DFA:



对于每个状态，输入a, b后转移到的状态如下表所示

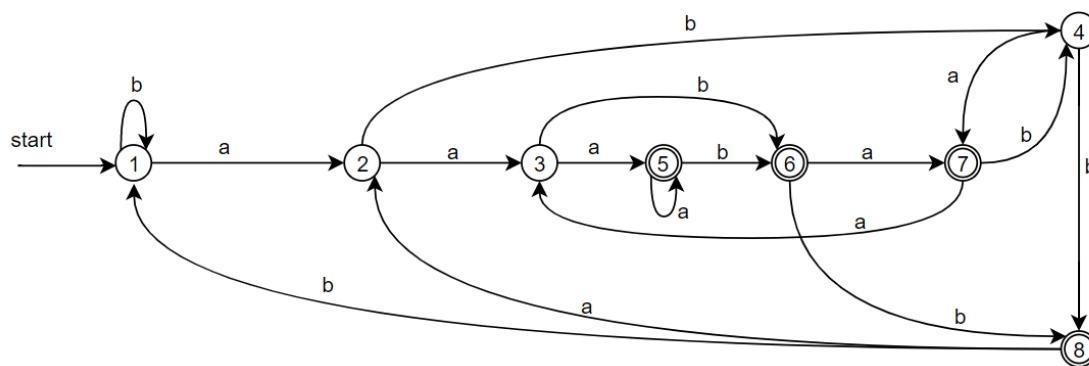
State	a	b
A	B	C
B	D	E
C	B	C
D	F	G
E	H	I
F	F	G
G	H	I
H	D	E
I	B	C

- 将{A, B, C, D, E, F, G, H, I}按非接收态和接收态分为{A, B, C, D, E} 和{F, G, H, I}， $\Pi new = \{A, B, C, D, E\} \{F, G, H, I\}$
- 对于{A, B, C, D, E}， 在输入b上， 状态A, B, C都转移到{A, B, C, D, E}的某个成员上， 状态D, E都转移到{F, G, H, I}的某个成员上， 故这一轮 $\Pi new = \{A, B, C\} \{D, E\} \{F, G, H, I\}$
- 对于{F, G, H, I}， 在输入b上， 状态F, G都转移到{F, G, H, I}的某个成员上， 状态H转移到{D, E}的成员E上， I转移到{A, B, C}的成员C上， 故这一轮 $\Pi new = \{A, B, C\} \{D, E\} \{F, G\} \{H\} \{I\}$
- 对于{A, B, C}， 在输入b上， 状态A, C都转移到{A, B, C}的成员C上， 状态B转移到{D, E}的成员E上， 故这一轮 $\Pi new = \{A, C\} \{B\} \{D, E\} \{F, G\} \{H\} \{I\}$
- 对于{D, E}， 在输入b上， 状态D转移到{F, G}中的成员G上， 状态E转移到{I}的成员I上， 故这一轮 $\Pi new = \{A, C\} \{B\} \{D\} \{E\} \{F, G\} \{H\} \{I\}$
- 对于{F, G}， 在输入a上， 状态F转移到{F, G}中的成员F上， 状态G转移到{H}的成员H上， 故 $\Pi final = \{A, C\} \{B\} \{D\} \{E\} \{F\} \{G\} \{H\} \{I\}$

根据 $\Pi final = \{A, C\} \{B\} \{D\} \{E\} \{F\} \{G\} \{H\} \{I\}$ 将状态A,C合并， 得到状态最小化的DFA

DFA State	Min-DFA State	a	b
A, C	1	2	1
B	2	3	4
D	3	5	6
E	4	7	8
F	5	5	6
G	6	7	8
H	7	3	4
I	8	2	1

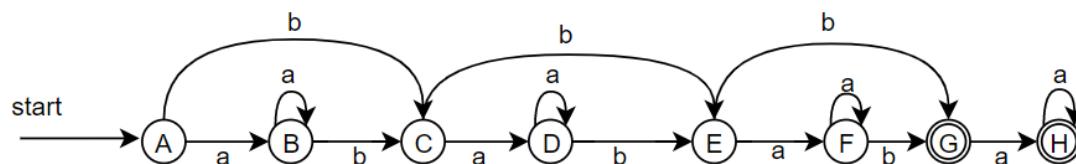
状态最小化DFA:



[在线验证](#)

2. $a^*ba^*ba^*ba^*$

初始DFA:



对于每个状态，输入a, b后转移到的状态如下表所示

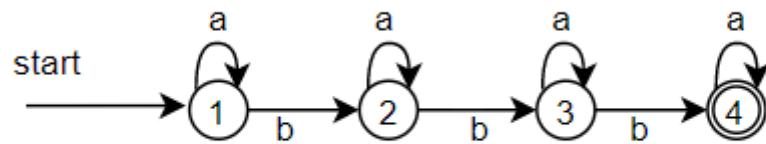
State	a	b
A	B	C
B	B	C
C	D	E
D	D	E
E	F	G
F	F	G

State	a	b
G	H	
H	H	

1. 将{A, B, C, D, E, F, G, H}按非接收态和接收态分为{A, B, C, D, E, F} 和{G, H}， $\Pi new = \{A, B, C, D, E, F\} \{G, H\}$
2. 对于{A, B, C, D, E, F}， 在输入b上， 状态A, B, C, D都转移到{A, B, C, D, E, F}的某个成员上， 状态E, F都转移到{G, H}的成员G上， 故这一轮 $\Pi new = \{A, B, C, D\} \{E, F\} \{G, H\}$
3. 对于{A, B, C, D}， 在输入b上， 状态A, B都转移到{A, B, C, D}的成员C上， 状态C, D都转移到{E, F}的成员E上， 故 $\Pi final = \{A, B\} \{C, D\} \{E, F\} \{G, H\}$

根据 $\Pi final = \{A, B\} \{C, D\} \{E, F\} \{G, H\}$ 将状态合并， 得到状态最小化的DFA

DFA State	Min-DFA State	a	b
A, B	1	1	2
C, D	2	2	3
E, F	3	3	4
G, H	4	4	



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