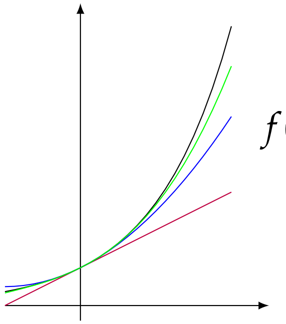
Développements limités usuels

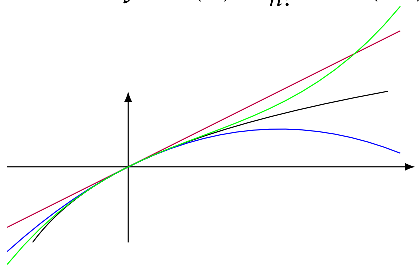
On trouvera ci-dessous des approximations utiles au voisinage de 0. Ces approximations permettent d'obtenir des équivalents simples dans de nombreuses situations. Les graphiques en regard de certaines fonctions permettent de se rendre compte de la qualité de l'approximation suivant l'ordre auquel on pousse le développement limité.

Approximation affine — Développement limité d'ordre n

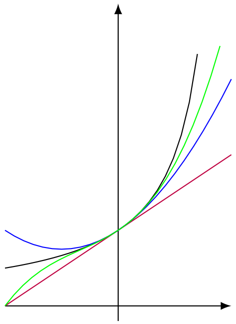


$$f(x) = f(0) + f'(0)\cdot x + o(x) = f(0) + f'(0)\cdot x + f''(0)\cdot \tfrac{x^2}{2!} + f'''(0)\cdot \tfrac{x^3}{3!} + \cdots + f^{(n)}(0)\cdot \tfrac{x^n}{n!} + o(x^n)$$

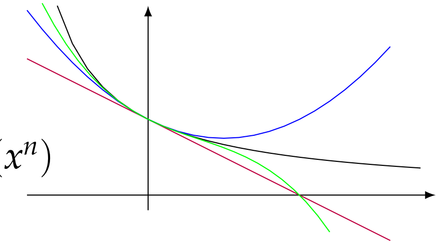
$$e^x = 1 + x + o(x) = 1 + x + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} - \cdots + \tfrac{x^n}{n!} + o(x^n)$$



$$\ln(1+x) = x + o(x) = x - \tfrac{x^2}{2} + \tfrac{x^3}{3} - \cdots + (-1)^{n+1}\cdot \tfrac{x^n}{n} + o(x^n)$$

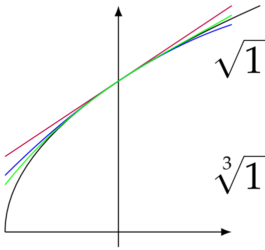


$$\frac{1}{1-x} = 1 + x + o(x) = 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n)$$



$$\frac{1}{1+x} = 1 - x + o(x) = 1 - x + x^2 - x^3 + \cdots + (-1)^n\cdot x^n + o(x^n)$$

$$(1+x)^\alpha = 1 + \alpha x + o(x) = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}\cdot x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}\cdot x^3 + \cdots + \frac{\alpha\cdots(\alpha-n+1)}{n!}\cdot x^n + o(x^n)$$



$$\sqrt{1+x} = 1 + \tfrac{1}{2}x + o(x) = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 + \cdots + (-1)^{n+1}\cdot \frac{1\times 3\times\cdots\times(2n-3)}{2\times 4\times\cdots\times 2n}\cdot x^n + o(x^n)$$

$$\sqrt[3]{1+x} = 1 + \tfrac{1}{3}x + o(x) = 1 + \tfrac{1}{3}x - \tfrac{1}{9}x^2 + \tfrac{5}{81}x^3 + \cdots + (-1)^{n+1}\cdot \frac{2\times 5\times 8\times\cdots\times(3n-4)}{3^n\cdot n!}\cdot x^n + o(x^n)$$

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