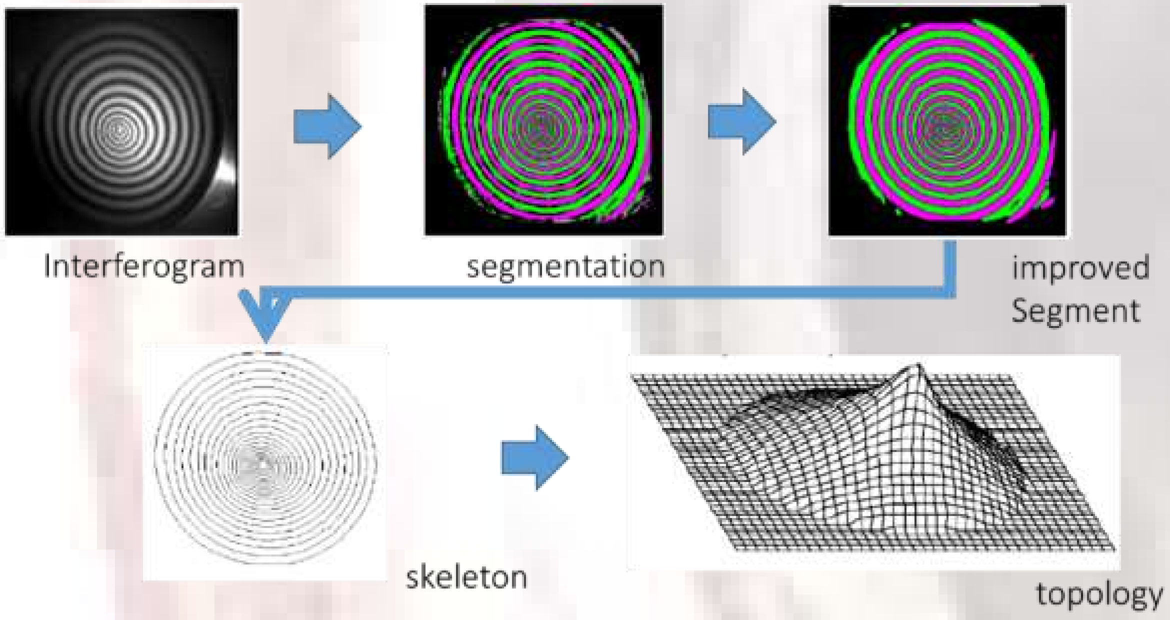
Fringe Pattern Analysis

Fringe Skeleton Method

1) Intensity distribution
   1. Identification of local extrema
   2. Fringe sampling points for interpolation
2) determination of points with integer or half-integer order of interference
3) absolute order has to be identified additionally
4) Relatively low accuracy of phase measurements



Processing:

① improvement of SNR by spatial and temporal filtering
② creation of the skeleton (segmentation)
③ improvement of the skeleton shape
④ numbering the fringes
⑤ reconstruction of the phase by interpolation

Phase estimation methods

General Form of Fringe Patterns

$$\boxed{I(x,y)} = \boxed{a(x,y)} + \boxed{b(x,y)} \cos \boxed{\delta(x,y)}$$

Fringe Pattern	Averaged/ Background	Contrast of the fringes	Phase to be Determined
Measurable	Unknown	Unknown	Unknown

Imaging System & Object transmittance, Reflectivity…

Parameters to be Measured

Generating More Information (measureable) for determining unknown parameters

FRINGE ANALYSIS

PHASE ESTIMATION METHODS

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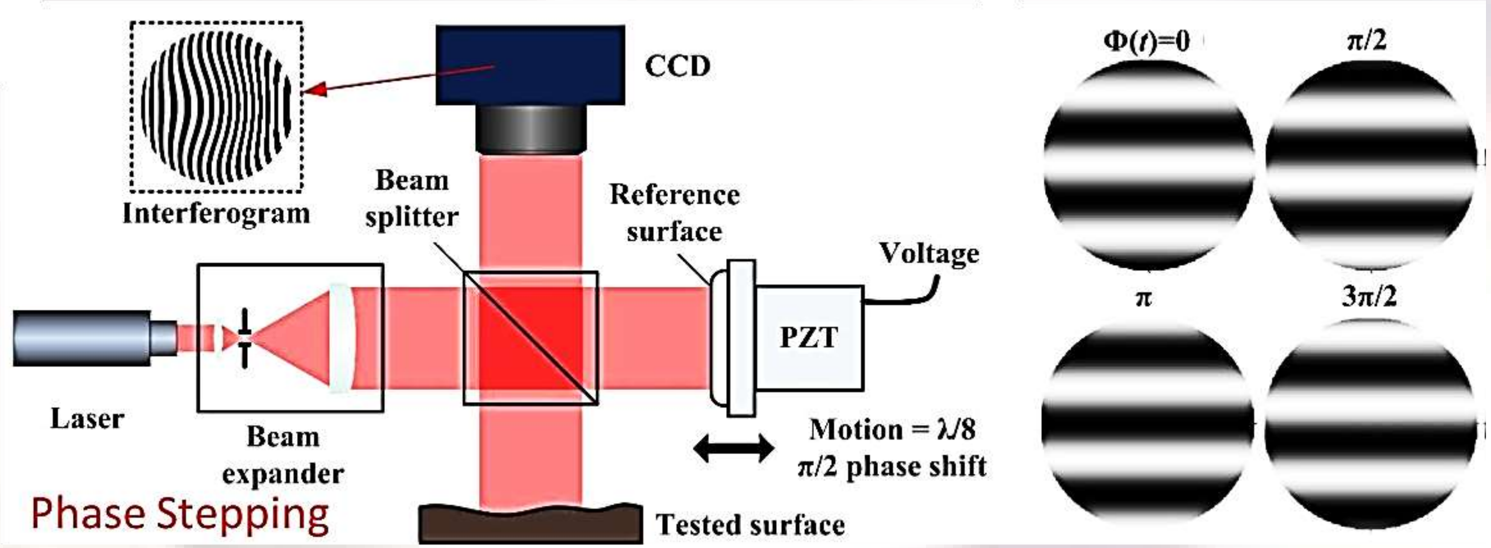
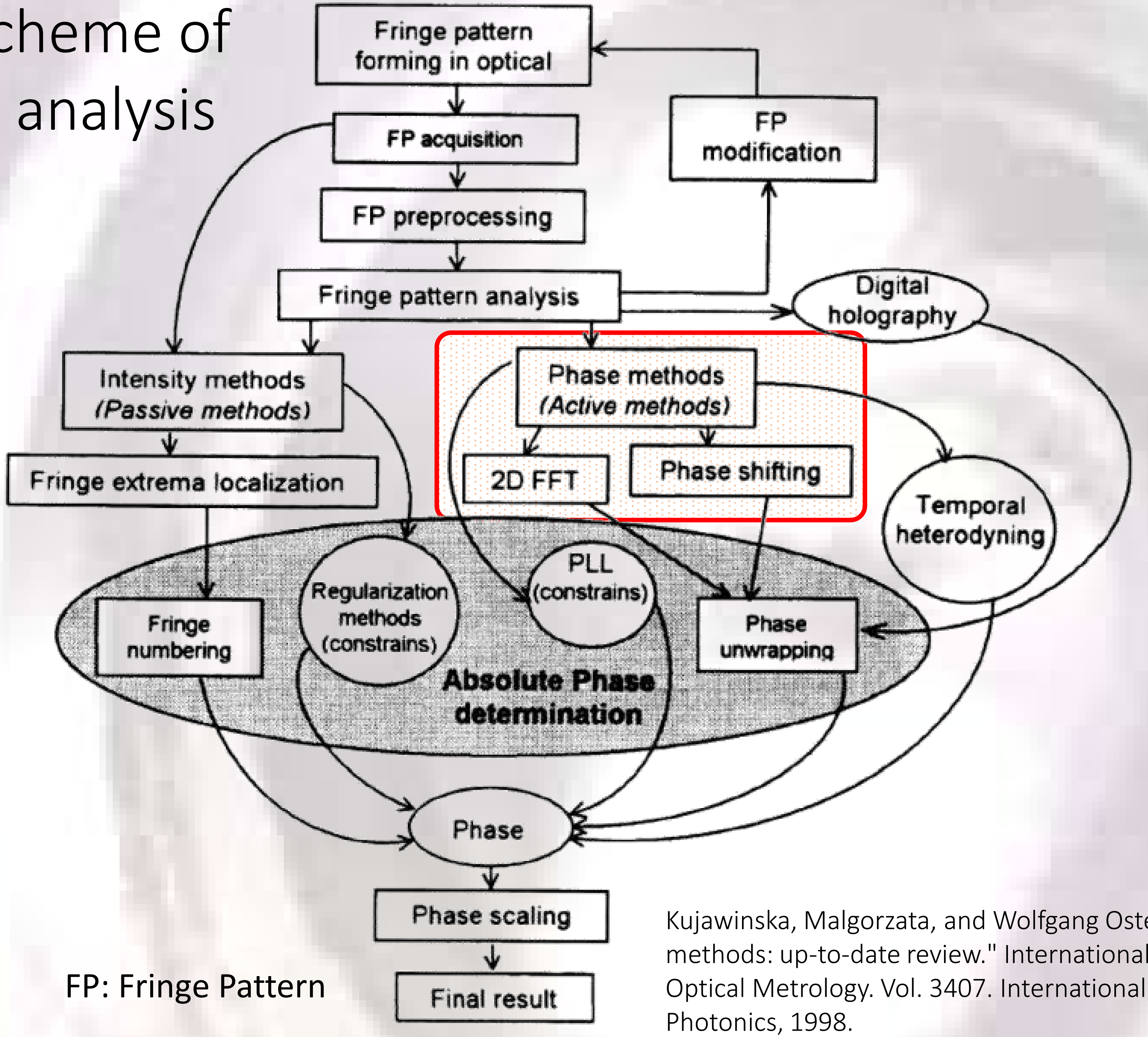


The general scheme of fringe pattern analysis process



FP: Fringe Pattern

Kujawinska, Malgorzata, and Wolfgang Osten. "Fringe pattern analysis methods: up-to-date review." International Conference on Applied Optical Metrology. Vol. 3407. International Society for Optics and Photonics, 1998.

Direct Method

$$\boxed{I(x,y)} = \boxed{a(x,y)} + \boxed{b(x,y)} \cos \boxed{\delta(x,y)}$$

Fringe Pattern	Averaged/ Background	Contrast of the fringes	Phase to be Determined
Measurable	**Unknown**	**Unknown**	**Unknown**

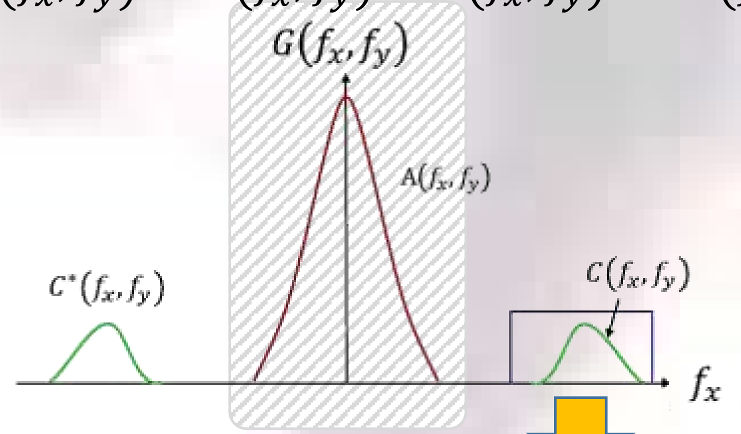
$$I(x,y) = a(x,y) + b(x,y)\cos\delta(x,y)$$
$$= a(x,y) + \boxed{\frac{1}{2}b(x,y)e^{i\delta(x,y)}} + \frac{1}{2}b(x,y)e^{-i\delta(x,y)}$$

$$\frac{1}{2}b(x,y)(\cos\delta + i\sin\delta) = Re\left[\frac{b(x,y)}{2}e^{i\delta}\right] + Im\left[\frac{b(x,y)}{2}e^{i\delta}\right]$$

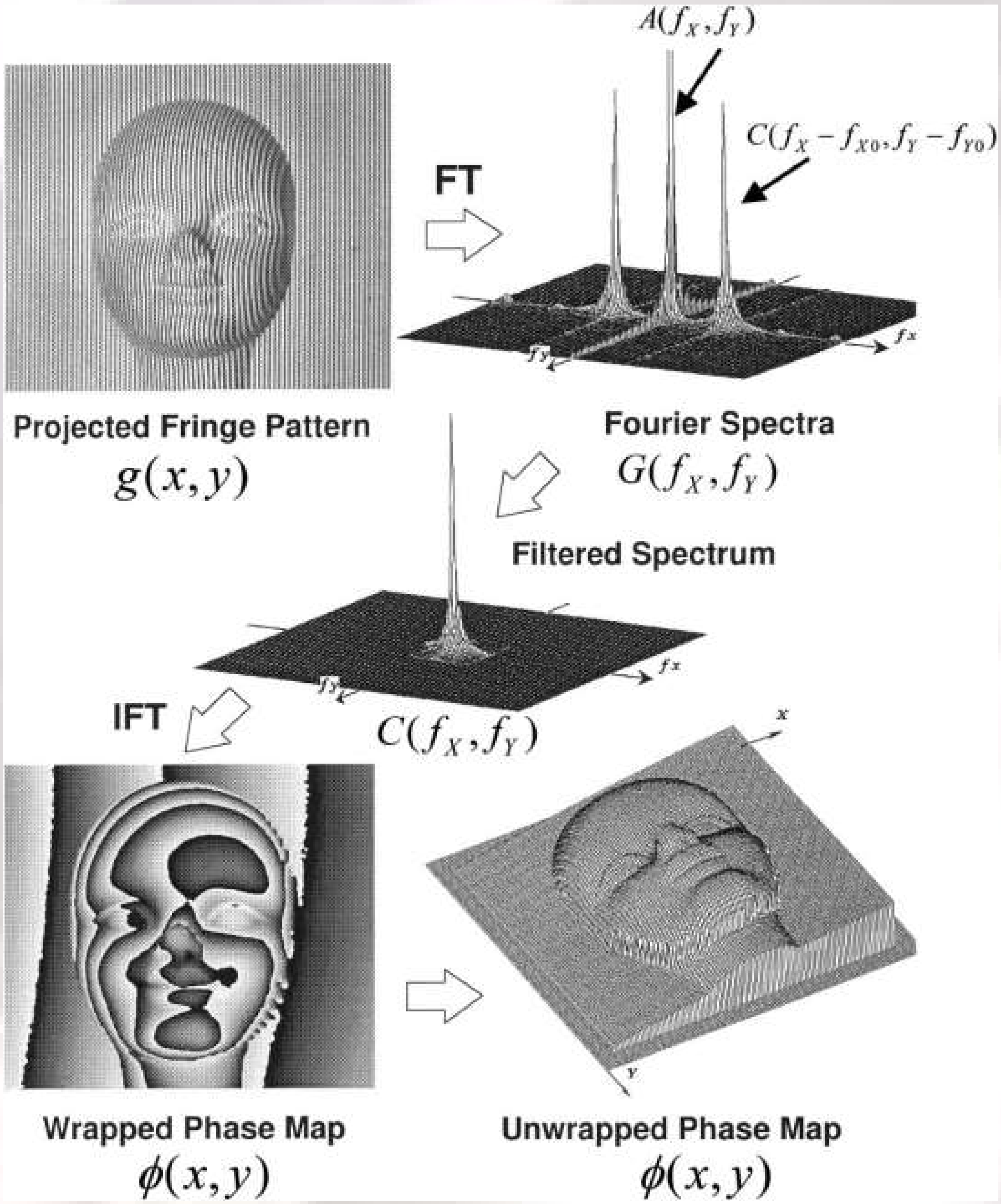
$$\delta = \tan^{-1}\frac{Im\left[\frac{b(x,y)}{2}e^{i\delta}\right]}{Re\left[\frac{b(x,y)}{2}e^{i\delta}\right]}$$

Fourier Transform to 2D frequency Domain

$$\mathcal{F}(I(x,y)) = G(f_x, f_y)$$
$$\Rightarrow$$
$$G(f_x, f_y) = \mathrm{A}(f_x, f_y) + C(f_x, f_y) + C^*(f_x, f_y)$$



Shifting &
Inverse Fourier Transform



DOI: 10.1364/AO.52.000020

Direct Method

$$\boxed{I(x,y)} = \boxed{a(x,y)} + \boxed{b(x,y)} \cos \boxed{\delta(x,y)}$$

Fringe Pa

Measu

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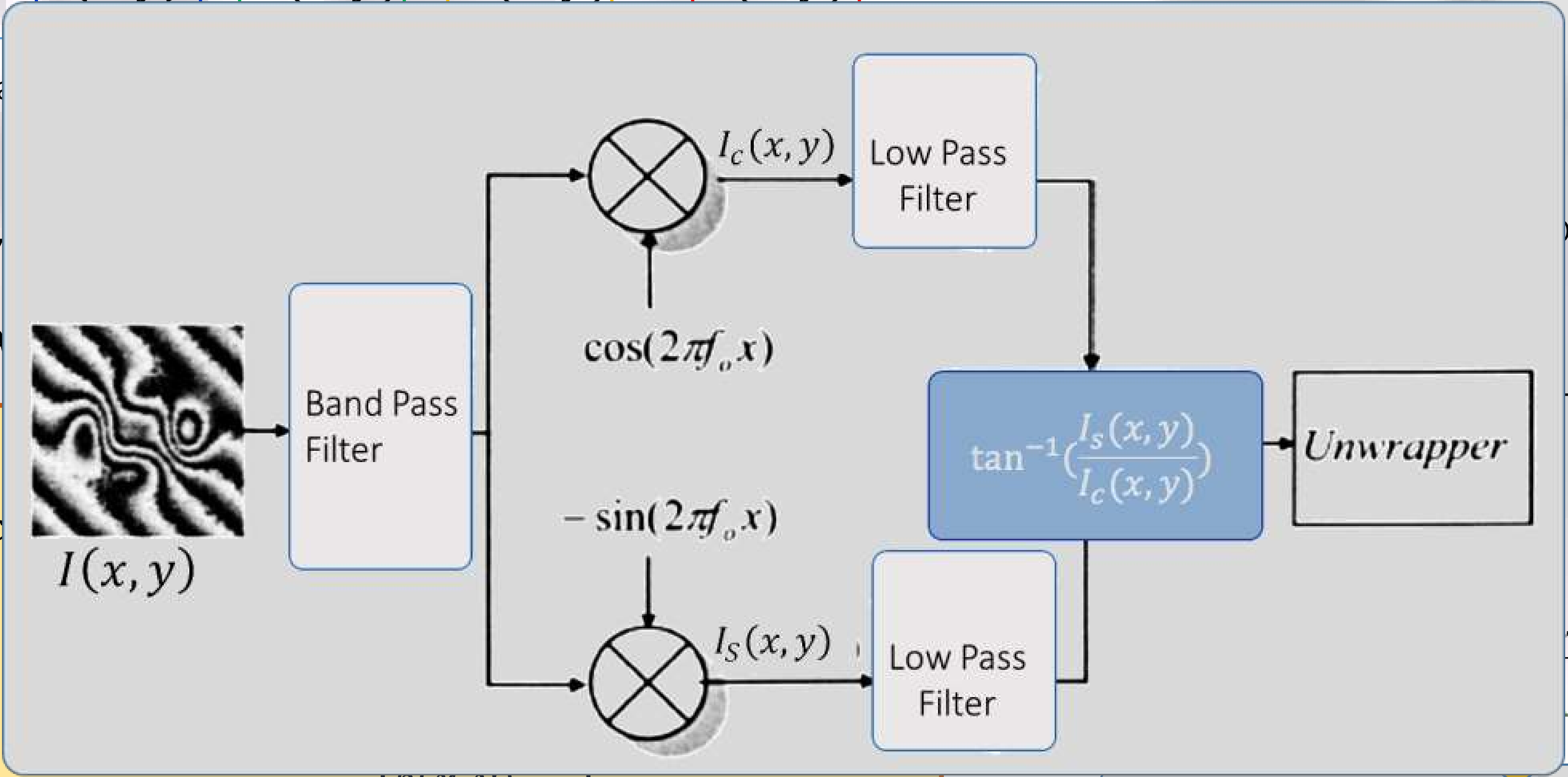
omain

$C^*(f_x, f_y)$

$\frac{1}{2} b(x,y)(c$



$f_y)$

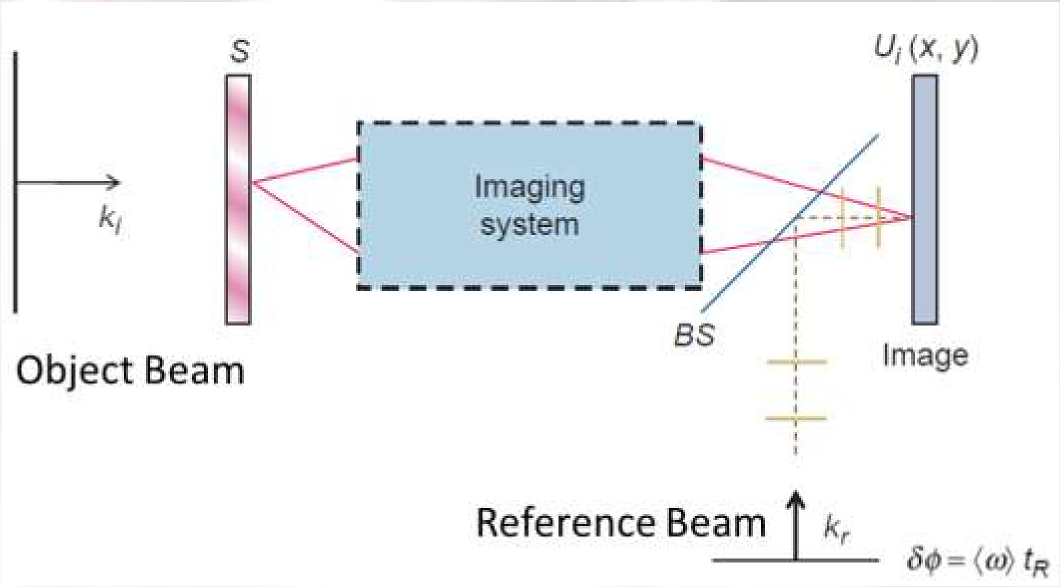
f_x

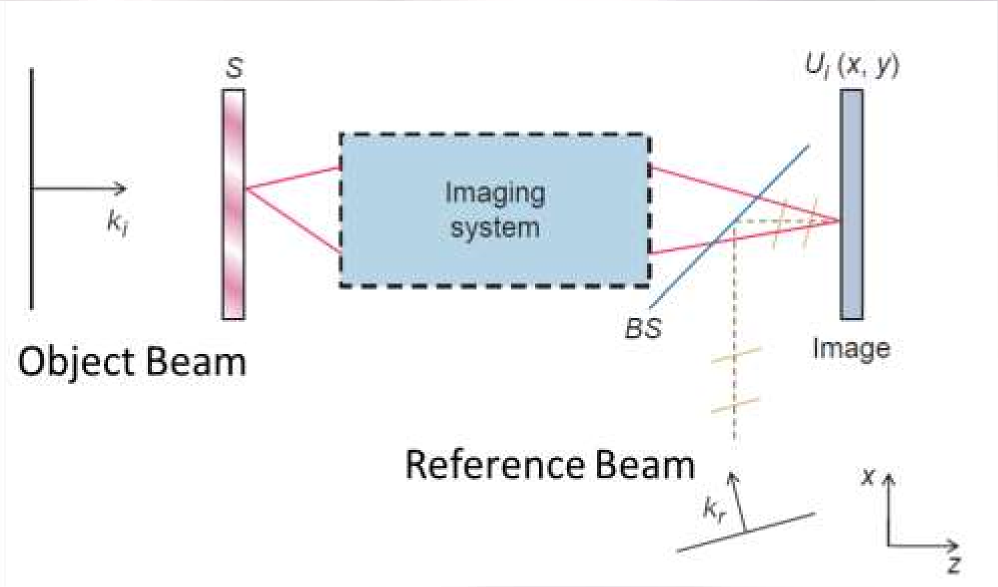
$$Re\left[\frac{b(x,y)}{2} e^{i\delta}\right]$$

Shifting &
Inverse Fourier Transform

Phase estimation methods

- **Temporal Phase Shifting**
  Phase shift introduced between the test and reference beams as a function of time
- **Spatial Phase Shifting**
  1) Phase shift data obtained from a single interferogram that requires a carrier pattern of almost straight fringes to either compare phases of adjacent pixels or to separate orders while performing operations in the Fourier domain.
  2) Simultaneously record multiple interferograms with appropriate relative phase shift differences separated spatially in space.





Temporal phase shifting

The phase modulation needs to generate linear and uniform phases over the field of view during the exposure time of the detector.

$$\varphi = 2\pi f t$$

$$\Longrightarrow I(x,y;t) = a(x,y) + b(x,y)\cos(\delta(x,y) + \varphi(t))$$

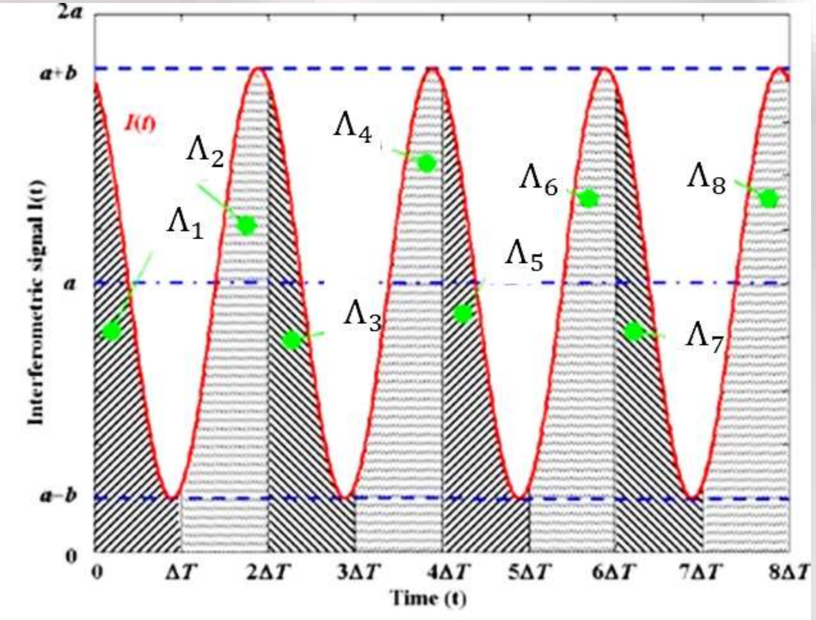
Each image is obtained by integrating over a time interval ΔT

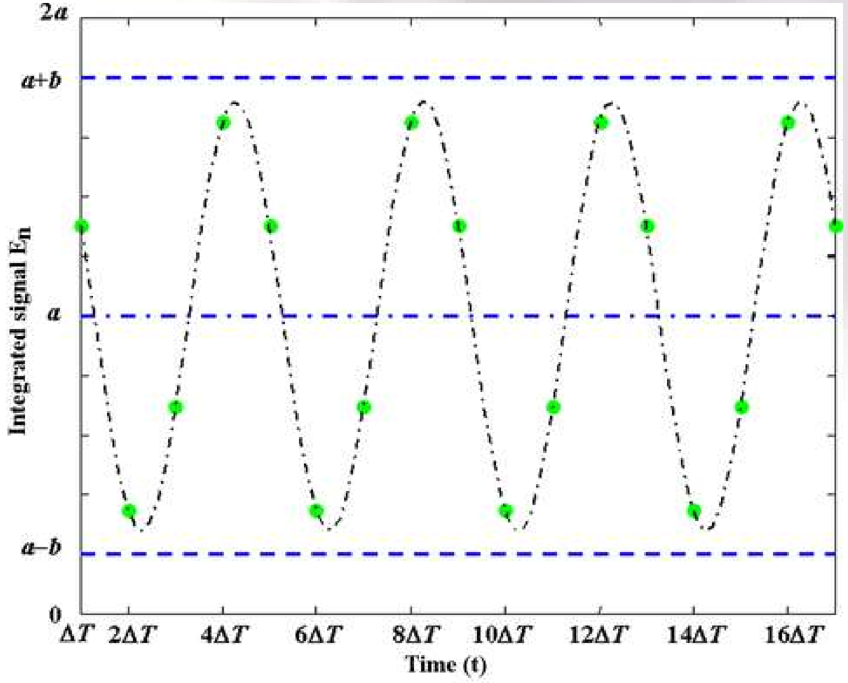
$$\Longrightarrow \Lambda_n = \frac{1}{\Delta T}\int_{(n-1)\Delta T}^{n\Delta T} I(x,y;t)dt$$

$$= a(x,y) + b(x,y)\frac{1}{\Delta T}\int_{(n-1)\Delta T}^{n\Delta T} cos(\delta(x,y) + \varphi(t))dt$$

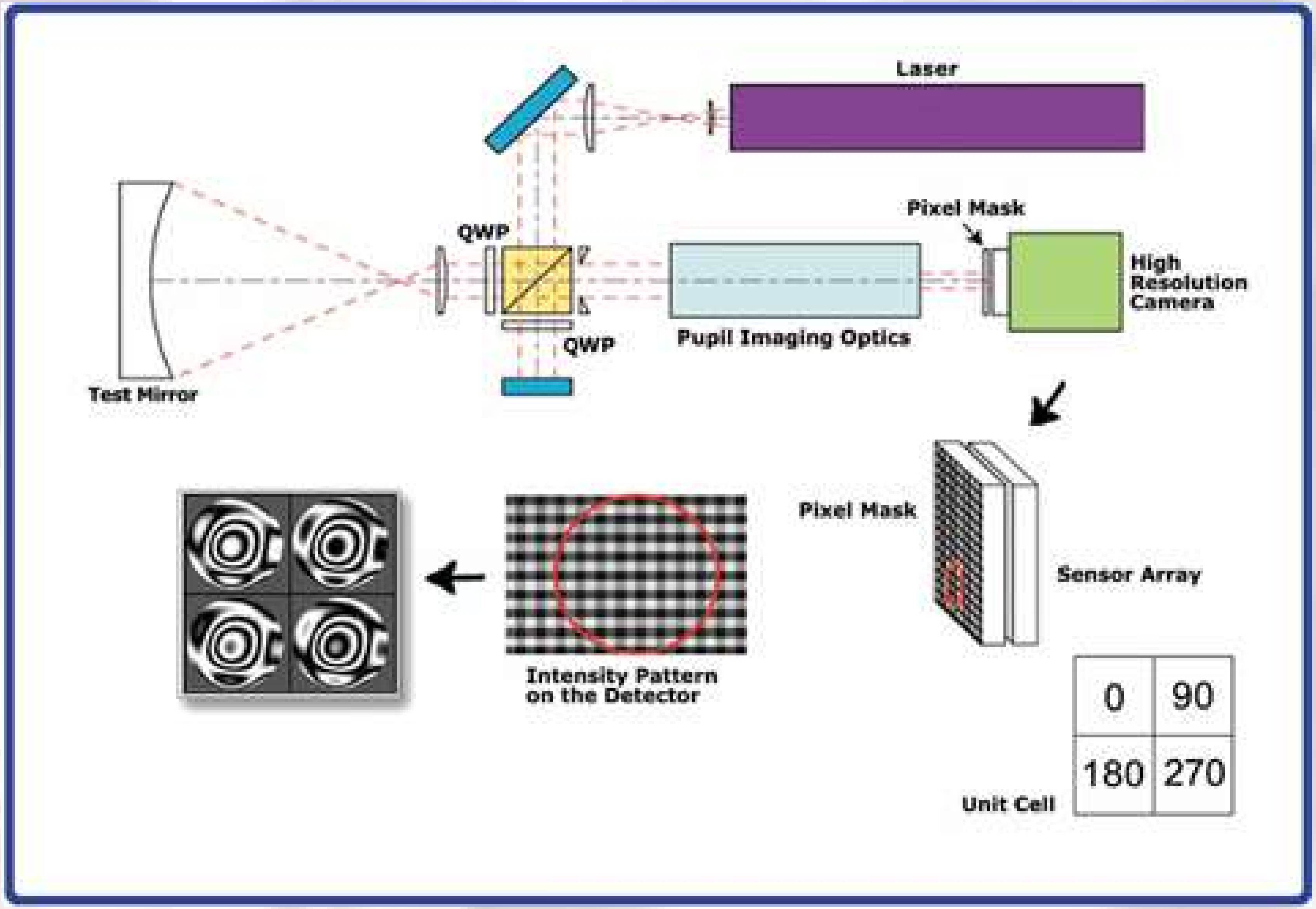
$$= a(x,y) + b(x,y)\boxed{sinc(\pi f \Delta T)}\cos\left(\delta + 2\pi\left(n - \frac{1}{2}\right) f\Delta T\right)$$

φ must be selected according to Shannon theorem thus $\varphi < \pi$





Spatial Phase-Shifting



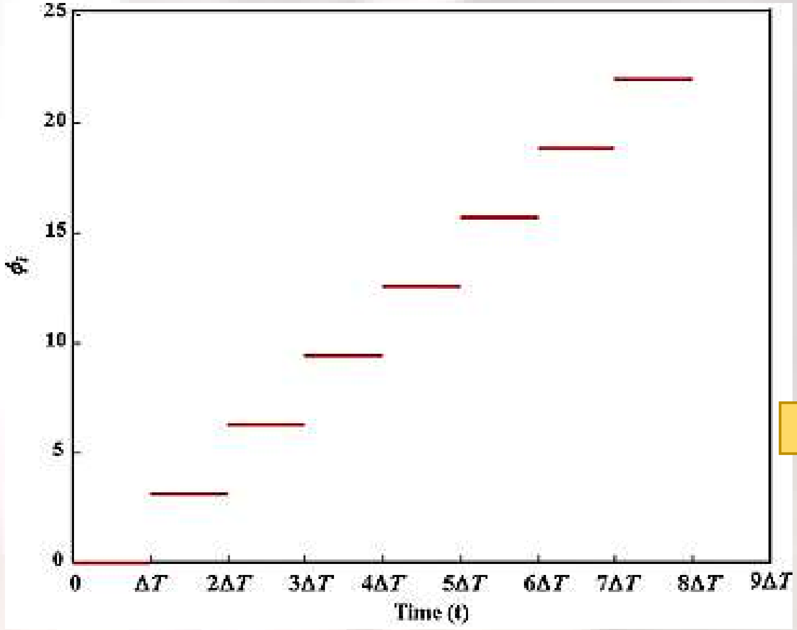
http://optiworks.jp/products_4d_phasecam6000_e.html

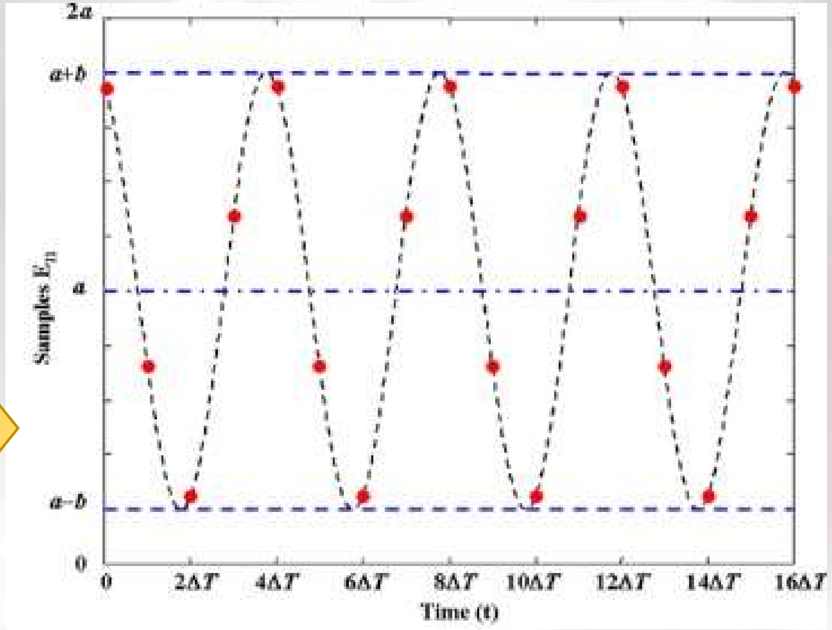
Phase Stepping

The phase modulation is a step and maintains constant during the integration time of the detector

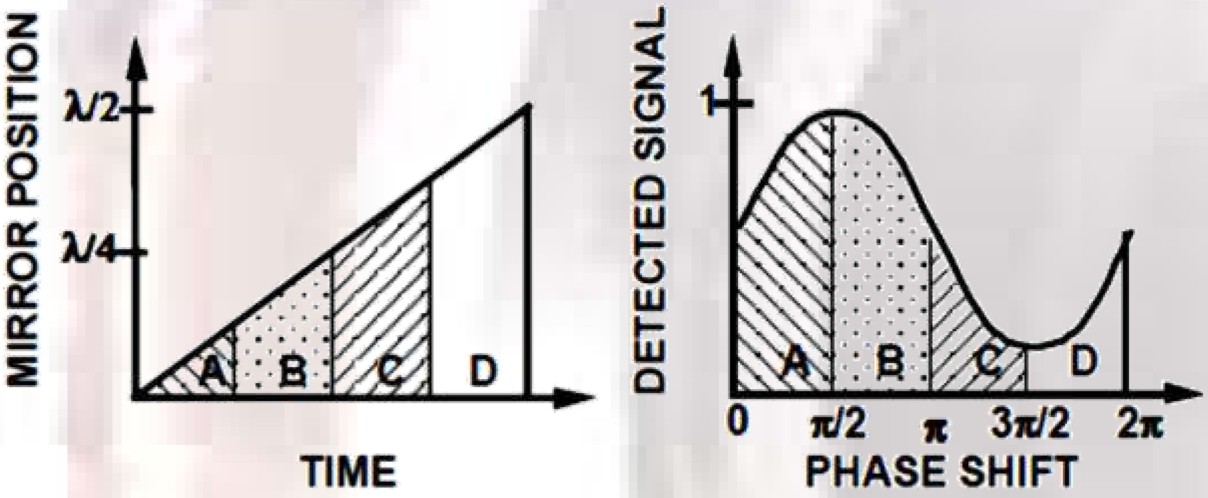
$$\varphi_n = \frac{2\pi}{N}(n-1)[u(t-(n-1)\Delta T) - u(t-n\Delta T)], n = 1, 2, \dots$$

$$\Lambda_n = a(x,y) + b(x,y)\cos(\delta + \varphi_n), n = 1, 2, \dots$$

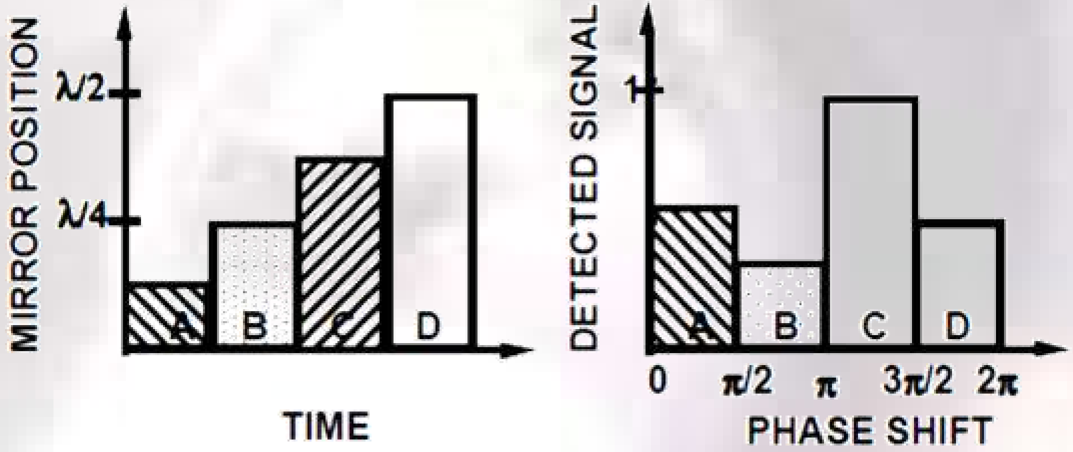




Phase Shifting vs. Phase Stepping

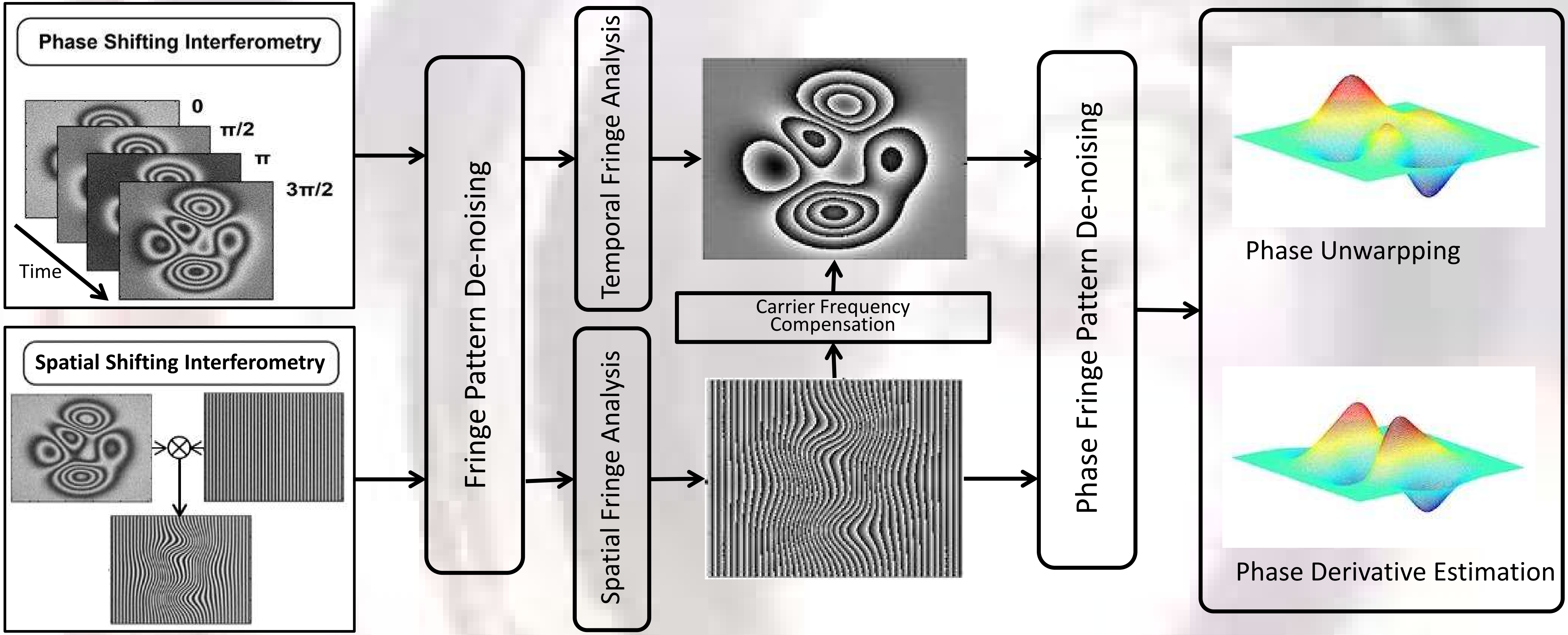


Phase Shifting

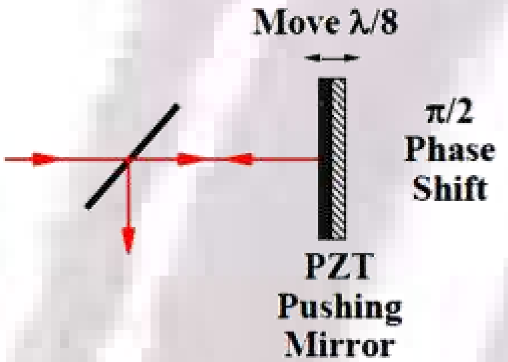


Phase Stepping

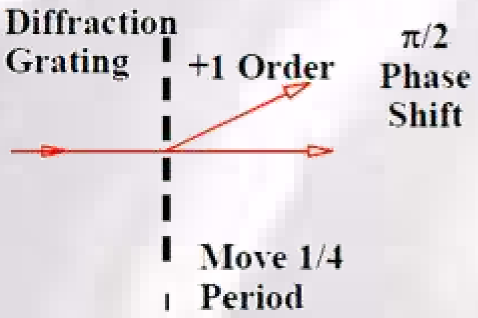
Phase estimation methods



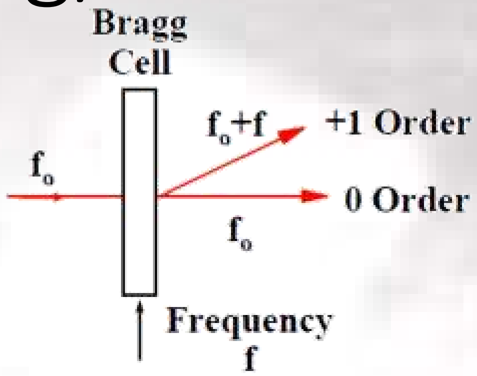
Mechanisms for Phase Shifting/ Phase Stepping



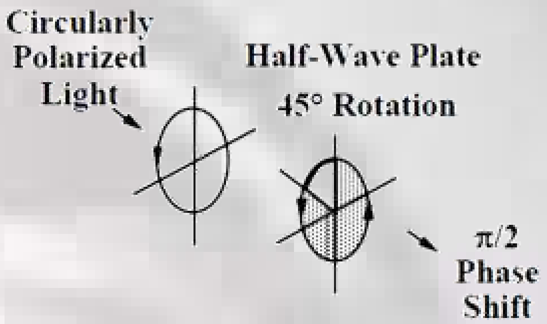
Moving Mirror
$\delta = 2 \times \lambda/8$



Diffraction Grating
$\delta = p/4$

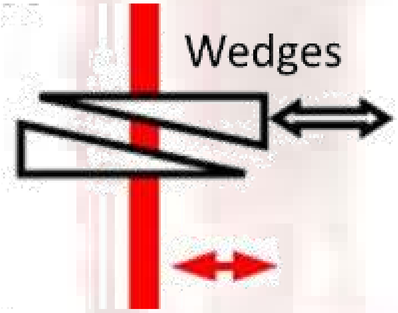


Bragg Cell/ Acousto-optical modulator

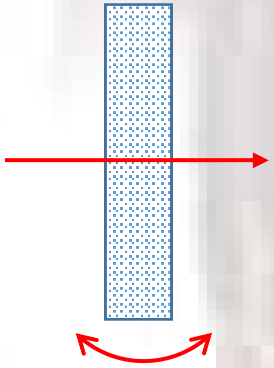


Rotating Half-Wave Plate

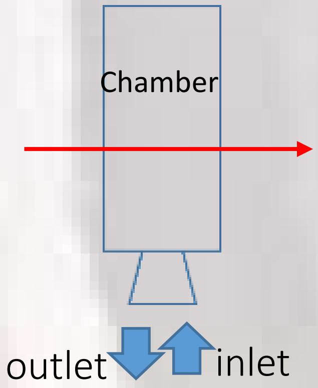
$\Delta\emptyset = 2\theta$



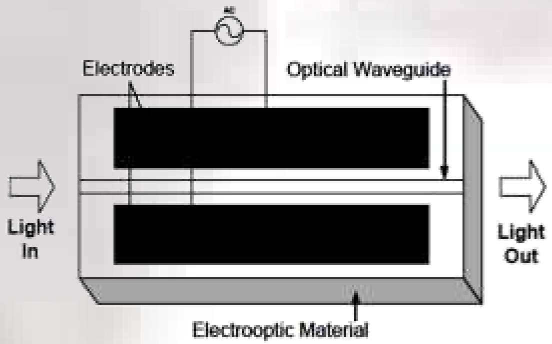
Wedge Compensator
$\delta = (n-1)\Delta l \tan\theta$



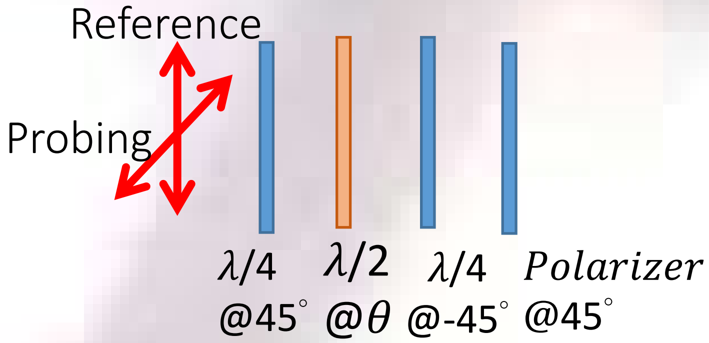
Titling Plane-Parallel plate



Pressure Chamber



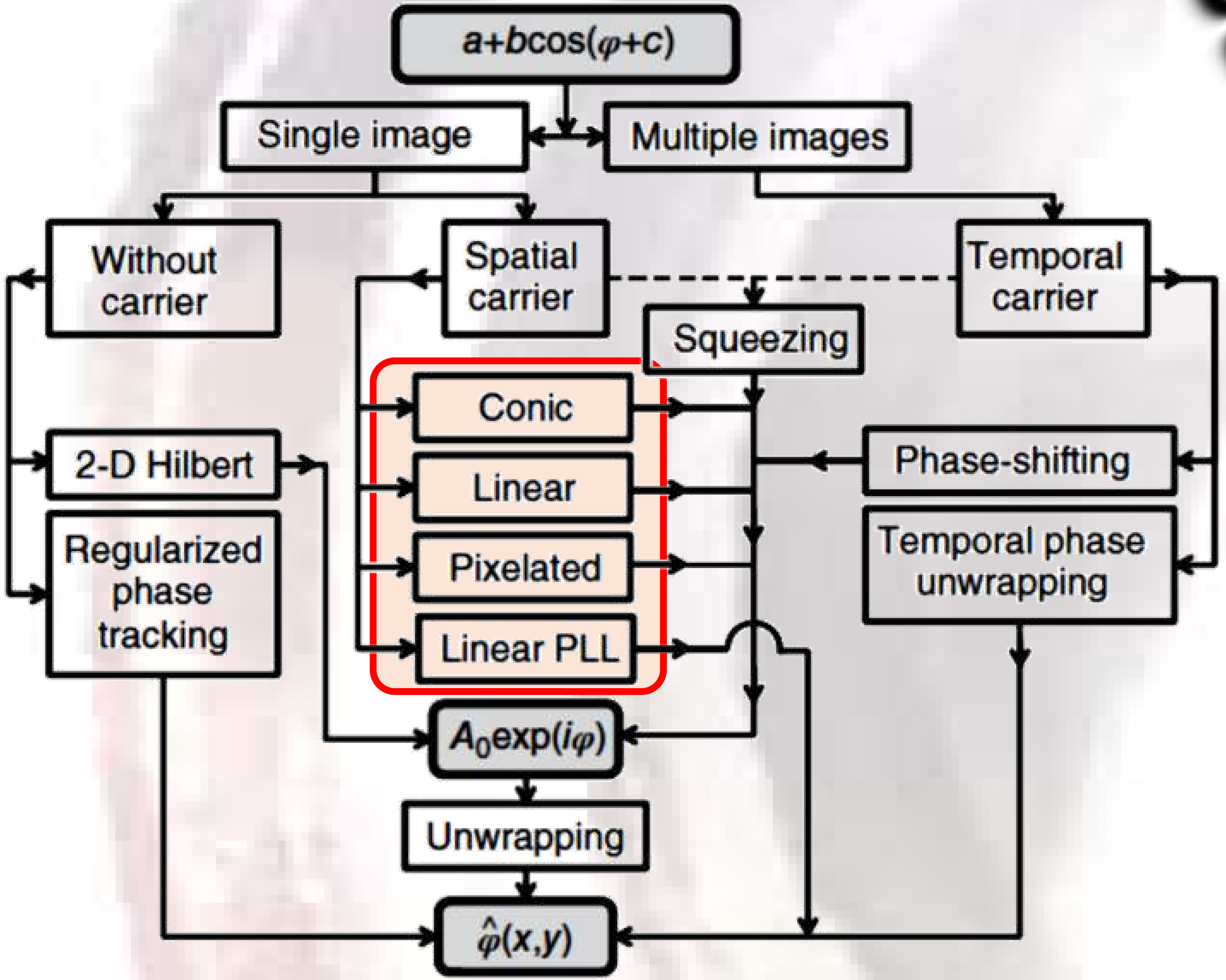
electro-optic modulator
$$\varphi = n(E)koL = 2\pi n(E)L/\lambda o$$

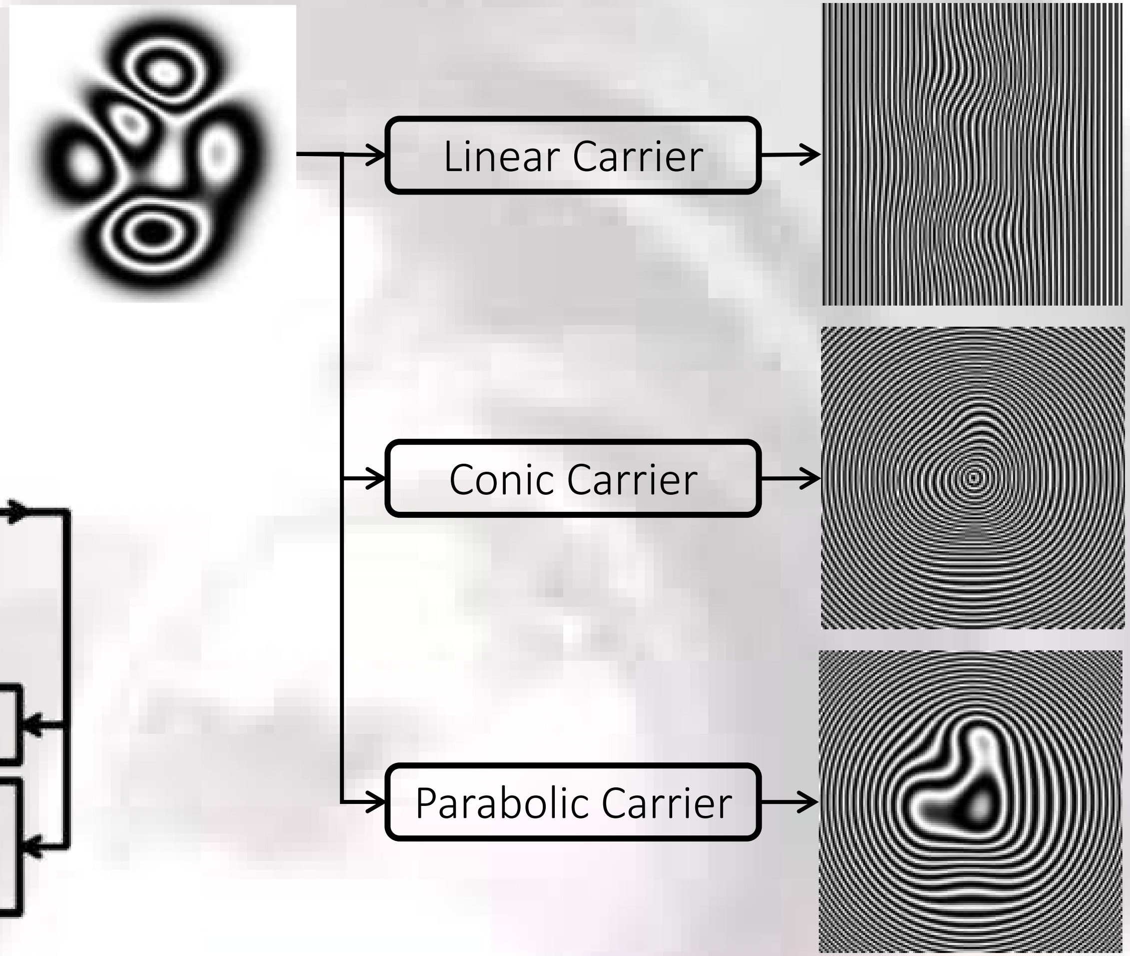


$\Delta\emptyset = 4\theta$

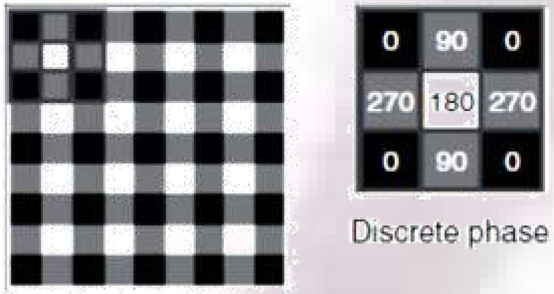
Geometric Phase Shifter

Classification of phase estimation methods



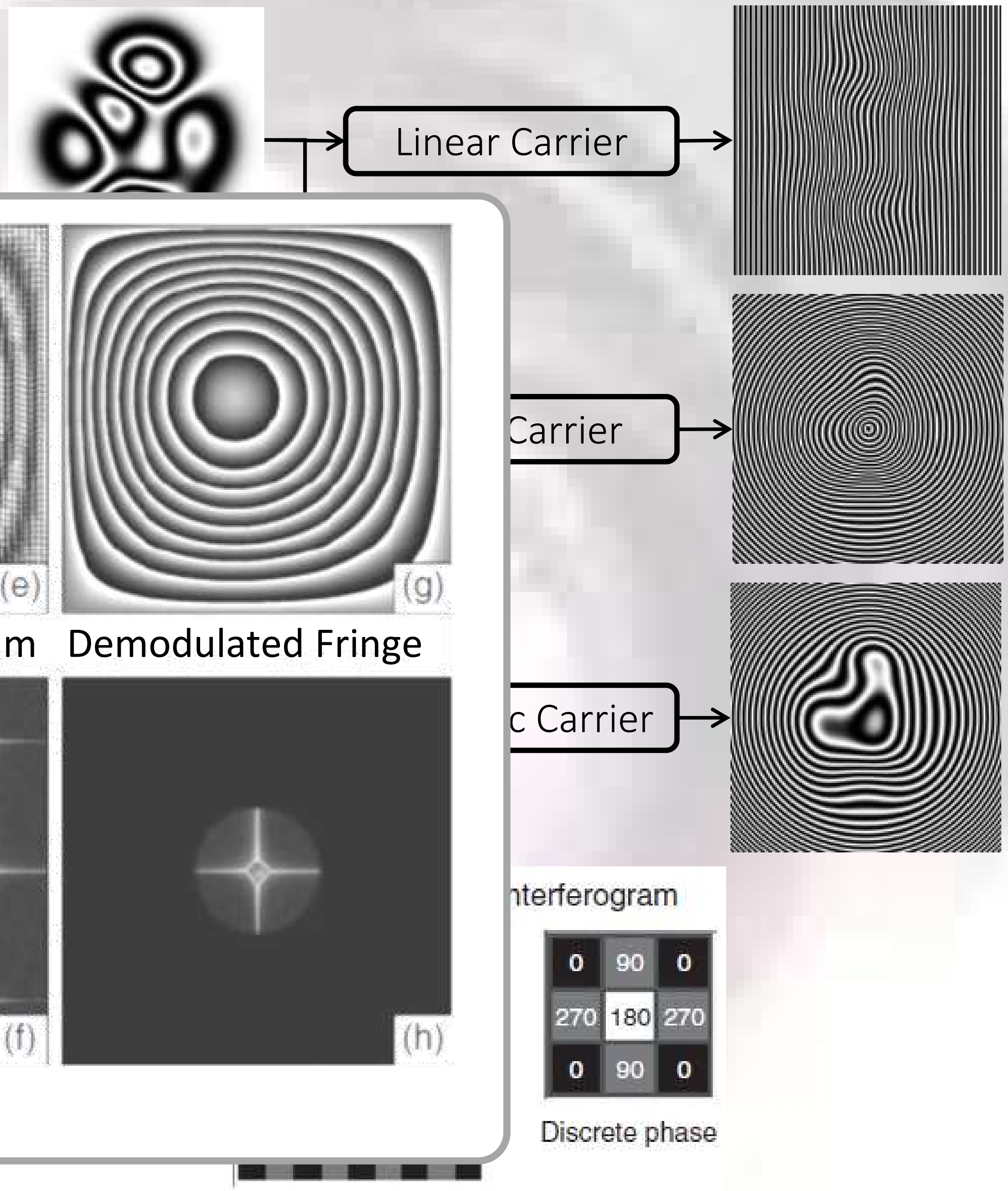


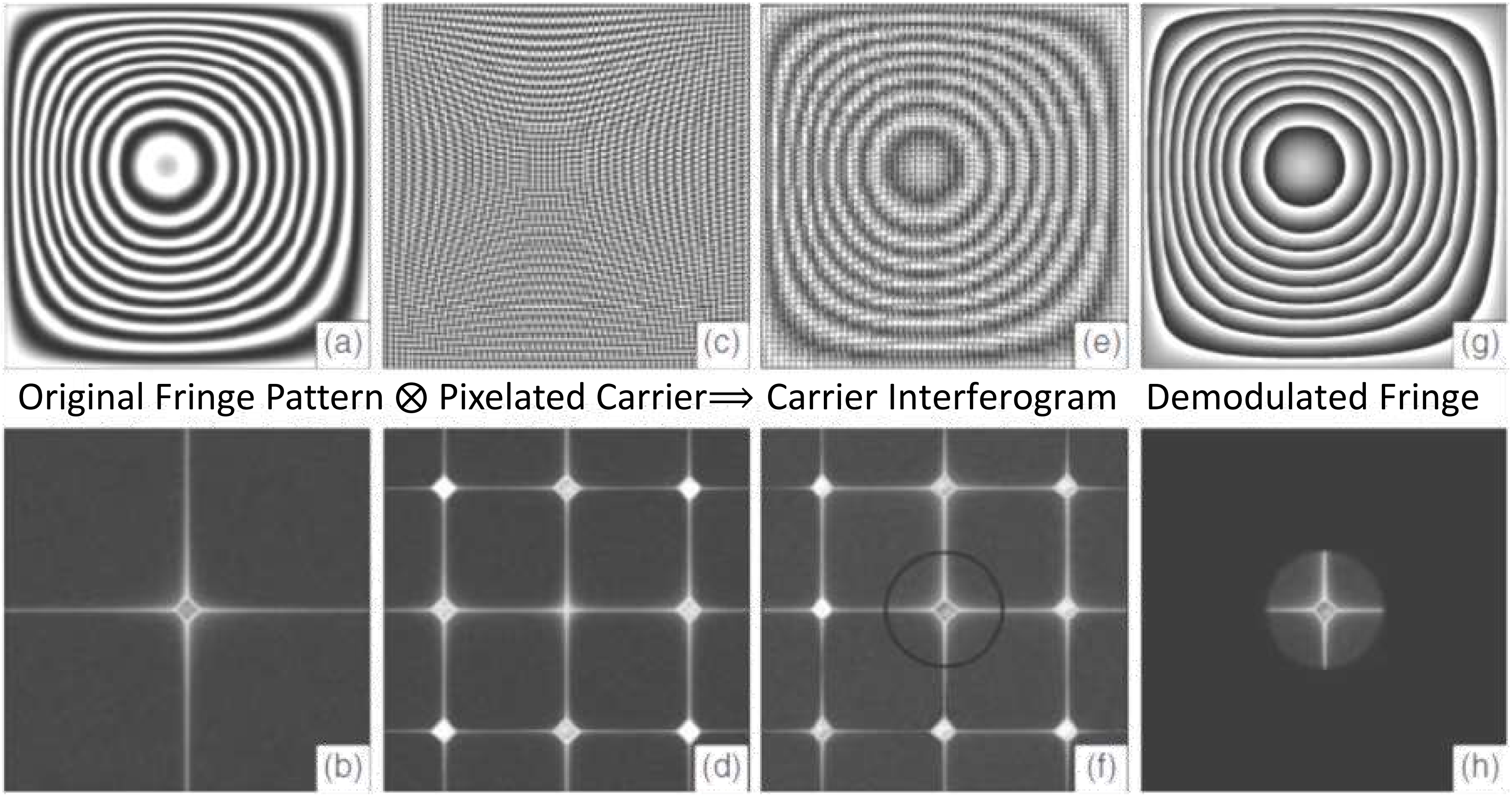
Pixelated carrier interferogram



Discrete phase

Classification of phase estimation methods





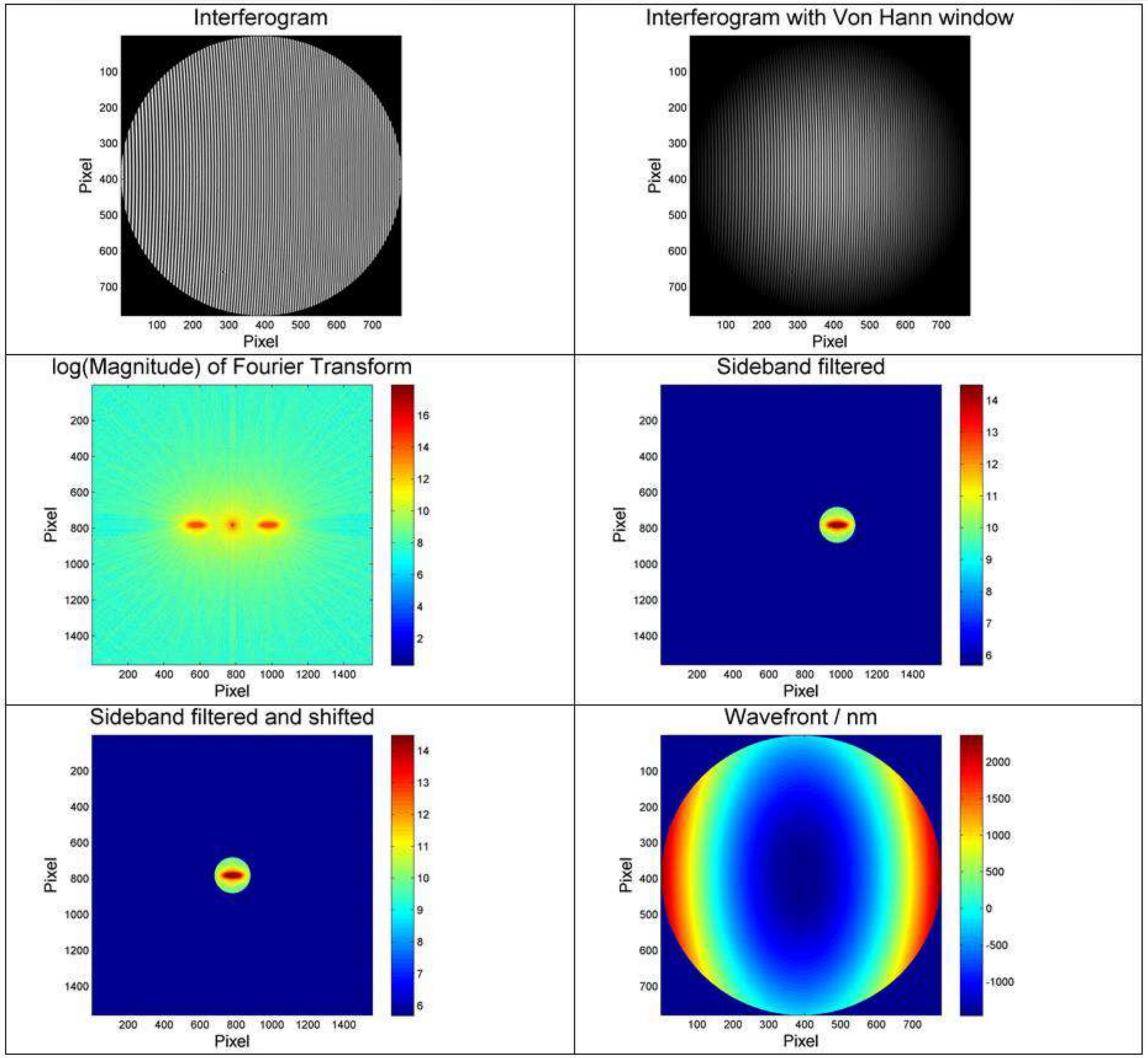
Original Fringe Pattern ⊗ Pixelated Carrier⟹ Carrier Interferogram Demodulated Fringe

Corresponding Phase Map

Procedures for Extracting Phase Map from Spatial Carrier Fringes

① Capturing Fringe Patterns With Spatial Carrier

$$I(x,y) = a(x,y) + b(x,y)\cos\delta\,(x,y)$$



② Fringe Pattern De-noising

③ Fourier transform

$$\mathcal{F}(I(x,y)) = G(f_x,f_y) = \mathrm{A}(f_x,f_y) + C(f_x,f_y) + C^*(f_x,f_y)$$

④ Filtering with bandpass

$$G'(f_x,f_y) = \begin{cases} C(f_x,f_y); \; \|(f_x,f_y)-(\mu,\nu)\| \le \mathrm{R} \\ 0; \text{otherwise} \end{cases}$$

⑤ Centering the fileted Spectrum

$$G'(f_x,f_y) = \begin{cases} C(f_x,f_y); \; \|(f_x,f_y)-(0,0)\| \le \mathrm{R} \\ 0; \text{otherwise} \end{cases}$$

⑥ reconstructed phase and Unwrapped

$$\mathcal{F}^{-1}\left(G'(f_x,f_y)\right) = C(x,y) = \frac{1}{2}b(x,y)e^{i\delta}$$

$$\Longrightarrow \delta(x,y) = \tan^{-1}\left(\frac{ImC(x,y)}{ReC(x,y)}\right)$$

Phase-Measurement Algorithms		
# of Frames	Phase Shift	Phase
3	$\pi/2$	$\emptyset = \tan^{-1}(\frac{I_1 - I_2}{I_2 - I_3})$
4	$\pi/2$	$\emptyset = \tan^{-1}(\frac{I_2 - I_4}{I_2 - I_3})$
Carré Equation	$\pi/2$	$\emptyset = \tan^{-1}(\frac{\sqrt{3[(I_2 - I_3) - (I_1 - I_4)][(I_2 - I_3) + (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)})$
5 (Schwider-Hariharan)	$\pi/2$	$\emptyset = \tan^{-1}(\frac{-2I_2 + 2I_4}{I_1 - 2I_3 + I_5})$
7	$\pi/3$	$\emptyset = \tan^{-1}(\frac{\sqrt{3}(I_2 + I_3 - I_5 - I_6)}{-I_1 - I_2 + I_3 + 2I_4 + I_5 - I_6 - I_7})$
8	$\pi/2$	$\emptyset = \tan^{-1}(\frac{I_1 + 5\, I_2 - 11I_3 - 15I_4 + 15I_5 + 11I_6 - 5I_7 - I_8}{I_1 - 5\, I_2 - 11I_3 + 15I_4 + 15I_5 - 11I_6 - 5I_7 + I_8})$
12	$\pi/3$	$\emptyset = \tan^{-1}(\frac{\sqrt{3}(-3I_2 - 3I_3 + 3I_4 + 9I_5 + 6I_6 - 6I_7 - 9I_8 - 3I_9 + 3I_{10} + 3I_{11}}{2I_1 + I_2 - 7I_3 - 11I_4 - I_5 + 16I_6 + 16I_7 - I_8 - 11I_9 - 7I_{10} + I_{11} + 2I_{12}})$
N (synchronous detection)	$\alpha_i = \frac{2\pi i}{N}$, $i = 1,2 \ldots\ldots$	$\emptyset = -\tan^{-1}\left[\frac{\sum_{i=1}^{N} \sin \alpha_i}{\sum_{i=1}^{N} \cos \alpha_i}\right]$