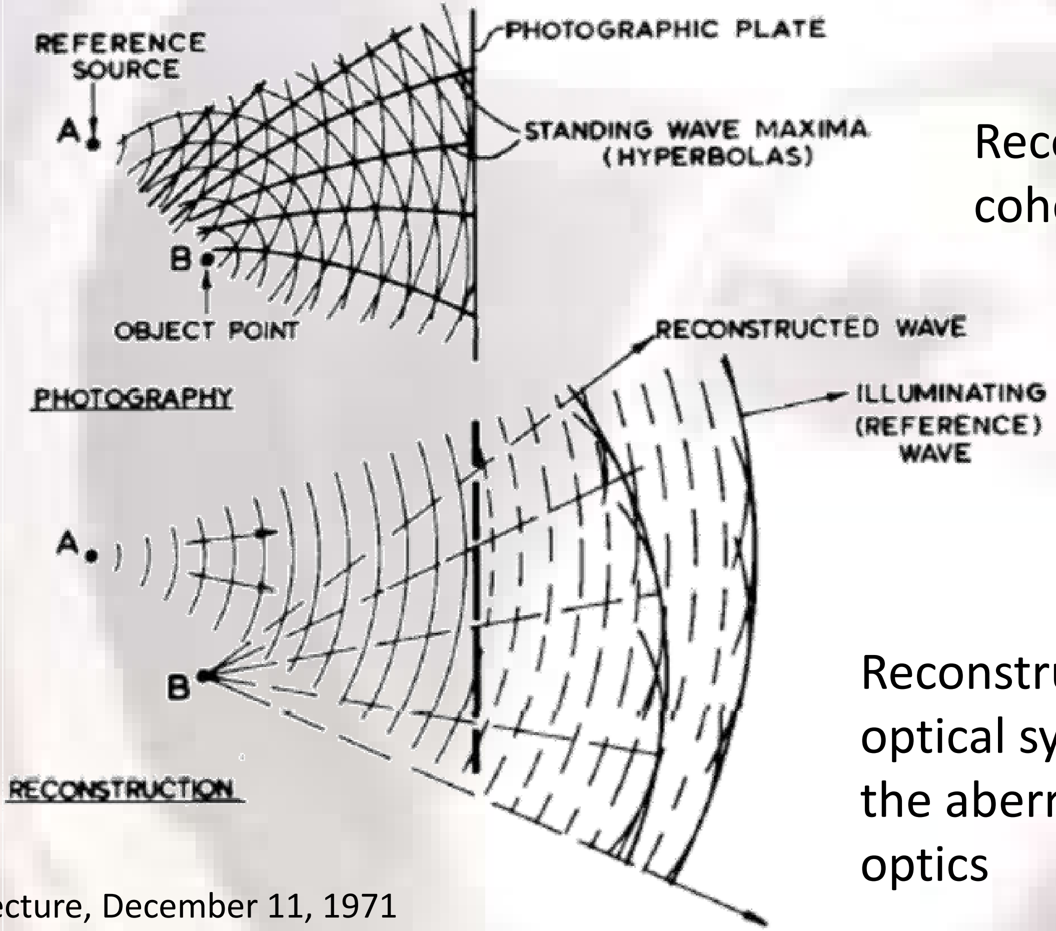
Holography


Fraunhofer
Holographic
measurement technology
at production speed

Introduction to holography

Holography is proposed by D. Gabor in 1948, it initially aimed at solving the resolution problem of electron microscope caused by the abreactions of electron optics.

D. Gabor

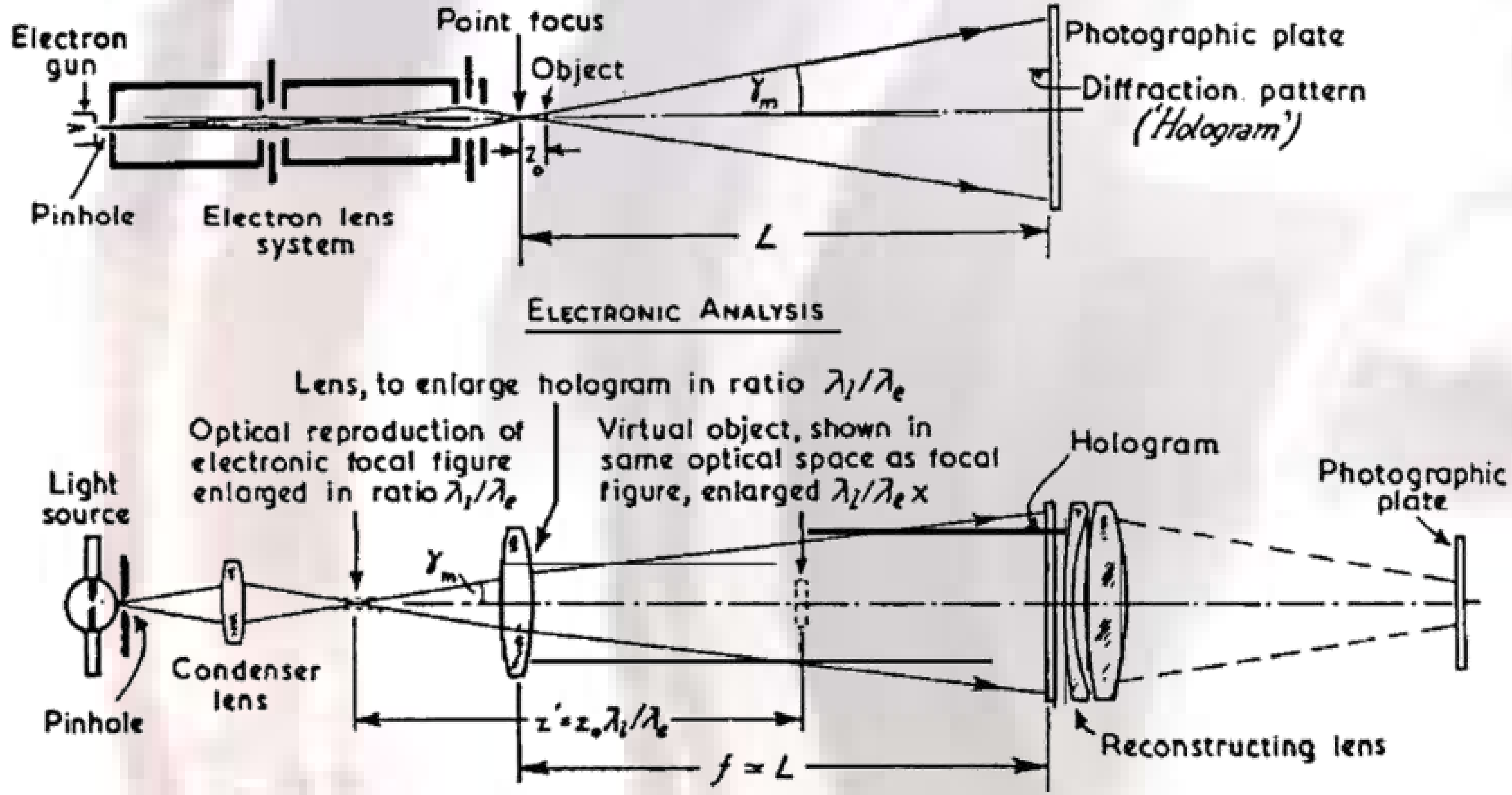


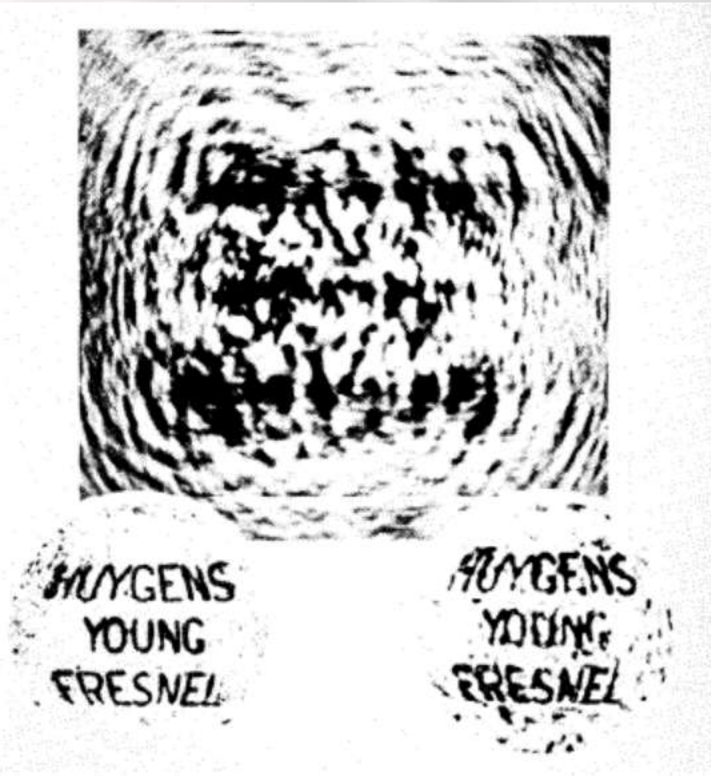
Recording image with coherent background

Reconstructed with light, in an optical system which corrected the aberrations of the electron optics

DENNIS GABOR, HOLOGRAPHY, 1948-1971, Nobel Lecture, December 11, 1971

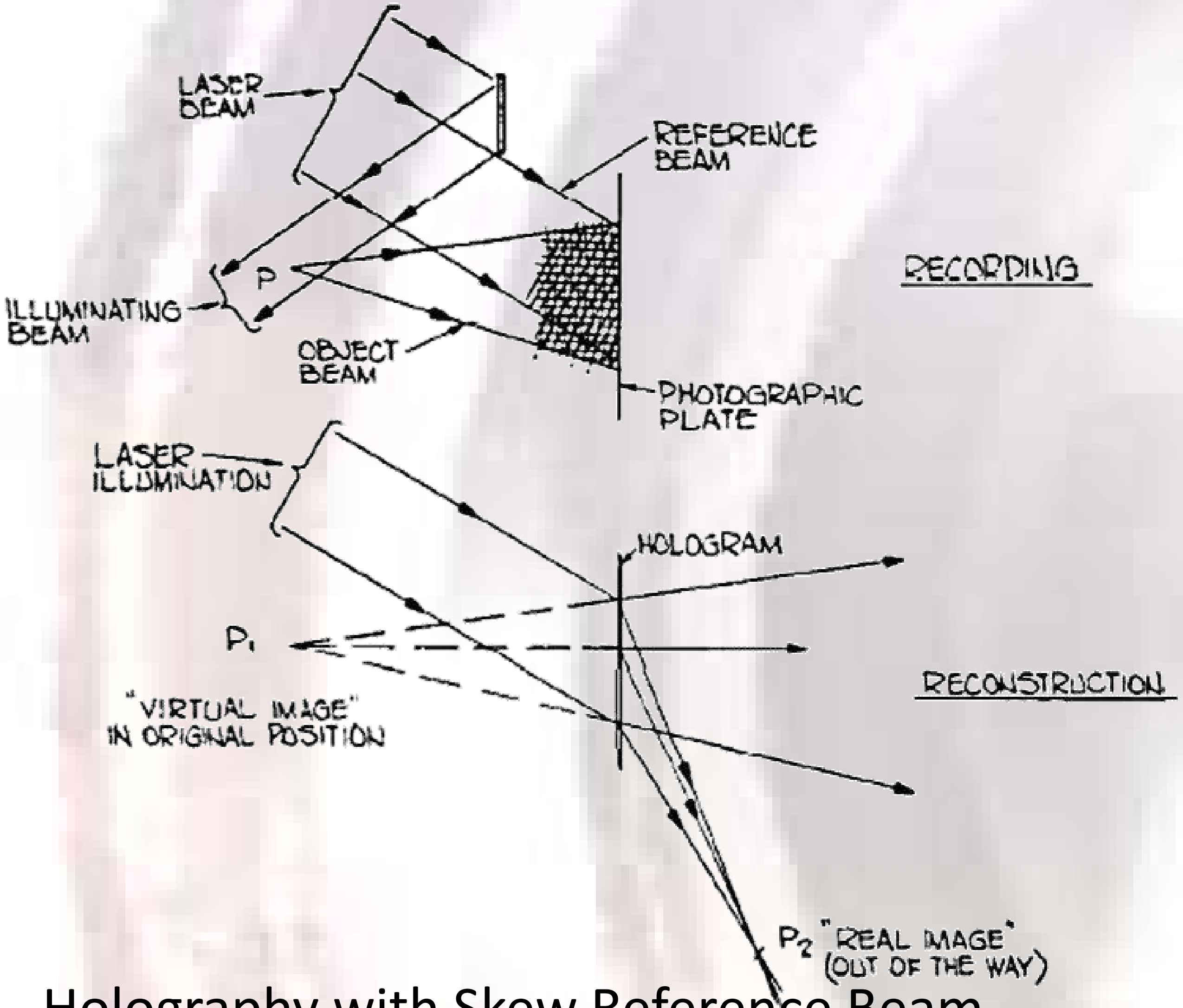
Introduction to holography





The Principle of Electron Microscopy by Recon structured Wavefronts (@1949)

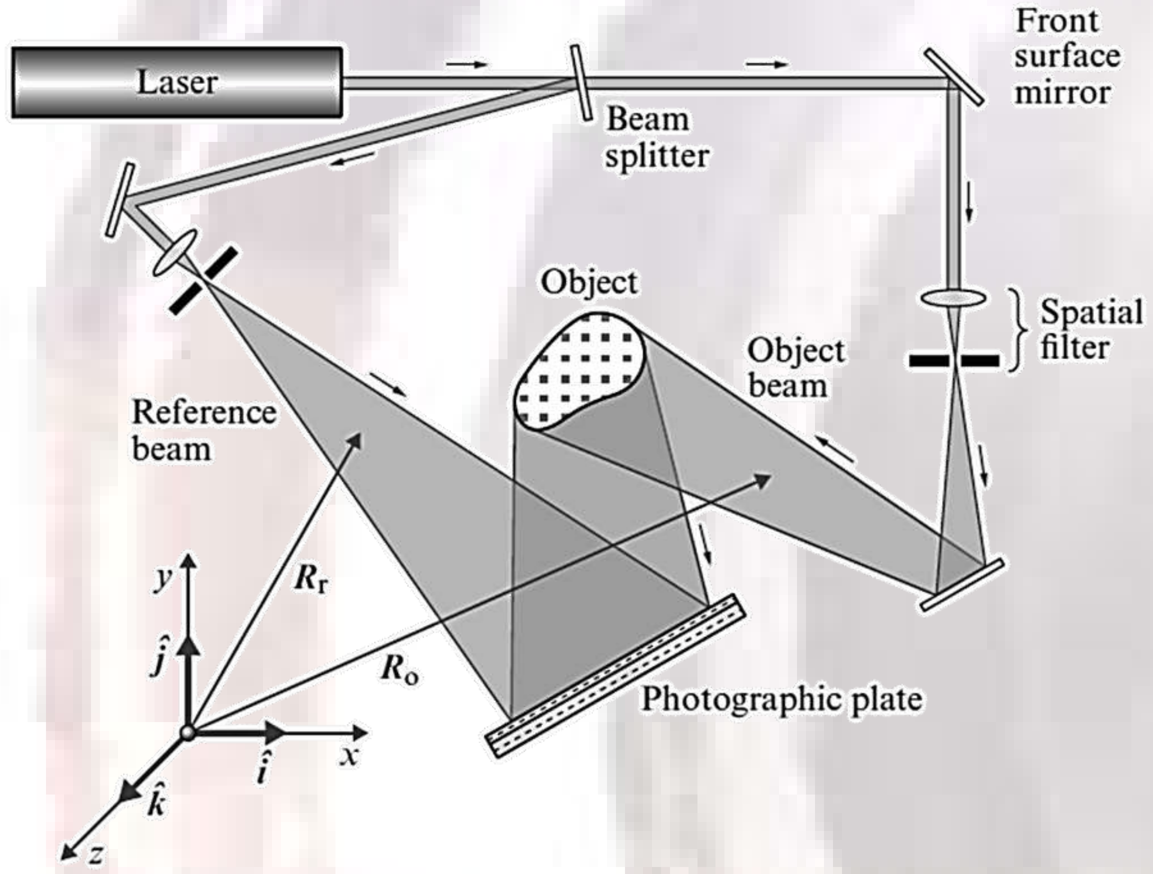
Introduction to holography



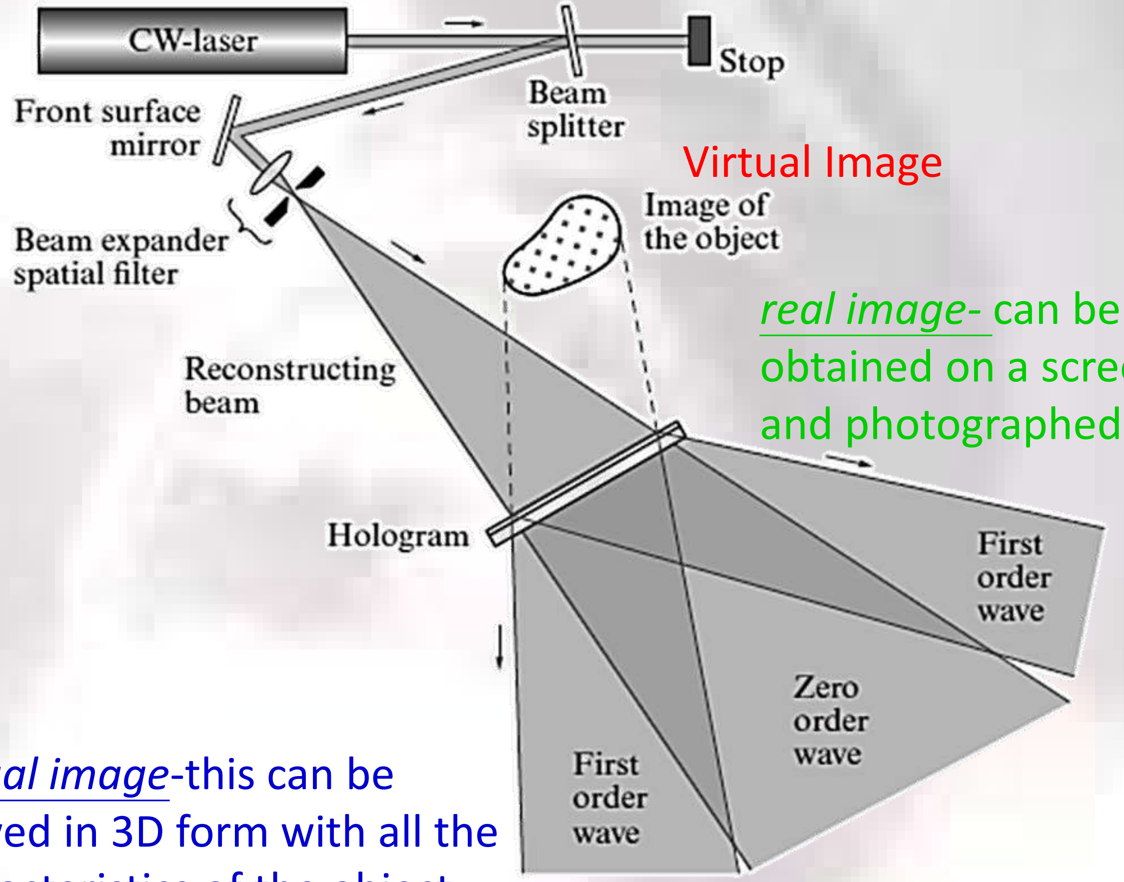
Holography with Skew Reference Beam.
E. N. Leith and J. Upatnieks, 1963

"Train and Bird" is the first hologram ever made with a laser using the off-axis technique in 1964 by Emmett Leith and Juris Upatnieks at the University of Michigan

Recording/ Reconstructing of Hologram



A typical setup for recording of a hologram



Virtual Image

real image- can be obtained on a screen and photographed

virtual image-this can be viewed in 3D form with all the characteristics of the object

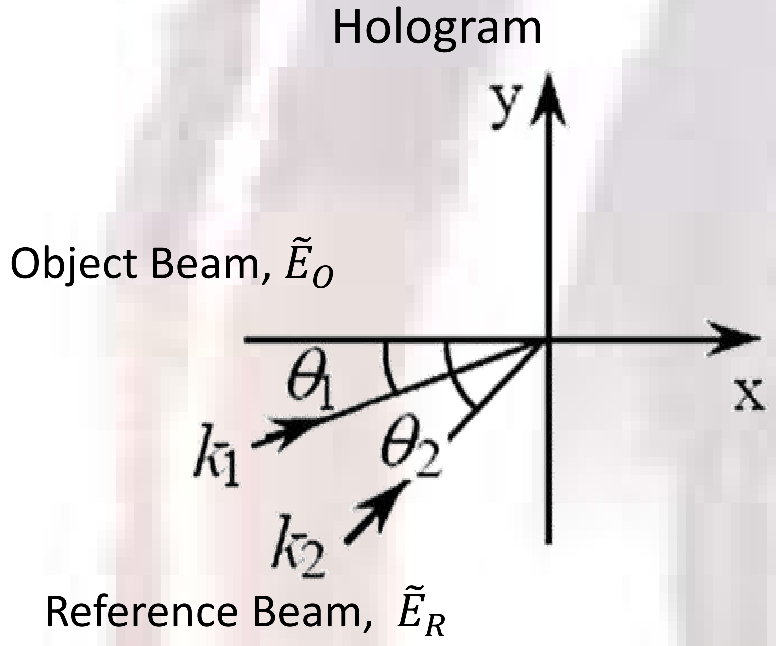
Transmission Layout

A typical setup used for reconstruction of a hologram

Photography vs. Holography

Photography	Holography
• 2D Intensity map for 3D object • Lenses are needed for imaging • Recorded film is called photograph • Resemblance between object and photograph • One image recorded on a film (typical), Information capacity is less • Part of photograph gives partial information only • Point to point light intensity recorded • Lacks depth perception or parallax • No special illumination required to view a photograph • Intensity variation only recorded	• Intensity and Phase for 3D object • Lenses not needed for imaging • Recorded film is called hologram • No resemblance between the objects and hologram • (Multi-)Holograms can be recorded on a film, Information capacity is much more • Part of hologram gives full information of object • Intensity and phase variation recorded • Provides depth perception and parallax • To view a hologram, the wavefront is reconstructed • Record the light-wavefront that carries all information of the scene

Simple model for Holography



$$E_O = e_O e^{i(\tilde{\kappa}_1 \cdot \tilde{r} + \omega t + \emptyset_1)}; E_R = e_R e^{i(\tilde{\kappa}_2 \cdot \tilde{r} + \omega t + \emptyset_2)}$$

where

$$\tilde{r} = r_1 \tilde{e}_1 + r_2 \tilde{e}_2$$

$$\tilde{\kappa}_1 = \frac{2\pi}{\lambda}(cos\theta_1 \tilde{e}_1 + sin\theta_1 \tilde{e}_2); \tilde{\kappa}_2 = \frac{2\pi}{\lambda}(cos\theta_2 \tilde{e}_1 + sin\theta_2 \tilde{e}_2)$$

$$\Delta = (\tilde{\kappa}_1 - \tilde{\kappa}_2) \cdot \tilde{r} + (\emptyset_1 - \emptyset_2)$$

Intensity $I(x,y)$ distribution on the hologram,

$$I(x,y) = (E_O + E_R) \times (E_O^* + E_R^*)$$

$$= (E_O E_O^* + E_R E_R^* + E_R E_O^* + E_O E_R^*)$$

$$= I_O + I_R + E_R E_O^* + E_O E_R^*$$

For Exposure time t, the total Energy Recorded, P,

$$P = It$$

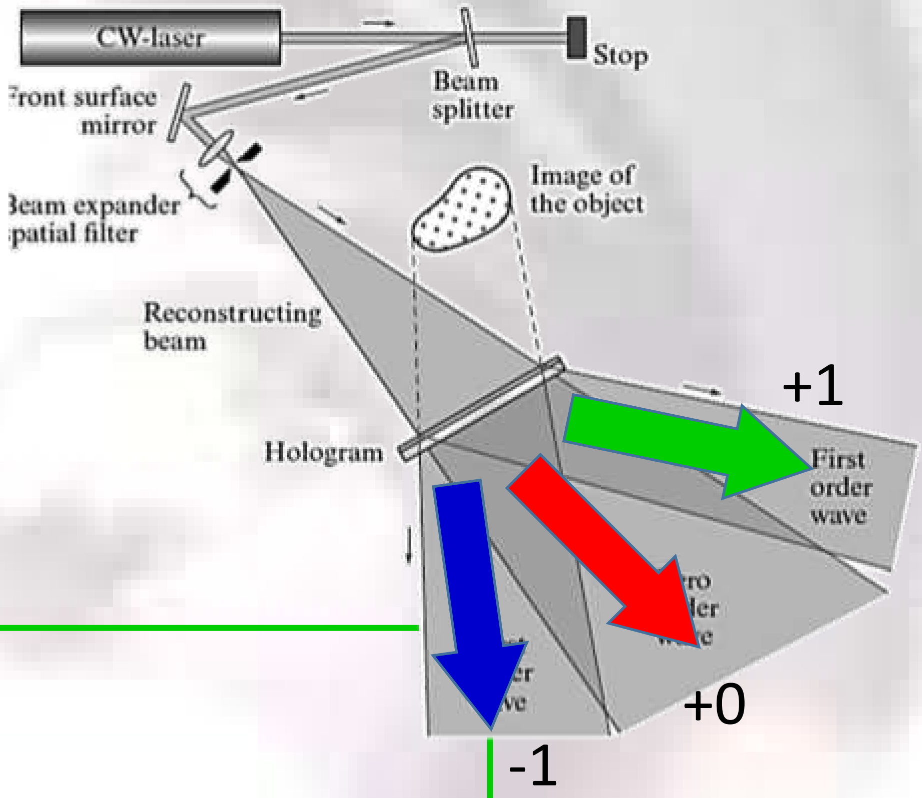
Simple model for Holography

Considering the transmission of the hologram is β, that is

$$\beta = \frac{E_{pass\ through}}{E_{incident}}$$

$\Rightarrow$ illuminating the hologram with Reference light

$$\Rightarrow P\beta E_R = \beta t E_R(E_O E_O^* + E_R E_R^* + E_R E_O^* + E_O E_R^*)$$
$$= \beta t E_R(I_O + I_R) + \beta t I_R(E_O^* e^{2\emptyset_r} + E_O)$$



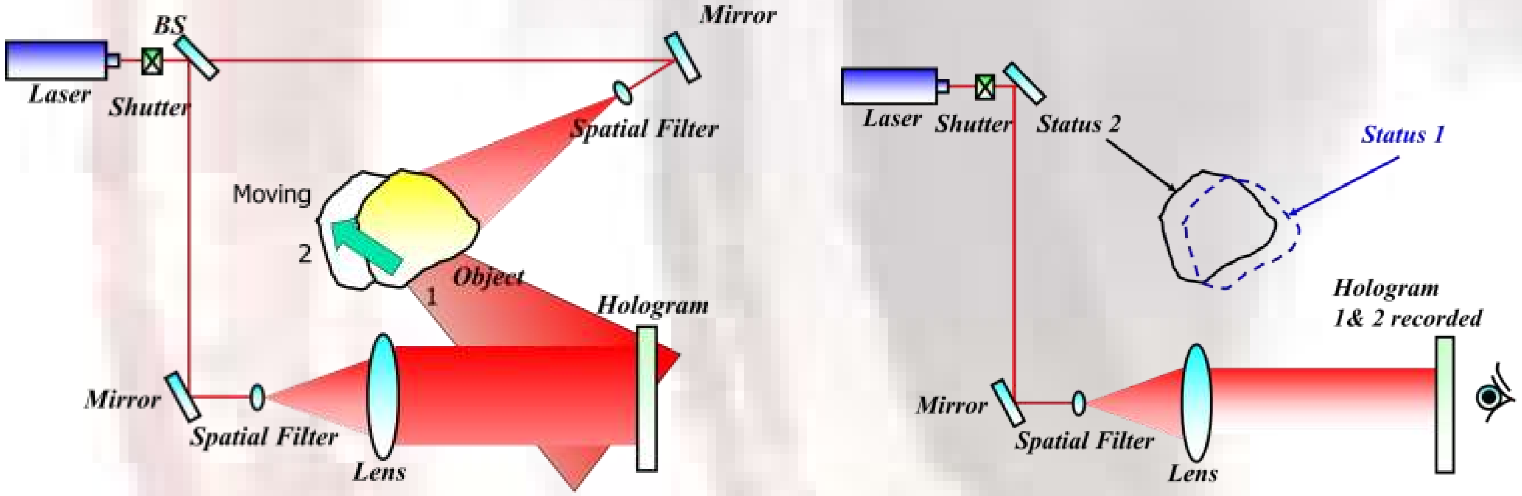
Direct wave –
identical to reference wave except for an overall change in amplitude

Conjugate wave –
complex conjugate of object wave displaced

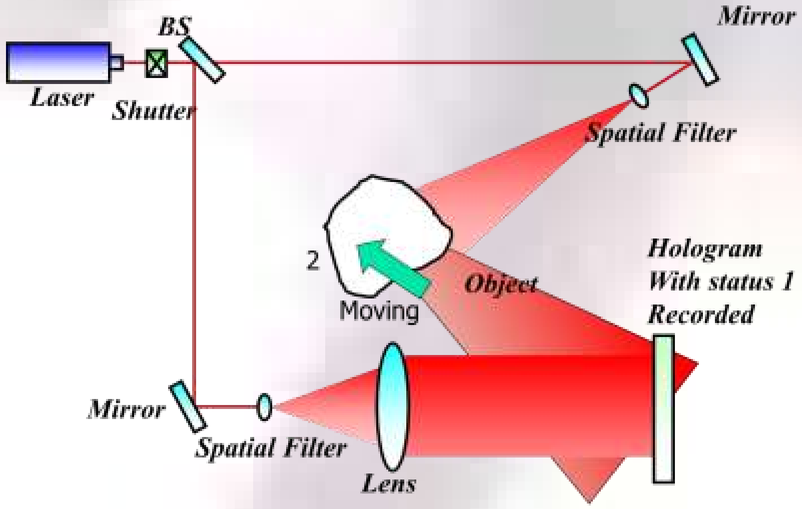
Object wave –
identical to object wave except for a change in intensity

Holographic Interferometry

1. Double-exposure hologram interferometry,
2. Real-time hologram interferometry,
3. Time-average hologram interferometry
   - Stroboscopic time-average hologram interferometry,
   - Continuous time-average hologram interferometry



Double-exposure hologram interferometry



Real-time hologram interferometry

Double-exposure hologram interferometry

First Exposure

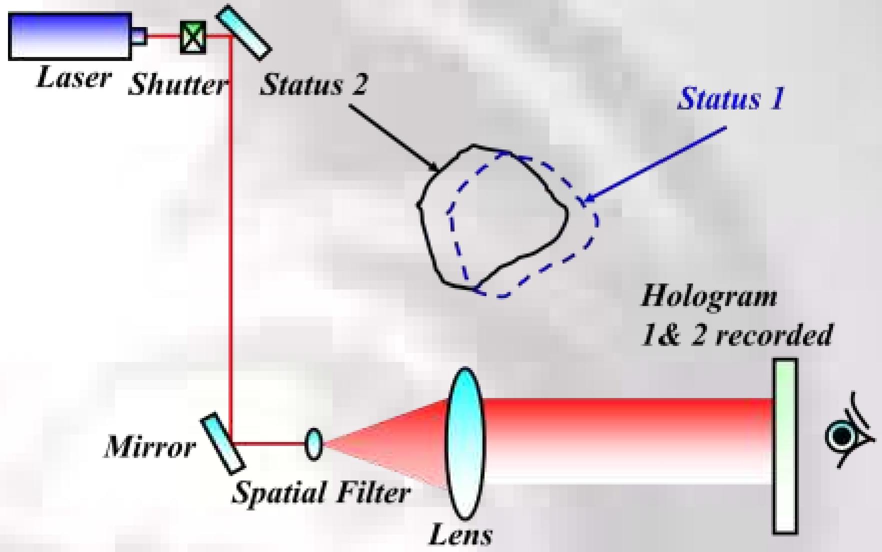
$$E_O = e_O e^{i(\tilde{\kappa}_1\cdot\tilde{r}+\omega t+\emptyset_1)}$$

$$\Rightarrow I_1 = I_O + I_R + E_R E_O^* + E_O E_R^*$$

Second Exposure for a unit time interval

$$E'_O = e_O e^{i(\tilde{\kappa}_1\cdot\tilde{r}+\omega t+\emptyset'_1)}$$

$$\Rightarrow I_2 = I'_O + I_R + E_R E'^*_O + E'_O E_R^*$$



Exposure for twice (at different Status)

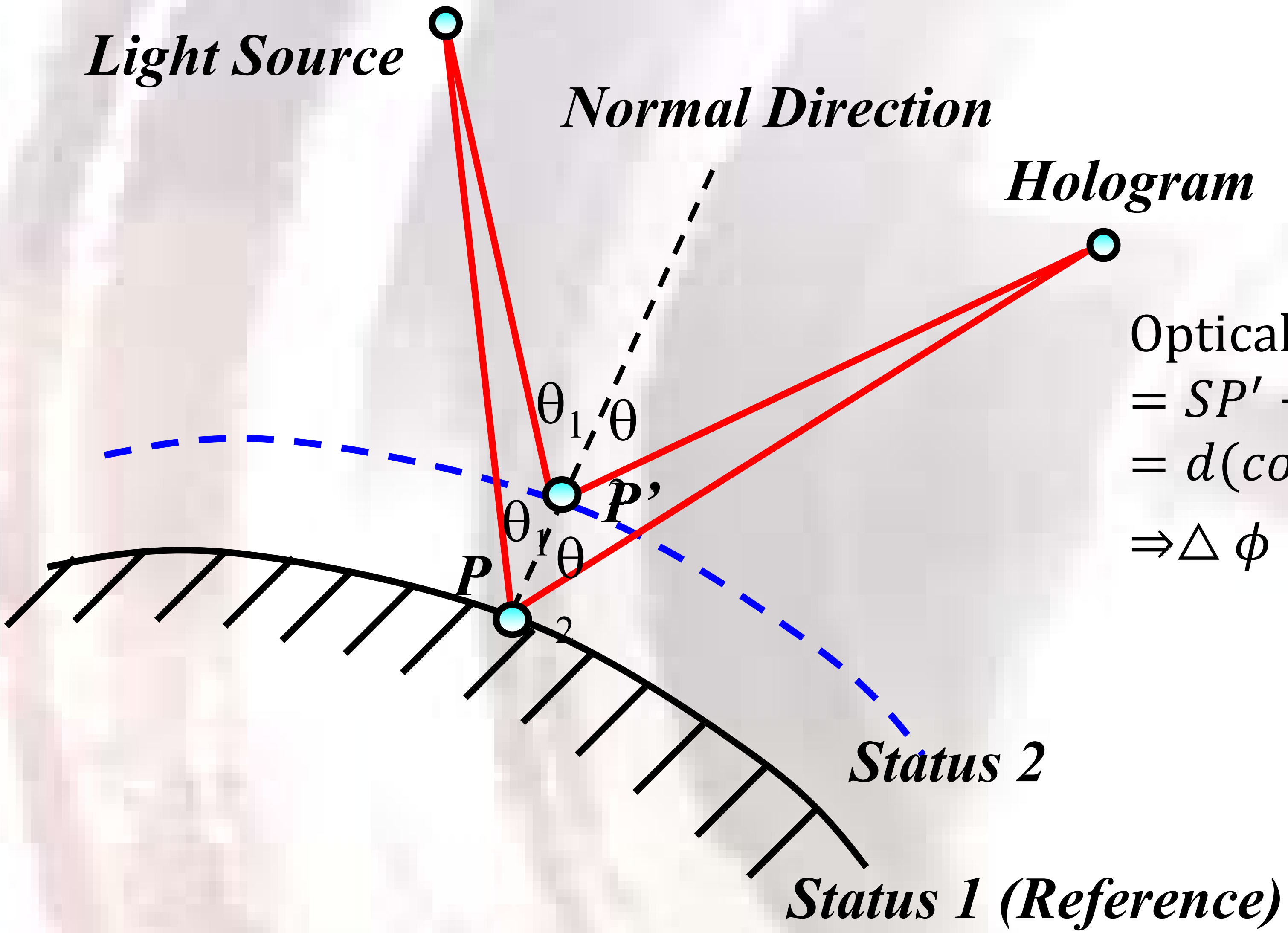
$$\Rightarrow \Xi = I_1 + I_2 = I_O + I_R + E_R E_O^* + E_O E_R^* + I'_O + I_R + E_R E'^*_O + E'_O E_R^*$$

Illuminating the hologram with Reference light

$$\Rightarrow E_R \beta \Xi = 2\beta E_R(2I_R + I_O + I'_O) + \beta I_R e^{2\emptyset_r}(E_O^* + E'^*_O) + \beta I_R(E_O + E'_O)$$

$$\text{Recording Intensity} \propto I_R^2\left(I_O + I'_O + \boxed{2\sqrt{I_O I'_O}\cos(\emptyset_1 - \emptyset'_1)}\right)$$

Fringe Formula



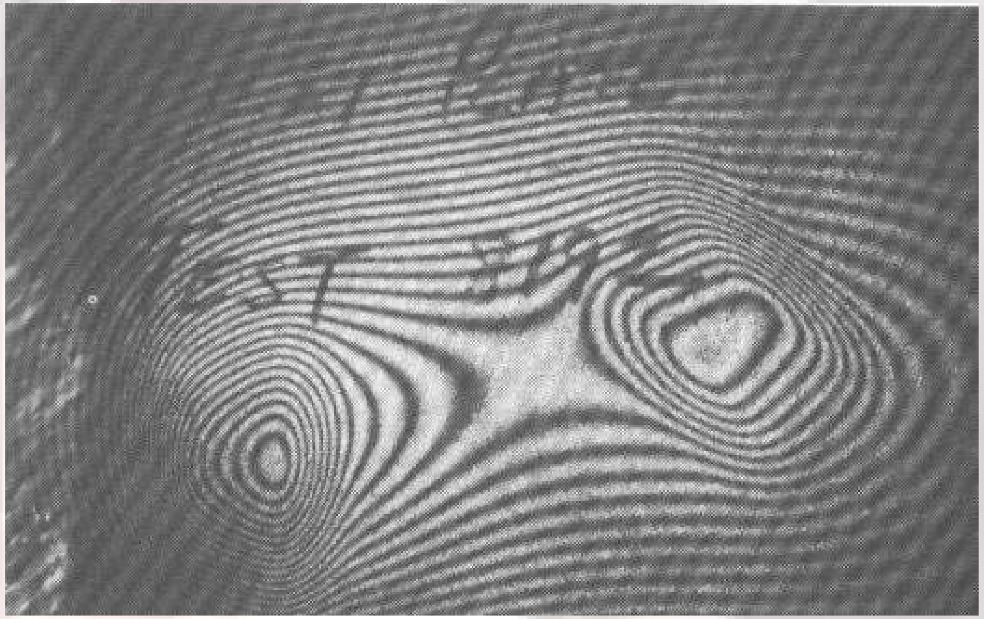
Optical Path Difference

$$= SP' + P'H - (SP + PH)$$

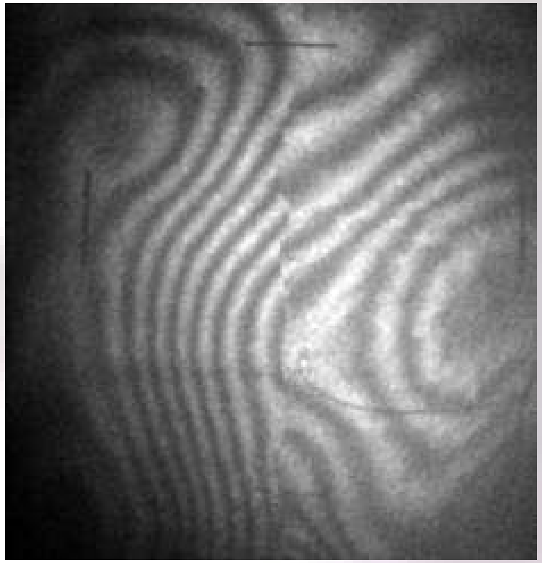
$$= d(cos\theta_1 + cos\theta_2)$$

$$\Rightarrow \triangle \phi = \frac{2\pi}{\lambda} d(cos\theta_1 + cos\theta_2)$$

Cases for Double Exposure

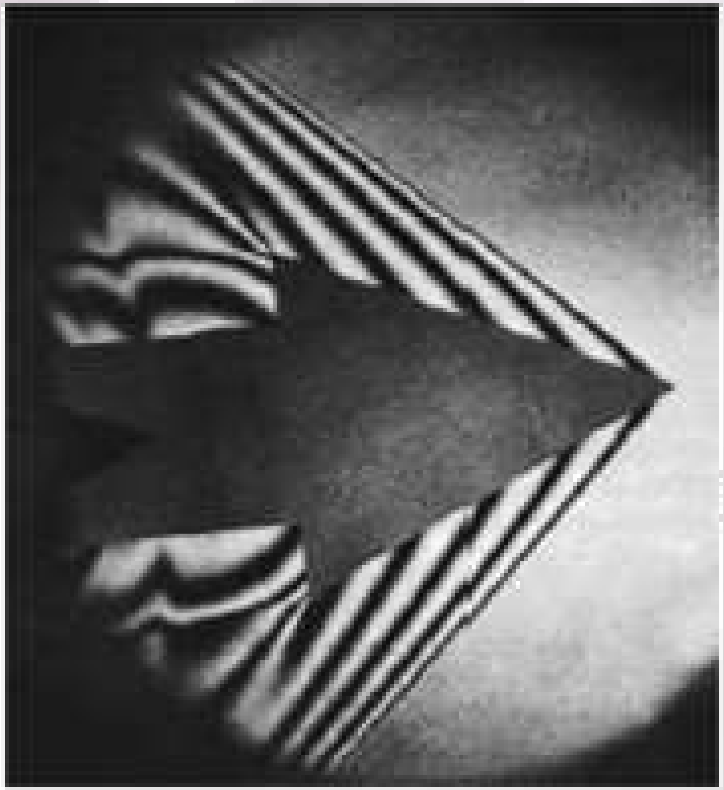


Double exposure hologram of a flat metal plate which was stressed between exposures from Hilton & Mayville, Opt. Eng 24 757-768 (1985)

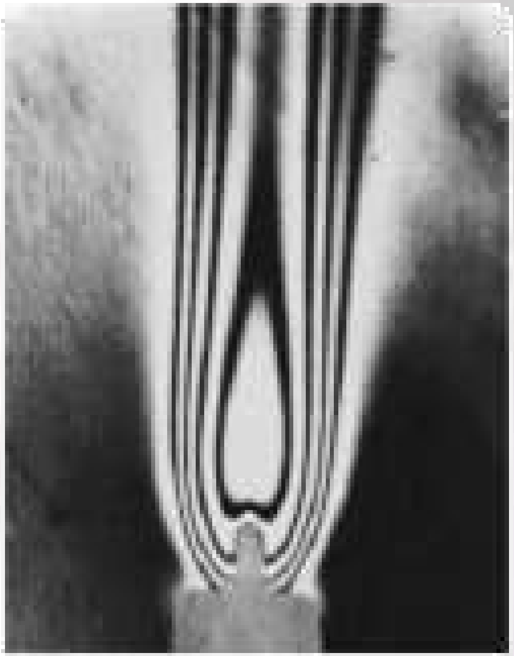


A concrete wall with a defect excited by an impact

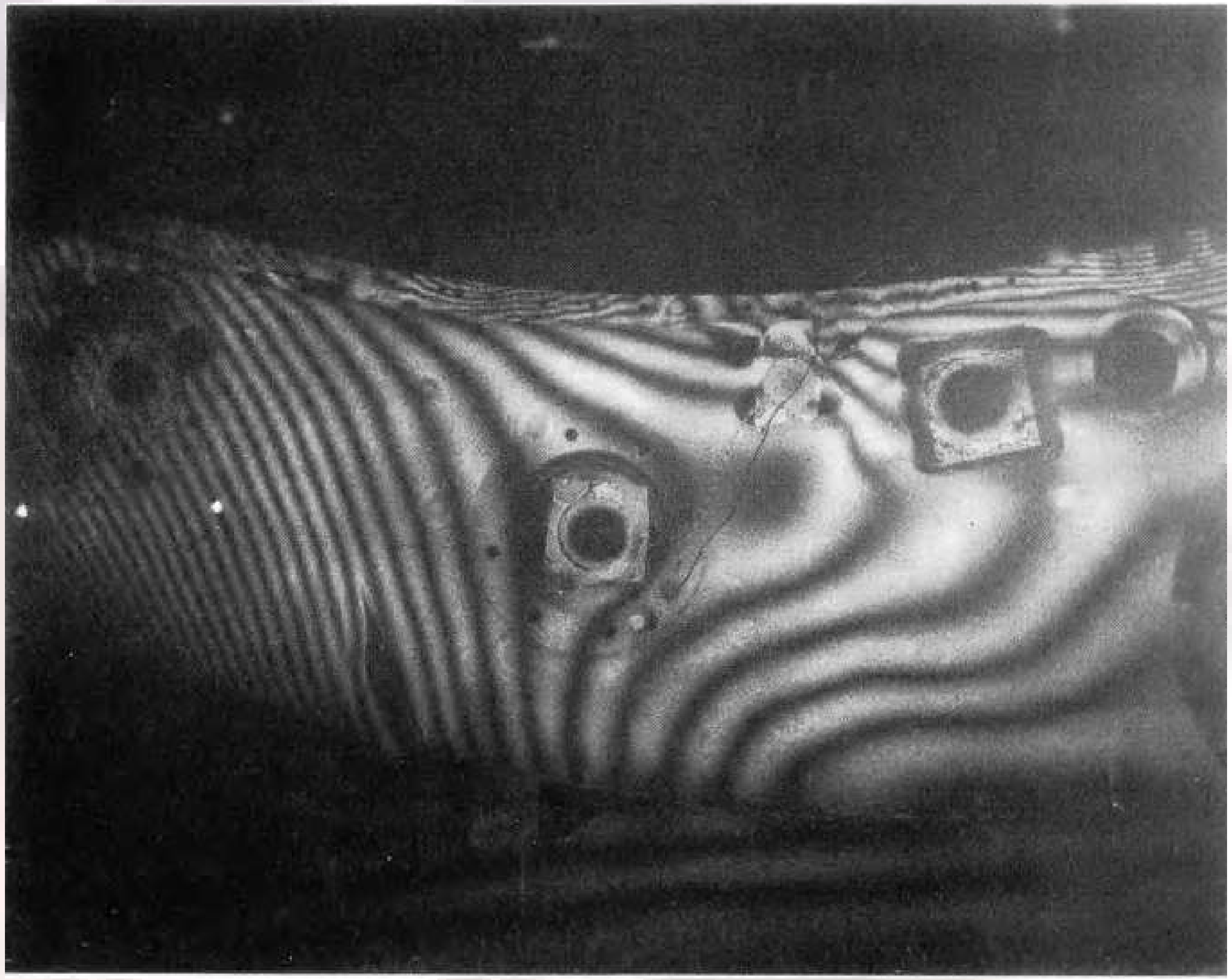
Cases for Double Exposure



Air Flow Past Cone



Candle



Double-exposure hologram showing interference fringes due to deformation. The object was a gearbox housing and deformation was produced by local heating between holographic exposures.

Real Time Method

First Exposure

$E_O = e_O e^{i(\tilde{\kappa}_1 \cdot \tilde{r} + \omega t + \emptyset_1)}$

$\Rightarrow I_1 = I_O + I_R + E_R E_O^* + E_O E_R^*$

After Developing, the hologram is considered as a transmission medium

$\Rightarrow \mathcal{L} = \beta(I_O + I_R + E_R E_O^* + E_O E_R^*)$

Relocating the hologram back to the same position and then the scenario observed through the hologram become

$E_{Real\ Time} = \beta(E_o' + E_R)(I_O + I_R + E_R E_O^* + E_O E_R^*)$

$= \beta[I_O\left(E_o' + E_R + E_R\, e^{(-\emptyset_o + \emptyset_o')} + \mathrm{E}_R^* e^{(\emptyset_o + \emptyset_o')}\right)$

$+ I_R\left(E_o' + E_R + E_O + E_O^* e^{2\emptyset_r}\right)]$

Observing the living interference through the hologram along reference beam

$$= \beta^2[(I_O+I_R)E_R + I_O\left(e_R e^{(-\emptyset_o+\emptyset'_o+\emptyset_r))}\right)][(I_O+I_R)E_R^* + I_O\left(e_r e^{(\emptyset_o-\emptyset'_o-\emptyset_r)}\right)]$$

$$= \beta^2[(I_O+I_R)^2 I_R + 2(I_O + I_R)I_O I_R \boxed{\cos(\emptyset'_o - \emptyset_o)} + I_O^2 I_R]$$

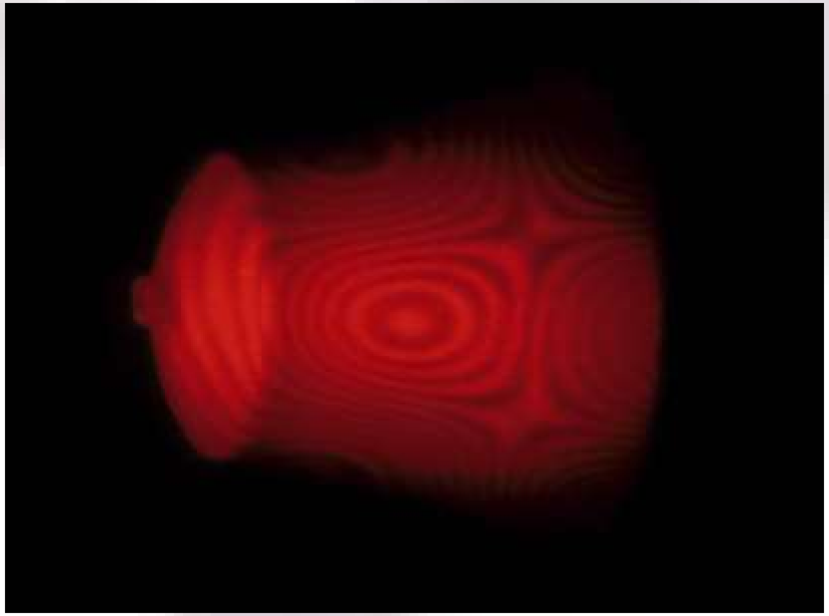
Fringe Formula,
in general, same as traditional interferometry

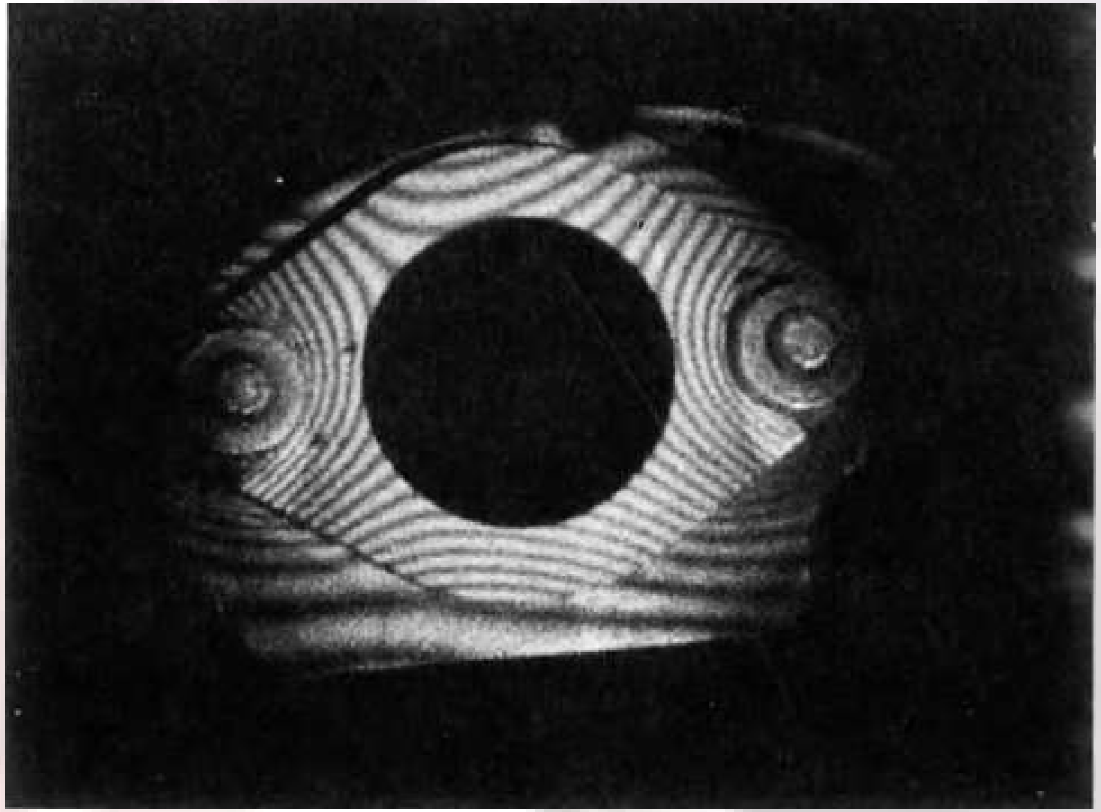
Pose:

The change of the fringe pattern can be observed in real-time whenever the condition changed

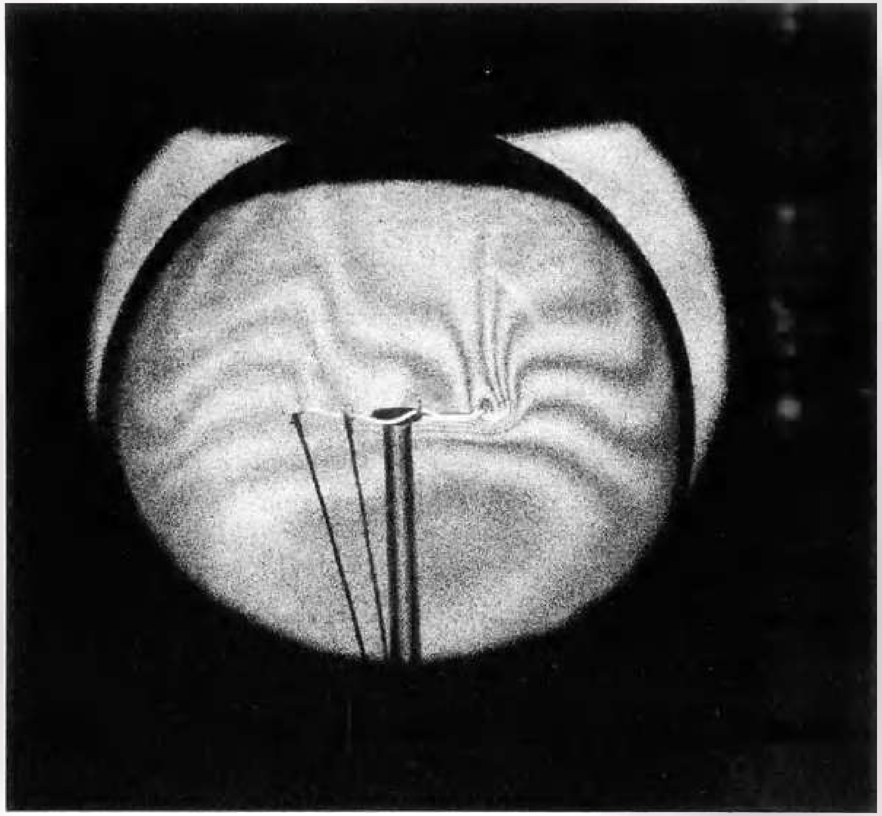
Cons:

1. Relocating the hologram back to the same location as it was under exposition is not easy
2. The reconstructed fringe pattern is relatively poor; typically the fringe visibility is low in compared to the double exposure method





Photograph of real-time interference pattern observed during bolt tightening. It shows the asymmetric deformation of a small bearing housing



Gas flow inside a gas-tilled light bulb is observed in real time using thermoplastic recording materials

Time Averaged Method

Stroboscopic time-average hologram interferometry

If the movement of an object can be described as

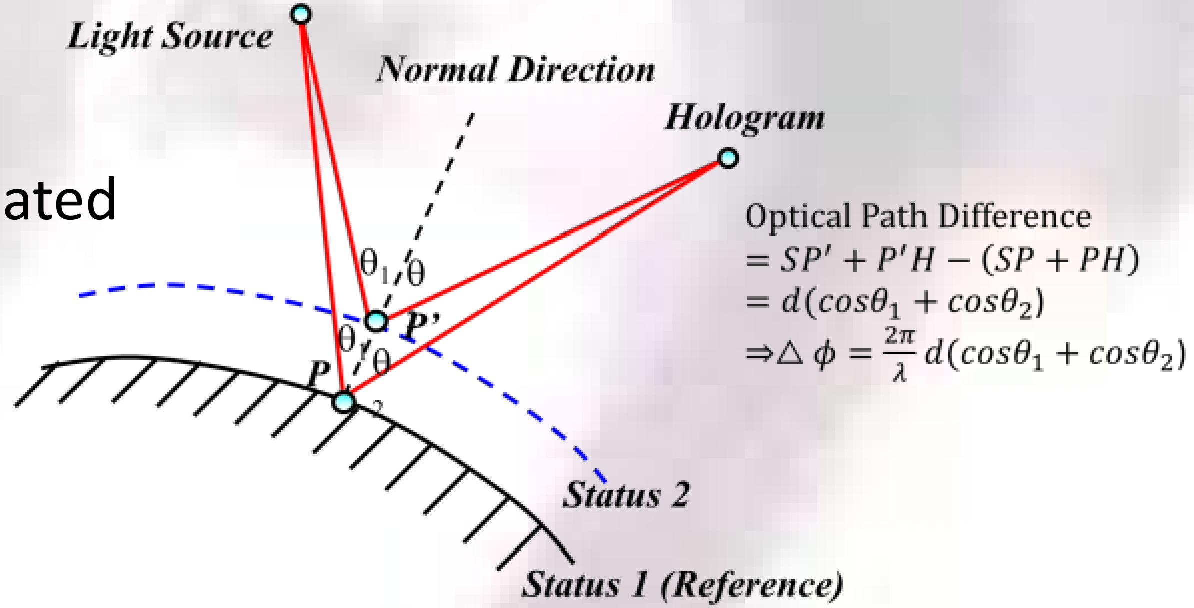
$$d = A_0 cos\omega t$$

Then the object beam will be of an additional phase term

$$\triangle\phi = \frac{2\pi}{\lambda} A_0 cos\omega t(cos\theta_1 + cos\theta_2)$$

Recalling the hologram can record the scenery many times, then the hologram will record the object as $\sum_i E_o(t_i)$, and then intensity accumulated on the hologram can be expressed as

$$I = (E_R + \sum_i E_o(t_i)) \times \left(E_R^* + \sum_i E_O^*(t_i)\right)$$



Time Averaged Method

Stroboscopic time-average hologram interferometry

$$I = (E_R + \sum_i E_o(t_i)) \times \left(E_R^* + \sum_i E_O^*\,(t_i) \right)$$

$$I = I_R + \sum_i I_o(t_i) + E_R \sum_i E_O^*\,(t_i) + E_R^* \sum_i E_o(t_i)$$

Again, the hologram is developed and then reconstructing the scenery with reference beam, then light intensity observed from hologram, $\mathcal{L}_r$ will become

$$\mathcal{L}_r = \Delta t \beta E_R(I_R + \sum_i I_o(t_i) + E_R \sum_i E_O^*\,(t_i) + E_R^* \sum_i E_o(t_i))$$

$$\Rightarrow \mathcal{L}_r = \beta \Delta t E_R(\sum_i I_o(t_i) + I_R) + \beta \Delta t I_R(\sum_i E_O^*\,(t_i)\, e^{2\emptyset_r} + \sum_i E_o(t_i))$$

Object beam

If t_i is select to be $\cos \omega t_i = \pm 1$, then light intensity of the object beam can be simplified to

$$\propto \sum_i E_O(t_i) \sum_i E_O^*\,(t_i) = \cos^2(\frac{2\pi}{\lambda} A_0 cos\omega t(cos\theta_1 + cos\theta_2))$$

Time Averaged Method

Stroboscopic time-average hologram interferometry

light intensity of the object beam

$$\propto \sum_i E_O(t_i) \sum_i E_O^*(t_i) = \cos^2\left(\frac{2\pi}{\lambda} A_0 cos\omega t(cos\theta_1 + cos\theta_2)\right)$$

$\Rightarrow \frac{2\pi}{\lambda} A_0 cos\omega t(cos\theta_1 + cos\theta_2) = \frac{(2m-1)\pi}{2},\ m = 1,2,\ldots$ → dark fringe will occure

The vibration amplitude become

$$A_0 = \frac{\lambda(2m-1)}{4(\cos\theta_1 + \cos\theta_2)}$$

Note: for a give time, the phase term is

Time Averaged Method

Continuous time-average hologram interferometry

For objects subjected to steady-state periodic vibration, recording the periodic motion with exposure time much longer than the motion period.

For holographic interferometry, the are different ways to perform the time-averaged method.

(1) using a hologram to record the object while the object is undergoing periodic motion, and reconstructing with reference beam;

(2) recording object with holography while it is still, developing the hologram, relocating the hologram, recording living vibration fringe pattern with CCD/ CMOS camera but with longer exposure time as compared to the motion period.

Time Averaged Method

Continuous time-average hologram interferometry

Considering a hologram continuously records the object while the object is undergoing periodic motion, and reconstructing with the reference beam, then the light intensity of The reconstructed scenery can be expressed as

$$\mathcal{L}_r = \beta\Delta t E_R(\int_t^{t+\Delta t} I_o(t)dt + I_R) + \beta\Delta t I_R(\int_t^{t+\Delta t} E_O^*(t)dt\, e^{2\emptyset_r} + \int_t^{t+\Delta t} E_O(t)dt)$$

Object beam

And the light intensity become

$$\propto \int_t^{t+\Delta t} E_O(t)dt \int_t^{t+\Delta t} E_O^*(t)dt$$

$$= \int_t^{t+\Delta t} e^{\frac{2\pi}{\lambda}A_0 cos\omega t(cos\theta_1+cos\theta_2)}\, dt \int_t^{t+\Delta t} e^{-\frac{2\pi}{\lambda}A_0 cos\omega t(cos\theta_1+cos\theta_2)}\, dt$$

$$= \Delta t^2 J_0^2\left(\frac{2\pi}{\lambda}A_0 cos\omega t(cos\theta_1 + cos\theta_2)\right)$$

Time Averaged Method

Continuous time-average hologram interferometry

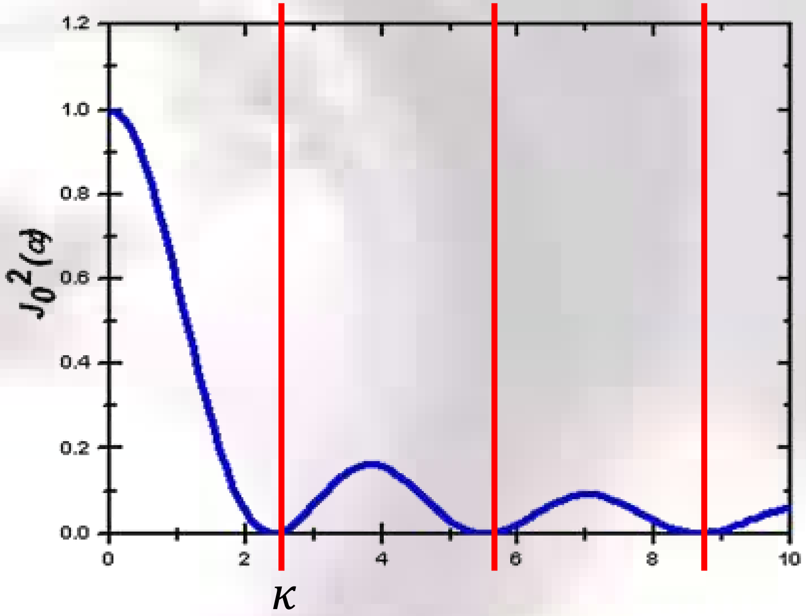
Fringe Formula

Since the light intensity $\propto \Delta t^2 J_0^2\boxed{(\frac{2\pi}{\lambda}A_0 cos\omega t(cos\theta_1 + cos\theta_2))}$

IF $\kappa \rightarrow J_0^2(\frac{2\pi}{\lambda}A_0 cos\omega t(cos\theta_1 + cos\theta_2)) = 0 \Leftrightarrow$ Dark Fringe

$$\Longrightarrow A_0 = \frac{\lambda\kappa}{2\pi(cos\theta_1 + cos\theta_2)}$$

If Dense Fringes are introduced



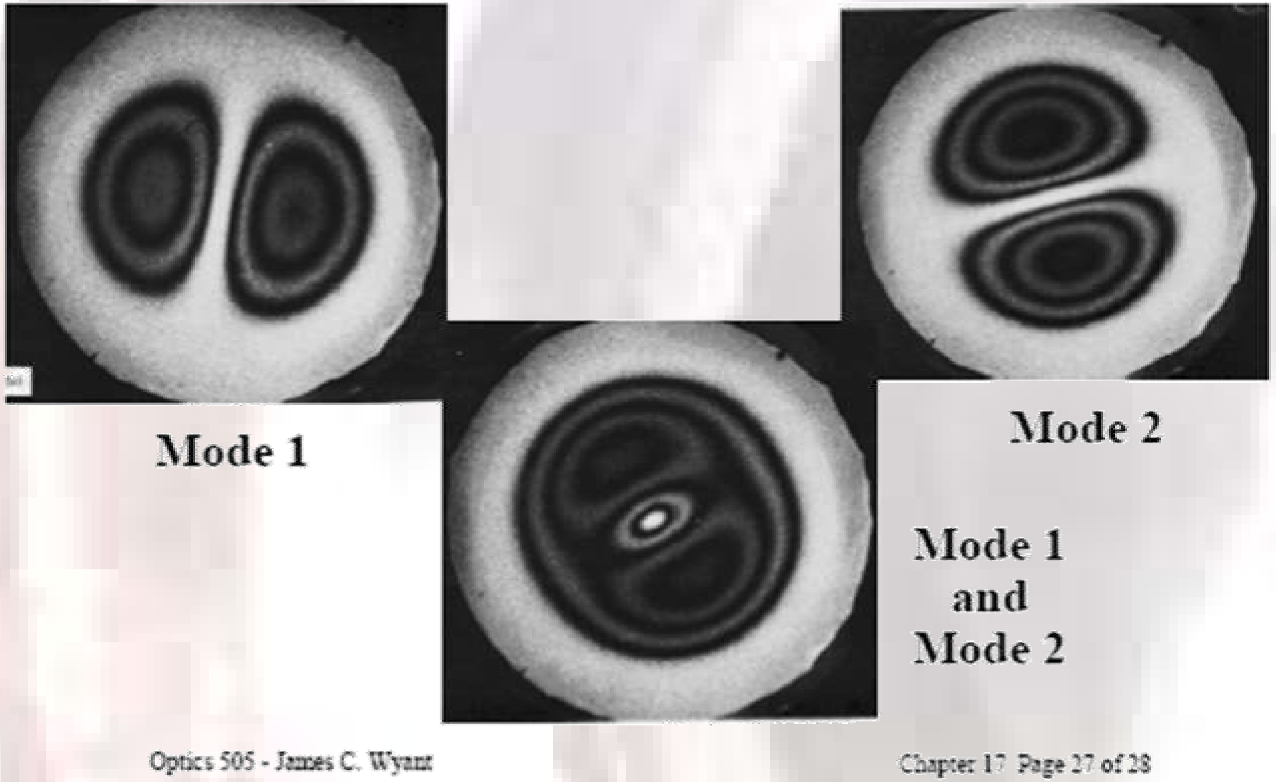
Alternative

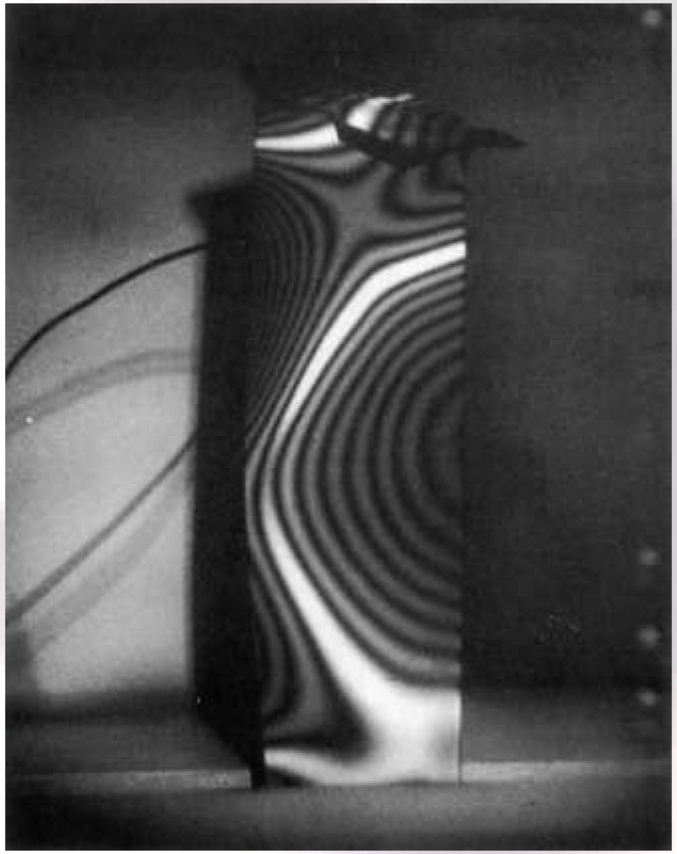
1. Double Exposure, taking hologram while object is still and subjected to vibration with exposure time t1 and t2)

   Fringe Formula $=\left\{J_0\left(\frac{2\pi}{\lambda}A_0 cos\omega t(cos\theta_1 + cos\theta_2)\right) + \frac{t_1}{t_2}\right\}^2$

2. Real time method

   Fringe Formula $=\left\{J_0\left(\frac{2\pi}{\lambda}A_0 cos\omega t(cos\theta_1 + cos\theta_2)\right) + 1\right\}^2$





Time-averaged hologram of a vibrating turbine blade. The brightest fringes are the stationary nodal lines

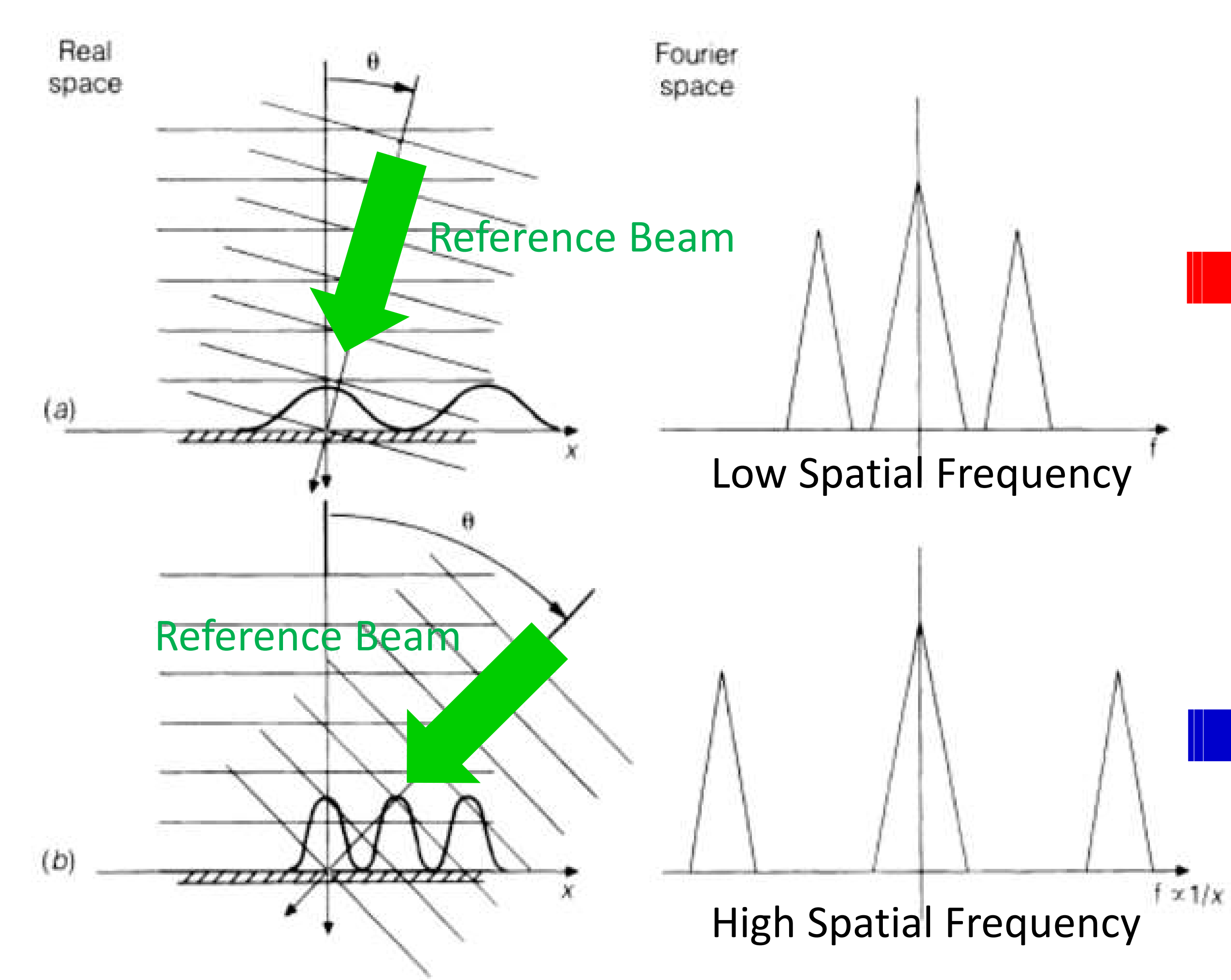
Limitation of Traditional Holography

Typical limitations or drawbacks of traditional holography are

1. Slow,
2. Photography based;
3. High-power Laser required;
4. Complex optical arrangement
5. Complex and expensive facilities;
6. Qualitative analogue Output;

Would CCD can replace the traditional photographic recording medium?

Would CCD can replace the traditional photographic recording medium?

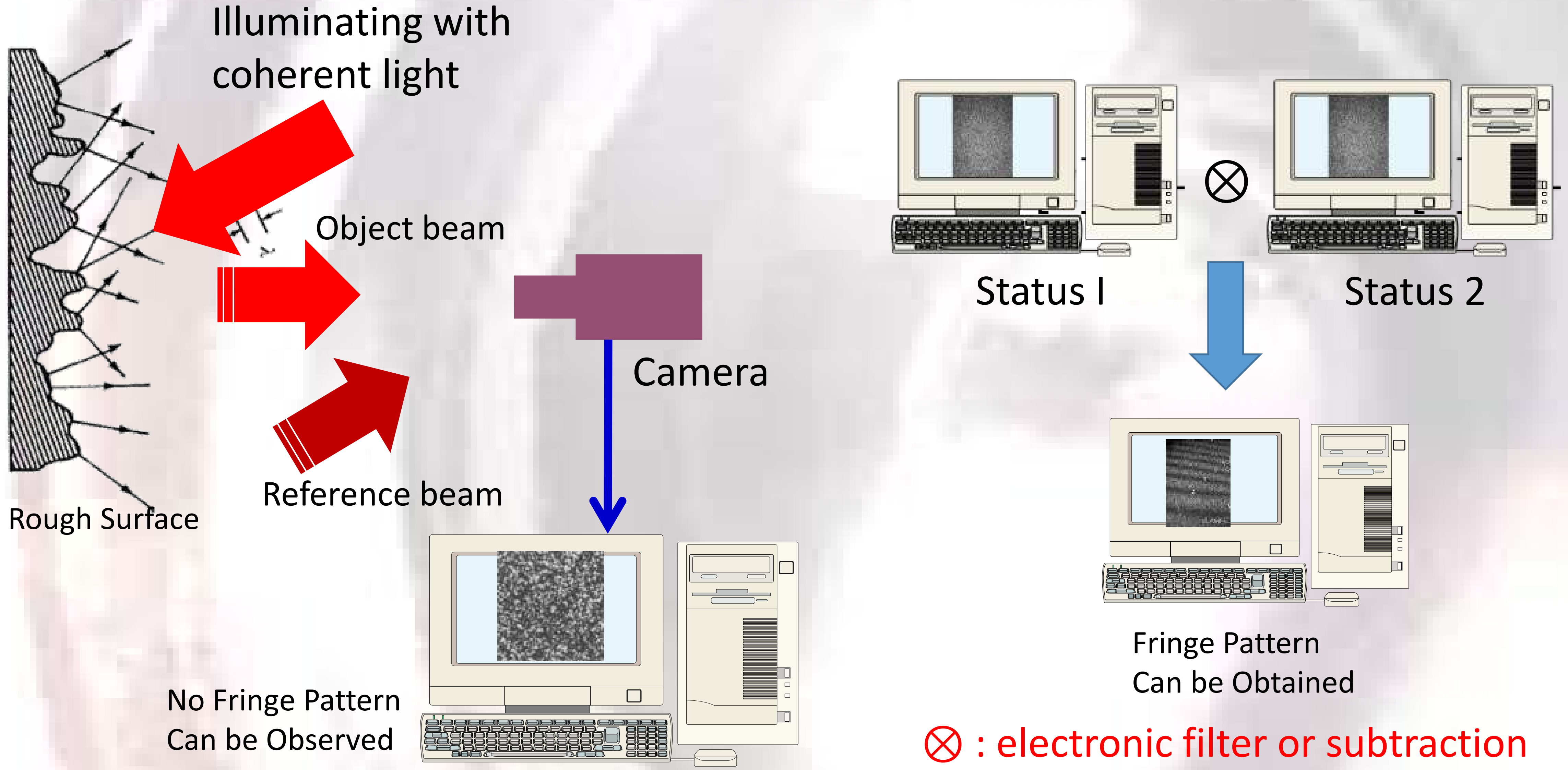


Fewer Fringes within a unit area

Off-axis (Skew Reference Beam) For Holography layout to avoid "the Second Image" problem for Optical hologram reconstruction
→Dense Fringe
→High spatial resolution recoding medium needed

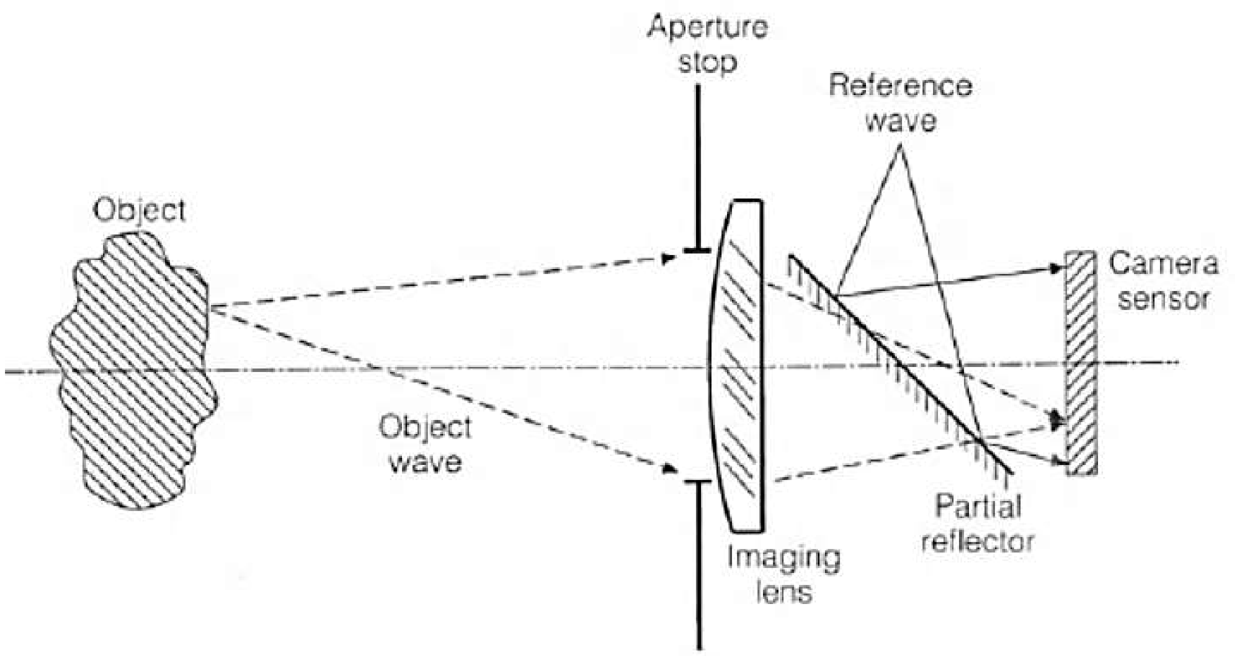
Television Holography

(Electronic/ Digital Speckle Pattern Interferometry or electronic holography)

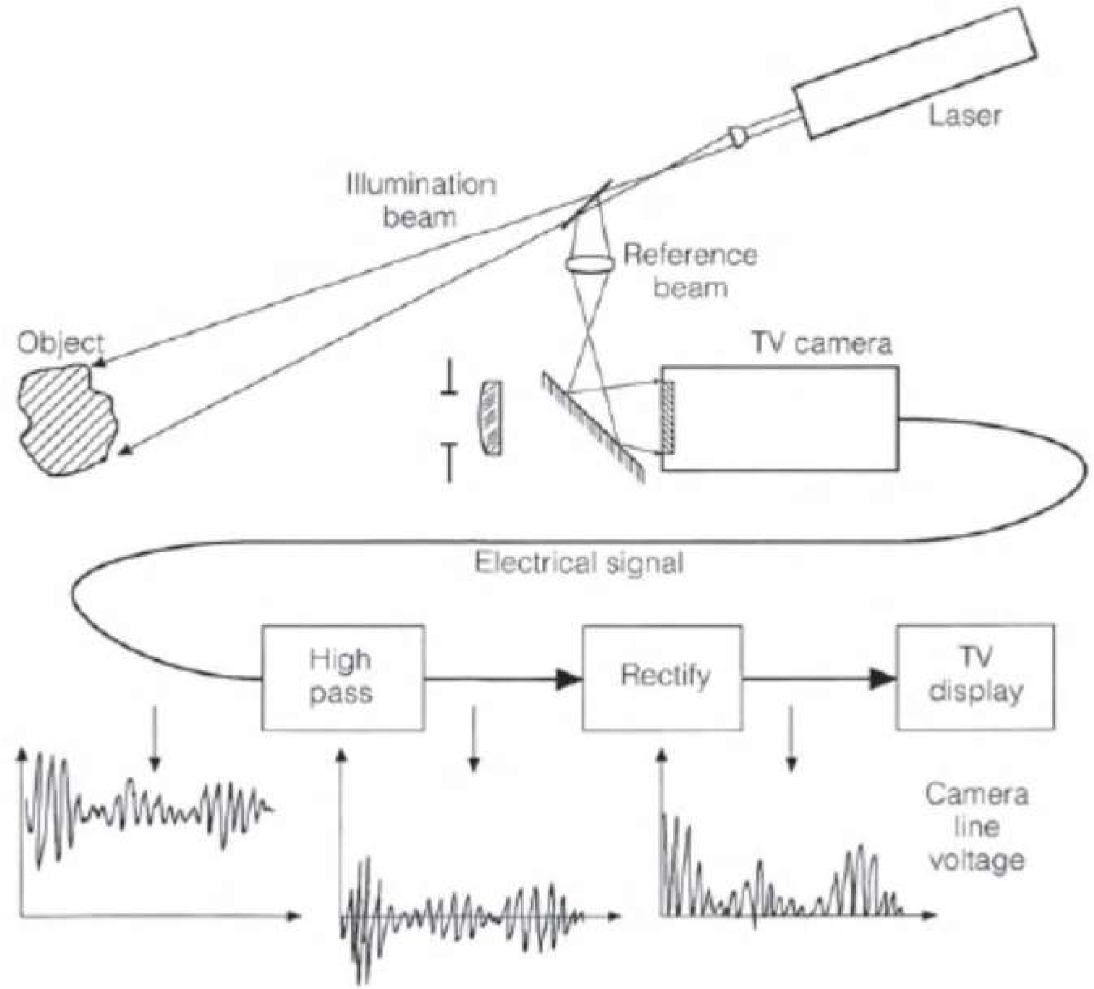


Basic Structure for TV Holography

Aperture Stop –
reduce the lateral resolution
➔ Intensity resolution is reduced

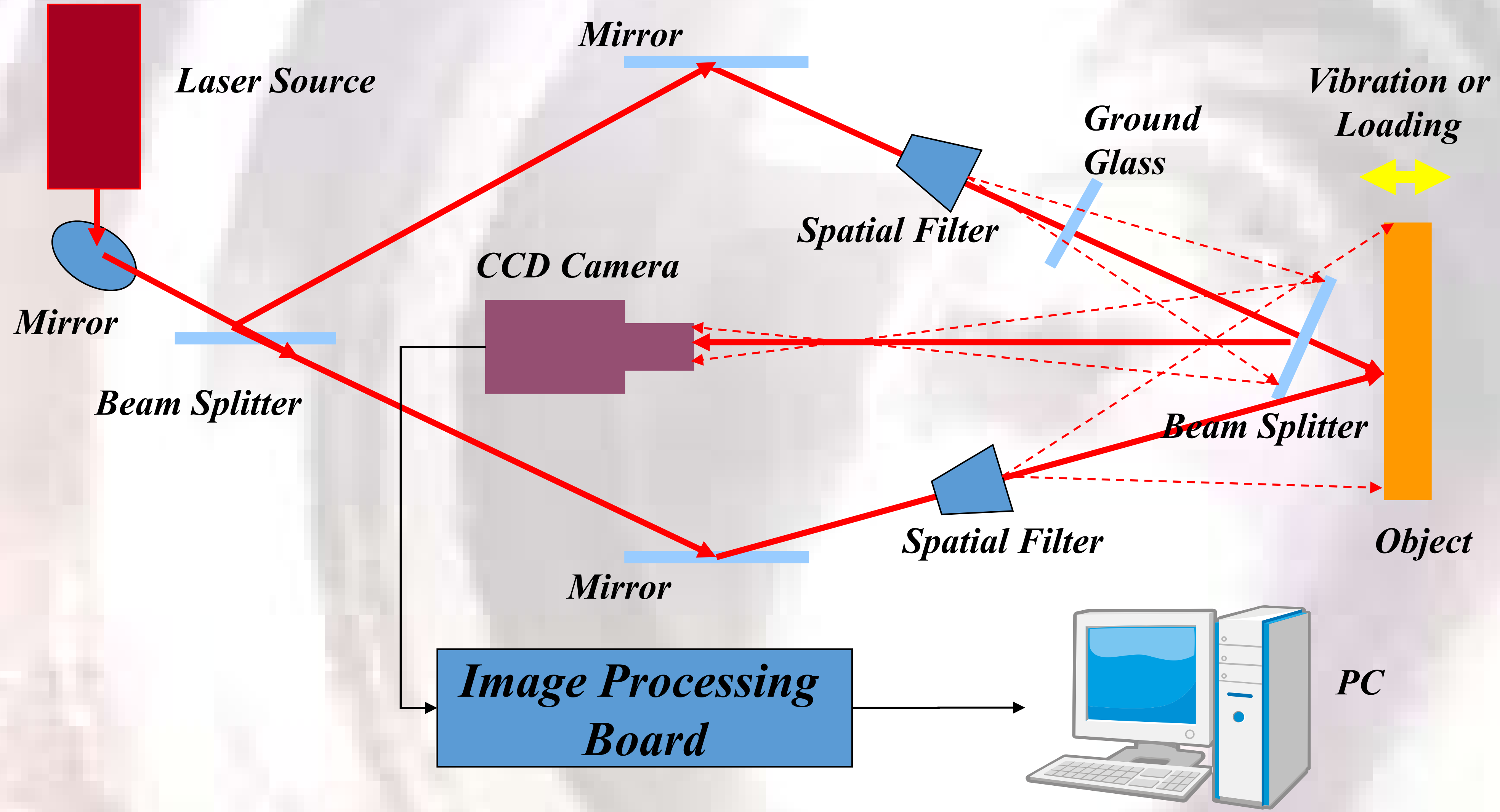


Optical Head for TV Holography based on on-axis Hologram Reconstructing Optical Arrangement

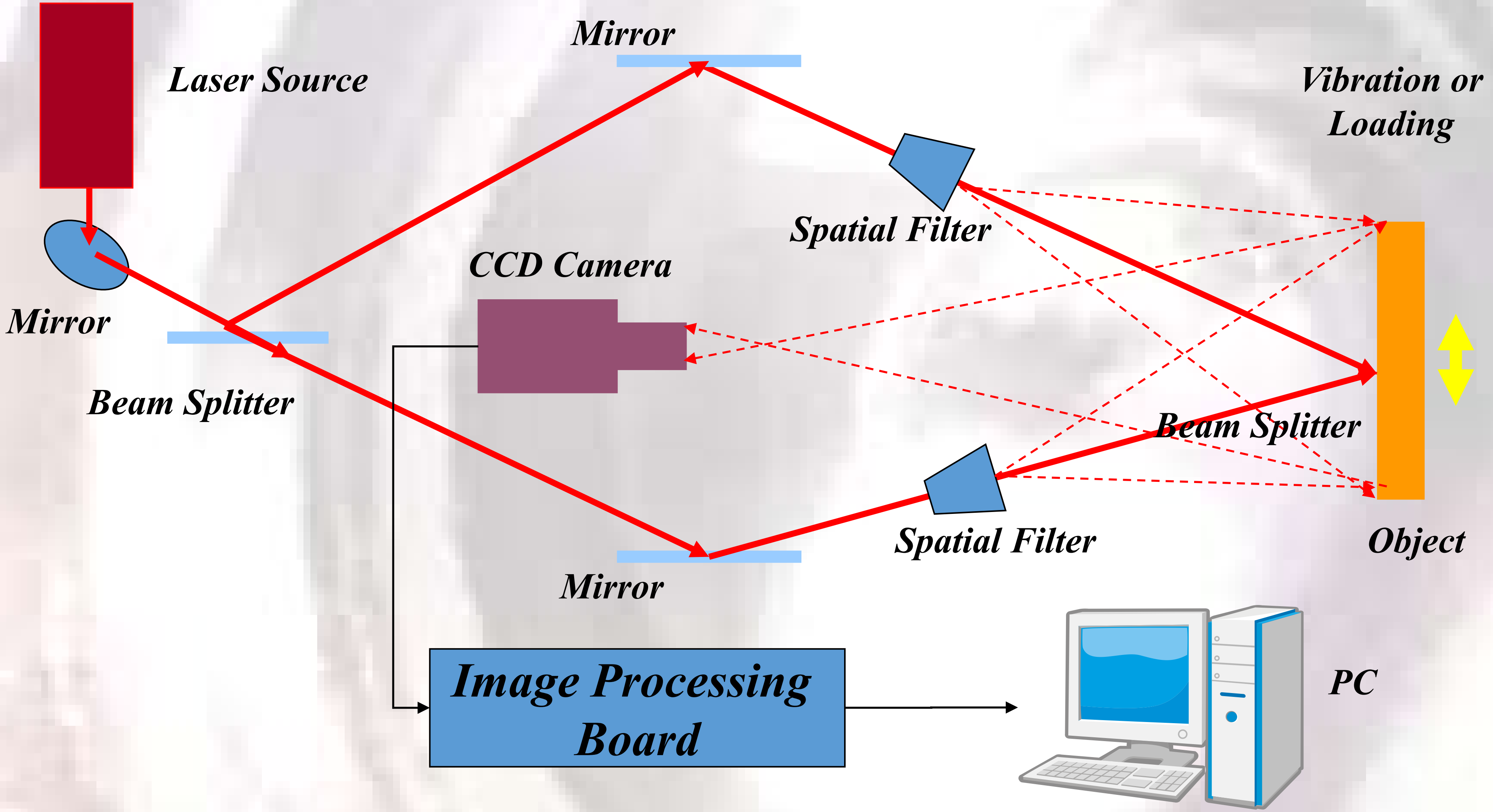


Basic Structure for TV Holography with electronic filter introduced

Out-of-Plan ESPI



In-Plan ESPI



Laser Source
Mirror
Mirror
Beam Splitter
CCD Camera
Spatial Filter
Vibration or Loading
Beam Splitter
Spatial Filter
Object
Mirror
Image Processing Board
PC

The light intensity detected by CCD at time t is:

$$Before\ Load : I(x,y,t) = I_o + I_r + 2\sqrt{I_o I_r}\cos\varphi(x,y)$$

$$After\ Load : I(x,y,t) = I_o + I_r + 2\sqrt{I_o I_r}\cos[\varphi(x,y) + \Delta(x,y,t)]$$

Where

$I_o = The\ Object\ Light\ Beam$

$I_r = The\ Reference\ Light\ Beam$

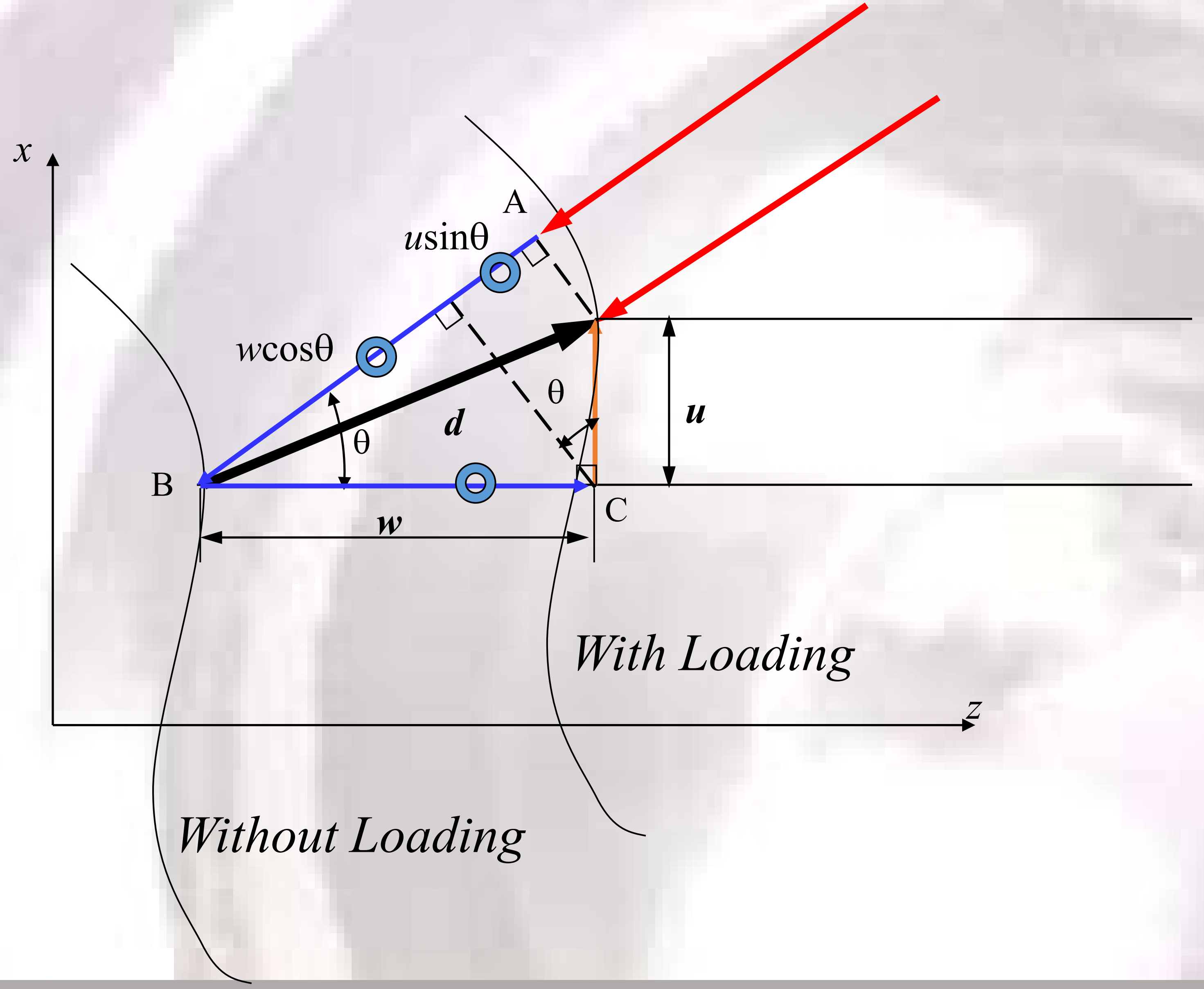
$\lambda = Wave\ Length$

$\theta = The\ Illumination\ Angle\ of\ the\ Objective\ Light\ Beam$

$\varphi = Random\ Phase\ Caused\ by\ Surface\ Roughness$

$\Delta = 2\pi\delta/\lambda \quad (\delta \equiv Optical\ Path\ Length\ Difference, OPD)$

The optical setup of the ESPI/DSPI method is always designed to be sole of in-plane or out-of-plane sensitivity.



The Displacement and Optical Path Length Difference can be related by

$$\delta = w(1+\cos\theta) + u\sin\theta \quad (\textit{for displacement on xz plan, Observed from z-axis})$$

$$\delta = w(1+\cos\theta) + u\sin\theta$$

$1.\ Out\text{-}of\text{-}plan\ Displacement\ Measurement$

$To\ achive\ high\ out\text{-}of\text{-}plan\ displacment\ sensitivity\ measurment$

$$\sin\theta \to 0 \Rightarrow 1+\cos\theta \cong 2$$

$That\ is\ the\ objective\ beam\ should\ be\ parallel\ to$

$line\text{-}of\text{-}view\ of\ CCD\ Camera$

$$\therefore \Delta_{out-of-plan} = \frac{4\pi}{\lambda} w$$

$2.\ In\text{-}plan\ Displacement\ Measurement$

$$\Delta_{in-plan} = \frac{2\pi}{\lambda}\{[w(1+\cos\theta_r) + u\sin\theta_r] + [w(1+\cos\theta_o) + u\sin\theta_o]\}$$

$$while\ \theta_r + \theta_o = 0$$

$$\Rightarrow \Delta_{in-plan} = \frac{4\pi}{\lambda} u\sin\theta$$

Subtracted Image Signal (SIS) ESPI

The reference image is subtracted from the object image, the output signal

$$V(x,y) = 4\alpha\sqrt{I_o I_r}\,\underbrace{\sin(\varphi + \Delta/2)}_{\text{Due to speckle effect}}\,\underbrace{\sin(\Delta/2)}_{\text{Due to Physical Change}}$$

$\alpha \equiv$ *The slope of the CCD camera's sensitivity*

$I_o \equiv$ *The light intensity of the object light*

$I_r \equiv$ *The light intensity of the reference light*

$\varphi \equiv$ *Random phase from surface roughness*

$\Delta \equiv$ *Displacement induced Phase change*

$\Delta = 2\pi(1+\cos\theta)d(x,y,t)/\lambda,$

Out-of-Plan (Rotation) Deformation

Apply ESPI on Vibration Measurement

Reviewing Time Averaged Method

Accumulate the light intensity from t_1 to $t_1+\tau$, then the output voltage, which was converted from the CCD/ CMOS camera detected light intensity, will be

$$V_{CCD} = \alpha \int_{t_1}^{t_1+\tau} [I_o + I_r + 2\sqrt{I_o I_r} \cos(\varphi + \Delta)]dt$$

Where

α = the slope of the CCD/ CMOS camera sensitivity curve

τ = the shutter opening time interval or the total exposure time

$\Delta = 4A\pi(1 + cos\theta)\sin(\omega t) / \lambda$

$$V_{CCD} = \alpha\tau \left[I_o + I_r + 2\sqrt{I_o I_r} J_0 \left(\frac{2A\pi(1 + \cos\theta)}{\lambda} \right) \cos\varphi \right]$$

This is true only the exposure time is equal to vibration period

Video Signal Addition Method

- **METHOD:** CCD camera itself will accumulate the incoming light intensity during each shutter opening time interval
- **NOTE:** If the CCD camera accumulates the light intensity from t_1 to t_1+t, and t is equal to $2N\pi/\omega$ (i.e., N times the vibration period)

$$V_{CCD} = \frac{2N\alpha\pi}{\omega}\left[I_o + I_r + 2\sqrt{I_o I_r}\,J_o(k)\cos\varphi\right]$$

$$\text{Where } k = \frac{2A\pi(1+\cos\theta)}{\lambda}$$

Visibility → poor
Image or Signal Processing required!!

Video Subtraction Method

- **METHOD**: *Subtracting two video signals before and at vibration by the image processing system*

$$Before\,Vibration \Rightarrow (V_{CCD})_{REF} = \frac{2N\alpha\pi}{\omega}[I_o + I_r + 2\sqrt{I_o I_r}\cos\varphi]$$

$$At\ Vibration \Rightarrow V_{CCD} = \frac{2N\alpha\pi}{\omega}\left[I_o + I_r + 2\sqrt{I_o I_r}J_o(k)\cos\varphi\right]$$

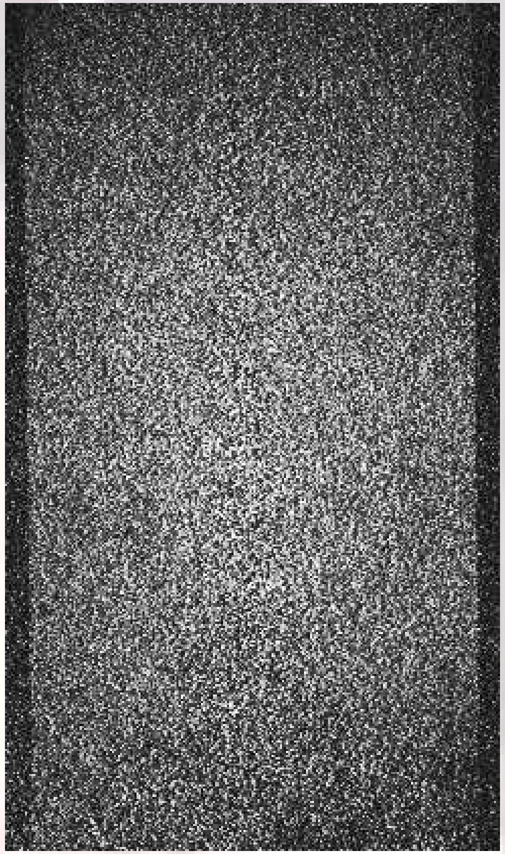
$$Subtracted\ Signal \Rightarrow V_{IM} = \frac{4N\alpha\pi}{\omega}\sqrt{I_o I_r}[J_o(k) - 1]\cos\varphi$$

$$Convert\ into\ Brightness \Rightarrow B_{IM} = \frac{4N\alpha\beta\pi}{\omega}\sqrt{I_o I_r}\left\{[J_o(k) - 1]^2\cos^2\varphi\right\}^{1/2}$$

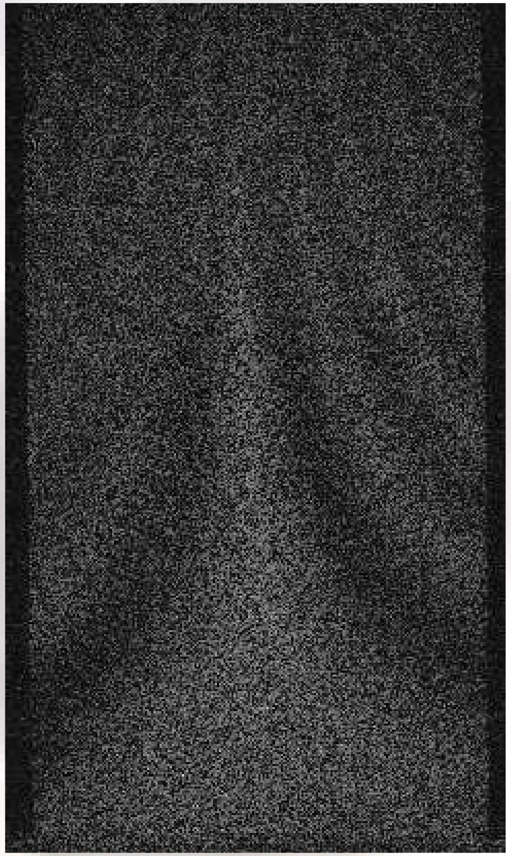
$$Vibration \Rightarrow A = \lambda\zeta_i^* / [2\pi(1 + \cos\theta)]$$

$$\forall \zeta_i^* \rightarrow B_{IM}(k = \zeta_i^*)\ is\ local\ minium$$

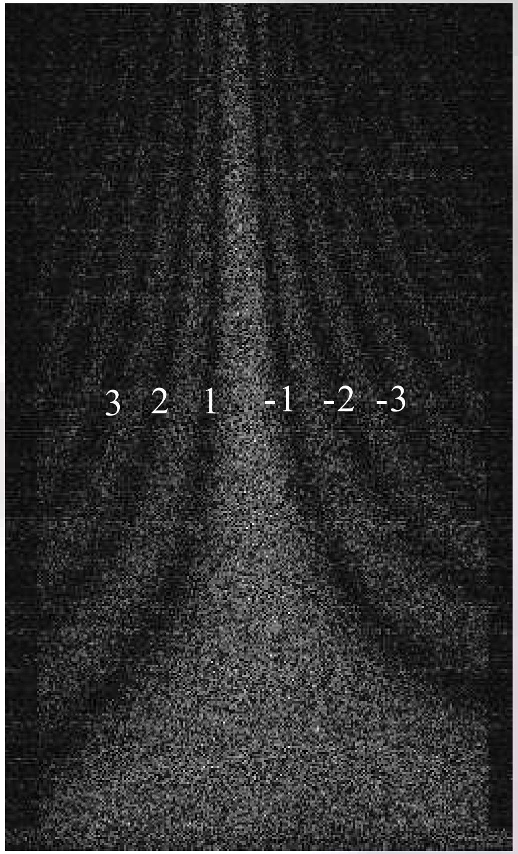
Three Methods to extract the fringe patterns of a edge-clamped plate vibration at second harmonic modes



ESPI
(ADDITION)



ESPI
(SUBTRACTION)



AF-ESPI

$$V_{CCD} = \alpha \int_{t_1}^{t_1+\tau} \left[I_o + I_r + 2\sqrt{I_o I_r} \cos(\varphi + \Delta)\, dt \right]$$

For the convenience of the following discussions, τ can be decomposed into two parts, that is,

$$\tau = \delta + 2N\pi/\omega$$

N= The number of cycles, (number that makes δ become a minimum value)

δ= The time difference between vibration period and shutter opening time.

$Taking\ \delta \leq \frac{2\pi}{\omega},$

$Then\ V_{CCD}\ becomes$

$$V_{CCD} = \frac{2\alpha N\pi}{\omega}\left[I_o + I_r + 2\sqrt{I_o I_r} J_o(k) \cos\varphi \right]$$

$$\boxed{+\alpha\delta[I_o + I_r + 2\sqrt{I_o I_r}(\cos\varphi - \frac{k\sin\omega\delta}{\delta\omega}\sin\varphi - \frac{(k)^2 \sin 2\omega\delta}{\delta\omega}\cos\varphi - \ldots)]}$$

$$V_{CCD} = \frac{2\alpha N\pi}{\omega}\left[I_o + I_r + 2\sqrt{I_o I_r}\, J_o(k)\cos\varphi\right]$$

$$+\alpha\delta[I_o + I_r + 2\sqrt{I_o I_r}\,(\cos\varphi - \frac{k\sin\omega\delta}{\delta\omega}\sin\varphi - \frac{(k)^2\sin 2\omega\delta}{\delta\omega}\cos\varphi - \ldots)]$$

where

$$k = \frac{2A\pi(1+\cos\theta)}{\lambda}$$

As $\delta \to 0$

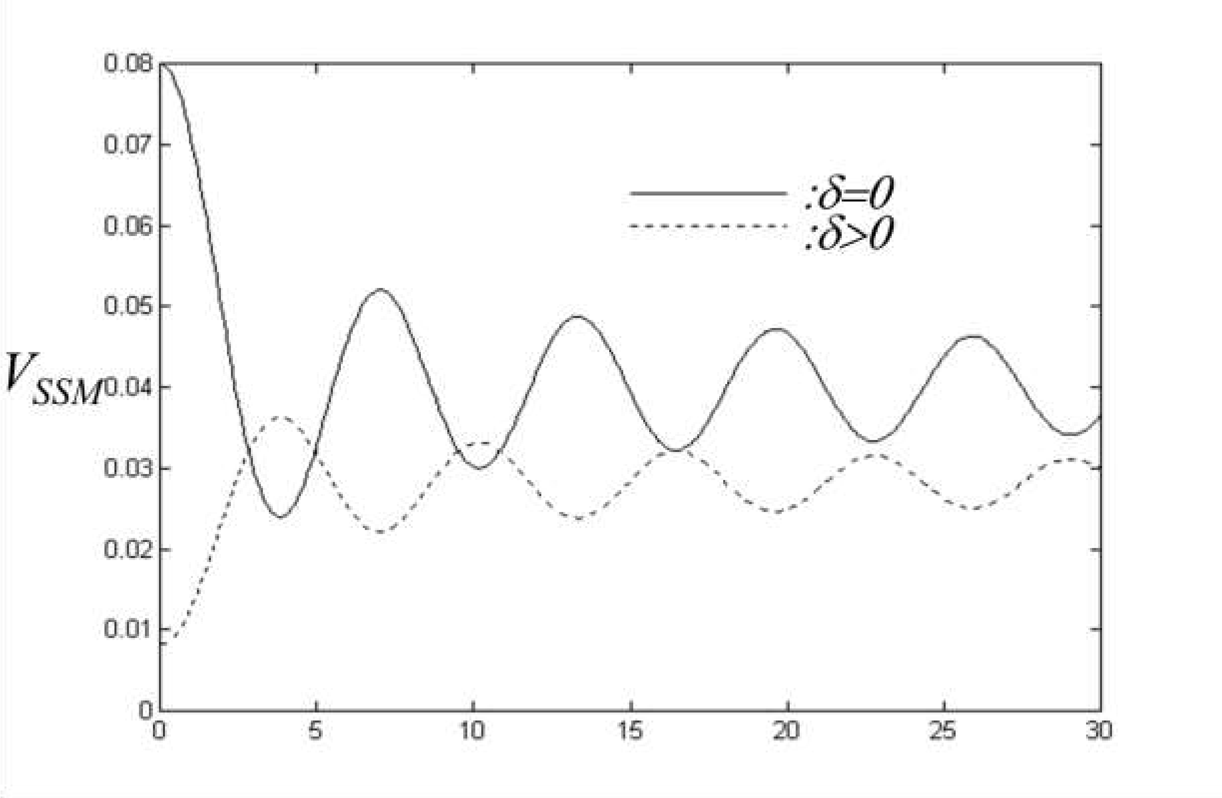
$$\Rightarrow V_{CCD}(\delta \to 0) = \frac{2N\alpha\pi}{\omega}\left[I_o + I_r + 2\sqrt{I_o I_r}\, J_o(k)\cos\varphi\right]$$

Subtraction Signal Method

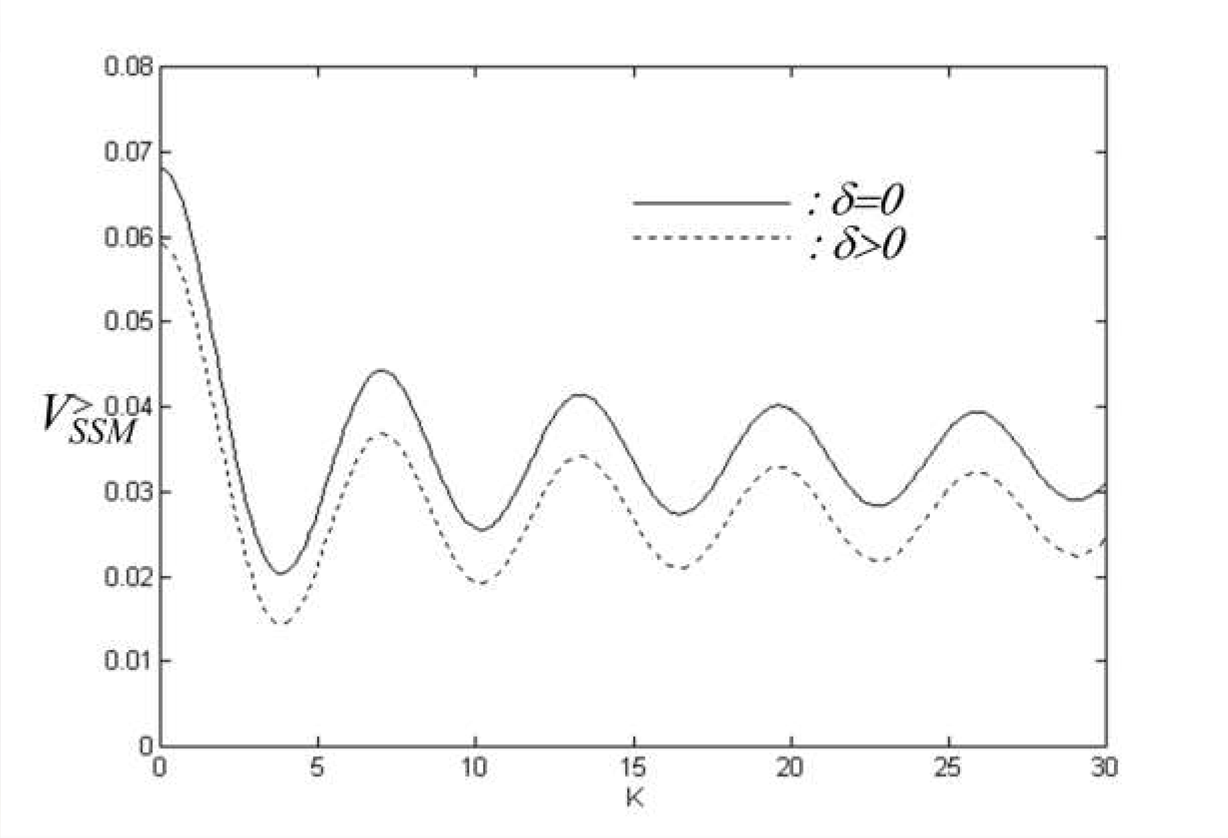
$$\Rightarrow V_{SSM} = \frac{2\alpha N\pi}{\omega}\left[2\sqrt{I_o I_r}\, J_o(k)\cos\varphi - 1\right]$$

$$+\alpha\delta[2\sqrt{I_o I_r}\,(\cos\varphi - \frac{k\sin\omega\delta}{\delta\omega}\sin\varphi - \frac{(k)^2\sin 2\omega\delta}{\delta\omega}\cos\varphi - \ldots)]$$

Mismatch between the shutter and vibration period



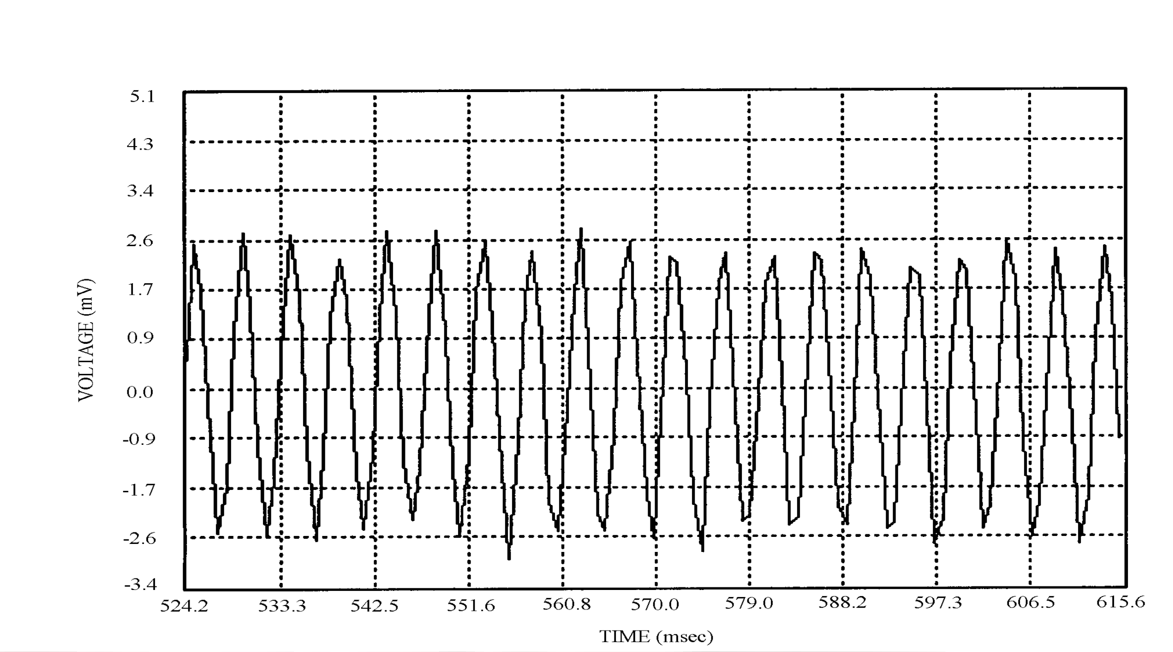
100 Hz (Low Frequency)



1000 Hz (High Frequency)

Amplitude fluctuation method (AF ESPI)

- **METHOD:** *Subtracting two images obtained both at vibration status by using image procession system*
- **REASON:** *The vibration amplitude would be slightly changed during each cycle (Driving Force Fluctuation Considered)*



A traditional Vibration Force Signal obtained during experimental

AF ESPI Method

Consider the reference video signal

$$(V_{CCD})_{REF} = \frac{2N\alpha\pi}{\omega}[I_o + I_r + 2\sqrt{I_o I_r} J_o(\frac{2\pi}{\lambda} A_o(1+\cos\theta))\cos\varphi]$$

The amplitude of second image is slightly changed, and the output video signal is:

$$\boxed{(V_{CCD})_{SEC} = \frac{2N\alpha\pi}{\omega}[I_o + I_r + 2\sqrt{I_o I_r} J_o([\frac{2\pi}{\lambda}(A_o + \Delta A)(1+\cos\theta)])\cos\varphi]}$$

Subtracting those two signals by image procession system:

$$\Delta V = \frac{2N\alpha\pi}{\omega}[2\sqrt{I_o I_r}\{J_o([\eta(A_o + \Delta A)]) - J_o(\eta A_o)\}\cos\varphi]$$

Where $\eta = 2(1+\cos\theta)\pi/\lambda$

AF ESPI Method

Since the amplitude is rather small, change the signal difference into:

$$\lim_{\Delta A\to 0}(\Delta V/\Delta A)=\lim_{\Delta A\to 0}\frac{2N\alpha\pi}{\omega}\left\{2\sqrt{I_oI_r}\left[\frac{J_o(\eta(A_o+\Delta A))-J_o(\eta A_o)}{\Delta A}\right]\cos\varphi\right\}$$

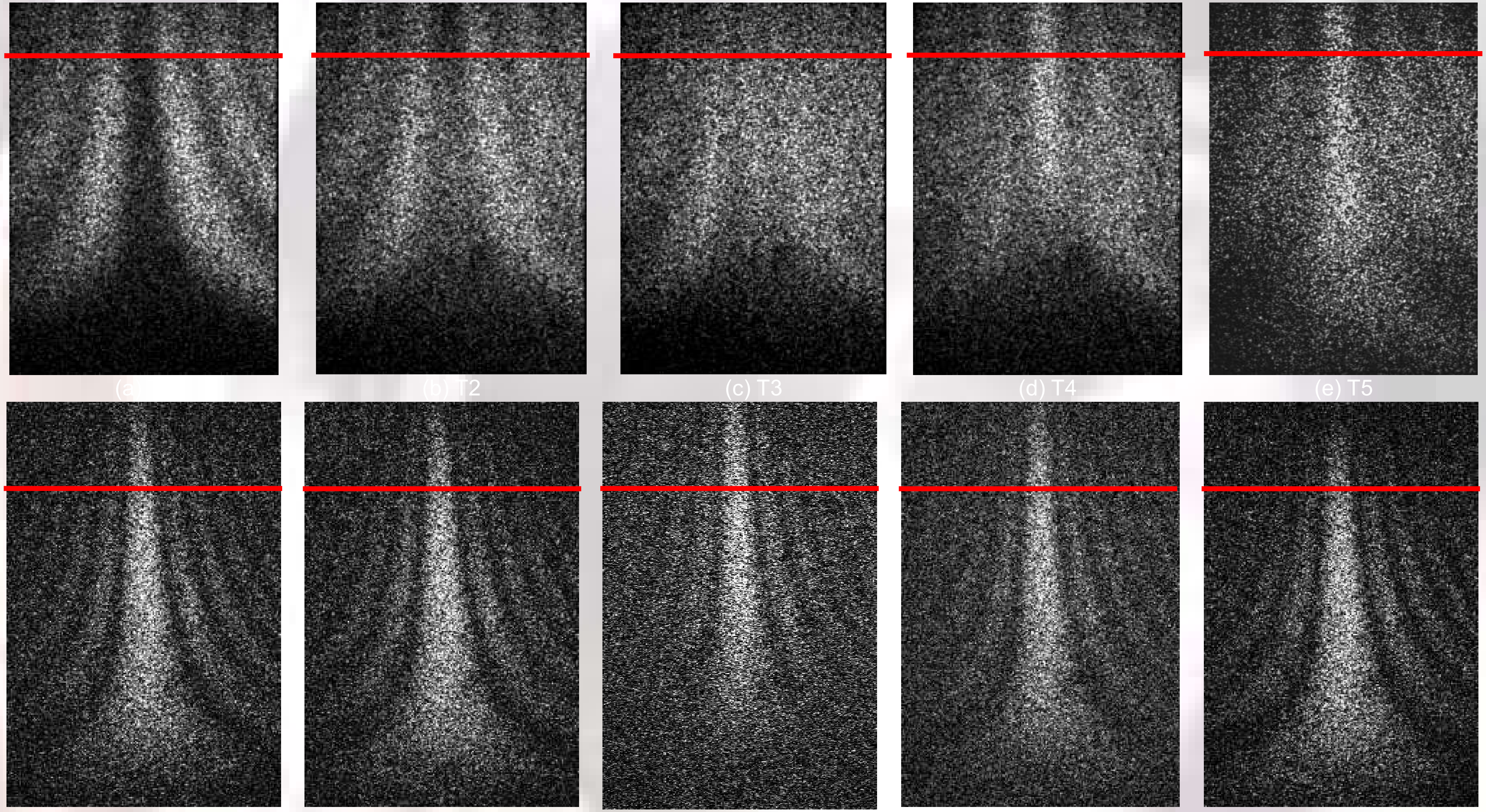
i.e., $$\frac{dV}{dA}=-\frac{2N\alpha\pi}{\omega}\left[2\sqrt{I_oI_r}J_1(\eta A_o)\cos\varphi\right]$$

Then by using LUT (look up table) of image procession system, the brightness becomes:

$$\boxed{B_{IM}=\frac{4N\alpha\beta\pi\sqrt{I_oI_r}}{\omega}[J_1^2(k)\cos^2\varphi]^{1/2}}$$

Relation equation: $$A_o=\frac{\lambda\zeta_i}{2\pi(1+\cos\theta)}$$

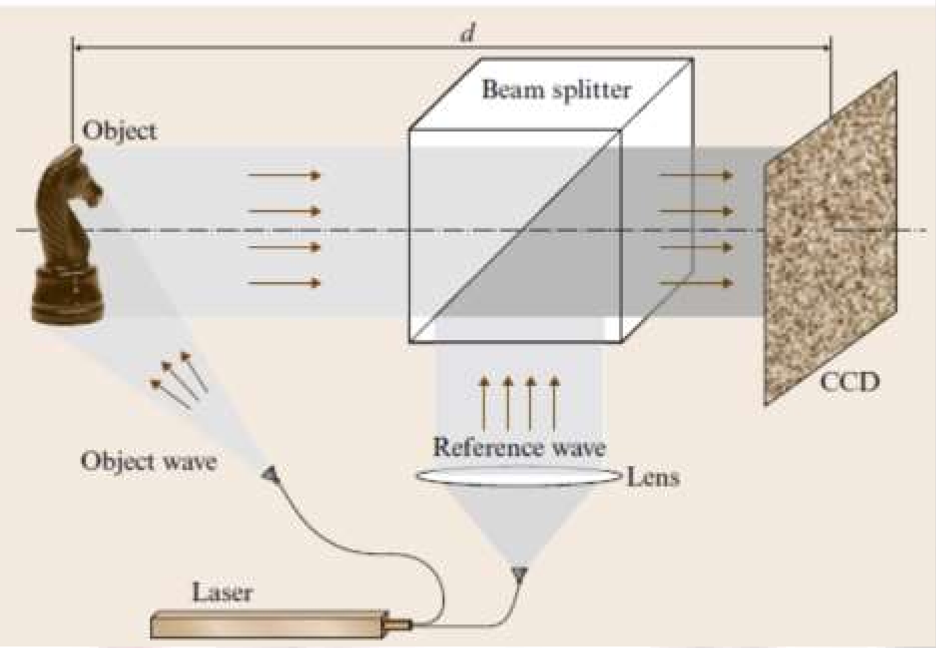
The time variation of TA ESPI/ AF ESPI fringe patterns of the 2nd modes



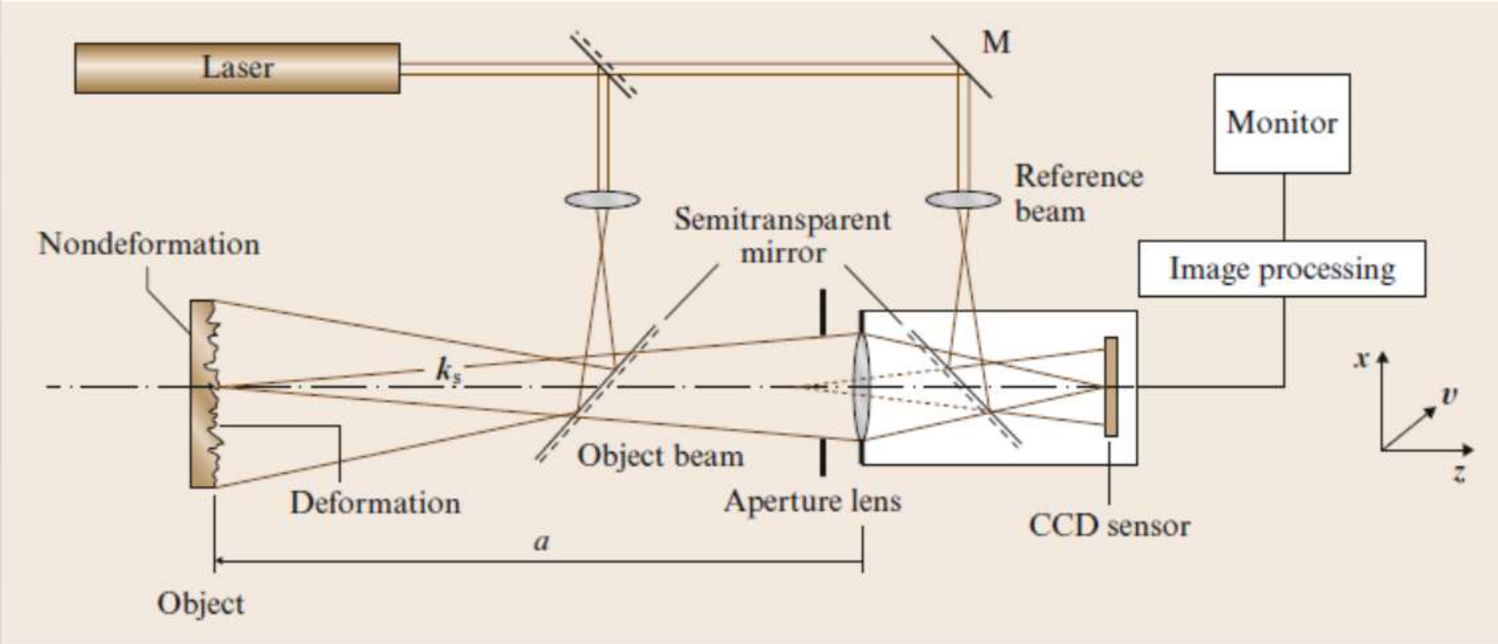
Digital Holography

DSPI: the object is focused onto the target of an electronic sensor. Thus an image-plane hologram is formed as result of the interference with an inline reference wave

Digital Hologram (DH): recording object without using the imaging lens. The sensor records the superposition of the reference beam and the object beam in the near-field region –a kind of Fresnel hologram



setup for recording a digital hologram with a CCD/CMOS

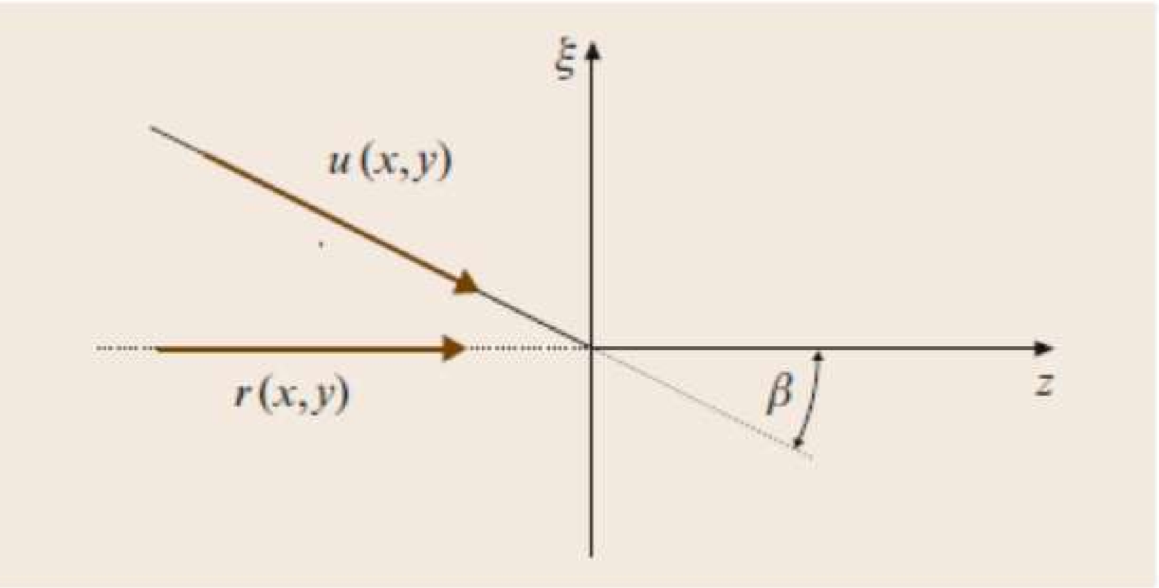


An typical optical setup of a Digital speckle pattern interferometer

Principle of Digital Holography

The spatial resolution of the sensor is limited, the spatial frequencies of the interference fringes in the hologram plane have to be considered. The fringe spacing g and the spatial frequency f_x , is function of angle β(between the object and the reference wave)

$$g = \frac{1}{f_x} = \frac{\lambda}{2\sin\left(\frac{\beta}{2}\right)}$$



If we assume that the discrete sensor has a pixel $\Delta\xi$; according to the sampling theorem, requiring at least two pixels per fringe for a correct reconstruction of the periodic function

$$2\Delta\xi \leq \frac{1}{f_x} = \frac{\lambda}{2\sin\left(\frac{\beta}{2}\right)}$$

$$\Rightarrow \beta < \frac{\lambda}{2\Delta\xi}$$

Considering the high-resolution CCD or CMOS chips have pixel size $\Delta\xi$ is about 4μm; the maximum allowable angle between the reference and the object wave is only $4°$.

Principle of Digital Holography

1. CCD/CMOS Sensor records the interferogram $h(x,y)$

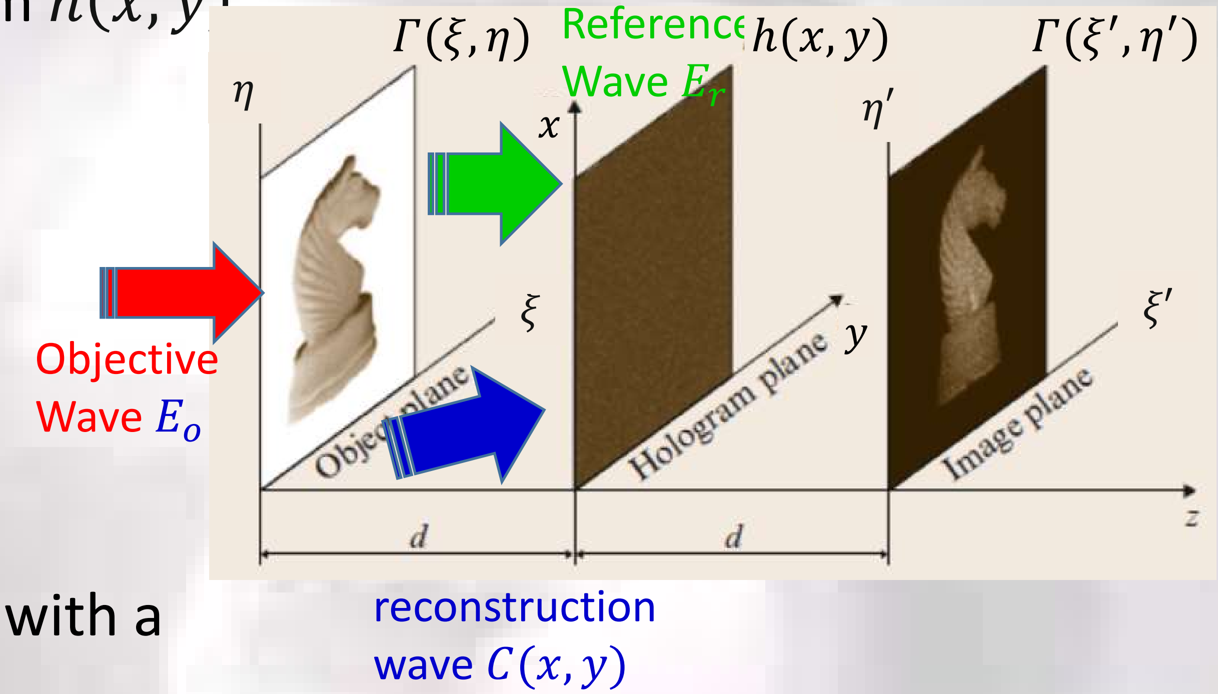
$$h(x,y) = |E_o(x,y) + E_r(x,y)|^2$$
$$h(x,y) = rr^* + ro^* + or^* + oo^*$$

2. transforming the intensity distribution into a gray value distribution

$$\Rightarrow T = t[h(x,y)]$$

3. reconstructing by illuminating the hologram with a reconstruction wave $E_r(x,y)$, then

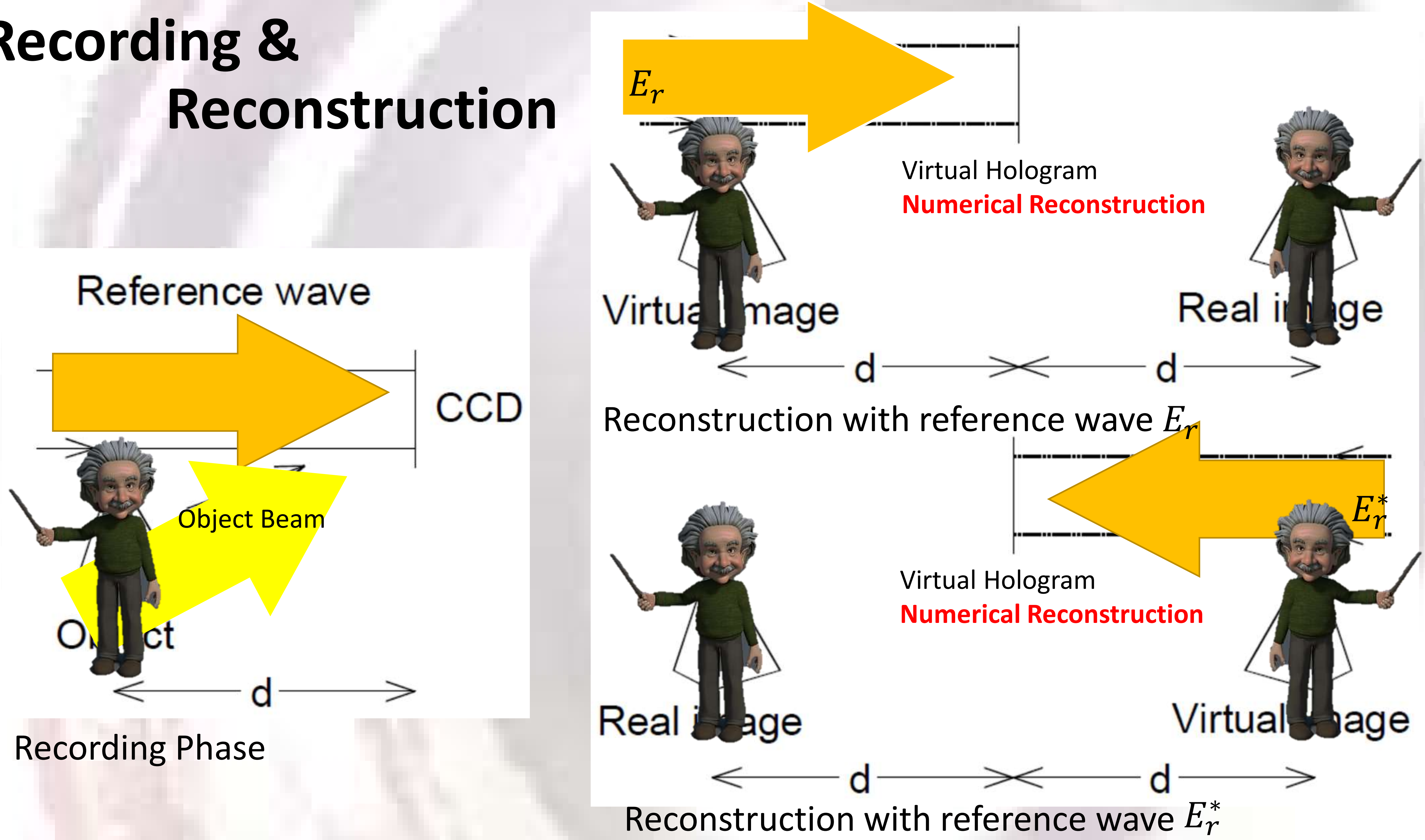
$$\Gamma(\xi',\eta') = t[h(x,y)]E_r(x,y)$$



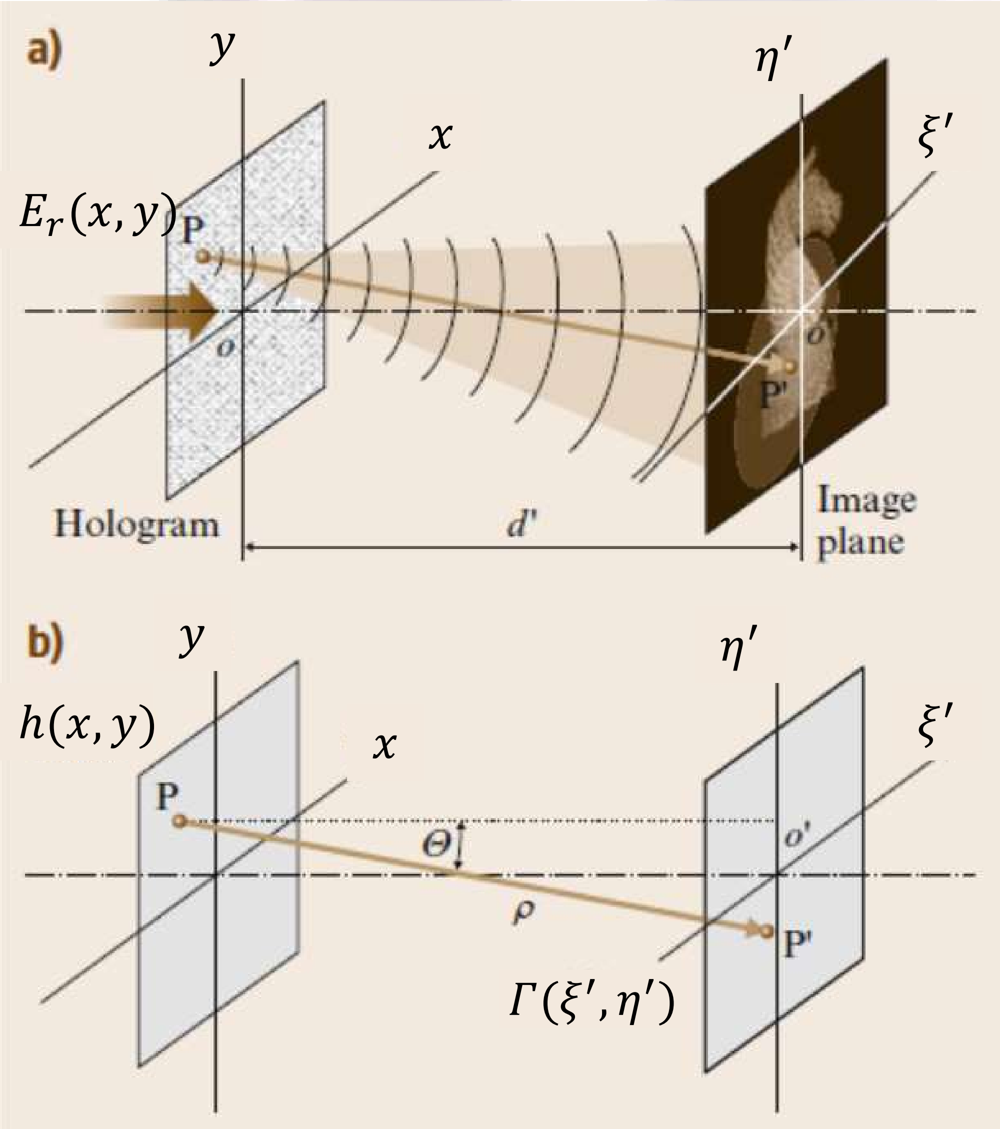
The hologram $t[h(x,y)]$ diffracts the wave $C(x,y)$in such, that images of the object wave are reconstructed. In general, **four terms are reconstructed** if the wave $\Gamma(x',y')$ propagates in space; i.e.,$\Gamma(x',y') = \alpha(co^2 + cr^2 + cur^* + cru^*) + ct_0$

In general the reference wave C = E_r or its conjugated version C = E_r^* is applied

Recording & Reconstruction



Principle of Digital Holography



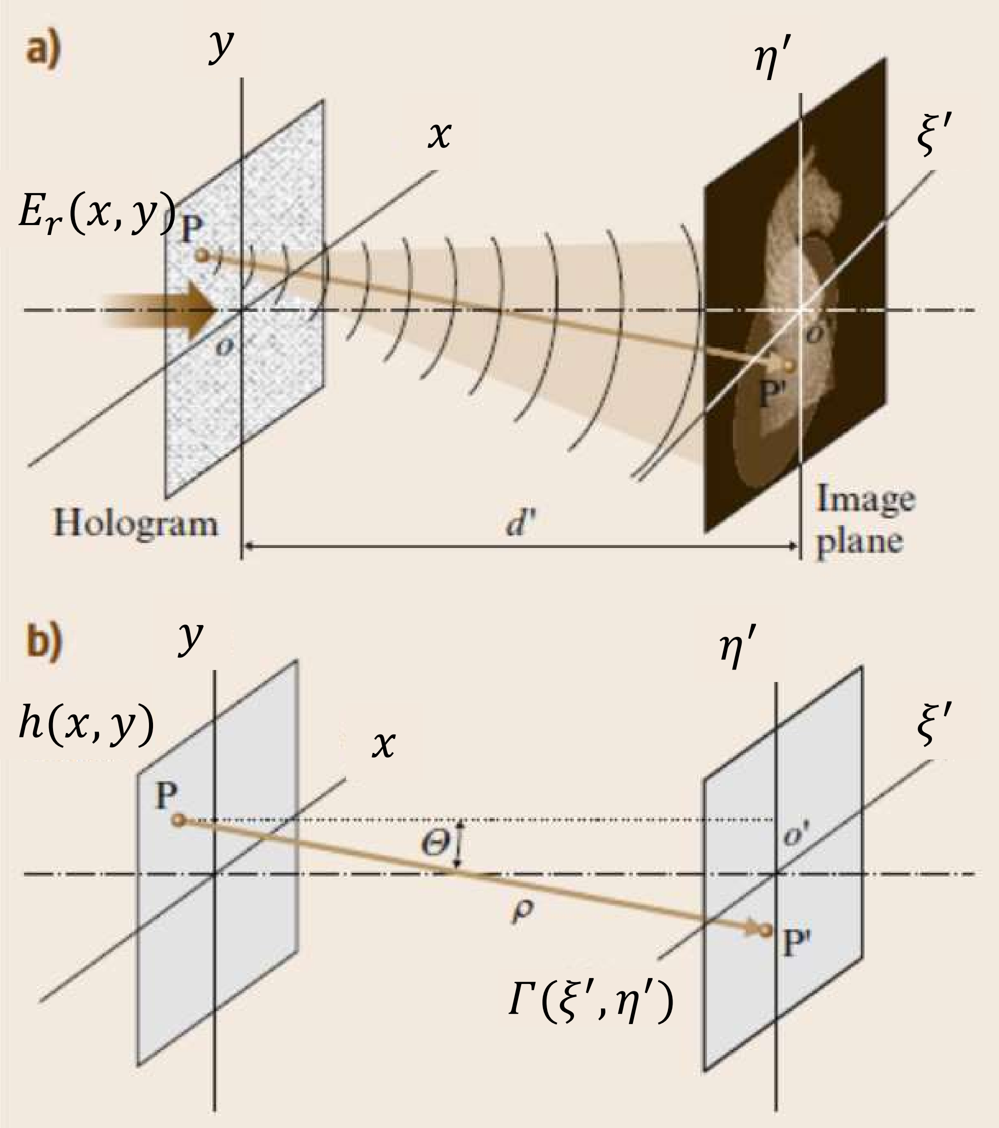
Digital holography the reconstruction of the object wave in the image plane $\Gamma(\xi',\eta')$ is done by numerical simulation of the physical process

The reconstruction wave is the reference wave $E_r(x,y)$ propagates through the hologram $h(x,y)$ obeys the Huygens principle.

each point $P(x,y)$ on the hologram illuminated by the reference beam will act as a point light source and emit a spherical wave

At a given distance $d' = d$ from the hologram a sharp real image of the object can be reconstructed as the superposition of all waves. For the a virtual image $-d$ is used.

Principle of Digital Holography



At a distance d' the diffracted field $\Gamma(\xi',\eta')$ can be

$$\Gamma(\xi',\eta') = \frac{i}{\lambda}\iint t[h(x,y)]E_r(x,y)\frac{e^{-ik\rho}}{\rho}\cos\theta dxdy$$

Where

$$\rho(x-\xi',y-\eta') = \sqrt{d^2+(x-\xi')^2+(y-\eta')^2}$$

$$k = \frac{2\pi}{\lambda}$$

Since $\cos\theta = d'/\rho$

$$\Rightarrow \Gamma(\xi',\eta') = \frac{id}{\lambda}\iint t[h(x,y)]E_r(x,y)\frac{e^{-ik\rho\prime}}{\rho'^2}dxdy$$

Principle of Digital Holography

The numerical reconstruction delivers the complex amplitude of the wavefront, the phase distribution $\phi(\xi',\eta')$ and the intensity $I\ (\xi',\eta')$ can be calculated directly from the reconstructed complex function $\Gamma(\xi',\eta')$

$$\phi(\xi',\eta') = \tan^{-1}\frac{Im|\Gamma(\xi',\eta')|}{Re|\Gamma(\xi',\eta')|}$$

$$I\ (\xi',\eta') = \Gamma(\xi',\eta')\Gamma^{*}(\xi',\eta')$$

Other Reconstruction

- **On the object side**

Real image can be produced by using the conjugate reference beam for reconstruction. To reconstruct an undistorted real image in Digital Holography

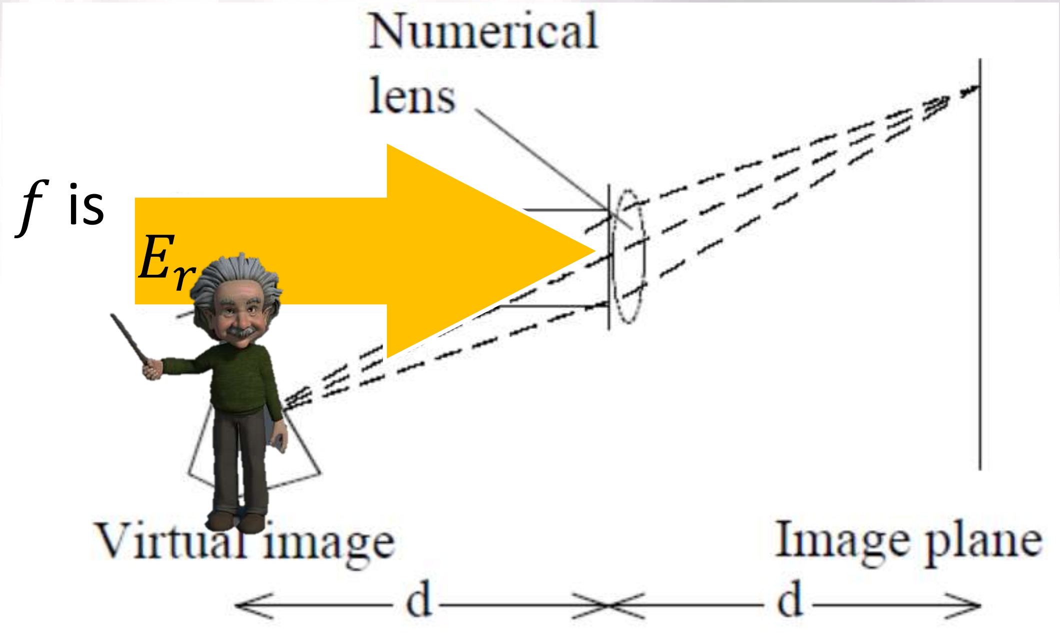
$$\Gamma(\xi,\eta) = \frac{i}{\lambda}\iint t[h(x,y)]E_r^*(x,y)\frac{e^{-ik\rho}}{\rho}\cos\theta dxdy$$

- **Viewing behind the virtual hologram**

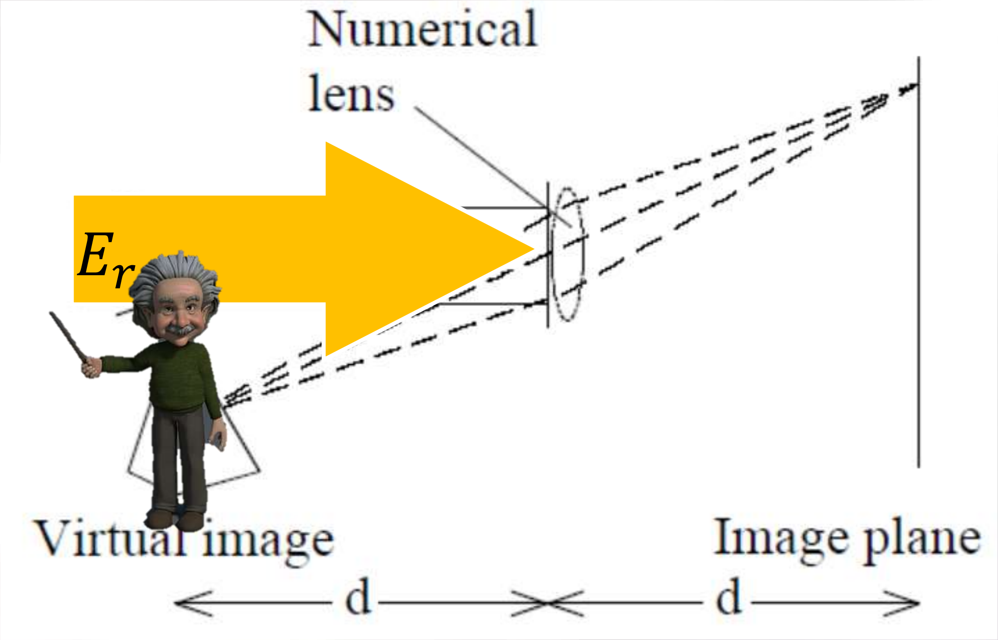
The imaging properties of a lens with focal length f is considered by a complex factor $L(x,y)$

$$L(x,y) = e^{i\frac{\pi}{\lambda f}(x^2+y^2)}$$

If the magnification is 1, then taking $f = d / 2$



● Viewing behind the virtual hologram



The imaging properties of a lens with focal length f is considered by a complex factor $L(x,y)$

$$L(x,y) = e^{i\frac{\pi}{\lambda f}(x^2+y^2)}$$

If the magnification is 1, then taking $f = d/2$

$$\Gamma(\xi',\eta') = \frac{i}{\lambda}\iint t[h(x,y)]E_r(x,y)L(x,y)\frac{e^{-ik\rho}}{\rho}\cos\theta dxdy$$

Considering $L(x,y)$ will cause phase aberrations, the problem can be eliminate by multiplying the reconstructed wave field by $P(\xi',\eta') = e^{i\frac{\pi}{\lambda f}(\xi'^2+\eta'^2)}$

$$\Gamma(\xi',\eta') = \frac{i}{\lambda}P(\xi',\eta')\iint t[h(x,y)]E_r(x,y)L(x,y)\frac{e^{-ik\rho}}{\rho}\cos\theta dxdy$$

Reconstructions are performed numerically

Numerical approach continuing

For x- and y-values as well as for ξ' and η' are small compared to the distance d between the reconstruction plane and the optical sensor (CCD/CMOS), then ρ can be expanded by Taylor series

$$\rho = d + \frac{(\xi - x)^2}{2d} + \frac{(\eta - y)^2}{2d}$$

The reconstruction formula becomes

$$\Gamma(\xi,\eta) = \frac{i}{\lambda d} e^{\frac{-2id\pi}{\lambda}} e^{\frac{-i\pi}{\lambda d}(\xi^2+\eta^2)} \iint t[h(x,y)]E_r^*(x,y) e^{\frac{-i\pi}{\lambda d}(x^2+y^2)} e^{\frac{2i\pi}{\lambda d}(x\xi+y\eta)} \cos\theta dxdy$$

By substituting $\nu = \dfrac{\xi}{\lambda d}$ $\mu = \dfrac{\eta}{\lambda d}$

$$\Rightarrow \Gamma(\xi,\eta) = \frac{i}{\lambda d} e^{\frac{-2id\pi}{\lambda}} e^{-i\pi\lambda d\,(\nu^2+\mu^2)} \iint t[h(x,y)]E_r^*(x,y) e^{\frac{-i\pi}{\lambda d}(x^2+y^2)} e^{i2\pi(x\nu+y\mu)} \cos\theta dxdy$$

$$\Rightarrow \Gamma(\xi,\eta) = \frac{i}{\lambda d} e^{\frac{-2id\pi}{\lambda}} e^{-i\pi\lambda d\,(\nu^2+\mu^2)} \mathfrak{F}^{-1}\left(t[h(x,y)]E_r^*(x,y) \cos\theta\, e^{\frac{-i\pi}{\lambda d}(x^2+y^2)}\right)$$

Numerical approach continuing

The function $\Gamma(\xi, \eta)$ can be digitized if the hologram function $h(x, y)$ is sampled from the CCD/ CMOS sensor with N× N pixels and the corresponding pixel size is Δx and Δy, then

$$\Gamma(m,n) = \frac{i}{\lambda d} e^{\frac{-2id\pi}{\lambda}} e^{-i\pi\lambda d\,(m^2\Delta\nu^2 + n^2\Delta\mu^2)} \times \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} t[h(k,l)]E_r^*(k,l) e^{\frac{-i\pi}{\lambda d}(k^2\Delta x^2 + l^2\Delta y^2)} e^{i2\pi(k\Delta x m\Delta\nu + l\Delta y n\Delta\mu)} \cos\theta$$

Where m=0,1,2… N and n=0,1,2… N

$$\Delta\nu = \frac{1}{N\Delta x} \qquad \Delta\mu = \frac{1}{N\Delta y}$$

Numerical approach continuing

By substituting $\Delta\xi = \dfrac{\lambda d}{N\Delta x}$ $\Delta\eta = \dfrac{\lambda d}{N\Delta y}$

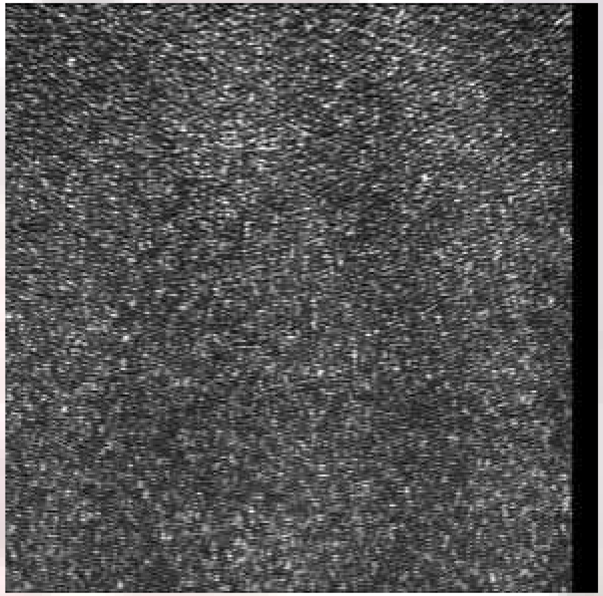
$$\Gamma(m,n) = \frac{i}{\lambda d} e^{\frac{-2id\pi}{\lambda}} e^{-i\pi\lambda d\,\left(\frac{m^2}{N^2\Delta x^2}+\frac{n^2}{N^2\Delta y^2}\right)} \times \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} \boxed{t[h(k,l)]E_r^*(k,l)e^{\frac{-i\pi}{\lambda d}(k^2\Delta x^2+l^2\Delta y^2)}\cos\theta}\, e^{i2\pi\left(\frac{km}{N}+\frac{ln}{N}\right)}$$

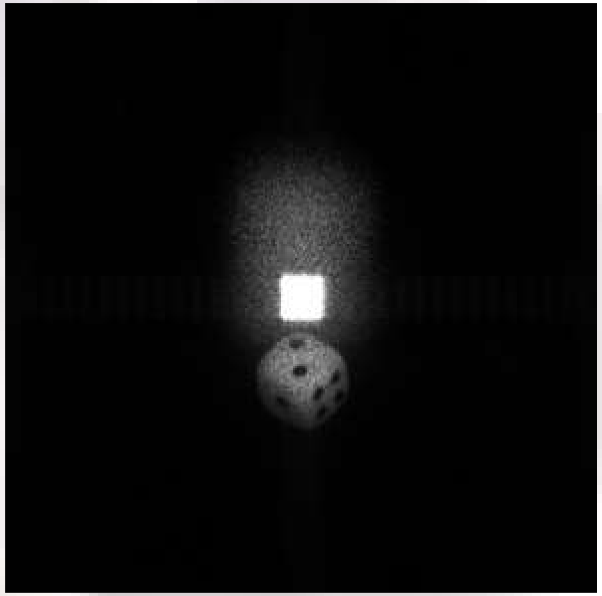
Inverse discrete Fourier transform

Calculating can be done by using the Fast Fourier Transform (FFT) algorithm

The corresponding discrete formula for reconstruction via a virtual lens with $f = d\,/\,2$ is

$$\Gamma(m,n) = \frac{i}{\lambda d} e^{\frac{-2id\pi}{\lambda}} e^{+i\pi\lambda d\,\left(\frac{m^2}{N^2\Delta x^2}+\frac{n^2}{N^2\Delta y^2}\right)} \times \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} t[h(k,l)]E_r(k,l)e^{\frac{+i\pi}{\lambda d}(k^2\Delta x^2+l^2\Delta y^2)}\cos\theta\, e^{i2\pi\left(\frac{km}{N}+\frac{ln}{N}\right)}$$



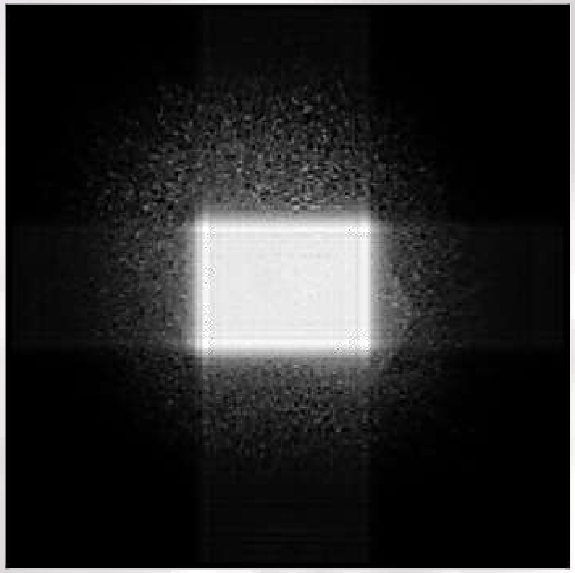


- The bright square in the center is the undiffracted reconstruction wave (zero order).
- The image is spatially separated from the zero order term.
- The other (virtual) image is out of focus in this reconstruction.

The object is placed d=1.054m apart from the CCD-array of 1024×1024 pixels with pixel size $\Delta x \ = \Delta y \ = 6.8\mu m$, with 632.8 nm He-Ne Laser

DC term

photograph
of the object

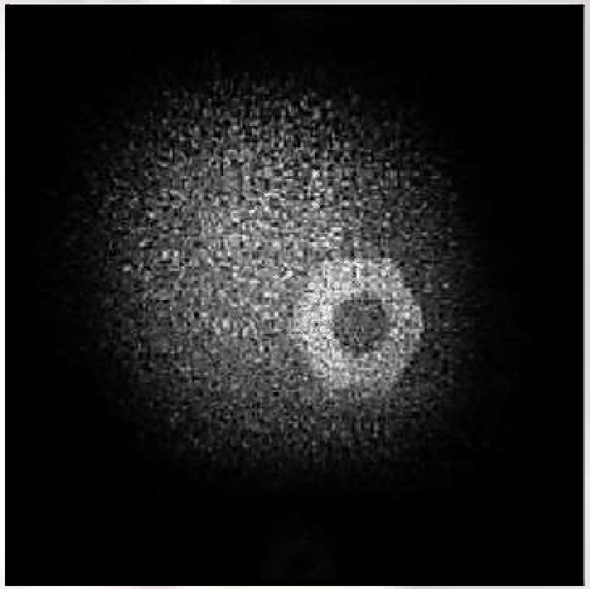


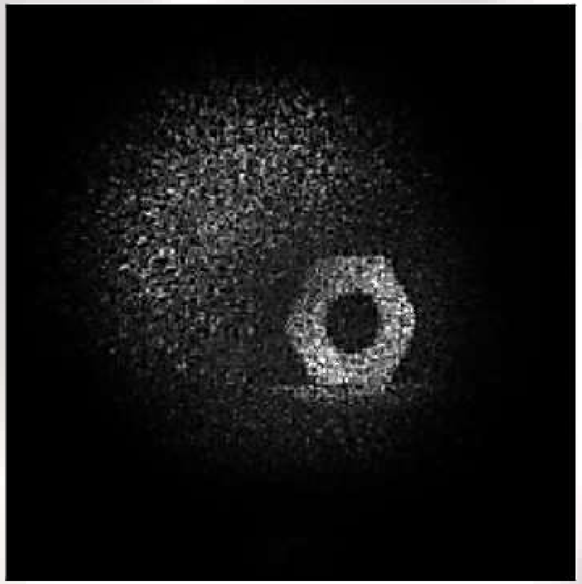
reconstruction without
DC term suppression

$$I(i,j) - I_m$$

Where

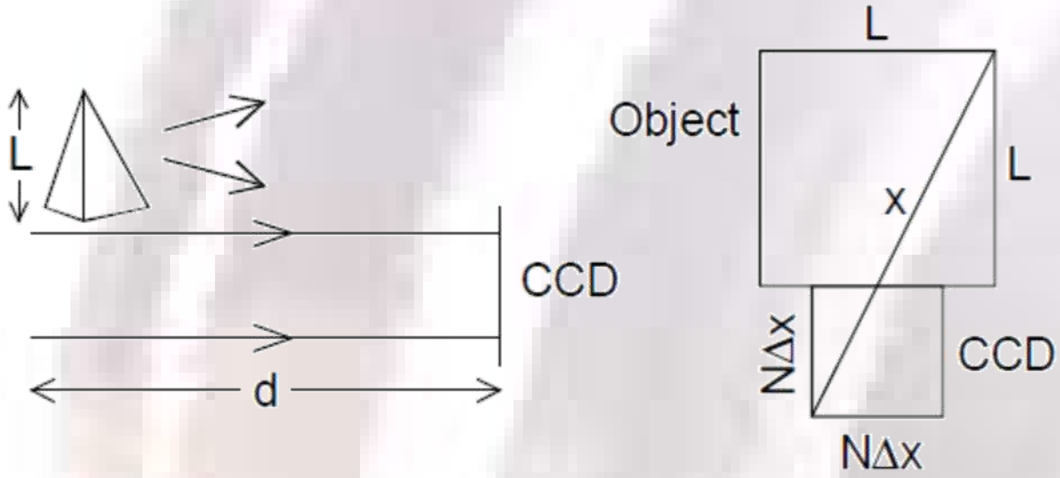
$$I_m = \frac{1}{N}\sum\sum I(i,j)$$



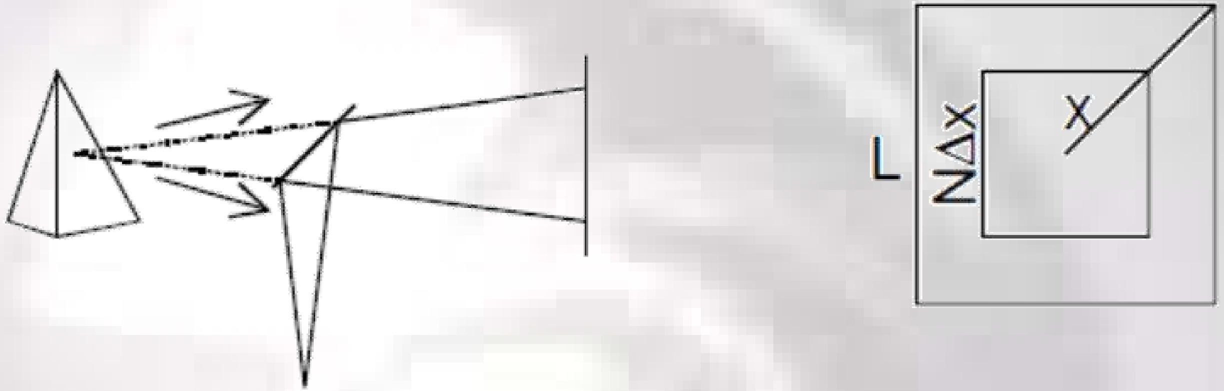


With high pass filter

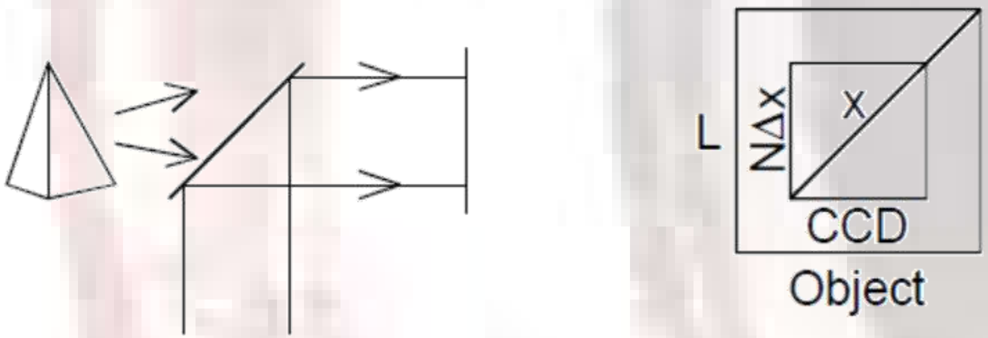
Typical Layout for DH Recording



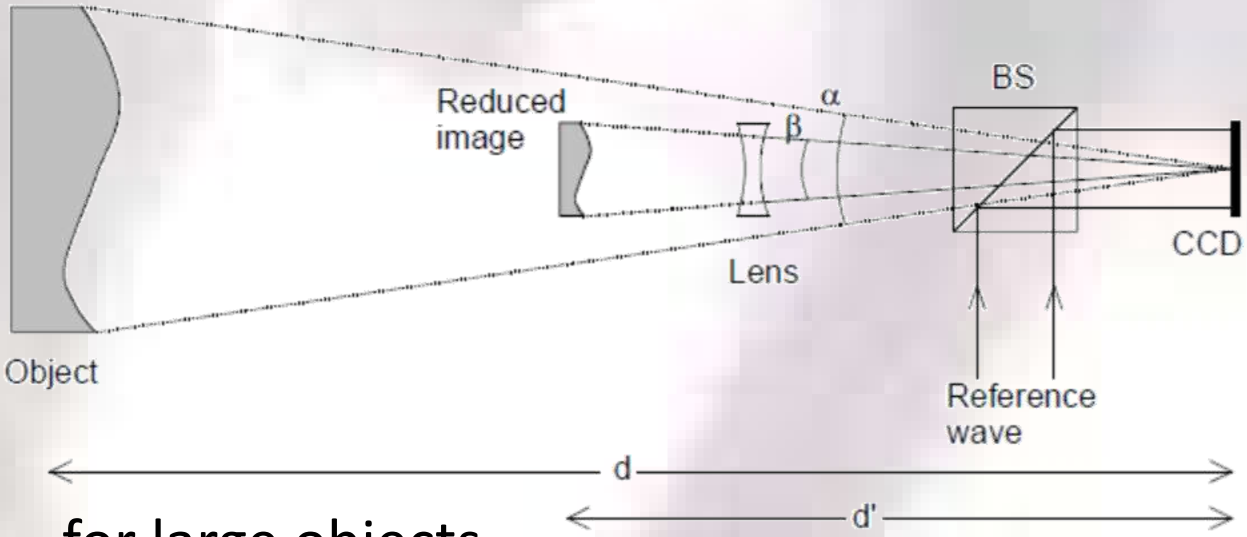
Off-Axis



In-Line/ lensless Fourier holography



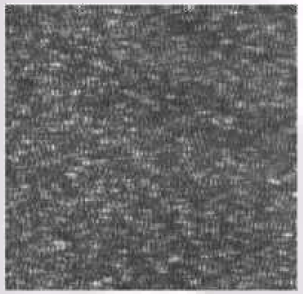
In-Line/ Collimating Beam



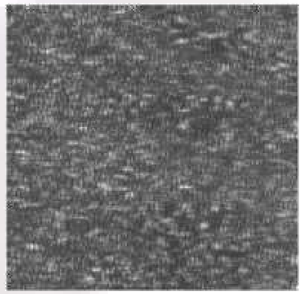
for large objects

Digital Holographic Interferometry (DHI)

$h_1(x, y)$



DH, Phase 1

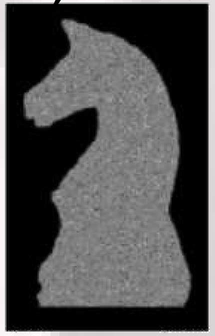


DH, Phase 2

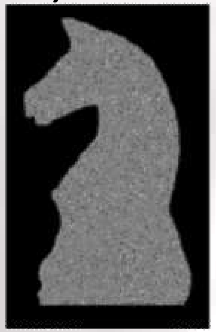
$h_2(x, y)$

$\Gamma_1(\xi, \eta)$

$$\phi_1(\xi', \eta') = \tan^{-1} \frac{Im|\mathit{\Gamma}_1\ (\xi', \eta')|}{Re|\mathit{\Gamma}_1(\xi', \eta')|}$$



Recon., Phase 1



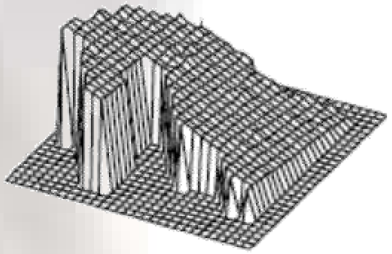
Recon., Phase 2

$\Gamma_2(\xi, \eta)$

$$\phi_2(\xi', \eta') = \tan^{-1} \frac{Im|\mathit{\Gamma}_2\ (\xi', \eta')|}{Re|\mathit{\Gamma}_2(\xi', \eta')|}$$

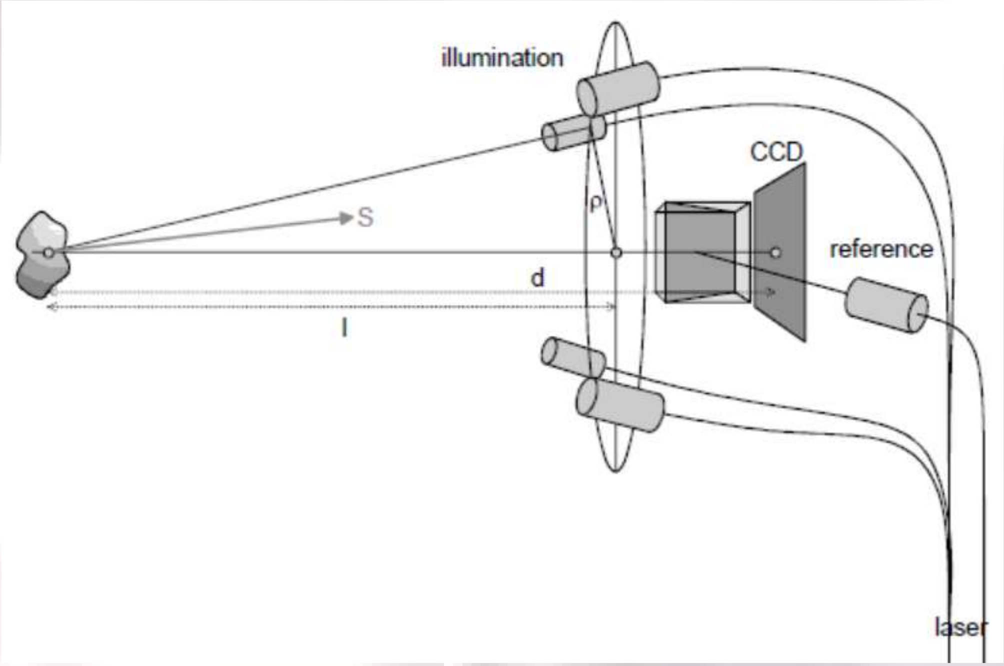
$\phi_2(\xi', \eta') - \phi_1(\xi', \eta')$

DHI



results from the interference phase

F$(\phi_2(\xi', \eta') - \phi_1(\xi', \eta');\ \lambda)$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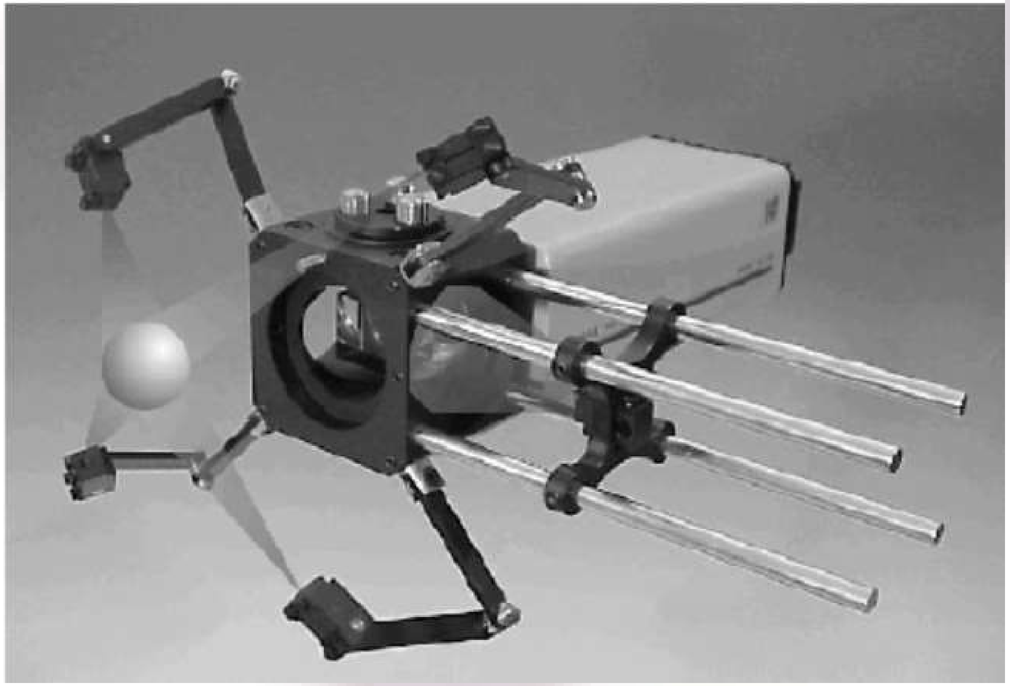

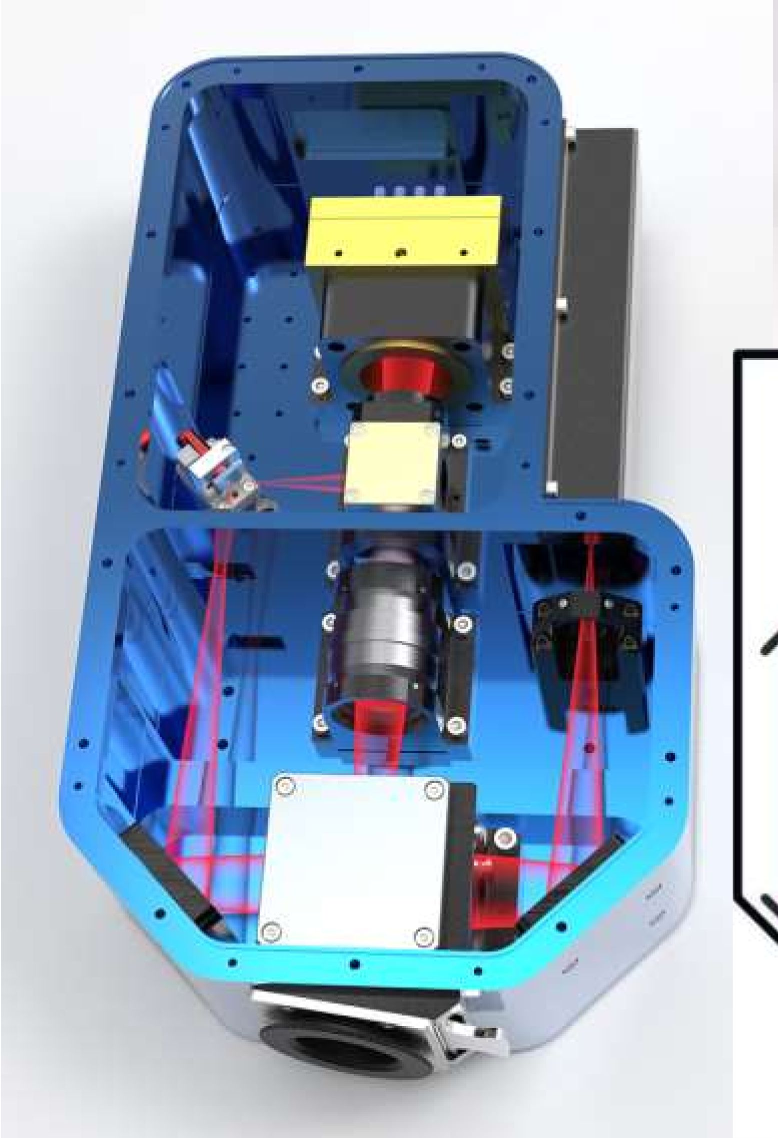

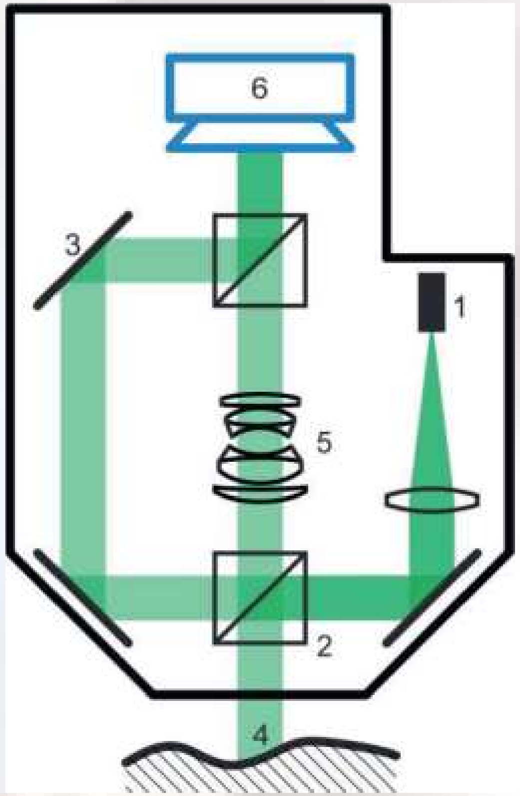
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