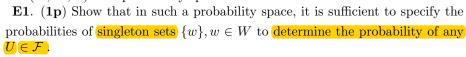


Exercises for the lecture: Conditioning, inference and probability logic

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1 Probability

Let (W, \mathcal{F}, P) be a probability space where $\mathcal{F} = 2^W$.





Given is a Anglo-American deck of 52 well-shuffled cards. The number-cards count as their natural value; the jack, queen and king (face cards) count as 10 and the aces are valued as 11.

E2. (3p) What is the probability to get a black jack (value of 21) after three drawings, given that the first card is an ace?

2 Probability logic

Let A, B be propositions. We showed that reasoning by modus ponens

$$\begin{array}{c}
A \Rightarrow B \\
\hline
A \\
\hline
B
\end{array}$$

with uncertain premises, i.e. $P(A \Rightarrow B) = p_I$ and $P(A) = p_A$ yields the following bound on the conclusion B:

$$P(B) \ge p_A + p_I - 1$$

E3. (2p) Since p_A and p_I are probabilities, it follows that $p_A + p_I \leq 1$. Show that $p_A + p_I - 1 \geq 0$, i.e. that it could a probability . Which event's probability is it? Also, discuss the relationship between p_A and p_I , i.e. the relationship between the probability of an implication and its premise.

Let A, B be propositions. Reasoning by modus tollens is



$$\begin{array}{c}
A \Rightarrow B \\
\sim B \\
\hline
\sim A
\end{array}$$



- **E4**. (**2p**) Formulate the probabilistic equivalent, assuming that you are certain of the premises. Show that the conclusion follows with probability 1.
- **E5**. (**3p**) Assume you are uncertain about the premises: let $P(A \Rightarrow B) = p_I$ and $P(\bar{B}) = p_{\bar{B}}$. What is the tightest bound on the probability of the conclusion, $P(\bar{A})$?
- Let A, B be propositions. The *weak syllogism*, which is unsound in deductive reasoning, is

$$\begin{array}{c}
A \Rightarrow B \\
B \\
\hline
A
\end{array}$$

Nevertheless, we might think that A becomes more plausible given B, because A is a possible cause for B. This kind of reasoning is often employed by scientists for hypothesis generation (not for validation!). For example, let A = "my hypothesis is correct" and B = "data X are observed". The weak syllogism then reads "If my hypothesis is correct, then data X will be observed. I observe X. Hence, my hypothesis becomes more plausible."

E6. (**4p**) Assume that the premises hold with probabilities $P(A \Rightarrow B) = p_I$, $P(B) = p_B$ and that $P(A) = p_A$. How certain do you have to be of the implication, so that A does indeed become more probable given B, i.e. $P(A|B) \ge P(A)$?