Exercises for the lecture: Conditioning, inference and probability logic

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1 Probability

Let $(W, \mathcal{F}, P)$ be a probability space where $\mathcal{F} = 2^W$.

E1. (**1p**) Show that in such a probability space, it is sufficient to specify the probabilities of singleton sets $\{w\}, w \in W$ to determine the probability of any $U \in \mathcal{F}$.

Given is a Anglo-American deck of 52 well-shuffled cards. The number-cards count as their natural value; the jack, queen and king (face cards) count as 10 and the aces are valued as 11.

E2. (**3p**) What is the probability to get a black jack (value of 21) after three drawings, given that the first card is an ace?

2 Probability logic

Let A, B be propositions. We showed that reasoning by *modus ponens*

$$\frac{\begin{array}{c} A \Rightarrow B \\ A \end{array}}{B}$$

with uncertain premises, i.e. $P(A \Rightarrow B) = p_I$ and $P(A) = p_A$ yields the following bound on the conclusion B:

$$P(B) \geq p_A + p_I - 1$$

E3. (**2p**) Since p_A and p_I are probabilities, it follows that $p_A + p_I \leq 1$. Show that $p_A + p_I - 1 \geq 0$, i.e. that it could a probability . Which event's probability is it? Also, discuss the relationship between p_A and p_I, i.e. the relationship between the probability of an implication and its premise.

Let A, B be propositions. Reasoning by *modus tollens* is

$$\frac{\begin{array}{c} A \Rightarrow B \\ \sim B \end{array}}{\sim A}$$

E4. (**2p**) Formulate the probabilistic equvialent, assuming that you are certain of the premises. Show that the conclusion follows with probability 1.

E5. (**3p**) Assume you are uncertain about the premises: let $P(A \Rightarrow B) = p_I$ and $P(\bar{B}) = p_{\bar{B}}$. What is the tightest bound on the probability of the conclusion, $P(\bar{A})$?

Let A, B be propositions. The *weak syllogism*, which is unsound in deductive reasoning, is

$$\frac{\begin{array}{c} A \Rightarrow B \\ B \end{array}}{A}$$

Nevertheless, we might think that A becomes more plausible given B, because A is a possible cause for B. This kind of reasoning is often employed by scientists for hypothesis generation (not for validation!). For example, let $A =$"my hypothesis is correct" and $B =$"data X are observed". The weak syllogism then reads "If my hypothesis is correct, then data X will be observed. I observe X. Hence, my hypothesis becomes more plausible."

E6. (**4p**) Assume that the premises hold with probabilities $P(A \Rightarrow B) = p_I$, $P(B) = p_B$ and that $P(A) = p_A$. How certain do you have to be of the implication, so that A does indeed become more probable given B, i.e. $P(A|B) \geq P(A)$?