Department Of Aerospace Engineering,
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AS2101 : Introduction to Aerospace Engineering

Report 7

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1 Aim

To Find the Integral

$$\int_{-1}^{1} e^{-x} sin^2(4x)$$

using Trapezoidal and Gaussian Quadrature and comparing the efficiency of both the techniques.

2 Theory

2.1 Fundamental Rule of Gaussian Quadrature

This is used to find the exact values of definite integrals of polynomials (degree = 2n-1, where n is the number of nodes solved.)

$$\int_{-1}^{1} f(x)dx = \sum_{i=0}^{n} f(x_i) \cdot w_i \tag{1}$$

Here, w is called the weighted sum.

2.2 Gauss-Legendre Quadrature

To find the weighted sum (w), we need a set of orthogonal polynomials called the **Legendre Polynomials** which are given by Rodrigue's formula, which states

$$P_n(x) = \frac{1}{2^n n!} \cdot \frac{d^n}{dx^n}(x^2 - 1)^n$$

The roots of $P_n(x)$ are the nodes used. After this, we calculate th weighted sum byt the formula :

$$w_i(x) = \frac{2}{(1 - x_i^2)[(P_n(x_i)]^2}$$

2.3 Trapeziodal Method of Integration

We divide the area under curve into several trapezoids and sum up the area to ultimately find the value of definite integral.

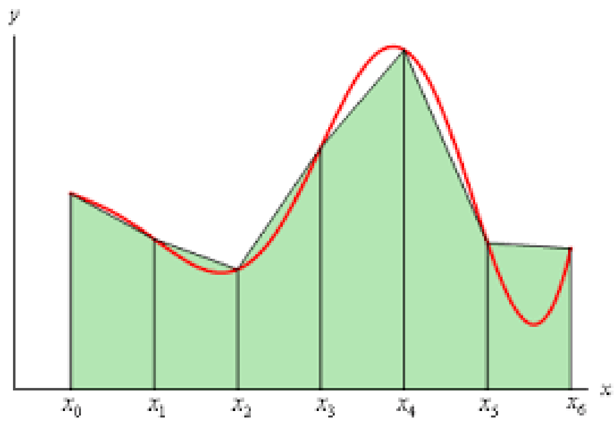

Figure 1: Trapezoidal Method of Integration

$$I = \sum_{i=0}^{n} \frac{(x_{i+1} - x_i)(f(x_i) + f(x_{i+1}))}{2}$$

Legendre Polynomials

n	$\widetilde{P}_n(x)$
0	1
1	$2x - 1$
2	$6x^2 - 6x + 1$
3	$20x^3 - 30x^2 + 12x - 1$
4	$70x^4 - 140x^3 + 90x^2 - 20x + 1$
5	$252x^5 - 630x^4 + 560x^3 - 210x^2 + 30x - 1$

Figure 2: First 5 Legendre Polynomials

Nodes and weights

Number of points, n	Points, x_i		Weights, w_i	
1	0		2	
2	$\pm\frac{1}{\sqrt{3}}$	±0.57735...	1	
3	0		$\frac{8}{9}$	0.888889...
	$\pm\sqrt{\frac{3}{5}}$	±0.774597...	$\frac{5}{9}$	0.555556...
4	$\pm\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}}$	±0.339981...	$\frac{18+\sqrt{30}}{36}$	0.652145...
	$\pm\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}}$	±0.861136...	$\frac{18-\sqrt{30}}{36}$	0.347855...
5	0		$\frac{128}{225}$	0.568889...
	$\pm\frac{1}{3}\sqrt{5 - 2\sqrt{\frac{10}{7}}}$	±0.538469...	$\frac{322+13\sqrt{70}}{900}$	0.478629...
	$\pm\frac{1}{3}\sqrt{5 + 2\sqrt{\frac{10}{7}}}$	±0.90618...	$\frac{322-13\sqrt{70}}{900}$	0.236927...

Figure 3: Nodes and Weight Sums of first 5 n

2.4 Change of Limits

As we can see in the Fundamental rule of Gaussian Quadrature, the limits give us integrals inly from -1 to 1.

Thus, in order to change the limits from (-1 to 1) to (a to b), we use $x = \frac{b-a}{2}u + \frac{b+a}{2}$ such that we can get the required integral by putting u from -1 to 1.

$$\int_a^b f(x)dx = \int_{-1}^1 f\left(\frac{b-a}{2}u + \frac{b+a}{2}\right) du$$

3 First 5 Legendre Polynomials - Graphical Representation

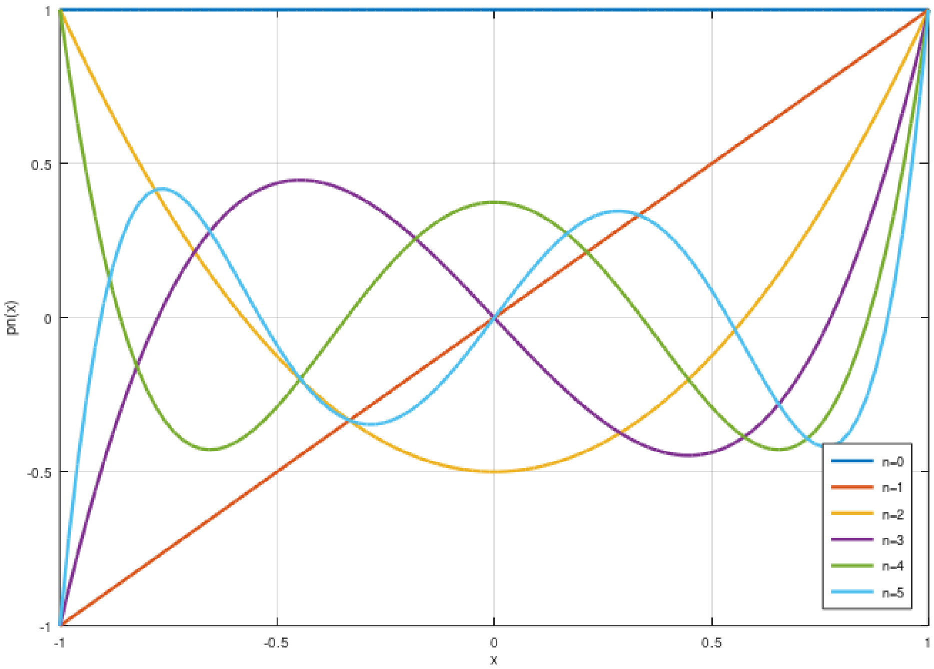

Figure 4: Graphical Representation of first 5 Legendre polynomials

4 Manual Integration

We will find the value by manual integration first, and then later tally with the results of the Trapezoidal and gaussian Method.

$$I = \int_{-1}^1 e^{-x}\sin^2(4x)dx$$

As we know, $f(x) \equiv f(a+b-x)$;

$$I = \int_{-1}^1 e^{x}\sin^2(4x)dx$$

Solving this, we get

$$I = [-\frac{e^x}{130}[8\sin(8x) + \cos(8x) - 65]_{-1}^1$$

Which on putting limits gives us
$I = 0.9899352767719962$

5 Results using Trapezoidal and Gaussian Methods

5.1 Gaussian Method

```
>> gaussian(10)
ans = 0.989935015655239

>> gaussian(30)
ans = 0.989935276771999

>> gaussian(50)
ans = 0.989935276771994
```

**Code attached in appendix*

5.2 Trapezoidal Method

```
>> trapezoidal(10)
ans = 1.036961178946498

>> trapezoidal(30)
ans = 0.994978738845161

>> trapezoidal(50)
ans = 0.991745973094080
```

**Code attached in appendix*

6 Errors(log(error) vs N : Graphical Analysis

6.1 Gaussian Method

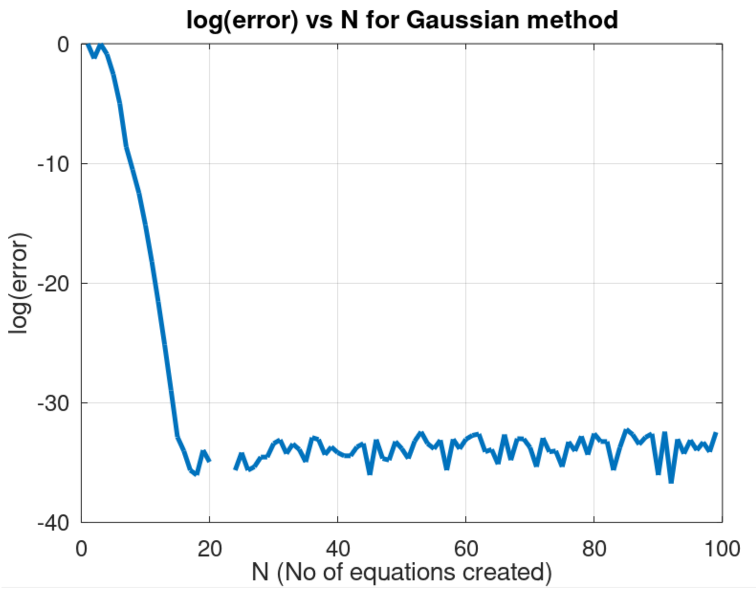

Figure 5: log(error) vs N graph for Gaussian Method

6.2 Trapezoidal Method

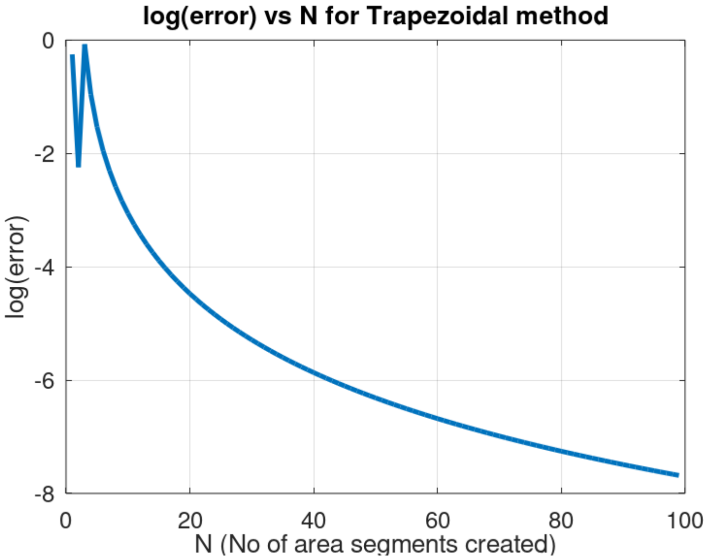

Figure 6: log(error) vs N graph for Trapezoidal Method

A .m code for Gaussian Method

```
# Pranit Zope
# AE20B046
# Task 06

function retval = gaussian (n)
format long
x=zeros(99);
w=zeros(100);
for i=1:99
file_name=int2str(i);
a=strcat(file_name,"roots.txt");
b=strcat(file_name,"weights.txt");
temp_1=importdata(a);
temp_2=importdata(b);
  for j=1:size(temp_1)(1,2)
    x(i,j)=temp_1(1,j);
  endfor
  for j=1:size(temp_2)(1,2)
      w(i,j)=temp_2(1,j);
endfor
endfor
retval=0;
for k=1:99
  retval+=w(n,k)*f(x(n,k));
endfor
endfunction
```

B .m code for Trapezoidal Method

```
# Pranit Zope
# AE20B046
# Task 06

function retval = trapezoidal(n)
format long

x=linspace(-1,1,n+1);
retval=0;
for i=1:n
  retval+=0.50*(x(i+1)-x(i))*(f(x(i))+f(x(i+1)));
endfor
endfunction
```

C .m code for error analysis

C.1 Gaussian Method

```
# Pranit Zope
# AE20B046
# Task 06
```

```
format long;
x=1:99;
y1=[];
for i=1:99
  y1=[y1,log(error(gaussian(i)))];
endfor
gaussian(21)
plot(x,y1,"-",'linewidth',2.5)
grid on
title('log(error) vs N for Gaussian method')
xlabel('N (No of equations created)')
ylabel('log(error)')
set(gca,'fontsize',24)
```

C.2 Trapezoidal Method

```
# Pranit Zope
# AE20B046
# Task 06

format long
x=1:99
y1=[]
for i=1:99
  y1=[y1,log(error(trapezoidal(i)))]
endfor
trapezoidal(99)
plot(x,y1,"-",'linewidth',2.5)
grid on
title('log(error) vs N for Trapezoidal method')
xlabel('N (No of area segments created)')
ylabel('log(error)')
set(gca,'fontsize',24)
```