

# EFFICIENT DEEP LEARNING READING GROUP

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# SPDY: Accurate **Pruning** with speedup guarantees

ICML'22  
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# Motivation

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- Unstructured Pruning (Weight Pruning)
- Previous Work:
  - Minimize the number of remaining weights
- This Work
  - Minimize the inference time
- Goal:
  - Automatically determines layer-wise sparsity
- Methods:
  - Dynamic Programming
  - Local Search

# Introduction

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- Pruning Methods:
  - Structured Pruning
  - Unstructured Pruning
- Runtime Side, unstructured sparsity is important:
  - Algorithms provides speedup on CPUs, GPUs or Specialized hardware.
  - Commodity CPUs, AMD models only cares sparsity instead of quantization.
- Key issue of previous unstructured pruning works:
  - Not consider the acceleration methods.

# Introduction

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- Contribution:
  - learned efficient Sparsity Profiles via Dynamic programming search (SPDY). Determine layer-wise sparsity to achieve a desired speedup.
  - First, optimization problem → dynamic programming solver.
  - Second, learns the layer-wise error-scores automatically, based on calibration dataset.

# Optimization Problem

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- Constrained Optimization Problem:

$$\min_{s_1, \dots, s_L \in S} \sum_{\ell=1}^L e_\ell^{s_\ell} \quad \text{s.t.} \quad \sum_{\ell=1}^L t_\ell^{s_\ell} \leq T. \quad (1)$$

- Assumption:
  - Overall execution time = Sum of the individual layer runtimes.
  - Pruning a layer  $\ell$  to sparsity  $s$  ultimately incurs some model error  $e_\ell^{s_\ell}$ , which is additive.
- Integer linear program (ILP)
- However, NP-hard and requires exponential time to solve.

# Efficient Solver

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- Make time t as an integer-value
- Dynamic Programming
- Recursion:

$$E_\ell^t = \min_{s \in S} E_{\ell-1}^{t-t_\ell^s} + e_\ell^s \quad (2)$$

$$E_1^t = \min_{s \in S'} e_1^s \text{ if } S' = \{s \mid t_1^s = t\} \neq \emptyset \text{ else } \infty. \quad (3)$$

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**Algorithm 1** We efficiently compute the optimal layer-wise sparsity profile with execution time at most  $T$  given  $S$ ,  $e_\ell^s$ ,  $t_\ell^s$  and assuming that time is discretized, using bottom-up dynamic programming.

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```
D ← L × (T + 1) matrix filled with ∞
P ← L × (T + 1) matrix
for s ∈ S do
    if e_1^s < D[1, t_1^s] then
        D[1, t_1^s] ← e_1^s; P[1, t_1^s] ← s
    end if
end for
for ℓ = 2, …, L do
    for s ∈ S do
        for t = t_ℓ^s + 1, …, T do
            if e_ℓ^s + D[ℓ − 1, t − t_ℓ^s] < D[ℓ, t] then
                D[ℓ, t] ← e_ℓ^s + D[ℓ − 1, t − t_ℓ^s]; P[ℓ, t] ← s
            end if
        end for
    end for
end for
t ← argmin_t D[L, t] // return D[L, t] as optimal error
for ℓ = L, …, 1 do
    s ← P[ℓ, t] // return s as optimal sparsity for layer ℓ
    t ← t − t_ℓ^s
end for
```

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# Error Metric $e_\ell^s$

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- Previous:
  - Weight Magnitude ; Squared Weight Magnitude ; Loss change
- Ours: “learning” or “search”

$$e_\ell^s = c_\ell \cdot \left( \frac{i}{|S| - 1} \right)^2, \quad s = 1 - (1 - \delta)^i. \quad (4)$$

T=10,000, |S| = 42, L = 52 for ResNet50.

- How to check optimal “c”?

# Quickly Check the Quality of a Sparsity Profile

- This database stores for each layer and each sparsity the “reconstruction” of the remaining weights after pruning.

$$\operatorname{argmin}_{W^s} \|f_\ell(X, W) - f_\ell(X, W^s)\|_2^2, \text{ for layer } \ell.$$

- First, we query the database for the corresponding reconstructed weights of each layer, each at its target sparsity.
- Second, we “stitch together” the resulting model from the reconstructed weights, and evaluate it on a given small validation set.

# Determine sensitivity values $\mathbf{C}$ .

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**Algorithm 4** SPDY search for optimal sensitivity values  $\mathbf{c}^*$ .  
We use  $k = 100$  and  $\delta = 0.1$  in our experiments.

```
function eval( $\mathbf{c}$ )
     $e_\ell^s \leftarrow$  compute by formula (4) using  $\mathbf{c}$  for all  $\ell$ 
     $s_\ell \leftarrow$  run DP algorithm with  $e_\ell^s$  for all  $\ell$ 
     $M \leftarrow$  stitch model for  $s_\ell$  from database
    Return calibration loss of  $M$ .

     $\mathbf{c}^* \leftarrow$  sample uniform vector in  $[0, 1]^L$ 
    for  $k$  times do
         $\mathbf{c} \leftarrow$  sample uniform vector in  $[0, 1]^L$ 
        if  $eval(\mathbf{c}) < eval(\mathbf{c}^*)$  then
             $\mathbf{c}^* \leftarrow \mathbf{c}$ 
        end if
    end for
    for  $d = \lceil \delta \cdot L \rceil, \dots, 1$  do
        for  $k$  times do
             $\mathbf{c} \leftarrow \mathbf{c}^*$ 
            Randomly resample  $d$  items of  $\mathbf{c}$  in  $[0, 1]$ 
            if  $eval(\mathbf{c}) < eval(\mathbf{c}^*)$  then
                 $\mathbf{c}^* \leftarrow \mathbf{c}$ 
            end if
        end for
    end for
end for
```

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# Overall

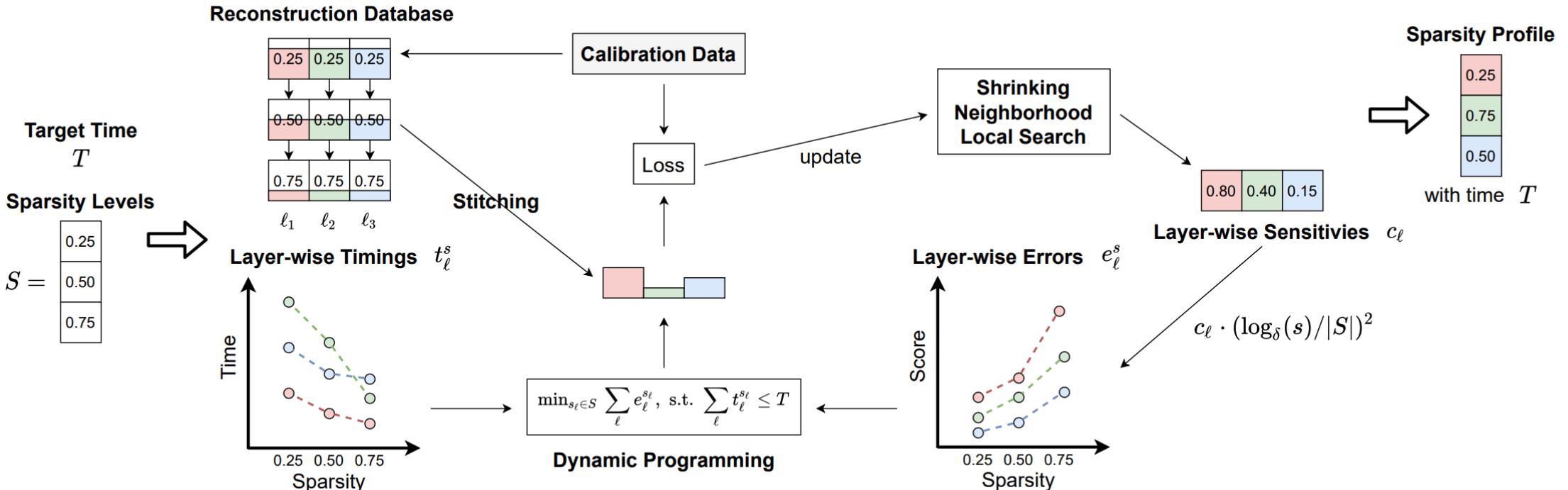


Figure 3. A visual overview of the full SPDY method.

$T=10,000$ ,  $|S| = 42$ ,  $L = 52$  for ResNet50.

# Result

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Model	Dense	Speed.	CPU	SPDY	Uni.	GMP
ResNet50	76.13	2.00×	AMD	<b>76.39</b>	76.01	75.85
ResNet50	76.13	2.50×	AMD	<b>75.56</b>	75.12	74.76
ResNet50	76.13	3.00×	AMD	<b>74.75</b>	74.02	73.44
ResNet50	76.13	3.50×	AMD	<b>73.06</b>	71.62	70.22
MobileNetV1	71.95	1.50×	Intel	<b>71.38</b>	61.33	70.63
YOLOv5s	56.40	1.50×	Intel	<b>55.90</b>	54.70	55.00
YOLOv5s	56.40	1.75×	Intel	<b>53.10</b>	50.90	47.20
YOLOv5m	64.20	1.75×	Intel	<b>62.50</b>	61.70	61.50
YOLOv5m	64.20	2.00×	Intel	<b>60.70</b>	58.30	57.20
BERT SQuAD	88.54	3.00×	Intel	<b>88.53</b>	88.22	87.98
BERT SQuAD	88.54	3.50×	Intel	<b>87.56</b>	87.23	87.22
BERT SQuAD	88.54	4.00×	Intel	<b>86.44</b>	85.63	85.13
BERT SQuAD*	88.54	4.00×	Intel	<b>87.14</b>	86.37	86.39

*Table 4.* Comparing accuracy metrics for sparsity profiles after gradual pruning models with respective state-of-the-art methods.

# Sparse Double Descent: Where Network Pruning Aggravates Overfitting

ICML'22  
Zheng He, et al.



# Motivation

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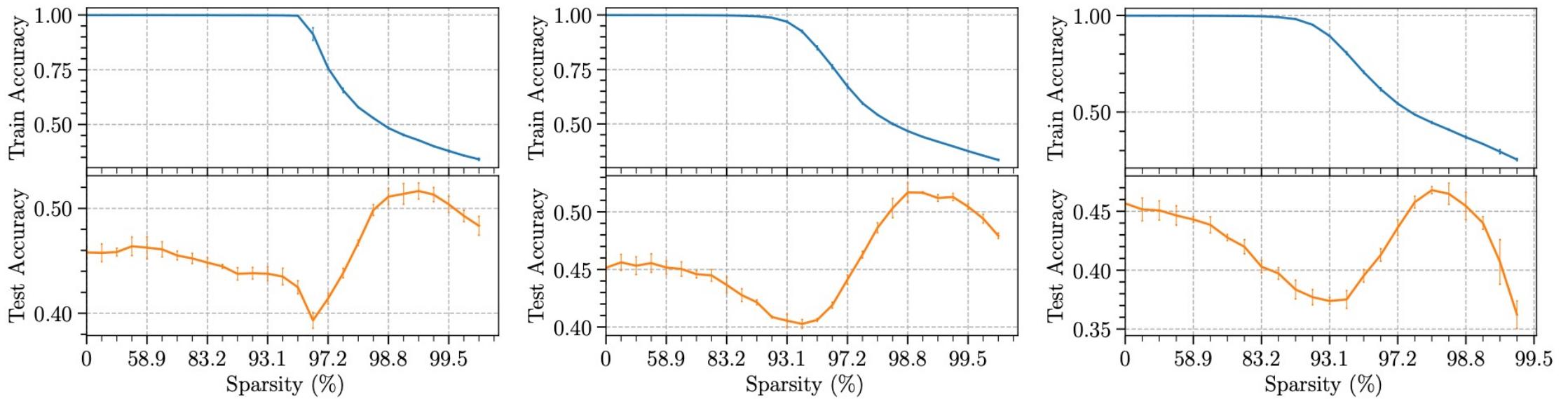
- Previous work:
  - Increase the model sparsity will prevents the overfitting.
- Sparse Double Descent:
  - Increase the model sparsity, test performance first gets worse (overfitting) then gets better (relieved overfitting).

# Introduction

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- Overparameterized DNNs are “good at” overfitting.
- In practice, DNNs often achieve higher generalization than smaller models.
- Recent, Deep Double Descent:
  - Model capacity increases, test performance first gets better then worse (overfitting) then gets better (relieved overfitting).
- Contribution:
  - Sparse Double Descent.
    - Increase the model sparsity, test performance first gets worse (overfitting) then gets better (relieved overfitting).
  - L2 learning distance
  - Contrary to the lottery ticket hypothesis.

# Sparse Double Descent



*Figure 2.* Sparse Double Descent of ResNet-18 on CIFAR-100 with 40% symmetric label noise, pruned using different strategies. We plot the train and test accuracy against sparsity. **Left:** Magnitude-based pruning. **Middle:** Gradient-based pruning. **Right:** Random pruning.

# Four phases of model sparsity

## 1. Low sparsity:

Pruned network = dense model

## 2. Critical phase:

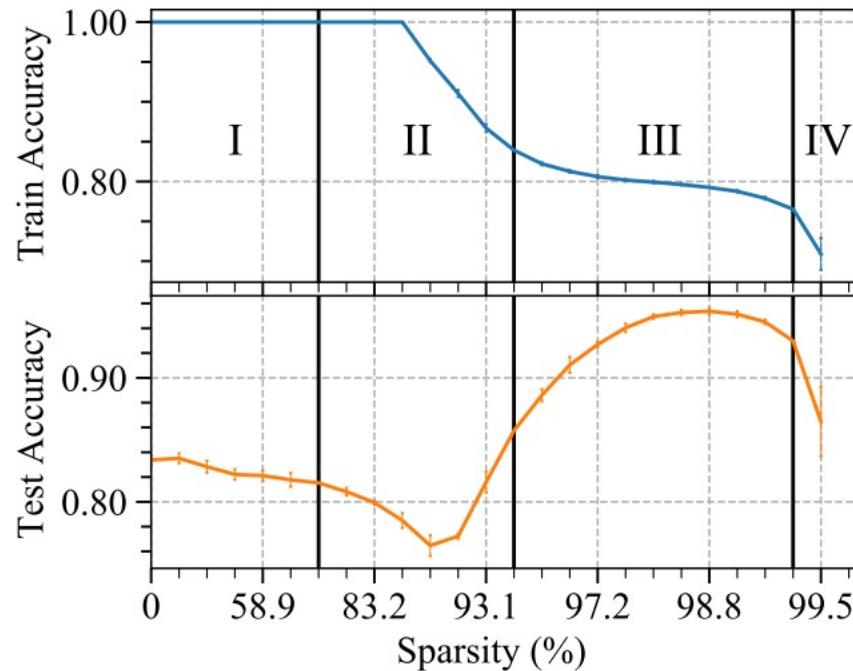
Severe Overfitting

## 3. Sweet Phase:

Boosted accuracy

## 4. Collapsed Phase:

Accuracy Drops



*Figure 5.* Illustration of four phases using the result of LeNet-300-100 on MNIST with 20% symmetric label noise. I: Light Phase. II: Critical Phase. III: Sweet Phase. IV: Collapsed Phase.

# Why Sparse Double Descent Occurs

- The Learning Distance Hypothesis for Sparse Double Descent

- L2 distance:

$$D(\mathbf{w}_{init}, \mathbf{w}_{learned}^i) = \|\mathbf{w}_{init} - \mathbf{w}_{learned}^i\|_2.$$

- Learning distance correlates  
the test accuracy

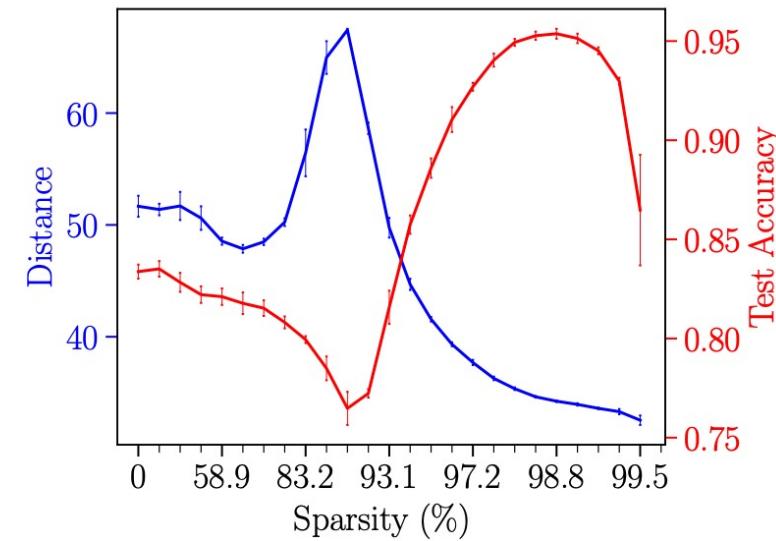
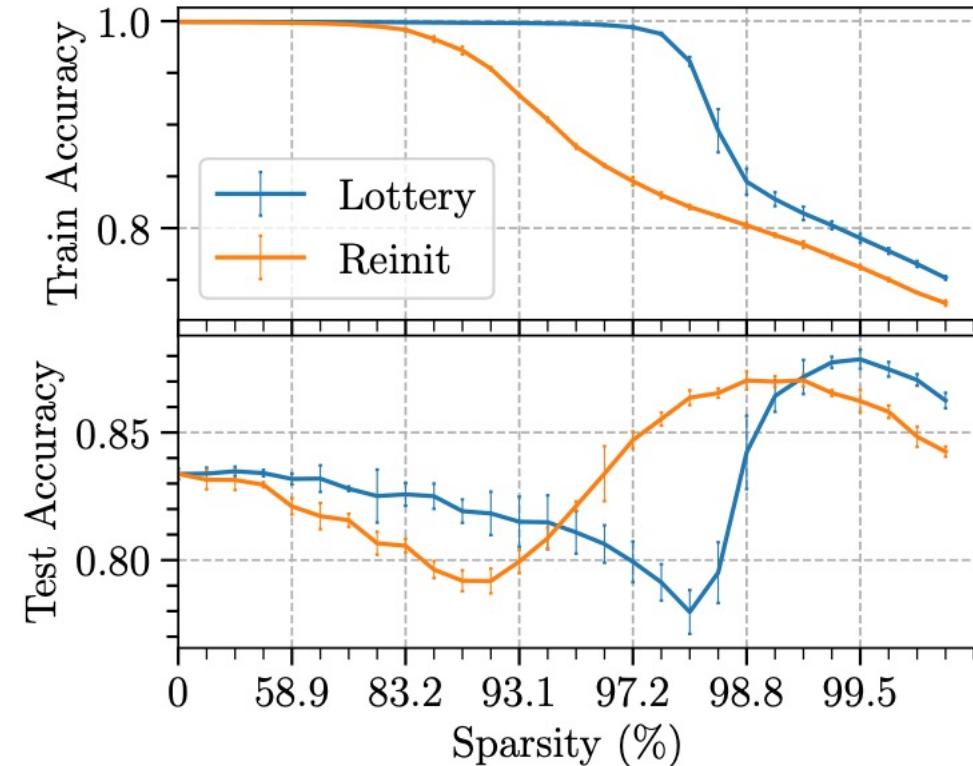


Figure 9. The curve of learning distance for LeNet-300-100 on MNIST with  $\epsilon = 20\%$  may explain the double descent of test accuracy. As model sparsity increases, learning distance coincides the changes of test accuracy. The blue lines refer to  $\ell_2$  learning distance and the red lines are test accuracy.

# Lottery tickets may not win at all time

- Reinitialized models could beat lottery ticket models at the same sparsity but different phases.
- Due to the Sparse Double Descent
- Add noise → fit noise.



*Figure 10.* Performance of ResNet-18 on CIFAR-10 with  $\epsilon = 20\%$  when retrained from either the original initialization (lottery tickets), or a random reinitialization. Reinitialization results sometimes surpass lottery results.

# CHEX: CHannel EXploration for CNN Model Compression

CVPR'22  
Zejiang Hou, et al.



# Motivation

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- Structured Pruning or Dense-to-Sparse training.
- Training from scratch
- Prune and regrow the channels throughout the training process.
  - tackle the channel pruning problem via a well-known column subset selection (CSS) formulation

# Introduction

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- Previous pruning:
  - pre-training a large model until convergence,
  - pruning a few unimportant channels by the pre-defined criterion
  - finetuning the pruned model to restore accuracy.
- long training time
- In this work, they dynamically adjust the importance of the channels via a **periodic pruning and regrowing process**
- allows the prematurely pruned channels to be recovered and prevents the model from losing the representation ability early in the training process.

# Overview

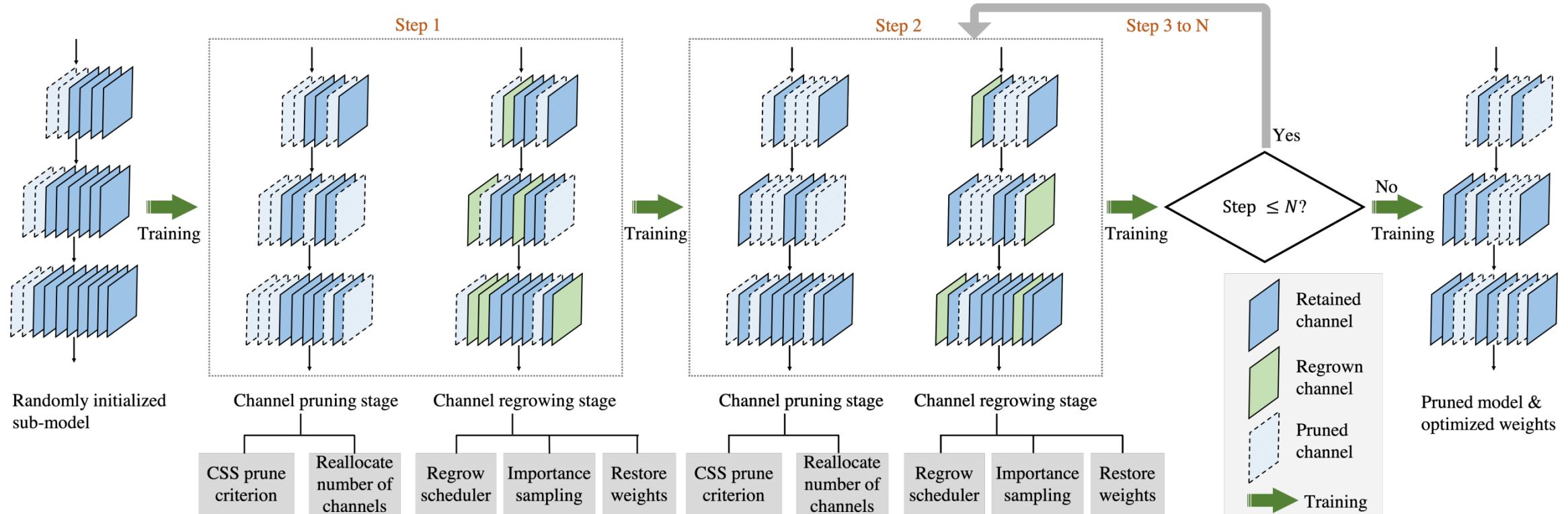


Figure 2. An illustration of our CHEX method, which jointly optimizes the weight values and explores the sub-model structure in one training pass from scratch. In CHEX, both retained and regrown channels in the sub-model are active, participating in the training iterations.

# Pruning Stage1: Reallocate number of channels

- learnable scaling factors in batch normalization (BN).
- ranking all scaling factors in descending order and preserving the top  $1 - S$  percent of the channels.

# Pruning Stage2: CCS-Criterion

- Leverage score
- Information of N-th row.

A handwritten note showing the following equations:

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

对方子  $\begin{bmatrix} m_{11} & m_{12} \end{bmatrix} = u_{11} \cdot \begin{bmatrix} v_{11} & v_{12} \end{bmatrix} + u_{12} \cdot \begin{bmatrix} v_{21} & v_{22} \end{bmatrix}$

知乎 @Cheng Wang

- $U_{11}$  and  $U_{12}$  present the importance of the row with  $m_{11}, m_{12}$ .

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## Algorithm 2: CSS-based channel pruning.

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- 1 **Input:** Model weights  $\mathbf{w}^l$ ; pruning ratios  $\kappa^l$  ;
  - 2 **Output:** The pruned layer  $l$  ;
  - 3 Compute the number of retained channels  
 $\tilde{C}^l = \lceil (1 - \kappa^l) C^l \rceil$  ;
  - 4 Compute the top  $\tilde{C}^l$  right singular vectors  $\mathbf{V}_{\tilde{C}^l}^l$  of  $\mathbf{w}^l$  ;
  - 5 Compute the leverage scores for all the channels in layer  $l$   
 $\psi_j^l = \|[\mathbf{V}_{\tilde{C}^l}^l]_{j,:}\|_2^2$  for all  $j \in [C^l]$  ;
  - 6 Retain the important channels identified as  
 $\mathcal{T}^l = \text{ArgTopK}(\{\psi_j^l\}; \tilde{C}^l)$  ;
  - 7 Prune channels  $\{\mathbf{w}_{:,j}^l, j \notin \mathcal{T}^l\}$  from layer  $l$  ;
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# Regrow Stage1: scheduler of number of regrown channels

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- Cosine decay scheduler to **gradually reduce** the number of regrown channels

$$\delta_t = \frac{1}{2} \left( 1 + \cos \left( \frac{t \cdot \pi}{T_{\max}/\Delta T} \right) \right) \delta_0$$

where  $\delta_0$  is the initial regrowing factor,  $T_{\max}$  denotes the total exploration steps, and  $\Delta T$  represents the frequency to invoke the pruning-regrowing steps.

## Regrow Stage2: determine the channels to regrow

- Orthogonal projection formula:

$$\epsilon_j^l = \|\mathbf{w}_j^l - \mathbf{w}_{\mathcal{T}^l}^l (\mathbf{w}_{\mathcal{T}^l}^{l^T} \mathbf{w}_{\mathcal{T}^l}^l)^{\dagger} \mathbf{w}_{\mathcal{T}^l}^{l^T} \mathbf{w}_j^l\|_2^2. \quad (2)$$

- A **higher orthogonality value** indicates that the channel is harder to approximate by others, and may have a better chance to be retained in the CSS pruning stage of the future steps.
- Be sampled with a relatively **higher probability**

# Regrow Stage3: assign weight

- *Most recently used* (MRU) parameters, which are the last values before they are pruned

# Overall algorithm

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**Algorithm 1:** Overview of the CHEX method.

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- 1 **Input:** An  $L$ -layer CNN model with weights  
 $\mathbf{W} = \{\mathbf{w}^1, \dots, \mathbf{w}^L\}$ ; target channel sparsity  $S$ ; total training iterations  $T_{\text{total}}$ ; initial regrowing factor  $\delta_0$ ; training iterations between two consecutive steps  $\Delta T$ ; total pruning-regrowing steps  $T_{\text{max}}$ ; training set  $\mathcal{D}$  ;
- 2 **Output:** A sub-model satisfying the target sparsity  $S$  and its optimal weight values  $\mathbf{W}^*$ ;
- 3 Randomly initialize the model weights  $\mathbf{W}$ ;
- 4 **for** *each training iteration*  $t \in [T_{\text{total}}]$  **do**
- 5     Sample a mini-batch from  $\mathcal{D}$  and update the model weights  $\mathbf{W}$  ;
- 6     **if**  $\text{Mod}(t, \Delta T) = 0$  *and*  $t < T_{\text{max}}$  **then**
- 7         Re-allocate the number of channels for each layer in the sub-model  $\{\kappa^l, l \in [L]\}$  by Eq.(4) ;
- 8         Prune  $\{\kappa^l C^l, l \in [L]\}$  channels by CSS-based pruning in Algorithm 2 ;
- 9         Compute the channel regrowing factor by a decay scheduler function ;
- 10         Perform importance sampling-based channel regrowing in Algorithm 3 ;

# Results

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Method	PT	FLOPs	Top-1	Epochs	Method	PT	FLOPs	Top-1	Epochs					
<b>ResNet-18</b>														
PFP [45]	Y	1.27G	67.4%	270	GBN [82]	Y	2.4G	76.2%	350					
SCOP [71]	Y	1.10G	69.2%	230	LeGR [4]	Y	2.4G	75.7%	150					
SFP [24]	Y	1.04G	67.1%	200	SSS [35]	N	2.3G	71.8%	100					
FPGM [26]	Y	1.04G	68.4%	200	TAS [9]	N	2.3G	76.2%	240					
DMCP [16]	N	1.04G	69.0%	150	GAL [48]	Y	2.3G	72.0%	150					
<b>CHEX</b>	N	1.03G	<b>69.6%</b>	250	Hrank [46]	Y	2.3G	75.0%	570					
<b>ResNet-34</b>														
Taylor [62]	Y	2.8G	72.8%	-	Taylor [62]	Y	2.2G	74.5%	-					
SFP [24]	Y	2.2G	71.8%	200	C-SGD [6]	Y	2.2G	74.9%	-					
FPGM [26]	Y	2.2G	72.5%	200	SCOP [71]	Y	2.2G	76.0%	230					
GFS [79]	Y	2.1G	72.9%	240	DSA [63]	N	2.0G	74.7%	120					
DMC [12]	Y	2.1G	72.6%	490	CafeNet [69]	N	2.0G	76.9%	300					
NPPM [11]	Y	2.1G	73.0%	390	<b>CHEX-1</b>	N	2.0G	<b>77.4%</b>	250					
SCOP [71]	Y	2.0G	72.6%	230	<b>ResNet-50</b>									
CafeNet [69]	N	1.8G	73.1%	300	SCP [37]	N	1.9G	75.3%	200					
<b>CHEX</b>	N	2.0G	<b>73.5%</b>	250	Hinge [44]	Y	1.9G	74.7%	-					
<b>ResNet-101</b>														
SFP [24]	Y	4.4G	77.5%	200	AdaptDCP [89]	Y	1.9G	75.2%	210					
FPGM [26]	Y	4.4G	77.3%	200	LFPC [23]	Y	1.6G	74.5%	235					
PFP [45]	Y	4.2G	76.4%	270	ResRep [8]	Y	1.5G	75.3%	270					
AOFP [7]	Y	3.8G	76.4%	-	Polarize [88]	Y	1.2G	74.2%	248					
NPPM [11]	Y	3.5G	77.8%	390	DSNet [41]	Y	1.2G	74.6%	150					
DMC [12]	Y	3.3G	77.4%	490	CURL [56]	Y	1.1G	73.4%	190					
<b>CHEX-1</b>	N	3.4G	<b>78.8%</b>	250	DMCP [16]	N	1.1G	74.1%	150					
<b>CHEX-2</b>	N	1.9G	<b>77.6%</b>	250	MetaPrune [52]	N	1.0G	73.4%	160					
					EagleEye [40]	Y	1.0G	74.2%	240					
					CafeNet [69]	N	1.0G	75.3%	300					
					<b>CHEX-2</b>	N	1.0G	<b>76.0%</b>	250					

(a)