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Correlation and Contagion: Measuring Systematic Risk with Semiparametric Methods

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ABSTRACT

The ability to adequately measure systematic risk in financial markets is a necessity for policy-makers to craft prudent and pragmatic regulation. This thesis introduces two novel semiparametric methods of quantifying systematic risk that do not rely on unrealistic distributional assumptions. These models are shown to significantly outperform commonly implemented parametric models in the measurement of downside systematic risk.

DECLARATION

I declare that the work presented in this Honours thesis is, to the best of my knowledge and belief, original and my own work, except as acknowledged in the text, and that material has not been submitted, either in whole or in part, for a degree at this or any other university.

S.J. Morrison

Stuart Morrison - 7 November 2016

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Two roads diverged in a wood, and I-

I took the one less traveled by,

And that has made all the difference.

—Robert Frost, *The Road Not Taken* (18-20)

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LIST OF ABBREVIATIONS, ACRONYMS AND INITIALISMS

AIC Akaike Information Criterion

ARMA Autoregressive Moving Average

ASX Australian Stock Exchange

BIC Bayesian Information Criterion

CAViaR Conditional Autoregressive Value at Risk

DGP Data Generating Process

EDF Empirical Distribution Function

GARCH Generalised Autoregressive Conditional Heteroskedasticity

i.i.d. independently and identically distributed

IFM Inference from Margins

MLE Maximum Likelihood Estimation

MSE Mean Squared Error

OLS Ordinary Least Squares

SAV-GARCH Symmetric Absolute Value GARCH

USD United States Dollars

VaR Value at Risk

1. INTRODUCTION

1.1 Motivation

Critical moments in recent economic history have been characterised by systematic financial instability and marked by the prevalence of financial crises. These crises led to periods of increased systematic risk that affected not only primary financial institutions such as banks but also integrated components of the financial system such as money markets and payment systems freezing much of the global economy. de Bandt and Hartmann (2000) note that global financial systems are more susceptible to systematic risk compared to other global markets due to the unified network of services between financial institutions.

The systematic risk literature examines the hazard precipitated by financial crises through the study of financial contagion. Financial contagion is defined in numerous ways in the literature. As an example, Forbes and Rigobon (2002) state that financial contagion is manifest in a significant increase of stochastic dependence between international financial markets during a time of financial crisis. Hartmann *et al.* (2004) adopt a similar definition but look instead at the increase in systematic dependence in a portfolio of financial assets in one financial market. We accept both definitions as equally important in the study of systematic risk and examine the risk of financial contagion both within and between financial markets accordingly.

Because financial contagion indicates an increase in systematic risk, even a financial institution with a well-diversified portfolio may still be overexposed or under-capitalised to excessive risk. The social cost to this exposure is often much larger than the private cost to the exposed institution creating significant room for a policymaker to improve expected social benefit by crafting policies such as capital adequacy requirements (de Bandt and Hartmann, 2000). Thus, the concept of financial contagion is an important issue for regulators and policymakers to comprehend to create financial policy and the ability to adequately quantify and measure the exposure to systematic risk is a necessity in this pursuit.

In practice, commonly used econometric techniques often have severe drawbacks in the accurate measurement of financial risk. Frequently implemented parametric methods often rely on unrealistic assumptions and can lead to a costly underestimation of downside risk (Chen *et al.*, 2009). Similarly, regularly employed semiparametric and nonparametric methods are often inadequate in estimating this downside risk in large financial portfolios (Panchenko, 2004). The specific limitations of these techniques will be discussed in the literature review in Section 2.

Hence, the primary motivations for this thesis are twofold; the social benefit gained in undertaking prudent systematic risk management of financial markets as well as the improvement of the inadequate tools currently employed in risk analysis. For a regulator or policymaker to make an informed decision in regards to systematic risk policy, they must be able to quantify the risk exposure accurately. This thesis introduces two novel methods of estimating systematic risk in financial applications that forgo unrealistic assumptions that are currently employed in the literature and in practice.

1.2 Measuring Systematic Risk

In this section we develop the relevant theory of the measurement of systematic risk. Systematic risk is quantified by estimating the probability of coexceedance, which describes the conditional probability that one market will fall below a Value at Risk (VaR) threshold, given another market falling below a certain VaR threshold. For the returns of two financial markets or financial assets X_j and X_k the probability of coexceedance G in time t is given by

$$G^{jk}_{\tau,t} = Pr\left(X_{jt} < \mathrm{VaR}_{X_{jt}}(\tau) | X_{kt} < \mathrm{VaR}_{X_{kt}}(\tau)\right) \tag{1.1}$$

In financial applications, the VaR is a useful measure of risk as it describes a threshold in the distribution of returns such that there is a probability equal to τ of making that loss, that is

$$\begin{aligned} &Pr\left(X_{jt} < \mathrm{VaR}_{X_{jt}}(\tau)\right) = \tau \\ &\text{for } \tau \in [0,1] \end{aligned} \tag{1.2}$$

Thus, Equation 1.1 captures the downside systematic risk between two financial markets or two financial assets.

In modelling the probability of coexceedance, the required stochastic information is encompassed entirely in the rank, or dependence, structure of the univariate distribution functions and not in the realisations of the returns themselves. The probability of coexceedance can be expressed in terms of the univariate marginal probability distributions of the returns F_j and F_k, or

$$G^{jk}_{\tau,t} = Pr\left(F_j(x_{jt}) < \tau \middle| F_k(x_{kt}) < \tau\right) \tag{1.3}$$

If we have a d-dimensional portfolio of financial assets, it is possible to estimate the conditional probability upon the value of the remaining assets in the portfolio by the following equation

$$\begin{aligned} &G^{jk}_{\tau,t}|\mathcal{F}_t = Pr\left(F_j(x_{jt}) < \tau \middle| F_k(x_{kt}) < \tau, \mathcal{F}_t\right) \\ &\text{where } \mathcal{F}_t = \{F_i(x_{it}) = u_{it}\}^d_{i=1,i\neq j,k} \end{aligned} \tag{1.4}$$

Equivalently, Bayes Theorem yields

$$G^{jk}_{\tau,t}|\mathcal{F}_t = \frac{Pr\left(F_j(x_{jt}) < \tau, F_k(x_{kt}) < \tau \middle| \mathcal{F}_t\right)}{Pr\left(F_k(x_{kt}) < \tau \middle| \mathcal{F}_t\right)} \tag{1.5}$$

The introduction of conditioning variables allows for a rich and dynamic estimation of exposure to systematic risk conditionally on the health of other financial markets. This allows the estimation methods we introduce to be used during tranquil financial times as well as times of financial crisis to demonstrate the additional systematic risk that markets are exposed to due to financial contagion. Although it is possible to include a wide variety of economic objects into the conditioning portfolio, in this thesis, we will only include the values of the remaining financial objects in the portfolio for our analysis.

1.3 Contribution

The core contribution of this thesis is the introduction two novel methodologies to measure systematic risk in financial portfolios through the modelling of the probability of coexceedance as per Equation 1.5. Motivated by the social benefit of prudent and pragmatic regulatory requirements for financial institutions, the approaches introduced in this thesis do not rely on any unrealistic distributional assumptions to ensure that downside risk is not misspecified.

The initial step in the introduced method is to estimate the time-varying distributions of returns with quantile regression techniques. Based on these estimates, the first method we introduce uses nonparametric regression techniques where the portfolio densities are modelled with kernel density functions in order to estimate the conditional probability of coexceedance. We expect this kernel-based method to be accurate in smaller portfolios but it will suffer from the curse of dimensionality as the data requirements from this estimation technique increase exponentially in the size of the portfolio.

Recognising that the first model we introduce is not scalable to high dimensional portfolios, the second method we introduce models the probability of coexceedance using logit regression techniques. By using the logit regression as the tool for the estimation of the relevant probabilities we are able to expand the size of the portfolio to higher dimensions as data requirements for this logit-based model increase linearly in the size of the portfolio while still forgoing strong distributional assumptions.

We apply these methods to two different datasets. The first portfolio contains the returns of equity market indices of Brazil, Chile, Argentina and Mexico over a fifteen year period. We construct the introduced models as well as commonly used parametric copula models for a comparison of performance. In a test of predicting instances of coexceedance over the sample period, the introduced methodologies significantly outperform the parametric models in the accuracy of prediction. The logit-based method we introduce shows approximately a 25% lower Mean Squared Error (MSE) of prediction compared to the parametric copula model. The kernel-based method we introduce demonstrates approximately a 55% decrease in the MSE of prediction compared to the parametric copula model. These results show that the models introduced in this thesis are far superior to commonly employed parametric copula models in the accuracy of measuring systematic risk in a portfolio of financial indices.

We also apply the logit-based method to a larger portfolio of stocks listed on the Australian Stock Exchange (ASX) to demonstrate its use in a higher dimensional application. We perform the same test of prediction of coexceedance and compare performance again to a parametric copula model. The logit-based model shows up to a 60% decrease in MSE of prediction in this test compared to the parametric copula model. This further demonstrates the ability for the logit-based model to be scalable in dimensions to larger portfolios while maintaining the accuracy in estimating systematic risk.

The application of the introduced methodologies to these two datasets shows the accuracy of the measurement of systematic risk in both small and large financial portfolios. Thus, both of the introduced methodologies are incredibly useful tools for policymakers and regulators to implement in the construction of prudent financial regulation.

1.4 Outline

The remainder of this thesis is organised as follows. Firstly, Section 2 outlines a detailed review of the current literature in the measurement of systematic risk. Section 3 sets out the methodology of the introduced estimation procedures. Section 4 shows the application of the introduced methodologies to the portfolio of the equity indices of the South American countries and details the relative tests of prediction performance. Finally, Section 5 demonstrates the logit-based estimation methodology on a large portfolio of Australian securities.

2. LITERATURE REVIEW

The literature on measuring systematic risk and financial contagion can be divided into three distinct sections. The first section comprises of early research into financial contagion which was concerned with the estimation of association measures between financial objects and the manner in which they change during times of financial volatility. Modern approaches to modelling the risk associated with financial contagion evolved out of this literature and took two distinct paths. The first of which implements parametric models to estimate risk during times of financial crisis in order to examine the extent of financial contagion. The second area of modern literature utilises nonparametric and semiparametric models for the same purpose. As many characteristics of systematic risk in financial markets were established in early research, it is important to view the evidence provided from this foundational literature. The remainder of this section will address each segment of the literature in turn.

2.1 Early Association Methods

The literature regarding dependence modelling has been concerned with systematic risk in portfolios in financial applications for many decades. Earlier works such as Longin and Solnik (1995) examined whether association measures such as linear correlation significantly changed through time. Longin and Solnik (1995) find evidence that linear correlation measures between the returns of several national equity indices increase during times of increased financial volatility. It should be noted that the use of linear correlations is now an outdated approach as it relies on the strong assumption of a joint elliptical distribution being the true data distribution, however Longin and Solnik's (1995) primary result is now a common motif in systematic risk literature and it is generally accepted that returns of financial assets are more strongly correlated in volatile financial times than in tranquil times.

In a similar vein, Ang and Chen (2002) estimate association measures of returns in different portfolios of equities listed on stock exchanges in the United States of America. Ang and Chen (2002) focus on conditional association measures, such as estimating linear correlation measures past a certain VaR threshold in the distribution of the returns in the portfolio. They discover that the linear correlation coefficient of returns below the conditional mean of each equity's return distribution is up to 30% larger than the linear correlation measure above the conditional mean. This result reiterates the evidence from Longin and Solnik (1995) of stronger stochastic dependence between financial assets in volatile times as well introducing the idea that there exists a large systematic downside risk in portfolios of financial assets that may not be adequately measured by global correlation measures. Although this finding came long before the prevalence of parametric copula models, the result of an asymmetric correlation coefficient should be a caution to any econometric practitioner who aims to fit a symmetrical parametric distribution over a portfolio of assets.

The inadequacy of directly implementing global correlation measures has led practitioners to examine and estimate the dependence structure between the returns of financial assets with parametric and nonparametric methods. Parametric estimation is often associated with estimating copula functions to model the global dependence structure. The semiparametric and nonparametric literature introduces many techniques but particularly emphasises methods such as measuring discrete quantiles of the distributions of financial returns directly through quantile regression techniques or through nonparametric techniques such as empirical distribution functions.

2.2 Parametric Methods

Patton (2006) provides a connection between the methodology of dependence modelling to the methodology of parametric estimation using copula functions. A copula is a statistical model that couples univariate distribution functions to create a multivariate distribution function. The copula function only contains information on the rank between random variables and does not depend on the realisations of the random variables themselves.

Many copula models have parameters that can be derived analytically from association measures such as monotonic correlation and tail dependence measures. Patton (2006) exploits this result to model the dependence structure between a portfolio of exchange rates, using a parametric copula model. The paper focuses on the estimation of dependence in the upper and lower tails of the respective distributions and an illustration of a global dependence structure is not shown. The copula parameters are estimated in two distinct periods of both financial tranquillity and financial volatility. The estimation results show a larger tail dependence parameter in the volatile financial period.

Patton (2009) provides several examples of employing copula models based on elliptical distribution functions, such as the Gaussian copula and the Student-t copula, to model financial data. These simpler models have widespread use in applied risk management applications; the use of the Gaussian copula was used to manage the risk of collateralized debt obligations in the time directly preceding the global financial crisis (Li, 2000).

More recent works on parametric copulas have involved the factorisation of copula parameters to create copula distributions known as factor copulas. Many manifestations of factor copulas were introduced in applications of measuring dependence structures with asymmetric tails, as the additional parametrisation allows the distributional form to be more flexible than the commonly used symmetrical curves. However, the benefit of asymmetries is burdened with the increase of parameters on a scale that in many instances makes fitting factor copulas too computationally intensive to be of much use in a financial setting. Very recent papers now aim to create factor copulas that are tractable in large portfolio applications.

Oh and Patton (2015) introduce a class of factor copulas that are parametrised linearly so as to be tractable in larger dimensions. Oh and Patton (2015) apply this class of copulas to a portfolio of securities on listed on the Standard and Poor's Top 100 index, the largest one hundred firms by market capitalisation in the United States. The copula model is shown to fit the global asymmetries of the portfolio well, however the estimation procedure used is that of simulated method of moments which is not a practical tool for an applied risk manager due to its computational intensity.

Bartels and Ziegelmann (2016) use the linearly factored copula models implemented in Oh and Patton (2015) but introduce a maximum likelihood estimation technique. Bartels and Ziegelmann (2016) use the estimated factor copula to compare goodness-of-fit based on a criterion of frequency of the financial assets breaking the threshold of estimated VaR. This manifestation of the linearly factored models is shown to outperform commonly used parametric copula models while remaining computational tractable. Further inference on systematic risk using this approach has not yet appeared in the literature.

2.3 Nonparametric and Semiparametric Methods

Commonly known semiparametric and nonparametric methods that rely on kernel densities are notably sparse in systematic risk literature. As Panchenko (2004) notes, these methods suffer from the curse of dimensionality and are practically unusable for a diversified portfolio in applied risk management. However, they remain useful tools in small portfolio applications, despite paucity of applications in recent research.

Engle and Manganelli (2004) make a significant contribution to the risk analysis literature by introducing the Conditional Autoregressive Value at Risk (CAViaR) quantile regression model. The CAViaR model allows accurate estimation of time-varying conditional quantiles while forgoing distributional assumptions. The CAViaR model is used extensively in time series analysis due to its precise estimation of asymmetries in conditional financial distributions.

Cappiello *et al.* (2008) use the CAViaR model to estimate the asymmetric distributions of several currencies in order to measure any increase in downside risk in currency valuations in global currency markets after the introduction of the Euro. Cappiello *et al.* (2008) are able to measure the asymmetric distributions of currency returns accurately and find that any changes in the systematic risk between currency pairs is not correlated with the introduction of the Euro and is likely caused by other economic factors.

Bouyé and Salmon (2009) use non-linear quantile regression techniques to estimate the univariate conditional distributions of a portfolio of currency exchange rates and then fit a parametric copula to model the multivariate dependence between the items in the portfolio. Bouyé and Salmon (2009) fit several commonly used parametric copula models on the multivariate distribution and compare results based on goodness-of-fit tests.

Cappiello *et al.* (2014) again utilise the CAViaR model to measure coexceedances between four Latin American countries' equity indices. The probabilities of coexceedance are estimated using a Ordinary Least Squares (OLS) regression technique on the instance of coexceedance to obtain a consistent estimate of the dependence structure at discrete intervals of the distribution of the returns. The estimation procedure is carried out in both tranquil and volatile financial times and statistical tests are implemented to test for the difference in systematic risk between these two time periods. By this method Cappiello *et al.* (2014) are able to measure systematic risk without distributional assumptions, though the measure is an average measure of the entire time period and cannot be used for inferential analysis.

While we recognise the potential for accurate systematic risk modelling by employing recently developed factor copulas, the literature has not yet advanced to a point where such models are practical to implement in systematic risk applications. Instead, this thesis applies methodology that uses semiparametric techniques to forgo distributional assumptions in modelling financial returns distributions and introduces new semiparametric methods of measuring systematic risk in a financial portfolio.

3. METHODOLOGY

This section outlines the methodology of the novel estimation procedures. Firstly, we describe the process of specifying the appropriate time-varying distributions for the returns series and the quantile regression models implied by the chosen time series model. Subsequently, we show the estimation steps for the novel estimators, the kernel-based model and the logit-based model, respectively.

We use quantile regression techniques to estimate the probability integral transform of the conditional returns distributions in order to specify the introduced estimators as functions of the stochastic information of the portfolio. Using this estimate of the rank structure, we construct the first novel method, the kernel-based model, that evaluates the probability of coexceedance as per Equation 1.5 with the conditioning variables modelled by kernel density functions. The second novel method, the logit-based model, applies the same estimate of the rank structure and quantifies the conditional probability of coexceedance as the ratio of two logit regression models.

3.1 Time Series Specification and Quantile Regression

The initial step of the estimation procedure requires the specification of appropriate time series models for the items in the portfolio in order to specify the suitable quantile regression models. Financial returns generally have conditional distributions with time-dependent moments, especially the mean and variance, which must be modelled appropriately (Engle, 1982). Commonly used time series specifications for modelling these characteristics include Autoregressive Moving Average (ARMA) models and Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models, but there remains a wide literature on extensions of these models. As an example, Orhan and Kksal (2012) provides a measured list of extensions to the GARCH model for VaR estimation, though there exist many other collections of similar time series specifications.

The ARMA model is a useful tool for modelling distributions with autocorrelated means which is often displayed in financial returns data. The specification for an ARMA(P, Q) model for a random time series variable X is shown as

$$\begin{aligned} X_t &= \alpha_0 + \sum_{p=1}^{P} \alpha_p x_{t-p} + \epsilon_t + \sum_{q=1}^{Q} \phi_q \epsilon_{t-q} \\ &\text{where } \epsilon \overset{iid}{\sim} \mathcal{N}(0,1) \end{aligned} \tag{3.1}$$

GARCH specifications parametrise both the mean and the variance of the conditional distribution, which is appropriate for modelling returns data that display volatility. The specification of a ARMA(P, Q)-GARCH(M, N) time series model is

$$\begin{aligned} X_t &= \alpha_0 + \sum_{p=1}^{P} \alpha_p x_{t-p} + \epsilon_t + \sum_{q=1}^{Q} \phi_q \epsilon_{t-q} \\ &\text{where } \epsilon_t = \sigma_t \eta_t,\ \eta \overset{iid}{\sim} \mathcal{N}(0,1) \\ \sigma_t^2 &= \delta_0 + \sum_{m=1}^{M} \delta_m \epsilon_{t-m}^2 + \sum_{n=1}^{N} \xi_{t-n} \sigma_{t-n}^2 \end{aligned} \tag{3.2}$$

Considerable evidence suggests that financial returns are more volatile in both times of extremely high and extremely low financial performance (Hong *et al.*, 2007). This motivates the specification of the Symmetric Absolute Value GARCH (SAV-GARCH) model as a common extension of ARMA-GARCH models that has the following specification

$$\begin{aligned} X_t &= \alpha_0 + \alpha_1 x_{t-1} + \epsilon_t \\ &\text{where } \epsilon_t = \sigma_t \eta_t,\ \eta \overset{iid}{\sim} \mathcal{N}(0,1) \\ \sigma_t &= \delta_0 + \delta_1 |x_{t-1}| + \delta_2 \sigma_{t-1} \end{aligned} \tag{3.3}$$

We choose the appropriate model specification based on the Box-Jenkins approach to model selection that includes diagnostics such as information criteria, parsimony and the significance of parameters (Jenkins and Box, 1970). Information criteria are log-likelihood diagnostic tools which indicate goodness-of-fit, penalised for over parametrisation and we employ information criteria specifications of Akaike Information Criterion (AIC) and Bayesian

Information Criterion (BIC) which have the functions of

$$\begin{aligned} \text{AIC} &= 2k - 2 \cdot \mathcal{L} \\ \text{BIC} &= k \cdot \log(T) - 2 \cdot \mathcal{L} \end{aligned} \tag{3.4}$$

Where $\mathcal{L}$ is the log-likelihood of the time series model, T denotes the sample size and k is the number of parameters in the model. For a goodness-of-fit evaluation, the lowest information criteria score between models is desirable.

ARMA and GARCH models require symmetrical and independently and identically distributed (i.i.d.) innovations with zero mean for the time series specification to be identified. Even extensions of the GARCH model that introduce asymmetries in volatility still require a symmetrical conditional distributions. By modelling quantiles directly through quantile regression techniques we are able to forgo this assumption of a symmetrical distribution.

The conditional quantile function q_t^X is equal to the inverse distribution function of X, $F_{Xt}^{-1}(\cdot)$, or

$$F_{Xt}^{-1}\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-1}\right) = q_t^X\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-1}\right) \tag{3.5}$$

Where $\boldsymbol{\beta}_\tau^X$ denotes the parameters of the quantile regression model at level $\tau \in [0,1]$ and $\mathcal{I}_{t-1}$ denotes the information set at time t. The parameters of the chosen quantile regression framework can be estimated by minimising the following check loss function (Koenker, 2005, p 15)

$$\begin{aligned} \hat{\boldsymbol{\beta}}_\tau^X &= \arg\min_{\boldsymbol{\beta}_\tau^X} T^{-1} \sum_{t=1}^{T} \left(\tau - \mathbb{1}\left(x_t < q_t^X\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-1}\right)\right)\right) \cdot \left(x_t - q_t^X\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-1}\right)\right) \\ \text{Where} \quad \mathbb{1}\left(x_t < q_t^X\left(\tau|\boldsymbol{\beta}_X^\tau, \mathcal{I}_{t-1}\right)\right) &= \begin{cases} 1 & \text{if } x_t < q_t^X\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-1}\right) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{3.6}$$

The quantile regression technique is effective in modelling the conditional distribution of returns as the only distributional assumption is that the chosen time series model is the true Data Generating Process (DGP) of the time series. This need not be a strong assumption as there are countless available time series specifications that can provide a good fit of the data.

Based on the quantile regression form, we estimate the probability integral transform of the univariate time series. The following equation, evaluated at the estimates of the quantile regression parameters, applies the probability integral transformation while forgoing any strong distributional assumptions which may lead to the misspecification of risk in the individual returns series.

$$F_{Xt}\left(x_t \middle| \hat{\boldsymbol{\beta}}_\tau^X, \mathcal{I}_{t-1}\right) = \int_0^1 \mathbb{1}\left(q_t^X\left(\tau \middle| \hat{\boldsymbol{\beta}}_\tau^X, \mathcal{I}_{t-1}\right) < x_t\right) d\tau \tag{3.7}$$

Employing the quantile regression estimation procedure allows to estimate the probabilistic information of the univariate distributions of returns without strong distributional assumptions. Using this information we model the rank structure of the portfolio of returns using the novel methodologies.

3.2 Kernel-Based Model

Based on the semiparametric estimation of the univariate probability integral transformations, this section introduces the first novel method of this thesis, the kernel-based model, to estimate the rank structure between the portfolio of returns. We define specifications similar to that of the Nadaraya-Watson estimator to model the appropriate probabilities to construct the probability of coexceedance as per Equation 1.5.

3.2.1 Construction of the Kernel-Based Model

To estimate the probability of coexceedance we define two kernel regression models, the "joint" and the "marginal" models, hereafter denoted by the scripts J and M respectively. For example, if we are interested in estimating the probability of coexceedance for financial markets X_j and X_k we define the following dependent variables

$$\begin{aligned} Y_{Jt}^{jk} &= \mathbb{1}\left(F_j(x_{jt}) < \tau, F_k(x_{kt}) < \tau\right) \\ Y_{Mt}^{k} &= \mathbb{1}\left(F_k(x_{kt}) < \tau\right) \end{aligned} \tag{3.8}$$

To estimate the probability of coexceedance, we estimate the conditional expectation of both joint and marginal variables using nonparametric regression techniques that model the densities of the conditioning variables with kernel density functions. For the joint model, we give the following specification for the conditional expectation

$$
\begin{aligned}
E\left[Y_{Jt}^{jk}\middle|\mathcal{F}_t\right] &= \frac{\sum_{t=1}^{T}\left[\mathbb{1}\left(F_j(x_{jt})<\tau, F_k(x_{kt})<\tau\right)\prod_{i=1,i\neq j,k}^{d} K_i\left(\frac{u_i - F_i(x_{it})}{h_i}\right)\right]}{\sum_{t=1}^{T}\left[\prod_{i=1,i\neq j,k}^{d} K_i\left(\frac{u_i - F_i(x_{it})}{h_i}\right)\right]} \\
&= Pr\left(F_j(x_{jt})<\tau, F_k(x_{kt})<\tau\middle|\mathcal{F}_t\right)
\end{aligned}
\tag{3.9}
$$

And analogously for the marginal model, the specification is

$$
\begin{aligned}
E\left[Y_{Mt}^{k}\middle|\mathcal{F}_t\right] &= \frac{\sum_{t=1}^{T}\left[\mathbb{1}\left(F_k(x_{kt})<\tau\right)\prod_{i=1,i\neq j,k}^{d} K_i\left(\frac{u_i - F_i(x_{it})}{h_i}\right)\right]}{\sum_{t=1}^{T}\left[\prod_{i=1,i\neq j,k}^{d} K_i\left(\frac{u_i - F_i(x_{it})}{h_i}\right)\right]} \\
&= Pr\left(F_k(x_{kt})<\tau\middle|\mathcal{F}_t\right)
\end{aligned}
\tag{3.10}
$$

Where K is a kernel density function and h is a bandwidth parameter that we describe with further detail in Appendix A. As this method utilises kernel density functions to model the conditioning variables we expect this estimator to be especially accurate in small portfolio applications (Panchenko, 2004). These specifications have the form of the Nadaraya-Watson estimator (Davidson and Mackinnon, 2004, p.689) that is used for nonparametric regression, though in this specific application this estimator is a semiparametric object as we estimate the marginal distribution functions semiparametrically.

3.2.2 Kernel-Based Prediction of Coexceedances

In this section we introduce the specification of the first novel model of this thesis, the kernel-based model. Using the specifications of the conditional probabilities the kernel-based estimator of the conditional probability of coexceedance as per Equation 1.5 is

$$
G_{\tau,t}^{jk}|\mathcal{F}_t = \frac{\sum_{t=1}^{T}\left[\mathbb{1}\left(F_j(x_{jt})<\tau, F_k(x_{kt})<\tau\right)\prod_{i=1,i\neq j,k}^{d} K_i\left(\frac{u_i - F_i(x_{it})}{h_i}\right)\right]}{\sum_{t=1}^{T}\left[\mathbb{1}\left(F_k(x_{kt})<\tau\right)\prod_{i=1,i\neq j,k}^{d} K_i\left(\frac{u_i - F_i(x_{it})}{h_i}\right)\right]}
\tag{3.11}
$$

The use of kernel density functions allows the estimation of conditional probabilities to be incredibly accurate while remaining free of distributional assumptions. The shortcoming of this kernel-based model is that it will suffer from the curse of dimensionality, specifically it requires a sample size that increases of order 2 in dimensions. This kernel-based estimator should perform quite well in small portfolio applications but it quickly grows pragmatically impossible to implement as the portfolio size increases.

3.3 Logit-Based Model

We identify that the kernel-based estimator has the limitation of the curse of dimensionality which motivates us to introduce the second novel method of this thesis, the logit-based model, which measures the same conditional probabilities as above and is scalable into higher dimensional portfolios.

3.3.1 Construction of the Logit-Based Model

We construct the logit-based models with the same dependent variables in Section 3.2.1, however we apply a logit regression specifications for the conditional expectations of the joint and the marginal variables. The logit regression has two main advantages that motivates its use in this risk analysis application. Firstly, the data requirements are linear in parameters which makes it a practical tool in applied risk management where large and diversified institutions have portfolios of hundreds of dimensions. Secondly, it constrains the expectation of the dependent variable between 0 and 1 by construction which is a strong advantage over an alternative model, the linear probability model, which models the same variables with OLS regression techniques. The conditional expectations from the linear probability model are not necessarily bounded in $[0, 1]$ and spurious results may occur near the boundary. As the main examination of conditional probabilities is concerned with risk close to the lower boundary of the probability space, the logit model has a significant advantage over the linear probability model which may not be accurate in such circumstances.

The conditional expectations of the joint and the marginal variables with a logit specification with parameter vector $\boldsymbol{\nu}$ are given by

$$
\begin{aligned}
E\left[Y^{jk}_{Jt}\middle|\mathcal{F}_t\right] &= \Lambda(\mathbf{X}'_t\boldsymbol{\nu}^{jk}_J) \\
&= Pr\left(F_j(x_{jt}) < \tau, F_k(x_{kt}) < \tau\middle|\mathcal{F}_t\right) \\
E\left[Y^{k}_{Mt}\middle|\mathcal{F}_t\right] &= \Lambda(\mathbf{X}'_t\boldsymbol{\nu}^{k}_M) \\
&= Pr\left(F_k(x_{kt}) < \tau\middle|\mathcal{F}_t\right) \\
\text{where } \Lambda(\mathbf{X}'_t\boldsymbol{\nu}) &= \frac{\exp\left(\mathbf{X}'_t\boldsymbol{\nu}\right)}{\left(1+\exp\left(\mathbf{X}'_t\boldsymbol{\nu}\right)\right)}
\end{aligned}
\tag{3.12}
$$

Where the matrix $\mathbf{X}_t$ is a matrix of explanatory variables given by

$$
\mathbf{X}'_t = \begin{bmatrix}1 & \{F_i(x_{it}) = u_i\}^{d}_{i=1, i\neq j,k}\end{bmatrix}
\tag{3.13}
$$

We estimate the logit regression parameters with Maximum Likelihood Estimation (MLE); computationally, we maximise the log-likelihood function with respect to logit parameters where the maximisation problem is given by

$$
\begin{aligned}
\hat{\boldsymbol{\nu}} &= \arg\max_{\nu} \mathcal{L}(\boldsymbol{\nu}|\mathbf{X}) \\
&= \arg\max_{\nu} \sum_{t=1}^{T} \left[Y_t \cdot \log\left(\Lambda\left(\mathbf{X}'_t\boldsymbol{\nu}\right)\right) + (1-Y_t)\cdot\log\left(1-\Lambda\left(\mathbf{X}'_t\boldsymbol{\nu}\right)\right)\right]
\end{aligned}
\tag{3.14}
$$

3.3.2 Logit-Based Prediction of Coexceedances

In this section we introduce the specification of the second novel model of this thesis, the logit-based model. Based on the above logit specifications, we model the conditional probability of coexceedance as per Equation 1.5 as the ratio of the joint and marginal logit regressions, or

$$
G^{jk}_{\tau,t}|\mathcal{F}_t = \frac{\Lambda(\mathbf{X}'_t\boldsymbol{\nu}^{jk}_J)}{\Lambda(\mathbf{X}'_t\boldsymbol{\nu}^{k}_M)}
\tag{3.15}
$$

We do not show mathematically that this specification is a correct representation of the conditional probability of coexceedance, thus we introduce this model only as a heuristic measure and test robustness of the subsequent estimates by comparing it against other commonly used econometric techniques.

4. APPLICATION TO A SOUTH AMERICAN PORTFOLIO

We first apply the introduced methodologies to a portfolio of the equity indices of the South American countries Brazil, Chile, Argentina and Mexico. We choose these countries specifically because they are known as emerging markets that are shown to exhibit significant periods of volatility, even outside of times of financial crisis (Cappiello *et al.*, 2014).

The methodology is applied in the `R` programming language (R Core Team, 2016) and the relevant code is provided in an online GitHub repository[1]. We use the introduced models to make predictions of coexceedance within the sample of the data range and compare our results to the equivalent predictions of a parametric copula model to test the robustness of the result and relative performances of the models as well as misspecifications caused by the parametric restrictions. When applied to the data, both of the introduced methodologies significantly outperform the parametric copula model in a test of prediction of coexceedances based on MSE criterion.

4.1 Data

The data has a date range from 5 January 1988 to 26 August 2016 and is sourced from the Bloomberg Online Subscription Service. There are some constraints to the availability of data over this range, most prominently there is no data available for the equity index of Mexico before 1994. On combining the series, we are left with 4198 days on which all markets are open and all data are available, ranging over the period from 26 January 1994 to 3 September 2012.

[1] The repository for the relevant code is available at `https://github.com/Ecnmtrx/CorrelationPlusContagion`

4.1.1 Summary Statistics and Data Characteristics

We use the daily closing price of the equity index P_t and calculate returns R_t denominated in United States Dollars (USD), with the formula

$$R_t = \log\left(\frac{P_t \cdot C_t}{P_{t-1} \cdot C_{t-1}}\right) \tag{4.1}$$

Where C_t denotes the exchange rate of the respective national currency to USD. The returns formula shown in Equation 4.1 is a widely used approximation of financial returns as it allows the returns to be continuous over the entire real line, a necessity for the use of most statistical models. Daily returns are denominated out of the respective national currency so the value of the return can be directly compared to the returns of the other countries in the portfolio.

Table 1 shows summary statistics of the returns. All four countries in the portfolio report measures of sample skewness close to 0. As a rule of thumb measure, if the sample skewness is between $-1/2$ and $1/2$, the data can be thought to be approximately symmetrical. All four countries display evidence of excess kurtosis. Compared to a normal distribution which has a kurtosis of 3, the distributions of all four countries display a higher kurtosis, indicating that it is extreme values in the tails of the distributions that contribute most to the variation in the data.

Tab. 1: Summary Statistics for South American Daily Returns Data

	Brazil	**Chile**	**Argentina**	**Mexico**
Days Available	5096	5904	5096	4637
Min	-0.2895	-0.0941	-0.1568	-0.2133
Mean	0.0007	0.0006	0.0003	0.0003
Max	0.3450	0.1483	0.1704	0.2002
Skewness	0.3224	-0.01800	-0.1371	-0.0802
Kurtosis	14.8540	9.1076	8.3967	15.7072
Date Range between 05/01/1988 to 26/08/2016				

Source: Bloomberg. (2016) Bloomberg Professional. [Online]. Available at: Subscription Service (Accessed: 27 August 2016)

The leptokurtic distributions of returns post-rationalises the semiparametric technique of estimating the probability of coexceedance without distributional assumptions. The use of any method that relies on Gaussian assumptions would not account for the extreme values that excess kurtosis implies. Although these assumptions are prevalent in applied econometric risk analysis, such excess kurtosis will introduce a potentially costly underestimation of extreme downside returns. The excess kurtosis coupled with approximate symmetry of the data may motivate the use of methods based on the Student-t distribution, but a practitioner must still be weary of the distributional assumptions this imposes.

Throughout the time period considered there were several significant financial crises. Given a wide range of criteria, specific dates of financial crises are often debated in literature. As a heuristic indication of where to expect times of high financial volatility we use the definitions of financial crises from Cappiello *et al.* (2014) that are based on significant increases in correlation measures and are given by the following dates.

1. Tequila Crisis: November 1, 1994 to March 31, 1995
2. Asian Crisis: June 2, 1997 to December 31, 1997
3. Russian Crisis: August 3, 1998 to December 31, 1998
4. Argentinian Crisis: March 26, 2001 to May 15, 2001
5. Subprime Crisis: February 15, 2007 to March 30, 2007
6. Lehman Crisis: September 1, 2008 to October 31, 2008
7. Euro Crisis: August 1, 2011 to September 30, 2011

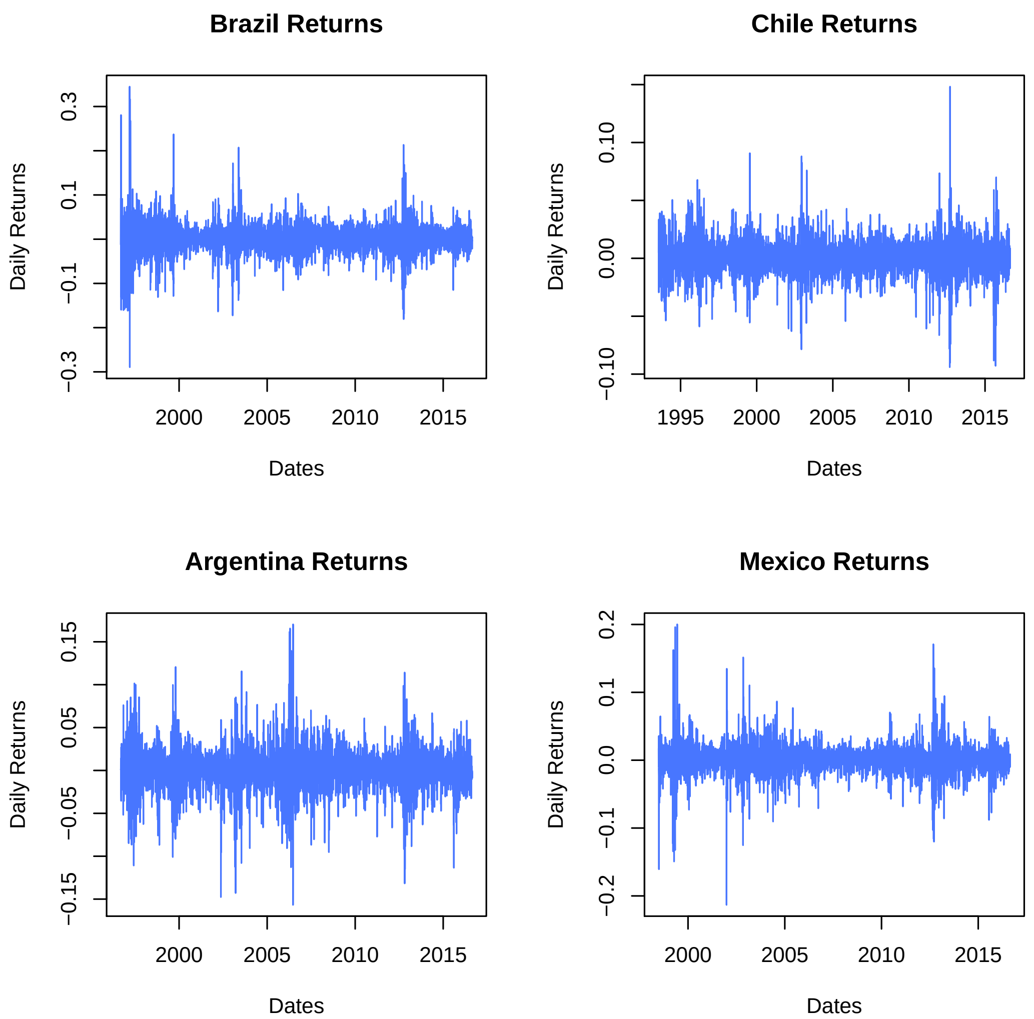
4.2 Estimation

Using the South American equity index returns, we perform the introduced estimation procedures. Firstly, we specify the univariate time series models and employ the relevant quantile regression techniques. Secondly, we estimate the logit regression parameters and lastly, we show the estimation of the probability of coexceedance with both the logit-based model and the kernel-based model.

4.2.1 Time Series Specification and Quantile Regression

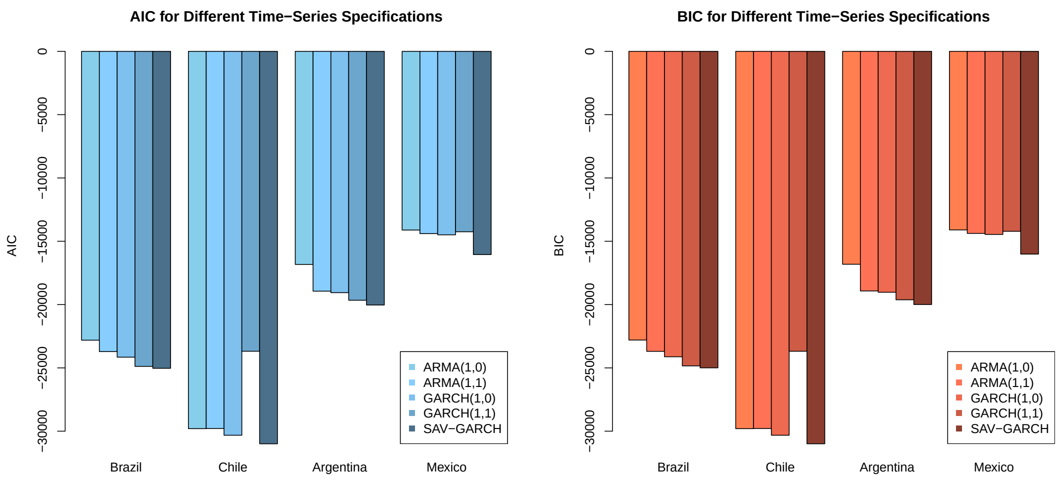
For each of the four countries in the South America portfolio, we obtain maximum likelihood estimates for several time series models. Figure 1 shows plots of the daily returns for the four countries' equity markets. A visual examination of the returns data shows initial evidence of volatility in all four returns series. The presence of volatility motivates the use of models such as ARMA-GARCH specifications that allow the volatility to be directly estimated, although models without volatility are also estimated for goodness of fit purposes.

Fig. 1: Daily Returns for the Equity Markets of Brazil, Chile, Argentina and Mexico



We estimate the following specifications: ARMA(1,0), ARMA(1,1), ARMA(1,0)-GARCH(1,0), ARMA(1,0)-GARCH(1,1) and SAV-GARCH and apply the Box-Jenkins approach for model selection. For all four countries, the SAV-GARCH specification has the lowest AIC and BIC, which indicates the best fit of the models that were implemented. Figure 2 shows a visual representation of the information criteria scores, with a full table of values available in Appendix B. We do not report the coefficients of the time series models at this stage in the estimation procedure as the estimation of the various time series model is used as a specification tool to obtain the appropriate quantile regression function for each country.

Fig. 2: Information Criteria for Time Series Specifications for the South American Portfolio



The time series specification of the SAV-GARCH model implies the CAViaR quantile regression specification to model the conditional distributions (Engle and Manganelli, 2004). We show a derivation of the CAViaR quantile regression model from the SAV-GARCH model in Appendix C. The CAViaR quantile regression has the form, for variable X

$$q_t^X\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-1}\right) = \beta_{0,\tau}^X + \beta_{1,\tau}^X x_{t-1} + \beta_{2,\tau}^X q_{t-1}^X\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-2}\right) - \beta_{1,\tau}^X\beta_{2,\tau}^X x_{t-2} + \beta_{3,\tau}^X|x_{t-1}| \quad (4.2)$$

The CAViaR specification models the conditional quantile at time t given the conditional quantile at time $t-1$. Thus, the CAViaR model allows for conditional quantiles to have an autocorrelated structure and to be a function of volatilities across time (Li, 2011). This autocorrelated form allows us to precisely estimate time-varying asymmetries in the conditional returns distribution in order to examine the underlying downside risk for a financial asset.

The inclusion of lags of the conditional quantiles in the CAViaR quantile regression model, which are themselves estimated from the quantile regression model, requires that we manually specify the value of the initial quantile for estimation purposes. For the estimation procedure we initialise each conditional quantile for $\tau \in [0, 1]$ with the τ-th unconditional quantile of the returns series for the respective variable. Robustness checks indicate that the value chosen for the initial quantile has little effect on the parameter estimates.

We estimate the parameters of the CAViaR model as per the check loss function in Equation 3.6. Table 2 shows the CAViaR parameter estimates for several quantile levels as an example of the parameter estimates. Most of the estimates are significant at the 1% level of significance, though we face a perplexing problem of non-global misspecification. The CAViaR specification is implied by the time series model, however the parameter estimates for β_0 for all countries are insignificant at most levels of τ but are significant around the median. Other parameters are also significant for some values of τ but not for others. Cappiello *et al.* (2014) also note the same issue in similar estimation technique, where there is an obvious misspecification issue, but not at the global level. Currently the literature is underwhelming in its approach to remedy this problem. As the CAViaR estimates are used as an input for the main analysis of coexceedances, we use all parameter estimates but acknowledge that further investigation is warranted.

The parameter estimates of the quantile regression function are asymptotically normal with a mean vector $\boldsymbol{\beta}_\tau^X$ with a variance covariance matrix $\mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{-1}$, where $\boldsymbol{\beta}_\tau^X$ is a vector of the true quantile regression parameter estimates and where $\mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{-1}$ is composed of matrices $\mathbf{A}$ and $\mathbf{B}$, defined as

$$\begin{aligned} \mathbf{A} &= \sum_{t=1}^{T} q_t^X\left(\boldsymbol{\beta}_\tau^X\right)(1 - q_t^X\left(\boldsymbol{\beta}_\tau^X\right))\mathbf{X}_t\mathbf{X}_t' \\ \mathbf{B} &= \sum_{t=1}^{T} f_{\rho_\tau}(0|\mathbf{X}_t)\mathbf{X}_t\mathbf{X}_t' \end{aligned} \tag{4.3}$$

Where $\mathbf{X}$ denotes the explanatory variables of the quantile regression formula and $f_{\rho_\tau}(0|\mathbf{X})$ denotes the conditional density of the quantile regression loss function evalued at 0 (Koenker, 2005, p.72). The variance-covariance matrix $\mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{-1}$ is cumbersome to estimate in practice; instead, we rely on bootstrapping techniques to find the standard errors of the quantile regression parameters.

We take $B = 750$ bootstrap samples of the data and estimate the CAViaR model parameters for each iteration. The variance-covariance matrix is estimated with the formula

$$V\left[\hat{\boldsymbol{\beta}}_\tau^X\right] = \frac{1}{B}\sum_{b=1}^{B}(\hat{\boldsymbol{\beta}}_\tau^{Xb} - \overline{\boldsymbol{\beta}}_\tau^{Xb})(\hat{\boldsymbol{\beta}}_\tau^{Xb} - \overline{\boldsymbol{\beta}}_\tau^{Xb})' \tag{4.4}$$

Where $\hat{\boldsymbol{\beta}}_\tau^{Xb}$ denotes the quantile regression parameter estimates for $b = 1, ..., B$ and $\overline{\boldsymbol{\beta}}_\tau^{Xb}$ is the mean of the parameter estimates from the bootstrapped sample. Based on this bootstrapping method, standard errors are reported accordingly in parentheses in Table 2.

Tab. 2: CAViaR Parameter Estimates for $\tau = \{0.05, 0.25, 0.75, 0.95\}$

$\boldsymbol{\tau}$**= 0.05**	Brazil	Chile	Argentina	Mexico
$\hat{\beta}_0$	-0.0011	-0.0010	-0.0011	-0.0010
	(0.0102)	(0.0033)	(0.0106)	(0.0057)
$\hat{\beta}_1$	0.2589***	0.3965***	0.2701***	0.4158***
	(0.0409)	(0.0351)	(0.0536)	(0.0452)
$\hat{\beta}_2$	0.8840***	0.8322***	0.8649***	0.8477***
	(0.2067)	(0.1500)	(0.2718)	(0.1821)
$\hat{\beta}_3$	-0.1926***	-0.2399***	-0.2235***	-0.2615***
	(0.0551)	(0.0511)	(0.0745)	(0.0567)
$\boldsymbol{\tau}$**= 0.25**	Brazil	Chile	Argentina	Mexico
$\hat{\beta}_0$	-0.0003	-0.0004	-0.0004	-0.0008
	(0.0026)	(0.0012)	(0.0026)	(0.0019)
$\hat{\beta}_1$	0.1198***	0.2545***	0.1313***	0.1647***
	(0.0166)	(0.0149)	(0.0166)	(0.0150)
$\hat{\beta}_2$	0.8897***	0.8250***	0.8966***	0.9021***
	(0.1731)	(0.1702)	(0.2383)	(0.2005)
$\hat{\beta}_3$	-0.0704***	-0.0908***	-0.0762***	-0.0681***
	(0.0227)	(0.0213)	(0.0236)	(0.0207)
$\boldsymbol{\tau}$**= 0.75**	Brazil	Chile	Argentina	Mexico
$\hat{\beta}_0$	0.0003	0.0015	0.0008	0.0005
	(0.0046)	(0.0025)	(0.0022)	(0.0031)
$\hat{\beta}_1$	-0.0602***	0.1810***	0.0053	0.0120
	(0.0145)	(0.0144)	(0.0140)	(0.0150)
$\hat{\beta}_2$	0.9127***	0.6258**	0.7995***	0.8605***
	(0.2765)	(0.3038)	(0.1810)	(0.3056)
$\hat{\beta}_3$	0.0613***	0.1721***	0.1154***	0.0793***
	(0.0209)	(0.0232)	(0.0192)	(0.0198)
$\boldsymbol{\tau}$**= 0.95**	Brazil	Chile	Argentina	Mexico
$\hat{\beta}_0$	0.0008	0.0011	0.0006	0.0002
	(0.0083)	(0.0051)	(0.0073)	(0.0062)
$\hat{\beta}_1$	-0.1219***	0.1400***	-0.0105	-0.0319
	(0.0407)	(0.0371)	(0.0432)	(0.0432)
$\hat{\beta}_2$	0.8911***	0.8071***	0.8982***	0.8969***
	(0.1805)	(0.2303)	(0.2070)	(0.2185)
$\hat{\beta}_3$	0.1878***	0.2943***	0.1730***	0.2075***
	(0.0540)	(0.0509)	(0.0646)	(0.0576)

Standard errors in parentheses.
$*\ p < 0.1$, $**\ p < 0.05$, $***\ p < 0.01$

For each country, we apply the probability integral transform in order to analyse the probability of coexceedance in terms of the rank structure of the portfolio. There is no closed form function for the probability integral transform as per Equation 3.7 for the CAViaR quantile regression model, instead we approximate the probability integral transformation with a numerical method outlined in Appendix D.

4.2.2 Logit-Based Model Estimation

Based on the methodology outlined in Section 3 we specify several logit regression models for each of the pairs of countries of Brazil, Chile, Argentina and Mexico. For the pairs of countries we will use the nomenclature that we are estimating the probability of coexceedance of the country that is named first in the pair conditional on the country named second in the pair having already fallen below a certain VaR threshold. For example, for the country pair Brazil-Chile, we are estimating the probability that Brazil falls below a certain VaR threshold conditionally that Chile has already fallen below its respective VaR threshold.

For notational simplicity, we denote Brazil, Chile, Argentina and Mexico with script 1,2,3 and 4, respectively. Table 3 shows the model specifications for each of the pairwise country logit regression models that we specify as per Equation 3.12 and Equation 3.13.

Tab. 3: Model Specification for the Logit Regressions

	Y_J	Y_M	$\mathbf{X}$
Brazil-Chile	$\mathbb{1}\left(F_{1_t}(\mathbf{x}_{1t}) < \tau, F_{2_t}(\mathbf{x}_{2t}) < \tau\right)$	$\mathbb{1}\left(F_{2_t}(\mathbf{x}_{2t}) < \tau\right)$	$[\mathbf{1}\ F_{3_t}(\mathbf{x}_{3t})\ F_{4_t}(\mathbf{x}_{4t})]'$
Brazil-Argentina	$\mathbb{1}\left(F_{1_t}(\mathbf{x}_{1t}) < \tau, F_{3_t}(\mathbf{x}_{3t}) < \tau\right)$	$\mathbb{1}\left(F_{3_t}(\mathbf{x}_{3t}) < \tau\right)$	$[\mathbf{1}\ F_{2_t}(\mathbf{x}_{2t})\ F_{4_t}(\mathbf{x}_{4t})]'$
Brazil-Mexico	$\mathbb{1}\left(F_{1_t}(\mathbf{x}_{1t}) < \tau, F_{4_t}(\mathbf{x}_{4t}) < \tau\right)$	$\mathbb{1}\left(F_{4_t}(\mathbf{x}_{4t}) < \tau\right)$	$[\mathbf{1}\ F_{2_t}(\mathbf{x}_{2t})\ F_{4_t}(\mathbf{x}_{4t})]'$
Chile-Argentina	$\mathbb{1}\left(F_{2_t}(\mathbf{x}_{2t}) < \tau, F_{3_t}(\mathbf{x}_{3t}) < \tau\right)$	$\mathbb{1}\left(F_{3_t}(\mathbf{x}_{3t}) < \tau\right)$	$[\mathbf{1}\ F_{1_t}(\mathbf{x}_{1t})\ F_{4_t}(\mathbf{x}_{4t})]'$
Chile-Mexico	$\mathbb{1}\left(F_{2_t}(\mathbf{x}_{2t}) < \tau, F_{4_t}(\mathbf{x}_{4t}) < \tau\right)$	$\mathbb{1}\left(F_{4_t}(\mathbf{x}_{4t}) < \tau\right)$	$[\mathbf{1}\ F_{1_t}(\mathbf{x}_{1t})\ F_{3_t}(\mathbf{x}_{3t})]'$
Argentina-Mexico	$\mathbb{1}\left(F_{3_t}(\mathbf{x}_{3t}) < \tau, F_{4_t}(\mathbf{x}_{4t}) < \tau\right)$	$\mathbb{1}\left(F_{4_t}(\mathbf{x}_{4t}) < \tau\right)$	$[\mathbf{1}\ F_{1_t}(\mathbf{x}_{1t})\ F_{2_t}(\mathbf{x}_{2t})]'$

As an example of a model specification, drawing on Table 3, the logit regression models for the country pair Brazil-Chile is as follows

$$
\begin{aligned}
E\left[Y_{Jt}^{12}|\mathbf{X}_t^{12}\right] &= \Lambda\left(\mathbf{X}_t^{12'}\boldsymbol{\nu}_J^{12}\right) = \Lambda\left(\nu_{J0}^{12} + \nu_{J1}^{12}\cdot F_{3_t}(x_{3t}) + \nu_{J2}^{12}\cdot F_{4_t}(x_{4t})\right) \\
E\left[Y_M^{12}|\mathbf{X}^{12}\right] &= \Lambda\left(\mathbf{X}^{12'}\boldsymbol{\nu}_M^{12}\right) = \Lambda\left(\nu_{M0}^{12} + \nu_{M1}^{12}\cdot F_{3_t}(x_{3t}) + \nu_{M2}^{12}\cdot F_{4_t}(x_{4t})\right)
\end{aligned} \tag{4.5}
$$

The choice of the order of the country pairs is arbitrary for the purpose of this thesis, but a practitioner would be able to make a choice of appropriate models to specify. Our chosen specification allows us to look at the systematic risk of the Brazilian equity index conditional on the other countries in our portfolio. The combination of these models would be of interest to a Brazilian regulator or policy maker and allow risk to be analysed depending on the state of the markets in nearby countries of Chile, Argentina and Mexico.

Using the model specifications, as per Table 3, the logit models are estimated using maximum likelihood estimation, by maximising the log-likelihood function specified in Equation 3.14. We are interested in the VaR thresholds of 5% and 10% as commonly used measures in applied risk management and estimate the models for these thresholds.

The parameter estimates for the logit regressions are shown in Table 4. All but one parameter (ν_{M0} for the Argentina-Mexico specification) are significant at the 1% level of significance. We include the non-significant parameter in our estimation as it still allows for additional information as an input into the estimation of the probability of coexceedance. As part of the maximum likelihood estimation procedure we also numerically estimate the Hessian matrix $\mathbf{H}$ for each model. The Hessian is a matrix of second derivatives of the log-likelihood function with respect to the parameter estimates. The variance-covariance matrix of the parameter estimates for each logit regression can be found as a function of the Hessian matrix, where the relationship is shown as

$$\mathrm{V}\left[\hat{\boldsymbol{\nu}}\right] = -\mathbf{H}^{-1}(\hat{\boldsymbol{\nu}}) \tag{4.6}$$

When evaluated at the parameter estimates $\hat{\boldsymbol{\nu}}$, the variance covariance matrix is known as the empirical Hessian estimator of the variance covariance matrix. We show the structure of the Hessian matrix in Appendix E.

Tab. 4: Parameter Estimates for the MLE Logit Regression

Logit Parameter Estimates at the 5% Threshold

Joint Model	Brazil-Chile	Brazil-Argentina	Brazil-Mexico	Chile-Argentina	Chile-Mexico	Argentina-Mexico
ν_{J0}	-1.3672***	-1.2657***	-1.2395***	-1.6169***	-2.0003***	-1.7316***
	(0.2300)	(0.2388)	(0.2487)	(0.2629)	(0.2598)	(0.2452)
ν_{J1}	-3.8514***	-4.5202***	-7.4234***	-11.5936***	-8.7494***	-6.8278***
	(0.8469)	(0.9469)	(1.3283)	(2.0948)	(1.5229)	(1.2483)
ν_{J2}	-7.2185***	-7.4316***	-5.0975***	-2.3166**	-1.4504*	-3.1796***
	(1.1984)	(1.2820)	(1.0555)	(0.9166)	(0.7657)	(0.8486)
Marginal Model	**Brazil-Chile**	**Brazil-Argentina**	**Brazil-Mexico**	**Chile-Argentina**	**Chile-Mexico**	**Argentina-Mexico**
ν_{M0}	-1.2379***	-1.2999***	-1.1045***	-1.1499***	-0.9908***	-0.9710***
	(0.1344)	(0.1314)	(0.1321)	(0.1275)	(0.1278)	(0.1300)
ν_{M1}	-1.9344***	-1.9839***	-2.1731***	-3.4260***	-3.5098***	-3.6408***
	(0.3254)	(0.3105)	(0.3102)	(0.3704)	(0.3750)	(0.3689)
ν_{M2}	-2.6332***	-2.1211***	-2.5268***	-1.3886***	-1.8155***	-1.6893***
	(0.3391)	(0.3146)	(0.3231)	(0.3207)	(0.3322)	(0.3129)

Logit Parameter Estimates at the 10% Threshold

Joint Model	Brazil-Chile	Brazil-Argentina	Brazil-Mexico	Chile-Argentina	Chile-Mexico	Argentina-Mexico
ν_{J0}	-0.7495***	-0.8111***	-0.7707***	-0.8445***	-1.1008***	-0.9259***
	(0.1599)	(0.1507)	(0.1485)	(0.1670)	(0.1652)	(0.1561)
ν_{J1}	-3.6010***	-3.1964***	-4.2286***	-6.5196***	-5.8737***	-5.5983***
	(0.5179)	(0.4510)	(0.4688)	(0.8062)	(0.6934)	(0.6054)
ν_{J2}	-5.2077***	-4.4452***	-3.1372***	-3.370***7	-2.3065***	-2.3055***
	(0.6076)	(0.5122)	(0.4246)	(0.6017)	(0.5012)	(0.4335)
Marginal Model	**Brazil-Chile**	**Brazil-Argentina**	**Brazil-Mexico**	**Chile-Argentina**	**Chile-Mexico**	**Argentina-Mexico**
ν_{M0}	-0.4875***	-0.4532***	-0.2935***	-0.2360**	-0.2707***	-0.1260
	(0.1026)	(0.1014)	(0.1029)	(0.0996)	(0.1001)	(0.1022)
ν_{M1}	-1.9338***	-1.7067***	-2.5215***	-3.2814***	-3.3262***	-3.2550***
	(0.2293)	(0.2224)	(0.2289)	(0.2665)	(0.2605)	(0.2558)
ν_{M2}	-2.3581***	-2.5946***	-2.1805***	-1.8383***	-1.6380***	-2.0686***
	(0.2333)	(0.2353)	(0.2268)	(0.2413)	(0.2350)	(0.2323)

Standard errors in parantheses.
$* \ p < 0.1$, $** \ p < 0.05$, $*** \ p < 0.01$

4.2.3 Probabilities of Coexceedance Using the Logit-Based Model

After the estimation of the logit regression parameters, we use the logit-based model to estimate the conditional probability of coexceedances for the country pairs in our portfolio. As mentioned above, we focus on the probabilities of coexceedance past the 5% and 10% level. Our estimation procedure is as follows. For each day in which the second named country in each pair falls below its respective threshold, we estimate the probability that the first named country will fall below its respective threshold, using the formula as per Equation 3.15. For example, the probability of coexceedance for the pair Brazil-Chile in time t is

$$G_{\tau,t}^{12}|\mathcal{F}_t = \frac{\Lambda\left(\mathbf{X}_t'\nu_J^{12}\right)}{\Lambda\left(\mathbf{X}_t'\nu_M^{12}\right)} \tag{4.7}$$

Figure 3 displays a visual representation of the dynamic predictive power of this model for Brazil-Chile at the 5% level. The black vertical lines in the left hand figure show the instances of coexceedance compared to the predicted probability of coexceedance from the logit regression models in orange. Where an orange line is high on the backdrop of a black, the model has predicted that instance of coexceedance to a high degree. While not a formal diagnostic tool, it is a powerful initial diagnostic check. Many of the instances of coexceedances are predicted with a high probability where as days during which Chile fell past the 5% threshold but Brazil did not are estimated with a low probability of occurrence. We provide a formal diagnostic tests of the accuracy of prediction in Section 4.5.

The right hand figure in Figure 3 shows another visual representation of the predictive power of the model. The scatter plot represents the same probabilities of coexceedance, but with a colour ramp graphic to represent the relative degree of certainty. The light grey boxes on the plot background represent the financial crises defined above in chronological order. Figure 4 shows the prediction of probability plots of the remainder of the country pairs. A visual analysis shows a difference in systematic dependence between different country pairs. Brazil tends to be highly dependent on the other countries in the portfolio, with conditional probabilities of coexceedance for Brazil approaching 100% in times of financial crisis. The probabilities of coexceedance do not tend to be as high for the other countries, indicating that Brazil tends to be more dependent on the health of the other countries compared to the rest of the portfolio.

Fig. 3: Prediction of the Probability of Coexceedance for Brazil-Chile at $\tau = 0.05$ using the Logit-Based Model

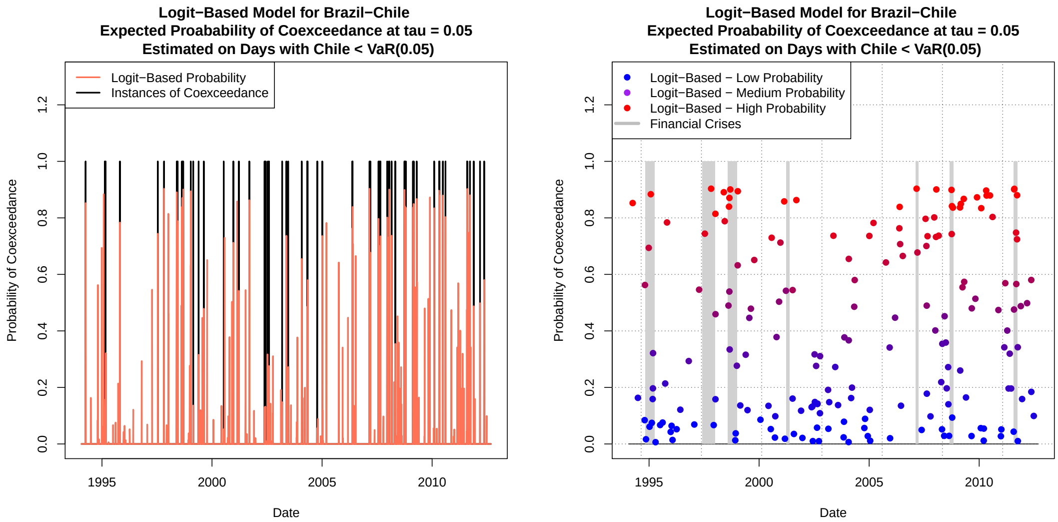
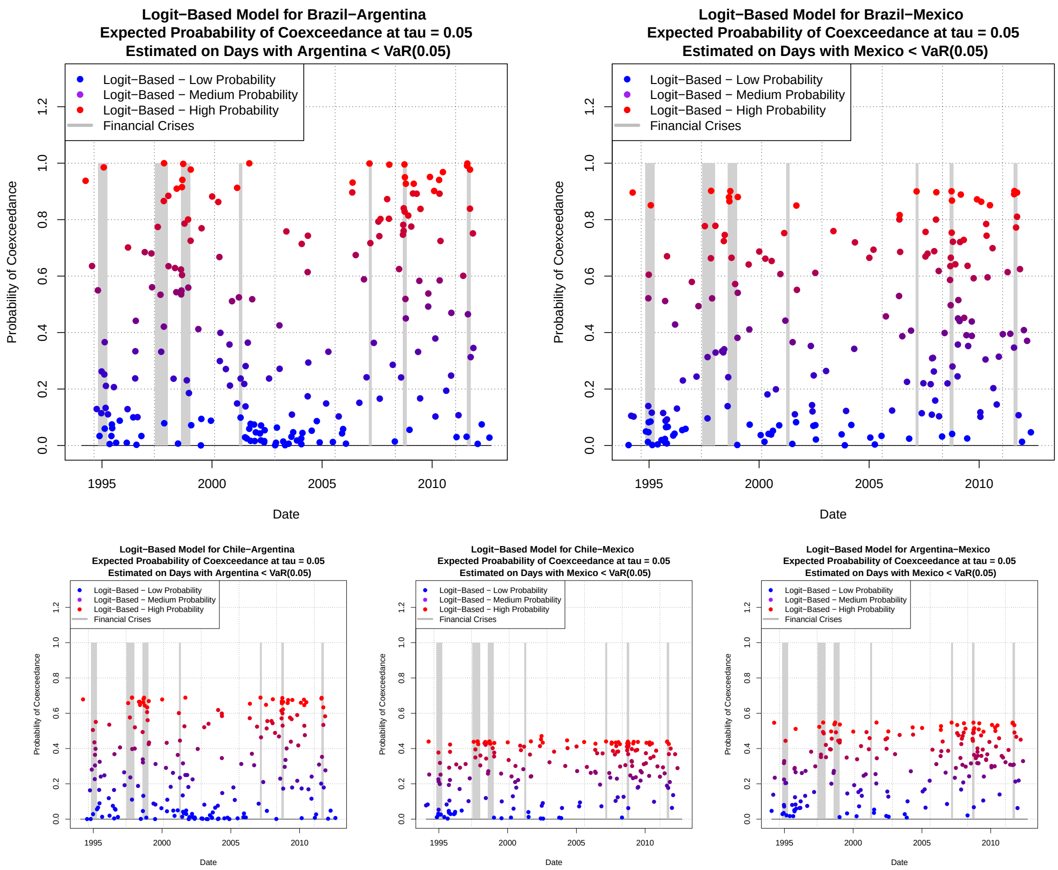


Fig. 4: Prediction of the Probability of Coexceedance for the Portfolio at $\tau = 0.05$ using the Logit-Based Model

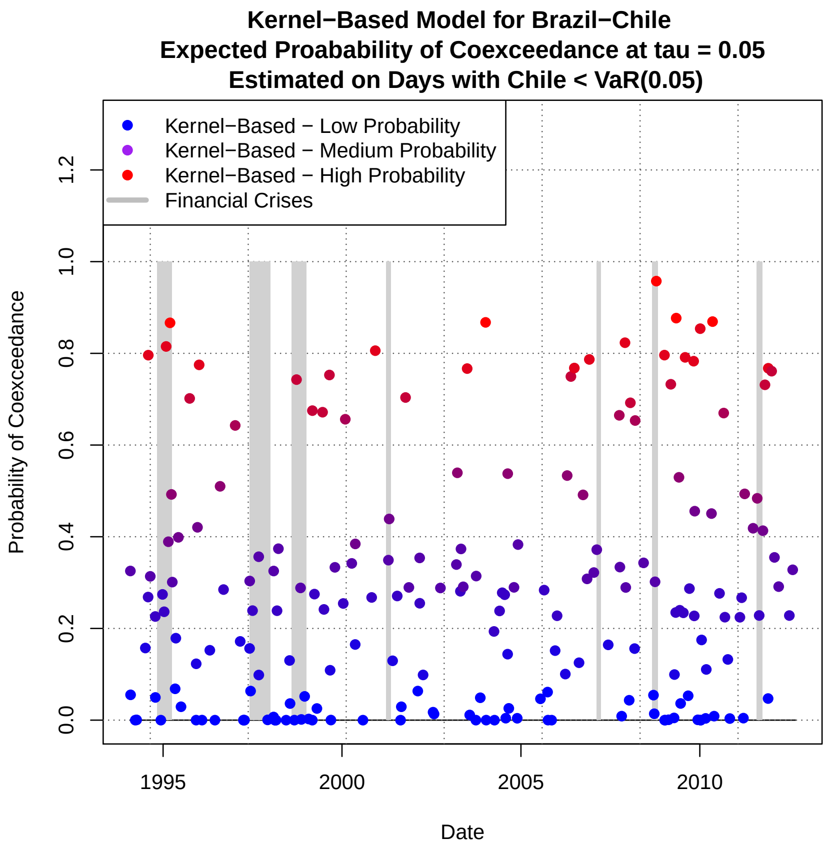


4.2.4 Probabilities of Coexceedance using the Kernel-Based Model

We also apply the kernel-based model to the same set of probability integral transformations and estimate the probability of coexceedances as per Equation 3.11 between the pairs of countries as outlined above. For this application, we use the Gaussian kernel and apply Silverman's Rule-of-Thumb estimator for the respective bandwidth parameters, as previously described in Appendix A. Figure 5 shows a visual example of the prediction of coexceedances for the country pair, Brazil-Chile, at the 5% level using the kernel-based model.

Although a visual inspection may show that a large portion of the predicted probabilities on the lower bound of 0, an inspection of the predicted probabilities themselves show that these predicted values are in fact a very low probability value close to 0. The plots for the prediction of coexceedances for the remaining country pairs are shown in Appendix F.

Fig. 5: Prediction of the Probability of Coexceedance for Brazil-Chile at $\tau = 0.05$ using the Kernel-Based Method



4.3 Robustness Checks

To test the performance of the methodology proposed in the Section 3, we estimate commonly used econometric models to heuristically check the robustness of our estimation procedure. We estimate parametric models, using copula distributions as the joint distribution of dependence. After the estimation procedures of the parametric model we test for misspecifications in applying the parametric model and then we compare the different model specifications on mean squared error MSE criterion based on prediction of coexceedance within the sample.

4.3.1 Copula Notation

This section introduces the copula notation that we use in the estimation procedure of the parametric models. A copula is a function that links a portfolio of univariate distributions together as one joint distribution. For an arbitrary probability distribution function, Sklar's theorem (Sklar, 1959) guarantees the existence of a copula function under some common regularity conditions and allows a d-dimensional multivariate distribution function $\mathbf{F}$ to be represented as a copula function $\mathcal{C}$ which is a function of the univariate distribution functions, F_i for $i = 1, ..., d$, or

$$\mathbf{F}(\mathbf{x}) = \mathcal{C}\left((F_1(\mathbf{x}_1), ..., F_d(\mathbf{x}_d)\right) \tag{4.8}$$

An important aspect of copula functions that is widely exploited in econometric modelling is that the univariate marginal distributions of the copula need not be restricted to any particular distribution, or in anyway related to each other. This allows complete flexibility in specifying the marginal distributions with parametric or nonparametric methods. The copula distribution function contains all the dependence information between the marginals, where as all the univariate information is contained by the univariate distribution functions themselves.

If the joint distribution function is differentiable, by taking the d partial derivatives with respect to the margins, we obtain the joint density function $\mathbf{f}$ which can be expressed as a product of the copula density function c and the univariate density functions f_i, given by

$$\begin{aligned}\frac{\partial^k \mathbf{F}(\mathbf{x})}{\partial x_1 ... \partial x_k} &= \mathbf{f}(\mathbf{x}) \\ &\equiv c\left(F_1(x_1), ..., F_k(k)\right) \cdot \prod_{i=1}^{k} f_i(x_i)\end{aligned} \tag{4.9}$$

We exploit this result directly in the maximum likelihood estimation of the parametric models we implement.

4.3.2 Parametric Estimation

We estimate several, commonly used parametric copula models jointly with the time series specifications of the four countries. We use the result from Equation 4.9 directly in the joint maximum likelihood estimation for the time series and parametric copula estimation where we computationally maximise the joint log-likelihood function with respect to the joint parameter matrix $\mathbf{\Theta}$, where the log-likelihood function $\mathcal{L}$ is given by

$$\mathcal{L}\left(\mathbf{\Theta}|\mathbf{X}\right) = \sum_{t=1}^{T}\left[\log\left(c\left(F_1(x_{1t}), F_2(x_{2t}), F_3(x_{3t}), F_4(x_{4t})\middle|\boldsymbol{\theta}_c\right)\right) + \sum_{i=1}^{4}\log\left(f_i\left(x_{it}\middle|\boldsymbol{\theta}_i\right)\right)\right] \tag{4.10}$$

where $\boldsymbol{\theta}_i$ denotes the parameter vectors for the appropriate densities. Based on the diagnostic tests of the time series specifications in Section 4.2 we restrict our joint estimation procedure to using the SAV-GARCH model as the univariate time series model for each country. We make the parametric restriction that the innovations and thus the conditional returns distributions are Gaussian as per Equation 3.3. We choose five commonly used copula models with specifications as shown in Table 5.

Tab. 5: Distribution Functions of Copula Specifications

Copula	Copula Distribution Function	Parameters
Independence	$\prod_{i=1}^{d} F_i(\mathbf{x}_i)$	
Clayton	$\left(\sum_{i=1}^{d} F_i(\mathbf{x}_i)^{-\theta} - d + 1\right)^{-1/\theta}$	θ
Gumbel	$\exp\left(-\left[\sum_{i=1}^{d}\left(-\log\left(F_i(\mathbf{x}_i)\right)\right)^{\theta}\right]^{1/\theta}\right)$	θ
Gaussian	$\mathbf{\Phi}_{\mathbf{R}}\left(\mathbf{\Phi}_{\mathbf{R}}^{-1}(\mathbf{F}(\mathbf{x}))\right)$	Correlation matrix $\mathbf{R}$
Student-t	$\mathbf{t}_{\mathbf{R},df}\left(\mathbf{t}_{\mathbf{R},df}^{-1}(\mathbf{F}(\mathbf{x}))\right)$	$\mathbf{R}$ and degrees of freedom df

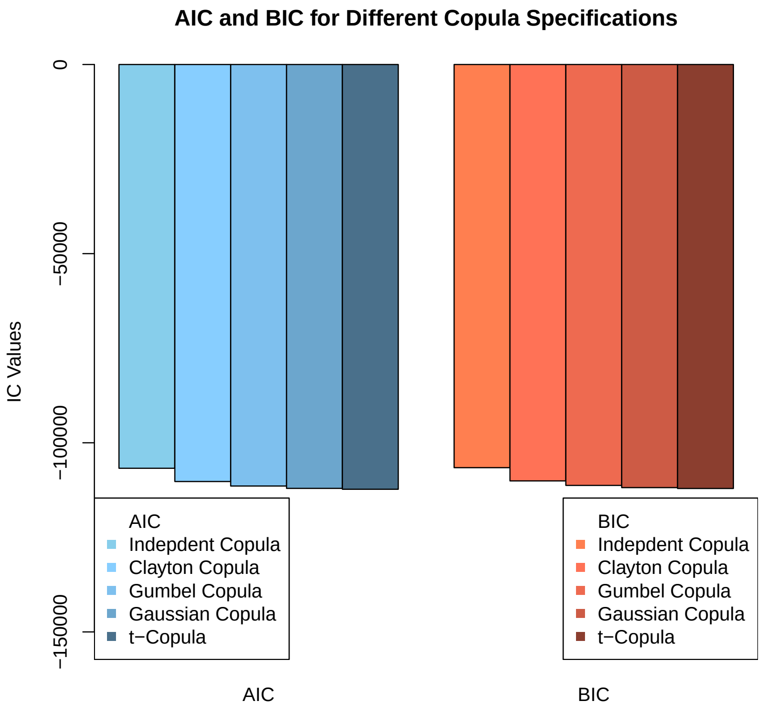
We have specific motivations in modelling the rank structure of the portfolio with each of the copula specifications listed above. The independence copula has a distribution function equal to that of the product of the univariate distribution functions and it is included in the estimation procedure in order to determine the additional information obtained by placing a parametric restriction on the dependence structure of the joint distribution. The Clayton and Gumbel copula are employed as they allow positive lower and upper tail dependence, respectively, and are useful measures for estimating extreme dependencies of the joint distribution. If the dependence structure is characterised by extreme values then these objects should fit well.

The Gaussian and the Student-t copulas are elliptical in distribution and are derived from the probability distributions of the same name. The Gaussian copula does not allow for either lower or upper extreme tail dependence, whereas the Student-t copula allows for both lower and upper tail dependence but restricts the upper and lower tail dependence to be symmetrical. The extent of extreme tail dependence by the degrees of freedom parameter and as the degrees of freedom parameter approaches infinity, the Student-t copula approaches the Gaussian copula.

We undertake the joint MLE procedure and based on information criteria and significance of parameters the model that employs the Student-t copula is deemed the best fit. Figure 6 shows a visual representation of the AIC and BIC results for the different models, with full information criteria values shown in Appendix G. Table 6 shows the parameter estimates for the joint time series model using the Student-t copula with the time series parameters are denoted as per Equation 3.3. **R** denotes the matrix of correlation coefficients of the Student-t copula and the Degrees of Freedom is the degrees of freedom parameter of the Student-t copula.

All but three of the parameter estimates for the joint model using the Student-t copula are significant at the 1% level of significance with the remainders significant at least at the 10% level. The standard errors, denoted in parentheses, are obtained from the numerically derived hessian matrix. The econometric benefit of estimating the time series models and the copula model jointly is that the parameter estimates have non-zero covariance, compared to the approach of inference from margins where the models are estimated independently of each

Fig. 6: Information Criteria Tests for the Joint Time Series Estimates



other, requiring that standard errors be modified for the two-stage estimation procedure.

The performance of the different copula specifications allows us to obtain an initial heuristic view of the true rank structure of the four equity markets. The independence copula was the worst-performing model based on information criteria, showing that there is improved explanatory power by imposing parametric restrictions on the dependence structure. The Clayton and Gumbel copulas were the next worst performing, respectively, indicating the portfolio is not described well by the the extreme values.

The Student-t copula performs marginally better than the Gaussian copula by information criteria. The two elliptical copulas performing well suggests that most of dependence structure is explained by the mean values of these bell-shaped curves. It is no surprise that the Student-t copula is the best performer as it restricts the rank of the portfolio to the same correlation structure as the Gaussian copula but allows for extreme values to be modelled adequately through its degrees of freedom parameter. The restriction that the upper and lower tail dependence are equal for the Student-t copula model is somewhat restrictive, but in light of the low skewness statistic for each of the countries' returns data, it remains a good fit.

Tab. 6: Parameter Estimates for the SAV-GARCH Models and t-Copula

Time Series Parameters

	Brazil	Chile	Argentina	Mexico
$\hat{\alpha}_0$	0.0007**	0.0005***	0.0003*	0.0004*
	(0.0003)	(0.0001)	(0.0002)	(0.0002)
$\hat{\alpha}_1$	0.1123***	0.2219***	0.0708***	0.0604***
	(0.0130)	(0.0129)	(0.0130)	(0.0138)
$\hat{\delta}_0$	0.0038***	0.0016***	0.0013***	0.0017***
	(0.0002)	(0.0001)	(0.0001)	(0.0002)
$\hat{\delta}_1$	0.2230***	0.1883***	0.1318***	0.1976***
	(0.0090)	(0.0093)	(0.0076)	(0.0096)
$\hat{\delta}_2$	0.7074***	0.7397***	0.8457***	0.7729***
	(0.0105)	(0.0128)	(0.0088)	(0.0122)

Correlation Matrix

$\hat{\mathbf{R}}$	Brazil	Chile	Argentina	Mexico
Brazil	1	0.5690***	0.6138***	0.6474***
		(0.0117)	(0.0109)	(0.0102)
Chile	0.5690***	1	0.4908***	0.5247***
	(0.0117)		(0.0130)	(0.0124)
Argentina	0.6138***	0.4908***	1	0.5633***
	(0.0109)	(0.0130)		(0.0119)
Mexico	0.6474***	0.5247***	0.5633***	1
	(0.0102)	(0.0124)	(0.0119)	

Degrees of Freedom

Degrees of Freedom	10.0143***
	(0.5337)

Standard errors in parantheses.
$*\ p < 0.1$, $**\ p < 0.05$, $***\ p < 0.01$

4.3.3 Probabilities of Coexceedances Using the Copula Model

Using the estimated Student-t copula specification, we are able to estimate the probability of coexceedance, conditional on the remainder on the portfolio. The copula distribution function has the characteristics of a probability distribution function that we can evaluate by integrating the copula density function over the relevant interval. For example, the conditional probability of coexceedance for the pair Brazil-Chile in time t is given by

$$G^{12}_{\tau,t}|\mathcal{F}_t = \frac{\displaystyle\int_0^\tau \int_0^\tau c\left(u_1, u_2 \middle| F_{3t}(x_{3t}), F_{4t}(x_{4t})\right) du_1 du_2}{\displaystyle\int_0^\tau \int_0^1 c\left(u_1, u_2 \middle| F_{3t}(x_{3t}), F_{4t}(x_{4t})\right) du_1 du_2} \tag{4.11}$$

Where the denominator integrates to the 3-dimensional Student-t copula, marginal of Brazil. For computation, we use a numerical integration technique in the R package `R2cuba` (Hahn *et al.*, 2015) for the two integration steps. We use this computational intensive method because most statistical packages give only approximations of the above equation which can result in spurious probabilities when values are close to the lower boundary of the probability space. We use this method in the comparison of performance in Section 4.5.1.

4.4 Misspecification Tests

In this section we examine whether the distributional assumptions of the Student-t copula create global misspecifications of the dependence structure in the portfolio. We test for significant distances between the Student-t copula and an Empirical Distribution Function (EDF) constructed from the parametric probability integral transform over the diagonal sections and the lower tail dependence functions. The EDF does not rely on any global distribution assumptions and will show the underlying dependence structure more accurately than the parametric dependence structure. A significant distance between the two indicate a global misspecification of the rank of the portfolio.

We further test for misspecification between the probability integral transform between the quantile regression method and the parametric method by comparing EDFs constructed with the two methods. Significant distances between these functions indicate that the parametric models are inadequate at modelling the conditional distributions of the returns and further motivates quantile regression techniques.

4.4.1 Diagonal Sections

Firstly, we test whether the bivariate diagonal sections for each model specification are significantly different from each as a test of global misspecification of the rank structure. The diagonal section F_{jk}^{D} of a bivariate distribution function describes the following probability

$$\begin{aligned} &F_{jk}^{D}(\tau) = Pr\left(F_j(x_j) < \tau, F_k(x_k) < \tau\right) \\ &\tau \in [0,1] \end{aligned} \tag{4.12}$$

Estimating the diagonal section reduces the bivariate margin to a two-dimensional object where the notion of distance is more easily described. The bivariate margins have easily obtained functional forms from the joint dependence structure. Bivariate margins of the

Student-t copula are also Student-t copula with the same degrees of freedom parameter as the joint model (Kotz and Nadarajah, 2004, pp 15). We model the probability integral transform from the CAViaR quantile regression with an EDF and we also employ an EDF for the probability integral transform estimated from the parametric time series models as estimated in Section 4.3.2. We show the plots of the diagonal sections for each country pair in Appendix H.

To estimate the distance d_{jk} between two diagonal sections we use the second generalised mean function on a fine grid of size N, given by

$$\begin{aligned} d_{jk} &= \left(N^{-1}\sum_{n=1}^{N}\epsilon_n^2\right)^{1/2} \\ \text{where } \epsilon_n &= F_j^D(n\cdot\Delta\tau) - F_k^D(n\cdot\Delta\tau) \\ \text{and where } \Delta\tau &= N^{-1} \end{aligned} \tag{4.13}$$

Where F_j^D and F_k^D are two diagonal sections for the same country pair but of different model specifications. We obtain standard errors of the distance between the diagonal sections through a bootstrap procedure. We run a bootstrap algorithm simultaneously on the joint maximum likelihood estimation of the joint time series model with a Student-t copula and on the CAViaR model estimation procedure to obtain $B = 250$ different bootstrapped model specifications and diagonal sections. For each of the bootstrapped samples, we compute the distance between diagonal sections as per the procedure above and find the standard error of the second generalised mean distance as

$$SE_{d_{jk}} = \left(\frac{1}{B}\sum_{b=1}^{B}\left(d_{jk}^b - \overline{d}_{jk}^b\right)^2\right)^{1/2} \tag{4.14}$$

Where d_{jk}^b is the second generalised mean distance of bootstrap iteration $b = 1, ..., 250$ and where $\overline{d}_{jk}^b$ is the average of the bootstrapped distances.

Table 7 show the estimates of distance between the diagonal sections. The distance between the diagonal sections for the Student-t copula and the EDF based on the parametric probability integral transform are significantly different from zero for the country pairs Brazil-Mexico, Chile-Argentina and Argentina-Mexico. As the EDF does not rely on any global distributional assumptions these significant differences indicate that the parametric

Tab. 7: Second Generalised Mean Distances of Diagonal Sections

	Student-t and Empirical	CAViaR and Empirical
Brazil-Chile	0.0249	0.0299**
	(0.0189)	(0.0126)
Brazil-Argentina	0.03209	0.0364***
	(0.0163)	(0.0129)
Brazil-Mexico	0.0318***	0.0348***
	(0.0085)	(0.0125)
Chile-Argentina	0.0265**	0.0303**
	(0.0118)	(0.0124)
Chile-Mexico	0.02630	0.0290***
	(0.0109)	(0.0096)
Argentina-Mexico	0.0336***	0.0359***
	(0.0111)	(0.0118)
Standard errors in parantheses. $* \ p < 0.1$, $** \ p < 0.05$, $*** \ p < 0.01$		

structure imposed by the Student-t copula does not fit the data adequately due to the unrealistic assumptions of symmetry and kurtosis. This results reveals that the use of parametric copula models in risk management scenarios may produce costly misspecification of risk and motivates the use of semiparametric methods to model the dependence structure without distributional assumptions.

The distances between the diagonal sections of the EDFs constructed from the parametric probability integral transform and from the semiparametric estimation from the CAViaR specification are significantly different from zero which indicates the Gaussian assumptions required to model the time series specifications parametrically are unrealistic and are not a good fit for the data further motivating the use of quantile regression techniques to evaluate the probability integral transform.

4.4.2 Lower Tail Dependence

The same test of distance is also performed on the the bivariate tail dependence functions of the above specifications. The tail dependence function describes the probability that extreme lower tail events will happen jointly between two variables. The lower tail dependence λ_L is given by

$$\lambda_L^{jk} = \lim_{\tau \to 0} \frac{Pr(F_j(x_j) < \tau, F_k(x_k) < \tau)}{\tau} \tag{4.15}$$

Figure 7 shows plots the estimates of the lower tail dependence for $\tau = [0, 0.20]$ and we perform the calculation of second generalised mean distance on the lower tail dependence as per the procedure above. Table 8 show the results of the distances between the lower tail dependence functions. Standard errors of the distances are estimated with a bootstrapping technique as above.

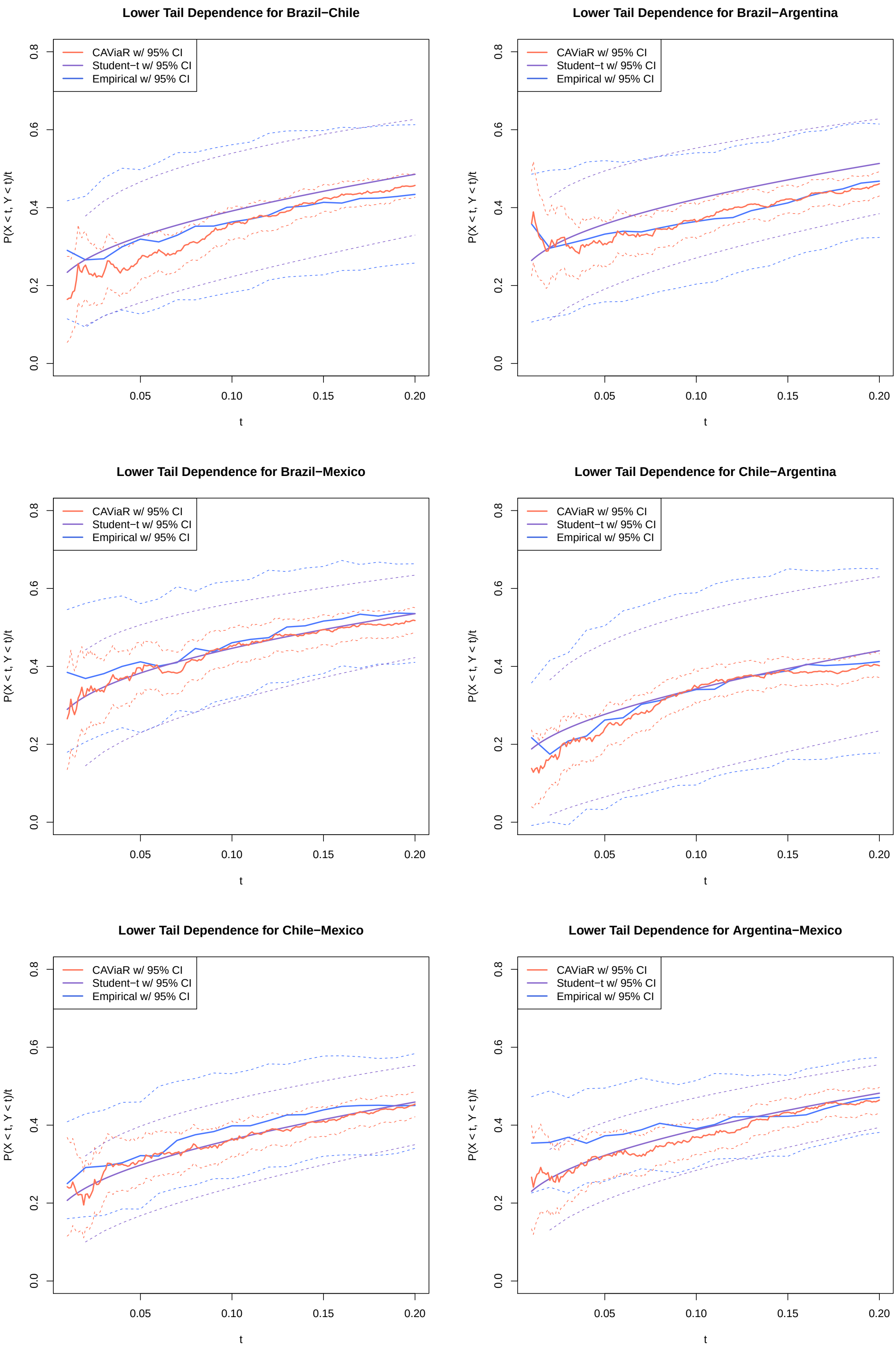
Tab. 8: Second Generalised Mean Distances of Lower Tail Dependence

	Student-t and Empirical	CAViaR and Empirical
Brazil-Chile	0.0420	0.0517***
	(0.0630)	(0.0148)
Brazil-Argentina	0.0532	0.0192
	(0.0579)	(0.0144)
Brazil-Mexico	0.0294	0.0306*
	(0.0658)	(0.0147)
Chile-Argentina	0.0198	0.0215***
	(0.0180)	(0.0064)
Chile-Mexico	0.0291	0.0306*
	(0.0629)	(0.0146)
Argentina-Mexico	0.0478	0.0464***
	(0.0409)	(0.0102)

Standard errors in parantheses.
$* \, p < 0.1$, $** \, p < 0.05$, $*** \, p < 0.01$

For a majority of the country pairs, the distances between the lower tail dependence functions of the CAViaR and the parametric-based EDF specifications are significantly different from zero. This indicates the semiparametric procedure of estimating the probability integral transform is more precise in modelling the conditional distributions of returns. None of the distances between the lower tail dependence functions of the Student-t copula and the empirical copula are significant indicating that the Student-t copula is modelling the tail dependence adequately, despite having the distributional restriction that the upper and lower tail dependencies are symmetrical. These results further motivate the use of the introduced semiparametric methods as they do not rely parametric assumptions in the modelling of extreme lower tail values of the returns distributions.

Fig. 7: Lower Tail Dependence Plots



4.5 Coexceedance Tests

In this section we test the relative performances of the novel models against the Student-t copula in predicting instances of coexceedance. Accuracy in predicting instances of coexceedances directly demonstrates the adeptness of the respective models in quantifying systematic risk as it indicates that the dependence structure in the portfolio is modelled precisely. We test the models based on MSE criterion and find the introduced models significantly outperform the Student-t copula in measuring systematic risk in the portfolio.

4.5.1 Method of Performance Tests

We introduce the method of the relative performance tests before discussing results in Section 4.5.2. We are interested in the coexceedances past the thresholds of 5% and 10% VaR and the test procedure is as follows. For each day in which the second named country in each pair falls below its respective threshold, we estimate the probability that the first named country will fall below its respective threshold, using the formula as per Equation 3.15 with the logit-based model, the kernel-based model and the Student-t copula. For each time period that we estimate the probability of coexceedance, we evaluate the squared error of the prediction of each model in order to calculate the MSE over the entire date range considered, where MSE is given by

$$\begin{aligned}
&\text{MSE} = \frac{1}{M}\sum_{m=1}^{M}\left(\hat{Y}_m - Y_m\right)^2 \\
&\text{where } Y_m = \begin{cases} 1 & \text{if } F_1(x_{1m}) < \tau \text{ and } F_2(x_{2m}) < \tau \\ 0 & \text{otherwise} \end{cases} \\
&\text{and where } \hat{Y}_m = G^{12}_{\tau,t}\middle|\mathcal{F}_t
\end{aligned} \tag{4.16}$$

where $m = 1, ..., M$ describes the time periods where the second named country of each pair falls below its respective threshold. This procedure demonstrates the accuracy of quantifying systematic risk in both times of financial tranquility and volatility to display the performance of each model.

4.5.2 Results of Performance Tests

We apply the test procedure outlined above and present the evaluated MSE values in Table 9. Both of the introduced models display significantly lower MSE for the prediction of coexceedances compared to the Student-t copula demonstrating the accuracy that these models quantify systematic risk in the portfolio.

The logit-based model displays MSE on average 25% and 27% lower than the Student-t copula at the 5% and 10% VaR thresholds, respectively. As expected, the kernel-based model displays an even stronger relative performance exhibiting MSE on average 55% and 35% lower than the Student-t copula at the 5% and 10% VaR thresholds, respectively.

Tab. 9: Comparison of MSE of Prediction of Coexceedance for South American Portfolio

MSE of Prediction of Coexceedences		MSE $\tau = 0.05$	MSE $\tau = 0.10$
Brazil-Chile	Kernel-Based	0.1047*	0.1415*
	Logit-Based	0.1495	0.1560
	Student-t Copula	0.2370	0.2869
Brazil-Argentina	Kernel-Based	0.0942*	0.1628
	Logit-Based	0.1485	0.1596*
	Student-t Copula	0.2620	0.2417
Brazil-Mexico	Kernel-Based	0.0955*	0.1625*
	Logit-Based	0.1680	0.1712
	Student-t Copula	0.2269	0.2433
Chile-Argentina	Kernel-Based	0.1015*	0.1480*
	Logit-Based	0.1632	0.1726
	Student-t Copula	0.2386	0.2058
Chile-Mexico	Kernel-Based	0.0961*	0.1463*
	Logit-Based	0.1820	0.1829
	Student-t Copula	0.2162	0.2187
Argentina-Mexico	Kernel-Based	0.1160*	0.1687*
	Logit-Based	0.1884	0.1894
	Student-t Copula	0.2002	0.2336

Lowest MSE values indicated with *

These results demonstrate that the introduced models are far superior to the commonly used parametric alternatives in quantifying systematic risk in a financial portfolio. As the introduced models do not rely on any unrealistic distributional assumptions, they accurately measure systematic risk in extreme lower tail events that are not captured by parametric models. The kernel-based model significantly outperformed the other models in this small portfolio in the measurement of systematic risk. The logit-based model also outperformed the parametric model in this small portfolio and is scalable into higher dimensional portfolios.

5. APPLICATION TO AN AUSTRALIAN PORTFOLIO

To show the application of the logit-based model in a higher dimensional portfolio, we perform the estimation procedure on a large portfolio of securities listed on the Australian Stock Exchange (ASX). We test the performance in quantifying systematic risk by constructing a test of the prediction of coexceedances for a selection of securities conditional on Commonwealth Bank Australia (CBA) and we compare to a Student-t copula as a parametric alternative. In this test the logit-based model on average displays a 60% lower MSE than the Student-t copula, demonstrating its scalability and accuracy in a higher dimension portfolio.

5.1 Data Description

The portfolio is comprised of equities currently in the ASX100, the top one hundred firms by total market capitalisation listed on the ASX, that consistently have available data over a period between 5 January 2000 to 6 April 2016. The securities listed on the ASX100 are important to analyse as they contain approximately 40% of the total market capitalisation of listed corporations in Australia (Market Index, 2016). Table 10 lists the equities in the chosen portfolio with full company names and industries.

We source our data for the ASX equities from the Bloomberg Online Subscription Service. We start with an initial portfolio of every equity that is currently listed in the ASX100 and remove stocks from this initial portfolio that do not have data available over the period to obtain the portfolio of 57 securities. Our sample consists of 4196 days where data is available for every stock over this period.

The final portfolio is overrepresented by certain industries with 17 of the securities belonging to financial and real estate industries. These securities are likely to be more susceptible to financial contagion than other industries due to their reliance on efficient financial markets and are valuable to examine for changes in their exposure to systematic risk.

Tab. 10: Portfolio of Australian Securities

ASX Code	Company	Industry	ASX Code	Company	Industry
ABC	Adelaide Brighton	Materials	IOF	Investa Office Fund	Real Estate
ALL	Aristocrat Leisure	Consumer	JHX	James Hardie Industries	Materials
ALQ	ALS Ltd.	Industrials	LLC	Lendlease Group	Real Estate
AMC	Amcor	Materials	MGR	Mirvac Group	Real Estate
AMP	AMP Ltd.	Financials	MQG	Macquarie Group	Financials
ANN	Ansell	Health	NAB	National Aust. Bank	Financials
ANZ	ANZ Banking Group	Financials	NCM	Newcrest Mining	Mining
ASX	ASX Ltd	Financials	ORG	Origin Energy	Energy
AWC	Alumina	Mining	ORI	Orica	Materials
BEN	Bendigo Bank	Financials	OSH	Oil Search Ltd.	Energy
BHP	BHP Billiton Ltd.	Mining	PPT	Perpetual Ltd.	Financials
BKL	Blackmores Ltd.	Consumer	PRY	Primary Health Care Ltd.	Health
BOQ	Bank of QLD	Financials	QAN	QANTAS Airways	Industrials
CBA	Commonwealth Bank	Financials	QBE	QBE Insurance	Financials
CCL	Coca-Cola Amatil	Consumer	RHC	Ramsay Health Care	Health
CGF	Challenger	Financials	RIO	Rio Tinto	Mining
CIM	Cimic Group	Industrials	RMD	Resmed	Health
COH	Cochlear	Health	SGP	Stockland	Real Estate
CPU	Computershare	Info. Tech.	SHL	Sonic Healthcare	Health
CSL	CSL	Health	STO	Santos	Energy
CSR	CSR	Materials	SUN	Sun Resources	Energy
CTX	Caltex Aust.	Energy	TAH	Tabcorp Holdings	Consumer
DOW	Downer EDI	Industrials	TCL	Transurban	Industrials
FLT	Flight Centre Travel	Consumer	TLS	Telstra Corp.	Communication
FXJ	Fairfax Media	Consumer	WBC	Westpac Banking	Financials
GNC	Graincorp	Consumer	WES	Wesfarmers	Consumer
GPT	GPT	Real Estate	WOW	Woolworths	Consumer
HVN	Harvey Norman	Consumer	WPL	Woodside Petroleum	Energy
ILU	Iluka Resources	Materials			

5.1.1 Summary Statistics and Characteristics

Table 11 shows summary statistics for the returns of the portfolio of ASX securities. For all stocks, there is evidence of excess kurtosis with the sample kurtosis being exceptionally high for some securities, namely QBE Insurance with a sample kurtosis of approximately 146. This is stark evidence of excess extreme values in returns which would not be modelled adequately by a Gaussian curve. A majority of the securities exhibit little to mild skewness (sample skewness between -1 and 1) with only a handful of securities displaying high skewness statistics, namely QBE Insurance with a skewness statistic of approximately -3.9. As most of the securities are approximately symmetrical we may be motivated to use symmetrical curves such as the Student-t distribution to model the joint distribution of the portfolio, but must be weary of the parametric restrictions this will impose.

Tab. 11: Summary Statistics of the ASX Portfolio

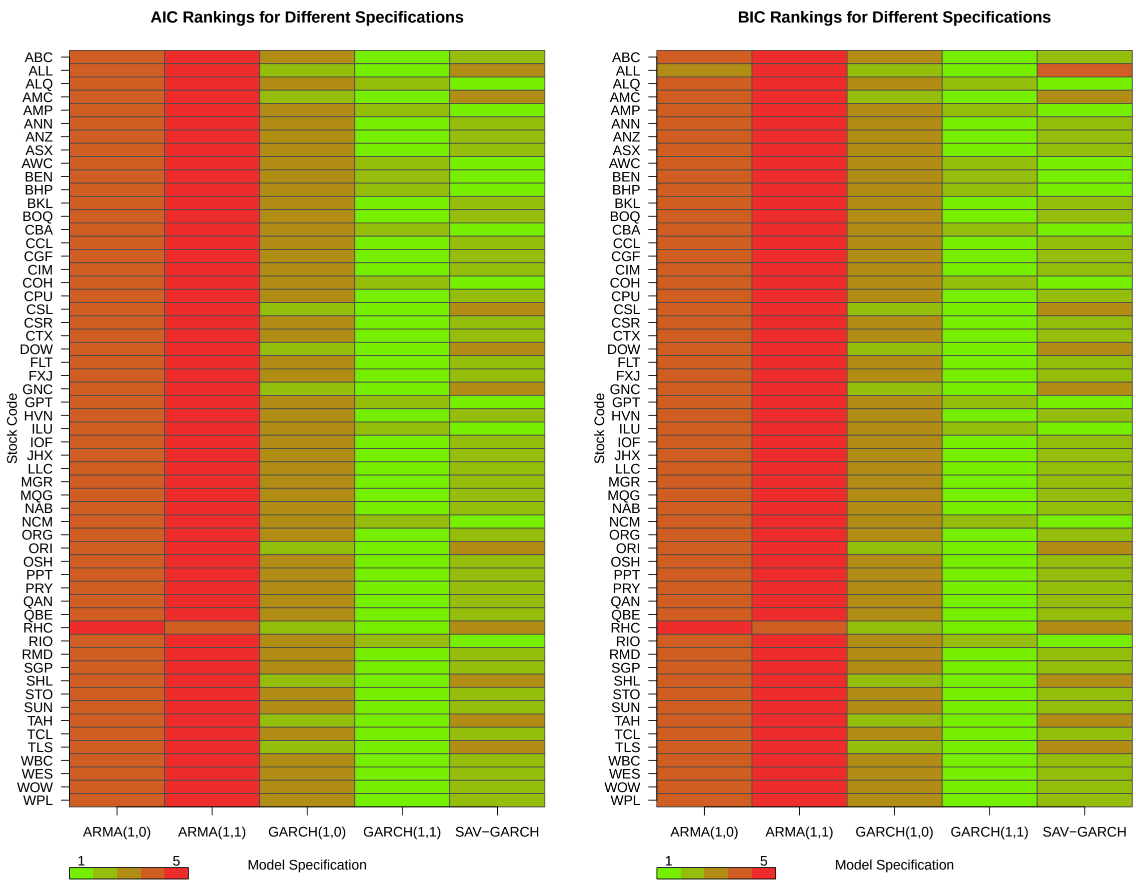
ASX Code	Mean	Skewness	Kurtosis	ASX Code	Mean	Skewness	Kurtosis
ABC	0.0005	0.4013	15.5435	IOF	0.0001	-0.7748	21.9498
ALL	0.0002	-4.0843	81.7260	JHX	0.0004	0.2595	8.2493
ALQ	0.0004	-0.3028	12.7949	LLC	-0.0001	-0.8735	11.0214
AMC	0.0003	0.1179	7.2795	MGR	-0.0001	-0.2224	31.2633
AMP	-0.0002	-2.7471	74.4212	MQG	0.0002	0.2335	22.9364
ANN	0.0001	-0.6274	14.1833	NAB	0.0001	-0.3876	12.2188
ANZ	0.0002	-0.0099	10.7067	NCM	0.0003	-0.0955	6.0311
ASX	0.0003	0.2820	13.7577	ORG	0.0002	0.0160	26.0724
AWC	-0.0002	-0.2602	7.8330	ORI	0.0002	-0.3544	13.4182
BEN	0.0001	0.9381	18.8712	OSH	0.0003	-0.4009	13.8829
BHP	0.0002	-0.1571	6.5781	PPT	0.0002	0.1076	11.4146
BKL	0.0008	0.4973	13.0745	PRY	0.0001	0.2721	9.9198
BOQ	0.0002	-0.0807	6.7731	QAN	0.0001	-0.0380	14.1927
CBA	0.0003	-0.0689	8.6593	QBE	0.0001	-3.8793	145.5163
CCL	0.0002	-0.2626	10.3604	RHC	0.0010	0.1118	5.6288
CGF	0.0003	-0.3140	16.5043	RIO	0.0001	-1.8772	38.0620
CIM	0.0005	-0.3492	10.3682	RMD	0.0005	-0.1566	12.6848
COH	0.0004	-0.9377	26.1460	SGP	0.0001	-0.2613	11.2850
CPU	0.0001	-1.3723	38.1907	SHL	0.0002	-0.7735	16.3250
CSL	0.0006	0.8253	18.4971	STO	0.0001	-0.4546	9.6619
CSR	0.0001	-1.1732	19.0599	SUN	0.0001	-0.8052	15.9189
CTX	0.0006	-0.3952	9.1321	TAH	0.0001	-1.3167	22.4390
DOW	0.0003	-0.4194	28.7829	TCL	0.0003	0.4419	16.8790
FLT	0.0003	-0.1846	14.3362	TLS	-0.0001	-0.6401	9.0422
FXJ	-0.0004	0.1393	7.6781	WBC	0.0003	-0.1287	7.1341
GNC	0.0001	0.9452	37.9664	WES	0.0003	-0.4137	11.7870
GPT	-0.0001	-1.8594	48.9735	WOW	0.0003	-0.3993	8.0847
HVN	0.0001	0.1500	6.3052	WPL	0.0002	-0.0607	7.2910
ILU	0.0001	-0.2464	9.7474				

Source: Bloomberg. (2016) Bloomberg Professional. [Online].
Available at: Subscription Service (Accessed: 4 October 2016)

5.2 Time Series Specification and Quantile Regression

To choose the appropriate time series specification for each security, we perform the estimation procedure as per Section 3.1. We estimate the same time series specifications as above in the South American portfolio in order to choose the appropriate quantile regression model based on information criteria. Figure 8 shows a visual representation of the AIC and BIC ranks for each equity in the portfolio for the given time series specifications. The bright green cells indicate the best model based on information criteria for the respective stock and the bright red cells indicate the worst performing model based on information criteria for the respective stock. The GARCH$(1, 1)$ model is the best performing model on this criterion for a majority of the shares in the portfolio and the SAV-GARCH model is the best fit for the remainders. Appendix I shows the values of the information criteria in full.

Fig. 8: Colour Matrix of Time Series Model Ranks on AIC and BIC



Based on the appropriate time series for each shock, we the perform quantile regression estimation. For the quantile regression specification implied by the GARCH$(1,1)$ process, we use the following specification (Lee and Noh, 2013)

$$\begin{aligned} & q_t^X\left(\tau|\boldsymbol{\beta}_\tau^X,\mathcal{I}_{t-1}\right) = \beta_{0,\tau}^X + \beta_{1,\tau}^X x_{t-1} + \beta_{2,\tau}^X \sigma_t \\ & \text{Where } \sigma_t^2 = \delta_0 + \delta_1 \epsilon_{t-1}^2 + \xi_1 \sigma_{t-1}^2 \end{aligned} \tag{5.1}$$

Due to the specification of the GARCH model we have chosen, there is no closed form expression of the quantile regression model. We estimate the volatility term σ_t using the time series parameters as per Equation 3.2 and include it as an exogenous variable in the quantile regression function above (Lee and Noh, 2013). After the estimation of quantile regression parameters, we estimate the semiparametric probability integral transformation as per the numerical approximation method in Appendix D.

5.2.1 Logit Regression

To adequately test for changes in systematic risk between a diversified portfolio of stocks, we choose to estimate the probability of coexceedance conditional on CBA and estimate the logit regression parameters accordingly. CBA is Australia's largest publicly listed firm by market capitalisation and consequently, Australia's largest commercial bank. We suggest this financial status makes CBA rather robust to normal financial volatility, yet not immune to financial contagion and makes it a central element in examining excess risk due to financial contagion.

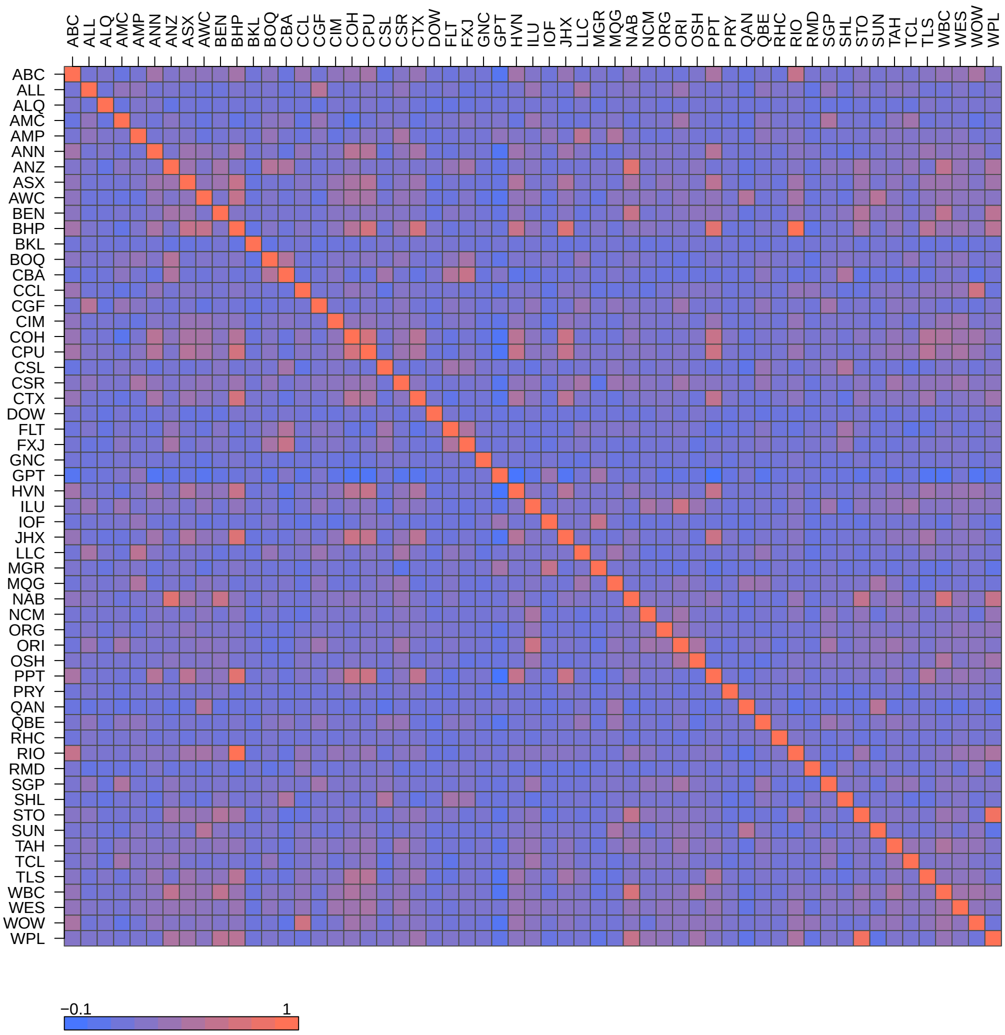
For each security in the portfolio we estimate the logit regression parameters based on the specification in Equation 3.13 where CBA is the coexceedance conditioning variable. As an example of logit regression parameters, we display the estimates for CBA and ANZ Bank (ANZ) at the 5% VaR threshold in Appendix J with standard errors obtained for the empirical Hessian. Many of the parameter estimates for the logit regressions are not significant, however we still include these variables in the specification as an input for the estimation of the probability of coexceedance.

5.2.2 Student-t Copula

To compare the logit-based model against a parametric alternative we fit a Student-t copula to model the rank structure of the portfolio. Due to the computational burden of estimating such a large model by joint MLE we implement an Inference from Margins (IFM) approach. The IFM approach is a two-stage estimation procedure, where firstly we estimate the parameters of the times series models individually and use the respective probability integral transformations to estimate the copula parameters as a second step.

The fitted Student-t copula has an estimated degrees of freedom parameter of 19.7 and Figure 9 shows a visual representation of the estimated correlation matrix. The values in the matrix range from approximately -0.1, indicated in blue, to 1, indicated in orange. Most of the matrix has correlation values close to 0, but we find stocks in the same industry have higher correlation measures; for example BHP and RIO, both mining companies, have a correlation measure of approximately 0.411. There are $1,597$ estimated parameters for the Student-t copula specification and we refrain from displaying the estimates in full as we employ the parameters only as an input for the main objective of quantifying systematic risk.

Fig. 9: Visual Representation of the Student-t Correlation Matrix



5.3 Coexceedance Tests

To test the accuracy of measuring systematic risk between a diversified portfolio of stocks, we estimate the probability of coexceedance for a selection of securities from different industry sectors conditional on CBA. The securities we select to examine financial risk are BHP Billiton (BHP; mining), ANZ Bank (ANZ; financials) and Computershare (CPU; information technology). This selection will allow us to examine not only the relative performance of the methods applied but also the differences in systematic risk manifested in these different industry sectors. We apply the test procedure of predicting instances of coexceedance as outlined in Section 4.5.1.

5.3.1 Results of Performance Tests

We apply the test procedure outlined above and present the evaluated MSE values in Table 12. In this test, the logit-based model significantly outperforms the Student-t copula in predicting coexceedances and demonstrates its accuracy in measuring systematic risk in higher dimensional portfolios. At the 5% VaR threshold level, the logit-based model has on average a 60% lower MSE than the Student-t copula and at the 10% VaR threshold it has on average a 45% lower MSE than the Student-t copula.

Tab. 12: Comparison of MSE of Prediction of Coexceedance for ASX Portfolio

MSE of Prediction of Coexceedances		MSE $\tau = 0.05$	MSE $\tau = 0.10$
BHP-CBA	Logit-Based	0.1492*	0.2408*
	Student-t Copula	0.4955	0.5491
ANZ-CBA	Logit-Based	0.1579*	0.3686*
	Student-t Copula	0.3493	0.3838
CPU-CBA	Logit-Based	0.1864*	0.1275*
	Student-t Copula	0.4164	0.4962
Lowest MSE values indicated with *			

These results further indicate that the logit-based model is an excellent model for quantifying systematic risk in financial portfolios and is able to model a high dimensional portfolio accurately. As the logit-based model does not rely on distributional assumptions to quantify the dependence structure of the portfolio it is able to model the underlying rank structure of the securities.

The Student-t copula model also incorporates a significant amount of estimation error in prediction of coexceedance due to the high number of parameters required for the copula. Each of the 1,597 parameters permit estimation error which is encompassed in the predictions of coexceedance which are shown to be far more inaccurate than the logit-based model that requires only 57 parameter estimates.

The estimation procedure for the logit-based model is also significantly less computationally intensive than the Student-t copula. The number of parameters of the logit-based model increase linearly in the size of the portfolio compared to the Student-t copula that increases its parameter dimensions and computational time in higher orders with the portfolio size. This further motivates the use of the logit-based model in high dimensional applications as it is more accurate and less computationally burdensome than the parametric alternative.

6. CONCLUSION

This thesis was motivated by the social benefit of prudent financial regulation in systematic risk applications. Accurate measurement of systematic risk is a necessity in crafting pragmatic financial regulation and many commonly used methods rely on unrealistic assumptions to quantify this risk. The core contribution of this thesis was the introduction of two semiparametric methodologies of quantifying systematic risk in financial applications that outperform commonly used parametric methods.

6.1 Primary Results

We employed the probability of coexceedance as a measure of systematic risk that captures the probability of one financial market falling below a certain VaR threshold conditional on another financial market falling below its respective VaR threshold.

Current measures implemented in the literature to evaluate the probability of coexceedance in financial applications are often inadequate as they tend to rely on unrealistic distributional assumptions. Parametric methods of measuring systematic risk impose distributional assumptions on the global dependence structure that are often too restrictive. Although there exists a new branch of research on parametric methods that are more flexible, the current manifestations are inadequate for examining systematic risk.

The approach we introduced in this thesis followed on from the semiparametric methods employed in literature where we modelled the dependence structure without any distributional assumptions with the introduced methodology outlined in Section 3. The initial step of our procedure is to model the univariate conditional distributions of the financial returns with quantile regression techniques in order to model rich asymmetries such that downside risk is not misspecified.

Based on the estimates of the quantile regression we introduce two novel semiparametric methodologies for quantifying systematic risk in financial portfolios. The first model we introduce, the kernel-based model, estimates the densities of the conditioning variables with kernel density functions. We expect this model to be accurate in small portfolio applications but is impractical to implement in higher dimensional portfolios. The second model we introduce, the logit-based model, quantifies the probability of coexceedance using logit regression techniques. This model is scalable into higher dimension portfolios as the sample size required increases linearly in the number of dimensions.

We applied these novel methodologies to a portfolio of the returns of equity indices of Brazil, Chile, Argentina and Mexico and performed a test of prediction of coexceedances that directly demonstrates the ability for these models to accurately quantify systematic risk. We compared these results to a parametric copula model as a relative performance measure and based on this test we demonstrated that the introduced models significantly outperform the parametric copula model in measuring systematic risk in this portfolio.

We also applied the logit-based model to a large portfolio of Australian equities to demonstrate its accuracy in measuring systematic risk in a higher dimensional application. We performed a similar test of prediction of coexceedance and compared again to a parametric copula for a comparison of performance. Based on this test we demonstrated that the logit-based model considerably outperforms the parametric copula in the measurement of systematic risk in this higher dimensional portfolio.

The application of the introduced methodologies to the South American and the Australian portfolios demonstrated that the novel models in this thesis are able to outperform commonly used parametric models in measuring systematic risk.

6.2 Further Research

The methodologies we introduced in thesis lend themselves to avenues of further research. Predominately, a promising pursuit would be the application of factor copula models outlined in Section 2 to model the global dependence structure of a portfolio. Although the models we introduced in this thesis were accurate in the prediction of the probability of coexceedance, the semiparametric methods we employed resulted in us being naive of the rank structure of the portfolio. Developing factor copula models for this application would allow us to examine the rank structure in financial portfolios which allows us a rich understanding of the underlying dependence. The current manifestations of factor copula models are inadequate for the purposes we pursue in this thesis but there appears great potential in the development of these models.

6.3 Concluding Remarks

The methods we introduced in this thesis to model systematic risk in financial portfolios are new to the literature and outperformed commonly used parametric techniques in both small portfolio and higher dimensional portfolio applications. The methods we introduced are useful tools for a policymaker to employ to create prudent financial regulation for the benefit of society.

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Appendices

Appendix A

KERNEL DENSITY FUNCTIONS AND BANDWIDTH ESTIMATION

This section outlines the relevant theory for the kernel density functions and the estimation of the bandwidth parameter.

A.1 Kernel Density Functions

A non-negative function $K(x)$ is called a kernel function if its integral over the real line is equal to 1 (Carmona, 2014, p.288), or

$$\int_{-\infty}^{\infty} K(u)du = 1$$

Thus, the kernel density function has similar characteristics to a probability density function. Davidson and Mackinnon (2004, p.679) note that the specification of the kernel density function employed often has little effect on the accuracy of the density estimation and further note that the Gaussian kernel is a popular choice due to its mathematical tractability. The Gaussian kernel has the functional form of the standard normal density (Carmona, 2014, p.288), given by

$$K_{\text{Gaussian}}(x) = \frac{1}{(2\pi)^{1/2}} \cdot \exp\left(\frac{x^2}{2}\right)$$

A.2 Bandwidth Estimation

The bandwidth parameter determines the neighbourhood of datapoints where positive densities are described. The Mean Squared Error (MSE) of the density estimate is susceptible of changes in the bandwidth parameter. Too small a bandwidth parameter creates a noisy density estimate with high variance. As a trade off, too large a bandwidth parameter creates a biased density estimate that is uninformative (Davidson and Mackinnon, 2004, p.680). The bandwidth parameter should be estimated to minimise the global MSE of the density estimate. A commonly used estimation procedure for the bandwidth parameter h is the Silverman's Rule-of-Thumb estimator where the optimal bandwidth is estimated as a function of the sample standard deviation s and sample size n (Silverman, 1986, p.87), or

$$h = 1.06 s n^{-1/5}$$

A modern configuration of Silverman's Rule-of-Thumb estimator that we employ in this thesis compares the spread of the sample data to the normal distribution to further prevent oversmoothing (Davidson and Mackinnon, 2004, p.681), or

$$h = 0.9 \min \left(s, \frac{\mathrm{IQR}}{1.349} \right) n^{-1/5}$$

Where IQR is the interquartile range of the sample and 1.349 is the interquartile range of the normal distribution.

Appendix B

SOUTH AMERICAN TIME SERIES INFORMATION CRITERIA VALUES

Tab. 13: AIC and BIC for Time Series Specification for South American Portfolio

		ARMA(1,0)	ARMA(1,1)	ARMA(1,0)GARCH(1,0)	ARMA(1,0)GARCH(1,1)	SAV-GARCH
Brazil	AIC	-22,814.8144	-23,713.7014	-24,1455.5608	-24,882.3459	-25,034.4021*
	BIC	-22,801.6966	-23,694.0247	-24,122.7662	-24,842.9925	-24,995.0486*
Chile	AIC	-29,799.9517	-29,788.6401	-30,,321.5840	-23,688.9518	-30,996.9446*
	BIC	-29,786.8828	-29,769.0367	-30,288.9118	-23,649.7452	-30,957.7379*
Argentina	AIC	-16,829.8684	-18,942.7138	-19,060.8344	-19,660.8560	-20,033.2986*
	BIC	-16,817.1685	-18,293.6639	-19,029.0846	-19,622.7563	-19,995.1988*
Mexico	AIC	-14,114.7085	-14,396.2591	-14,491.4606	-14,249.0552	-16,050.1938*
	BIC	-14,102.8632	-14,378.4913	-14,461.8474	-14,213.5195	-16,014.6581*

Lowest AIC and BIC values indicated with *

Appendix C

DERIVATION OF CAVIAR QUANTILE REGRESSION MODEL

We begin with the DGP of the SAV-GARCH time series specification, where, for a random time series variable X

$$x_t = \alpha_0 + \alpha_t x_{t-1} + \epsilon_t$$

$$\text{Where } \epsilon_t = \sigma_t \eta_t,\ \ \eta_t \sim \text{iid}(0,1)$$

$$\sigma_t = \delta_0 + \delta_1 |x_{t-1}| + \delta_2 \sigma_{t-1}$$

Letting F_t^X and F^η denote the probability distributions of X_t and η, respectively, for a $\tau \in [0,1]$

$$\begin{aligned} F_t^X(x_t) &= Pr\left(\alpha_0 + \alpha_t x_{t-1} + \sigma_t \eta_t \leq x_t | \mathcal{I}_{t-1}\right) \\ &= F^\eta\left(\frac{x_t - \alpha_0 - \alpha_1 x_{t-1}}{\sigma_t}\right) = \tau \end{aligned}$$

Taking the inverse of F^η, it follows that the quantile function for X_t, $q_t^X(\tau)$, is given by

$$q_t^X(\tau|\mathcal{I}_{t-1}) = \alpha_0 + \alpha_1 x_{t-1} + \sigma_t F^{\eta,-1}(\tau)$$

Where $F^{\eta,-1}$ is the inverse function of F^η

$$= \alpha_0 + \alpha_1 x_{t-1} + (\delta_0 + \delta_1|x_{t-1}| + \delta_2 \sigma_{t-1}) \cdot F^{\eta,-1}(\tau)$$

$$= \left(\alpha_0 + \delta_0 F^{\eta,-1}(\tau) - \alpha_0 \cdot \delta_2\right) + \alpha_1 x_{t-1} + \delta_2 q_{t-1}^X(\tau|\mathcal{I}_{t-2}) - \alpha_1 \cdot \delta_2 x_{t-2}$$

Letting $\beta_{0,\tau}^X = \left(\alpha_0 + \delta_0 F^{\eta,-1}(\tau) - \alpha_0 \cdot \delta_2\right)$

$$\beta_{1,\tau}^X = \alpha_1$$

$$\beta_{2,\tau}^X = \delta_2$$

$$\beta_{3,\tau}^X = \delta_1 \cdot F^{\eta,-1}(\tau)$$

$$q_t^X(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-1}) = \beta_{0,\tau}^X + \beta_{1,\tau}^X x_{t-1} + \beta_{2,\tau}^X \cdot q_{t-1}^X\left(\tau|\boldsymbol{\beta}_\tau^X, \mathcal{I}_{t-2}\right) - \beta_{1,\tau}^X \cdot \beta_{2,\tau}^X x_{t-1} + \beta_{3,\tau}^X |x_{t-1}|$$

Appendix D

METHOD FOR APPROXIMATING PROBABILITY INTEGRAL

There is no closed form solution to the probability integral in Equation 3.7 for the CAViaR quantile regression model. We use a numerical estimation technique to approximate the value of the probability integral transform.

Over an evenly spaced, fine grid of size S, we estimate a S-vector $\mathbf{Q}$ where each typical element of $\mathbf{Q}$, denoted Q_s, is the estimated conditional quantile of the returns distribution evaluated at the estimates for the parameters at time t, or

$$Q_s = q_t^X \left(\tau_s \middle| \hat{\boldsymbol{\beta}}_{\tau_s}^X, \mathcal{I}_{t-1} \right)$$

where $\tau_s = s \cdot \Delta\tau$

and where $= \Delta\tau = S^{-1}$

Where $\Delta\tau$ is the length of each grid interval. The elements of $\mathbf{Q}$ will be weakly monotonic, where

$$Q_{1,t} \leq Q_{2,t} \leq Q_{3,t} \leq ... \leq Q_{S,t}$$

The numerical approximation of the integral in Equation 3.7 is the Riemann sum over this fine grid, where

$$\hat{F}_{Xt}(x_t) = \sum_{s=1}^{S} \mathbb{1}\left(Q_{s,t} < x_t\right) \cdot S^{-1}$$

Appendix E

HESSIAN MATRIX FOR LOGIT LOG-LIKELIHOOD

The gradient vector $\mathbf{g}$ is a vector of first derivatives of the log-likelihood function $\mathcal{L}(\boldsymbol{\nu}|\mathbf{X})$ with respect to the parameters of the logit regression $\boldsymbol{\nu}$. For the South American portfolio the gradient vector is given by

$$\mathbf{g} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial \nu_0} \\ \frac{\partial \mathcal{L}}{\partial \nu_1} \\ \frac{\partial \mathcal{L}}{\partial \nu_2} \end{bmatrix}$$

The Hessian $\mathbf{H}$ is a matrix of second derivatives of the log-likelihood function, which is equivalent to a matrix of first derivatives of the gradient vector. For the South American portfolio, the Hessian matrix is given by

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \mathcal{L}}{\partial \nu_0^2} & \frac{\partial^2 \mathcal{L}}{\partial \nu_0 \cdot \partial \nu_1} & \frac{\partial^2 \mathcal{L}}{\partial \nu_0 \cdot \partial \nu_2} \\ \frac{\partial^2 \mathcal{L}}{\partial \nu_1 \cdot \partial \nu_0} & \frac{\partial^2 \mathcal{L}}{\partial \nu_1^2} & \frac{\partial^2 \mathcal{L}}{\partial \nu_1 \cdot \partial \nu_2} \\ \frac{\partial^2 \mathcal{L}}{\partial \nu_2 \cdot \partial \nu_0} & \frac{\partial^2 \mathcal{L}}{\partial \nu_2 \cdot \partial \nu_1} & \frac{\partial^2 \mathcal{L}}{\partial \nu_2^2} \end{bmatrix}$$

The variance covariance matrix is the negative of the inverse of the Hessian matrix. When the Hessian matrix is evaluated at $\hat{\boldsymbol{\nu}}$ is know as the empirical Hessian estimator of variance covariance matrix and has the form

$$\hat{V}(\hat{\boldsymbol{\nu}}) = -\mathbf{H}^{-1}(\hat{\boldsymbol{\nu}})$$

Appendix F

PREDICTION OF COEXCEEDANCE PLOTS FOR THE KERNEL-BASED MODEL

Fig. 10: Prediction of Coexceedance Plots for the Kernel-Based Model (1)

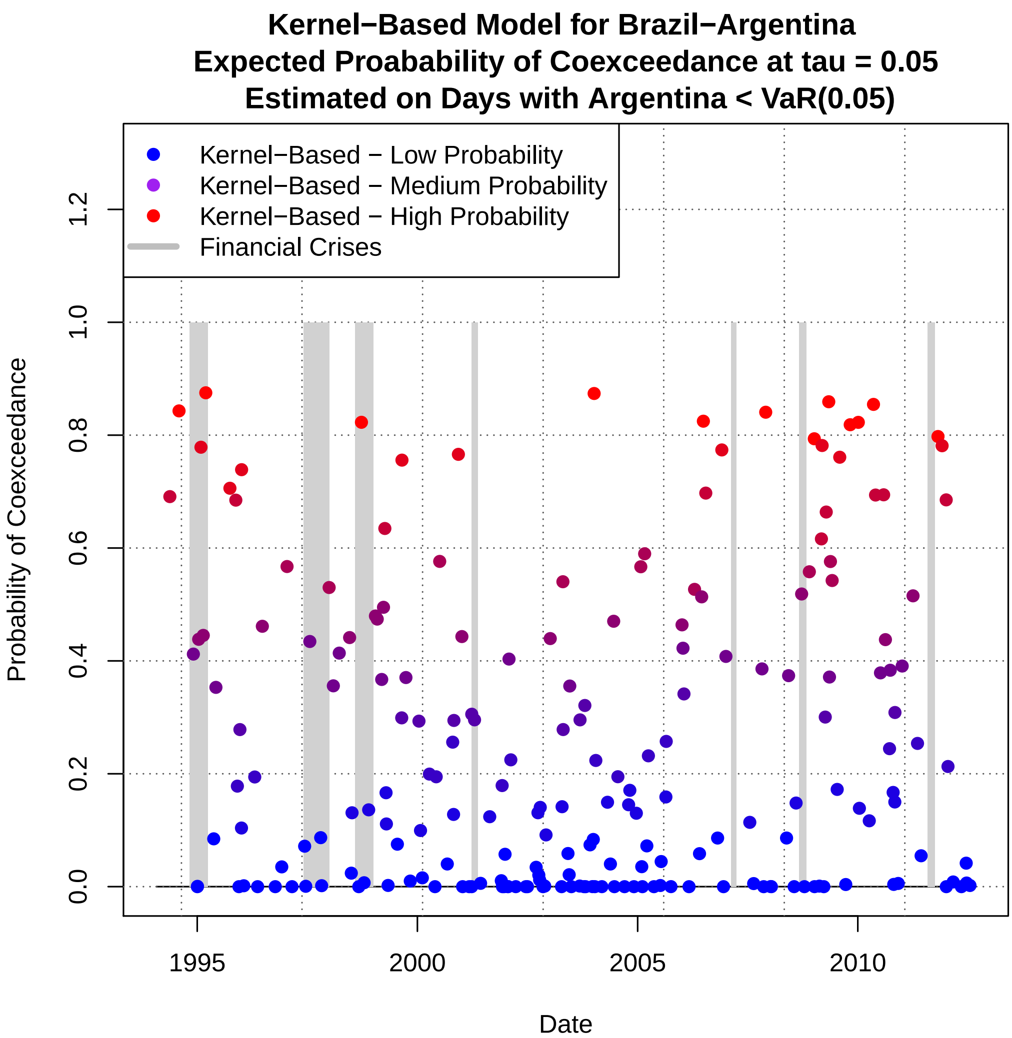
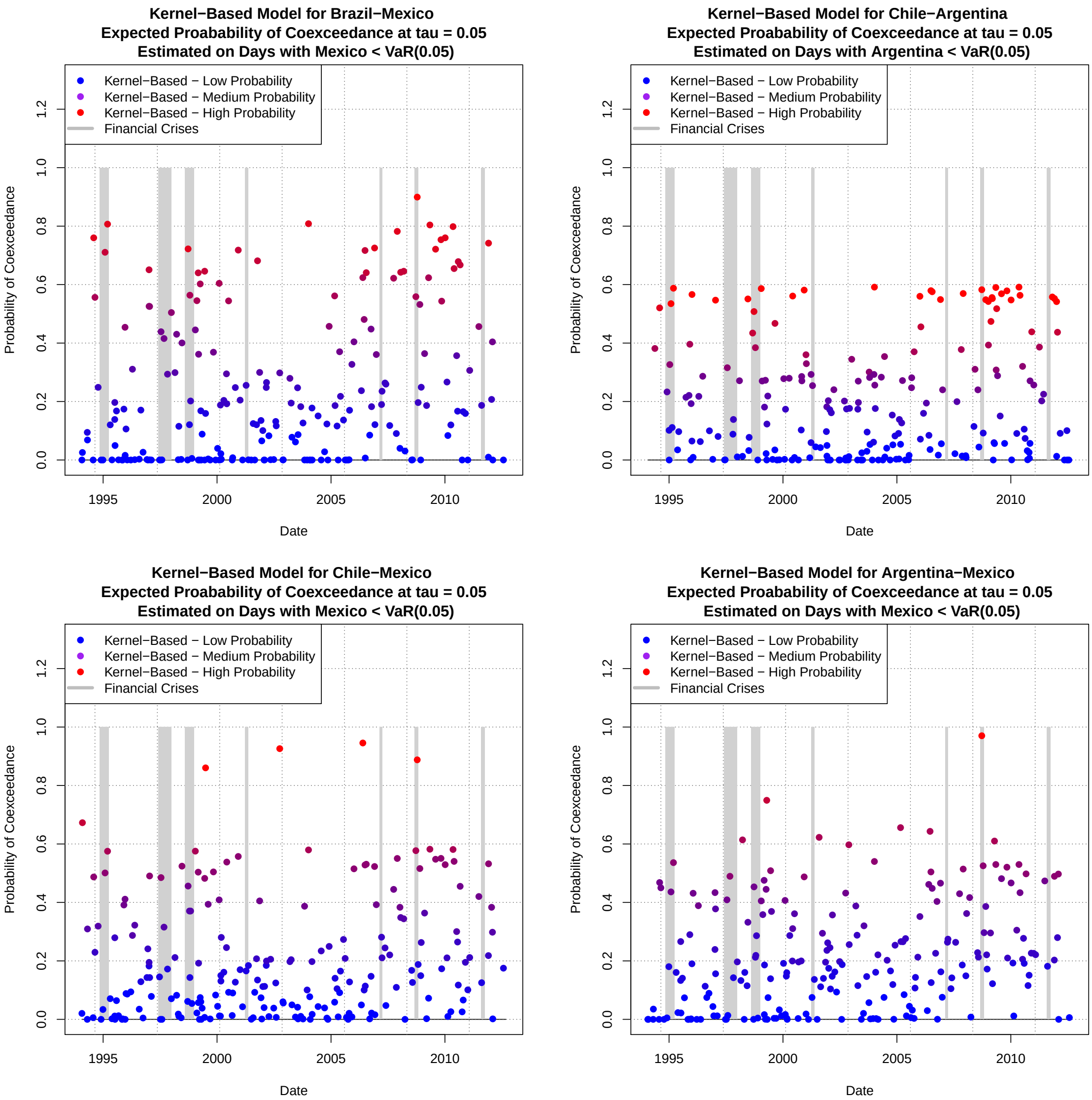


Fig. 11: Prediction of Coexceedance Plots for the Kernel-Based Model (2)



Appendix G

INFORMATION CRITERIA VALUES FOR THE COPULA MODELS FOR THE SOUTH AMERICAN PORTFOLIO

Tab. 14: Information Critera Values for the Copula Specifications

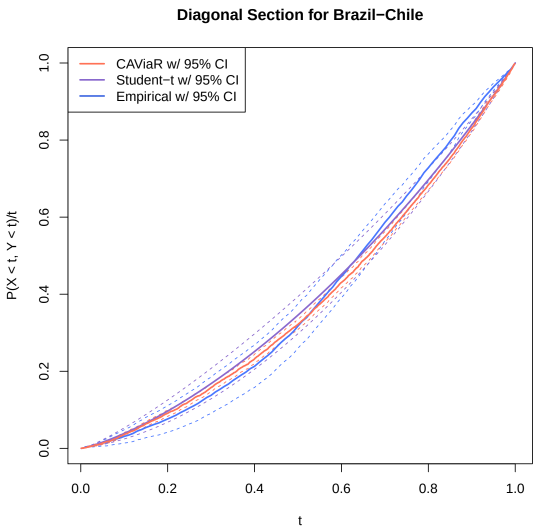
Copula Specification	AIC	BIC
Independent	-106736.5876	-106582.0144
Gumbel	-111431.4868	-111269.1850
Clayton	-110239.8287	-110077.5269
Gaussian	-112050.7370	-111849.7919
Student-t	-112286.3678*	-112077.6941*
Lowest AIC and BIC values indicated with *		

Appendix H

DIAGONAL SECTION PLOTS FOR THE SOUTH AMERICAN PORTFOLIO

Figure 12 and 13 show the plots of the bivariate diagonal sections for each pair of countries in the South American portfolios. The 95% confidence interval is indicated in dashed line next to the point estimate of the diagonal section. Standard errors for the confidence interval are estimated from the bootstrap estimation procedure of the joint maximum likelihood of the time series model and of the CAViaR model, respectively.

Fig. 12: Diagonal Section Plots for the South American Portfolio (1)



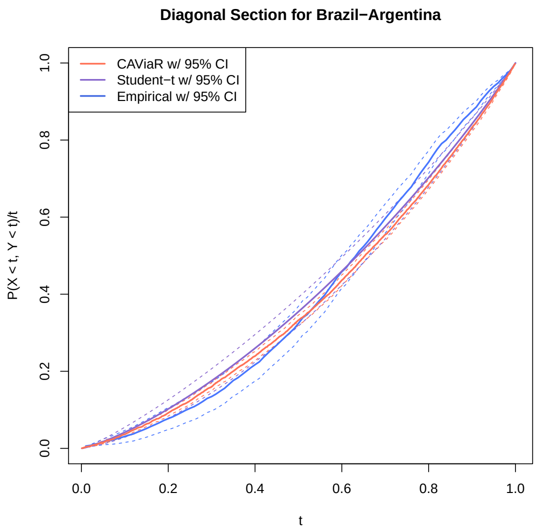
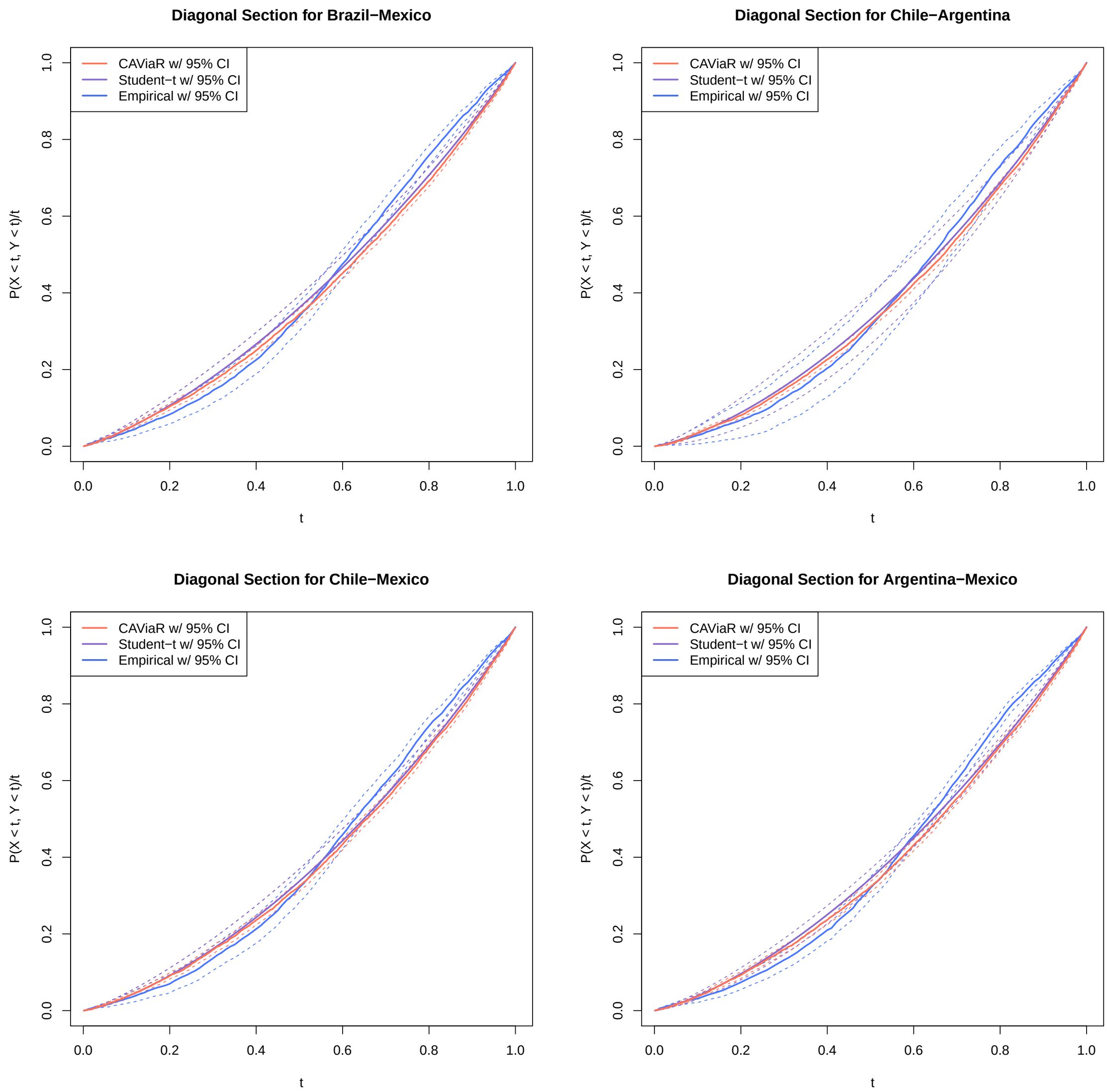


Fig. 13: Diagonal Section Plots for the South American Portfolio (2)



Appendix I

INFORMATION CRITERIA VALUES FOR ASX PORTFOLIO TIME SERIES SPECIFICATIONS

Tab. 15: AIC Values for ASX Portfolio Time Series Specifications

ASX Code	ARMA(1,0)	ARMA(1,1)	ARMA(1,0)GARCH(1,0)	ARMA(1,0)GARCH(1,1)	SAV-GARCH
ABC	-19319.0181	-19312.8973	-19476.3736	-19695.7592*	-19592.5541
ALL	-17903.5257	-17896.9631	-18142.2298	-18299.0533*	-17914.8011
ALQ	-20175.3789	-20171.6231	-20472.2896	-21064.0160	-21122.8437*
AMC	-22843.4402	-22835.2952	-22989.2343	-23201.9326*	-22972.5121
AMP	-20713.4702	-20707.5815	-20935.5974	-21735.6272	-21859.3713*
ANN	-21252.0186	-21245.9650	-21436.1631	-21735.5723*	-21731.4724
ANZ	-22548.3209	-22544.4157	-23008.7196	-23957.6444*	-23030.8769
ASX	-22416.0994	-22411.9392	-22757.0915	-23438.3423*	-23359.8597
AWC	-18453.3101	-18445.9444	-18765.8157	-19016.7455	-19239.6016*
BEN	-21491.9188	-21484.3108	-21735.0972	-22387.8339	-22396.3462*
BHP	-20721.3715	-20713.3987	-21026.5353	-21336.6334	-21604.4276*
BKL	-21501.5700	-21493.4140	-21891.0287	-22192.7518*	-21988.1952
BOQ	-21750.0260	-21741.7839	-22092.4121	-22696.4995*	-22549.0878
CBA	-23424.3330	-23417.7065	-23868.1864	-24684.7837	-24790.4243*
CCL	-22268.4890	-22261.5836	-22486.6851	-22832.4378*	-22679.2585
CGF	-17937.3130	-17933.4831	-18895.3155	-19686.4156*	-19506.0015
CIM	-19325.9294	-19318.2462	-19703.3182	-19924.7631*	-19788.8238
COH	-20617.2097	-20610.4949	-20687.9684	-20751.4875	-20790.9347*
CPU	-19534.9675	-19527.3102	-19686.3293	-19772.1179*	-19704.7406
CSL	-20988.9502	-20982.3196	-21243.9317	-21487.4860*	-21040.3110
CSR	-20126.1685	-20118.2863	-20305.9821	-20497.9024*	-20459.0673
CTX	-19429.6043	-19421.9688	-19604.6207	-19854.3559*	-19730.8996
DOW	-17952.8573	-17952.3235	-18086.5415	-18204.7326*	-18006.4087
FLT	-19797.3808	-19791.1682	-20108.6446	-20323.7696*	-20122.7869
FXJ	-20107.6851	-20099.6854	-20256.0780	-20440.5602*	-20280.0492
GNC	-21064.5422	-21056.3779	-21260.1949	-21524.2643*	-21250.8784
GPT	-19822.6945	-19815.3239	-22675.9095	-22892.6006	-23198.7204*
HVN	-20435.2531	-20428.6646	-20620.8406	-20813.7933*	-20736.2432
ILU	-18907.3105	-18900.0954	-19048.1123	-19274.2614	-19415.5621*
IOF	-20256.6695	-20252.0295	-22029.6922	-23323.4106*	-23321.8515
JHX	-19756.5627	-19748.9097	-19881.6057	-20176.9515*	-20083.4708
LLC	-21174.1156	-21166.1449	-21421.3385	-21800.6316*	-21438.2335
MGR	-20149.6144	-20141.2265	-21650.8665	-23567.2831*	-23400.4745
MQG	-19572.7358	-19565.0583	-20272.9348	-21172.5404*	-21052.6386
NAB	-22335.2807	-22328.7365	-22873.9087	-23820.5251*	-23534.7126
NCM	-18321.0650	-18318.8162	-18452.5752	-18641.0478	-18760.0319*
ORG	-20534.5044	-20526.6095	-20927.2765	-21327.0376*	-21147.4136
ORI	-21049.5720	-21041.5980	-21391.9584	-21569.6787*	-21257.2242
OSH	-18586.3646	-18579.2669	-18940.7517	-19454.8596*	-19027.2436
PPT	-20159.1610	-20151.2825	-20467.4589	-21035.9673*	-20905.4567
PRY	-21271.3474	-21263.1631	-21473.3340	-21632.5454*	-21537.5684
QAN	-19871.5417	-19864.6590	-20186.3682	-20568.0495*	-20454.7696
QBE	-19409.4622	-19404.6156	-20589.5705	-21070.0039*	-20783.4987
RHC	-21401.8136	-21418.9718	-21630.1406	-21694.7942*	-21518.1681
RIO	-19608.5983	-19601.2213	-20243.5892	-21083.7140	-21135.0620*
RMD	-19839.1353	-19831.8624	-20240.0005	-20651.1202*	-20376.6290
SGP	-21558.9485	-21551.3411	-22526.4307	-24042.3372*	-23913.0831
SHL	-22073.8491	-22066.3434	-22207.8800	-22243.2275*	-22107.3002
STO	-19865.6977	-19858.0721	-20232.9260	-21071.4921*	-20880.1832
SUN	-21551.4877	-21548.3871	-21944.0821	-23081.2871*	-22718.1488
TAH	-22809.8386	-22802.1449	-23063.9808	-23376.2938*	-22875.3946
TCL	-22232.5710	-22226.4816	-22589.8357	-23201.8213*	-23172.7945
TLS	-23994.9186	-23992.6023	-24200.6477	-24291.1535*	-24180.2226
WBC	-22856.4379	-22847.8992	-23179.0880	-23957.8413*	-23905.4131
WES	-22277.8764	-22269.4567	-22611.0219	-23293.2818*	-23074.2795
WOW	-23986.2407	-23977.4408	-24190.2669	-24519.9354*	-24295.2200
WPL	-21270.6910	-21264.1475	-21553.7633	-22098.3005*	-22036.8295

Lowest AIC and BIC values indicated with *

Tab. 16: BIC Values for ASX Portfolio Time Series Specifications

ASX Code	ARMA(1,0)	ARMA(1,1)	ARMA(1,0)GARCH(1,0)	ARMA(1,0)GARCH(1,1)	SAV-GARCH
ABC	-19306.3753	-19293.9330	-19451.0879	-19664.1521*	-19560.9470
ALL	-17890.8829	-17877.9989	-18116.9441	-18267.4462*	-17883.1940
ALQ	-20162.7361	-20152.6588	-20447.0039	-21032.4089	-21091.2366*
AMC	-22830.7973	-22816.3310	-22963.9486	-23170.3255*	-22940.9050
AMP	-20700.8273	-20688.6173	-20910.3117	-21704.0201	-21827.7642*
ANN	-21239.3757	-21227.0007	-21410.8775	-21703.9652*	-21699.8653
ANZ	-22535.6781	-22525.4514	-22983.4339	-23926.0373*	-22999.2698
ASX	-22403.4566	-22392.9750	-22731.8058	-23406.7352*	-23328.2526
AWC	-18440.6672	-18426.9801	-18740.5300	-18985.1384	-19207.9945*
BEN	-21479.2760	-21465.3465	-21709.8115	-22356.2268	-22364.7391*
BHP	-20708.7286	-20694.4344	-21001.2497	-21305.0263	-21572.8205*
BKL	-21488.9272	-21474.4497	-21865.7430	-22161.1447*	-21956.5881
BOQ	-21737.3831	-21722.8197	-22067.1264	-22664.8924*	-22517.4807
CBA	-23411.6902	-23398.7422	-23842.9007	-24653.1766	-24758.8172*
CCL	-22255.8462	-22242.6194	-22461.3994	-22800.8307*	-22647.6513
CGF	-17924.6702	-17914.5188	-18870.0299	-19654.8085*	-19474.3944
CIM	-19313.2865	-19299.2819	-19678.0325	-19893.1560*	-19757.2167
COH	-20604.5669	-20591.5307	-20662.6827	-20719.8804	-20759.3276*
CPU	-19522.3247	-19508.3460	-19661.0436	-19740.5108*	-19673.1335
CSL	-20976.3073	-20963.3553	-21218.6460	-21455.8789*	-21008.7039
CSR	-20113.5257	-20099.3220	-20280.6964	-20466.2953*	-20427.4602
CTX	-19416.9615	-19403.0046	-19579.3350	-19822.7488*	-19699.2925
DOW	-17940.2145	-17933.3593	-18061.2558	-18173.1255*	-17974.8016
FLT	-19784.7379	-19772.2040	-20083.3589	-20292.1625*	-20091.1798
FXJ	-20095.0422	-20080.7212	-20230.7924	-20408.9531*	-20248.4421
GNC	-21051.8994	-21037.4136	-21234.9092	-21492.6572*	-21219.2713
GPT	-19810.0517	-19796.3596	-22650.6239	-22860.9935	-23167.1133*
HVN	-20422.6102	-20409.7004	-20595.5549	-20782.1862*	-20704.6361
ILU	-18894.6677	-18881.1311	-19022.8266	-19242.6543	-19383.9550*
IOF	-20244.0267	-20233.0652	-22004.4065	-23291.8035*	-23290.2444
JHX	-19743.9198	-19729.9454	-19856.3200	-20145.3444*	-20051.8636
LLC	-21161.4727	-21147.1806	-21396.0528	-21769.0245*	-21406.6264
MGR	-20136.9716	-20122.2623	-21625.5808	-23535.6760*	-23368.8674
MQG	-19560.0930	-19546.0940	-20247.6491	-21140.9333*	-21021.0315
NAB	-22322.6379	-22309.7723	-22848.6231	-23788.9180*	-23503.1055
NCM	-18308.4222	-18299.8520	-18427.2895	-18609.4407	-18728.4248*
ORG	-20521.8615	-20507.6452	-20901.9908	-21295.4305*	-21115.8065
ORI	-21036.9292	-21022.6338	-21366.6727	-21538.0716*	-21225.6171
OSH	-18573.7217	-18560.3026	-18915.4660	-19423.2525*	-18995.6365
PPT	-20146.5182	-20132.3183	-20442.1732	-21004.3601*	-20873.8496
PRY	-21258.7045	-21244.1988	-21448.0483	-21600.9383*	-21505.9612
QAN	-19858.8988	-19845.6947	-20161.0825	-20536.4424*	-20423.1625
QBE	-19396.8194	-19385.6513	-20564.2848	-21038.3968*	-20751.8916
RHC	-21389.1708	-21400.0075	-21604.8549	-21663.1871*	-21486.5610
RIO	-19595.9554	-19582.2570	-20218.3035	-21052.1069	-21103.4549*
RMD	-19826.4924	-19812.8981	-20214.7148	-20619.5131*	-20345.0219
SGP	-21546.3057	-21532.3768	-22501.1450	-24010.7301*	-23881.4760
SHL	-22061.2063	-22047.3792	-22182.5943	-22211.6204*	-22075.6931
STO	-19853.0548	-19839.1078	-20207.6403	-21039.8850*	-20848.5761
SUN	-21538.8449	-21529.4228	-21918.7964	-23049.6800*	-22686.5417
TAH	-22797.1957	-22783.1807	-23038.6951	-23344.6867*	-22843.7875
TCL	-22219.9281	-22207.5173	-22564.5500	-23170.2142*	-23141.1874
TLS	-23982.2757	-23973.6380	-24175.3620	-24259.5464*	-24148.6155
WBC	-22843.7950	-22828.9350	-23153.8023	-23926.2342*	-23873.8060
WES	-22265.2335	-22250.4925	-22585.7363	-23261.6747*	-23042.6724
WOW	-23973.5979	-23958.4765	-24164.9812	-24488.3283*	-24263.6129
WPL	-21258.0481	-21245.1832	-21528.4776	-22066.6934*	-22005.2224

Lowest AIC and BIC values indicated with *

Appendix J

LOGIT REGRESSION PARAMETERS FOR ANZ-CBA AT THE 5% THRESHOLD

Tab. 17: Joint Logit Regression Parameters for ANZ-CBA at the 5% Threshold

ASX Code	$\hat{\nu}_J$		ASX Code	$\hat{\nu}_J$	
Intercept	9.3849*	(4.8019)	IOF	2.8683	(2.4959)
ABC	2.4080	(2.5655)	JHX	8.3670**	(3.3962)
ALL	-1.2598	(2.3106)	LLC	-2.0678	(2.2282)
ALQ	0.4788	(2.3280)	MGR	-2.2872	(2.2543)
AMC	-6.1638**	(2.7891)	MQG	-3.6773	(2.4223)
AMP	1.1629	(2.1208)	NAB	-3.0941	(3.0157)
ANN	2.0537	(2.5470)	NCM	-7.2419**	(3.1755)
ASX	2.6786	(2.6434)	ORG	-6.4147**	(3.2083)
AWC	-4.3988	(2.8594)	ORI	-2.6753	(2.5147)
BEN	-1.4423	(2.6391)	OSH	6.0260**	(2.6634)
BHP	-0.0445	(2.5863)	PPT	-6.8196*	(3.5979)
BKL	-7.8203***	(2.9428)	PRY	6.7058**	(2.9370)
BOQ	-16.3365***	(5.5878)	QAN	6.3842**	(2.9384)
CCL	4.4255*	(2.6147)	QBE	4.0298	(2.4835)
CGF	2.1856	(2.1376)	RHC	-3.7038	(2.4204)
CIM	-4.8694**	(2.2202)	RIO	-5.9538*	(3.4444)
COH	-3.8413	(3.4186)	RMD	1.0174	(2.3840)
CPU	6.1886**	(3.1346)	SGP	1.2643	(2.6181)
CSL	5.8784**	(2.5501)	SHL	-8.6425**	(3.9058)
CSR	2.0605	(2.3386)	STO	0.2498	(1.7432)
CTX	-0.6942	(1.9507)	SUN	0.6273	(2.2358)
DOW	-0.3341	(1.8132)	TAH	0.6972	(1.8715)
FLT	-8.4108**	(3.4324)	TCL	-5.7233**	(2.5779)
FXJ	-14.2011***	(4.7615)	TLS	-5.0438*	(2.8345)
GNC	-2.3210	(2.1700)	WBC	-0.1953	(2.7720)
GPT	-0.4805	(2.0946)	WES	0.4646	(2.1676)
HVN	-2.6487	(2.4381)	WOW	-0.5965	(2.2825)
ILU	-8.1659**	(4.0920)	WPL	0.5074	(2.5718)

Standard errors in parentheses.
$* \ p < 0.1$, $** \ p < 0.05$, $*** \ p < 0.01$

Tab. 18: Marginal Logit Regression Parameters for ANZ-CBA at the 5% Threshold

ASX Code	$\hat{\nu}_M$		ASX Code	$\hat{\nu}_M$	
Intercept	-0.1818	(0.4290)	IOF	0.0510	(0.4686)
ABC	1.3764***	(0.4212)	JHX	-0.4046	(0.4474)
ALL	-1.9286***	(0.4324)	LLC	-2.1203***	(0.4417)
ALQ	0.1416	(0.4155)	MGR	-2.3944***	(0.4617)
AMC	0.9139**	(0.4225)	MQG	-1.0151**	(0.4620)
AMP	-2.5565***	(0.4805)	NAB	0.5613	(0.4180)
ANN	0.1074	(0.4755)	NCM	1.9264***	(0.4355)
ASX	0.7429*	(0.4449)	ORG	0.7959*	(0.4446)
AWC	-3.0492***	(0.4774)	ORI	-0.4491	(0.4062)
BEN	0.1280	(0.5240)	OSH	-2.0447***	(0.4963)
BHP	1.1482***	(0.4267)	PPT	-0.5300	(0.4080)
BKL	-1.3507***	(0.4450)	PRY	1.7811***	(0.4320)
BOQ	-0.3356	(0.4242)	QAN	-1.9990***	(0.4376)
CCL	-1.1758***	(0.4210)	QBE	-0.7280*	(0.3997)
CGF	-0.0231	(0.4313)	RHC	-1.8650***	(0.4927)
CIM	4.9397***	(0.5408)	RIO	-1.3474***	(0.4138)
COH	-0.5279	(0.4757)	RMD	-2.0647***	(0.4474)
CPU	-3.0596***	(0.4645)	SGP	1.2577***	(0.4146)
CSL	-1.8131***	(0.4550)	SHL	-1.4157***	(0.4634)
CSR	1.0770**	(0.4647)	STO	-2.0284***	(0.4432)
CTX	-0.8170*	(0.4373)	SUN	-0.8407**	(0.4221)
DOW	-1.9472***	(0.4603)	TAH	-1.8309***	(0.4564)
FLT	-4.0722***	(0.5341)	TCL	-2.0315***	(0.4787)
FXJ	-1.7014***	(0.4293)	TLS	2.9037***	(0.4532)
GNC	-0.2181	(0.4262)	WBC	1.0610**	(0.4422)
GPT	0.6890	(0.4538)	WES	-0.9960**	(0.4458)
HVN	0.2214	(0.4264)	WOW	0.8494*	(0.4697)
ILU	-1.8123*	(0.4523)	WPL	7.4184***	(1.0477)

Standard errors in parentheses.
$* \ p < 0.1$, $** \ p < 0.05$, $*** \ p < 0.01$